MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS DEPARTMENT

MTE 506 DIGITAL CONTROL

Lab 8 – SPRING 2019

Lab 8

Goals of The Lab

- Discrete PID
- Zero Order Hold

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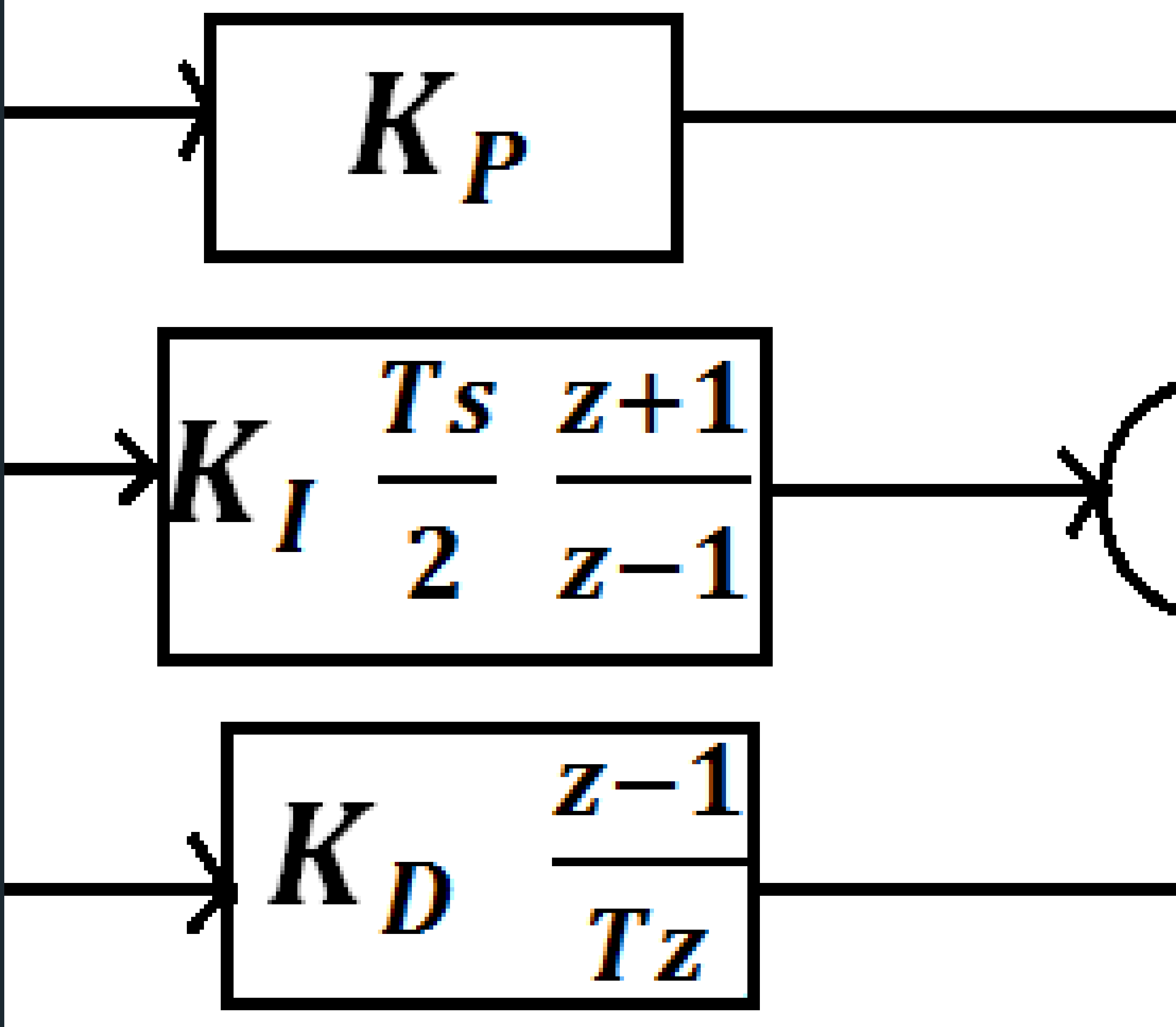
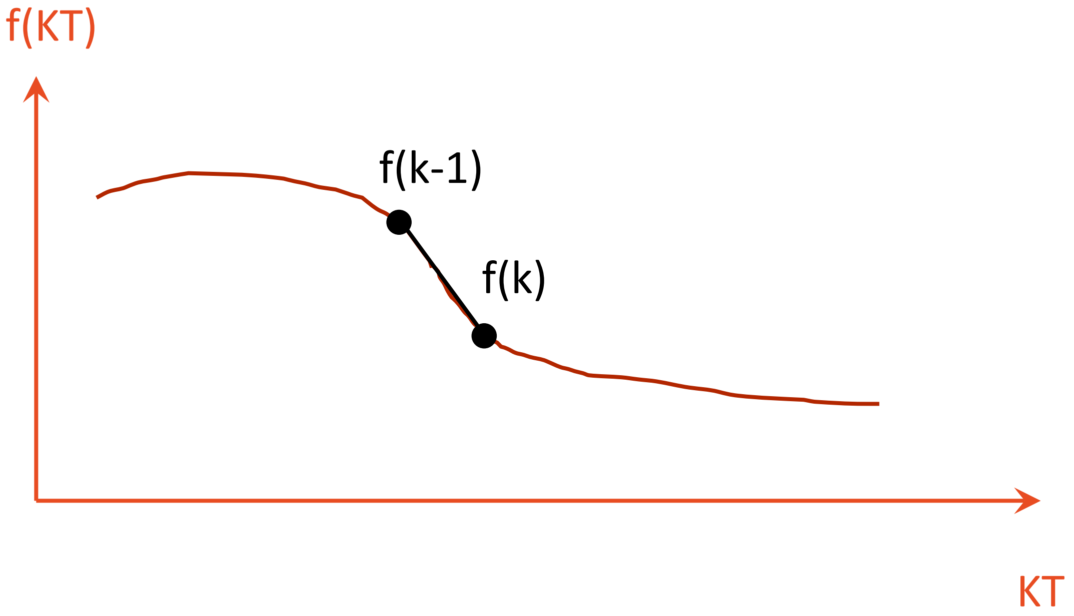
Discrete PID



