Fluctuations in a Dual Labor Market

Normann Rion*

December 12, 2019

Abstract

JEL Classification:

Keywords:

1 Model

The model follows a discrete timing and embeds 4 types of agents ; the households, the firms, the fiscal authority and the central bank. Households can be either unemployed or employed through a fixed-term or an open-ended contract. Three types of firms coexist. Perfectly competitive final good producers generate the homogenous good valued by households for consumption and investment. This final good producers aggregate the differentiated goods produced by the retailers. The latter retailers are monopolistically competitive and transform the homegeneous intermediate good into a differentiated retail good. Intermediate firms produce the associated intermediate good and experience perfect competition. These intermediate firms, which we will simply call firms in what follows, use labor as their only input. In the following subsections, I describe the behavior of these agents in more detail.

1.1 Households

Households are identical and constitute a continuum represented by the interval $(0, 1)$. Perfect pooling of revenues leads to an equal consumption for all households, as in Merz (1995) and Andolfatto (1996). Households hold the firms, consume the homogeneous good produced by final good firms and save using one-period nominal bonds. They earn wages, unemployment benefits, interests on their savings and pay lump-sum taxes. As a result, the households' program boils down to

$$\max_{\{c_t, B_{t+1}\}_{t=0}^{+\infty}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

$$\text{s.t} \quad c_t + \frac{B_{t+1}}{P_t} = R_{t-1}\frac{B_t}{P_t} + \overline{w^p_t}_t n^p_t + \overline{w^f_t} n^f_t + bu_t + \Pi_t - \tau_t$$

C_t marks down consumption.[1] B_t is the amount of nominal bond holdings at the beginning of period t, with the associated nominal interest rate R_t between t and $t+1$. $\overline{w^p_t}$, $\overline{w^f_t}$ and $\overline{w_t}$ are respectively the average real wages for permanent jobs, temporary jobs and all workers. b denotes the unemployment benefits. $n^p_t = \int_0^1 n^p_{i,t} di$ is the aggregate permanent employment and n^f_t denotes its temporary counterpart. u_t marks down the measure of unemployed households. Firms transfer their profits to households through Π_t, while the government taxes τ_t to finance the payment of unemployment benefits.

*Ecole Normale Supérieure and Paris School of Economics. E-mail: normann.rion@psemail.eu

[1] I omit the subscript i since all households share the same allocations for consumption and savings.

The first order conditions with respect to c_t and B_{t+1} lead to the following Euler equation

$$\lambda_t = \beta \mathbb{E}_t \left[R_t \frac{P_t}{P_{t+1}} \lambda_{t+1} \right] \tag{1}$$

$$\lambda_t = u'(c_t) \tag{2}$$

The resulting firms' discount factor is $\beta_{t,s} = \beta^{s-t}\lambda_s/\lambda_t$ since households own firms.

1.2 Final good firms

Final good firms are identical and compete to produce the good consumed by households. They use a Dixit-Stiglitz aggregator as technology to put together retail goods and produce Y_t

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right)^{\frac{\epsilon_t}{\epsilon_t} - 1} \tag{3}$$

where ϵ_t is the elasticity of substitution between retail goods.

The firm takes as given the price of the retail goods $P_{i,t}$ and the price of the final good P_t and maximizes its profits with respect to the components of its input $\{Y_{i,t}\}_{i\in(0,1)}$ under the constraint (3). Therefore, the program of the final good firm boils down to

$$\max_{\{Y_{i,t}\}_{i\in[0,1]}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$
$$\text{subject to (3)}$$

The subsequent first order condition provides an expression for the demand of the retail good i

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_t} Y_t \tag{4}$$

1.3 Retailers

Retailers buy goods from intermediate firms and sell the obtained production to final good producer[2]. They are monopolistically competitive and lie on the interval $(0, 1)$. Retailers accomplish the one-to-one transformation of the intermediate good into a retail good. Denoting $X_{i,t}$ the retailer i's input in the intermediate good, the production technology writes

$$Y_{i,t} = X_{i,t}$$

As a result, retailers face a marginal cost that equals the relative price of the intermediate good ϕ_t. I assume that these firms face a staggered price adjustment à-la Calvo (1983).

$$P_{i,t} = \begin{cases} P_{i,t-1} & \text{with probability } \psi \\ P_{i,t}^* & \text{with probability } 1-\psi \end{cases}$$

A fraction ψ of retailers is able to adjust its prices to the optimal value $P_{i,t}^*$, whereas the remaining retailers stick to their former prices. There is no indexation of non-adjusted princes on inflation in this model. Therefore, the price-setting retail firm i at period t experiences the following program

[2] At this step, it is possible to introduce capital to extend the model.

$$
\begin{aligned}
&\max_{P^*_{i,t}} \mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T} \psi^{T-t} \left(\frac{P^*_{i,t}}{P_T} - \phi_T \right) Y_{i,T} \\
&\text{subject to } Y_{i,T} = \left(\frac{P^*_{i,t}}{P_T} \right)^{-\epsilon_t} Y_T
\end{aligned}
$$

where $\beta_{t,s}$ is the retailer's discount factor on time t for a unit of profit earned at time s. This leads to the following first order condition

$$
\mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T} \psi^{T-t} P_T^{\epsilon_T} Y_T \left(\frac{P^*_{i,t}}{P_T} - \mu_T \phi_T \right) = 0 \tag{5}
$$

where $\mu_t = \epsilon_t/(\epsilon_t - 1)$ is a mark-up shock such that $log\left(\mu_t\right) = (1-\rho_\mu)\, log\left(\epsilon/(\epsilon-1)\right) + \rho_\mu log\left(\mu_{t-1}\right) + \epsilon_t^\mu$, with $\epsilon_t^\mu \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\mu^2\right)$.

1.4 Intermediate good firms and the labor market

I mainly rely on Rion (2019) as for the general shaping of the labor market, which relates intermediate-good firms and households. Workers can be either employed or unemployed and are identical. There is no on-the-job search, which implies that only unemployed workers search for a job. Intermediate good firms are in perfect competition, sell their output to retailers at price ϕ_t and use labor as input. They can employ one worker or maintain one vacancy. These firms also face an aggregate productivity shock A_t. A_t follows the process $log\left(A_t\right) = (1-\rho_A) log\left(\overline{A}\right) + \rho_A log\left(A_{t-1}\right) + \epsilon_t^A$, with $\epsilon_t^A \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_A^2\right)$. The number of firm-worker contacts per period is $m(e, v)$, where e is the number of job-seekers and v is the number of vacancies. A classic measure of the matching activity is the labor market tightness $\theta = v/e$. The matching function m has constant returns to scale, which enables the definition of the firm-worker meeting probability $p(\theta)$ on the job seekers' side and its counterpart $q(\theta)$ on the firms' side.

$$
\begin{aligned}
q &= \frac{m\left(e, v\right)}{v} = m\left(\theta^{-1}, 1\right) \\
p &= \frac{m\left(e, v\right)}{e} = m\left(1, \theta\right) = \theta q(\theta)
\end{aligned}
$$

The workers' firm-worker meeting probability is increasing, whereas the firms' firm-worker meeting probability is decreasing. Note that these meeting probabilities are not the classic job-finding and vacancy-filling probabilities. Indeed, in this paper, a firm-worker meeting does not necessarily lead to a productive firm-worker association. When a firm and a worker meet at period t, the idiosyncratic productivity of the match z_t is revealed. Importantly, I assume that this idiosyncratic productivity is i.i.d across time and drawn from a distribution with cdf G. This assumption departs from the framework of Rion (2019), where the idiosyncratic productivity of matches is persistent. I reluctantly make this assumption for the sake of simplicity on behalf of realism. Indeed, persistent idiosyncratic shocks require to keep track of the whole distribution of idiosyncratic productivities. Since this paper is, as far as I know, the first to describe fluctuations in a dual labor market with an endogenous contractual choice at the hiring stage, I prefer to leave the potentially complex distributional mechanisms for future research.

I now describe the timing in the economy. Figure 1.4 sums it up. At the beginning of the period, agents learn the value of shocks. According to these new information, firms manage their workforce ; they may lay off workers and post vacancies. Next, new matches are revealed. Importantly, a paired firm and worker may return to search if the revealed productivity of the match is disappointing. Moreover, workers fired in the current period are able to participate to the present meeting round. This assumption is important to avoid an underestimation of the labor market flows, especially when the length of one periods is long compared with the average length of a job. Indeed, temporary jobs last 1.5 months on average in France (Dares, 2018), which might become a problem when the model is estimated on a quarterly basis and the workers must stay unemployed at least one period to be able to find a job again. Then, production is carried out, firms pay for wages and firing costs. At this point, households consume, and so ends the period.

