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QUESTION 1

1.1 truth table: F = (p ˅ q) ^ (p → r) ^ (¬r) → q

p	**q**	**r**	(p ∨ q)	(p → r)	(p ∨ q) ∧ (p → r)	(¬r)	(¬r) → q	[(p ∨ q) ∧ (p → r)] ∧ (¬r)	(p ∨ q) ∧ (p → r) ∧ (¬r) → q
T	T	F	T	F	F	T	T	F	T
T	F	T	T	T	T	F	T	F	T
T	T	F	T	F	F	T	T	F	T
T	F	T	T	T	T	F	T	F	T
F	T	F	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	F	T
F	T	F	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	F	T

∴ The result for F is always true for every combination of truth value for p, q, and r.

Thus, the entire expression is a tautology, meaning it is always true regardless of the truth values of the individual variables p, q, and r

1.2Logical Equivalence Laws: $\neg (q \to p) \vee (p \wedge q) \equiv q$

$\neg (\neg q \to q) \vee (p \wedge q) \equiv$ De Morgans Law

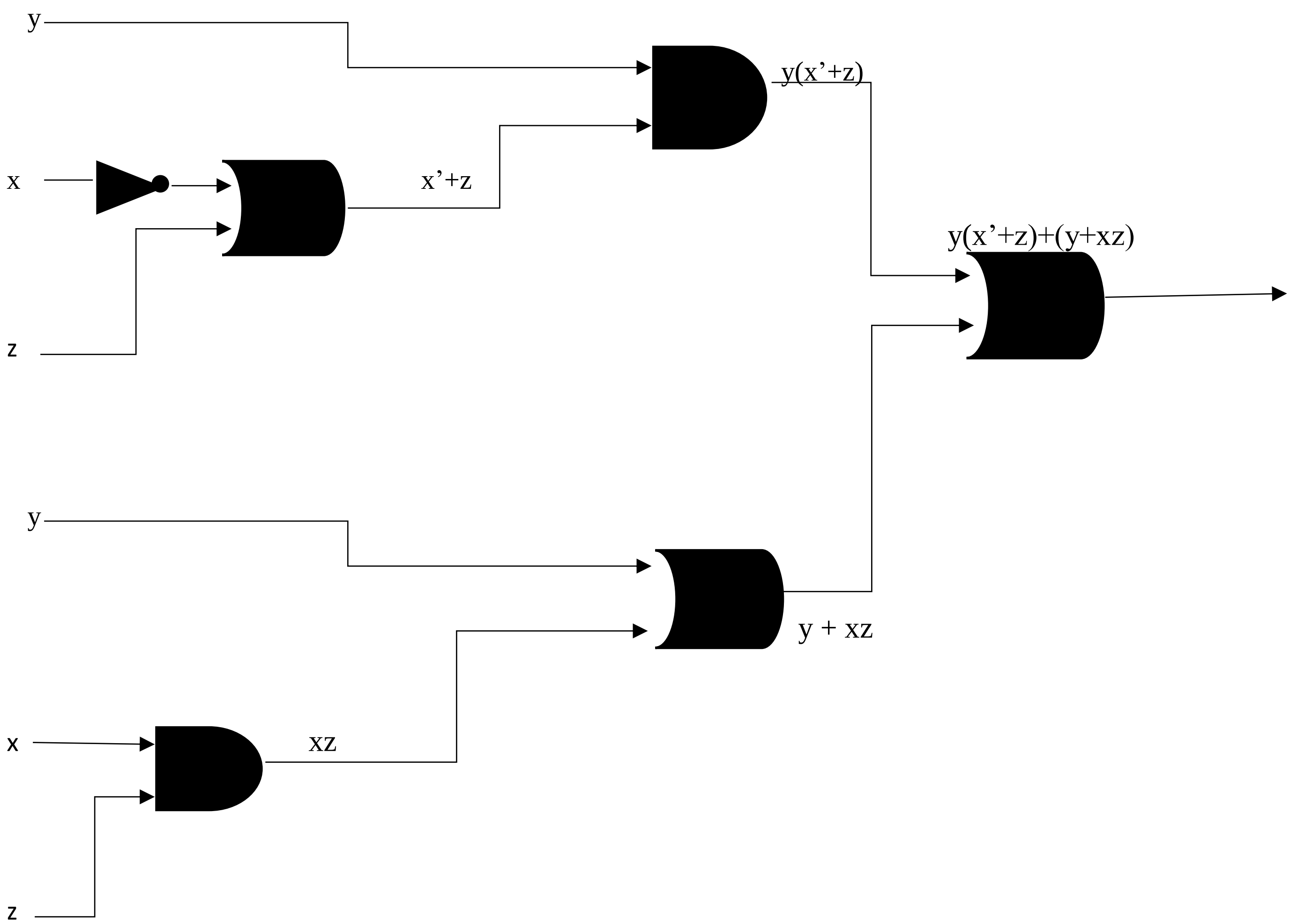
$(q \wedge \neg p) \vee (p \wedge q) \equiv$