Mathematical Derivation



Discrete Derivative

$$y(k) = \frac{df}{dt} \cong \frac{f(k) - f(k-1)}{T_s}$$

$$Y(z) = \frac{F(z) - z^{-1}F(z)}{T_s} = \frac{F(z)[1 - z^{-1}]}{T_s} = \frac{z}{z}\frac{F(z)[1 - z^{-1}]}{T_s}$$

$$Y(z) = \frac{z - 1}{T_s z}F(z)$$

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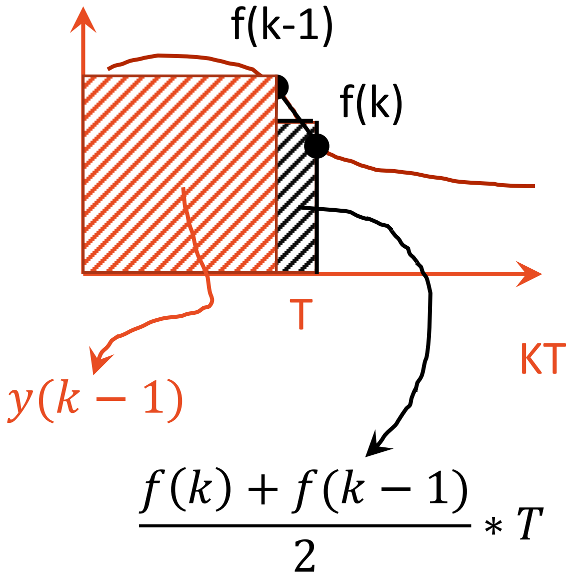
Mathematical Derivation

Discrete Integral (using Trapezoidal rule)



$$y(k) = \int_0^{k*T_s} f(t)dt = y(k-1) + \frac{f(k)+f(k-1)}{2} T_s$$

$$Y(z) = z^{-1}Y(z) + \frac{F(z)[1+z^{-1}]}{2} T_s \rightarrow Y(z)[1-z^{-1}] = \frac{T_s}{2}[1+z^{-1}]F(z)$$

$$Y(z) = \frac{T_s}{2}\frac{1+z^{-1}}{1-z^{-1}}F(z) = \frac{T_s}{2}\frac{z+1}{z-1}F(z)$$

