Waiting for the Prince Charming: Fixed-Term Contracts as Stopgaps*

Normann Rion†

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Abstract

In this paper, I build a simple Mortensen-Pissarides model embedding a dual labor market. I derive conditions for the existence of an equilibrium with both protected open-ended contracts and exogenously short fixed-term contracts. I also study dynamics after a change in firing costs. Fixed-term contracts play the role of fillers while open-ended contracts are used to lock up most productive matches. High firing costs favor the emergence of a dual equilibrium and encourage the resort to fixed-term employment in job creation. This substitution scheme is intertwined with a general-equilibrium effect. Open-ended contracts represent the bulk of employed workers; higher firing costs reduces aggregate job destruction, which pushes down unemployment and in turn reduces job creation flows through fixed-term contracts. I calibrate the model on the French labor market. Policy experiments demonstrate that there is no joint gain in employment and social welfare through changes on firing costs around the baseline economy. Welfare and employment improving policies consist in large cuts in firing costs, where fixed-term employment eventually disappears. Increases in firing costs within a dual labor market lead to a sluggish adjustment, while large cuts in firing costs lead to a quicker one. Still, the adjustment time is highly non-monotonous between these two extremes. Uncertainty over firing costs significantly strengthens fixed-term employment on behalf of open-ended employment.

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1 Introduction

Since the end of the 1980s and the introduction of fixed-term contracts, the segmentation of European labor markets, opposing strongly protected open-ended contracts and short fixed-term contracts, has worsened. In France, fixed-term contracts represented more than 80 % of created jobs in 2018. Academics often point at firing costs as encouraging the resort to fixed-term contracts.

Should a dual labor market be reformed? Which post-reform dynamic behavior should we expect? To address these questions, I extend the classic model of Mortensen and Pissarides (1994) to

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†Le Mans Université (GAINS-TEPP, IRA) & Cepremap (ENS-Paris). E-mail: normann@dynare.org

include both open-ended and fixed-term contracts. As in the latter, firms and workers meet following a matching function and negotiate wages through Nash bargaining. Matches are heterogeneous in productivity and face i.i.d productivity shocks. When a firm-worker pair forms, it draws a productivity from a given distribution and assesses accordingly whether an open-ended or a fixed-term contract is preferable. The latter differ only over one dimension; open-ended contracts stipulate a red-tape firing cost, while fixed-term contracts have a high exogenous job destruction rate. I calibrate the model to the French labor market.

My contribution includes theoretical and numerical insights. I design the model to be the most tractable possible allowing an endogenous choice between open-ended contracts and fixed-term contracts at the hiring stage, which facilitates its integration into more complex frameworks, as is done in Rion (2020). In numerical terms, I analyze the dynamic response of the dual labor market following reforms on employment protection legislation, which, as far as I know, has not been done yet. Overall, the resulting model provides rich theoretical insights about the economic schemes at stake in a dual labor market. It also provides results on the desirability of employment protection legislation reforms in terms of welfare and employment. The numerical results I present must be taken with a grain of salt considering the assumptions made to ensure tractability: productivity shocks are i.i.d, agents are risk neutral and there is no on-the-job search.

At the equilibrium, when a firm-worker pair meets and draws a high productivity, the best way to maximize benefits consists in locking up the pair through an open-ended contract. Otherwise, agents can commit to a short productive relationship through a fixed-term contract or go back to searching for a better match on the labor market. The meeting process being costly, agents opt for a fixed-term contract if the productivity draw is not too disappointing. Thus, fixed-term contracts provide a productive interlude, while enabling to go back to seeking a high-productivity match before long: fixed-term contracts act as fillers before meeting the Prince Charming.

I find that two opposed effects mix up to shape the behavior of the labor market with respect to firing costs. The first effect is a substitution effect towards fixed-term employment as firing costs increase. Higher firing costs make open-ended contracts more rigid and fosters the resort to less productive fixed-term contracts for the sake of flexibility. Thus, higher firing costs provide an incentive to hire more fixed-term contracts and favor the emergence of an equilibrium with coexisting fixed-term and open-ended contracts. The second effect is a general-equilibrium effect. Higher firing costs reduce open-ended job destruction. Since most workers operate under open-ended contracts, aggregate job destruction decreases and so does unemployment. In turn, the depressed unemployment rate reduces fixed-term job creation flows.

As for social welfare and employment, I find that there is no free lunch when the dual nature of the labor market is preserved: a benevolent planner cannot both decrease unemployment and improve welfare around the baseline economy. The optimal policy consists in a large cut in firing costs, which both increases welfare and decreases unemployment. If the cut in firing costs is large enough, fixed-term employment is no longer relevant and only open-ended employment remains at the equilibrium. Consequently, this model supports a transition towards a unique open-ended contract, a reform the literature has extensively tackled[1].

In terms of dynamics, an increase in firing costs with respect to the baseline calibration leads to a several-year long adjustment. Indeed, flows are reduced on the open-ended side of the labor market, which tends to slow down transitions. Conversely, a strong decrease in firing costs enhances open-ended job creation and destruction flows, which reduces the adjustment duration. Between these two endpoints, the behavior of the adjustment time after a moderate cut in firing posts is non-monotonous and intricate. Transitions toward a unique-contract equilibrium with a cut in firing

[1]See Lepage-Saucier et al. (2013) and Amable (2014) for reviews

costs end by 2 years. Taking account of frequent marginal employment protection legislation reforms — one every 7 months on average in France — I extend the model to include uncertainty with respect to firing costs. I find that such a policy risk significantly bolsters fixed-term employment on behalf of open-ended employment and pushes up unemployment.

My paper relates to three strands in the literature. First, it contributes to explain the dynamic interaction between fixed-term and open-ended contracts and labor market institutions as Saint-Paul (1996) and Dolado et al. (2002) do. Many papers assume that the new firm-worker pairs' types of contracts is primarily dictated by an exogenous assignment probability[2]. The Lucas critique applies here, as changing firing costs does not lead to a change in the contractual assignment probability. The contractual assignment probability strongly constrains the share of fixed-term contracts in job creation and, thereof, the composition of job creation and job destruction flows. I leave more leeway to the new matches for the choice of their contract, as the only criterion is the expected surplus of the match. In this manner, the Lucas critique has less ground left to undermine my results.

Second, it delineates the mechanisms underpinning the choice between fixed-term and open-ended contracts[3]. Smith (2007) has first highlighted the role of fixed-term contracts as stopgaps. In his stock-flow matching model, firms hire poorly productive workers on limited duration to be ready to hire highly productive workers when they appear. Its framework is very different, though; search is directed and labor market institutions are not explicitly modeled.

Thirdly, my paper contributes to the literature assessing policy measures using structural models of the labor market. The closest papers in that regard are Dolado et al. (2018) and Cahuc et al. (2019).

Cahuc et al. (2016, 2019) build a matching model where firm-worker pairs face heterogeneous arrival rates in adverse productivity shocks. Firms-worker pairs opt for fixed-term contracts when adverse productivity shocks are frequent, whereas open-ended contracts are beneficial when adverse productivity shocks are unusual. Expiring fixed-term contracts can be converted into open-ended contracts. In addition, firm-worker pairs optimize hired fixed-term contracts' duration. Cahuc et al. (2016) calibrate the model on French data and find that higher firing costs do not change aggregate employment much but dramatically increase turnover on the fixed-term side of the market . Cahuc et al. (2019) extends the model of Cahuc et al. (2016) and assume that the resort to fixed-term contracts is taxed depending on the duration of contracts. Taxes may be refunded if the fixed-term contract is converted into an open-ended contract one at expiry. The authors estimate the model on French data and find a detrimental effect of taxes on the duration of fixed-term contracts. My paper departs from Cahuc et al. (2016, 2019) in several aspects; new matches are heterogeneous in productivity and not in productivity shock arrival rate. Moreover, I consider transitions between states while Cahuc et al. (2016, 2019) limit to steady-state analyses.

Siassi et al. (2015) is the closest paper in terms of ambitions when it comes to studying post-reform dynamics, but it is rather a complement than a substitute. Siassi et al. (2015) builds a discrete-time stochastic life-cycle model where workers are risk-averse. One-job firms and workers meet according to a standard matching function. When matches split, firms pay severance payments to the workers that enables them to smooth consumption while unemployed. The severance payment is a function of the worker's tenure. The model is calibrated on Spanish data with the 2008 severance payment scheme. The authors derive the optimal linear severance scheme with respect new entrants' utility. They find that the actual severance payment scheme is too generous and increases unemployment. The authors then derive transitions from the actual severance payment scheme towards the optimal one.

[2]See Cahuc and Postel-Vinay (2002), Bentolila et al. (2012), Sala and Silva (2009), Sala et al. (2012)

[3]See Cahuc et al. (2016) and the general introduction for a review

I depart from Siassi et al. (2015) in tree important dimensions. First, I model labor both fixed-term and open-ended contracts explicitly. In Siassi et al. (2015), the workers with low-tenure workers stand for fixed-term contracts while high-tenure workers embody open-ended contracts. This approach overlooks the rigidity of fixed-term contracts before their stipulated end date. Secondly, I focus more on theoretical mechanisms than numerical results. Siassi et al. (2015) uses computer-intensive methods to derive equilibria and transitions. The transition paths I derive have an analytic expression, which provides a transparent appraisal of the phenomena at stake. Thirdly, I consider red-tape firing costs instead of severance payments. Severance payments are a pure transfer between the firm and the worker. Thus, they do not impact jointly efficient job destruction decisions[4]. Severance payments indirectly impact job creation through the stronger workers' bargaining position and, thus, higher wages. Note that the impact of severance payments can be offset through a bargaining process with full commitment[5]. On the contrary, red-tape firing costs impact directly the surplus of open-ended matches and distort job destruction decisions. They also push up wages, which alters job creation incentives. Unlike severance payments, a full-commitment bargaining of wages before the hiring decision cannot offset the distortion red-tape firing costs induce. Beyond the fact that red-tape firing costs have an allocative role in contrast with severance payments, red-tape firing costs are substantial. Kramarz and Michaud (2010) shows that the sole law-prescribed severance payment severely understates the extent of termination costs.

The paper is structured as follows. Section 2 describes the model and derives the main theoretical results. In Section 3, I calibrate the model on French data, study employment protection legislation reforms, the associated post-reform dynamics and the impact of firing-cost uncertainty. Section 4 concludes.

2 The model

In this model a-la Mortensen and Pissarides (1994), there are two continua of risk neutral firms and households and time is continuous. The interest rate is denoted r. Firms are numerous and can either employ one worker or maintain a vacancy opened. They face i.i.d idiosyncratic productivity shocks that occur with a probability λ per unit of time and are drawn from a log-Normal distribution $log\ \mathcal{N}\left(0, \sigma_z^2\right)$ with cumulative distribution function G and support $(0, \infty)$. Our model mainly departs from the classic Mortensen-Pissarides framework by assuming that workers are either employed through open-ended contracts or fixed-term contracts.

The matching function is standard, with constant returns to scale over the number of vacancies v and the number of job seekers e. The number of matches per unit of time is denoted $m(e, v)$, while the job market tightness θ is classically set as the number of vacancies over the number of job seekers $\theta = v/e$. The constant-return-to-scale feature of the matching function entails that the job-meeting probability $p = m(e, v)/e$ and the seeker-meeting probability $q = m(e, v)/v$ only depend on the labor market tightness. q is a non-increasing function, whereas p is a non-decreasing one. Both q and p are convex. When paired with a worker, firms can hire through a fixed-term contract or hire through a open-ended contract. They are also able to resume searching for a worker if they are not satisfied with the productivity of the match.

As in the classic Mortensen-Pissarides model with Nash bargaining, the surplus of matches entirely determine hiring and firing decisions. Joint surpluses are defined as the sum of workers' and firms' surpluses. The surplus of a firm with a continuing open-ended contract with productivity z is

[4]Hiring and firing decisions are jointly efficient if firms and workers negotiate wages using Nash bargaining, which is the case in both the current paper and Siassi et al. (2015).

[5]Lazear (1990) detail this idea.

