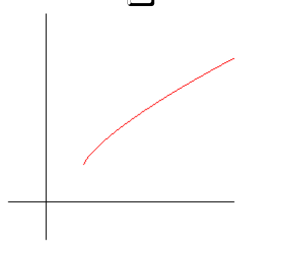
CIR2 2018-2019 QUIZZ 6 Courbes - Surfaces

Durée 30 minutes
Pas de document, ni calculatrice, ni téléphone portable
Inscrire les réponses sur la feuille-réponse jointe (il peut y avoir plusieurs réponses correctes, ou aucune)

1. Reconnaître l'allure de la courbe $\begin{cases} x = t^2 + 1 \\ y = t^4 + 1 \end{cases}$, $t \in \mathbb{R}_+$ x est négligeable devant y en +infini

1


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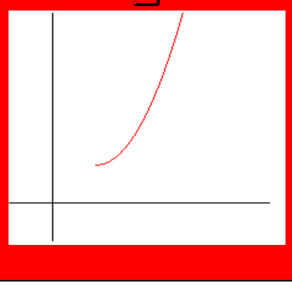

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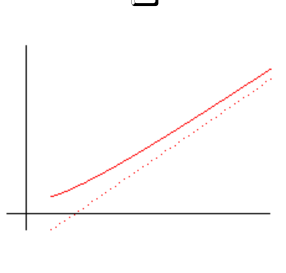

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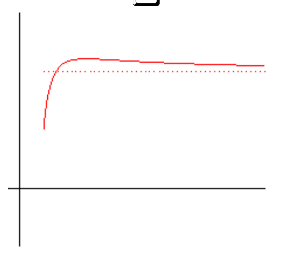
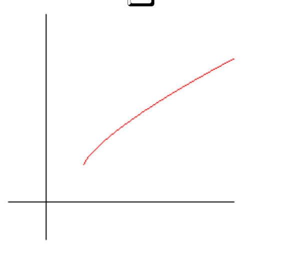


5
autre chose

2. Reconnaître l'allure de la courbe $x = \dfrac{t^2 + t + 1}{t + 1}$, $y = \dfrac{2t^2 + t + 1}{t^2 + 1}$, $t \in \mathbb{R}_+$ y tend vers 2 en +infini

1


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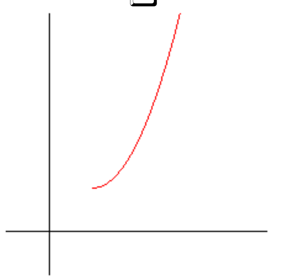

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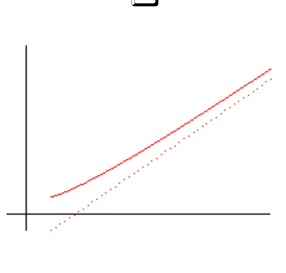

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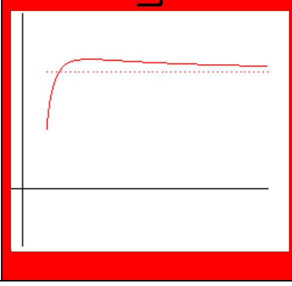
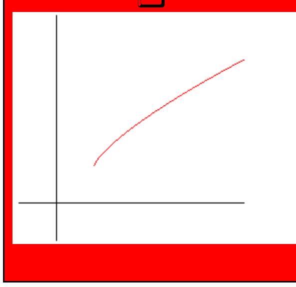


5
autre chose

3. Reconnaître l'allure de la courbe $\begin{cases} x = t^2 + 1 \\ y = t^{3/2} + 1 \end{cases}$, $t \in \mathbb{R}_+$

1


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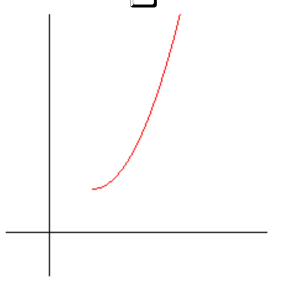

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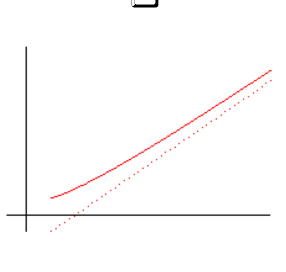

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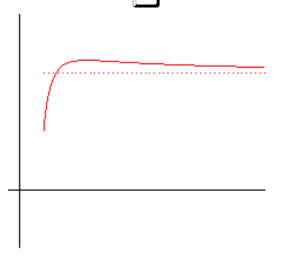
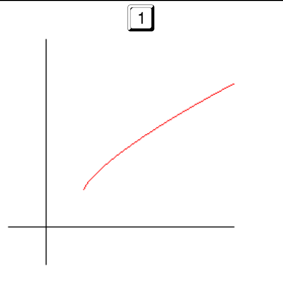