Discrete PID controller

Analog PID

$$\mu(t) = K_p e(t) + K_i \int_{t_0}^{t_f} e(t)dt + K_d \frac{de}{dt}$$

Discrete PID

$$\mathrm{M}(z) = K_p E(z) + K_i \frac{T_s}{2} \frac{z+1}{z-1} E(z) + K_d \frac{z-1}{T_s z} E(z)$$

Discrete PID controller

Implemented PID

$$\underbrace{U(t)}_{} = \underbrace{U(t - T_s)}_{\text{Previous Controller Action}} + \underbrace{K_p * e(t)}_{\text{Proportional Controller}} + \underbrace{K_i * \left[\frac{e(t) + e(t - T_s)}{2} * T_s\right]}_{\text{Integral Controller}} + \underbrace{K_d * \frac{e(t) - e(t - T_s)}{T_s}}_{\text{Differential Controller}}$$

A Low Pass Filter Must Be Added If Differential Controller Is Used

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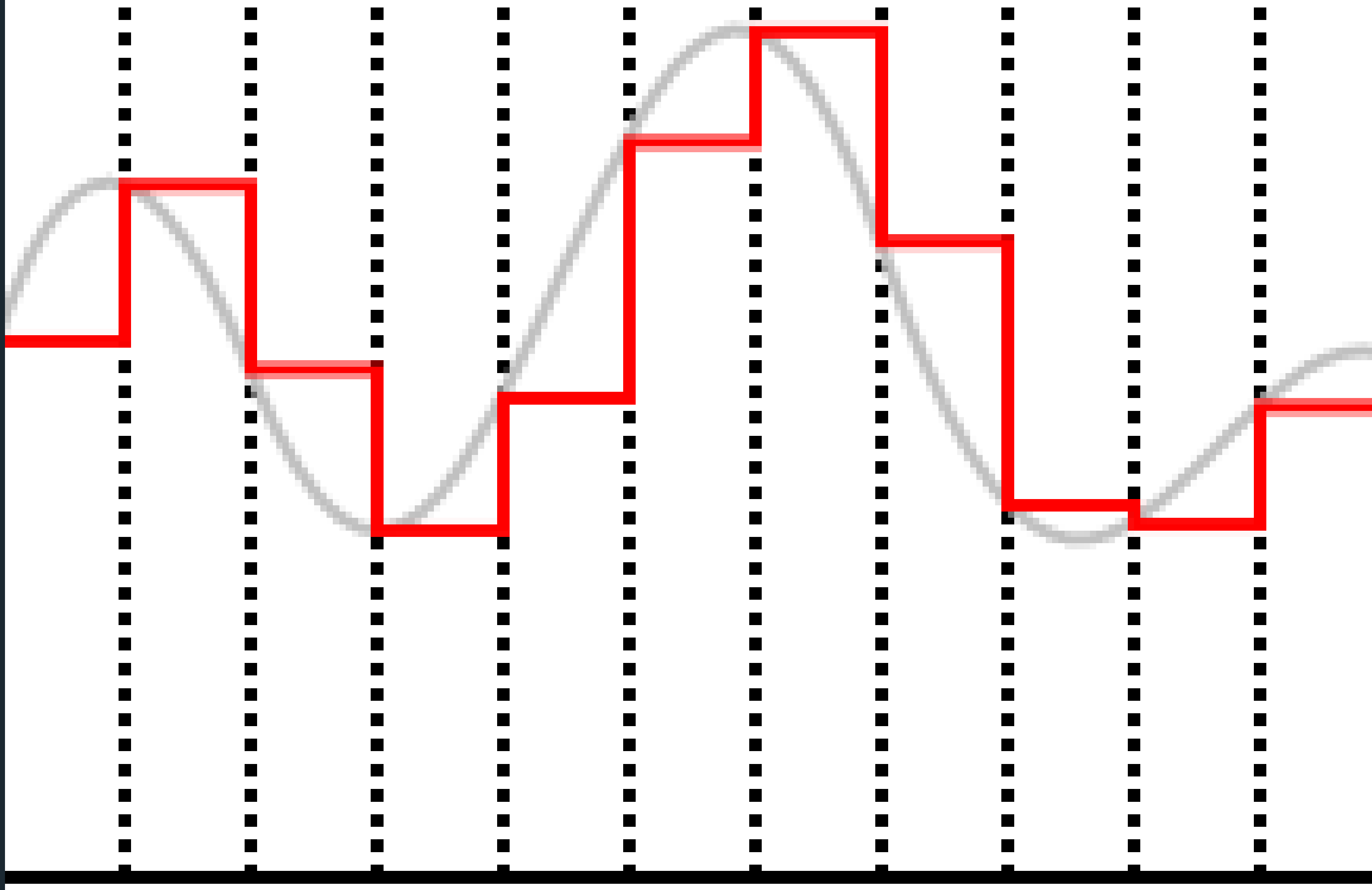
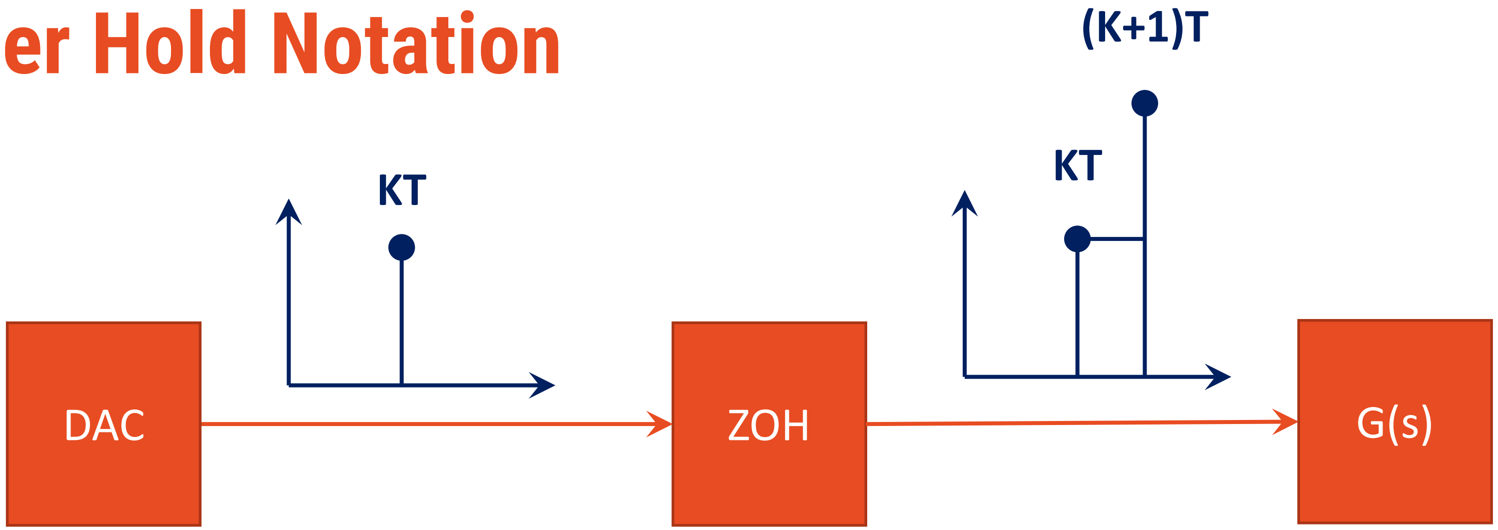
Zero Order Hold



Zero Order Hold Notation



Modeing the pulse

$$G_{ZOH}(s) = \mathcal{L}\big(1(t) - 1(t-T)\big) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1-e^{-sT}}{s}$$