$J^p(z) - (V - F) = J^p(z) - V + F$, where $J^p(z)$ is the firm's surplus from a continuing open-ended contract with productivity z, V the firm's surplus from an unfilled vacancy and F is the firing cost. Interestingly, the firm's surplus for a new open-ended match with productivity z is $J_0^p(z) - V$, where $J_0^p(z)$ is the firm's surplus from a new open-ended contract with productivity z. Indeed, when a contact occurs between a firm and a worker, the firm's outside option does not include the payment of a firing cost if no contract is signed. The open-ended and fixed-term workers' surpluses are standard. We denote $W^p(z)$ the worker's surplus from a continuing open-ended contract with productivity z, $W_0^p(z)$ the worker's surplus from a new open-ended contract with productivity z, $W^f(z)$ the worker's surplus from a fixed-term contract with a productivity z and U the unemployed's surplus.

$$
\begin{aligned}
S^f(z) &= \left(J^f(z) - V\right) + \left(W^f(z) - U\right) \\
S^p(z) &= \left(J^p(z) - [V - F]\right) + \left(W^p(z) - U\right) \\
S_0^p(z) &= \left(J_0^p(z) - V\right) + \left(W_0^p(z) - U\right)
\end{aligned}
$$

Wages are determined following a Nash bargaining. I mark down the workers' bargaining power as η. It is common to both open-ended and fixed-term contracts. Indeed, analyzing the role of employment protection legislation implies shutting down all differences between open-ended and fixed-term contracts, with the exception of genuinely legal ones.

$$J^p(z) - (V - F) = (1 - \eta)\, S^p(z) \tag{2.0.1}$$

$$J_0^p(z) - V = (1 - \eta)\, S_0^p(z) \tag{2.0.2}$$

$$J^f(z) - V = (1 - \eta)\, S^f(z) \tag{2.0.3}$$

Nash-bargaining makes endogenous separations as well as hiring decisions jointly efficient. In other words, there is no conflict over the hiring and firing choices between firms and workers. The formulas above enable the computation of the workers' rents.

$$
\begin{aligned}
W^p(z) &= U + \frac{\eta}{1-\eta}\left(J^p(z) - V\right) + \frac{\eta}{1-\eta} F \\
W_0^p(z) &= U + \frac{\eta}{1-\eta}\left(J_0^p(z) - V\right) \\
W^f(z) &= U + \frac{\eta}{1-\eta}\left(J^f(z) - V\right)
\end{aligned}
$$

Workers appropriate a fraction of the firms' surplus because of the sunk hiring costs firms pawn. The matching procedure can be considered as a production function with two inputs: job seekers and vacancies. The involvement of firms through recruiting costs in this process before any production takes place generates a hold-up situation favoring workers at the moment of wage bargaining[6] and enables them to extract a rent. Moreover, continuing open-ended workers benefit from a supplementary rent $\eta F/(1-\eta)$ when compared with fixed-term workers and new open-ended workers. The firing cost pushes up the open-ended workers' bargaining position by enhancing the threat of a costly separation. The firm will reward the worker through the wage for avoiding the separation and its associated cost F. Thus, the firing cost influences labor market outcomes through two channels: as a pure firing tax and through wages. The firing cost is not involved in Nash bargaining for a new open-ended match since the worker is not yet an insider.

[6] See Grout (1984) for a thorough development of this question

2.1 The agents' value functions

Vacancies and unemployed workers A firm-worker contact occurs with probability $q(\theta)$ per unit of time. The cost of a vacancy is γ per unit of time regardless of the contract type. When the idiosyncratic productivity of the match reveals, the firm chooses between hiring the worker through an open-ended contract or a fixed-term contract and letting the worker go back into the unemployed's pool.

$$rV = -\gamma + q(\theta) \int \max \left[J_0^p(z) - V, J^f(z) - V, 0 \right] dG(z) \tag{2.1.1}$$

Firing costs only apply if the match is validated in the first place. The potentially ephemeral constitution of a match does not boil down to the payment of firing costs if immediate separation is preferable. Thus, the role of firing costs is not unrealistically exacerbated in job creation.

The unemployed workers' value function embeds the unemployment benefit b and the various possibilities stemming from the eventual contact with a firm, which occurs with probability $p(\theta)$ per unit of time.

$$rU = b + p(\theta) \int \max \left[W_0^p(z') - U, W^f(z') - U, 0 \right] dG(z') \tag{2.1.2}$$

Open-ended contracts The firm's capital value of a continuing open-ended match consists in an immediate profit from production z net of the worker's wage $w^p(z)$. When the match exogenously separates with probability s, the firm goes back to searching a worker without paying the firing cost. In real life, this probability is associated with events such as quits, retirements or probationary employment terminations. In this case, firms do not have to pay firing costs. The firm may also face a productivity shock with probability λ and assess whether it keeps or lays off the worker regarding the new idiosyncratic productivity of the match. Firms laying off workers because of adverse productivity shocks have to pay firing costs.

$$rJ^p(z) = z - w^p(z) + s(V - J^p(z)) + \lambda \int \left(max\left[J^p(z'), V - F \right] - J^p(z) \right) dG(z') \tag{2.1.3}$$

J^p being increasing in the idiosyncratic productivity of the match, there exists a job destruction margin for continuing open-ended contracts z^p defined as

$$J^p(z^p) = V - F$$

The subsequent probability of separation for open-ended contracts is

$$\xi = s + \lambda G(z^p) \tag{2.1.4}$$

The workers' value function when he is under a continuing open-ended contract firstly consists in the wage w^p. The different alternatives mentioned above apply.

$$rW^p(z) = w^p(z) + \lambda \int \left(\max\left[W^p(z'), U \right] - W^p(z) \right) dG(z') + s(U - W^p(z)) \tag{2.1.5}$$

The value function associated with new open-ended contracts is analogous. The only notable difference originates from a specific wage function w_0^p and the transformation into a continuing open-ended contract when a productivity shock occurs. The new productivity either entails a costly separation or a renegotiation of wages, which include the payment of firing costs as a possible outcome of the negotiation. Consequently, the reassessed relationship corresponds to a continuing open-ended contract.

$$rJ_0^p(z) = z - w_0^p(z) + \lambda \int \left(\max\left[J^p\left(z'\right), V - F\right] - J_0^p(z)\right) dG\left(z'\right) + s\left(V - J_0^p(z)\right) \tag{2.1.6}$$

$$rW_0^p(z) = w_0^p(z) + \lambda \int \max\left(\left[W^p\left(z'\right), U\right] - W_0^p(z)\right) dG\left(z'\right) + s\left(U - W_0^p(z)\right) \tag{2.1.7}$$

Note that new open-ended matches become continuing open-ended matches after the first productivity shock. It is counter-factual as the probationary employment period is legally prescribed. In France, as far as open-ended contracts are concerned, probationary employment lasts at most 4 months, 6 months and 8 months for employees, supervisors and executives respectively. Fixed-term contracts that last more than 6 months generally have a 1-month probationary employment. As for fixed-term contracts that last less than 6 months, probationary employment lasts at most 2 weeks. Probationary employment enables both the firm and the worker to terminate the contractual relationship at zero cost. Theoretically speaking, the end of a contract during the probationary employment needs not to be justified. However, it is subject to an advance notice and a worker may legally challenge it and claim for compensation if he suspects his employer's decision is not bound to his lack of skill. Economic reasons cannot justify a termination during probationary employment, for example.

A match in probationary employment learning about its disappointing productivity may choose to split. In my framework, new firm-worker pairs immediately learn about their productivity. The probationary employment period is skipped in this case. I include probationary employment terminations in the cost-less separation rate s. Introducing uncertainty over the productivity of the match and the associated learning mechanisms would go beyond the scope of this paper. Bucher (2010) and Faccini (2014) do so in a dual labor market.

Fixed-term contracts Similarly to open-ended contracts, the firm employing a fixed-term worker gets an immediate profit from a match with productivity z. It also pays a wage $w^f(z)$. The match may split if the stipulated termination date is reached, which happens with probability δ per unit of time. In this case, the firm gets back to the labor market and earns the net return $V - J^f(z)$. A productivity shock may also take place with probability λ per unit of time.

$$rJ^f(z) = z - w^f(z) + \lambda \int \left(J^f\left(z'\right) - J^f(z)\right) dG(z') + \delta\left(V - J^f(z)\right) \tag{2.1.8}$$

The fixed-term worker's value function embeds the wage $w^f(z)$ and the expectations about next-period outcomes. If the match is not separated, the worker expects to get the average worker's value from a fixed-term contract. If a split occurs, the worker returns searching for a job.

$$rW^f(z) = w^f(z) + \lambda \int \left(W^f\left(z'\right) - W^f(z)\right) dG\left(z'\right) + \delta\left(U - W^f(z)\right) \tag{2.1.9}$$

Note that fixed-term matches split because of expiry shocks only. The probability of resignation, retirement or probationary employment termination s does not apply here as it is not empirically relevant. According to Milin (2018), only 2 % of fixed-term contracts terminated before their stipulated date in 2017.

Two serious discrepancies between the model and the actual French labor code need to be highlighted. The desire for simplicity guides these two strong assumptions.

First, the duration of fixed-term contracts is exogenous. Endogenous separations for fixed-term contracts are not possible. At first view, a certain lack of realism must be pointed at here. Indeed, according to the French legislation, a firm may fire a fixed-term worker before the stipulated date through the payment of a firing cost. Nevertheless, this makes the analysis vainly cumbersome. The main trade-off between fixed-term and open-ended contracts is set between long-lasting rigid open-ended contracts and very short fixed-term contracts. Consequently, the advantage of immediately paying the firing cost when fixed-term contracts are very short is thin when the firm can wait for the stipulated termination date to get rid of the worker. For the sake of simplicity, I assume that firms always prefer to wait for the end of fixed-term contracts instead of paying firing costs. The results barely change when this subtlety is accounted for. Here, the separation probability δ implicitly models the average duration of fixed-term contracts.

Secondly, the model does not embed the possibility of conversion into an open-ended contract when the stipulated duration of the fixed-term contract is reached, whereas the convertibility option actually exists. French data shows that fixed-term contracts seldom convert into open-ended contracts. The quarterly probability of transition from fixed-term to open-ended employment constitutes an upper bound of the conversion probability and amounts to 7.4 % according to Hairault et al. (2015). Meanwhile, the distribution of the duration of fixed-term contracts has its mean at 1.5 months and a median located around 5 days as Milin (2018) mentions. Thus, these 7.4 % more likely include multiple round trips between fixed-term employment and unemployment somehow ending into open-ended employment rather than unique and direct fixed-term-to-open-ended-employment trajectories. Consequently, the conversion rate probably lies far below transition rates from fixed-term to open-ended employment. Our calibration being carried out on a monthly basis, the value of the conversion probability becomes insignificant. The no-convertibility assumption is not innocuous in theoretical terms, as I demonstrate in Rion (2021).

2.2 Joint surpluses and wages

At the equilibrium, we assume that there is free-entry in the vacancy posting activity. The present discounted value of a vacancy is zero. Competition between firms depletes the profit opportunities from new jobs. In other words, the rent provided by the posting of vacancies attracts new entrants until its disappearance at the equilibrium.

$$V = 0$$

The free-entry condition leads to a condition for job creation, which states that the firm's expected cost for a firm-worker contact $\gamma/q(\theta)$ equals the expected profit it yields.

$$\frac{\gamma}{q(\theta)} = \int \max\left[J_0^p\left(z\right), J^f\left(z\right), 0\right] dG(z) \tag{2.2.1}$$

Using the Nash-bargaining rules (2.0.1)–(2.0.3) and the job creation condition (2.2.1), I get

$$\frac{\gamma}{(1-\eta)q(\theta)} = \int \max\left[S_0^p\left(z\right), S^f\left(z\right), 0\right] dG(z) \tag{2.2.2}$$

Deriving the expression for surpluses using the proper value functions with (2.2.2), I get

$$
\begin{aligned}
(r+s+\lambda)S^{p}\left(z\right)=&z-rU+\left(r+s\right)F+\lambda\int\max\left[S^{p}\left(z'\right),0\right]dG\left(z'\right) &(2.2.3)\\
S_{0}^{p}\left(z\right)=&S^{p}\left(z\right)-F &(2.2.4)\\
(r+\delta+\lambda)S^{f}\left(z\right)=&z-rU+\lambda\int S^{f}\left(z'\right)dG\left(z'\right) &(2.2.5)\\
rU=&b+\frac{\eta\gamma\theta}{1-\eta} &(2.2.6)
\end{aligned}
$$

The unemployed value the benefit b and the rent they obtain from finding a job on the next period. A common rent for all ex-post insiders is a share of the firms' recruitment cost. γ is lost for the firm at the very moment of agreement over the formation of a match and the subsequent wage bargaining. The asset value of being unemployed is also the employed's outside option.

Nash-sharing rules as well as workers' and joint surpluses pinpoint wages.

$$
\begin{aligned}
w^{p}(z) &= \eta\left(z+(r+s)F+\gamma\theta\right)+(1-\eta)b &(2.2.7)\\
w_{0}^{p}(z) &= \eta\left(z-\lambda F+\gamma\theta\right)+(1-\eta)b &(2.2.8)\\
w^{f}(z) &= \eta\left(z+\gamma\theta\right)+(1-\eta)b &(2.2.9)
\end{aligned}
$$

Open-ended workers' wages verify that $w_0^p(z)=w^p(z)-\eta(r+s+\lambda)F$. The wages of new open-ended workers are lower than continuing open-ended workers' ones. It reflects the lower threat point of outsiders. On one hand, at a given labor market tightness θ, the new open-ended workers' wages decrease with the firing cost. After the signature of the open-ended contract, firms have to pay the firing cost in case of an adverse productivity shock. Consequently, before the occurrence of any productivity shock, firms compensate the new worker's expected gain of bargaining position and the expected loss in profits by decreasing the current wage proposal, which does not entail the payment of firing costs in case of disagreement. On the other hand, continuing open-ended workers benefit from firing costs in the firms' outside option. Thus, continuing open-ended workers' wages increase with firing costs. Overall, wages increase with unemployment benefits and recruitment cost: they enhance the workers' outside option. Similarly, a higher labor market tightness encompasses greater job-finding opportunities for unemployed workers, which raises their outside option: wages increase.