5
autre chose

4. Reconnaître l'allure de la courbe $\begin{cases} x = \dfrac{t^2 + t + 1}{t + 1} \\ y = \dfrac{t^2 + t + 3}{t + 3} \end{cases}$, $t \in \mathbb{R}_+$

1


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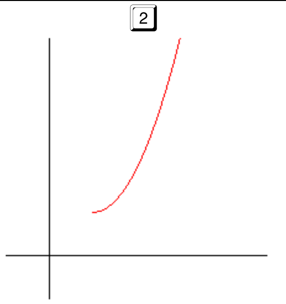

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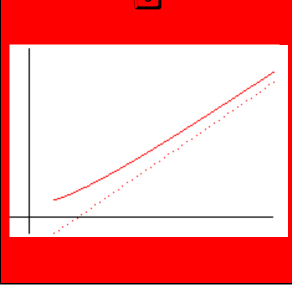

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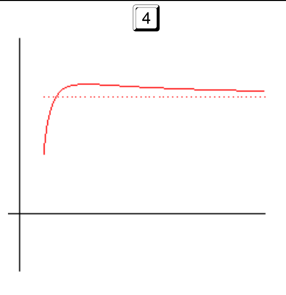
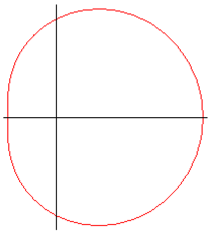
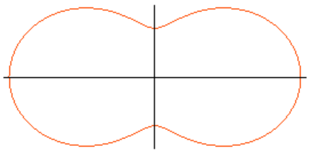
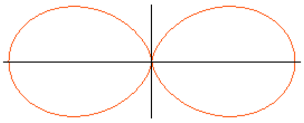
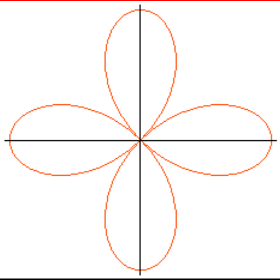
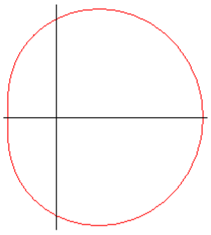
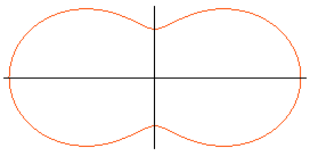
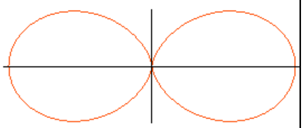
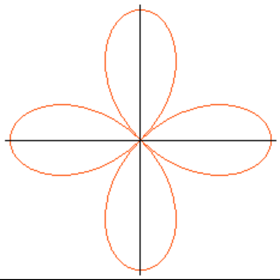
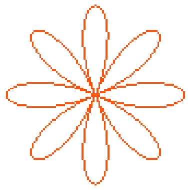


5
autre chose

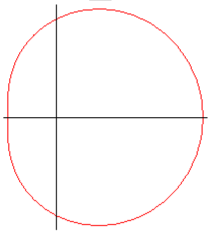
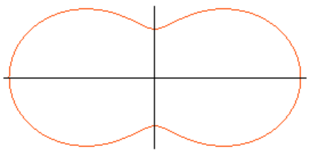
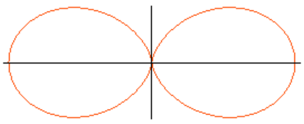
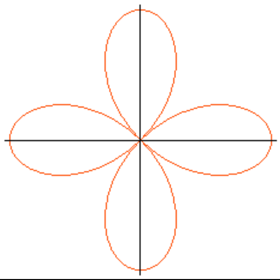
5. Reconnaître l'allure de la courbe polaire $r=\cos(2\theta)$ r s'annule et change de signe en $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ et $\frac{7\pi}{4}$

1	2	3	4	5
				autre chose

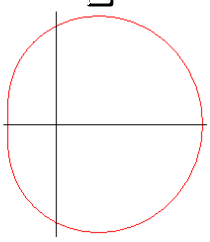
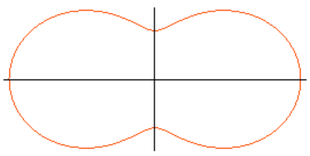
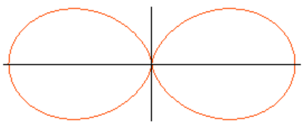
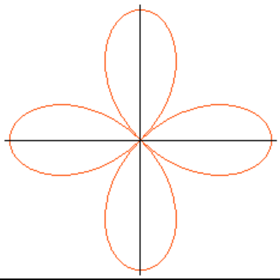
6. Reconnaître l'allure de la courbe polaire $r=\cos(4\theta)$ r s'annule et change de signe 8 fois sur $[0,2\pi]$

