

Ce quiz comporte **3** questions équipondérées; répondez directement sur la feuille.

Nom:

CORRIGÉ

1. Existe-t-il des valeurs de $c, d \in \mathbf{R}$ pour lesquelles les fonctions suivantes sont continues ?

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & (x, y) \neq (0, 0) \\ c & (x, y) = (0, 0) \end{cases} \quad g(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ d & (x, y) = (0, 0) \end{cases}$$

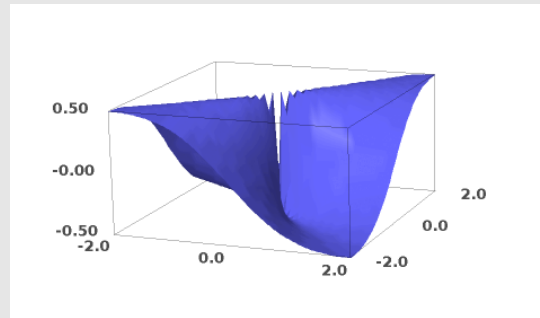
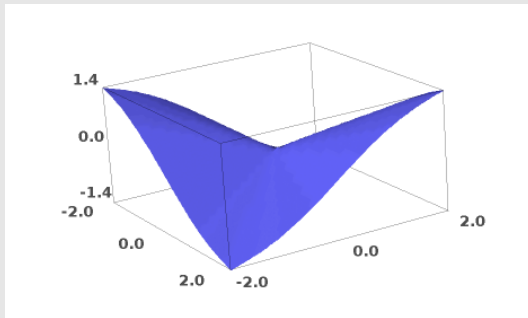
Pour étudier les limites de f et g au voisinage de l'origine, passons en coordonnées polaires :

- $\lim_{\substack{r \rightarrow 0 \\ r \neq 0}} f(r \cos \theta, r \sin \theta) = \lim_{\substack{r \rightarrow 0 \\ r \neq 0}} r \cos \theta \sin \theta = 0$ indépendamment de θ puisque $|\sin \theta \cos \theta| \leq 1$.

On peut donc rendre f continue en prenant $c = 0$.

- $\lim_{\substack{r \rightarrow 0 \\ r \neq 0}} g(r \cos \theta, r \sin \theta) = \lim_{\substack{r \rightarrow 0 \\ r \neq 0}} \cos \theta \sin \theta = \cos \theta \sin \theta$ dépend de l'angle d'approche.

Aucune valeur de d ne permet de rendre g continue (la limite n'existe pas).



2. Calculer $\frac{\partial f}{\partial x}$ et $\frac{\partial f}{\partial y}$ en tout point du plan pour $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$.

[attention de bien accorder à l'origine l'attention particulière qu'elle mérite]

En $(x, y) \neq (0, 0)$, on dérive simplement le quotient de fonctions dérivables dont le dénominateur ne s'annule pas pour obtenir

$$\frac{\partial f}{\partial x} = \frac{x^4 + 3x^2y^2 + 2xy^3}{(x^2 + y^2)^2}, \quad \frac{\partial f}{\partial y} = \frac{-y^4 - 3x^2y^2 - 2x^3y}{(x^2 + y^2)^2} \quad (x, y) \neq (0, 0).$$

À l'origine, on doit considérer les fonctions partielles :

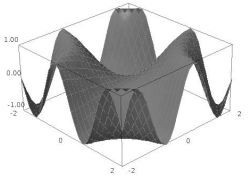
$$\begin{aligned} f|_{x=0}(y) &= -y \quad \text{même si } y = 0 \implies \frac{\partial f}{\partial y}(0, 0) = \frac{d}{dy}(-y) = -1, \\ f|_{y=0}(x) &= x \quad \text{même si } x = 0 \implies \frac{\partial f}{\partial x}(0, 0) = \frac{d}{dx}(x) = +1. \end{aligned}$$

3. Associez à chacune des fonctions de deux variables suivantes son graphe ainsi que ses courbes de niveau.

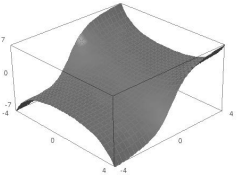
$f(x, y)$	graphe	courbes
$x^2 - y^2$	9	E
$\frac{15x^2y^2}{(x^2 + y^2)e^{x^2+y^2}}$	7	C
$\sin x + \sin y$	5	H

$f(x, y)$	graphe	courbes
$e^{-x^2} + e^{-y^2}$	3	A
$\cos \sqrt{x^2 + y^2}$	6	G
$\sin xy$	1	B

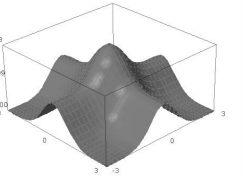
$f(x, y)$	graphe	courbes
$y^4 - 8y^2 - 4x^2$	8	D
$e^{x^2/10} \operatorname{Arctan} y$	2	I
$3y - 2x$	4	F



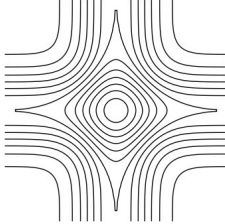
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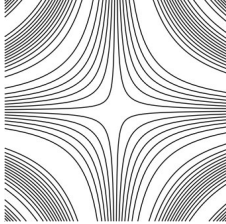
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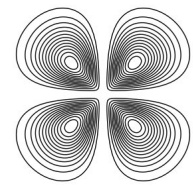
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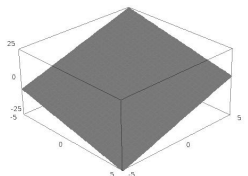
A)



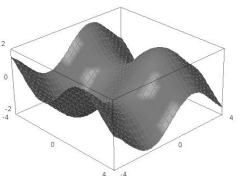
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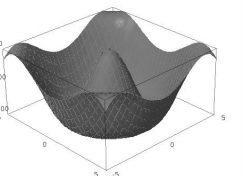
C)



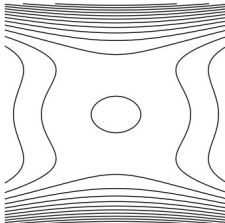
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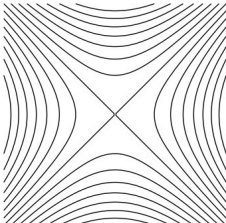
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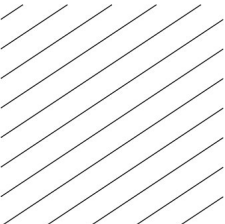
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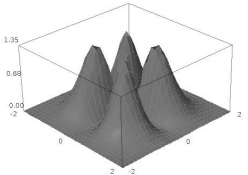
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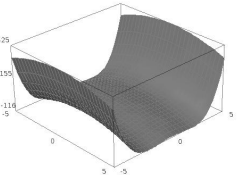
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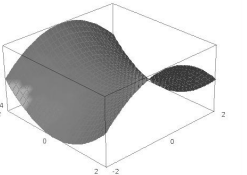
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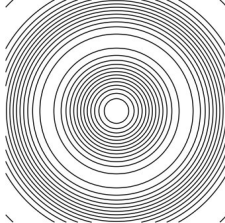
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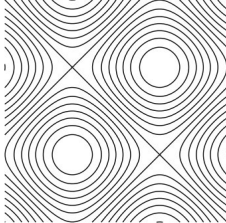
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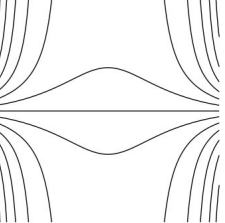
9)



G)



H)



I)