2.3 Job creation and job destruction

Joint surpluses of matches are enough to pinpoint hiring and firing decisions. Joint surpluses are linear and increasing in match productivities. Consequently, I define the job destruction margin for continuing open-ended contracts z^p as

$$S^{p}\left(z^{p}\right)=0 \qquad (2.3.1)$$

An open-ended match separates when the pair prefers paying the firing cost instead of continuing. The productivity must be sufficiently low to fill this requirement; the worker and the firm benefit from a separation if $z<z^p$. Using an integration by part in the definition of z^p (2.3.1) jointly with the definition of S^p (2.2.3), the job destruction condition for open-ended contracts is

$$z^{p}-b+(r+s)F+\frac{\lambda}{r+s+\lambda}\int_{z^{p}}^{+\infty}\left(1-G(z)\right)dz=\frac{\eta\gamma\theta}{1-\eta} \qquad (2.3.2)$$

The above equation defines a positive relationship between θ and z^p. The intuition behind this result is classic in the literature. A looser market tightness implies a higher job-finding probability for the unemployed, which makes their outside option stronger at the moment of wage bargaining: wages raise. It encourages firms to be more demanding in terms of match productivities and to increase the job destruction margin. An enhanced firing cost diminishes the job destruction margin; firms are more reluctant to pay firing costs and accept to maintain matches with worse productivities than before. Interestingly, a continuing open-ended match behaves as if it held a bond with face value F and yield $(r+s)$ that it used to pay firing costs in case of an endogenous separation.

In the same manner, I define the threshold z^f as follows

$$S^f\left(z^f\right) = 0 \tag{2.3.3}$$

The interpretation of this threshold is twofold. As far as a contact between a vacancy and a job-seeker is concerned, it states whether a fixed-term contract is profitable. A fixed-term contract is profitable as soon as $z \geq z^f$[7]. Conversely, when $z < z^f$, an existing match would have interest into splitting. The firm and the worker would prefer to get back into searching. (2.3.3) jointly with the Nash-bargaining rule (2.0.3) entail

$$z^f = \left(1 + \frac{\lambda}{r+\delta}\right)\left(b + \frac{\eta\gamma\theta}{1-\eta}\right) - \frac{\lambda}{r+\delta}Ez \tag{2.3.4}$$

where $Ez = \int z dG(z)$. As previously, a higher labor market tightness enlarges wages because of the stronger workers' outside option. Fixed-term contracts are profitable on a thinner range and z^f increases. A higher average productivity Ez encourages the resort to fixed-term contracts: the profitability margin decreases as fixed-term contracts become more often beneficial.

An analogous threshold z^c can be defined for the desirability of new open-ended contracts

$$S_0^p\left(z^c\right) = 0 \tag{2.3.5}$$

Meanwhile, since $S^p\left(z^p\right) = 0$ and $\partial S^p/\partial z = 1/(r+s+\lambda)$, I can rewrite S^p as

$$S^p(z) = \frac{z - z^p}{r+s+\lambda} \tag{2.3.6}$$

(2.3.5) and (2.3.6) enable a convenient writing for z^c.

$$z^c = z^p + (r+s+\lambda)F \tag{2.3.7}$$

Similarly, the joint surpluses of fixed-term matches and new open-ended matches verify

$$S^f(z) = \frac{z - z^f}{r+\delta+\lambda} \tag{2.3.8}$$

$$S_0^p(z) = \frac{z - z^c}{r+s+\lambda} \tag{2.3.9}$$

Note that $\partial S_0^p/\partial z = 1/(r+s+\lambda) > 1/(r+\delta+\lambda) = \partial S^f/\partial z$ if and only if $s < \delta$. I assume the validity of the latter condition, which states that the destruction rate of fixed-term contract is higher than the exogenous separation rate of open-ended contracts. It is undoubtedly the case in the data

[7] Of course, if a fixed-term contract is profitable, but still less beneficial than a open-ended contract, then the hire takes place through a open-ended contract.

as I show in the calibration section. As a result, S_0^p and S^f being increasing and linear in z, there exists z^* such that

$$S_0^p(z^*) = S^f(z^*) \tag{2.3.10}$$

For all $z \geq z^*$, $S_0^p(z) \geq S^f(z)$. I use (2.3.8), (2.3.9) and (2.3.10) to derive z^*.

$$\left(\frac{1}{r+s+\lambda} - \frac{1}{r+\delta+\lambda}\right) z^* = \frac{z^c}{r+s+\lambda} - \frac{z^f}{r+\delta+\lambda} \tag{2.3.11}$$

The following proposition characterizes the behavior of the thresholds.

Proposition 1. *These assertions are equivalent*

1. $z^* > z^f$
2. $z^* > z^c$
3. $z^c > z^f$

Proof. See Appendix B □

In the same manner, the equality of two of the thresholds z^f, z^c or z^* is equivalent to the equality between all of them $z^f = z^c = z^*$.

One ambition of this paper is to describe the endogenous choice between fixed-term contracts and open-ended contracts in job creation. With the thresholds previously defined and the increasing character of S_0^p and S^f, the job creation condition (2.2.2) becomes

$$\frac{\gamma}{(1-\eta)q(\theta)} = \int_{\max[z^c,z^*]}^{+\infty} S_0^p(z)\, dG(z) + \int_{z^f}^{\max[z^f,z^*]} S^f(z)\, dG(z) \tag{2.3.12}$$

Using two integrations by parts, the definitions of thresholds and proposition 1, the above equation writes

$$\frac{\gamma}{(1-\eta)q(\theta)} = \frac{1}{r+s+\lambda}\int_{\max[z^c,z^*]}^{+\infty} (1-G(z))\, dz + \frac{1}{r+\delta+\lambda}\int_{z^f}^{\max[z^f,z^*]} (1-G(z))\, dz \tag{2.3.13}$$

Now, I spell out the formal definition of a steady-state equilibrium in this model.

Definition 1. *A steady-state equilibrium in this economy is characterized by the tuple* $\left(\theta, z^p, z^c, z^f, z^*\right)$ *verifying equations* (2.3.2)*,* (2.3.4)*,* (2.3.7)*,* (2.3.11) *and* (2.3.13).

This job creation condition (2.3.13) heavily depends on the distribution of idiosyncratic shocks and the subsequent value of thresholds. It is possible to have no fixed-term contracts as well as both contracts at the hiring stage. Thus, a formal definition of dual job creation is useful.

Definition 2. *Job creation is said to be dual if one kind of contracts is not systematically preferred to the other at the hiring stage.*

In this model, job creation obeys the following proposition

Proposition 2. *Considering an equilibrium* $\left(\theta, z^p, z^c, z^f, z^*\right)$.

- *Job creation only occurs through open-ended contracts if and only if $z^* \leq z^f \leq z^c$. Open-ended contracts are hired when $z \in (\max[0, z^c], +\infty)$ as figure 2.1 displays.*

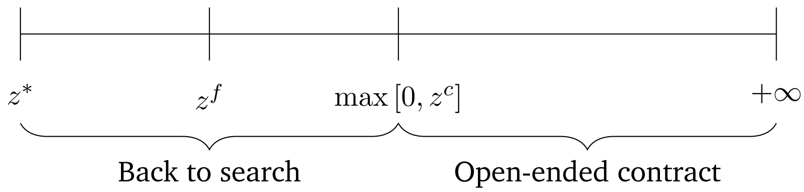

Figure 2.1: Hiring open-ended contracts only

- *Job creation is dual if and only if $\max\left[0, z^f\right] < z^*$. Fixed-term contracts are hired when $z \in (\max\left[0, z^f\right], z^*)$ and open-ended contracts are hired when $z \in (z^*, +\infty)$. Figure 2.2 sums it up.*

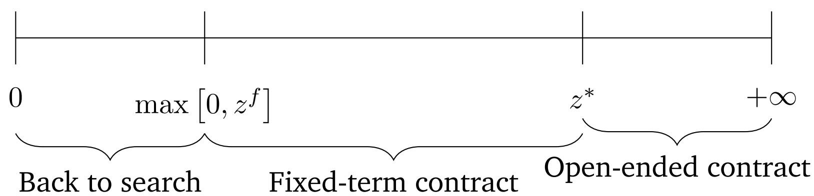

Figure 2.2: Dual job creation

Proof. See Appendix B □

The proposition above states that dual job creation requires the immediate gains of hiring an open-ended contract in good times to overcome the losses due to firing costs in future bad times. A new firm-worker pair drawing a high idiosyncratic productivity may want to take advantage of this opportunity in full. To this extent, the best way to make the most of the situation consists in hiring through an open-ended contract, which lasts longer and thus provides a higher surplus than a fixed-term contract. The match willingly locks itself through an open-ended contract in order to maximize the expected surplus. Otherwise, when the firm-worker pair draws a low productivity, both the worker and the firm go back searching. Fixed-term contracts serve as a compromise between a rigid contract or no productive relationship at all. Hiring through a fixed-term contract appears as an intermediate action: it generates a surplus through production while enabling to go back to searching for a better match before long. Fixed-term contracts are expedients to still take advantage of some positive surplus while waiting for better days to come. When, finally, a high productivity shock arises in a firm-worker meeting, an open-ended contract is signed.

In the current model, the exogenous separation rate s accounts for quits in open-ended matches among other costless termination channels.[8]. However, most quits are job-to-job transitions. What would happen if we explicitly modeled on-the-job search in the current framework? Assume workers review offers at the same rate as the unemployed. A workers quits his current position if and only if he finds a match that delivers a higher surplus. Note that, according to the law, fixed-term contracts can leave their position before the stipulated expiry date if they are hired under an open-ended contract. Thus, fixed-term workers can search on the job too. If there is no search cost, every worker

[8]Quits are marginal in fixed-term matches: only 2% of fixed-term contracts terminate before their stipulated end (Milin, 2018).

searches on the job. In this case, on-the-job search simply increase the workers' outside option compared to the baseline case. Now, assume that workers search at a cost. The higher the workers' surplus, the lower the eventual gain from on-the-job search. As continuing open-ended contracts deliver higher wages than fixed-term jobs all else equal, fixed-term workers would tend to search on the job more than open-ended workers do. A highly productive new firm-worker pair would sign an open-ended contract to lock up the match and avoid losing the worker through on-the-job search. Thus, a priori, on-the-job search would not change the ranking of contracts in job creation. Cao et al. (2010) studies in depth this question and builds a dual labor market with on-the-job search.

2.4 Aggregate flows and stocks on a dual labor market

The share of open-ended workers that moves into unemployment falls into exogenous separations, which occur with probability s, and endogenous separations, which occur when an adverse productivity shock takes place. Thus, the associated probability is $\xi = s + \lambda G(z^p)$. Unemployed workers' probability to become employed under open-ended contracts is $\mu^p = p(\theta)(1 - G(\max[z^*, z^c]))$. On the fixed-term side of the labor market, matches come to their stipulated end date with probability δ. An unemployed worker finds a fixed-term job if he contacts a firm and the productivity of the resulting match is in the proper region. It occurs with probability $\mu^f = p(\theta)\left(G\left(\max\left[z^*, z^f\right]\right) - G\left(z^f\right)\right)$. Denoting u the measure of non-employed workers, n^p the measure of open-ended workers and n^f the measure of fixed-term workers, fixed-term and open-ended employments evolve according to the following system.

$$\dot{n}^p = -\xi n^p + \mu^p u \tag{2.4.1}$$

$$\dot{n}^f = -\delta n^f + \mu^f u \tag{2.4.2}$$

Normalizing the workers' population to 1 leads to the following expressions for steady-state open-ended employment, fixed-term employment and non-employment.

$$n^p = \frac{\mu^p \delta}{\mu^p \delta + \xi \delta + \mu^f \xi} \tag{2.4.3}$$

$$n^f = \frac{\mu^f \xi}{\mu^p \delta + \xi \delta + \mu^f \xi} \tag{2.4.4}$$

$$u = \frac{\xi \delta}{\mu^p \delta + \xi \delta + \mu^f \xi} \tag{2.4.5}$$

As previously, an increase in θ has ambiguous consequences. On one hand, it strengthens the workers' outside option, which in turn raises wages, encourages the destruction of open-ended jobs and makes firms more demanding at the hiring stage in terms of productivity. On the other hand, job creation is bolstered by the enlarged probability of contact. Therefore, the impact on labor market stocks is ambiguous.

2.5 Comparative statics

In this subsection, I carry out comparative-statics exercises to describe the steady-state behavior of the model.

Figure 2.3 diagrammatically sums up the movements of the different loci equations (JD^p) (2.3.2), $\left(z^f\right)$ (2.3.4), (z^*) (2.3.11) and (JC) (2.3.13) after an increase in firing costs.

