Fluctuations in a Dual Labor Market

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April 8, 2020

1 Model

The model follows a discrete timing and embeds 4 types of agents; the households, the firms, the fiscal authority and the central bank. Households can be unemployed or employed through a fixed-term or an open-ended contract. Three types of firms coexist. Perfectly competitive firms produce the final good valued by households for consumption and investment. Final good producers aggregate the differentiated goods produced by the retailers. The latter retailers are in monopolistic competition and transform the homogeneous intermediate good into a differentiated retail good. Intermediate firms produce the associated intermediate good and experience perfect competition. These intermediate firms use labor as their only input. I now describe the behavior of different types of agents in more detail.

1.1 Households

Households are identical and constitute a continuum represented by the interval $(0, 1)$. They can be employed under a fixed-term or an open-ended contract, or unemployed. They earn wages or unemployment benefits accordingly. Households also hold firms, consume the homogeneous good produced by final good firms, save using one-period nominal bonds, earn interests on their savings and pay lump-sum taxes. Hence, I assume that taxes do not distort the choice of households over consumption and investment. I do not consider the interplay between payroll taxes and labor market dualism.

If households are identical *ex ante*, their different employment histories make them heterogeneous *ex post*. How labor market dualism impacts the consumption and saving behavior of households is beyond the scope of this paper. As **?** and **?** first did, I assume that households pool revenues and that capital markets are perfect. Thereby, I rule out the complication heterogeneity brings on. Households share equal consumption and investments. This assumption is not innocuous: unemployed, open-ended and fixed-term workers have different borrowing constraints and **?** shows that assets shape the search behavior. I leave these issues for future research.

The household's program boils down to

$$
\begin{aligned}
&\max_{\{c_t, B_{t+1}\}_{t=0}^{+\infty}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t u(c_t) \\
&\text{s.t} \quad c_t + \frac{B_{t+1}}{P_t} = R_{t-1}\frac{B_t}{P_t} + \overline{w_t^p} n_t^p + \overline{w_t^f} n_t^f + \rho^b \overline{w_t} u_t + \Pi_t - \tau_t
\end{aligned}
$$

C_t marks down consumption. B_t is the amount of nominal bond holdings at the beginning of period t, with the associated nominal interest rate R_t between t and $t+1$. $\overline{w_t^p}$, $\overline{w_t^f}$ and $\overline{w_t}$ denote respectively

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the average real wages for open-ended jobs, temporary jobs and all workers. ρ^b denotes the replacement rate of unemployment benefits over the average wage. n_t^p is the aggregate open-ended employment and n_t^f denotes its fixed-term counterpart. u_t marks down the measure of unemployed households. Firms transfer their profits to households through Π_t, while the government taxes τ_t to finance the payment of unemployment benefits.

The first order conditions with respect to c_t and B_{t+1} lead to the following Euler equation

$$u'(c_t) = \beta \mathbb{E}_t \left[R_t \frac{P_t}{P_{t+1}} u'(c_{t+1}) \right] \tag{1}$$

As households own firms, the firms' discount factor is $\beta_{t,s} = \beta^{s-t} u'(c_s) / u'(c_t)$.

1.2 Final good firms

Final good firms are identical and compete to produce the good consumed by households. They use a Dixit-Stiglitz aggregator as technology to put together retail goods and produce Y_t.

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon_t}} di \right)^{\frac{\epsilon}{\epsilon}-1} \tag{2}$$

where ϵ is the elasticity of substitution between retail goods.

The firm takes as given the price of the retail goods $P_{i,t}$ and the price of the final good P_t and maximizes its profits with respect to the components of its input $\{Y_{i,t}\}_{i\in(0,1)}$ under the constraint (2). The program of the final good firm boils down to

$$\max_{\{Y_{i,t}\}_{i\in[0,1]}} \quad P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

subject to (2)

The subsequent first order condition provides an expression for the demand of the retail good i

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t \tag{3}$$

1.3 Retailers

Retailers buy goods from intermediate firms and sell the obtained production to final good producer[1]. They are in monopolistic competition and lie on the interval $(0, 1)$. Retailers accomplish the one-to-one transformation of the intermediate good into a retail good. Denoting $X_{i,t}$ the retailer's input in the intermediate good, the production technology writes

$$Y_{i,t} = X_{i,t}$$

As a result, retailers face a real marginal cost that equals the relative price of the intermediate good ϕ_t. I assume that retailers adjust prices as **?** describe.

$$P_{i,t} = \begin{cases} P_{i,t-1} & \text{with probability } \psi \\ P_{i,t}^* & \text{with probability } 1-\psi \end{cases}$$

[1]At this step, it is possible to introduce capital to extend the model.

A fraction ψ of retailers is able to adjust its prices to the optimal value $P^*_{i,t}$, whereas the remaining retailers stick to their former prices. There is no indexation of non-adjusted princes on inflation in this model. The price-setting retailer i at period t has the following program

$$\max_{P^*_{i,t}} \mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T}\psi^{T-t}\left(\frac{P^*_{i,t}}{P_T} - \phi_T\right) Y_{i,T}$$

$$\text{subject to } Y_{i,T} = \left(\frac{P^*_{i,t}}{P_T}\right)^{-\epsilon_T} Y_T$$

This leads to the following first order condition

$$\mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T}\psi^{T-t} P_T^{\epsilon_T} Y_T \left(\frac{P^*_{i,t}}{P_T} - \mu\phi_T\right) = 0 \tag{4}$$

where $\mu = \epsilon/(\epsilon - 1)$ is the mark-up of retailers.

1.4 Intermediate good firms and the labor market

Intermediate-good firms use labor as input. They can employ one worker or maintain one vacancy. Workers can be unemployed or employed under a fixed-term or an open-ended contract. They are identical. There is no on-the-job search, which implies that only unemployed workers search for a job. When a firm and a worker meet, the idiosyncratic productivity of the match z is revealed. I assume that idiosyncratic productivity is i.i.d across time and drawn from a distribution with cumulative distribution function G. I reluctantly make this assumption for the sake of simplicity. With persistent idiosyncratic productivity, the computation of the equilibrium requires keeping track of the productivity distribution of matches as a state variable. Since the literature considering cycles and dual labor market is in early stages, I prefer to leave the distributional issues for future research.

The number of firm-worker contacts per period is $m(e, v)$, where e is the number of job-seekers and v is the number of vacancies. A classic measure of the matching activity is the labor market tightness $\theta = v/e$. The matching function m has constant returns to scale, which enables the definition of the firm-worker meeting probability $p(\theta)$ on the job seekers' side and its counterpart $q(\theta)$ on the firms' side.

$$q = \frac{m(e, v)}{v} = m\left(\theta^{-1}, 1\right)$$

$$p = \frac{m(e, v)}{e} = m(1, \theta) = \theta q(\theta)$$

p is increasing in labor market tightness, whereas q is decreasing in labor market tightness. Note that the meeting probabilities are not the classic job-finding and vacancy-filling probabilities. A firm-worker meeting does not lead to a production if the idiosyncratic productivity is too low.

The timing in the economy is summed up in Figure 1.4. At the beginning of the period, agents learn the value of shocks and firms manage their workforce accordingly. They lay off poorly productive workers and post vacancies. Next, new matches are revealed. Workers fired in the current period are able to participate to the present meeting round. Hence, I avoid understating labor market flows as most fixed-term contracts last less than a quarter: fixed-term jobs last 1.5 months on average in France (**?**). Finally, production is carried out, firms pay for wages and firing costs, households consume, and the period ends.

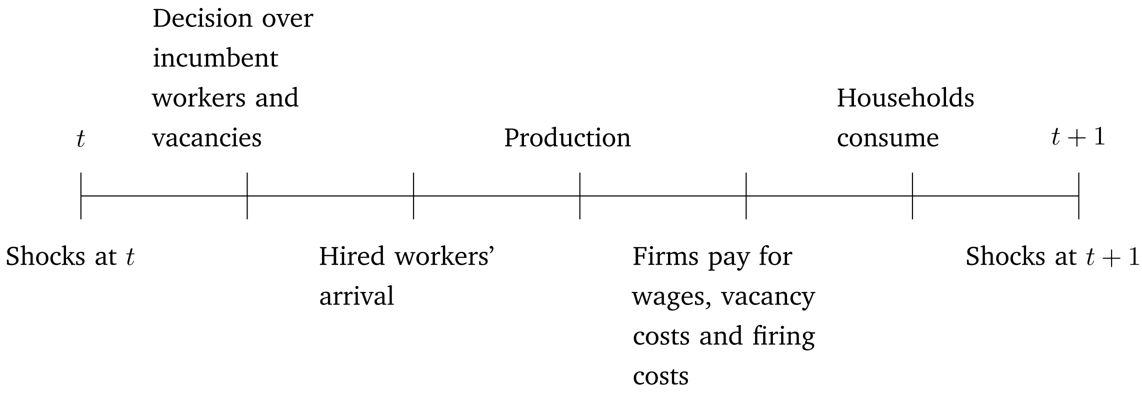

Figure 1: The timing of the economy

Vacancies I depart from the literature when it comes to job creation. **?** assumes that job creation occurs through fixed-term contracts only; open-ended jobs all come from converted fixed-term jobs, which is counterfactual. As **?** first did, **?** assume that new matches face hiring restrictions. Firms are allowed to hire either through an open-ended or a fixed-term contract according to an exogenous probability. Otherwise, all hires would take place through fixed-term contracts. As roughly 20% of hires are open-ended in France, different reasons than legal constraints on fixed-term hires explain open-ended hires.

Thereby, I assume that no constraints bind job creation. When paired with a worker, firms hire through a fixed-term contract or through an open-ended contract. They can also return searching for a worker and get the chance to be matched with another one on the next period. The present discounted value of a vacancy V_t bears witness of these different options:

$$V_t = -\gamma + q\left(\theta_t\right) \int max \left[J_t^{0,p}\left(z\right), J_t^f\left(z\right), \mathbb{E}_t \beta_{t,t+1} V_{t+1}\right] dG(z) \tag{5}$$

where γ is the cost of a vacancy, $J_t^{0,p}$ is the firm's surplus with a new open-ended match and $J_t^{0,f}$ is the firm's surplus with a new fixed-term match. The counterpart of these surpluses for continuing open-ended matches and continuing fixed-term matches are J_t^p and J_t^f. The associated surpluses of workers are denoted W, while the total surpluses are denoted S.

Surplus sharing I assume that wages are set each period by Nash bargaining. It is not realistic considering the evidence supporting rigidity in wages. In addition, wage flexibility seems inconsistent with the stickiness of retailers' prices. Still, I leave rigid wages for future research. Tractability motivates the use of Nash bargaining. It makes hiring and firing decisions jointly efficient and only dependent on the total surplus of a match. Denoting η the worker's share of the match surplus, the sharing rules write

$$W_t^p\left(z_t\right) = \eta S_t^p\left(z_t\right) \tag{6}$$

$$W_t^{0,p}\left(z_t\right) = \eta S_t^{0,p}\left(z_t\right) \tag{7}$$

$$W_t^f\left(z_t\right) = \eta S_t^f\left(z_t\right) \tag{8}$$

Total surpluses verify

$$S_t^p\left(z_t\right) = J_t^p\left(z_t\right) - \left(V_t - F_t\right) + W_t^p\left(z_t\right) - U_t \tag{9}$$

$$S_t^{0,p}(z_t) = J_t^{0,p}(z_t) - V_t + W_t^{0,p}(z_t) - U_t \tag{10}$$

$$S_t^f(z_t) = J_t^f(z_t) - V_t + W_t^f(z_t) - U_t \tag{11}$$

where U_t is the value of unemployment at the beginning of period t. The outside option of workers includes taking part in current period's market trades and take the unemployment benefit if no beneficial meeting occurred.

$$U_t = p(\theta_t) \int \max\left(W_t^{0,p}(z), W_t^f(z), U_t^0\right) dG(z) + (1 - p(\theta_t)) U_t^0 \tag{12}$$

$$U_t^0 = b_t + \mathbb{E}_t \beta_{t,t+1} U_{t+1} \tag{13}$$

where $b_t = \rho^b \overline{w_t} + h$ is the unemployed's flow of utility. It includes unemployment benefits as well as home production h.