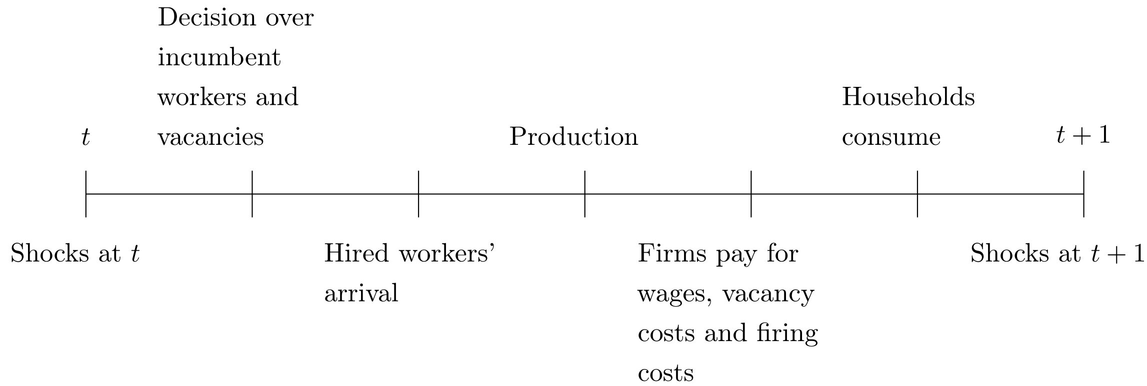

Figure 1: The timing of the economy

Now, I delineate the different possible situations on the labor market and the associated firms' and workers' surpluses.

Open-ended contracts A continuing open-ended contract delivers the wage w_t^p and stipulates a firing tax $F_t = \phi_t A_t F$. On each period, an open-ended match may exogenously separate with probability s. In this case, the firing cost needs not to be paid. Otherwise, the firm chooses whether it keeps or lays off the worker regarding the idiosyncratic productivity of the match. An endogenous separation entails the payment of the firing cost. The firm's surplus with a continuing permanent match being increasing in z_t, I define the threshold z_t^p below which paying the firing cost and dismissing the worker is preferrable. If I denote the firm's surplus with a continuing permanent match J_t^p and the present discounted value of a vacancy V_t, I get the equation

$$J_t^p(z_t) = \phi_t A_t z_t - w_t^p(z_t) + \mathbb{E}_t \beta_{t,t+1}(1-s)\left\{\int_{z_{t+1}^p}^{+\infty} J_{t+1}^p(z)\, dG(z) - G\left(z_{t+1}^p\right) F_{t+1} + sV_{t+1}\right\} \tag{6}$$

I denote ξ_t the permanent job destruction probability, which verifies

$$\xi_t = s + (1-s)G\left(z_t^p\right) \tag{7}$$

A worker benefitting from a continuing permanent contract earns the wage w_t^p and may either exogenously leave with probability s. If he does not, he faces a productivity shock and the match may split if the latter shock is adverse enough. As stated in the timing assumptions above, laid-off or quitting workers go back to the job seekers' pool immediately and are therefore eligible to participate to the current's period firm-worker meetings. I designate as $\widehat{U_t}$ the workers' associated present discounted value.

Consequently, a continuing permanent worker's present discounted utility W_t^p verifies

$$W_t^p(z_t) = w_t^p(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-s)\int_{z_{t+1}^p}^{+\infty} W_{t+1}^p(z)dG(z) + \xi_{t+1}\widehat{U_{t+1}}\right\} \tag{8}$$

New permanent matches stipulate different wages as their outside option does not include the payment of the firing cost in case of disagreement during the wage-bargaining process. Consequently, the surpluses of new open-ended matches involve the wage function $w_t^{0,p}$. After one-period, if there is no separation, a wage renegotiation occurs because idiosyncratic productivity changed. Firing costs now intervene in case of disagreement and the surpluses of continuing permanent contracts weigh in. Thus, the relevant functions of firm's and worker's surplus $J_t^{0,p}$ and $W_t^{0,p}$ follow

$$J_t^{0,p}(z_t) = \phi_t A_t z_t - w_t^{0,p}(z_t) + \mathbb{E}_t\beta_{t,t+1}(1-s)\left\{\int_{z_{t+1}^p}^{+\infty} J_{t+1}^p(z)\,dG(z) - G\left(z_{t+1}^p\right)F_{t+1} + sV_{t+1}\right\} \tag{9}$$

$$W_t^{0,p}(z_t) = w_t^{0,p}(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-s)\int_{z_{t+1}^p}^{+\infty} W_{t+1}^p(z)dG(z) + \xi_{t+1}\widehat{U_{t+1}}\right\} \tag{10}$$

Fixed-term contracts A fixed-term contract stipulates the wage function $w_{i,t}^f$ and the exogenous destruction probability δ[3]. Importantly, it yields a lower productivity with respect to permanent contracts by factor $\rho < 1$. The latter assumption is an important departure with respect to Rion (2019), where there is no productivity wedge between open-ended and fixed-term contracts *a priori*. I need this assumption to preserve the properties derived in Rion (2019), which essentially stem from the higher slope of open-ended matches' surplus with respect to the idiosyncratic productivity compared to fixed-term matches' surplus. In Rion (2019), the latter is a consequence of the persistence of productivity shocks. Temporary matches experience a higher job destruction probability than permanent contracts while facing the same frequency of productivity shocks. As a result, temporary matches discount the current production at a higher rate than permanent ones, which reduces their slope with respect to the current productivity. Here, this mechanism does not intervene as idiosyncratic productivities of firm-worker pairs are i.i.d across time.

A thought experiment provides better insights about the crucial nature of my assumption. For a given idiosyncratic productivity, if temporary and permanent contracts produced the same quantity of intermediate goods, there would be no room for labor market dualism at the equilibrium. One contract would be systematically preferred to the other at the hiring stage over the whole support of G. However, the assumption that temporary contracts are *per se* less productive than permanent ones is not far-fetched in most cases. Hagen (2002), Mertens et al. (2007), Pfeifer (2012) and Pfeifer (2014) estimate that German and Spanish temporary workers experience wages 5 to 10 % lower than their permanent counterparts. If wages reflect productivity, our assumption makes sense. Moreover, fixed-term positions are mainly filled by low-skilled or unexperienced workers (Fontaine and Malherbet, 2016) and benefit less from on-the-job training (Arulampalam and Booth, 1998; Arulampalam et al., 2004; Albert et al., 2005; Cutuli and Guetto, 2012).

Firms value the immediate production net of the wage. As for the continuation value, it simply embeds the occurence or non-occurence of an exogenous separation shock. I denote J_t^f the present discounted value of a temporary contract from the firm's point of view.

$$J_t^f(z_t) = \rho A_t z_t \phi_t - w_t^f(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-\delta)\int J_{t+1}^f(z)\,dG(z) + \delta V_{t+1}\right\} \tag{11}$$

[3]For an extensive discussion about the possibility of conversion into an open-contract contract at the expiry time and the endogenous choice of temporary jobs' durations, see Rion (2019)

Temporary workers immediatly value their wage. In case of separation, they can participate to the currrent period's labor market trades. I designate as W_t^f the worker's present discounted value of a temporary match, which follows

$$W_t^f(z_t) = w_t^f(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-\delta)\int W_{t+1}^f(z)\,dG(z) + \delta\widehat{U_{t+1}}\right\} \tag{12}$$

Vacancies In opposition to most of the literature, the choice between a temporary and a permanent contract at the hiring step is explicitly embedded and in endogenous. When paired with a worker, firms can hire through a temporary contract or hire through a permanent contract. They are also able to return searching for a worker and get the chance to be matched with another one on the next period. The following equation bears witness of these possible options.

$$V_t = -\gamma + q(\theta_t)\int max\left[J_t^{0,p}(z), J_t^f(z), E_t\beta_{t,t+1}V_{t+1}\right]dG(z) \tag{13}$$

I assume that there is free entry for firms which post vacancies at the equilibrium.

$$V_t = 0 \tag{14}$$

The hiring regions of permanent contracts and temporary contracts are denoted H_t^p and H_t^f, which are defined as follows

$$H_t^p = \left\{z_t : J_t^{0,p}(z_t) \geq \max\left[J_t^f(z_t), E_t\beta_{t,t+1}V_{t+1}\right]\right\} \tag{15}$$
$$H_t^f = \left\{z_t : J_t^f(z_t) \geq \max\left[J_t^{0,p}(z_t), E_t\beta_{t,t+1}V_{t+1}\right]\right\} \tag{16}$$

Firms go back to searching for a worker when the drawn idiosyncratic productivity belongs to $H_t^u = \overline{H_t^p \cup H_t^f}$. As a result, the present discounted value of unemployment U_t writes

$$U_t = b + \mathbb{E}_t\beta_{t,t+1}\widehat{U_{t+1}} \tag{17}$$

where b is the unemployment benefit and $\widehat{U_t}$ is the present discounted value of the expected current period's trades on the labor market. It is the workers' outside option just before shocks hit.

$$\begin{aligned}\widehat{U_t} = {} & p(\theta_t)\left(\int_{H_t^p} W_t^{0,p}(z)dG(z) + \int_{H_t^f} W_t^f(z)dG(z) + \left(\int_{H_t^u} dG(z)\right)U_t\right) \\ & + (1-p(\theta_t))\,U_t\end{aligned} \tag{18}$$

1.5 Wage bargaining

I assume that wages are set each period through a Nash-bargaining process. Thus, denoting η the worker's share of the surplus of the match, the sharing rules write

$$W_t^p(z_t) = \eta S_t^p(z_t) \tag{19}$$
$$W_t^{0,p}(z_t) = \eta S_t^{0,p}(z_t) \tag{20}$$
$$W_t^f(z_t) = \eta S_t^f(z_t) \tag{21}$$

where S_t^p is the total surplus of a continuing permanent match, $S_t^{0,p}$ is the total surplus from a new permanent match and S_t^f is the total surplus from a temporary match. These joint surpluses verify

$$S_t^p(z_t) = J_t^p(z_t) - (V_t - F_t) + W_t^p(z_t) - U_t \tag{22}$$
$$S_t^{0,p}(z_t) = J_t^{0,p}(z_t) - V_t + W_t^{0,p}(z_t) - U_t \tag{23}$$
$$S_t^f(z_t) = J_t^f(z_t) - V_t + W_t^f(z_t) - U_t \tag{24}$$

As intended, the continuing permanent worker's surplus includes the firing cost in case of endogenous separation. This bolsters his threat point in the Nash-bargaining process and pushes up his wage. The new permanent workers does not benefit from this effect since a failure in the bargaining process does not entail the payment of F_t.