$q \vee (\neg p \wedge p) \equiv$ Associative Law

$q \vee F \equiv$ Negation Law

$\therefore q \equiv \neg (\neg q \to q) \vee (p \wedge q)$

1.3 circuit diagram: F = y (x’ + z) + (y + xz)



QUESTION 2

2.1 functions:

$F(x) = 3x + 6$ and $g(x) = 4x + k$

$F(g(x)) = 3(4x+k) + 6$

$=12x + 3k + 6$

$G(f(x)) = 4(3x + 6)$

$= 12x + 24 + k$

$F(g(x)) = G(f(x))$

$12x + 3k + 6 = 12x + 24 + k$

$(12x - 12x) + (3k - k) + (6 - 24) = 0$

$2k - 18 = 0$

$2k = 18$

$2k/2 = 18/2$

$\therefore k = 9$

2.2 Sets

2.2.1 A – (B ∩ C) = (A – B) U (A - C)

A – (B Ꮖ C) = {a; e; i} – {u}

= {a; e; i}

(A – B) U (A - C) = {a; i; u} U {a; e; i; u}

= {a; e; i}

∴ A – (B Ꮖ C) = (A – B) U (A - C)

2.2.2 A ∩ (B - C) = (A ∩ B) – (A ∩ C)

A Ꮖ (B - C) = {a; u; i} ∩ {e}

= {a, i}

(A Ꮖ B) - (A Ꮖ C) = {a, i} –{e}

={a; i}

∴ A Ꮖ (B - C) = (A Ꮖ B) – (A Ꮖ C)

2.3 Arithmetic series

Tn = a + (n-1)d

178 = -2 + (n – 1)d

178 = -2 + (9n -9)

178 + 9 = 9n

189/9 = 9n/9

∴n= 21

$S_n = \frac{n}{2}(a_1 + a_1)$

$= \frac{21}{2}(-2 + 178)$

∴S_n= 1848

QUESTION 3

3.1 Crammers Rule

-x -2y -3z = 8

2x + z = 1

3x – 4y + 4z = 4

Coefficient of matrix

$$\begin{pmatrix} a = -1 & b = -2 & c = -3 \\ d = 2 & e = 0 & f = 1 \\ g = 3 & h = -4 & i = 4 \end{pmatrix}$$

Main determinant:

D = a(ei – fh) -b(di – fg) +c(dh – eg)

a(ei – fh) = -1(0 x 4 – 1 x -4) = -4

-b(di – fg) = 2(0 x 4 -1 x -4) = 8

+c(dh – eg) = -3(2 x 4 – 0 x 3) = -24

D = -4 + 8 – 24 = -20

Finding D_x , D_y , D_z:

$D_x = a(ei - fh) - b(di - fg) + c(dh - eg)$

$$\begin{pmatrix} a = 8 & b = -2 & c = -3 \\ d = 1 & e = 0 & f = 1 \\ g = 4 & h = -4 & i = 4 \end{pmatrix}$$

a(ei – fh) = 8(0 x 4 – 1 x 4) = -32

-b(di – fg) = 2(1 x 4 – 1x 4) = 0

+c(dh – eg) = 3(1 x -4 – 0 x 4) = -12

$D_x = -32 + 0 - 12 = -44$

$D_y = a(ei - fh) - b(di - fg) + c(dh - eg)$

$$\begin{pmatrix} a = -1 & b = 8 & c = -3 \\ d = 2 & e = 1 & f = 1 \\ g = 3 & g = 4 & i = 4 \end{pmatrix}$$

a(ei – fh) = -1(2 x 4 – 1 x 3) = -5

-b(di – fg) = -1(2 x 4 – 1 x 3) = -32

+c(dh – eg) = 3(2 x 4 – 1 x 3) = 15

$$D_y = -5 - 32 + 15 = -22$$

$$D_z = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$\begin{pmatrix} a = -1 & b = -2 & c = 8 \\ d = 2 & e = 0 & f = 1 \\ g = 3 & g = -4 & i = 4 \end{pmatrix}$$

a(ei – fh) = -1 (0 x 4 – 1 x 4) = 4

-b(di – fg) = -2(2 x 4 – 1 x 3) = -10

+c(dh – eg) = 8(2 x 4 – 0 x 3) = 40

$$D_z = 4 - 10 + 40 = 34$$

Finding x, y, z:

$$x = \frac{D_X}{D} = \frac{-44}{-20}$$

$$y = \frac{D_y}{D} = \frac{-22}{-20}$$

$$z = \frac{D_z}{D} = \frac{34}{-20}$$

3.2 Geometric progression

$T_n = a.r^{n-1}$

$1536 = 6.2^{n-1}$

$\frac{1536}{6} = \frac{6.2^{n-1}}{6}$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$8 = n - 1$