In (9)-(11), note that the surplus of incumbent open-ended matches includes firing costs, while the one of new open-ended workers does not. In case of disagreement during wage bargaining, a newly paired worker goes back to the pool of job seekers and the firm does not pay firing costs. That way, I do not overstate the role of firing costs.

Now, I describe the open-ended and the fixed-term contract and the associated surpluses.

Open-ended contracts A continuing open-ended contract delivers the wage w_t^p and stipulates a firing tax F_t. An open-ended match may separate with the exogenous probability s. In this case, the separation bears no cost. Otherwise, the firm chooses whether it keeps or lays off the worker regarding the idiosyncratic productivity of the match. An endogenous separation entails the payment of the firing cost and the opening of a new vacancy. Hence, a firm with an incumbent open-ended worker has the following surplus

$$J_t^p(z_t) = \phi_t A_t z_t - w_t^p(z_t) + \mathbb{E}_t \beta_{t,t+1} \left\{ (1-s) \int \max\left[J_{t+1}^p(z), V_{t+1} - F_{t+1}\right] dG(z) + sV_{t+1} \right\} \tag{14}$$

An incumbent open-ended worker earns a wage and may leave with probability s. If he does not, he faces a productivity shock and the match may split if the latter shock is adverse enough. Laid-off or quitting workers go back to the job seekers' pool immediately and are therefore eligible to participate to the current's period firm-worker meetings. Otherwise, the match goes on.

$$W_t^p(z_t) = w_t^p(z_t) + \mathbb{E}_t \beta_{t,t+1} \left\{ (1-s) \int \max\left[W_{t+1}^p(z), U_{t+1}\right] dG(z) + sU_{t+1} \right\} \tag{15}$$

New open-ended workers have a different wage function $w_t^{0,p}$: their outside option does not include the payment of the firing cost in case of disagreement during the wage bargaining and the possibility to search for a new job in case of disagreement. After one-period, if there is no separation, a wage renegotiation occurs because idiosyncratic productivity changed. The firm pays firing costs if an endogenous split occurs. Otherwise, the surplus of incumbent open-ended contracts weighs in.

$$J_t^{0,p}(z_t) = \phi_t A_t z_t - w_t^{0,p}(z_t) + \mathbb{E}_t \beta_{t,t+1} (1-s) \left\{ \int_{z_{t+1}^p}^{+\infty} J_{t+1}^p(z) \, dG(z) - G\left(z_{t+1}^p\right) F_{t+1} + sV_{t+1} \right\} \tag{16}$$

$$W_t^{0,p}(z_t) = w_t^{0,p}(z_t) + \mathbb{E}_t \beta_{t,t+1} \left\{ (1-s) \int \max\left(W_{t+1}^p(z), U_{t+1}\right) dG(z) + sU_{t+1} \right\} \tag{17}$$

Fixed-term contracts A fixed-term contract stipulates the wage function w_t^f. Fixed-term matches split with the exogenous probability δ. I assume that fixed-term contracts yield a lower productivity than open-ended matches with a factor $\rho < 1$. I discuss this assumption in detail below. Firms value the immediate production net of the wage. As for the continuation value, it simply embeds the possibility of an exogenous separation shock.

$$J_t^f(z_t) = \rho A_t z_t \phi_t - w_t^f(z_t) + \mathbb{E}_t \beta_{t,t+1} \left\{ (1-\delta) \int J_{t+1}^f(z)\, dG(z) + \delta V_{t+1} \right\} \tag{18}$$

Fixed-term workers immediately value their wage. In case of separation, they can immediately indulge in labor market trades.

$$W_t^f(z_t) = w_t^f(z_t) + \mathbb{E}_t \beta_{t,t+1} \left\{ (1-\delta) \int W_{t+1}^f(z)\, dG(z) + \delta U_{t+1} \right\} \tag{19}$$

Using the different definitions of the firms' and the workers' surpluses, the total surpluses write[2]

$$\begin{aligned} S_t^p(z_t) &= A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + F_t - \mathbb{E}_t \beta_{t,t+1}(1-s) F_{t+1} \\ &\quad + \mathbb{E}_t \beta_{t,t+1}(1-s) \int \max\left(S_{t+1}^p(z), 0\right) dG(z) \end{aligned} \tag{20}$$

$$S_t^{0,p}(z_t) = S_t^p(z_t) - F_t \tag{21}$$

$$S_t^f(z_t) = \rho A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + \rho \mathbb{E}_t \beta_{t,t+1}(1-\delta) \int S_{t+1}^f(z) dG(z) \tag{22}$$

As intended, the surplus of continuing open-ended workers includes the firing cost in case of endogenous separation. This bolsters his threat point in Nash bargaining and pushes up his wage. The new open-ended workers does not benefit from this effect since a failure in the bargaining process does not entail the payment of F_t. The total surplus of fixed-term workers shows the productivity gap ρ in the flows and the exogenous termination of the contract in the continuation value. Fixed-term contracts hit with a job destruction shock split regardless the productivity of the match.

Using the firms' surpluses, joint surpluses and the surplus sharing rules, wages verify

$$w_t^p(z_t) = \eta\left(A_t z_t \phi_t + F_t - E_t \beta_{t,t+1}(1-s) F_{t+1} + \gamma\theta_t\right) + (1-\eta) b_t \tag{23}$$

$$w_t^{0,p}(z_t) = \eta\left(A_t z_t \phi_t - E_t \beta_{t,t+1}(1-s) F_{t+1} + \gamma\theta_t\right) + (1-\eta) b_t \tag{24}$$

$$w_t^f(z_t) = \eta\left(\rho A_t z_t \phi_t + \gamma\theta_t\right) + (1-\eta) b_t \tag{25}$$

$$\tag{26}$$

The new open-ended worker is penalized with higher firing costs to compensate the future gains of wages in case of continuation. Moreover, labor market tightness increases the outside option of workers through the higher probability of finding a job. Wages increase with labor market tightness.

1.5 Job creation and job destruction

I assume there is free entry on vacancy posting.

$$V_t = 0 \tag{27}$$

[2] Detailed calculations are available in Appendix B.1

The job creation condition stems from the free-entry condition (27) and the Bellman equation defining the present discounted value of an unfilled vacancy (5). Using the Nash-sharing rules (7)-(8), the job creation condition becomes

$$\frac{\gamma}{(1-\eta)q\left(\theta_t\right)} = \int max\left[S_t^{0,p}\left(z\right), S_t^f\left(z\right), 0\right] dG(z)$$

The choice between fixed-term and open-ended contracts lies in the comparison of joint surpluses across contracts. Notice that $\partial S_t^{0,p}/\partial z_t = A_t \phi_t A_t > \rho A_t \phi_t = \partial S_t^f/\partial z_t$. Thus, there exists a productivity threshold z_t^* above which open-ended contracts are preferable to fixed-term ones.

$$S_t^{0,p}\left(z_t^*\right) = S_t^f\left(z_t^*\right)$$

As a result, job creation with (32) and (33), one finds that z_t^* verifies

$$(1-\rho)z_t^* = z_t^c - \rho z_t^f \tag{28}$$

I also define the threshold z_t^p below which an incumbent open-ended match splits. Similarly, fixed-term contracts become profitable when productivity exceeds z_t^f. I denote z_t^c the analogous threshold for new open-ended contracts.

$$S_t^p\left(z_t^p\right) = 0 \tag{29}$$

$$S_t^f\left(z_t^f\right) = 0 \tag{30}$$

$$S_t^{0,p}\left(z_t^c\right) = 0 \tag{31}$$

Joint surpluses are linear in z_t. I can rewrite them using (29)-(30).

$$S_t^p(z) = A_t \phi_t \left(z - z_t^p\right) \tag{32}$$

$$S_t^f(z) = \rho A_t \phi_t \left(z - z_t^f\right) \tag{33}$$

Using (20)-(31) and with integrations by part, equations profitability thresholds.

$$\begin{aligned} &A_t z_t^p \phi_t + F_t - \mathbb{E}_t \beta_{t,t+1}(1-s)F_{t+1} + \mathbb{E}_t \beta_{t,t+1}(1-s)A_{t+1}\phi_{t+1} \int_{z_{t+1}^p}^{+\infty} (1-G(z))\, dG(z) \\ &= b_t + \frac{\eta\gamma\theta_t}{1-\eta} \end{aligned} \tag{34}$$

$$A_t \phi_t z_t^c = A_t \phi_t z_t^p + F_t \tag{35}$$

$$\rho A_t z_t^f \phi_t + \rho \mathbb{E}_t \beta_{t,t+1}(1-\delta)A_{t+1}\phi_{t+1}\left(\mathbb{E}z - z_{t+1}^f\right) = b_t + \frac{\eta\gamma\theta_t}{1-\eta} \tag{36}$$

(37)

Using these thresholds and integrations by part, I rewrite the job creation condition.

$$\frac{\gamma}{(1-\eta)q\left(\theta_t\right)A_t\phi_t} = \int_{\max[z_t^c, z_t^*]}^{+\infty} (1-G(z))\,dz + \rho \int_{z_t^f}^{\max[z_t^c, z_t^*]} (1-G(z))\,dz \tag{38}$$

The following proposition sums up job creation.

Proposition 1. *Given* $\left(z_t^p, z_t^c, z_t^f, z_t^*\right)$*,*

- *if* $z_t^* \leq z_t^f \leq z_t^c$*, job creation is carried out through open-ended contracts only. it occurs when* $z \geq z_t^c$ *as figure 2 shows.*

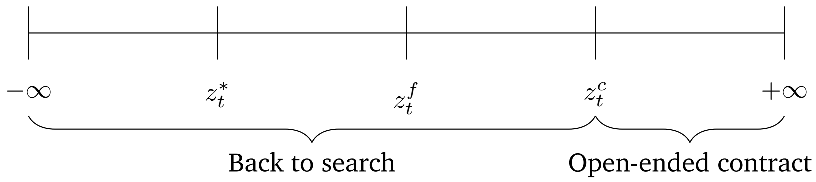

Figure 2: Open-ended hires only

- *Otherwise, necessarily,* $z_t^c \leq z_t^f \leq z_t^*$*: job creation is carried out through both open-ended contracts and fixed-term contracts. Fixed-term contracts are hired when* $z \in (z^f, z^*)$ *and open-ended contracts are hired when* $z \in (z^*, +\infty)$*. Figure 3 sums it up.*

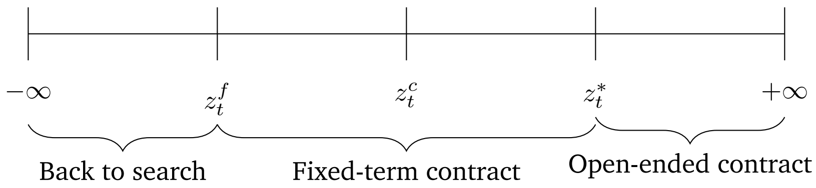

Figure 3: Dual job creation

Proof. See Appendix A □

Demonstrating proposition 1 follows the same steps as in **?**. However, the choice between open-ended contract and a fixed-term contract does not stem from the same mechanisms. Here, the trade-off opposes flexibility and productivity. An open-ended contract delivers a full productivity but may lead to the payment of firing costs if an adverse productivity shock hits. Thus, hiring an open-ended worker is worth it if productivity is high enough to overcome the expected firing costs. A fixed-term contract delivers a lower productivity, but it is shorter and separation costs zero. The option of hiring a fixed-term contract makes agents short-sighted: it pushes up the minimum productivity to hire an open-ended contract and tightens the hiring window for open-ended contracts.