Using the different definitions of the firms' and the workers' surpluses, the total surpluses write[4]

$$\begin{aligned} S_t^p(z_t) = {} & A_t z_t \phi_t - b + F_t - \mathbb{E}_t\beta_{t,t+1}(1-s)F_{t+1} - \mathbb{E}_t\beta_{t,t+1}(1-\xi_{t+1})\frac{\eta\gamma\theta_{t+1}}{1-\eta} \\ & + \mathbb{E}_t\beta_{t,t+1}(1-s)A_{t+1}\phi_{t+1}\int_{z_{t+1}^p}^{+\infty}(1-G(z))\,dG(z) \end{aligned} \tag{25}$$

$$S_t^{0,p}(z_t) = S_t^p(z_t) - F_t \tag{26}$$

$$S_t^f(z_t) = \rho A_t z_t \phi_t - b - \mathbb{E}_t\beta_{t,t+1}(1-\delta)\frac{\eta\gamma\theta_{t+1}}{1-\eta} + \rho\mathbb{E}_t\beta_{t,t+1}(1-\delta)A_{t+1}\phi_{t+1}\left(\mathbb{E}z - z_{t+1}^f\right) \tag{27}$$

Using the expressions for the firms' surpluses with the expressions for the total surpluses above and the surplus sharing rules, the wages verify

$$w_t^p(z_t) = \eta\left(A_t z_t\phi_t + F_t - E_t\beta_{t,t+1}(1-s)F_{t+1} + E_t\beta_{t,t+1}(1-\xi_{t+1})\gamma\theta_{t+1}\right) + (1-\eta)b$$
$$w_t^{0,p}(z_t) = \eta\left(A_t z_t\phi_t - E_t\beta_{t,t+1}(1-s)F_{t+1} + E_t\beta_{t,t+1}(1-\xi_{t+1})\gamma\theta_{t+1}\right) + (1-\eta)b$$
$$w_t^f(z_t) = \eta\left(\rho A_t z_t\phi_t + E_t\beta_{t,t+1}(1-\delta)\gamma\theta_{t+1}\right) + (1-\eta)b$$

The wage of the continuing permanent worker increases with the firing cost. The firing cost acts as a tax on separation but also as an upward force in the continuing permanent worker's threat point in the wage bargaining process. The new permanent worker is penalized with higher firing costs to compensate the future gains of wages in case of continuation. As usual in the Mortensen-Pissarides literature, the labor market tightness increases the outside option of the workers through the enlarged chances of finding a job and the wages raise.

In the case of Nash-bargaining, the hiring and firing decisions are efficient for both the worker and the firm. This may not be the case if wages are rigid.

1.6 Job creation and job destruction

The destruction of permanent contracts occurs when the surplus of a continuing match reaches zero. Thus, z_t^p follows

[4] Detailed calculations are available in Appendix A.1

$$S_t^p\left(z_t^p\right)=0 \tag{28}$$

Using (25), this definition is equivalent to

$$\begin{aligned} & A_t z_t^p \phi_t - b + F_t - \mathbb{E}_t \beta_{t,t+1}(1-s)F_{t+1} + \mathbb{E}_t \beta_{t,t+1}(1-s)A_{t+1}\phi_{t+1} \int_{z_{t+1}^p}^{+\infty} (1-G(z))\, dG(z) \\ & = \mathbb{E}_t \beta_{t,t+1}\left(1-\xi_{t+1}\right) \frac{\eta\gamma\theta_{t+1}}{1-\eta} \end{aligned} \tag{29}$$

At the hiring stage, temporary contracts become profitable when the productivity shock exceeds z_t^f such that

$$S_t^f\left(z_t^f\right)=0$$

Using (27), z_t^f verifies

$$\rho A_t z_t^f \phi_t - b + \rho \mathbb{E}_t \beta_{t,t+1}(1-\delta)A_{t+1}\phi_{t+1}\left(\mathbb{E}z - z_{t+1}^f\right) = \mathbb{E}_t \beta_{t,t+1}(1-\delta)\frac{\eta\gamma\theta_{t+1}}{1-\eta} \tag{30}$$

Analogously, new permanent contracts reach zero probitability at the threshold z_t^c, such that

$$S_t^{0,p}\left(z_t^c\right)=0$$

Since $\partial S_t^p/\partial z_t = A_t\phi_t$ and (28) is verified, one can rewrite the function S_t^p as follows using an integration

$$S_t^p(z) = A_t\phi_t\left(z - z_t^p\right) \tag{31}$$

In the same manner, the surplus associated with temporary contracts writes

$$S_t^f(z) = \rho A_t \phi_t\left(z - z_t^f\right) \tag{32}$$

Consequently, (31) provides a simple redefinition of z_t^c.

$$A_t\phi_t z_t^c = A_t\phi_t z_t^p + F_t \tag{33}$$

The job creation condition stems from the free-entry condition (14) and the Bellman equation defining the present discounted value of an unfilled vacancy (13). Using the Nash-sharing rules (20)-(21), the job creation condition becomes

$$\frac{\gamma}{(1-\eta)q\left(\theta_t\right)} = \int max\left[S_t^{0,p}\left(z\right), S_t^f\left(z\right), 0\right] dG(z)$$

The economic mechanisms that follow are extensively described in Rion (2019). The choice between temporary and permanent contracts lies in the comparison of joint surpluses across contracts. Interestingly, since $\partial S_t^{0,p}/\partial z_t = A_t\phi_t A_t > \rho A_t\phi_t = \partial S_t^f/\partial z_t$, there may exist a productivity threshold z_t^* above which

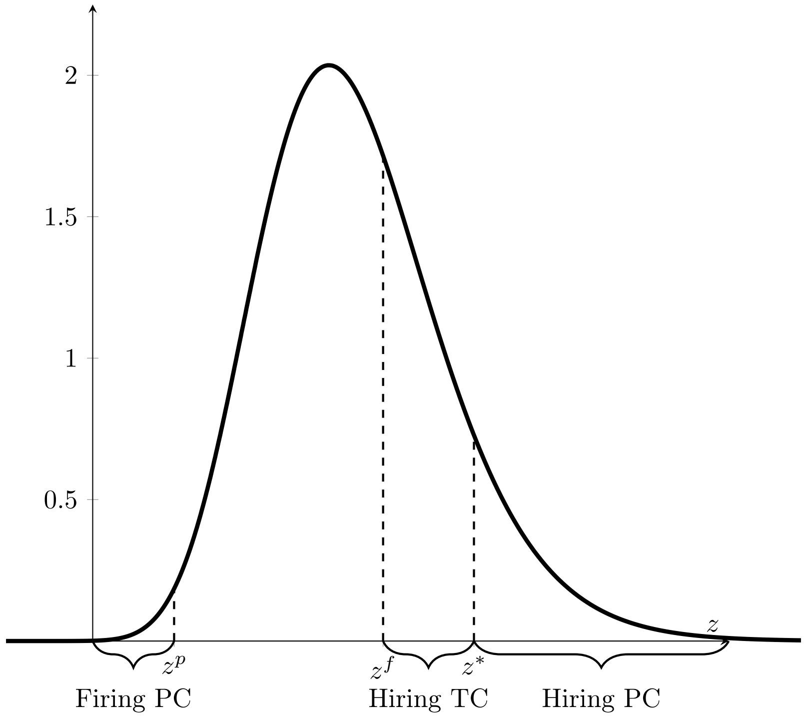

Figure 2: The probability distribution function of idiosyncratic shocks and the location of thresholds

The displayed pdf belongs to a log-Normal low with zero mean and standard deviation 0.2.

permanent contracts are preferable upon temporary ones. It will be the case in our calibration and estimation procedure. The discrimination between contracts is rooted in flexibility-productivity trade-off. Permanent contracts are more productive but are expensive during the downturns, whereas temporary contracts deliver a lower productivity but the associated workforce is more malleable because of the short stipulated durations. If job creation involves both temporary and permanent contracts, then Figure 2 sums up the hiring and firing policies.