Zero Order Hold Notation

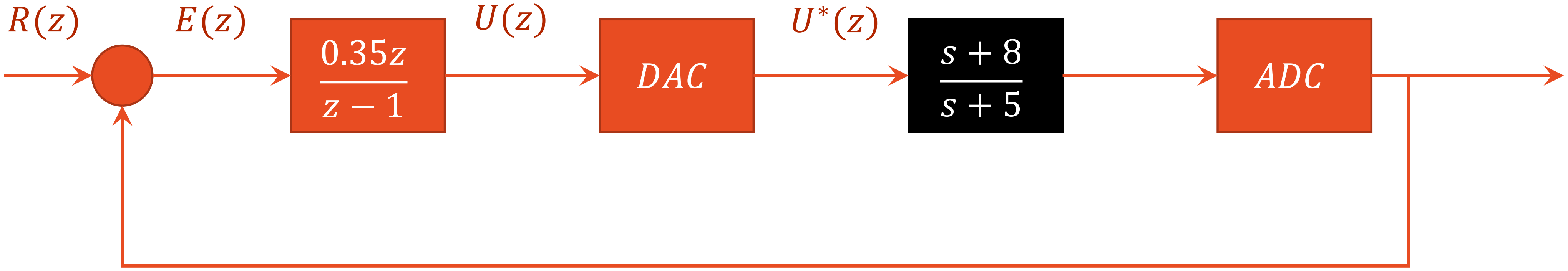
Modeing the pulse

$$G_{ZOH}(s) = \mathcal{L}\big(1(t) - 1(t-T)\big) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1-e^{-sT}}{s}$$

$$G_{ZA}(s) = G(s)G_{ZOH}(s) = (1-e^{-sT})\frac{G(s)}{s}$$

$$G_{ZA}(z) = \ (1-z^{-T})\mathcal{Z}\left[\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right]$$

Zero Order Hold for Closed Loop System



Find the z − transform function for the analog system with DAC and ADC

$$\frac{G(s)}{s} = \frac{s+8}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} = 1.6\frac{1}{s} - 0.6\frac{s}{s+5}$$

Zero Order Hold for Closed Loop System

$$Find\ the\ z - transform\ function\ for\ the\ analog\ system\ with\ DAC\ and\ ADC$$

$$\frac{G(s)}{s} = \frac{s+8}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} = 1.6\frac{1}{s} - 0.6\frac{s}{s+5}$$

$$G_{ZAS}(z) = (1 - z^{-1})\mathcal{Z}\left(\frac{G(s)}{s}\right) = \frac{z-1}{z}\mathcal{Z}(1.6 - 0.6e^{-5}) = \frac{z-1}{z}\left(1.6\frac{z}{z-1} - 0.6\frac{z}{z-e^{-5}}\right)$$

$$G_{ZAS}(z) = 1.6 - 0.6\frac{z-1}{z}\frac{z}{z-0.0067} = 1.6 - \frac{z-1}{z-0.0067} = \frac{0.6z + 0.989}{z-0.0067}$$

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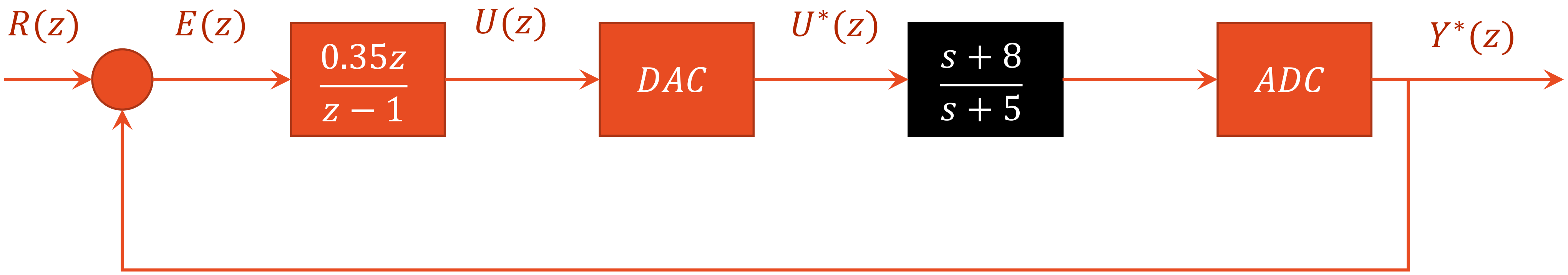
Zero Order Hold for Closed Loop System



Find the closed loop transfer function (paper work and using MATLAB)

Assignment

Don’t forget to pull the lab update from.

http://github.com/wbadry/mte506

Waleed El-Badry

END OF Lab 8