TD CIR2 . T2 . 22/01/21.

$\boxed{\text{EXO8}}$ . $u_{n+2} + u_{n+1} - 2u_n = 0$ . $u_0 = 1$

$u_1 = 2$ .

1) Méthode de l'équation caractéristique.

$$1\,u_{n+2} + 1\,u_{n+1} - 2u_n = 0.$$

$$1\,X^2 + 1\,X - 2 \times 1 = 0.$$

$$X^2 + X - 2 = 0.$$

$$\Delta = 1^2 - 4(-2) = 9$$

$$\begin{matrix} r_1 \\ r_2 \end{matrix} = \frac{-1 \pm \sqrt{\Delta}}{2} = \begin{matrix} -2 \\ \text{et} \\ 1 \end{matrix}$$

$$u_n = \lambda(-2)^n + \mu(1)^n$$

$$u_0 = \lambda(-2)^0 + \mu(1)^0 = 1 = \lambda + \mu.$$

$$u_1 = \lambda(-2)^1 + \mu 1^1 = 2 = -2\lambda + \mu.$$

$$u_0 - u_1 \Rightarrow -1 = \lambda + 2\lambda = 3\lambda \Rightarrow \lambda = -1/3.$$

$$\mu = 1 - \lambda = 1 + 1/3 = 4/3.$$

$$u_n = -\frac{1}{3}(-2)^n + \frac{4}{3}$$

## 2) Transformée de $Z$

a) Appliquer la transformée de $Z$ aux membres de l'équation sur un.

$$u_{n+1} \longrightarrow z f(z) - u_0 z = z f(z) - z$$

$$u_{n+2} \longrightarrow z \underbrace{(z f(z) - z)}_{u_{n+1}} - u_1 z = z^2 f(z) - z^2 - 2z$$

$$u_n \longrightarrow f(z).$$

$$u_{n+2} = -u_{n+1} + 2u_n.$$

$$z^2 f(z) - z^2 - 2z = -z f(z) + z + 2 f(z).$$

b) Calculer $f(z)$.

$$z^2 f(z) + z f(z) - 2 f(z) = z + 2z + z^2$$

$$f(z)(z^2 + z - 2) = z^2 + 3z = z(z+3).$$

$$f(z) = \frac{z(z+3)}{z^2+z-2} = \frac{z(z+3)}{(z+2)(z-1)}$$

c) Décomposer $\frac{1}{z} f(z)$ en éléments simples.

$$\frac{1}{z} f(z) = \frac{z+3}{(z+2)(z-1)} = \frac{A}{z+2} + \frac{B}{z-1}$$