Using the fact that $S_t^{0,p}(z_t^*) = S_t^f(z_t^*)$ with (31) and (32), one finds that z_t^* verifies

$$(1-\rho)z_t^* = z_t^c - \rho z_t^f \tag{34}$$

The behavior of the integrand of the job creation condition is summed up as follows

$$max\left[S_t^{0,p}(z_t), S_t^f(z_t), 0\right] = \begin{cases} S_t^{0,p}(z_t) & \text{if } z_t \geq z_t^* \\ S_t^f(z_t) & \text{if } z_t^f \leq z_t \leq z_t^* \end{cases}$$

Using the expressions of the derivatives of total surpluses with integration by parts, the job creation condition becomes

$$\frac{\gamma}{(1-\eta)A_t\phi_t q(\theta_t)} = \int_{z_t^*}^{+\infty} [1-G(z)]\,dz + \rho\int_{z_t^f}^{z_t^*} [1-G(z)]\,dz \tag{35}$$

1.7 Fiscal and monetary policy

In this framework, I assume that the government's role is to tax the households in order to provide for the unemployment benefits. The revenues from the firing tax get back to the government. Therefore, the budget of the government is

$$\tau_t + (1-s)G\left(z_t^p\right)n_t^p F = bu_t + g_t \tag{36}$$

where g_t is the government expenditure and follows the AR(1) log-process $log\left(g_t\right) = (1-\rho_g)\, log\left(\overline{g}\right) + \rho_g log\left(g_{t-1}\right) + \epsilon_t^g$, $\epsilon_t^g \sim \mathcal{N}\left(0, \sigma_g^2\right)$.

The monetary policy is decided in accordance with a simple Taylor rule

$$log\left(R_t/R\right) = \rho_R log\left(R_{t-1}/R\right) + (1-\rho_R)\left[\rho_\pi \mathbb{E}_t log\left(\frac{P_{t+1}}{P_t}\right) + \rho_y log\left(\frac{y_t}{y}\right)\right] + \epsilon_t^m \tag{37}$$

where y is the steady-state output and $\epsilon_t^m \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_m^2\right)$.

Fixed-term contracts are known to behave as buffers in front of workload fluctuations, as Saint-Paul (1996) Saint-Paul (1996) documented. Thus, the case of indeterminacy in the Taylor rule and the subsequent appearance of sunspot equilibria may prove interesting. For the sake of simplicity, though, I shall restrain the present analysis to the determinate case with $\rho_\pi > 1$ and $\rho_y > 0$ and leave indeterminacy and its consequences on dual labor markets for future research.

1.8 Market clearing conditions and the equilibrium

This paragraph sums up the different conditions enabling an utter closing of the model. The employment values sum to the measure of households, namely 1.

$$n_t^p + n_t^f + u_t = 1 \tag{38}$$

where n_t^p designates the measure of open-ended employees, n_t^f the measure of fixed-term employees and u_t unemployment.

The aggregate stock of job-seekers includes the formerly unemployed households and the new entrants in the unemployment pool from the current period.

$$e_t = u_{t-1} + \delta n_{t-1}^f + \xi_t n_{t-1}^p \tag{39}$$

The labor market tightness is the ratio between aggregate number of vacancies $v_t = \int_0^1 v_{i,t} di$ and the number of job seekers.

$$\theta_t = \frac{v_t}{e_t}$$

The employment values evolve according to the following law of motions. The employment stocks drop by the job destruction flow and augment by the job creation flow.

$$n_t^p = \left(1-\xi_t\right)n_{t-1}^p + \mu_t^p e_t \tag{40}$$

$$n_t^f = \left(1-\delta\right)n_{t-1}^f + \mu_t^f e_t \tag{41}$$

where $\mu_t^p = p\left(\theta_t\right)\left(1 - G\left(z_t^*\right)\right)$ and $\mu_t^f = p\left(\theta_t\right)\left(G\left(z_t^*\right) - G\left(z_t^f\right)\right)$ are the permanent and temporary job-finding rates respectively. The aggregate demand for final goods is

$$Y_t = c_t + g_t + \gamma v_t \tag{42}$$

The retailers face only one real marginal cost from the intermediate firms, which entails a unique equilibrium value for the optimal price-setting program, *id est* $P_{i,t}^* = P_t^*$.

Meanwhile, the market clearing from intermediate goods writes

$$\begin{aligned}\int_0^1 Y_{i,t} di = & A_t E_z \left[z \mid z \geq z_t^p\right] (1-\xi_t)\, n_{t-1}^p + (1 - G(z_t^*))\, q(\theta_t)\, v_t A_t E_z \left[z \mid z \geq z_t^*\right] \\ & + \rho A_t \mathbb{E}_z[z] (1-\delta)\, n_{t-1}^f + \rho \left(G(z_t^*) - G\left(z_t^f\right)\right) q(\theta_t)\, v_t A_t \mathbb{E}_z \left[z \mid z_t^* \geq z \geq z_t^f\right]\end{aligned}$$

Using the first order condition from the final good firm's program, I get

$$\begin{aligned}Y_t \Delta_t = & A_t E_z \left[z \mid z \geq z_t^p\right] (1-\xi_t)\, n_{t-1}^p + (1 - G(z_t^*))\, q(\theta_t)\, v_t A_t E_z \left[z \mid z \geq z_t^*\right] \\ & + \rho A_t E_z[z] (1-\delta)\, n_{t-1}^f + \rho \left(G(z_t^*) - G\left(z_t^f\right)\right) q(\theta_t)\, v_t A_t E_z \left[z \mid z_t^* \geq z \geq z_t^f\right]\end{aligned} \tag{43}$$

with $\Delta_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_t} di$ which measures price dispersion. Yun (1996) demonstrated that the associated law of motion is

$$\Delta_t = (1-\psi) \left(\frac{P_t^*}{P_t}\right)^{-\epsilon_t} + \psi \left(\frac{P_t}{P_{t-1}}\right)^{\epsilon_t} \Delta_{t-1} \tag{44}$$

while the price level follows

$$P_t = \left[\psi P_{t-1}^{1-\epsilon_t} + (1-\psi) \left(P_t^*\right)^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}} \tag{45}$$

Given the path of exogenous shocks $\left\{\epsilon_t^A, \epsilon_t^\mu, \epsilon_t^g, \epsilon_t^m\right\}_{t=0}^{+\infty}$, laws of motions of exogenous shocks $\left\{A_t, \mu_t, g_t\right\}_{t=0}^{+\infty}$ and initial values for the state variables $\left\{i_{-1}, n_{-1}^p, n_{-1}^f, \Delta_{-1}, P_{-1}\right\}$, the equilibrium sums up into the set of endogenous variables $\left\{i_t, c_t, Y_t, n_t^p, n_t^f, u_t, \Delta_t, z_t^p, z_t^c, z_t^f, z_t^*, \theta_t, \phi_t, v_t, P_t, P_t^*\right\}_{t=0}^{+\infty}$ pinned down by equations (1)-(2), (5), (29)-(30), (34) and (37)-(45).

2 Calibration and estimation procedure

The model is bridged to the data through two steps. Some of the parameters are firstly calibrated to match moments shaping typical dual European labor markets. Then, parameters driving shock processes and nominal rigidities are estimated through a Bayesian technique.

2.1 Calibration

The intended calibration exercise needs to meet two requirements so as to lead to relevant results from an heuristic point of view. A natural objective is the faithful reproduction of the main features of labor markets in the Euro area. Moreover, the unprecedented modelisation of a dual labor market in a DSGE model adds the requirement of numerical comparability with a classic labor market embedding firing costs within the Euro Area. I shall rely on modelisation choices Thomas and Zanetti (2009) made to compel with the latter demand.

The central role of the distribution for idiosyncratic productivity shocks in the shaping of hiring and separation decisions falls reluctantly within the need for comparability, in absence of a proper estimation procedure. Consequently, I assume that the standard deviation for these shocks amounts to 0.2. I follow Thomas and Zanetti (2009) in several other dimensions. The discount factor is simarly set to 0.99. The matching function is assumed to be in a Cobb-Douglas form $m(e, v) = me^{\sigma}v^{1-\sigma}$ with σ set to 0.6, which stands in the middle of the 0.5-0.7 range Burda and Wyplosz (1994) estimated for some European countries. The Hosios condition being still verified, I set the workers' bargaining power to 0.6 as well so that the congestion externalities do not weigh in. The elasticity of demand curves is set to 6. I also assume that the government-spending-to-gdp ratio is 20 % .

β	σ	η	σ_z	ϵ
0.99	0.6	0.6	0.2	6

Table 1: Initial parameters

I depart from Thomas and Zanetti (2009) in several dimensions. I assume that the vacancy-worker meeting probability from the firm's point of view is 0.7 instead of 0.9. The latter figure replicates flows on the US labor market, which are known to be bigger than the European ones. One important feature of the labor market is its size, since it influences labor market tightness and job-finding rates. Should we consider ILO-defined unemployment *stricto sensu* or include the inactive population ? Elsby et al. (2015) and Fontaine (2016) demonstrated the importance of the participation margin to explain unemployment fluctuations respectively in the United States and in France. This is all the more true with precarious employment, which involves people at the blurred frontier between unemployment and inactivity. According to data from Eurostat extending between 2002 and 2017, the participation rate in the Euro Area rate is around 67 %. Thus, I set steady-state employment to 0.67. In the same manner, I target a ratio of temporary contracts over total employment of 13 % in accordance with Eurostat estimates from 2006 to 2017. An important factor is the contractual composition of creation flows, which influences the rate of turnover in the labor force. Data from *Acoss - Dares* witness that 80 % of job creations occur through temporary contracts in France, a share that reaches 90 % in Spain as Felgueroso et al. (2017) document. At the other end of the spectrum, Addison et al. (2019) report a 45 % share of temporary contracts in German job creation. I target a 70 % share of fixed-term contracts in job creation to strike a balance. I also set the quarterly separation probability of permanent matches ξ^p to 3 % consistently with the magnitude of data from Eurostat. This leads to a steady-state separation probability of temporary matches of 40 %, which is equivalent to an average duration of 7.5 months. This estimate is in line with Eurostat data. Among separations involving permanent contracts, an essential factor is the probability of paying the firing cost. Jolivet et al. (2006) explain that more sclerotic markets lead to a higher share of voluntary quits, whereas lay-offs tend to be more significant in countries with more flexible labor markets. The French case is described as the most representative of the former phenomenon, with 80 % of separations involving permanent matches happening through voluntary channels according to Dares (2018). A reasonable value for the Euro Area is thus inferior. As a result, I target a rate of 60 % for the ratio of exogenous separations among total separations for permanent matches. The resulting value of s is 2.1 %.

