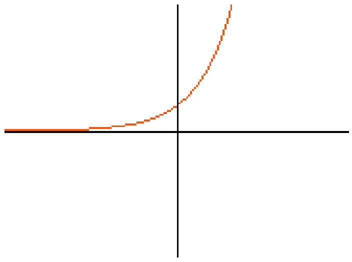
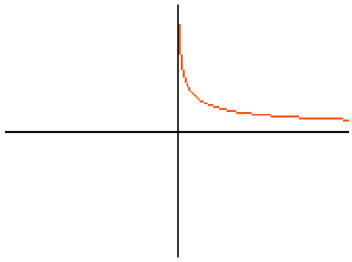
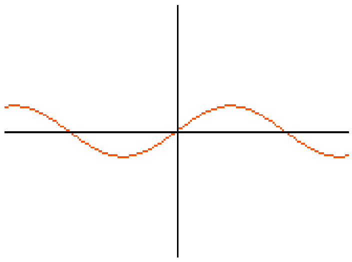
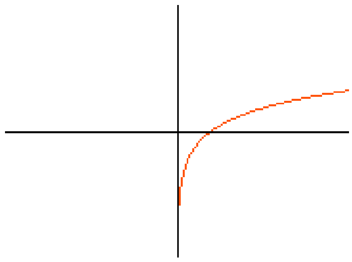
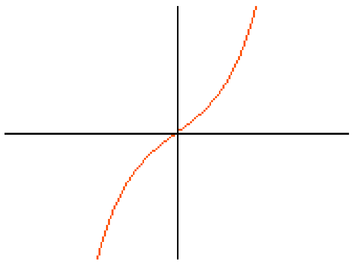
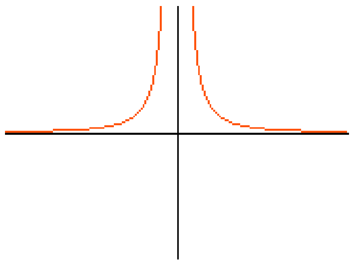
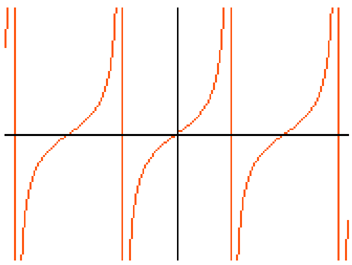
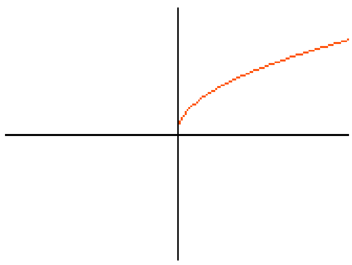
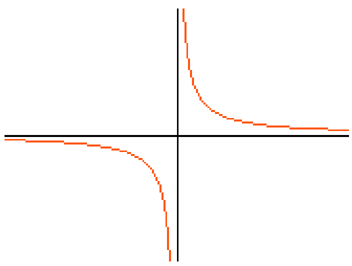
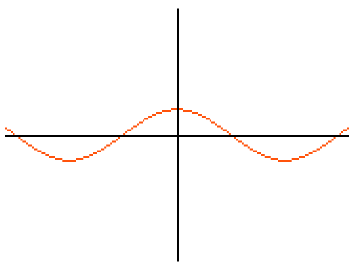
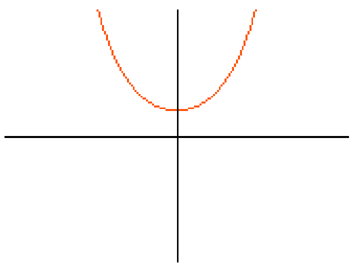
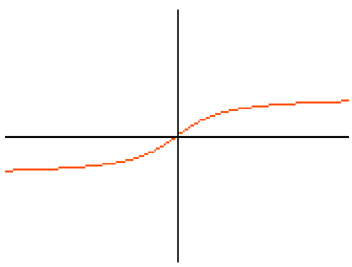
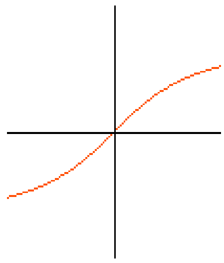
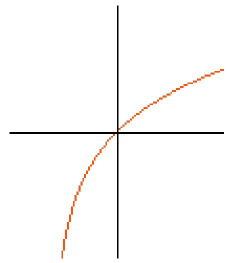