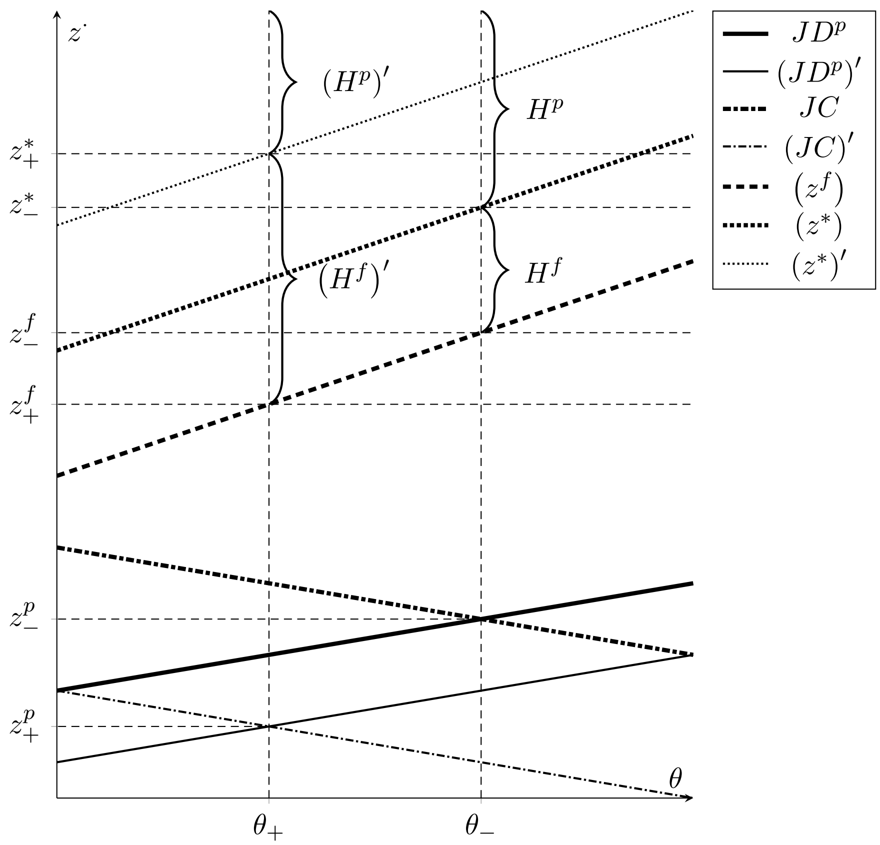

Figure 2.3: An increase in firing costs in the dual Mortensen-Pissarides model

The plus $+$ and minus $-$ subscripts respectively denote the equilibrium values before and after the change in firing costs. H^p and $(H^p)'$ respectively are the hiring regions under open-ended contracts before and after the change in firing costs. H^f and $\left(H^f\right)'$ are their counterparts for fixed-term contracts.

Enlarged firing costs decrease the job destruction margin for a given labor market tightness. The job destruction curve (JD^p) moves downward to reach $(JD^p)'$. Firm-worker pairs compensate their losses in the expected surplus of a continuing match by demanding more productive newcomers: (z^*) shifts upwards to $(z^*)'$. The locus for profitability of the fixed-term contracts $\left(z^f\right)$, as for it, remains unchanged. Consequently, higher firing costs necessarily increase the share of fixed-term contracts in job creation. This phenomenon causes a shift of the job creation condition downward: at a given θ, the expected profit from a firm-worker contact decreases as the signatures of open-ended contracts dwindle and are replaced by fixed-term contracts at the margin. (JC) shifts downward to reach its new position $(JC)'$. The job creation curve shifts further than the job destruction curve and the equilibrium labor market tightness decreases. Note that the subsequent reduction in the labor market tightness is stronger when only open-ended contracts are allowed. The replacement of open-ended hires with fixed-term ones at the margin mitigates the fall in the expected surplus from a contact and alleviates the fall in the posting of vacancies. Indeed, at the margin, matches with marginal productivity lower than z^c delivered no opportunities of a positive profit in the classic framework, whereas the matches with marginal productivity slightly lower than z^* deliver a positive profit through fixed-term employment in the dual framework.

Overall, the evolution of open-ended employment is ambiguous : job creation reduces as well as job destruction. As for fixed-term employment, job creation probability increases but the evolution of unemployment is unclear. Therefore, the response of the fixed-term job creation flow to an increase in firing costs is ambiguous.

An important alternative situation where the reasoning above is no longer valid consists in the insensitivity of the steady-state equilibrium to firing costs. This happens if there is no endogenous destruction of open-ended jobs. Proposition 3 mathematically recapitulates the previous results.

Proposition 3. *At the steady-state equilibrium,*

- *with endogenous job destruction of open-ended contracts*

$$\frac{\partial \theta}{\partial F} < 0, \frac{\partial z^p}{\partial F} < 0, \frac{\partial z^c}{\partial F} > 0, \frac{\partial z^f}{\partial F} < 0, \frac{\partial z^*}{\partial F} > 0$$

- *otherwise*

$$\frac{\partial \theta}{\partial F} = \frac{\partial z^c}{\partial F} = \frac{\partial z_i^f}{\partial F} = \frac{\partial z^*}{\partial F} = 0$$

Proof. See Appendix B □

An increase in firing costs entails a substitution towards fixed-term employment on behalf of open-ended employment, while the classic result of an ambiguous response of unemployment remains. The enhanced relative flexibility of fixed-term contracts makes them significantly more attractive than open-ended contracts, which are progressively replaced by fixed-term contracts at the margin. When there is no endogenous destruction of open-ended jobs, the equilibrium becomes insensitive to firing costs.

The above exercise shows that the relative desirability of each contract at the hiring stage heavily depends on firing costs. Intuitively, sufficiently low firing costs lead to a complete shutdown of fixed-term employment, the corner case being firing costs such that $z^c = z^f = z^*$ as propositions 1 and 2 suggest. Similarly, prohibitively high firing costs should drive open-ended employment to zero through the utter disappearance of job creation through open-ended contracts, which seems pretty unrealistic. It may occur if the distribution of idiosyncratic shocks is bounded upwards. In our case, an intermediate case arises. When firing costs are high enough, endogenous open-ended job destruction vanishes. It makes the equilibrium insensitive to further increases in firing costs. The knife-edge value of firing costs solution corresponds to the equilibrium where $z^p = 0$. The following proposition formalizes this intuition.

Proposition 4. *There exists $\widehat{F}$ and $\widetilde{F}$ such that the steady-state equilibria verify*

- *If $F \leq \widehat{F}$, there are only open-ended contracts*
- *If $\widehat{F} < F < \widetilde{F}$, open-ended and fixed-term contracts coexist*
- *If $\widetilde{F} \leq F$, there are no endogenous job destruction of open-ended contracts. The equilibrium becomes insensitive to F*

Figure 2.4 sums up the different sort of equilibria in terms of contractual composition depending on firing costs.

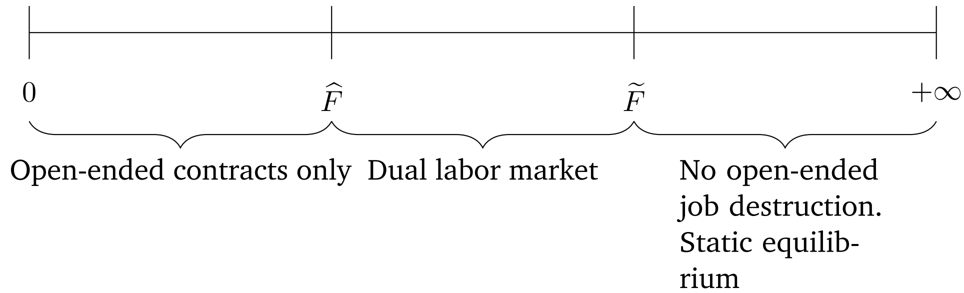

Figure 2.4: Firing costs and equilibrium employment

Proof. See Appendix B. □

The result above demonstrates the adaptability of our approach. The productivity-flexibility trade-off may deliver a classic labor market with a rigid side only, a dual labor market or a fixed-term-oriented labor market. This is ideal to study transitions and steady-state outcomes related large-scale policies, such as the ban of fixed-term contracts or large cuts in firing costs. The insensitive equilibrium associated with $F \geq \widetilde{F}$ is interesting too. The data suggests that only 30% of open-ended job destruction is currently endogenous, which is not so far from the 0% of our extreme case. To this extent, the economic schemes at stake in the region close to $\widetilde{F}$ might be insightful.

3 Numerical analyses

As we saw in the comparative statics section, quantitative analyses are necessary to circumscribe the behavior of employments and unemployment in response to changes in firing costs.

3.1 Calibration

In this section, I calibrate a model to mimic the behavior of the French labor market. Availability of data concerning dualism on the labor market guided this choice. The scope of the following analyses can be widened to many Western European countries, which share quite similar labor market institutions even though the strength of employment protection legislation may differ. One may quote Germany, Italy, Spain or Portugal among others[9].

I set the time unit period to one month in accordance with the average duration of 1.5 months for temporary contracts in 2017 documented by Milin (2018)[10]. Interestingly, the median duration associated with temporary contracts is much lower and amounts to 5 days. The interest rate is set to 5 % annually, which boils down to 0.4 % on a monthly basis. The matching function is specified as a Cobb-Douglas function $m(u, v) = mu^{\sigma}v^{1-\sigma}$. The elasticity of the matching function with respect to unemployment σ is set to 0.6, which stands in the middle of the range 0.5-0.7 estimated as reasonable by Burda and Wyplosz (1994) for Western Europe economies. In order to avoid the effect of congestion externalities and the complexity of their interlacing with the labor market institutions, we set the bargaining power η to 0.6[11]. As I mention in the theoretical discussion above, one important question is the distribution of idiosyncratic shocks. Uniform distributions are used in

[9] OECD (2013) and ILO detail the employment protection legislations of many countries.

[10] I essentially rely on this note to target moments characterizing the French labor market. It is based on the *Enquête sur les mouvements de main d'œuvre* carried out by *Dares*

[11] The Hosios condition is still valid in our framework.

seminal papers [12] and include important benefits in terms of tractability. However, Tejada (2017) shows that log-normal distributions replicate well the distribution of wages in dual labor markets. Results are qualitatively robust to changes in distributions of idiosyncratic shocks. Table 3.1 sums up the choice of parameters.

r	σ	η	λ	δ
0.4 %	0.6	0.6	0.083	0.67

Table 3.1: Parameters

$F/\overline{w^p}$	$\mu^f/\left(\mu^p+\mu^f\right)$	n^f/n	s/ξ	u	$q(\theta)$
1.33	0.83	0.12	0.70	0.26	0.33

Table 3.2: Targets for a calibration of the French labor market

F	b	s	σ_z	m	γ
1.30	1.00	0.013	0.05	0.53	3.78 $\cdot 10^{-3}$

Table 3.3: Calibrated parameters

Table 3.2 outlines the targeted labor markets moments. Fontaine et al. (2016) suggests that inactive workers significantly contribute to job creations through fixed-term contracts. As a result, I target an unemployment rate of 26 %, which corresponds to the 2016's French inactivity rate[13]. I set the steady state share of fixed-term contracts in employment to 12 %. I target an average share of fixed-term contracts in job creation of 83 %[14]. I consider that exogenous splits of open-ended matches constitute 70 % of separations in accordance with Milin (2018). As for the calibration of the French firing costs, I rely on Kramarz and Michaud (2010). Individual lay-offs marginally cost 4 months of the median wage, while the marginal cost of lay-off within a collective-termination plan represents 12 months of the median wage[15]. The former being the most frequent case, I reckon that total firing costs represent 4 months of the open-ended workers' average wage. As in Bentolila and Saint-Paul (1992) and Cahuc et al. (2016), I assume that red-tape costs actually embodied by firing costs only represent one third of total firing costs. Thus, I target a ratio of 4/3 for firing costs F with respect to the monthly open-ended workers' average wage. I also target the quarterly vacancy filling rate to 70 %, which is equivalent to a monthly rate of 33 %. As for the productivity shock arrival λ, there is no consensus stemming from the empirical literature. I assume a yearly average frequency. The calibration results in the determination of $(\sigma_z, F, b, s, m, \gamma)$, whose values are detailed in table 3.3.

[12] Among others, Mortensen and Pissarides (1994, 1999); Cahuc and Postel-Vinay (2002)

[13] Source: *Recensement de la population* 2016 - *Insee*

[14] Average from Q1-2000 to Q2-2019 computed with data from *Acoss - Urssaf* (*Déclarations préalables à l'embauche*)

[15] To be accurate, Kramarz and Michaud (2010) assesses that firms with more than 50 employees face a marginal cost of 97,727 FFr (Table 1b), which represents 14 months of the workers' median wage. Consequently, the associated median wage of fired workers is 6980 FFr. Thus, Table 2 shows that individual terminations cost 27,389 FFr, which amounts to 4 months of the fired workers' median wage, while the termination within a collective firing plan marginally costs 81,850 FFr, which equals 12 months of the median wage.

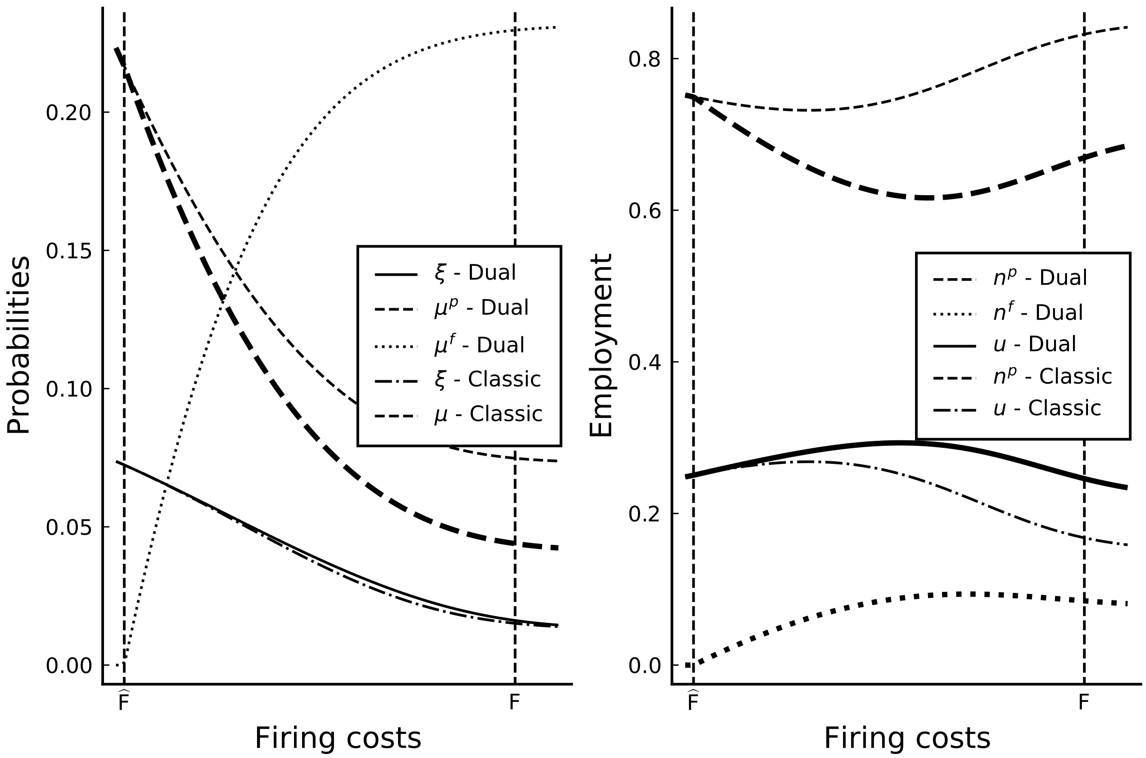

Figure 3.1: Steady-state transition probabilities, employments and firing costs

On the x-axis, $\widehat{F}$ is the threshold beyond which firing costs generate an equilibrium with dual job creation and is defined in Proposition 4. F denotes the value of firing costs as specified in the baseline calibration (see Table 3.3).