The calibration of firing costs constitutes a challenge. The data is scarce about this issue, which makes reasonable proposals difficult to spell. The latter is all the more true since heterogeneity between countries is high, whether it be from a legal or an economic point of view. The 0-to-5 OECD indicator for employment protection legislation enables a cursory comparison between Euro area countries. While French, German and Italian indexes of EPL against collective and individual dismissals of permanent contracts are close to 2.8, the Spanish index is roughly 2.4. Employment protection legislation in the

Euro Area seems closer to the French case, whose firing costs are examined by Kramarz and Michaud (2010). The latter assess that individual lay-offs marginally cost 4 months of the median wage, while the marginal cost of lay-off within a collective-termination plan represents 12 months of the median wage[5]. The former being the most frequent case, we reckon that total firing costs represent 4 months of the permanent workers' average wage . As in Bentolila et al. (2012) and Cahuc et al. (2016), we assume that red-tape costs actually embodied by F only represent one third of total firing costs for the firm[6]. Thus, we target a ratio of 4/9 for firing costs with respect to the quarterly permanent workers' average wage.

$F/\overline{w^p}$	$\mu^f/\left(\mu^p+\mu^f\right)$	n^f/n	ξ	s/ξ	n	$q(\theta)$
4/9	0.7	0.15	0.03	0.6	0.67	0.7

Table 2: Targets for a calibration of the labor market in the Euro area

The last parameters to be pinpointed are F, b, m, ρ and γ. The empirical evidence concerning b is thin. Indeed, the flow value of non-employment is a highly debated issue in labor economics. Hagedorn and Manovskii (2008) advocate a high relative value of non-employment with respect to employment to make the Mortensen-Pissarides model able to replicate faithfully fluctuations in unemployment and vacancies following productivity shocks of a realistic magnitude. The high replacement ratios of unemployment insurance in Western Europe combined with the non-monetary benefits of working support this view. The obtained b is coherent with this view. In the same manner, no proper empirical evidence is available to assess the productivity difference between fixed-term contracts and open-ended contracts in European countries, which explains why I prefer leave it as a free parameter in the calibration. I find a 3% productivity deficit among temporary contracts with respect to permanent contracts. The vacancy cost represents 1.5 % of the average wage, which is coherent with the (scarce) available evidence put forward by Kramarz and Michaud (2010).

F	b	s	δ	ρ	m	γ
0.38	0.83	0.02	0.4	0.97	0.53	0.02

Table 3: Calibrated parameters

2.2 Estimation

The unknown parameters are related to the shock processes and the nominal rigidities. I estimate them using a Sequential Monte Carlo procedure[7]. Following Thomas and Zanetti (2009), I choose the same time period extending from 1997-Q1 to 2007-Q4 and a nearly similar set of observables (Y_t, π_t, R_t, n_t) and quarterly data series, namely GDP at constant prices, employment, EONIA rates in annual terms and the GDP deflator. All concern the 19-country Euro area and are obtained from the ECB Data Warehouse. The demeaned growth rate of the GDP deflator is plugged to inflation π_t. Since R_t corresponds to the real interest rates, they correspond to the demeaned quarterly equivalent of EONIA rates diminished by estimated inflation. Real GDP and employment are logged and reluctantly linearly detrended. As a matter of fact, the use of any detrending method or filter as well as the comprehension of adequate observable equations to match growth rates in the data is mainly arbitrary and may deliver heterogeneous

[5]To be accurate, Kramarz and Michaud (2010) assess that firms with more than 50 employees face a marginal cost of 97,727 FFr (Table 1b), which represents 14 months of the workers' median wage. Consequently, the associated median wage of fired workers is 6980 FFr. Thus, Table 2 shows that individual terminations cost 27,389 FFr, which amounts to 4 months of the fired workers' median wage, while the termination within a collective firing plan marginally costs 81,850 FFr, which equals 12 months of the median wage.

[6]Transfers between separated firms and workers are not taken into account in firing costs because they play no allocational role, in contrast with red-tape firing costs

[7]See Herbst and Schorfheide (2016) for an extensive treatment of this method.

and mispecified estimation results. Filippo (2011) and Canova (2014) roughly propose to simultaneously estimate the parameters of flexible specifications for trends along deep parameters so as to let the model explain the part of the data it is able to account for. However, as Canova and Sala (2009) and Iskrev (2010); Iskrev et al. (2010) testify, identification issues are real in the estimation of DSGE models. The resort to the methods advocated by Filippo (2011) and Canova (2014) would magnify these problems through the introduction of new parameters. For this reason, I stick to the detrending method employed by Thomas and Zanetti (2009), which remains simple and enables comparisons between my model and their own.

Most chosen priors are chosen to be diffuse, reflecting the void of the DSGE literature with respect to dual labor markets. Thus, the priors for autocorrelations of the shock processes are uniform laws on $[0, 1]$, while standard deviations follow inverse gamma distributions with mean 0.5 and standard deviation 4[8]. As for the Taylor parameters r_π and r_y, the prior needs to be vague and embed the fact that $r_\pi > 1$ and $r_y > 0$. As a result, truncated normal laws with large standard deviations are employed. Druant et al. (2012) assess the average duration of firms' prices in the Euro Area to 10 months, which corresponds to a value of ψ around 0.7. A tight normal law around this value is subsequently chosen.

Identification issues in DSGE models are well known. My model does not escape this observation. Following the method Iskrev (2010) makes a case for, I find that the estimated parameters are identified with the available data. Nevertheless, the implementation of methods presented by Iskrev et al. (2010) reveal a few pitfalls already known in the literature. ρ_μ and σ_μ are jointly weakly identified. This problem also concerns ρ_R, ρ_π, ρ_y and σ_m, whose estimation is highly intertwined. Indeed, for given data, the higher the autocorrelation of interest rates in the Taylor rule, the stronger the needed policy adjustments. In turn, this pushes up the standard deviation of monetary policy shocks.

Table 4: Prior and posterior distributions of structural parameters.

	Prior distribution			Posterior distribution			
	Distr.	Para (1)	Para(2)	Mean	Std. Dev.	5%	95%
ρ_A	Uniform	0.00	1.00	0.84	0.08	0.69	0.95
ρ_μ	Uniform	0.00	1.00	0.04	0.04	0.00	0.14
ρ_g	Uniform	0.00	1.00	0.90	0.04	0.83	0.96
ρ_R	Uniform	0.00	1.00	0.76	0.06	0.65	0.85
ρ_π	Normal	1.50	0.75	1.82	0.60	1.08	3.01
ρ_y	Normal	0.50	0.75	0.66	0.17	0.43	0.98
ψ	Beta	0.7	0.1	0.79	0.04	0.85	0.99
σ_A	IGamma	0.15	1.00	0.13	0.01	0.11	0.15
σ_μ	IGamma	0.15	1.00	0.26	0.03	0.22	0.32
σ_g	IGamma	0.15	1.00	4.75	1.70	2.89	8.44
σ_m	IGamma	0.15	1.00	0.08	0.01	0.06	0.10

Para(1) and Para(2) correspond to mean and standard deviation of the prior distribution if the latter is Normal or Inverse Gamma. Para(1) and Para(2) correspond to lower and upper bound of the prior distribution when the latter is uniform

I run the Sequential Monte Carlo procedure with 1,000 likelihood-tempering steps, which involve a swarm of 100,000 particles and one Random-Walk-Metropolis-Hastings step each. The proper identification of parameters is checked for each draw from the prior distributions[9] and the Metropolis-Hastings steps

[8] Standard deviations are expressed in percentage
[9] Prior distributions are involved in the initialization of the swarm of particles.

along the method delineated by Iskrev (2010). The results are displayed in Table 4. Overall, the estimates are pretty similar to those obtained by Thomas and Zanetti (2009). The cost-push shocks displays virtually no autocorrelation, in opposition to productivity and government-spending shocks. The interest-rate-smoothing parameter and the price stickiness present the same order of magnitude as well. However, the standard deviations of the shocks differ significantly from their estimates, with the notable exception of monetary policy shocks. The cost-push shock displays a standard deviation of 0.26 % instead of 10%, while productivity shocks and government spending shocks are half as volatile. The high volatility of government-spending shocks may seem surprising. As a matter of fact, the current specification of government-spending shock could embody all demand shocks that are not explicitly modelled here but have a sizable effect on output. This may include shocks on investment, on exports or on prices of imported products for example. This problem vanishes when investment, capital, risk-aversion and habit formation are introduced in the same manner as Gertler et al. (2008). I prefer to display a simple model to insist on the economic phenomena specific to dual labour markets, which are relatively new in the DSGE literature. Interestingly, a marginal likelihood comparison between the classic Mortensen-Pissarides model and the current dual specification elects the latter as the most probable with respect to the data.