? is similar to the present model with two notable exceptions. Firstly, open-ended and fixed-term workers have the same productivity. Secondly, matches face i.i.d productivity shocks, whose occurrence follows a Poisson law. A new match with a high productivity chooses a contract to last as long as possible: it would be a pity to split before any productivity shock hits and to lose the advantage of a high productivity draw. Thus, on average, an open-ended contract lasts longer than a fixed-term contract and enables to lock up highly productive matches. A new match with a lower initial productivity may not find it optimal to face the risk of paying firing costs in the doldrums to secure current gains. In worst cases, going back to search is the best option. In the middle ground, though, fixed-term contracts are relevant; they enable both production and a quick return to search for a better match.

In this paper, productivity shocks no longer hit at random according to a memory-less Poisson law, which cuts out the incentive to secure the most productive matches through open-ended contracts. When meeting, a firm and a worker know that the current productivity draw will last one period. They do not hope a high productivity draw to last forever. Thereby, Fixed-term contracts lose their role of median solution between producing and searching for more productive matches. One contract would be systematically preferred to the other without the contractual productivity gap. Still, the assumption that fixed-term contracts are *per se* less productive than open-ended ones is not far-fetched. Fixed-term positions are mainly filled by low-skilled or inexperienced workers (**?**) and benefit less from on-the-job training (**????**). **?** estimate that fixed-term workers are 20% less productive than open-ended workers.

The main departure with **?** and **?** is the appearance of the threshold z_t^*. The movements in thresholds z_t^p, z_t^c, z_t^f and z_t^* shape the behavior of the labor market and its fluctuations. If job creation involves both fixed-term and open-ended contracts — as will be the case in my calibration — then Figure 4 sums up the hiring and firing policies and the position of the thresholds if idiosyncratic productivity shocks are drawn from a log-Normal distribution.

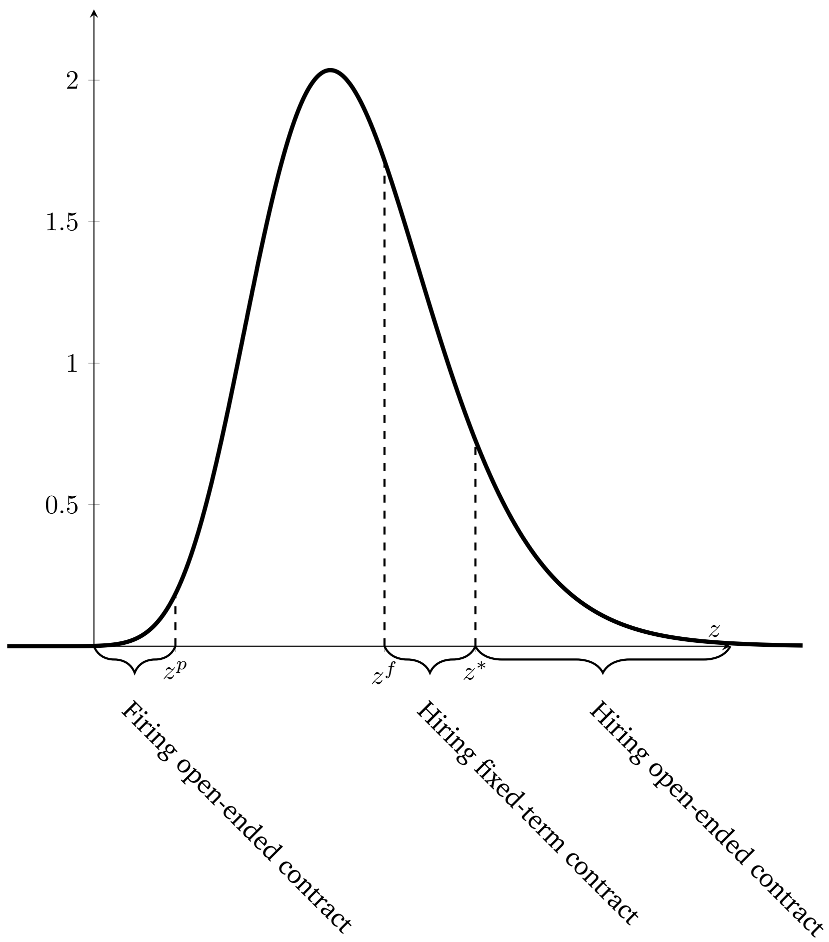

Figure 4: The probability distribution function of idiosyncratic shocks and the location of thresholds

The displayed pdf belongs to a log-Normal low with zero mean and standard deviation 0.2.

1.6 Fiscal and monetary policy

The government taxes households in order to provide for the unemployment benefits. The revenues from the firing tax get back to the government. Thus, the budget constraint of the government is

$$\tau_t + (1-s)G\left(z_t^p\right)n_t^p F_t = \rho^b \overline{w_t} u_t + g_t \tag{39}$$

where g_t is the government expenditure and follows the AR(1) log-process $log\left(g_t\right) = \left(1-\rho_g\right) log\left(\overline{g}\right) + \rho_g log\left(g_{t-1}\right) + \epsilon_t^g$, $\epsilon_t^g \sim \mathcal{N}\left(0, \sigma_g^2\right)$.

The monetary policy is decided in accordance with the Taylor rule

$$log\left(R_t/R\right) = \rho_R log\left(R_{t-1}/R\right) + \left(1-\rho_R\right)\left[\rho_\pi \mathbb{E}_t log\left(\frac{P_{t+1}}{P_t}\right) + \rho_y log\left(\frac{y_t}{y}\right)\right] + \epsilon_t^m \tag{40}$$

where y is the steady-state output and $\epsilon_t^m \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_m^2\right)$.

Fixed-term contracts are known to behave as buffers in front of workload fluctuations, as **?** documented. Thus, the case of indeterminacy in the Taylor rule and the subsequent appearance of sunspot equilibria may prove interesting. For the sake of simplicity, though, I shall restrain the present analysis to the determinate case with $\rho_\pi > 1$ and $\rho_y > 0$ and leave indeterminacy and its consequences on dual labor markets for future research.

1.7 Aggregate values and the equilibrium

This paragraph sums up the different conditions enabling an utter closing of the model. The employment values sum to the measure of households, namely 1.

$$n_t^p + n_t^f + u_t = 1 \tag{41}$$

n_t^p, n_t^f and u_t are the measures of open-ended employees, fixed-term employees and unemployed workers.

The aggregate stock of job-seekers includes the formerly unemployed households and the new entrants in the unemployment pool from the current period.

$$e_t = u_{t-1} + \delta n_{t-1}^f + \xi_t n_{t-1}^p \tag{42}$$

The labor market tightness is the ratio between aggregate number of vacancies $v_t = \int_0^1 v_{i,t} di$ and the number of job seekers.

$$\theta_t = \frac{v_t}{e_t}$$

The employment variables drop by the job destruction flow and augment by the job creation flow.

$$n_t^p = \left(1-\xi_t\right) n_{t-1}^p + \mu_t^p e_t \tag{43}$$

$$n_t^f = (1-\delta) n_{t-1}^f + \mu_t^f e_t \tag{44}$$

where $\mu_t^p = p\left(\theta_t\right)\left(1 - G\left(z_t^*\right)\right)$ and $\mu_t^f = p\left(\theta_t\right)\left(G\left(z_t^*\right) - G\left(z_t^f\right)\right)$ are the open-ended and fixed-term job-finding rates. $\xi_t = s + (1-s)G\left(z_t^p\right)$ is the the job destruction rate of open-ended contracts.

As for firms, the aggregate demand for final goods is

$$Y_t = c_t + g_t + \gamma v_t \tag{45}$$

The retailers face only one real marginal cost from the intermediate firms, which entails a unique equilibrium value for the optimal price-setting program, *id est* $P^*_{i,t} = P^*_t$.

The market clearing condition for intermediate goods states that intermediate goods are produced by incumbent workers or new workers through either open-ended or fixed-term contracts.

$$\begin{aligned} \int_0^1 Y_{i,t} di = & A_t E_z \left[z \mid z \geq z^p_t\right] (1-\xi_t)\, n^p_{t-1} + (1 - G(z^*_t))\, q(\theta_t)\, v_t A_t E_z \left[z \mid z \geq z^*_t\right] \\ & + \rho A_t \mathbb{E}_z [z] (1-\delta)\, n^f_{t-1} + \rho \left(G(z^*_t) - G\left(z^f_t\right)\right) q(\theta_t)\, v_t A_t \mathbb{E}_z \left[z \mid z^*_t \geq z \geq z^f_t\right] \end{aligned}$$

Using the first order condition from the final good firm's program (3), I get

$$\begin{aligned} Y_t \Delta_t = & A_t E_z \left[z \mid z \geq z^p_t\right] (1-\xi_t)\, n^p_{t-1} + (1 - G(z^*_t))\, q(\theta_t)\, v_t A_t E_z \left[z \mid z \geq z^*_t\right] \\ & + \rho A_t E_z [z] (1-\delta)\, n^f_{t-1} + \rho \left(G(z^*_t) - G\left(z^f_t\right)\right) q(\theta_t)\, v_t A_t E_z \left[z \mid z^*_t \geq z \geq z^f_t\right] \end{aligned} \tag{46}$$

with $\Delta_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_t} di$ which measures price dispersion. **?** demonstrated that the associated law of motion is

$$\Delta_t = (1-\psi) \left(\frac{P^*_t}{P_t}\right)^{-\epsilon_t} + \psi \left(\frac{P_t}{P_{t-1}}\right)^{\epsilon_t} \Delta_{t-1} \tag{47}$$

while the price level follows

$$P_t = \left[\psi P_{t-1}^{1-\epsilon_t} + (1-\psi) \left(P^*_t\right)^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}} \tag{48}$$

Average wages pinpoint unemployment benefits as well as firing costs. Firing costs are a share of the average wage of incumbent open-ended workers. Denoting $\widetilde{w^p_t}$ the average wage of incumbent workers with open-ended contracts, firing costs verify

$$F_t = \rho^F \widetilde{w^p_t} \tag{49}$$

The average wage of incumbent open-ended workers is simply the expected wage of open-ended matches with a productivity higher than the firing threshold.

$$\widetilde{w^p_t} = \mathbb{E}_t \left[w^p_t(z) \mid z \geq z^p_t\right]$$

Using (23), the average wage of incumbent open-ended workers boils down to

$$\widetilde{w^p_t} = \eta \left(A_t \phi_t \mathbb{E}_t \left[z \mid z^p_t \leq z\right] + F_t - E_t \beta_{t,t+1} (1-s) F_{t+1} + \gamma \theta_t\right) + (1-\eta) b_t \tag{50}$$

The average wage of open-ended workers includes the wage that are not fired and with productivity higher than the firing threshold.

$$\overline{w_t^p} = \frac{(1-\xi_t)\, n_{t-1}^p \mathbb{E}_t \left[w_t^p(z) \mid z \geq z_t^p\right] + \mu_t^p e_t \mathbb{E}_t \left[w_t^{0,p}(z) \mid z \geq z_t^*\right]}{n_t^p}$$

$$\overline{w_t^f} = \frac{(1-\delta)\, n_{t-1}^f \mathbb{E}_t \left[w_t^f(z)\right] + \mu_t^f e_t \mathbb{E}_t \left[w_t^f(z) \mid z_t^f \leq z \leq z_t^*\right]}{n_t^f}$$

Given the path of exogenous shocks $\{\epsilon_t^A, \epsilon_t^\mu, \epsilon_t^g, \epsilon_t^m\}_{t=0}^{+\infty}$, laws of motions of exogenous shocks $\{A_t, \mu_t, g_t\}_{t=0}^{+\infty}$ and initial values for the state variables $\left\{R_{-1}, n_{-1}^p, n_{-1}^f, \Delta_{-1}, P_{-1}\right\}$, the equilibrium sums up into the set of endogenous variables $\left\{R_t, c_t, Y_t, n_t^p, n_t^f, u_t, \Delta_t, z_t^p, z_t^c, z_t^f, z_t^*, \theta_t, \phi_t, v_t, P_t, P_t^*, b_t, F_t, \overline{w_t^p}, \overline{w_t^f}, \widetilde{w_t^p}\right\}_{t=0}^{+\infty}$ pinned down by equations (1), (4), (34)-(36), (28) and (40)-(48).