3.2 Steady-state employment and welfare

Employment Left-hand side of Figure 3.1 displays the steady-state transition probabilities and their evolution with respect to firing costs. As firing costs increase, the separation probability of open-ended contracts decreases. In the same manner, the open-ended-contract job-finding rate decreases with firing costs while the fixed-term-contract job-finding rate increases. Magnified firing costs encourage the substitution towards fixed-term contracts. When firing costs are higher than $\widehat{F}$, the substitution effect is responsible for the lower open-ended job finding rate in a dual labor market compared with a classic labor market with open-ended contracts only. Employees' outside option is bolstered by the possibility to find a job through fixed-term contracts for a given labor market tightness. Accordingly, the open-ended job destruction margin is higher in the dual than in the classic case and so is the open-ended job destruction probability.

The right-hand side of Figure 3.1 displays the evolution of employments in a classic and a dual Mortensen-Pissarides model. On the left-hand part of the graph, firing costs are lower than $\widehat{F}$. Job creation only happens through open-ended contracts and there is no fixed-term employment. The dual and the classic Mortensen-Pissarides models coincide when firing costs are in $\left[0, \widehat{F}\right]$. Theoretically, firing costs have a relatively ambiguous effect on employment. On one hand, firing costs discourage lay-offs because of a magnified separation cost. On the other hand, the workers' enhanced threat point in the wage-bargaining process discourages hires. As firing costs exceed $\widehat{F}$, an equilibrium with dual job creation becomes possible. The classic model delivers lower and lower employment levels. At one point, the increase in firing costs ends up jeopardizing hiring incentives more than it discourages lay-offs: unemployment reaches a minimum level and begins increasing.

In the dual labor market, the same phenomenon takes place regarding overall employment but the outcome differs. The difference between the classic and dual cases stem from the emergence of an additional mechanism in the dual case. When dual job creation is an equilibrium outcome, higher firing costs encourage the substitution of open-ended employment towards fixed-term employment. Starting from $\widehat{F}$, an increase in firing costs widens the spectrum of situations where fixed-term contracts are profitable. As fixed-term workers operate a larger share of new jobs, the job destruction flow enlarges. In my calibration, the widened possibilities of hires brought in by fixed-term contracts decrease less unemployment than magnified job destruction pushes it up : unemployment increases up to a maximum value of 0.28. The substitution effect is quantitatively substantial. Open-ended employment drops to a minimum of 0.62, while fixed-term employment increases from 0 to a maximum of 0.10.

When the firing cost takes even higher values, the behavior of employments reverts. Open-ended employment increases from 0.62 to 0.65, while fixed-term employment decreases from 0.10 to 0.09. At first view, however, fixed-term employment should still increase according to the substitution effect we previously described. A general-equilibrium effect is responsible for this behavior. Since open-ended employment represents the bulk of workers, unemployment tends to decrease as the open-ended job destruction probability shrinks. Even though higher firing costs push up the fixed-term jobs' hiring probability, the latter reduction in unemployment is strong enough to push down the job creation flow into fixed-term employment. Since fixed-term job destruction rates are constant, fixed-term employment shrinks. In the French data, a minor share of open-ended matches' separations induce the payment of a firing cost. The calibrated economy is close to the corner equilibrium with no endogenous open-ended job destruction. The French economy is set in the region where the general-equilibrium effect of an increase in firing costs outreaches the substitution effect. In graphic terms, open-ended employment increases with firing costs in the neighborhood of the baseline firing costs materialized by the dashed vertical line F in Figure 3.1.

Welfare Beyond employment, important considerations include the impact of firing costs on welfare. The comparative statics analysis carried out earlier as well as proposition 3 provide preliminary results. The unemployed would prefer the highest possible value for θ, which is associated with $F = 0$. Indeed, the unemployed workers value high job-finding probabilities. As for the fixed-term and open-ended matches, the results are theoretically ambiguous and a numerical analysis is necessary. Social welfare SW is defined as the sum of production from open-ended matches, production from fixed-term matches, the unemployed's home production net of the vacancy costs. Therefore, the steady-state social welfare SW verifies

$$rSW = n^p\overline{z^p} + n^f\overline{z^f} + bu - \gamma v - \lambda G\left(z^p\right) n^p F$$

where $\overline{z^p}$ and $\overline{z^f}$ are the average productivities of open-ended and fixed-term matches respectively. Figure 3.2 displays steady-state social welfare on the left panel and average productivities on the right panel as firing costs vary.

Many competing mechanisms intervene in the variation of social welfare with respect to firing costs. One may divide them in two categories: employment effects and productivity effects. The vacancy cost being tiny, one may neglect its quantitative effect in the comparative-statics analysis of welfare. As employment variations with firing costs are studied above, I now consider the evolution of productivities.

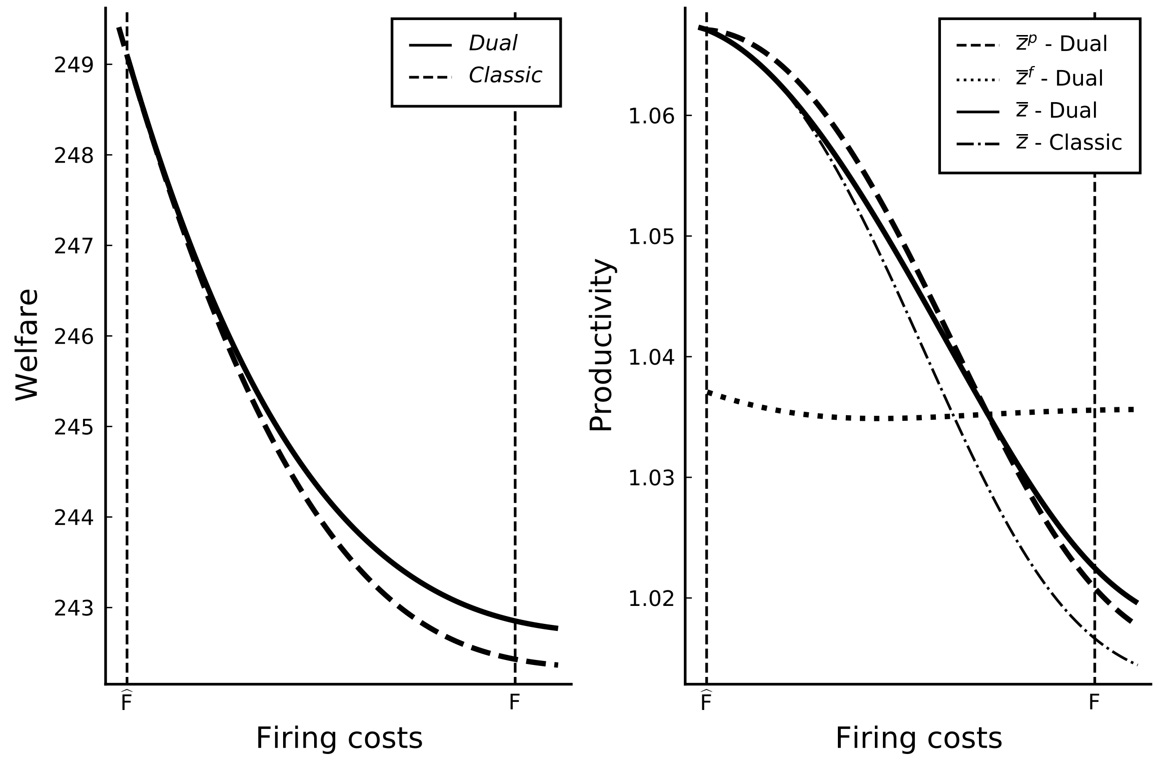

Figure 3.2: Evolution of social welfare and productivities with respect to firing costs

$\overline{z^p}$ and $\overline{z^f}$ are the average productivities of open-ended and fixed-term matches respectively. $\overline{z}$ is the overall average productivity in the economy.

On the open-ended side of the labor market, the higher the firing costs, the lower the rate of open-ended job destruction. As firing costs increase, continuing open-ended contracts become less and less productive. In contrast, augmented firing costs push up the job creation thresholds z^c and z^*. New open-ended contracts become more productive as firing costs enlarge. Overall, though, the stock of continuing open-ended matches is bigger than the stock of new open-ended matches and the former effect prevails. The average productivity of open-ended contracts $\overline{z^p}$ in the dual model and the average productivity of open-ended contracts in the classic model $\overline{z}$ decrease with firing costs. The productivity of open-ended contracts is higher in the dual model than in the classic model. It essentially stems from the fact that fixed-term employment replaces open-ended employment as firing costs increases beyond $\widehat{F}$. The productivity of new open-ended matches drives the difference between the dual and the classic specification. Indeed, z^* increases faster with firing costs in the dual equilibrium than z^c does in the classic case, and so does the average productivity of new open-ended matches in the dual labor market compared with the average productivity of new open-ended matches in the classic labor market. The average productivity of new open-ended contracts is thus higher in the dual specification than in the classic specification. The open-ended job destruction threshold z^p stays pretty similar in both the dual and the classic specification. As a result, the average productivity of continuing open-ended matches does not differ much across both specifications as firing costs change. The average productivity of new open-ended contracts accounts for the difference between both specifications in that regard.

On the fixed-term side of the labor market, an increase in firing costs has two opposite effects.

On the one hand, the higher the firing cost, the lower the labor market tightness and the lower the profitability threshold z^f, which reduces fixed-term jobs' productivity. On the other hand, higher firing costs encourage the substitution towards fixed-term contracts on behalf of open-ended contracts in the neighborhood of z^* because z^* increases with firing costs. This makes fixed-term contracts more productive on average. The right-hand side graph of figure 3.2 shows that those two effects cancel out over the chosen interval of firing costs.

The average productivity is higher in the dual framework than in the classic framework. The substitution account for this phenomenon. In the dual framework, most job creations occur through fixed-term contracts, and these new jobs are necessarily productive enough to generate a positive joint surplus. Moreover, fixed-term jobs are short enough to remain productive throughout their whole life span. In a classic framework, new open-ended jobs are more and more productive as well, but job creation flows are smaller because of firing costs and the continuing open-ended jobs are less and less productive as job destruction flows tighten. The higher the firing costs, the more the average productivity in the dual framework dominates the average productivity in the classic framework.

The overall behavior of welfare results from the intertwined productivity-related and employment-related mechanisms. As open-ended contracts cover most workers, the negative impact of firing costs on the average productivity of open-ended jobs dominates and the total average productivity decreases with firing costs. Since employment moves between 0.7 and 0.8 in response to changes in firing cost, the decreasing average productivity drives the welfare as the left panel of Figure 3.2. Why is the steady-state welfare higher in a dual labor market than in a classic labor market? The unemployment rate and the average productivity are higher in the dual framework. Theses two opposite effects lead to an ambiguous result in terms of social welfare. The difference between both frameworks is to be found on the open-ended job destruction side and the welfare loss firing costs induce. Open-ended job destruction rates are comparable between the dual and the classic framework. However, open-ended employment is higher in the classic framework. The welfare loss associated with open-ended job destruction is thus higher in the classic framework than in the dual framework. The latter mechanism tilts the balance in favor of the dual labor market, which dominates the classic labor market in terms of welfare.

A benevolent social planner willing to reduce unemployment and improve welfare faces a dilemma if it wants to keep the dual structure of the labor market at minimum political cost. Indeed, starting from the baseline firing costs, there is no free lunch for small changes in firing costs. In the neighborhood of baseline firing costs F, increasing firing costs decreases unemployment and welfare, while decreasing firing costs increases unemployment and welfare. A reform improving both unemployment and welfare needs a large cut in firing costs. The post-reform firing costs need to be close to or below $\widehat{F}$. A large welfare gain and a small unemployment decrease come at a high political cost since the optimal reform consists in strongly cutting firing costs, which leads to a persistent increase in unemployment and a persistent decrease in open-ended employment. Interestingly, the shape of the latter reform roughly resembles a unique-contract reform as there are no longer fixed-term contracts with firing costs below $\widehat{F}$.

3.3 Dynamics

In this section, we study dynamics after a change in firing costs as well as transitions between dual and classic labor markets. The mathematical aspect of these dynamics is discussed in Appendix C. I also study the consequences of regulatory uncertainty.