References

Addison, J.T., Teixeira, P., Grunau, P., Bellmann, L., 2019. Worker representation and temporary employment in germany: The deployment and extent of fixed-term contracts and temporary agency work. Journal of Participation and Employee Ownership 2, 24–46.

Albert, C., García-Serrano, C., Hernanz, V., 2005. Firm-provided training and temporary contracts. Spanish Economic Review 7, 67–88. URL: https://doi.org/10.1007/s10108-004-0087-1, doi:10.1007/s10108-004-0087-1.

Andolfatto, D., 1996. Business cycles and labor-market search. The american economic review , 112–132.

Arulampalam, W., Booth, A.L., 1998. Training and labour market flexibility: Is there a trade-off? British Journal of Industrial Relations 36, 521–536. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-8543.00106, doi:10.1111/1467-8543.00106, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/1467-8543.00106.

Arulampalam, W., Booth, A.L., Bryan, M.L., 2004. Training in Europe. Journal of the European Economic Association 2, 346–360. URL: https://doi.org/10.1162/154247604323068041, doi:10.1162/154247604323068041, arXiv:http://oup.prod.sis.lan/jeea/article-pdf/2/2-3/346/10317643/jeea0346.pdf.

Bentolila, S., Dolado, J.J., Jimeno, J.F., 2012. Reforming an insider-outsider labor market: the spanish experience. IZA Journal of European Labor Studies 1, 4.

Burda, M., Wyplosz, C., 1994. Gross worker and job flows in europe. European economic review 38, 1287–1315.

Cahuc, P., Charlot, O., Malherbet, F., 2016. Explaining the spread of temporary jobs and its impact on labor turnover. International Economic Review 57, 533–572. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/iere.12167, doi:10.1111/iere.12167, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/iere.12167.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of monetary Economics 12, 383–398.

Canova, F., 2014. Bridging dsge models and the raw data. Journal of Monetary Economics 67, 1 – 15. URL: http://www.sciencedirect.com/science/article/pii/S030439321400083X, doi:https://doi.org/10.1016/j.jmoneco.2014.06.003.

Canova, F., Sala, L., 2009. Back to square one: Identification issues in dsge models. Journal of Monetary Economics 56, 431 – 449. URL: http://www.sciencedirect.com/science/article/pii/S0304393209000439, doi:https://doi.org/10.1016/j.jmoneco.2009.03.014.

Cutuli, G., Guetto, R., 2012. Fixed-Term Contracts, Economic Conjuncture, and Training Opportunities: A Comparative Analysis Across European Labour Markets. European Sociological Review 29, 616–629. URL: https://doi.org/10.1093/esr/jcs011, doi:10.1093/esr/jcs011, arXiv:http://oup.prod.sis.lan/esr/article-pdf/29/3/616/1270854/jcs011.pdf.

Dares, 2018. Cdd, cdi : comment évoluent les embauches et les ruptures depuis 25 ans ? Dares analyses .

Druant, M., Fabiani, S., Kezdi, G., Lamo, A., Martins, F., Sabbatini, R., 2012. Firms' price and wage adjustment in europe: Survey evidence on nominal stickiness. Labour Economics 19, 772 – 782. URL: http://www.sciencedirect.com/science/article/pii/S0927537112000267, doi:https://doi.org/10.1016/j.labeco.2012.03.007. special Section on: Price, Wage and Employment Adjustments in 2007-2008 and Some Inferences for the Current European Crisis.

Elsby, M.W., Hobijn, B., Şahin, A., 2015. On the importance of the participation margin for labor market fluctuations. Journal of Monetary Economics 72, 64–82.

Felgueroso, F., García-Pérez, J.I., Jansen, M., Troncoso-Ponce, D., 2017. Recent trends in the use of temporary contracts in Spain. Studies on the Spanish Economy eee2017-25. FEDEA. URL: https://ideas.repec.org/p/fda/fdaeee/eee2017-25.html.

Filippo, F., 2011. Trend Agnostic One-Step Estimation of DSGE Models. The B.E. Journal of Macroeconomics 11, 1–36. URL: https://ideas.repec.org/a/bpj/bejmac/v11y2011i1n25.html.

Fontaine, F., Malherbet, F., 2016. CDD vs CDI: les effets d'un dualisme contractuel. Presses de Sciences Po.

Fontaine, I., 2016. French unemployment dynamics: a "three-state" approach. Revue d'économie politique 126, 835–869.

Gertler, M., SALA, L., TRIGARI, A., 2008. An estimated monetary dsge model with unemployment and staggered nominal wage bargaining. Journal of Money, Credit and Banking 40, 1713–1764. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1538-4616.2008.00180.x, doi:10.1111/j.1538-4616.2008.00180.x, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1538-4616.2008.00180.x.

Hagedorn, M., Manovskii, I., 2008. The cyclical behavior of equilibrium unemployment and vacancies revisited. American Economic Review 98, 1692–1706. URL: http://www.aeaweb.org/articles?id=10.1257/aer.98.4.1692, doi:10.1257/aer.98.4.1692.

Hagen, T., 2002. Do temporary workers receive risk premiums? assessing the wage effects of fixed–term contracts in west germany by a matching estimator compared with parametric approaches. LABOUR 16, 667–705. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-9914.00212, doi:10.1111/1467-9914.00212, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/1467-9914.00212.

Herbst, E., Schorfheide, F., 2016. Bayesian Estimation of DSGE Models. 1 ed., Princeton University Press. URL: https://EconPapers.repec.org/RePEc:pup:pbooks:10612.

Iskrev, N., 2010. Local identification in dsge models. Journal of Monetary Economics 57, 189 – 202. URL: http://www.sciencedirect.com/science/article/pii/S0304393209001883, doi:https://doi.org/10.1016/j.jmoneco.2009.12.007.

Iskrev, N., et al., 2010. Evaluating the strength of identification in dsge models. an a priori approach, in: Society for Economic Dynamics, 2010 Meeting Papers.

Jolivet, G., Postel-Vinay, F., Robin, J.M., 2006. The empirical content of the job search model: Labor mobility and wage distributions in Europe and the US. European Economic Review 50, 877–907. URL: https://hal.archives-ouvertes.fr/hal-00279066, doi:10.1016/j.euroecorev.2006.02.005.

Kramarz, F., Michaud, M.L., 2010. The shape of hiring and separation costs in france. Labour Economics 17, 27–37.

Mertens, A., Gash, V., McGinnity, F., 2007. The cost of flexibility at the margin. comparing the wage penalty for fixed-term contracts in germany and spain using quantile regression. LABOUR 21, 637–666. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9914.2007.00396.x, doi:10.1111/j.1467-9914.2007.00396.x, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-9914.2007.00396.x.

Merz, M., 1995. Search in the labor market and the real business cycle. Journal of monetary Economics 36, 269–300.

Pfeifer, C., 2012. Fixed-term contracts and wages revisited using linked employer-employee data. Journal for Labour Market Research 45, 171–183. URL: https://doi.org/10.1007/s12651-012-0107-9, doi:10.1007/s12651-012-0107-9.

Pfeifer, C., 2014. A note on dual internal labor markets and wages of temporary workers: Evidence from linked-employer-employee data. Journal of Labor Research 35, 133–142. URL: https://doi.org/10.1007/s12122-013-9173-1, doi:10.1007/s12122-013-9173-1.

Rion, N., 2019. Waiting for the Prince Charming: Fixed-Term Contracts as Stopgaps. URL: https://halshs.archives-ouvertes.fr/halshs-02331887. working paper or preprint.

Saint-Paul, G., 1996. Dual Labor Markets: A Macroeconomic Perspective. MIT Press. URL: https://books.google.fr/books?id=DVsTIar1iqwC.

Thomas, C., Zanetti, F., 2009. Labor market reform and price stability: An application to the euro area. Journal of Monetary Economics 56, 885–899.

Yun, T., 1996. Nominal price rigidity, money supply endogeneity, and business cycles. Journal of monetary Economics 37, 345–370.

A Detailed Calculations

A.1 Nash-bargaining joint surpluses

Using the different definitions of surpluses with the free-entry condition -namely equations (6), (8), (14) and (22) - I get

$$
\begin{aligned}
S_t^p\left(z_t\right)= & z_t-U_t+F_t-\mathbb{E}_t \beta_{t, t+1}(1-s) F_{t+1} \\
& +\mathbb{E}_t \beta_{t, t+1}\left\{(1-s) \int_{z_{t+1}^p}^{+\infty} S_{t+1}^p(z) d G(z)+\left(1-\xi_{t+1}\right) U_{t+1}+\xi_{t+1} \widehat{U_{t+1}}\right\}
\end{aligned} \tag{46}
$$

Meanwhile, the definition of $\widehat{U_t}$ (18) yields

$$
\widehat{U_t}=U_t+p\left(\theta_t\right)\left(\int_{H_t^p}\left(W_t^{0, p}(z)-U_t\right) d G(z)+\int_{H_t^f}\left(W_t^f(z)-U_t\right) d G(z)\right) \tag{47}
$$

Nash-sharing rules (20) imply that

$$
\widehat{U_t}=U_t+\frac{\eta}{1-\eta} p\left(\theta_t\right)\left(\int_{H_t^p} J_t^{0, p}(z) d G(z)+\int_{H_t^f} J_t^f(z) d G(z)\right) \tag{48}
$$