2 Calibration and estimation procedure

The model is bridged to the data through two steps. Some of the parameters are firstly calibrated to match moments shaping typical dual European labor markets. Then, parameters driving shock processes and the nominal rigidity are estimated through a Bayesian technique.

2.1 Calibration

The intended calibration exercise needs to meet two requirements so as to lead to relevant results from an heuristic point of view. A natural objective is the faithful reproduction of the main features of labor markets in the Euro area. Moreover, the unprecedented modelisation of a dual labor market in a DSGE model adds the requirement of numerical comparability with a classic labor market embedding firing costs within the Euro Area. I shall rely on modelisation choices **?** made to compel with the latter demand.

The central role of the distribution for idiosyncratic productivity shocks in the shaping of hiring and separation decisions falls reluctantly within the need for comparability, in absence of a proper estimation procedure. Consequently, I assume that the standard deviation for these shocks amounts to 0.2. I follow **?** in several other dimensions. The discount factor is simarly set to 0.99. The matching function is assumed to be in a Cobb-Douglas form $m(e, v) = me^\sigma v^{1-\sigma}$ with σ set to 0.6, which stands in the middle of the 0.5-0.7 range **?** estimated for some European countries. The Hosios condition being still verified, I set the workers' bargaining power to 0.6 as well so that the congestion externalities do not weigh in. The elasticity of demand curves is set to 6. I also assume that the government-spending-to-gdp ratio is 20 % .

β	σ	η	σ_z	ϵ
0.99	0.6	0.6	0.2	6

Table 1: Initial parameters

I depart from **?** in several dimensions. I assume that the vacancy-worker meeting probability from the firm's point of view is 0.7 instead of 0.9. The latter figure replicates flows on the US labor market, which are known to be bigger than the European ones. One important feature of the labor market is its size, since it influences labor market tightness and job-finding rates. Should we consider ILO-defined unemployment *stricto sensu* or include the inactive population ? **?** and **?** demonstrated the importance of the participation margin to explain unemployment fluctuations respectively in the United States and in France. This is all the more true with precarious employment, which involves people at the blurred frontier between unemployment and inactivity. According to data from Eurostat extending between 2002 and 2017, the participation rate in the Euro Area rate is around 67 %. Thus, I set steady-state employment to 0.67. In the same manner, I target a ratio of temporary contracts over total employment of 13 % in accordance with Eurostat estimates from 2006 to 2017. An important factor is the contractual composition of creation flows, which influences the rate of turnover in the labor force. Data from *Acoss - Dares* witness that 80 % of job creations occur through temporary contracts in France, a share that reaches 90 % in Spain as **?** document. At the other end of the spectrum, **?** report a 45 % share of temporary contracts in German job creation. I target a 70 % share of fixed-term contracts in job creation to strike a balance. I also set the quarterly separation probability of permanent matches ξ^p to 3 % consistently with the magnitude of data from Eurostat. This leads to a steady-state separation probability of temporary matches of 40 %, which is equivalent to an average duration of 7.5 months. This estimate is in line with Eurostat data. Among separations involving permanent contracts, an essential factor is the probability of paying the firing cost. **?** explain that more sclerotic markets lead to a higher share of voluntary quits, whereas lay-offs tend to be more significant in countries with more flexible labor markets. The French case is described as the most representative of the former phenomenon, with 80 % of separations involving permanent matches happening through voluntary channels according to **?**. A reasonable value for the Euro Area is thus inferior. As a result, I target a rate of 60 % for the ratio of exogenous separations among total separations for permanent matches. The resulting value of s is 2.1 %.

The calibration of firing costs constitutes a challenge. The data is scarce about this issue, which makes reasonable proposals difficult to spell. The latter is all the more true since heterogeneity between countries is high, whether it be from a legal or an economic point of view. The 0-to-5 OECD indicator for employment protection legislation enables a cursory comparison between Euro area countries. While French, German and Italian indexes of EPL against collective and individual dismissals of permanent contracts are close to 2.8, the Spanish index is roughly 2.4. Employment protection legislation in the Euro Area seems closer to the French case, whose firing costs are examined by **?**. The latter assess that individual lay-offs marginally cost 4 months of the median wage, while the marginal cost of lay-off within a collective-termination plan represents 12 months of the median wage[3]. The former being the most frequent case, we reckon that total firing costs represent 4 months of the permanent workers' average wage . As in **?** and **?**, we assume that red-tape costs actually embodied by F only represent one third of total firing costs for the firm[4]. Thus, we target a ratio of 4/9 for firing costs with respect to the quarterly permanent workers' average wage.

$F/\overline{w^p}$	$\mu^f/\left(\mu^p+\mu^f\right)$	n^f/n	ξ	s/ξ	n	$q(\theta)$
4/9	0.7	0.15	0.03	0.6	0.67	0.7

Table 2: Targets for a calibration of the labor market in the Euro area

The last parameters to be pinpointed are F, b, m, ρ and γ. The empirical evidence concerning b is thin. Indeed, the flow value of non-employment is a highly debated issue in labor economics. **?** advocate a high relative value of non-employment with respect to employment to make the Mortensen-Pissarides model able to replicate faithfully fluctuations in unemployment and vacancies following productivity shocks of a realistic magnitude. The high replacement ratios of unemployment insurance in Western Europe combined with the non-monetary benefits of working support this view. The obtained b is coherent with this view. In

[3]To be accurate, **?** assess that firms with more than 50 employees face a marginal cost of 97,727 FFr (Table 1b), which represents 14 months of the workers' median wage. Consequently, the associated median wage of fired workers is 6980 FFr. Thus, Table 2 shows that individual terminations cost 27,389 FFr, which amounts to 4 months of the fired workers' median wage, while the termination within a collective firing plan marginally costs 81,850 FFr, which equals 12 months of the median wage.

[4]Transfers between separated firms and workers are not taken into account in firing costs because they play no allocational role, in contrast with red-tape firing costs

the same manner, no proper empirical evidence is available to assess the productivity difference between fixed-term contracts and open-ended contracts in European countries, which explains why I prefer leave it as a free parameter in the calibration. I find a 3% productivity deficit among temporary contracts with respect to permanent contracts. The vacancy cost represents 1.5 % of the average wage, which is coherent with the (scarce) available evidence put forward by **?**.

F	b	s	δ	ρ	m	γ
0.38	0.83	0.02	0.4	0.97	0.53	0.02

Table 3: Calibrated parameters

2.2 Estimation

The unknown parameters are related to the shock processes and the nominal rigidities. I estimate them using a Sequential Monte Carlo procedure[5]. Following **?**, I choose the same time period extending from 1997-Q1 to 2007-Q4 and a similar set of observables (Y_t, π_t, R_t, n_t) and quarterly data series, namely GDP at constant prices, employment, EONIA rates in annual terms and the GDP deflator[6]. All concern the 19-country Euro area and are obtained from the ECB Data Warehouse. The demeaned growth rate of the GDP deflator is plugged to inflation π_t. Since R_t corresponds to the real interest rates, they correspond to the demeaned quarterly equivalent of EONIA rates diminished by estimated inflation. Real GDP and employment are logged and reluctantly linearly detrended. As a matter of fact, the use of any detrending method or filter as well as the comprehension of adequate observable equations to match growth rates in the data is mainly arbitrary and may deliver heterogeneous and mispecified estimation results. **?** and **?** roughly propose to simultaneously estimate the parameters of flexible specifications for trends along deep parameters so as to let the model explain the part of the data it is able to account for. However, as **?** and **??** testify, identification issues are real in the estimation of DSGE models. The resort to the methods advocated by **?** and **?** would magnify these problems through the introduction of new parameters. For this reason, I stick to the detrending method employed by **?**, which remains simple and enables comparisons between my model and their own. In Appendix **??**, I report the estimation results using different filtering methods and observables ranging from 1995Q3 to 2019Q2. Overall, the estimates are pretty robust.

Most chosen priors are chosen to be diffuse, reflecting the void of the DSGE literature with respect to dual labor markets. Thus, the priors for autocorrelations of the shock processes are uniform laws on $[0, 1]$, while standard deviations follow inverse gamma distributions with mean 0.5 and standard deviation 4[7]. As for the Taylor parameters r_π and r_y, the prior needs to be vague and embed the fact that $r_\pi > 1$ and $r_y > 0$. As a result, truncated normal laws with large standard deviations are employed. **?** assess the average duration of firms' prices in the Euro Area to 10 months, which corresponds to a value of ψ around 0.7. A tight normal law around this value is subsequently chosen.

Identification issues in DSGE models are well known. My model does not escape this observation. Following the method **?** makes a case for, I find that the estimated parameters are identified with the available data. Nevertheless, the implementation of methods presented by **?** reveal a few pitfalls already known in the literature. ρ_μ and σ_μ are jointly weakly identified. This problem also concerns ρ_R, ρ_π, ρ_y and σ_m, whose estimation is highly intertwined. Indeed, for given data, the higher the autocorrelation of interest rates in the Taylor rule, the stronger the needed policy adjustments. In turn, this pushes up the standard deviation of monetary policy shocks.

[5]See **?** for an extensive treatment of this method.
[6]Table C.2 details the series used for estimation
[7]Standard deviations are expressed in percentage

Table 4: Prior and posterior distributions of structural parameters.

Parameter	Prior distribution			Posterior distribution			
	Distr.	Para (1)	Para(2)	Mean	Std. Dev.	5%	95%
ρ_A	Uniform	0.0	1.0	0.73	0.09	0.58	0.91
ρ_μ	Uniform	0.0	1.0	0.11	0.1	0.01	0.3
ρ_g	Uniform	0.0	1.0	0.89	0.04	0.82	0.94
ρ_R	Uniform	0.0	1.0	0.78	0.05	0.69	0.86
ρ_π	Normal	1.5	0.75	2.06	0.63	1.11	3.01
ρ_y	Normal	0.12	0.15	0.28	0.07	0.17	0.4
ψ	Beta	0.7	0.05	0.83	0.03	0.77	0.88
σ_A	IGamma	0.5	4.0	0.31	0.04	0.26	0.38
σ_μ	IGamma	0.5	4.0	0.27	0.04	0.2	0.34
σ_g	IGamma	0.5	4.0	5.46	1.59	3.41	7.35
σ_m	IGamma	0.5	4.0	0.09	0.02	0.07	0.12

Para(1) and Para(2) correspond to mean and standard deviation of the prior distribution if the latter is Normal or Inverse Gamma. Para(1) and Para(2) correspond to lower and upper bound of the prior distribution when the latter is uniform

I run the Sequential Monte Carlo procedure with 500 likelihood-tempering steps, which involve a swarm of 30,000 particles and one Random-Walk-Metropolis-Hastings step each. The proper identification of parameters is checked for each draw from the prior distributions[8] and the Metropolis-Hastings steps along the method delineated by **?**. The results are displayed in Table **??**. Overall, the estimates are pretty similar to those obtained by **?**. The cost-push shocks displays virtually no autocorrelation, in opposition to productivity and government-spending shocks. The interest-rate-smoothing parameter and the price stickiness present the same order of magnitude as well. However, the standard deviations of the shocks differ significantly from their estimates, with the notable exception of monetary policy shocks. The cost-push shock displays a standard deviation of 0.26 % instead of 10%, while productivity shocks and government spending shocks are half as volatile. The high volatility of government-spending shocks may seem surprising. As a matter of fact, the current specification of government-spending shock could embody all demand shocks that are not explicitly modelled here but have a sizable effect on output. This may include shocks on investment, on exports or on prices of imported products for example. This problem vanishes when investment, capital, risk-aversion and habit formation are introduced in the same manner as **?**. I prefer to display a simple model to insist on the economic phenomena specific to dual labor markets, which are relatively new in the DSGE literature. Interestingly, as details Appendix D estimating the model without fixed-term contracts leads to similar parameter values and a non-significantly different marginal likelihood.

[8]Prior distributions are involved in the initialization of the swarm of particles.