Post-reform adjustment speed The speed of adjustment of an economy after a reform constitutes an important factor from the policy-maker's point of view. It depends both on employment values and employment transition probabilities. Employment transition probabilities are forward looking because they are functions of the labor market tightness and the thresholds pinpointing job destruction and job creation are forward-looking variables. Therefore, they jump immediately to the new steady-state values after an unexpected change in the parameters[16]. As for employment values, they are stock variables. These results are valid for both the classic and dual Mortensen-Pissarides model. Figure 3.3 displays the time at which 99 % of the adjustment is completed for each type of employment.

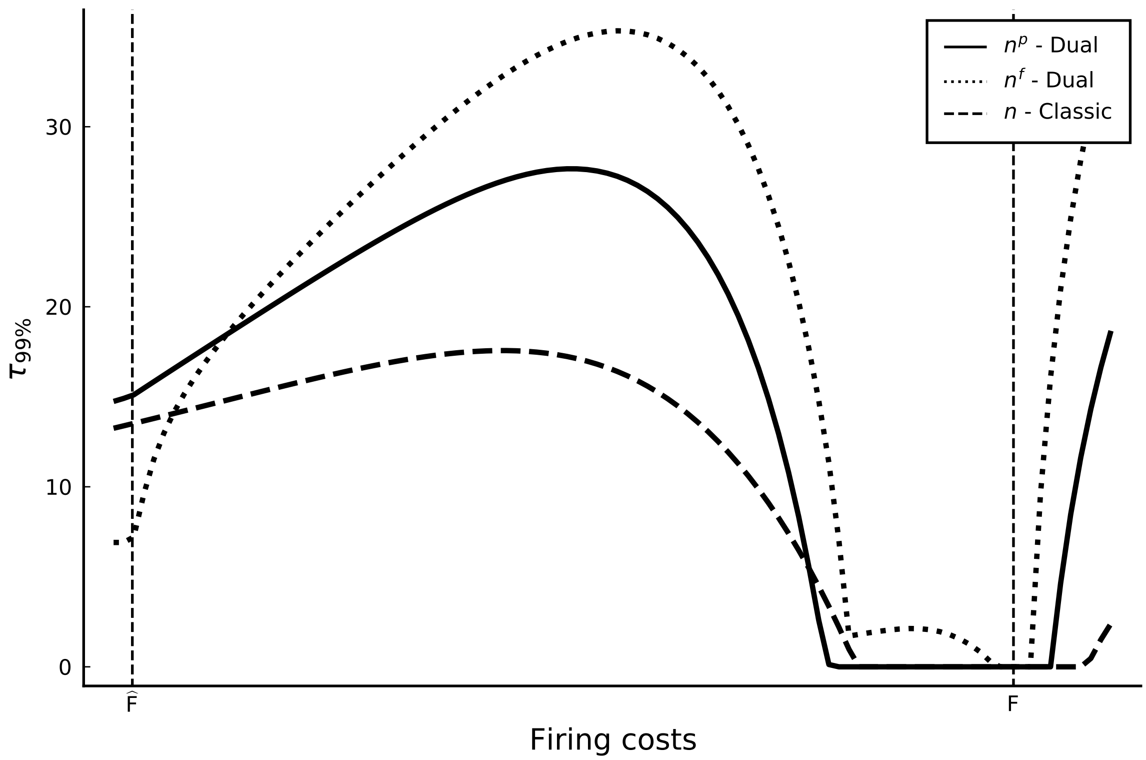

Figure 3.3: $\tau_{99\%}$ of different reforms with respect to the post-reform firing costs in months

The x-axis represents post-reform firing costs and the F-labeled vertical dashed stands for the baseline firing costs. The y-axis represents the time by which 99% of the adjustment is done when considering steady-state values.

Starting from the baseline value, a cut in firing costs immediately increases open-ended job creation and destruction probabilities. Flows thicken in the open-ended side of the labor market. Meanwhile, on impact, the least productive continuing open-ended matches now deliver a negative surplus and split; open-ended employment dives and unemployment soars.

In the classic framework, adjustment time reflects the interaction between the impact reaction and its absorption through bigger job creation and destruction flows. On the right-hand side of $\left[F, \widehat{F}\right]$, as post-reform firing costs move father and farther from the baseline, the impact reaction brings employment farther and farther from the new steady-state and transition rates do not increase enough to quickly absorb the impact reaction: adjustment time increases and reaches a maximum.

[16]This result is extensively developed in Pissarides et al. (2000) page 59-63.

As post-reform firing costs decrease, transition rates become high enough to quickly absorb the initial response of employment and adjustment time decreases. Overall, adjustment takes up to 1.5 years in the classic framework. Note that if, by a happy coincidence, the impact response of employment sets employment close to its new steady-state, adjustment would be much shorter. It does not take place in the current calibration, but may occur with other specifications. Figure A.6 in Appendix A shows such a case when the average duration between productivity shocks is 3 years: in the classic framework, employment immediately gets close to its new steady-state value when post-reform firing costs are half way between the baseline firing costs and $\widehat{F}$.

In the dual framework, a similar balance between the new steady-state employment values, the new transition probabilities and the employment impact responses shape dynamics. A novel force emerges in contrast with the classic framework; the fixed-term job creation rate decreases as firing costs decrease. Starting from the baseline economy, as post-reform firing costs decrease, the impact response sets open-ended employment further and further from its new steady-state value. The higher open-ended job transition rates cannot overcome this mechanism when firing costs are close to the baseline. The presence of fixed-term contracts that capture a decreasing but still important share of job creations extends the adjustment time. Adjustment times reach a maximum. As post-reform firing costs further decrease, the prominence of fixed-term contracts reduces and open-ended transition rates increase enough to overcome the impact response of open-ended employment. Adjustment takes up to 3 years. In the same manner as in the classic framework, if by chance the impact response brings employment close to its new steady state value, transitions are much shorter. As figure A.3 in Appendix A shows, when the average duration between two productivity shocks is one month, the impact response of open-ended employment brings it close to its new steady state value when post-reform firing costs are a bit higher than $\widehat{F}$.

Starting from the baseline value, an increase in firing costs reduces the transition probabilities on the open-ended side of the labor market. Since open-ended matches represent the bulk of employees in the baseline calibration, the new unemployment value depends on open-ended employment to reach its new steady-state value. The sluggish motion of open-ended employment slows down the adjustment of unemployment. The higher fixed-term job creation rate is not high enough to overcome the low transition rates on the open-ended side of the labor market. It reflects on the adjustment of fixed-term employment, which essentially relies on job creation. A 3-per-cent increase in firing costs needs an adjustment of 30 months, which represents 2.5 years.

Overall, the policy maker faces a high uncertainty with respect to adjustment time when it implements a reform. In the following paragraph, we describe the post-reform dynamics in two specific cases.

Post-reform dynamics The left-hand side of Figure 3.4 shows the transition from the baseline dual labor market to a classic labor market with half of the baseline firing costs. The labor market fully adjusts by roughly 24 months. It reaches a new equilibrium with higher open-ended employment, no fixed-term employment and a roughly similar unemployment. Temporary employment vanishes by 10 months. Unemployment first increases for two reasons. First, the aggregate job-finding rate drops; some of the unemployed who initially found a fixed-term job now remain unemployed. Second, the drop in firing costs pushes up the open-ended job destruction margin z^p, which entails the split of the least productive open-ended matches. Importantly, the impact reaction of open-ended employment is at odds with the final steady-state increase in the latter. The policy-maker has to deal with a magnified unemployment rate as well as a depleted open-ended employment on the first year of the transition. The cost of such a transition is prominent in employment terms.

The right-hand side of figure 3.4 represents the transition from a classic to a dual labor market.

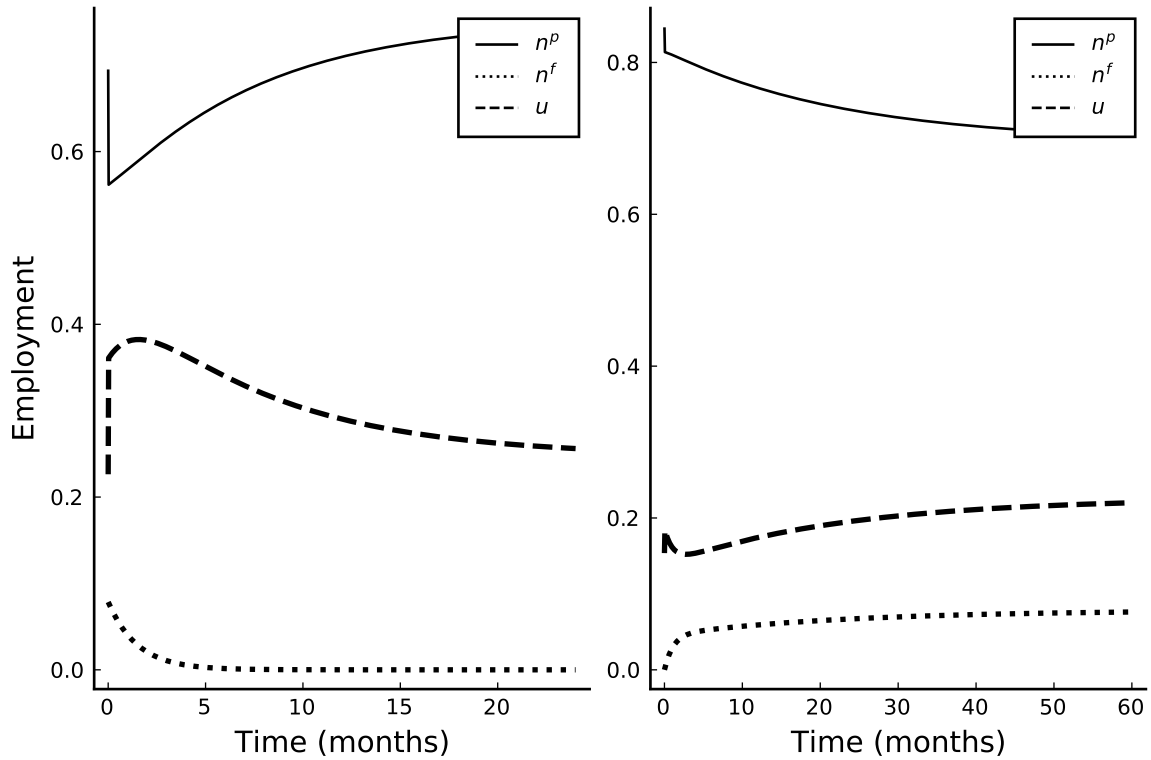

Figure 3.4: Post-reform dynamics

On the left, transitions of employments and unemployment following a unique-contract reform. On the right, transitions of employments and unemployment following the introduction of fixed-term contract

The adjustment is much slower and takes around 5 years to be 99 % complete. On impact, a few new open-ended matches split as the profitability threshold for open-ended jobs jumps. The expected value of a firm-worker contact increases thanks to the newly available fixed-term contracts. The vacancy-posting activity is more profitable and the labor market tightens. In response, firm-worker pairs are more demanding in terms of surplus, which explains why the profitability threshold of open-ended contract increases as fixed-term contract become available. Fixed-term employment quickly weighs in job creation as the main force, but the fixed-term job creation flows remain small as unemployment is stable. The thin job destruction flow from open-ended employment is the only force that may drive up unemployment. Thus, the substitution of open-ended employment towards fixed-term employment is pretty slow. These sluggish dynamics after the introduction of fixed-term contracts might bring an explanation to the expansion of fixed-term employment in the Western economies over the last decades. At the end of the 1970s, fixed-term contracts were introduced with a restricted scope and unemployment was low. Progressively, the legal constraints on the employability of fixed-term contracts loosened. In our model, an equivalent situation would embed an exogenous probability to accept or refuse a fixed-term match[17]. Initially, fixed-term matches are introduced, but most of them are refused. Imagine that the acceptance rate of fixed-term contracts is gradually extended. Considering the 5-year convergence duration as a trustworthy figure, fixed-term employment would take years to reach its equilibrium value if it had to adjust to each of the reforms

[17] Cahuc and Postel-Vinay (2002); Sala and Silva (2009); Bentolila et al. (2012) and Sala et al. (2012) make this assumption.

on the acceptance rate.

In both examples of reforms, the impact response of employments and the post-reform transition probabilities shape the dynamics. As far as I know, there is no exploitable data to verify my findings. Earliest time series of open-ended and fixed-term employents are annual and start on 1982, while fixed-term contracts were introduced in 1979. As for in-depth reforms, there were virtually none about employment protection in France in the last few years. The biggest reform was the *Accord national interprofessionnel* concluded on 2013 January 11 and was about introducing a pinch of experience rating in payroll taxes. Relevant data may stem from 2014's Italian Jobs Act, which imposes simpler termination procedures and caps on compensations[18].

Regulatory uncertainty If studying once-for-all in-depth reforms is relevant from a policy point of view, it remains a purely theoretical exercise. According to Fontaine and Malherbet (2016), reforms are actually frequent and often marginal in Western and Southern Europe. Between 2005 and 2013, they count 17 employment protection legislation reforms in France, 49 in Italy, 38 in Spain, 23 in Greece and 17 in Portugal. Thus, a natural question to address concerns the impact of regulatory uncertainty on the labor market equilibrium.