But (13) and (14) imply that

$$
\int_{H_t^p} J_t^{0, p}(z) d G(z)+\int_{H_t^f} J_t^f(z) d G(z)=\frac{\gamma}{q\left(\theta_t\right)} \tag{49}
$$

Since $p\left(\theta_t\right)=\theta_t q\left(\theta_t\right)$, the definition of $\widehat{U_t}$ boils down to

$$
\widehat{U_t}=U_t+\frac{\eta \gamma \theta_t}{1-\eta} \tag{50}
$$

Consequently, the present discounted value of unemployment equals

$$
U_t=b+\mathbb{E}_t \beta_{t, t+1} U_{t+1}+\mathbb{E}_t \beta_{t, t+1} \frac{\eta \gamma \theta_{t+1}}{1-\eta} \tag{51}
$$

Reintroducing (50) and (51) into (25) leads to the following expression for the surplus of continuing permanent contracts

$$
\begin{aligned}
S_t^p\left(z_t\right)= & A_t z_t \phi_t-b+F_t-\mathbb{E}_t \beta_{t, t+1}(1-s) F_{t+1}-\mathbb{E}_t \beta_{t, t+1}\left(1-\xi_{t+1}\right) \frac{\eta \gamma \theta_{t+1}}{1-\eta} \\
& +\mathbb{E}_t \beta_{t, t+1}(1-s) \int_{z_{t+1}^p}^{+\infty} S_{t+1}^p(z) d G(z)
\end{aligned} \tag{52}
$$

Following the same steps, I find that

$$
S_t^{0, p}\left(z_t\right)=S_t^p\left(z_t\right)-F_t \tag{53}
$$

As for temporary contracts, equations (11), (12) and (14) boil down to

$$
S_t^f\left(z_t\right)=\rho A_t z_t \phi_t-U_t+\mathbb{E}_t \beta_{t, t+1}\left\{(1-\delta) \int S_{t+1}^f(z) d G(z)+(1-\delta) U_{t+1}+\delta \widehat{U_{t+1}}\right\} \tag{54}
$$

Using (50) and (51), the surplus of temporary contracts is

$$S_t^f(z_t) = \rho A_t z_t \phi_t - b - \mathbb{E}_t \beta_{t,t+1}(1-\delta)\frac{\eta\gamma\theta_{t+1}}{1-\eta} + \mathbb{E}_t \beta_{t,t+1}(1-\delta)\left\{\int S_{t+1}^f(z)dG(z)\right\} \tag{55}$$

Notice that $\partial S_t^p/\partial z_t = A_t\phi_t$ and $\partial S_t^f/\partial z_t = \rho A_t \phi_t$ Moreover, $S_t^p(z_t^p) = 0$ and $S_t^f\left(z_t^f\right) = 0$ by definition. Thus, integration by parts lead to the following expressions for continuing permanent matches and temporary matches

$$\begin{aligned} S_t^p(z_t) &= A_t z_t \phi_t - b + F_t - \mathbb{E}_t \beta_{t,t+1}(1-s)F_{t+1} - \mathbb{E}_t \beta_{t,t+1}\left(1-\xi_{t+1}\right)\frac{\eta\gamma\theta_{t+1}}{1-\eta} \\ &\quad + \mathbb{E}_t \beta_{t,t+1}(1-s)A_{t+1}\phi_{t+1}\int_{z_{t+1}^p}^{+\infty}(1-G(z))\,dG(z) \end{aligned} \tag{56}$$

$$S_t^f(z_t) = \rho A_t z_t \phi_t - b - \mathbb{E}_t \beta_{t,t+1}(1-\delta)\frac{\eta\gamma\theta_{t+1}}{1-\eta} + \rho\mathbb{E}_t \beta_{t,t+1}(1-\delta)A_{t+1}\phi_{t+1}\left(\mathbb{E}z - z_{t+1}^f\right) \tag{57}$$

A.2 Steady-state equations

Given the parameters $\beta, \sigma, \eta, \sigma_z, \epsilon, F, b, , \delta, \rho, m, \gamma$, we need to derive the steady-state values of the variables $R, c, Y, n^p, n^f, u, \Delta, z^p, z^c, z^f, z^*, \theta, \phi, v, \pi, A, \mu, g$. Some of them can be directly computed through the following equations

$$\begin{aligned} R &= 1/\beta \\ \mu &= 1/\phi \\ \phi &= \frac{\epsilon - 1}{\epsilon} \\ P &= P^* = \Delta = 1 \\ A &= 1 \end{aligned}$$

Given (z^p, θ), $\left(q, \xi, z^f, z^c, z^*\right)$ can be directly derived.

$$\begin{aligned} q &= m\theta^{-\sigma} \\ \xi &= s + (1-s)G\left(z^p\right) \\ z^c &= z^p + \frac{F}{\phi} \\ z^f &= \frac{b + \beta(1-\delta)\left(\frac{\eta\gamma\theta}{1-\eta} - \rho\phi\mathbb{E}z\right)}{\rho\phi(1-\beta(1-\delta))} \\ z^* &= \frac{z^c - \rho z^f}{1-\rho} \end{aligned}$$

It is then possible to solve the following system in (z^p, θ).

$$\begin{aligned} &\phi z^p + [1-(1-s)\beta]\,F + (1-s)\beta\phi\int_{z^p}^{+\infty}(1-G(z))\,dz = b + \beta\left(1-\xi\right)\frac{\eta\gamma\theta}{1-\eta} \\ &\frac{\gamma}{(1-\eta)\phi q\left(\theta\right)} = \int_{z^*}^{+\infty}\left[1-G(z)\right]dz + \rho\int_{z^f}^{z^*}\left[1-G(z)\right]dz \end{aligned}$$

At this point, the values $\left(z^p, \theta, q, \xi, z^f, z^c, z^*\right)$ are known. One can compute right away p, μ^p and μ^f.

$$
\begin{aligned}
p &= \theta q \\
\mu^p &= p\left(1 - G\left(z^*\right)\right) \\
\mu^f &= p\left(G\left(z^*\right) - G\left(z^f\right)\right)
\end{aligned}
$$

Pinpointing $\left(n^p, n^f, u, e\right)$ involves the following linear system.

$$
\begin{aligned}
& n^p + n^f + u = 1 \\
& e = u + \delta n^f + \xi n^p \\
& n^p = \left(1 - \xi\right) n^p + \mu^p e \\
& n^f = \left(1 - \delta\right) n^f + \mu^f e
\end{aligned}
$$

Solving this system yields

$$
\begin{aligned}
n^p &= \frac{\delta \mu^p}{\xi(1-\delta)\mu^f + \delta(1-\xi)\mu^p + \xi\delta} \\
n^f &= \frac{\xi \mu^f}{\xi(1-\delta)\mu^f + \delta(1-\xi)\mu^p + \xi\delta} \\
u &= 1 - n^p - n^f \\
e &= u + \xi n^p + \delta n^f
\end{aligned}
$$

Now, the steady-state values v, Y, g and c can be obtained.

$$
\begin{aligned}
& v = \theta e \\
& Y = \left(1 - \xi\right) \mathbb{E}_z\left[z \mid z \geq z^p\right] n^p + \mu^p \mathbb{E}_z\left[z \mid z \geq z^*\right] e \\
& \quad + \rho\left(1 - \delta\right) \mathbb{E}_z\left[z\right] n^f + \rho \mu^f \mathbb{E}_z\left[z \mid z^* \geq z \geq z^f\right] e \\
& g = Y\left(\frac{g}{Y}\right) \\
& c = Y - g - \gamma v
\end{aligned}
$$

A.3 Log-linearization

The Euler equation can be derived from (1)-(2).

$$u'(c_t) = \beta R_t \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} u'(c_{t+1}) \right]$$

which becomes

$$\widehat{c_t} = \widehat{E_t c_{t+1}} - \left[-\frac{cu''}{u} \right]^{-1} \left(\widehat{R_t} - \widehat{E_t \pi_{t+1}} \right) \tag{58}$$

where $\pi_t = P_t / P_{t-1}$ denotes inflation.

The definition of price level dynamics (45) can be log-linearized as

$$\widehat{\pi_t} = (1 - \psi) \left(\widehat{P_t^*} - \widehat{P_{t-1}} \right) \tag{59}$$

The retailers' price-setting equation (5) becomes

$$\widehat{P_t^*} = (1 - \beta\psi) \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\psi)^k \left(\widehat{P_{t+k}} + \widehat{\mu_{t+k}} + \widehat{\phi_{t+k}} \right) \tag{60}$$

Substracting $\widehat{P_{t-1}}$ on each side of this equation, we get

$$\begin{aligned}
\widehat{P_t^*} - \widehat{P_{t-1}} &= (1 - \beta\psi) \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\psi)^k \left(\widehat{P_{t+k}} - \widehat{P_{t-1}} + \widehat{\mu_{t+k}} + \widehat{\phi_{t+k}} \right) && (61) \\
&= \sum_{k=0}^{+\infty} (\beta\psi)^k \mathbb{E}_t \widehat{\pi_{t+k}} + (1 - \beta\psi) \sum_{k=0}^{+\infty} (\beta\psi)^k \mathbb{E}_t \left(\widehat{\mu_{t+k}} + \widehat{\phi_{t+k}} \right) && (62) \\
&= \beta\psi \left(\mathbb{E}_t \widehat{P_{t+1}^*} - \widehat{P_t} \right) + (1 - \beta\psi) \left(\widehat{\mu_t} + \widehat{\phi_t} \right) + \widehat{\pi_t} && (63)
\end{aligned}$$

Using (59), we get

$$\widehat{\pi_t} = \beta \mathbb{E}_t \widehat{\pi_{t+1}} + \kappa \left(\widehat{\mu_t} + \widehat{\phi_t} \right) \tag{64}$$

where $\kappa = (1 - \beta\psi)(1 - \psi)/\psi$.