A Proofs

Proof Proposition 1 The behavior of the thresholds is characterized by the following proposition

Lemma 1. *These assertions are equivalent*

1. $z_t^* > z_t^f$
2. $z_t^* > z_t^c$
3. $z_t^c > z_t^f$

Proof.
- Assume that $z_t^* > z_t^f$. The definition of z_t^*(28) implies that $z_t^* = (1-\rho)\, z_t^* + \rho z_t^* = z_t^c + \rho\left(z_t^* - z_t^f\right)$. Since $z_t^* - z_t^f > 0$, the latter equality implies $z_t^* > z_t^c$.
- Assume that $z_t^* > z_t^c$. Again, jointly with algebraic manipulations, (28) implies that $\rho z_t^c = -(1-\rho)\, z_t^c + (1-\rho) z_t^* + \rho z_t^f > -(1-\rho)\, z_t^c + (1-\rho) z_t^c + \rho z_t^f > \rho z_t^f$, which entails that $z_t^c > z_t^f$.
- Assume that $z_t^c > z_t^f$. Algebraic manipulations and (28) imply that $(1-\rho)\, z_t^* = 1\left(z_t^c - z_t^f\right) + (1-\rho)\, z_t^f > (1-\rho)\, z_t^f$, which implies $z_t^* > z_t^f$. □

□

Referring to the job creation condition (38),

- If open-ended workers are the only ones hired, then $max\left[z^f, z^*\right] \leq z^f$, implying that $z^* \leq z^f$. Referring to Lemma 1, the latter inequality entails $z^f \leq z^c$. As a result, $z^* \leq z^f \leq z^c$.
- If job creation is dual, then

$$\begin{cases} 0 < \max\left[z^f, z^*\right] \\ z^f < z^* \end{cases}$$

Using Lemma 1, the latter system of inequalities boils down to $\max\left[0, z^f\right] < z^*$.

For each case, the converse propositions are straightforward using (38). □

B Detailed Calculations

B.1 Nash-bargaining joint surpluses

Using the different definitions of surpluses with the free-entry condition -namely equations (14), (15), (27) and (9) - I get

$$\begin{aligned} S_t^p\left(z_t\right) &= z_t - U_t + \mathbb{E}_t\beta_{t,t+1}U_{t+1} + F_t - \mathbb{E}_t\beta_{t,t+1}(1-s)F_{t+1} \\ &\quad + \mathbb{E}_t\beta_{t,t+1}(1-s)\int \max\left(S_{t+1}^p(z), 0\right) dG(z) \end{aligned} \tag{51}$$

Meanwhile, the definition of U_t (12) yields

$$U_t = p\left(\theta_t\right)\int \max\left(W_t^{0,p}(z) - U_t^0, W_t^f(z) - U_t^0, 0\right) dG(z) + U_t^0 \tag{52}$$

Nash-sharing rules (7) imply that

$$U_t = p(\theta_t) \frac{\eta}{1-\eta} \int \max\left(J_t^{0,p}(z), J_t^{0,f}(z), \mathbb{E}_t\beta_{t,t+1}V_{t+1}\right) dG(z) + (1-p(\theta_t))\, U_t^0 \tag{53}$$

But the definition of V_t (5) and the free entry condition (27) imply that

$$\int \max\left(J_t^{0,p}(z), J_t^{0,f}(z), \mathbb{E}_t\beta_{t,t+1}V_{t+1}\right) dG(z) = \frac{\gamma}{q(\theta_t)} \tag{54}$$

Since $p(\theta_t) = \theta_t q(\theta_t)$, the definition of U_t boils down to

$$U_t = \frac{\eta\gamma\theta_t}{1-\eta} + U_t^0 \tag{55}$$

Consequently, using (**??**) the outside value of an incumbent worker is

$$U_t = b + \frac{\eta\gamma\theta_t}{1-\eta} + \mathbb{E}_t\beta_{t,t+1}U_{t+1} \tag{56}$$

Reintroducing (56) into (20) leads to the following expression for the surplus of continuing permanent contracts

$$\begin{aligned} S_t^p(z_t) = {} & A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + F_t - \mathbb{E}_t\beta_{t,t+1}(1-s)F_{t+1} \\ & + \mathbb{E}_t\beta_{t,t+1}(1-s)\int \max\left(S_{t+1}^p(z), 0\right) dG(z) \end{aligned} \tag{57}$$

Following the same steps, I find that

$$S_t^{0,p}(z_t) = S_t^p(z_t) - F_t \tag{58}$$

As for temporary contracts, equations (18), (19) and (27) boil down to

$$S_t^f(z_t) = \rho A_t z_t \phi_t - U_t + \mathbb{E}_t\beta_{t,t+1}\left\{(1-\delta)\int S_{t+1}^f(z) dG(z) + U_{t+1}\right\} \tag{59}$$

Using (56), the surplus of incumbent fixed-term contracts is

$$S_t^f(z_t) = \rho A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + \mathbb{E}_t\beta_{t,t+1}(1-\delta)\left\{\int S_{t+1}^f(z) dG(z)\right\} \tag{60}$$

$$\begin{aligned} w_t^p = {} & \eta\left(A_t\phi_t\mathbb{E}_t\left[z \mid z \geq z_t^p\right] + F_t\right)\frac{(1-\xi_t)n_{t-1}^p}{n_t^p} + \eta A_t\phi_t\mathbb{E}_t\left[z \mid z \geq z_t^*\right]\frac{v_t q(\theta_t)\left(1-G(z_t^*)\right)}{n_t^p} \\ & + \eta\left(-\mathbb{E}_t\beta_{t,t+1}(1-s)F_{t+1} + \gamma\theta_t\right) + (1-\eta)b_t \end{aligned}$$

$$
\begin{aligned}
w_t^f &= \eta\rho A_t\phi_t\left((1-\delta)\mathbb{E}z\frac{n_{t-1}^f}{n_t^f}+\mathbb{E}_t\left[z \mid z_t^f\leq z\leq z_t^*\right]\frac{v_t q(\theta_t)\left(G\left(z_t^*\right)-G\left(z_t^f\right)\right)}{n_t^f}+\gamma\theta_t\right)+(1-\eta)b_t\\
\overline{w_t^p} &= \eta\left(A_t\phi_t\mathbb{E}_t\left[z \mid z\geq z_t^p\right]+F_t-\mathbb{E}_t\beta_{t,t+1}(1-s)F_{t+1}+\gamma\theta_t\right)+(1-\eta)b_t
\end{aligned}
$$

$$
\begin{aligned}
&\left(1-\rho^F(1-\xi)\eta\right)F\widehat{F_t}-\eta\left(A\phi\mathbb{E}\left[z \mid z\geq z^p\right]+F\right)(1-\xi)\widehat{n_{t-1}^p}\\
&+\eta\left((1-\xi)\left(A\phi\mathbb{E}\left[z \mid z\geq z^p\right]+F\right)+A\phi\mathbb{E}\left[z \mid z\geq z^*\right]\frac{vq(\theta)\left(1-G\left(z^*\right)\right)}{n^p}\right)\widehat{n_t^p}
\end{aligned}
$$

B.2 Steady-state equations

Given parameters $\beta,\sigma,\eta,\sigma_z,\epsilon,\rho^F,\rho^b,h,\delta,\rho,m,\gamma$, we need to derive the steady-state values of $R,c,Y,n^p,n^f,u,\Delta,z^p,z^c,z^f,$ Some of them can be directly computed through the following equations

$$
\begin{aligned}
R &= 1/\beta\\
\phi &= \frac{\epsilon-1}{\epsilon}\\
P &= P^* = \Delta = 1\\
A &= 1
\end{aligned}
$$

Given (z^p,θ,b), many variables can be derived in a tractable way.

$$
\begin{aligned}
q &= m\theta^{-\sigma}\\
p &= \theta q\\
U &= b+\frac{\eta\gamma\theta}{1-\eta}
\end{aligned}
$$

The following system in $(F,\overline{w^p})$ is tractable.

$$
\begin{aligned}
\overline{w^p} &= \eta\phi\mathbb{E}\left[z \mid z\geq z^p\right]+(1-\beta(1-s))F+(1-\eta)U\\
F &= \rho^F\overline{w^p}
\end{aligned}
$$

It delivers the following value for F.

$$
F=\frac{\rho^F\phi}{1-\rho^F(1-\beta(1-s))}\left(\eta\mathbb{E}\left[z \mid z\geq z^p\right]+(1-\eta)U\right)
$$

Some variables can then be computed directly.

$$
\begin{aligned}
z^c &= z^p+\frac{F}{\phi}\\
z^f &= \frac{U-\rho\phi(1-\delta)\beta\mathbb{E}z}{\rho\phi\left(1-\beta(1-\delta)\right)}
\end{aligned}
$$

$$
\begin{aligned}
z^* &= \frac{z^c - \rho z^f}{1-\rho} \\
\xi &= s + (1-s)G\left(z^p\right) \\
w^p &= \eta\phi\left((1-\xi)\mathbb{E}\left[z \mid z \geq z^p\right] + \xi\mathbb{E}\left[z \mid z \geq z^*\right]\right) + \eta\left((1-\xi)F - \eta\beta(1-s)F\right) + (1-\eta)U \\
w^f &= \eta\phi\rho\left((1-\delta)\mathbb{E}z + \delta\mathbb{E}\left[z \mid z^f \geq z \geq z^*\right]\right) + (1-\eta)U \\
\mu^p &= p\left(1 - G\left(z^*\right)\right) \\
\mu^f &= p\left(G\left(z^*\right) - G\left(z^f\right)\right)
\end{aligned}
$$

Solving the following linear system yields $\left(n^p, n^f, u, e\right)$.

$$
\begin{aligned}
n^p + n^f + u &= 1 \\
e &= u + \delta n^f + \xi n^p \\
n^p &= (1-\xi)\, n^p + \mu^p e \\
n^f &= (1-\delta)\, n^f + \mu^f e
\end{aligned}
$$

The resulting expressions are

$$
\begin{aligned}
n^p &= \frac{\delta\mu^p}{\xi(1-\delta)\mu^f + \delta(1-\xi)\mu^p + \xi\delta} \\
n^f &= \frac{\xi\mu^f}{\xi(1-\delta)\mu^f + \delta(1-\xi)\mu^p + \xi\delta} \\
u &= 1 - n^p - n^f \\
e &= u + \xi n^p + \delta n^f
\end{aligned}
$$

Average wages of open-ended and fixed-term workers w^p and w^f write

$$
\begin{aligned}
w^p &= \eta\phi\left((1-\xi)\mathbb{E}\left[z \mid z \geq z^p\right] + \xi\mathbb{E}\left[z \mid z \geq z^*\right]\right) + \eta\left((1-\xi)F - \eta\beta(1-s)F\right) + (1-\eta)U \\
w^f &= \eta\phi\rho\left((1-\delta)\mathbb{E}z + \delta\mathbb{E}\left[z \mid z^f \geq z \geq z^*\right]\right) + (1-\eta)U
\end{aligned}
$$

Consequently, the average wage for all workers is

$$
w = \frac{n^f}{n} w^f + \frac{n^p}{n} w^p
$$

Then, one can circumscribe (z^p, θ, b) by solving the following system numerically.

$$
\begin{aligned}
&\phi z^p + \left(1 - \beta(1-s)\right)F + (1-s)\beta\phi \int_{z^p}^{+\infty} (1 - G(z))\, dz = U \\
&\frac{\gamma}{(1-\eta)\phi q} = \int_{z^*}^{+\infty} (1 - G(z))\, dz + \rho \int_{z^f}^{z_t^*} (1 - G(z))\, dz \\
&b = \rho^b w + h
\end{aligned}
$$

Now, the steady-state values v, Y, g and c can be obtained.

$$
\begin{aligned}
&v = \theta e \\
&Y = (1-\xi)\,\mathbb{E}_z\left[z \mid z \geq z^p\right] n^p + \mu^p \mathbb{E}_z\left[z \mid z \geq z^*\right] e \\
&\quad + \rho\,(1-\delta)\,\mathbb{E}_z\left[z\right] n^f + \rho\mu^f \mathbb{E}_z\left[z \mid z^* \geq z \geq z^f\right] e \\
&g = Y\left(\frac{g}{Y}\right) \\
&c = Y - g - \gamma v
\end{aligned}
$$