1	2	3	4	5
				autre chose 

7. Reconnaître l'allure de la courbe polaire $r=1+\cos(2\theta)$ voir les valeurs de r en $0, \frac{\pi}{2}, \pi$

1	2	3	4	5
				autre chose

8. Reconnaître l'allure de la courbe polaire $r=2+\cos(2\theta)$ voir les valeurs de r en $0, \frac{\pi}{2}, \pi$

1	2	3	4	5
				autre chose

9. Soit une courbe $M(t)=\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, t\in\mathbb{R}$. Si $x(t_1)=-y(t_2)$ et $y(t_1)=-x(t_2)$, alors $M(t_1)$ et $M(t_2)$ sont :

1	2	3	4	5
symétriques par rapport à Ox	symétriques par rapport à Oy	symétriques par rapport à l'axe y=-x	symétriques par rapport à O	autre réponse

10. La longueur de la courbe $\begin{array}{l} x(t)=t+\sin t \\ y(t)=1+\cos t \end{array}$, $t\in[0,2\pi]$ est :

$$\frac{dM}{dt}=\begin{pmatrix}1+\cos t\\ -\sin t\end{pmatrix} \quad \left\|\frac{dM}{dt}\right\|^2=2+2\cos t=4\cos^2\left(\frac{t}{2}\right) \quad \left\|\frac{dM}{dt}\right\|=2\left|\cos\left(\frac{t}{2}\right)\right|=2\cos\left(\frac{t}{2}\right) \text{ sur } [0,\pi]$$

$$L=\int_0^1\left(1+2t^2\right)dt \quad L=\int_0^{2\pi}2\left|\cos\left(\frac{t}{2}\right)\right|dt=2\int_0^{\pi}2\cos\left(\frac{t}{2}\right)dt \text{ par symétrie}$$

1	2	3	4	5
2π	8	6	0	autre chose

11. La longueur de la courbe $\begin{array}{l} x(t)=t \\ y(t)=t^2 \\ z(t)=\frac{2}{3}t^3 \end{array}$, $t\in[0,1]$ est :

$$\frac{dM}{dt}=\begin{pmatrix}1\\ 2t\\ 2t^2\end{pmatrix},\ \left\|\frac{dM}{dt}\right\|^2=1+4t^2+4t^4=\left(1+2t\right)^2,$$

1	2	3	4	5
0	$\frac{1}{3}\sqrt{23}$	$\frac{5}{3}$	$\frac{47}{15}$	autre résultat

12. l'aire de la nappe paramétrée $\begin{array}{l} x=u\cos v \\ y=u\cos v, \\ z=u\sqrt{3} \end{array}$ $u\in[0,R\sqrt{3}], v\in[0,2\pi]$ est

$$\frac{\partial M}{du}=\begin{pmatrix}\cos v\\ \cos v\\ \sqrt{3}\end{pmatrix},\ \frac{\partial M}{dv}=\begin{pmatrix}-u\sin v\\ -u\sin v\\ 0\end{pmatrix},\ \frac{\partial M}{du}\wedge\frac{\partial M}{dv}=\begin{pmatrix}u\sqrt{3}\sin v\\ -u\sqrt{3}\sin v\\ 0\end{pmatrix},\ \left\|\frac{\partial M}{du}\wedge\frac{\partial M}{dv}\right\|=u\sqrt{6}\left|\sin v\right|$$

$$aire=\iint u\sqrt{6}\left|\sin v\right|du\,dv=\sqrt{6}\int_0^{R\sqrt{3}}u\,du\int_0^{2\pi}\left|\sin v\right|dv \quad \text{(variables séparées)}$$

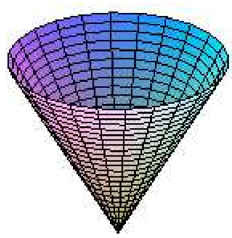
$$\int_0^{2\pi}\left|\sin v\right|dv=2\int_0^{\pi}\sin v\,dv=4\ .\ aire=6R^2\sqrt{6}$$ (remarque : il s'agit en fait d'une surface plane (y=x))

1	2	3	4	5
$6\pi R^2$	$2\pi R\sqrt{3}$	$4\pi R^2$	$4\pi^2R^2$	autre résultat

Si on avait $y=u\sin v$, la surface était plus intéressante (cône $z^3=3\left(x^2+y^2\right)$)

On aurait eu

$$\frac{\partial M}{du}=\begin{pmatrix}\cos v\\ \sin v\\ \sqrt{3}\end{pmatrix},\ \frac{\partial M}{dv}=\begin{pmatrix}-u\sin v\\ u\cos v\\ 0\end{pmatrix},\ \frac{\partial M}{du}\wedge\frac{\partial M}{dv}=\begin{pmatrix}-u\sqrt{3}\cos v\\ -u\sqrt{3}\sin v\\ u\end{pmatrix},\ \left\|\frac{\partial M}{du}\wedge\frac{\partial M}{dv}\right\|=2u$$



et $aire=\iint 2u\,du\,dv=\int_0^{R\sqrt{3}}2u\,du\int_0^{2\pi}dv\ =6\pi R^2$