I assume that firing costs undergo i.i.d shocks with probability ϵ per unit of time to study the impact of regulatory uncertainty. Firing costs follow a uniform distribution over $\{F_1, F_2\}$. The Bellman equations defining the firms' and workers' programs now include the shock in firing costs. Firing costs become a new state variable. When a shock in firing costs occurs, firing costs jumps to either F_1 or F_2 with an equal probability $1/2$ and the present discounted values associated with each type of contracts change accordingly. I detail the model in Appendix D. A steady-state no longer exists: the system is now a Markov Jump Non-Linear System[19]

Intuitively, uncertainty in firing costs should discourage hires through open-ended contracts. When $F = F_2$, matches with a productivity just over the job destruction threshold split if a firing cost shock leads to $F = F_1$. When $F = F_1$, matches anticipate that shocks may lead to an increase firing costs, which encourages substitution towards fixed-term contracts at the hiring stage.

A natural way to measure the impact of regulatory uncertainty on employments and welfare is to compare the true economy with regulatory uncertainty to its counterpart without regulatory uncertainty. In the proposed extension, the true economy is such that the firing shock arrival rate ϵ is positive, $F_1 < F_2$ and the expected value of firing costs $(F_1 + F_2)/2$ verifies the baseline target defined in Table 3.2. I set ϵ to match the French and Portuguese score of 17 reforms in 9 years, which seems to be a lower bound in Western and Southern European countries. Consequently, our results will understate the actual impact of regulatory uncertainty in the mentioned countries.

I also consider different values for the wedge between firing costs in both states 1 and 2 as there is no data to discipline it. To make sure the model with uncertainty over firing costs reflects the actual economy, I estimate the parameters that match the baseline moments in Table 3.2 using a simulated method of moments. The simulated method of moments minimizes the wedge between actual and simulated moments over the same set of parameters as the baseline calibration[20]. Figure 3.5 shows realizations of fixed-term and open-ended employments for a given path of firing cost shocks.

Once I have got the parameters associated with the true economy, I may delineate an analogous economy without shocks on firing costs; the shock arrival rate on firing costs ϵ is equal to zero and firing costs are set to their expected value $(F_1 + F_2)/2$. In that regard, I can compare the stochastic

[18]See Boeri and Garibaldi (2019) for an assessment of the Italian Jobs Act

[19]do Valle Costa et al. (2012) is an approachable introduction to the linear case.

[20]Appendix D.3 details the procedure

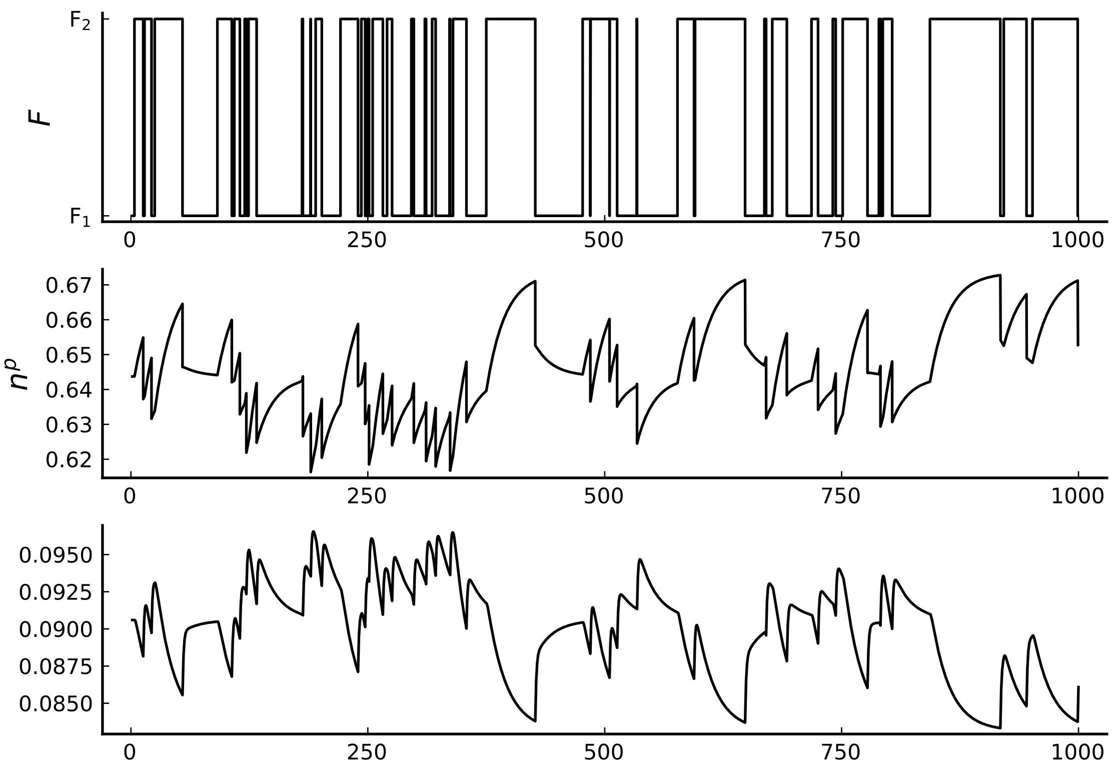

Figure 3.5: Employment dynamics with a given path of firing cost shocks

steady-state of the model with shocks in firing costs and the steady state of the model without shocks in firing costs.

Figure 3.6 compares steady-state employment values in the economies with and without shocks in firing costs. In other words, it compares the actual firing-cost-uncertain economy to its hypothetical firing-cost-certain equivalent. Taking the economy with shocks in firing costs as a reference, it displays the percentage deviation in employments when the uncertainty over firing costs vanishes. Starting from an equilibrium with 2.5 % changes of firing costs with respect to the firing cost value verifying the baseline target, shutting down uncertainty provides a 0.9 % increase in open-ended employment and 1.5% decrease in fixed-term employment. Policy uncertainty significantly fuels substitution towards fixed-term employment, which increases unemployment.

Robustness As previously mentioned, there is a diversity of views about the value of the productivity shock arrival rate λ. In Appendix A, I plot the same graphs for calibrations with significantly higher and lower productivity shock arrival rates. I carry out three calibrations with three different average durations between productivity shocks: a month, a year and the mid-length of a typical business cycle, namely 3 years. In the same manner, the red-tape costs associated with an endogenous job destruction are difficult to assess. I calibrate the model with a zero share of exogenous splits of open-ended contracts to assess the sensitivity of my results to this parameter. Assuming that all inactive workers search for a job may seem a pretty polar case despite the blurry frontier between employment, unemployment and inactivity among the workers patronizing the fixed-term side of the labor market. Thus, I also carry out a calibration targeting a 10% unemployment rate. The results stay the same.

A higher arrival rate of productivity shocks and a zero share of cost-less separations of open-

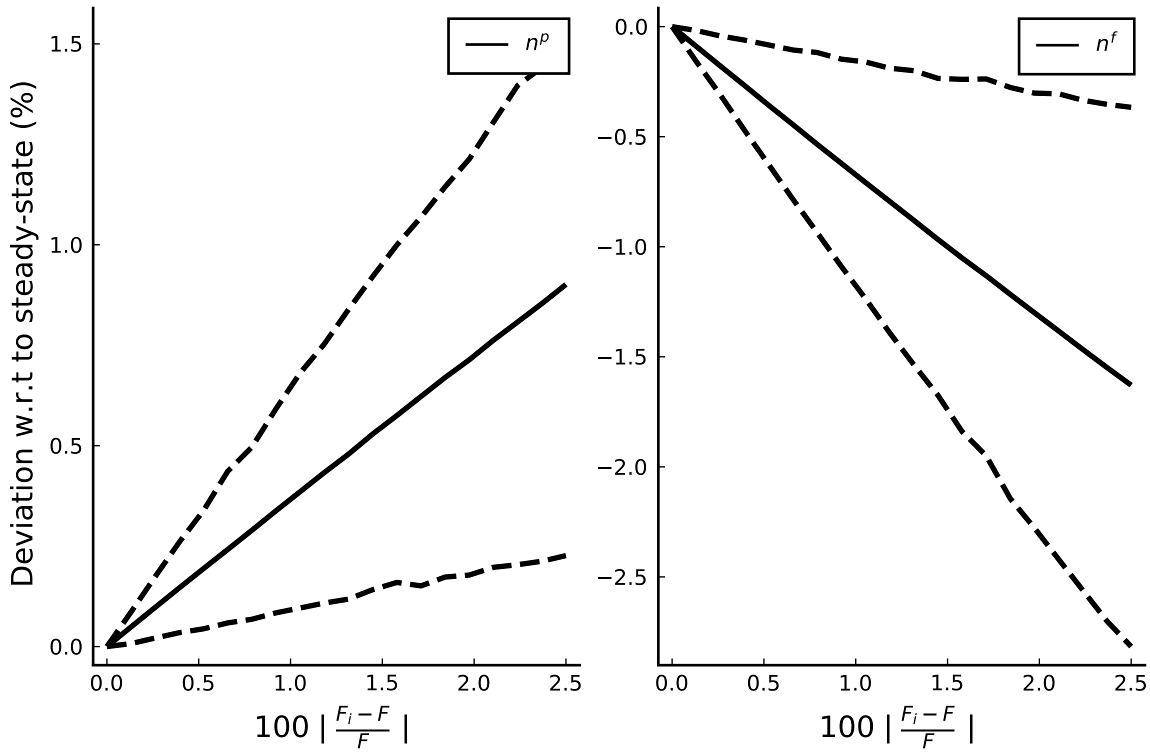

Figure 3.6: Extend of regulatory uncertainty and employment

Steady-state employments when $\epsilon = 0$ and the value of F is the value such that the targets in Table 3.2 is met on average when $F_2 = F(1+\Delta)$ and $F_1 = F(1-\Delta)$. Parameters are estimated for different values of $\mid \frac{F_1 - F}{F} \mid = \mid \frac{F_2 - F}{F} \mid$ using the model with regulatory uncertainty. A simulated method of moments with 1,000 paths of policy shocks with a 1,000-month duration each is carried. The average and 95-per-cent confidence interval appear as plain and dashed lines.

ended jobs magnify the curvature of transition probabilities, employments, average productivities, welfare and adjustment time with respect to firing costs. Open-ended job destruction flows are thicker and firing costs are more frequently paid. The differences between the dual and classic labor markets enlarge. In quantitative terms, changes in firing cost with a similar size have a stronger impact in terms of employment and welfare. The productivity shock arrival rate significantly impacts the steady-state outcome of a change in firing costs. If the value of the productivity shock arrival rate is not well known, a cut in firing costs leads to pretty unpredictable steady-state employment outcomes. Higher productivity shock arrival rate lead to longer adjustment durations up to 5 years.

4 Conclusion

In this paper, I have built a simple matching model with both fixed-term and open-ended contracts. The model provides a theoretical rationale to explain the contractual choice at the hiring step: fixed-term contracts act as stopgaps, offering both production and a possibility to return quickly on the labor market to fall onto a high-productivity match. In terms of policy, the removal of fixed-term contracts and a strong cut in firing costs leads to a gain in both welfare and employment within 18 months. Frequent marginal changes in firing costs push up unemployment and fixed-term employment and weaken open-ended employment.

Understanding the mechanisms underpinning dynamics in a dual labor market requires a few sacrifices. Still, tractability comes at a high cost: productivity shocks are i.i.d, there is no on-the-job search or conversion of fixed-term contracts into open-ended contracts or proper modeling of probationary employment periods. The next paper should be the quantitative counterpart of the present one.

References

Bruno Amable. Who wants the contrat de travail unique? social support for labor market flexibilization in france. *Industrial Relations: A Journal of Economy and Society*, 53(4):636–662, 2014.

Samuel Bentolila and Gilles Saint-Paul. The macroeconomic impact of flexible labor contracts, with an application to spain. *European Economic Review*, 36(5):1013 – 1047, 1992. ISSN 0014-2921. doi: http://dx.doi.org/10.1016/0014-2921(92)90043-V.

Samuel Bentolila, Pierre Cahuc, Juan J. Dolado, and Thomas Le Barbanchon. Two-tier labour markets in the great recession: France versus spain*. *The Economic Journal*, 122(562):F155–F187, 2012. doi: 10.1111/j.1468-0297.2012.02534.x.

Tito Boeri and Pietro Garibaldi. A tale of comprehensive labor market reforms: Evidence from the italian jobs act. *Labour Economics*, 59:33–48, 2019.

Anne Bucher. Hiring Practices, Employment Protection and Temporary Jobs. working paper or preprint, June 2010.

Michael Burda and Charles Wyplosz. Gross worker and job flows in europe. *European economic review*, 38(6):1287–1315, 1994.

Pierre Cahuc and Fabien Postel-Vinay. Temporary jobs, employment protection and labor market performance. *Labour Economics*, 9(1):63 – 91, 2002. ISSN 0927-5371. doi: https://doi.org/10.1016/S0927-5371(01)00051-3.

Pierre Cahuc, Olivier Charlot, and Franck Malherbet. Explaining the spread of temporary jobs and its impact on labor turnover. *International Economic Review*, 57(2):533–572, 2016. ISSN 1468-2354. doi: 10.1111/iere.12167.

Pierre Cahuc, Olivier Charlot, Franck Malherbet, Helène Benghalem, and Emeline Limon. Taxation of Temporary Jobs: Good Intentions with Bad Outcomes? *The Economic Journal*, 130(626): 422–445, 11 2019. ISSN 0013-0133. doi: 10.1093/ej/uez062.