B.3 Log-linearization

The Euler equation can be log-linearized starting from

$$
\widehat{c_t} = \widehat{E_t c_{t+1}} - \left[-\frac{cu''}{u}\right]^{-1}\left(\widehat{R_t} - \widehat{E_t \pi_{t+1}}\right) \tag{61}
$$

where $\pi_t = P_t/P_{t-1}$ denotes inflation.
The definition of price level dynamics (48) can be log-linearized as

$$
\widehat{\pi_t} = (1-\psi)\left(\widehat{P_t^*} - \widehat{P_{t-1}}\right) \tag{62}
$$

The retailers' price-setting equation (4) becomes

$$
\widehat{P_t^*} = (1-\beta\psi)\mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\psi)^k\left(\widehat{P_{t+k}} + \widehat{\phi_{t+k}}\right) \tag{63}
$$

Substracting $\widehat{P_{t-1}}$ on each side of this equation, we get

$$
\begin{aligned}
\widehat{P_t^*} - \widehat{P_{t-1}} &= (1-\beta\psi)\mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\psi)^k\left(\widehat{P_{t+k}} - \widehat{P_{t-1}} + \widehat{\phi_{t+k}}\right) && (64)\\
&= \sum_{k=0}^{+\infty}(\beta\psi)^k\mathbb{E}_t\widehat{\pi_{t+k}} + (1-\beta\psi)\sum_{k=0}^{+\infty}(\beta\psi)^k\mathbb{E}_t\widehat{\phi_{t+k}} && (65)\\
&= \beta\psi\left(\mathbb{E}_t\widehat{P_{t+1}^*} - \widehat{P_t}\right) + (1-\beta\psi)\widehat{\phi_t} + \widehat{\pi_t} && (66)
\end{aligned}
$$

Using (62), we get

$$
\widehat{\pi_t} = \beta\mathbb{E}_t\widehat{\pi_{t+1}} + \kappa\widehat{\phi_t} \tag{67}
$$

where $\kappa = (1-\beta\psi)(1-\psi)/\psi$.
The log-linearization of exogenous processes yields

$$
\begin{aligned}
\widehat{A_t} &= \rho_A\widehat{A_{t-1}} + \epsilon_t^A \\
\widehat{g_t} &= \rho_g\widehat{g_{t-1}} + \epsilon_t^g
\end{aligned}
$$

The other log-linearizations of equations (34)-(36), (28) and (40)-(46) yield

$$
\begin{aligned}
&A\phi z^p\left(\widehat{A_t}+\widehat{z_t^p}+\widehat{\phi_t}\right)+\beta(1-s)\left(A\phi\int_{z^p}^{+\infty}(1-G(z))\,dz-F\right)(c_t-\mathbb{E}_t\widehat{c_{t+1}})\\
&+\beta(1-s)A\phi\left(\int_{z^p}^{+\infty}(1-G(z))\,dz\left(\mathbb{E}_t\widehat{A_{t+1}}+\mathbb{E}_t\widehat{\phi_{t+1}}\right)-(1-G(z^p))\,z^p\mathbb{E}_t\widehat{z_{t+1}^p}\right)\\
&+F\widehat{F_t}-\beta(1-s)F\mathbb{E}_t\widehat{F_{t+1}}-b\widehat{b_t}-\frac{\eta\gamma\theta}{1-\eta}\widehat{\theta_t}=0\\
&\rho A\phi z^f\left(\widehat{A_t}+\widehat{z_t^f}+\widehat{\phi_t}\right)+\rho\beta(1-\delta)A\phi\left(\mathbb{E}z-z^f\right)\left(\mathbb{E}_t\widehat{A_{t+1}}+\mathbb{E}_t\widehat{\phi_{t+1}}+\widehat{c_t}-\mathbb{E}_t\widehat{c_{t+1}}\right)\\
&-\rho\beta(1-\delta)A\phi z^f\mathbb{E}_t\widehat{z_{t+1}^f}-b\widehat{b_t}-\frac{\eta\gamma\theta}{1-\eta}\widehat{\theta_t}=0\\
&(1-\rho)z^*\widehat{z_t^*}-z^p\widehat{z_t^p}+\frac{F}{A\phi}\left(\widehat{A_t}+\widehat{\phi_t}\right)+\rho z^f\widehat{z_t^f}=0\\
&\frac{\gamma}{(1-\eta)\phi q(\theta)}\left(-\widehat{A_t}-\widehat{\phi_t}+\sigma\widehat{\theta_t}\right)+(1-\rho)\left(1-G\left(z^*\right)\right)z^*\widehat{z_t^*}+\rho\left(1-G\left(z^f\right)\right)z^f\widehat{z_t^f}=0\\
&\widehat{R_t}=\rho_R\widehat{R_{t-1}}+(1-\rho_R)\left[\rho_\pi\mathbb{E}_t\widehat{\pi_{t+1}}+\rho_y\widehat{Y_t}\right]+\epsilon_t^m\\
&\widehat{v_t}=\widehat{\theta_t}-\frac{(1-\xi^p)\,\theta n^p}{v}\widehat{n_{t-1}^p}-\frac{(1-\delta)\,\theta n^f}{v}\widehat{n_{t-1}^f}+\frac{(1-s)\theta g\left(z^p\right)z^p n^p}{v}\widehat{z_t^p}\\
&\widehat{n_t^p}=(1-\xi)\,\widehat{n_{t-1}^p}-(1-s)z^p g\left(z^p\right)\widehat{z_t^p}+\frac{\left(1-G\left(z^*\right)\right)q(\theta)v}{n^p}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)-\frac{z^*g\left(z^*\right)q(\theta)v}{n^p}\widehat{z_t^*}\\
&\widehat{n_t^f}=(1-\delta)\,\widehat{n_{t-1}^f}+\frac{\left(G\left(z^*\right)-G\left(z^f\right)\right)q(\theta)v}{n^f}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)+\frac{q(\theta)v}{n^f}\left(z^*g\left(z^*\right)\widehat{z_t^*}-z^f g\left(z^f\right)\widehat{z_t^f}\right)\\
&\widehat{Y_t}=\frac{c}{Y}\widehat{c_t}+\frac{g}{Y}\widehat{g_t}+\frac{\gamma v}{Y}\widehat{v_t}\\
&\widehat{Y_t}=\widehat{A_t}+\frac{(1-s)\left(\int_{z^p}^{+\infty}zg(z)dz\right)n^p}{Y}\widehat{n_{t-1}^p}+\frac{\rho\,(1-\delta)\,\mathbb{E}zn^f}{Y}\widehat{n_{t-1}^f}-\frac{(1-s)n^p\left(z^p\right)^2g\left(z^p\right)}{Y}\widehat{z_t^p}\\
&-(1-\rho)\left(z^*\right)^2g\left(z^*\right)\frac{vq(\theta)}{Y}\widehat{z_t^*}-\rho\left(z^f\right)^2g\left(z^f\right)\frac{vq(\theta)}{Y}\widehat{z_t^f}\\
&+\left(\int_{z^*}^{+\infty}zg(z)dz+\rho\int_{z^f}^{z^*}zg(z)dz\right)\frac{vq(\theta)}{Y}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)\\
&F\left(1-\rho^F\eta\right)\widehat{F_t}-\rho^F(1-\eta)b\widehat{b_t}-\rho^F\eta\left(\gamma\theta\widehat{\theta_t}-\beta(1-s)F\left(\mathbb{E}_t\widehat{F_{t+1}}+\widehat{c_t}-\mathbb{E}_t\widehat{c_{t+1}}\right)\right)\\
&-\rho^F\eta\phi\left(\mathbb{E}\left[z\mid z\geq z^p\right]\widehat{\phi_t}+\frac{\int_{z^p}^{+\infty}(1-G(z))\,dz-z^p\left(1-G\left(z^p\right)\right)}{\left(1-G\left(z^p\right)\right)^2}z^p g\left(z^p\right)\widehat{z_t^p}\right)=0\\
&w^p\widehat{w^p}-\eta(1-s)\phi\left(\int_{z^p}^{+\infty}zg(z)dz\widehat{n_{t-1}^p}-\left(z^p\right)^2g\left(z^p\right)\widehat{z_t^p}\right)-\frac{\eta\phi qv}{n^p}\left(\int_{z^*}^{+\infty}zg(z)dz\left(-\sigma\widehat{\theta_t}+\widehat{v_t}\right)-\left(z^*\right)^2g\left(z^*\right)\widehat{z_t^*}\right)\\
&-\eta\gamma\theta\widehat{\theta_t}-\eta F\left((1-\xi)\widehat{F_t}-(1-s)z^p g\left(z^p\right)\widehat{z_t^p}\right)+\eta\beta(1-s)F\left(\widehat{c_t}-\mathbb{E}_t\widehat{c_{t+1}}+\mathbb{E}_t\widehat{F_{t+1}}\right)-(1-\eta)b\widehat{b_t}\\
&+\eta\left((1-s)\phi\int_{z^p}^{+\infty}zg(z)dz+(1-\xi)F+\frac{\phi qv}{n^p}\int_{z^p}^{+\infty}zg(z)dz\right)\widehat{n_t^p}\\
&-\eta\phi\left((1-s)\phi\int_{z^p}^{+\infty}zg(z)dz+\frac{\phi qv}{n^p}\int_{z^p}^{+\infty}zg(z)dz\right)\widehat{\phi_t}=0\\
&w^f\widehat{w_t^f}-\eta\rho\phi\left((1-\delta)\mathbb{E}z+\frac{qv}{n^f}\int_{z^f}^{z^*}zg(z)dz\right)\left(\widehat{\phi_t}-\widehat{n_t^f}\right)-\eta\gamma\theta\widehat{\theta_t}-\eta\rho\phi(1-\delta)\mathbb{E}z\widehat{n_{t-1}^f}\\
&-\frac{\eta\rho\phi qv}{n^f}\left(\int_{z^f}^{z^*}zg(z)dz\left(-\sigma\widehat{\theta_t}+\widehat{v_t}\right)+\left(z^*\right)^2g\left(z^*\right)\widehat{z_t^*}-\left(z^f\right)^2g\left(z^f\right)\widehat{z_t^f}\right)-(1-\eta)b\widehat{b_t}\\
&b\left(n^p+n^f\right)\widehat{b_t}+n^p\left(b-\left(\rho^b w^p+h\right)\right)\widehat{n_t^p}+n^f\left(b-\left(\rho^b w^f+h\right)\right)\widehat{n_t^p}-\rho^b n^p w^p\widehat{w_t^p}-\rho^b n^f w^f\widehat{w_t^f}=0
\end{aligned}
$$

C Estimation

C.1 Calibration

I carry out the calibration using known parameters and steady-state targets. A first block of steady-state values and parameters is tractable.

$$
\begin{aligned}
\phi &= \frac{\epsilon - 1}{\epsilon} \\
u &= 1 - n \\
n^f &= n\left(\frac{n^f}{n}\right) \\
n^p &= n - n^f \\
s &= \xi\left(\frac{s}{\xi}\right) \\
\delta &= \frac{\left(1-\left(\frac{n^f}{n}\right)\right)\left(\frac{\mu^f}{\mu^p+\mu^f}\right)}{\left(\frac{n^f}{n}\right)\left(1-\left(\frac{\mu^f}{\mu^p+\mu^f}\right)\right)}\xi \\
e &= u + \delta n^f + \xi n^p \\
\mu^p &= \frac{\xi n^p}{e} \\
\mu^f &= \frac{\delta n^f}{e}
\end{aligned}
$$

Knowing ξ, s and σ^z, it is possible to numerically solve for z^p using the fact that $\xi = s + (1-s)G\left(z^p\right)$. Then, the following system in $(\overline{w^p}, U, F)$ — where $U = b + \frac{\eta\gamma\theta}{1-\eta}$ — is tractable.

