

## Courbes paramétrées - Étude locale

3 méthodes :

\* Limite de la corde

$$\lim_{t \rightarrow t_0} \frac{y(t) - y(t_0)}{x(t) - x(t_0)}$$

\* Limite de la tangente

$$\lim_{t \rightarrow t_0} \frac{D(y)(t)}{D(x)(t)}$$

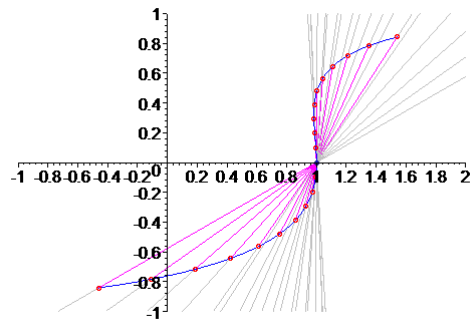
\* Développement limité

$$M(t) = M(t_0) + (t - t_0) \left( \frac{dM}{dt} \right) + \frac{1}{2} (t - t_0)^2 \left( \frac{d^2 M}{dt^2} \right) + \frac{1}{6} (t - t_0)^3 \left( \frac{d^3 M}{dt^3} \right) + (t - t_0)^3 \varepsilon(t - t_0)$$

Visualisation des cordes

$$x := t \rightarrow t^3 + \cos(t)$$

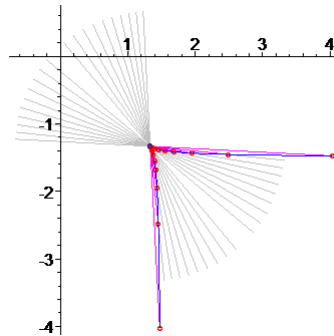
$$y := \sin$$



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$$x := t \rightarrow \frac{1}{3} \frac{(t+2)^2}{t+1}$$

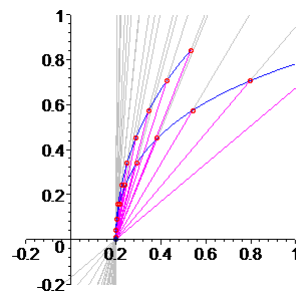
$$y := t \rightarrow \frac{1}{3} \frac{(t-2)^2}{t-1}$$



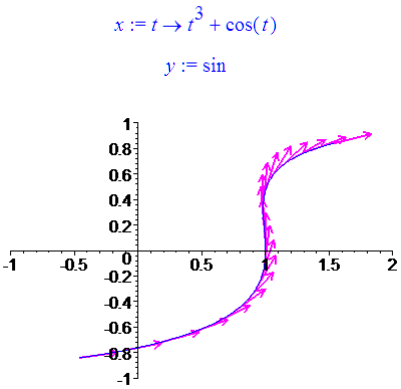
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$$x := t \rightarrow .2 + \frac{t^4}{2-t}$$

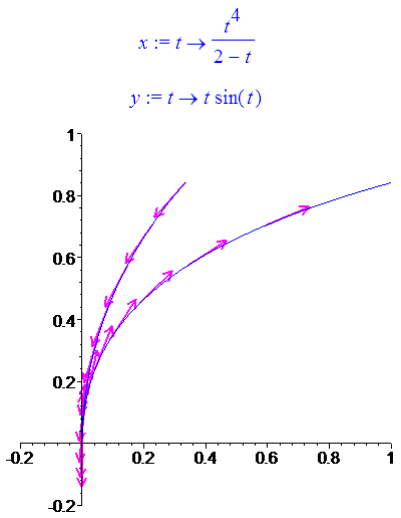
$$y := t \rightarrow t \sin(t)$$



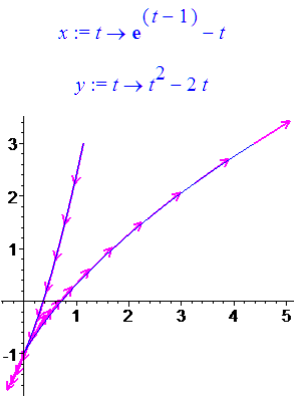
Visualisation des tangentes



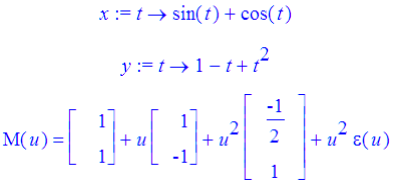
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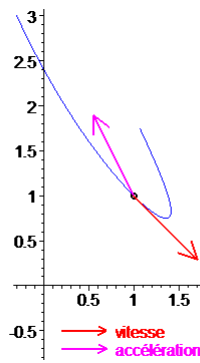


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Utilisation des développements liités