$n = 9$

$S_n = a.\frac{r^n - 1}{r - 1}$

$= 6.\frac{2^9 - 1}{2 - 1}$

= 3066

3.3 Inverse Matrix Method

$3x - 2y + 3z = 8$

$2x + y - 2 = 1$

$4x - 3y + 2z = 4$

$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad D = \begin{bmatrix} 8 \\ 18 \\ 16 \end{bmatrix}$

$D_A = a(ei - fh) - b(di - fg) + c(dh - eg)$

$a(ei - fh) = 3\big((-1)(2) - (-1)(-3)\big) = -3$

$-b(di - fg) = -(-2((2)(2) - (-1)(4) = 16$

$+c(dh - eg) = 3\big((2)(-3) - (1)(4)\big) = -30$

$D_a = (-3) + (16) + (-30) = -17$

Matrix of minors:

$for\ a_{11=3}$:

$A(a_{11}) = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = (1 \times 2) - (-1 \times -3) = -1$

$for\ a_{12} = 2$:

$A(a_{12}) = \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} = (2 \times 2) - (-1 \times -4) = 8$

$for\ a_{13} = 3$:

$A(a_{13}) = \begin{vmatrix} 2 & -1 \\ 4 & -3 \end{vmatrix} = (2 \times -3) - (1 \times 4) = -10$

$for\ a_{21} = 2$:

$A(a_{21}) = \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-2 \times 2) - (3 \times -3) = 5$

$for\ a_{22} = 1$:

$A(a_{22}) = \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = (3 \times 2) - (3 \times 4) = -6$

$for\ a_{23} = -1$:

$A(a_{23}) = \begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} = (3 \times -1) - (3 \times 2) = -9$

$for\ a_{31} = -3$:

$A(a_{31}) = \begin{vmatrix} 2 & -1 \\ 4 & -3 \end{vmatrix} = (2 \times -3) - (1 \times 4) = -10$

$for\ a_{32} = -3$:

$A(a_{32}) = \begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} = (3\ \times -1\ ) - (3\ \times 2) = -9$

$for\ a_{33} = 2$:

$A(a_{33}) = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = (3\ \ \times 1\ ) - (-2\ \times -2) = 7$

Cofactor matrix:

$$\begin{bmatrix} 1 & -8 & 10 \\ -5 & 6 & -9 \\ 10 & -9 & 7 \end{bmatrix}$$

Finding A^{-1}:

$$A^{-1} = \frac{1}{\det(A)} \times adj(A)$$

$$A^{-1} = \frac{1}{-17} \times \begin{bmatrix} 1 & -8 & 10 \\ -5 & 6 & -9 \\ 10 & -9 & 7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{17} & \frac{8}{17} & -\frac{10}{17} \\ \frac{5}{17} & -\frac{6}{17} & \frac{9}{17} \\ -\frac{10}{17} & \frac{9}{17} & -\frac{7}{17} \end{bmatrix}$$

Solving for X:

$X = \ A^{-1}.D$

$$X = \begin{bmatrix} -\frac{1}{17} & \frac{8}{17} & -\frac{10}{17} \\ \frac{5}{17} & -\frac{6}{17} & \frac{9}{17} \\ -\frac{10}{17} & \frac{9}{17} & -\frac{7}{17} \end{bmatrix} \times \begin{bmatrix} 8 \\ 18 \\ 16 \end{bmatrix}$$

$$X = \begin{bmatrix} (-\frac{1}{17}.8 + \frac{8}{17}.18 - \frac{10}{17}.16) \\ (\frac{5}{17}.8 - \frac{6}{17}.18 + \frac{9}{17}.16) \\ (-\frac{10}{17}.8 + \frac{9}{17}.18 - \frac{7}{17}.16) \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-24}{17} \\ \frac{76}{17} \\ \frac{-30}{17} \end{bmatrix}$$

$$x = \frac{-24}{17}$$

$$y = \frac{76}{17}$$

$$z = \frac{-30}{17}$$