$$
\begin{aligned}
&\overline{w^p} = \eta\phi\mathbb{E}\left[z \mid z \geq z^p\right] + \eta(1-\beta(1-s))F + (1-\eta)U \\
&\phi z^p + \left(1-\beta(1-s)\right)F + (1-s)\beta\phi\int_{z^p}^{+\infty}\left(1-G(z)\right)dz = U \\
&F = \rho^F\overline{w^p}
\end{aligned}
$$

Combining these three expressions, I get

$$
\begin{aligned}
F &= \frac{\rho^F\phi}{1-\rho^F(1-\beta(1-s))}\left(\eta\mathbb{E}\left[z \mid z \geq z^p\right] + (1-\eta)\left(z^p + \beta(1-s)\int_{z^p}^{+\infty}\left(1-G(z)\right)dz\right)\right) \\
U &= \phi z^p + \left(1-\beta(1-s)\right)F + (1-s)\beta\phi\int_{z^p}^{+\infty}\left(1-G(z)\right)dz
\end{aligned}
$$

It is possible to compute z^c.

$$
z^c = z^p + \frac{F}{\phi}
$$

A non-linear system in $\left(z^f, z^*, \rho\right)$ can then be solved numerically.

$$
\begin{aligned}
&\rho z^f \phi + \rho \mathbb{E}_t \beta (1-\delta) \phi \left(\mathbb{E} z - z^f\right) = U \\
&(1-\rho) z^* = z^c - \rho z^f \\
&\frac{\mu^f}{\mu^p + \mu^f} = \frac{G\left(z^*\right) - G\left(z^f\right)}{1 - G\left(z^f\right)}
\end{aligned}
$$

Then, one can derive (θ, γ, b, m).

$$
\begin{aligned}
&p = \frac{\mu^p}{1 - G\left(z^*\right)} \\
&\theta = \frac{p}{q} \\
&\gamma = (1-\eta) \phi q \left(\int_{z^*}^{+\infty} (1 - G(z))\, dz + \rho \int_{z^f}^{z^*} (1 - G(z))\, dz \right) \\
&b = U - \frac{\eta \gamma \theta}{1-\eta} \\
&m = q \theta^\sigma
\end{aligned}
$$

Average wages of open-ended and fixed-term workers w^p and w^f write

$$
\begin{aligned}
w^p &= \eta \phi \left((1-\xi) \mathbb{E}\left[z \mid z \geq z^p\right] + \xi \mathbb{E}\left[z \mid z \geq z^*\right]\right) + \eta \left((1-\xi) F - \eta \beta (1-s) F\right) + (1-\eta) U \\
w^f &= \eta \phi \rho \left((1-\delta) \mathbb{E} z + \delta \mathbb{E}\left[z \mid z^f \geq z \geq z^*\right]\right) + (1-\eta) U
\end{aligned}
$$

Consequently, the average wage for all workers is

$$
w = \frac{n^f}{n} w^f + \frac{n^p}{n} w^p
$$

h verifies

$$
h = \rho^b w - b
$$

The parameters $(F, h, s, \delta, \rho, m, \gamma)$ are all determined at this point

C.2 Data

Observable	Time range	Unique Identifier
GDP at constant prices	1995:Q1 - 2019:Q3	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N
GDP deflator	1995:Q2 - 2019:Q2	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.IX.D.N
Nominal interest rates	1994:Q1 - 2019:Q3	FM.M.U2.EUR.4F.MM.EONIA.HSTA
Employment	1995:Q1 - 2019:Q3	ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N

Table 5: Times series used for estimation

All these time series are drawn from the ECB Data Warehouse

Observable	Symbol	Time range	Source	Unique Identifier
Open-ended job creations	jc^p	2000:Q1 - 2019:Q3	Acoss	Acoss Stat 296
Fixed-term job creations	jc^f	2000:Q1 - 2019:Q3	Acoss	Acoss Stat 296
Share of fixed-term contracts in job creation	$\mu^f / (\mu^p + \mu^f)$	2000:Q1 - 2019:Q3	Acoss	Acoss Stat 296
Share of fixed-term employment	n^f	2003:Q1 - 2019:Q3	Insee	010605905
Endogenous open-ended job destruction	jd^p	2001:Q1 - 2017:Q4	DARES	DMMO - EMMO
Fixed-term job destruction	jd^f	1998:Q1 - 2017-Q4	DARES	DMMO - EMMO
Vacancies	v	1989:Q1 - 2019:Q2	OECD	LMJVTTNVFRQ647S

Table 6: Labor market times series

C.3 Additional Graphs

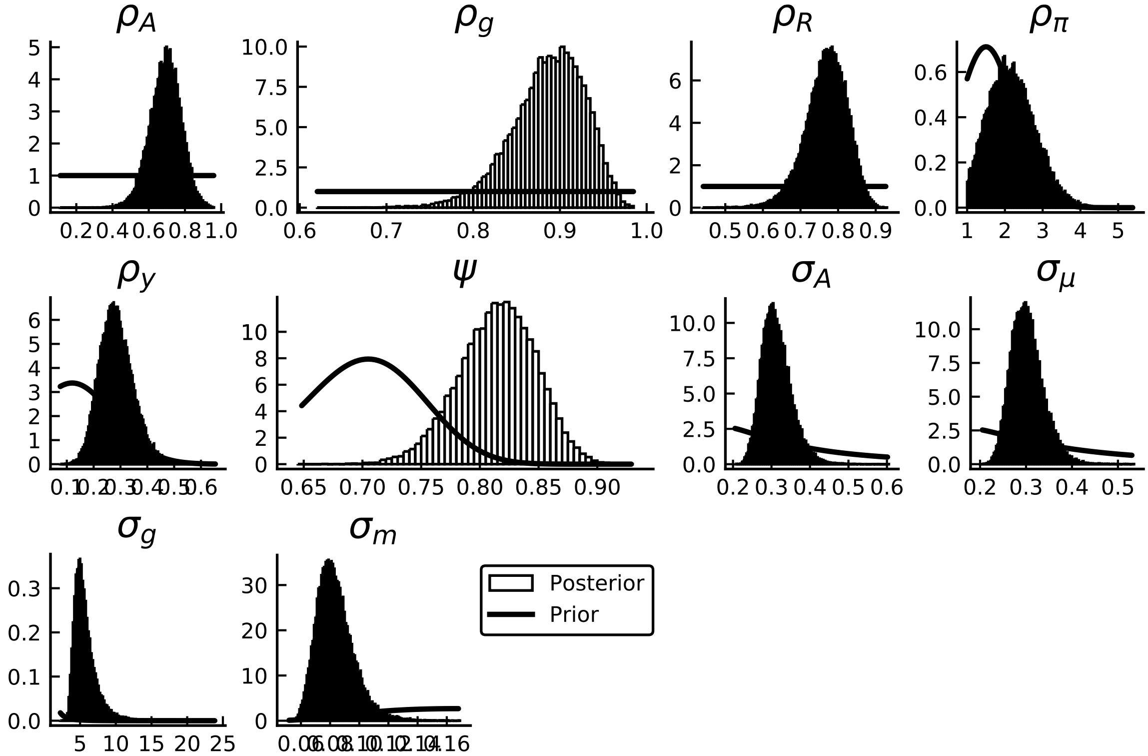

Figure 5: Prior and posterior distributions

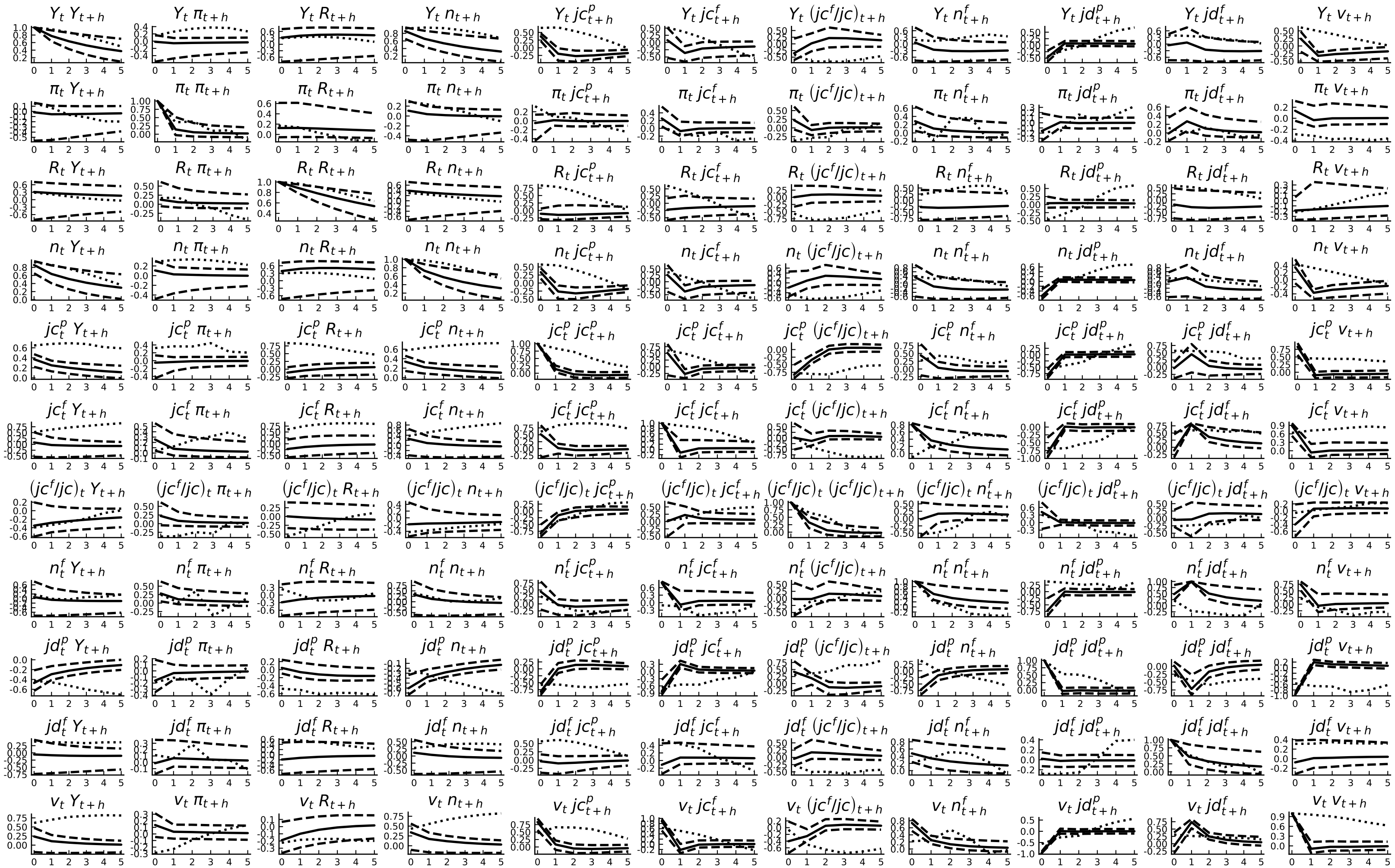

Figure 6: Simulated and data cross-correlations

The x-axis is the lag h and the y-axis is the correlation between the variable x_t and the variable x_{t+h}. The solid line is the simulated value, the dashed line is the 95 % confidence interval around the latter and the dotted line is the value from the data.