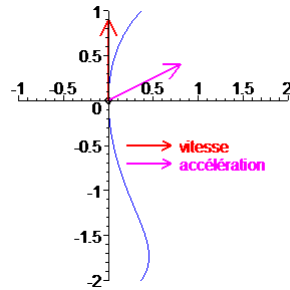
*point ordinaire*

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$$x := t \rightarrow \frac{\sin(t^2)}{1-t}$$

$$y := t \rightarrow 1 + 2t - \cos(t)$$

$$M(u) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 2 \end{bmatrix} + u^2 \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} + u^3 \varepsilon(u)$$



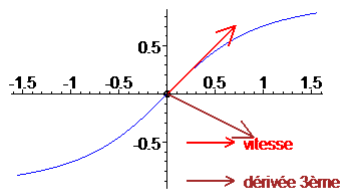
*point ordinaire à tangente verticale*

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$$x := \tan$$

$$y := \sin$$

$$M(u) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 1 \\ 1 \end{bmatrix} + u^2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^3 \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{6} \end{bmatrix} + u^3 \varepsilon(u)$$



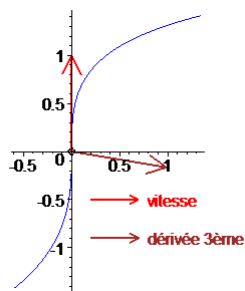
*point d'inflexion*

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$$x := t \rightarrow t^4 + 2t^3$$

$$y := t \rightarrow 2 \sin(t)$$

$$M(u) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 2 \end{bmatrix} + u^2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^3 \begin{bmatrix} 2 \\ -\frac{1}{3} \end{bmatrix} + u^3 \varepsilon(u)$$



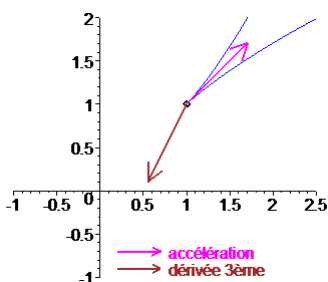
*point d'inflexion à tangente verticale*

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$$x := t \rightarrow t^2 + 2 \frac{1}{t} - 2$$

$$y := t \rightarrow 2t + \frac{1}{t^2} - 2$$

$$M(u+1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^2 \begin{bmatrix} 3 \\ 3 \end{bmatrix} + u^3 \begin{bmatrix} -2 \\ -4 \end{bmatrix} + u^3 \varepsilon(u)$$



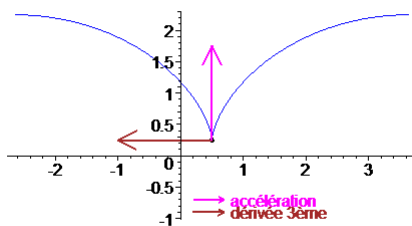
*rebroussement 1ere espèce*

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$$x := t \rightarrow \frac{1}{2} + \sin(t) - t$$

$$y := t \rightarrow \frac{5}{4} - \cos(t)$$

$$M(u) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^2 \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} + u^3 \begin{bmatrix} -\frac{1}{6} \\ 0 \end{bmatrix} + u^3 \varepsilon(u)$$



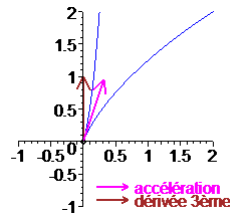
*rebroussement 1ere espèce tangente verticale*

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$$x := t \rightarrow (t+1)^2$$

$$y := t \rightarrow -2t + \frac{1}{t^2} - 3$$

$$M(u-1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + u^3 \begin{bmatrix} 0 \\ 4 \end{bmatrix} + u^3 \varepsilon(u)$$



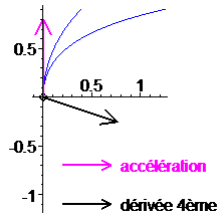
rebroussement 1ere espèce

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$$x := t \rightarrow \frac{t^4}{2-t}$$

$$y := t \rightarrow t \sin(t)$$

$$M(u) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + u^3 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^4 \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{6} \end{bmatrix} + u^4 \varepsilon(u)$$



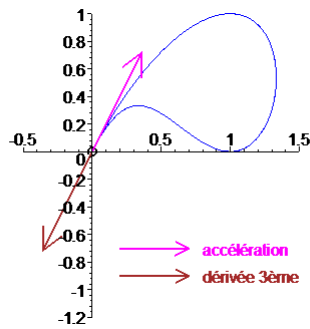
rebroussement 2eme espèce

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$$x := t \rightarrow 2 \frac{\sin(t)^2}{2 + \sin(2t)}$$

$$y := t \rightarrow \frac{\sin(2t)^2}{2 + \sin(2t)}$$

$$M(u) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + u^3 \begin{bmatrix} -1 \\ -2 \end{bmatrix} + u^4 \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix} + u^4 \varepsilon(u)$$



dérivée 3ème colinéaire à dérivée seconde

Il faut un DL au 4ème ordre