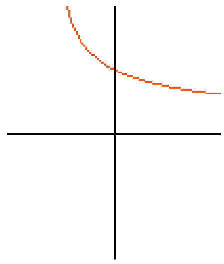
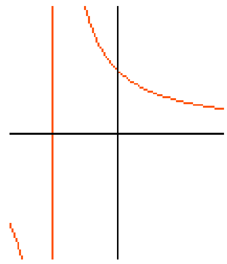
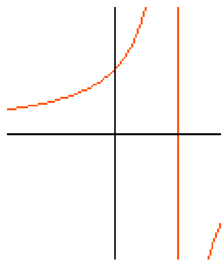
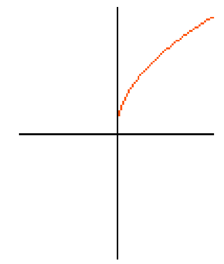
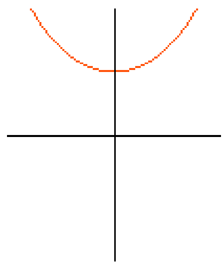
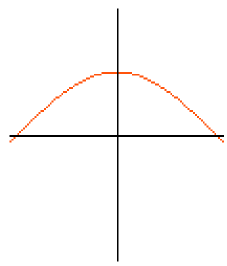
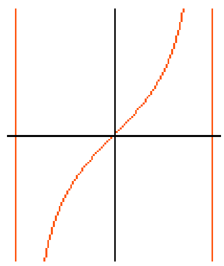
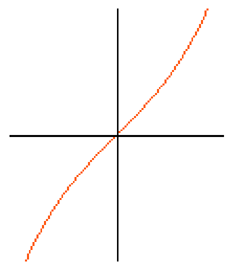
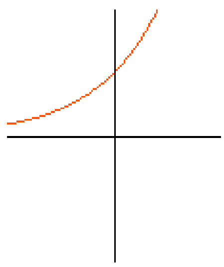
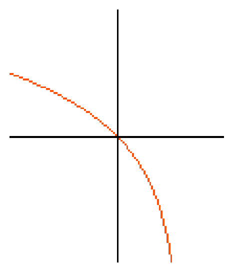
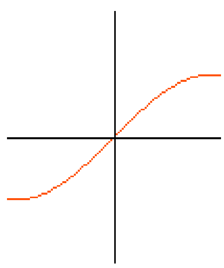
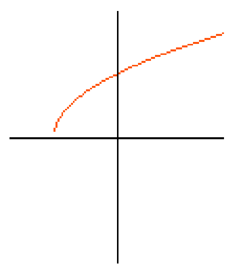
Écrire les dérivées - Associer chaque fonction et son graphe sur $\mathbb{R}$