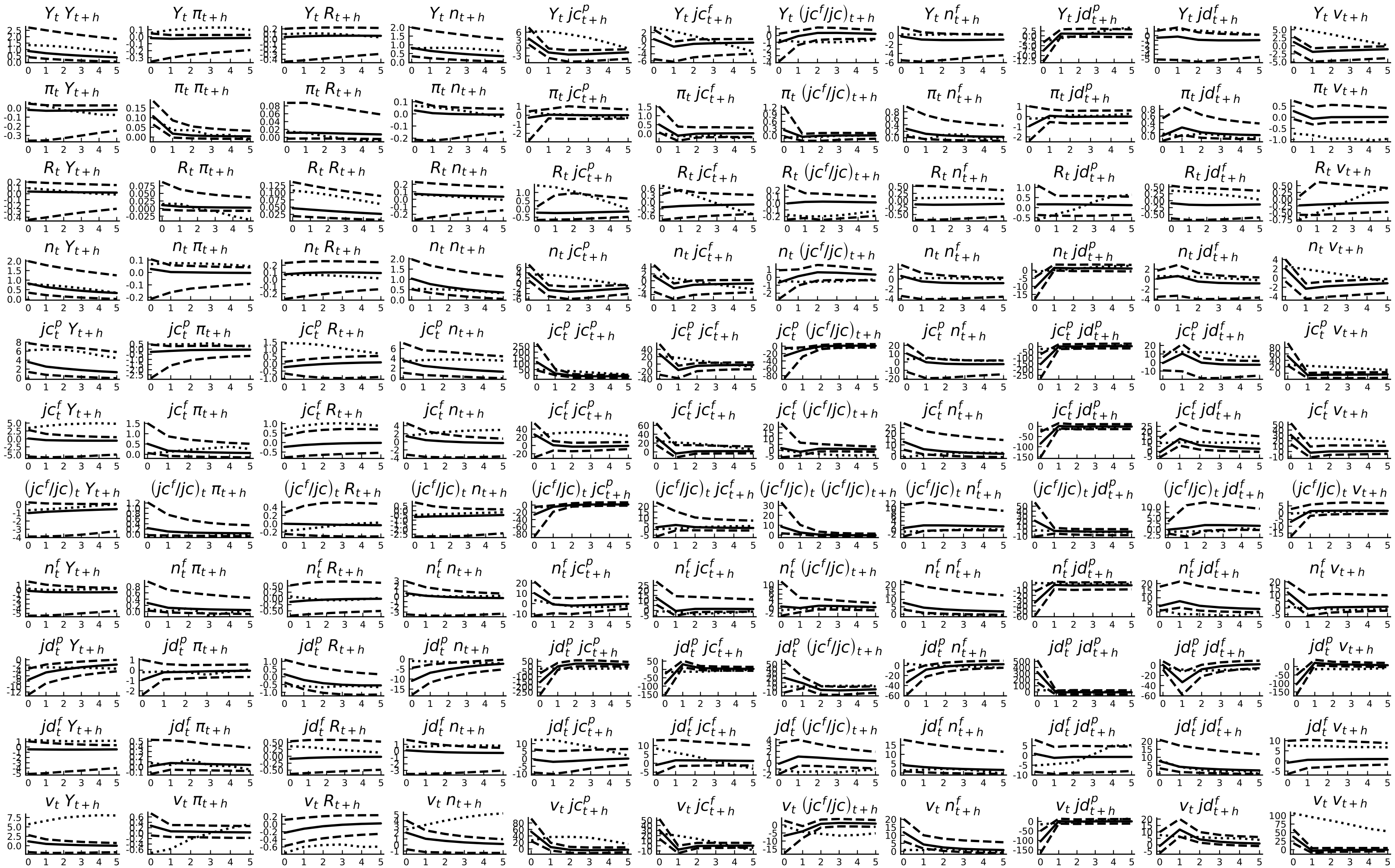

Figure 7: Simulated and data cross-covariances

The x-axis is the lag h and the y-axis is the covariance between the variable x_t and the variable x_{t+h}. The solid line is the simulated value, the dashed line is the 95 % confidence interval around the latter and the dotted line is the value from the data.

D Robustness checks

Parameters	Hamilton		Linear trend		First difference		Hodrick-Prescott	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ρ_A	0.61	0.09	0.71	0.08	0.17	0.1	0.63	0.08
ρ_g	0.87	0.05	0.88	0.04	0.81	0.06	0.87	0.04
ρ_R	0.79	0.04	0.77	0.05	0.82	0.04	0.69	0.05
ρ_π	1.97	0.56	2.2	0.63	1.65	0.41	2.28	0.6
ρ_y	0.24	0.05	0.29	0.06	0.48	0.1	0.42	0.05
ψ	0.85	0.02	0.82	0.03	0.67	0.05	0.79	0.03
σ_A	0.61	0.08	0.31	0.04	0.3	0.03	0.29	0.03
σ_μ	0.22	0.03	0.3	0.03	0.27	0.03	0.29	0.03
σ_g	7.59	2.22	5.38	1.19	3.55	0.72	5.4	1.4
σ_m	0.06	0.01	0.08	0.01	0.08	0.01	0.06	0.01

Table 7: Estimations with Hamilton, linear-trend, first-difference and Hodrick-Prescott filters

E Classic Model

E.1 Equilibrium equations

$$u'(c_t) = \beta R_t \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} u'(c_{t+1}) \right]$$

$$A_t z_t^p \phi_t + F_t - \mathbb{E}_t \beta_{t,t+1}(1-s)F_{t+1} + \mathbb{E}_t \beta_{t,t+1}(1-s)A_{t+1}\phi_{t+1} \int_{z_{t+1}^p}^{+\infty} (1 - G(z))\, dG(z)$$

$$= b_t + \frac{\eta\gamma\theta_t}{1-\eta}$$

$$z_t^c = z_t^p + \frac{F_t}{A_t \phi_t}$$

$$\frac{\gamma}{(1-\eta)A_t\phi_t q(\theta_t)} = \int_{z_t^c}^{+\infty} [1 - G(z)]\, dz$$

$$n_t^p = (1-\xi_t)\, n_{t-1}^p + \mu_t^p e_t$$

$$v_t = \theta_t \left(1 - (1-\xi_t)\, n_{t-1}^p\right)$$

$$Y_t = c_t + g_t + \gamma v_t$$

$$Y_t \Delta_t = A_t E_z \left[z \mid z \geq z_t^p\right](1-\xi_t)\, n_{t-1}^p + (1 - G(z_t^c))\, q(\theta_t)\, v_t A_t E_z\left[z \mid z \geq z_t^c\right]$$

$$\mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T} \psi^{T-t} P_T^{\epsilon_T} Y_T \left(\frac{P_{i,t}^*}{P_T} - \mu_T \phi_T \right) = 0$$

$$log\,(R_t/R) = \rho_R log\,(R_{t-1}/R) + (1-\rho_R) \left[\rho_\pi \mathbb{E}_t log \left(\frac{P_{t+1}}{P_t} \right) + \rho_y log \left(\frac{y_t}{y} \right) \right] + \epsilon_t^m$$

$$log\,(A_t/A) = \rho^A log\,(A_{t-1}/A) + \epsilon_t^A$$

$$log\,(\mu_t/\mu) = \rho^\mu log\,(\mu_{t-1}/\mu) + \epsilon_t^\mu$$

$$log\,(g_t/g) = \rho^g log\,(g_{t-1}/g) + \epsilon_t^g$$

E.2 Log-linearization

$$\widehat{c_t} = \widehat{E_t c_{t+1}} - \left[-\frac{cu''}{u}\right]^{-1}\left(\widehat{R_t} - \widehat{E_t \pi_{t+1}}\right)$$

$$A\phi z^p\left(\widehat{A_t} + \widehat{z_t^p} + \widehat{\phi_t}\right) + \beta(1-s)\left(A\phi\int_{z^p}^{+\infty}(1-G(z))\,dz - F\right)(c_t - \mathbb{E}_t\widehat{c_{t+1}})$$

$$+\beta(1-s)A\phi\left(\int_{z^p}^{+\infty}(1-G(z))\,dz\left(\mathbb{E}_t\widehat{A_{t+1}} + \mathbb{E}_t\widehat{\phi_{t+1}}\right) - (1-G(z^p))\,z^p\mathbb{E}_t\widehat{z_{t+1}^p}\right)$$

$$+F\widehat{F_t} - \beta(1-s)F\mathbb{E}_t\widehat{F_{t+1}} - b\widehat{b_t} - \frac{\eta\gamma\theta}{1-\eta}\widehat{\theta}_t = 0$$

$$z^c\widehat{z_t^c} = z^p\widehat{z_t^p} + \frac{F}{A\phi}\left(\widehat{F_t} - \widehat{A_t} - \widehat{\phi_t}\right)$$

$$\frac{\gamma}{(1-\eta)\phi q(\theta)}\left(-\widehat{A_t} - \widehat{\phi_t} + \sigma\widehat{\theta}_t\right) + (1-G(z^c))\,z^c\widehat{z_t^c} = 0$$

$$\widehat{n_t^p} = (1-\xi)\,\widehat{n_{t-1}^p} - (1-s)z^p g(z^p)\,\widehat{z_t^p} + \frac{(1-G(z^c))\,q(\theta)v}{n^p}\left(\widehat{v_t} - \sigma\widehat{\theta}_t\right) - \frac{z^c g(z^c)\,q(\theta)v}{n^p}\widehat{z_t^c}$$

$$\widehat{v_t} = \widehat{\theta}_t - \frac{(1-\xi)\,\theta n^p}{v}\widehat{n_{t-1}^p} + \frac{(1-s)\theta g(z^p)\,z^p n^p}{v}\widehat{z_t^p}$$

$$\widehat{Y}_t = \frac{c}{Y}\widehat{c_t} + \frac{g}{Y}\widehat{g_t} + \frac{\gamma v}{Y}\widehat{v_t}$$

$$\widehat{Y}_t = \widehat{A_t} + \frac{(1-s)\left(\int_{z^p}^{+\infty} zg(z)dz\right)n^p}{Y}\widehat{n_{t-1}^p} - \frac{(1-s)n^p\,(z^p)^2\,g(z^p)}{Y}\widehat{z_t^p}$$

$$-(z^c)^2\,g(z^c)\,\frac{vq(\theta)}{Y}\widehat{z_t^c} + \left(\int_{z^c}^{+\infty} zg(z)dz\right)\frac{vq(\theta)}{Y}\left(\widehat{v_t} - \sigma\widehat{\theta}_t\right)$$

$$\widehat{\pi_t} = \beta\mathbb{E}_t\widehat{\pi_{t+1}} + \frac{(1-\beta\psi)(1-\psi)}{\psi}\widehat{\phi}_t + \epsilon_t^\mu$$

$$\widehat{R_t} = \rho_R\widehat{R_{t-1}} + (1-\rho_R)\left[\rho_\pi\mathbb{E}_t\widehat{\pi_{t+1}} + \rho_y\widehat{Y}_t\right] + \epsilon_t^m$$

$$\widehat{A_t} = \rho_A\widehat{A_{t-1}} + \epsilon_t^A$$

$$\widehat{g_t} = \rho_g\widehat{g_{t-1}} + \epsilon_t^g$$

E.3 Estimation

Table 8: Prior and posterior distributions of structural parameters.

Parameter	Prior distribution			Posterior distribution			
	Distr.	Para (1)	Para(2)	Mean	Std. Dev.	5%	95%
ρ_A	Uniform	0.0	1.0	0.78	0.09	0.62	0.91
ρ_μ	Uniform	0.0	1.0	0.1	0.11	0.01	0.35
ρ_g	Uniform	0.0	1.0	0.95	0.02	0.91	0.97
ρ_R	Uniform	0.0	1.0	0.68	0.08	0.53	0.8
ρ_π	Normal	1.5	0.75	2.52	0.57	1.3	3.13
ρ_y	Normal	0.12	0.15	0.12	0.06	0.03	0.24
ψ	Beta	0.7	0.05	0.8	0.04	0.73	0.85
σ_A	IGamma	0.5	4.0	0.31	0.03	0.26	0.37
σ_μ	IGamma	0.5	4.0	0.27	0.04	0.2	0.33
σ_g	IGamma	0.5	4.0	7.14	2.36	3.12	9.64
σ_m	IGamma	0.5	4.0	0.1	0.03	0.07	0.15

Para(1) and Para(2) correspond to mean and standard deviation of the prior distribution if the latter is Normal or Inverse Gamma. Para(1) and Para(2) correspond to lower and upper bound of the prior distribution when the latter is uniform

Table 9: Posterior Odds

Model	$log\left(p\left(Y_{1:T}\right)\right)$	Std. Dev.
Classic	-38.2	1.3
Dual	-39.9	1.1