The log-linearization of exogenous processes yields

$$\begin{aligned}
\widehat{A_t} &= \rho_A \widehat{A_{t-1}} + \epsilon_t^A \\
\widehat{\mu_t} &= \rho_\mu \widehat{\mu_{t-1}} + \epsilon_t^\mu \\
\widehat{g_t} &= \rho_g \widehat{g_{t-1}} + \epsilon_t^g
\end{aligned}$$

The other log-linearizations of equations (29)-(30), (34) and (37)-(43) yield

$$
\begin{aligned}
&A\phi z^p\left(\widehat{A_t}+\widehat{z_t^p}+\widehat{\phi_t}\right)-\left(A\phi z^p+F-b\right)\left(c_t-\mathbb{E}_t\widehat{c_{t+1}}\right)\\
&+\beta(1-s)A\phi\int_{z^p}^{+\infty}\left(1-G(z)\right)dz\left(\mathbb{E}_t\widehat{A_{t+1}}+\mathbb{E}_t\widehat{\phi_{t+1}}\right)\\
&-\beta(1-\xi)\frac{\eta\gamma\theta}{1-\eta}\mathbb{E}_t\theta_{t+1}+\beta(1-s)z^p\left(-A\phi\left(1-G\left(z^p\right)\right)+\frac{\eta\gamma\theta}{1-\eta}g\left(z^p\right)\right)\mathbb{E}_t\widehat{z_{t+1}^p}=0\\
&\rho A\phi z^f\left(\widehat{A_t}+\widehat{z_t^f}+\widehat{\phi_t}\right)+\rho\beta(1-\delta)A\phi\left(\mathbb{E}z-z^f\right)\left(\mathbb{E}_t\widehat{A_{t+1}}+\mathbb{E}_t\widehat{\phi_{t+1}}\right)\\
&-\left(\rho A\phi z^f-b\right)\left(\widehat{c_t}-\mathbb{E}_t\widehat{c_{t+1}}\right)-\rho\beta(1-\delta)A\phi z^f\mathbb{E}_t\widehat{z_{t+1}^f}-\beta(1-\delta)\frac{\eta\gamma\theta}{1-\eta}\mathbb{E}_t\widehat{\theta_{t+1}}=0\\
&(1-\rho)z^*\widehat{z_t^*}-z^p\widehat{z_t^p}+\frac{F}{A\phi}\left(\widehat{A_t}+\widehat{\phi_t}\right)+\rho z^f\widehat{z_t^f}=0\\
&\frac{\gamma}{(1-\eta)\phi q(\theta)}\left(-\widehat{A_t}-\widehat{\phi_t}+\sigma\widehat{\theta_t}\right)+(1-\rho)\left(1-G\left(z^*\right)\right)z^*\widehat{z_t^*}+\rho\left(1-G\left(z^f\right)\right)z^f\widehat{z_t^f}=0\\
&\widehat{R_t}=\rho_R\widehat{R_{t-1}}+(1-\rho_R)\left[\rho_\pi\mathbb{E}_t\widehat{\pi_{t+1}}+\rho_y\widehat{Y_t}\right]+\epsilon_t^m\\
&\widehat{v_t}=\widehat{\theta_t}-\frac{\left(1-\xi^p\right)\theta n^p}{v}\widehat{n_{t-1}^p}-\frac{(1-\delta)\theta n^f}{v}\widehat{n_{t-1}^f}+\frac{(1-s)\theta g\left(z^p\right)z^p n^p}{v}\widehat{z_t^p}\\
&\widehat{n_t^p}=\left(1-\xi^p\right)\widehat{n_{t-1}^p}-(1-s)z^p g\left(z^p\right)\widehat{z_t^p}+\frac{\left(1-G\left(z^*\right)\right)q(\theta)v}{n^p}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)-\frac{z^*g\left(z^*\right)q(\theta)v}{n^p}\widehat{z_t^*}\\
&\widehat{n_t^f}=(1-\delta)\widehat{n_{t-1}^f}+\frac{\left(G\left(z^*\right)-G\left(z^f\right)\right)q(\theta)v}{n^f}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)+\frac{q(\theta)v}{n^f}\left(z^*g\left(z^*\right)\widehat{z_t^*}-z^f g\left(z^f\right)\widehat{z_t^f}\right)\\
&\widehat{Y_t}=\frac{c}{Y}\widehat{c_t}+\frac{g}{Y}\widehat{g_t}+\frac{\gamma v}{Y}\widehat{v_t}\\
&\widehat{Y_t}=\widehat{A_t}+\frac{(1-s)\left(\int_{z^p}^{+\infty}zg(z)dz\right)n^p}{Y}\widehat{n_{t-1}^p}+\frac{\rho\left(1-\delta\right)\mathbb{E}zn^f}{Y}\widehat{n_{t-1}^f}-\frac{(1-s)n^p\left(z^p\right)^2g\left(z^p\right)}{Y}\widehat{z_t^p}\\
&-(1-\rho)\left(z^*\right)^2g\left(z^*\right)\frac{vq(\theta)}{Y}\widehat{z_t^*}-\rho\left(z^f\right)^2g\left(z^f\right)\frac{vq(\theta)}{Y}\widehat{z_t^f}\\
&+\left(\int_{z^*}^{+\infty}zg(z)dz+\rho\int_{z^f}^{z^*}zg(z)dz\right)\frac{vq(\theta)}{Y}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)
\end{aligned}
$$

B Estimation

B.1 Calibration

The calibration of parameters is carried out as follows. A first block can be directly derived using tractable expressions.

$$
\begin{aligned}
\mu&=\frac{\epsilon}{\epsilon-1}\\
\phi&=\frac{1}{\mu}\\
u&=1-n\\
n^f&=n\left(\frac{n^f}{n}\right)\\
n^p&=n-n^f\\
s&=\xi\left(\frac{s}{\xi}\right)
\end{aligned}
$$

$$
\begin{aligned}
\delta &= \frac{\left(1-\left(\frac{n^f}{n}\right)\right)\left(\frac{\mu^f}{\mu^p+\mu^f}\right)}{\left(\frac{n^f}{n}\right)\left(1-\left(\frac{\mu^f}{\mu^p+\mu^f}\right)\right)}\xi \\
e &= u+\delta n^f+\xi n^p \\
\mu^p &= \frac{\xi n^p}{e} \\
\mu^f &= \frac{\delta n^f}{e}
\end{aligned}
$$

A second block needs some numerical solver to be pinpointed. Given $\left(z^p, F, z^*, z^f\right)$, it is possible to derive tractable expressions for $(z^c, \theta, \rho, \gamma)$ through the following calculations

$$
\begin{aligned}
z^c &= z^p+\frac{F}{\phi} \\
p &= \frac{\mu^p}{1-G\left(z^*\right)} \\
\theta &= \frac{p}{q} \\
\rho &= \frac{z^*-z^c}{z^*-z^f} \\
\gamma &= (1-\eta)\phi q\left(\int_{z^*}^{+\infty}(1-G(z))\,dz+\rho\int_{z^f}^{z^*}(1-G(z))\,dz\right)
\end{aligned}
$$

I also define the following intermediate variables

$$
\begin{aligned}
U^p &= \phi z^p+(1-\beta(1-s))F+\beta(1-s)\phi\int_{z^p}^{+\infty}(1-G(z))\,dz \\
U^f &= \rho\phi\left((1-\beta(1-\delta))z^f+\beta(1-\delta)\mathbb{E}z\right) \\
b^p &= U^p-\beta(1-\xi)\frac{\eta\gamma\theta}{1-\eta} \\
b^f &= U^f-\beta(1-\delta)\frac{\eta\gamma\theta}{1-\eta} \\
\overline{w^p} &= \eta\phi\left((1-\xi)\mathbb{E}\left[z \mid z\geq z^p\right]+\xi\mathbb{E}\left[z \mid z\geq z^*\right]\right)+(1-\xi)\eta F-\eta\beta(1-s)F+(1-\eta)U^p
\end{aligned}
$$

Using the latter expressions, I find $\left(z^p, F, z^*, z^f\right)$ by numerically solving the system

$$
\begin{aligned}
\xi &= s+(1-s)G\left(z^p\right) \\
\left(\frac{\mu^f}{\mu^p+\mu^f}\right) &= \frac{G\left(z^*\right)-G\left(z^f\right)}{1-G\left(z^f\right)} \\
F &= \overline{w^p}\left(\frac{F}{\overline{\overline{w^p}}}\right) \\
b^p &= b^f
\end{aligned}
$$

It is then possible to compute m and b as follows

$$
m = q\theta^\sigma
$$

$$b = b^p$$

The parameters $(F, b, s, \delta, \rho, m, \gamma)$ are all determined at this point

B.2 Data

Observable	Unique Identifier
GDP at constant prices	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N
GDP deflator - Quarterly growth rate	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.IX.D.N
Nominal interest rates	FM.M.U2.EUR.4F.MM.EONIA.HSTA
Employment	ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N

Table 5: Times series from ECB Data Warehouse