fonction	dérivée		
$\frac{1}{x}$			
$\frac{1}{x^2}$			
e^x			
$\cos x$			
$\sin x$			
$\mathrm{ch} x$			
$\mathrm{sh}\, x$			
$\tan x$			
$\ln(x)$			
$\arctan x$			
$\frac{1}{\sqrt{x^3}}$			
$\sqrt{x}$			

Associer à chaque fonction son DL en 0 et son graphe au voisinage de l'origine

$\frac{1}{1-x}$	$x+\frac{x^3}{6}+x^4\varepsilon(x)$		
$\frac{1}{1+x}$	$x+\frac{x^3}{3}+x^4\varepsilon(x)$		
e^x	$x-\frac{x^3}{6}+x^4\varepsilon(x)$		
$\cos x$	$1+\frac{x^2}{2}+\frac{x^4}{24}+x^5\varepsilon(x)$		
$\sin x$	$1+x+x^2+x^3+x^3\varepsilon(x)$		
$\mathrm{ch}x$	$-x-\frac{x^2}{2}-\frac{x^3}{3}+x^3\varepsilon(x)$		
$\mathrm{sh}\,x$	$1+x+\frac{x^2}{2}+\frac{x^3}{6}+x^3\varepsilon(x)$		
$\tan x$	$1-\frac{1}{2}x+x\,\varepsilon(x)$		
$\ln(1-x)$	$1+\frac{1}{2}x+x\,\varepsilon(x)$		
$\ln(1+x)$	$1-\frac{x^2}{2}+\frac{x^4}{24}+x^5\varepsilon(x)$		
$\frac{1}{\sqrt{1+x}}$	$x-\frac{x^3}{3}+x^3\varepsilon(x)$		
$\sqrt{1+x}$	$x-\frac{x^2}{2}+x^2\,\varepsilon(x)$		
$\arctan x$	$1-x+x^2-x^3+x^3\varepsilon(x)$		

Séries

1/ Soit (v_n) une suite de réels telle que $v_n \xrightarrow[n\to\infty]{} 0$. Soient a, b et c trois réels tels que $a+b+c=0$.

On étudie la série $[u_n]$ définie par $u_n = a\ v_n + b\ v_{n+1} + c\ v_{n+2}$

Calculer la somme partielle de rang N de cette série. Étudier sa convergence.

2/ Étudier la convergence et calculer éventuellement la somme de la série $[u_n]_{n\in\mathbb{N}}$ dans les cas suivants :

$$u_n = \frac{1}{5^n},\ u_n = \left(\frac{-1}{3}\right)^n,\ u_n = \frac{2^n}{3^{n+1}}\ ,\ u_n = \frac{n^2}{n^2+n+1}\ ,\ u_n = \frac{\exp(n)}{2^n}\ ,\ u_n = \frac{2^{n+1}}{n!}\ ,\ u_n = 2^n \exp(-2n)$$

$$u_n = \frac{1}{n!} + \frac{1}{(n+1)!}\ ,\ u_n = \frac{2^{n+1}+3^{n+2}}{5^n}\ ,\ u_n = \frac{9}{(3n+1)(3n+4)}\ ,\ u_n = \frac{1}{\sqrt{n+1}+\sqrt{n}}\ ,\ u_n = \sqrt{n^2+n} - n$$

3/ Soit z un complexe.

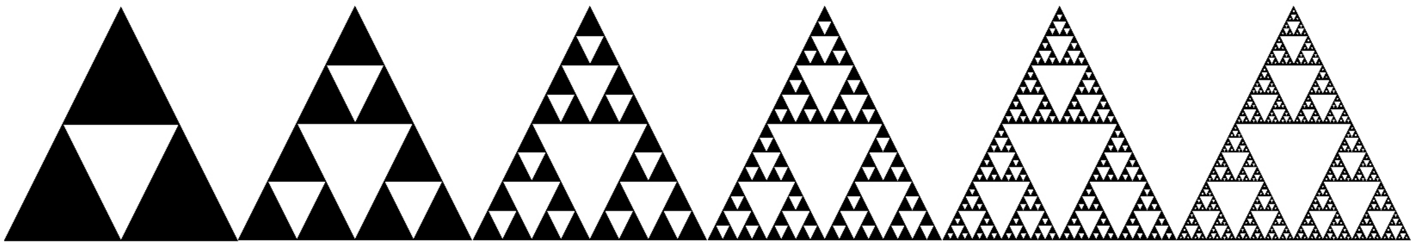
Étudier la série $\sum_{k=N}^{\infty} z^k$ (convergence somme éventuelle) de 2 manières :

- en mettant en facteur une puissance appropriée de z
- en étudiant le reste de rang ? ? ? de la série

4/ Discuter suivant les valeurs de α et β dans $\mathbb{R}$ la convergence de la série $\left[\sqrt{n} + \alpha\sqrt{n+1} + \beta\sqrt{n+2}\right]$

Indication : faire un développement limité de $\sqrt{n} + \alpha\sqrt{n+1} + \beta\sqrt{n+2}$ quand $n \longrightarrow +\infty$

5/ Pour chacune des 6 figures ci-dessous, quelle est l'aire en noir ? Si on continuait ? limite ?



Wacław Franciszek Sierpiński (1882-1969)