Shutao Cao, Enchuan Shao, and Pedro Silos. Fixed-Term and Permanent Employment Contracts: Theory and Evidence. Technical report, 2010.

Oswaldo Luiz do Valle Costa, Marcelo D Fragoso, and Marcos G Todorov. *Continuous-time Markov jump linear systems*. Springer Science & Business Media, 2012.

Juan J. Dolado, Carlos GarcíaSerrano, and Juan F. Jimeno. Drawing Lessons from the Boom of Temporary Jobs in Spain. *The Economic Journal*, 112(480):F270–F295, 06 2002. ISSN 0013-0133. doi: 10.1111/1468-0297.00048.

Juan Jose Dolado, Etienne Lalé, and Nawid Siassi. From dual to unified employment protection: Transition and steady state. *Available at SSRN 2702404*, 2018.

Renato Faccini. Reassessing labour market reforms: Temporary contracts as a screening device. *The Economic Journal*, 124(575):167–200, 2014. doi: 10.1111/ecoj.12072.

François Fontaine and Franck Malherbet. *CDD vs CDI: les effets d'un dualisme contractuel*. Presses de Sciences Po, 2016.

Idriss Fontaine et al. French unemployment dynamics: a three-state approach. *Revue d'économie politique*, 126(5):835–869, 2016.

Paul A. Grout. Investment and wages in the absence of binding contracts: A nash bargaining approach. *Econometrica*, 52(2):449–460, 1984. ISSN 00129682, 14680262.

Jean-Olivier Hairault, Thomas Le Barbanchon, and Thepthida Sopraseuth. The cyclicality of the separation and job finding rates in france. *European Economic Review*, 76:60 – 84, 2015. ISSN 0014-2921. doi: https://doi.org/10.1016/j.euroecorev.2015.01.013.

ILO. Employment protection legislation database.

Francis Kramarz and Marie-Laure Michaud. The shape of hiring and separation costs in france. *Labour Economics*, 17(1):27 – 37, 2010. ISSN 0927-5371. doi: https://doi.org/10.1016/j.labeco.2009.07.005.

Edward P Lazear. Job security provisions and employment. *The Quarterly Journal of Economics*, pages 699–726, 1990.

Nicolas Lepage-Saucier, Juliette Schleich, and Etienne Wasmer. 14. le contrat de travail unique: quid pro quo ou simple quiproquo? *Regards croises sur l'economie*, (1):263–277, 2013.

Kévin Milin. Cdd, cdi : comment évoluent les embauches et les ruptures depuis 25 ans ? *Dares analyses*, (26), June 2018.

Dale T. Mortensen and Christopher A. Pissarides. Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3):397–415, 1994. ISSN 00346527, 1467937X.

Dale T Mortensen and Christopher A Pissarides. New developments in models of search in the labor market. *Handbook of labor economics*, 3:2567–2627, 1999.

OECD. Detailed description of employment protection legislation, 2012–2013. 2013.

Christopher A Pissarides et al. Equilibrium unemployment theory. *MIT Press Books*, 1, 2000.

Anne E Polivka. Contingent and alternative work arrangements, defined. *Monthly Lab. Rev.*, 119:3, 1996.

Project Jupyter, Matthias Bussonnier, Jessica Forde, Jeremy Freeman, Brian Granger, Tim Head, Chris Holdgraf, Kyle Kelley, Gladys Nalvarte, Andrew Osheroff, M Pacer, Yuvi Panda, Fernando Perez, Benjamin Ragan Kelley, and Carol Willing. Binder 2.0 - Reproducible, interactive, sharable environments for science at scale. In Fatih Akici, David Lippa, Dillon Niederhut, and M Pacer, editors, *Proceedings of the 17th Python in Science Conference*, pages 113 – 120, 2018. doi: 10.25080/Majora-4af1f417-011.

Normann Rion. Fluctuations in a Dual Labor Market. working paper or preprint, May 2020.

Normann Rion. Job Creation in a Dual Labor Market: a Constructivist Approach. working paper or preprint, 2021.

G. Saint-Paul. *Dual Labor Markets: A Macroeconomic Perspective*. MIT Press, 1996. ISBN 9780262193764.

Hector Sala, José I. Silva, and Manuel Toledo. Flexibility at the margin and labor market volatility in oecd countries*. *The Scandinavian Journal of Economics*, 114(3):991–1017, 2012. doi: 10.1111/j.1467-9442.2012.01715.x.

Héctor Sala and José I. Silva. Flexibility at the margin and labour market volatility: The case of Spain. *Investigaciones Economicas*, 33(2):145–178, May 2009.

Nawid Siassi, Juan J. Dolado, and Etienne Lalé. Moving Towards a Single Labor Contract: Transition vs. Steady State. Technical report, 2015.

Eric Smith. Limited duration employment. *Review of Economic Dynamics*, 10(3):444 – 471, 2007. ISSN 1094-2025. doi: https://doi.org/10.1016/j.red.2007.01.001.

Mauricio M Tejada. Dual labor markets and labor protection in an estimated search and matching model. *Labour Economics*, 46:26–46, 2017.

A Robustness Checks

A.1 $\lambda = 1$

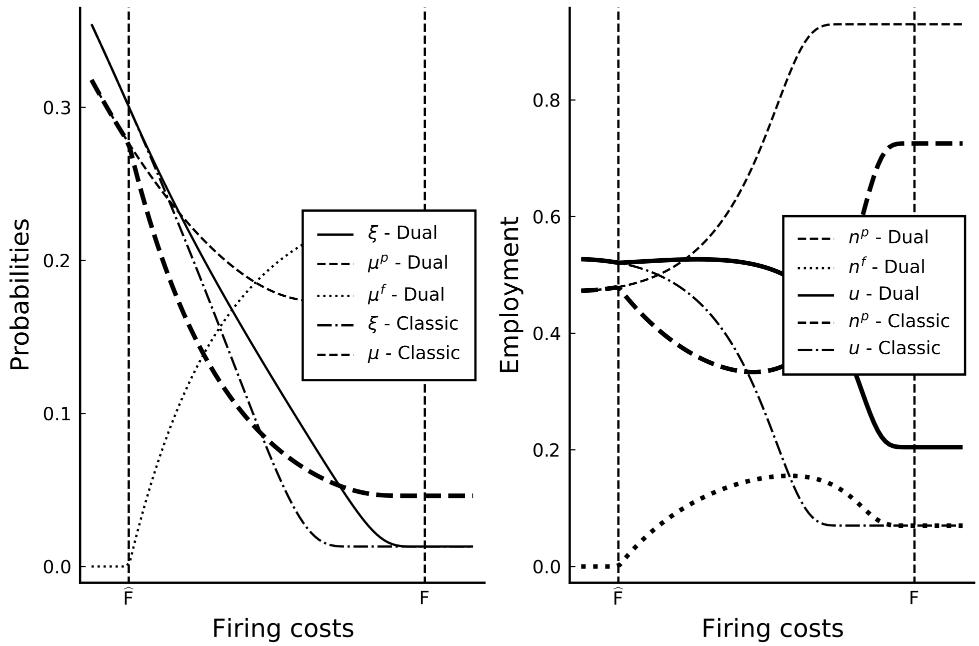

Figure A.1: Steady-state employment values and firing costs

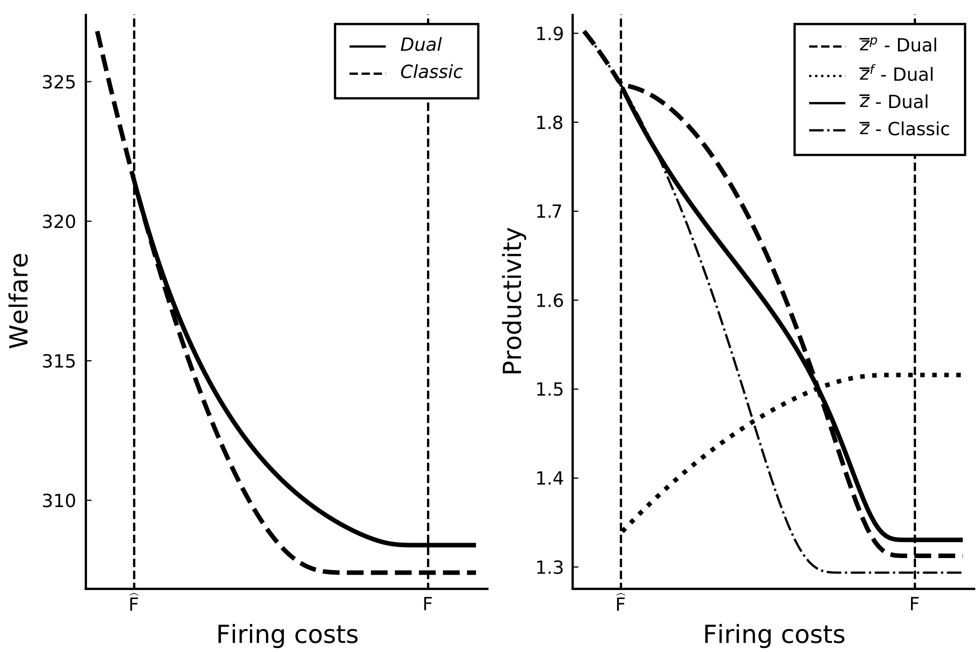

Figure A.2: Social welfare and firing costs

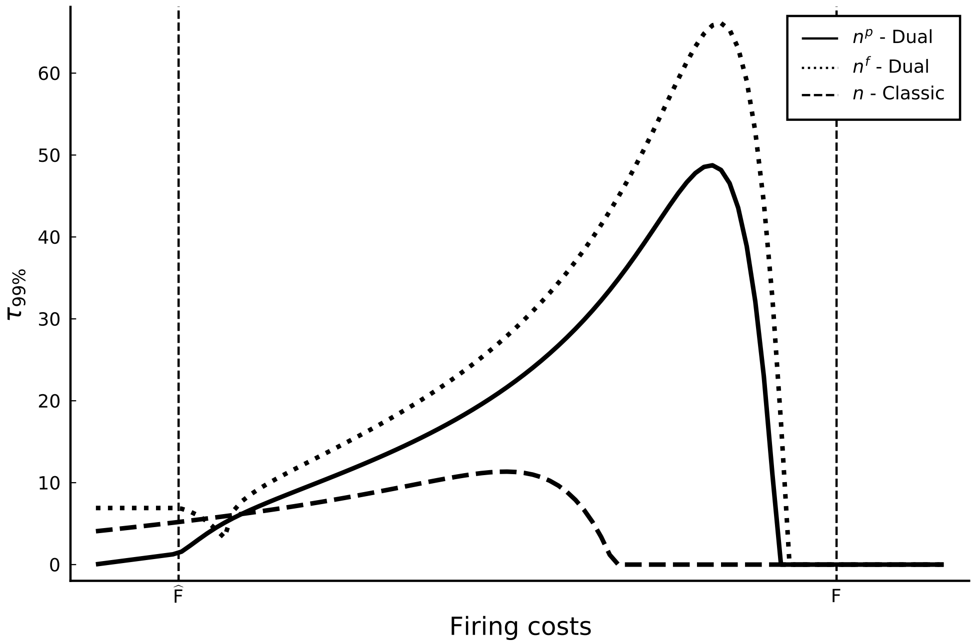

Figure A.3: $\tau_{99\%}$ of different reforms with respect to the post-reform firing costs in months

A.2 $\lambda = 1/36$

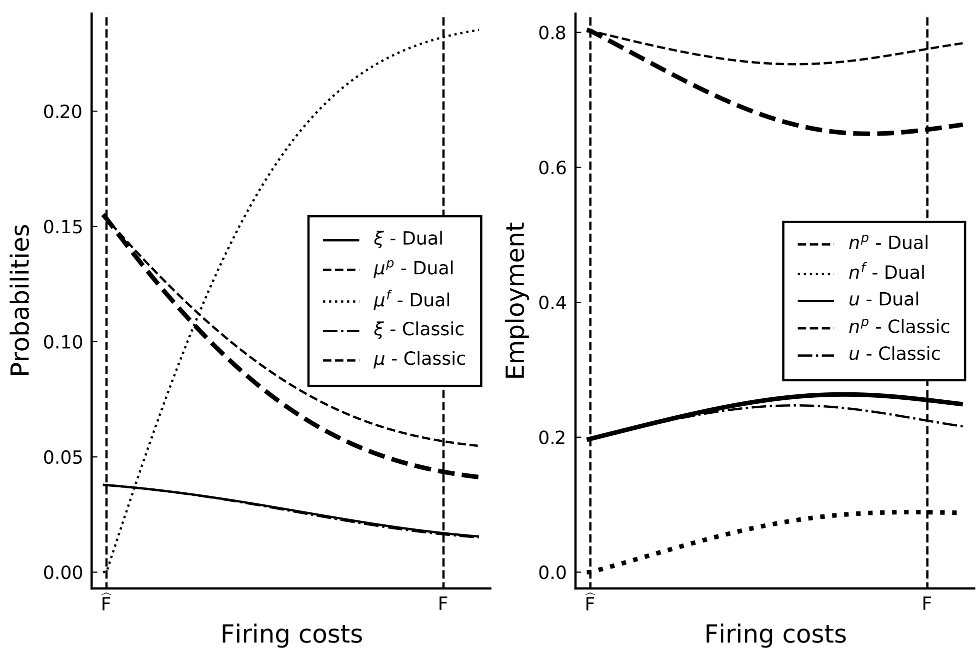

Figure A.4: Steady-state employment values and firing costs

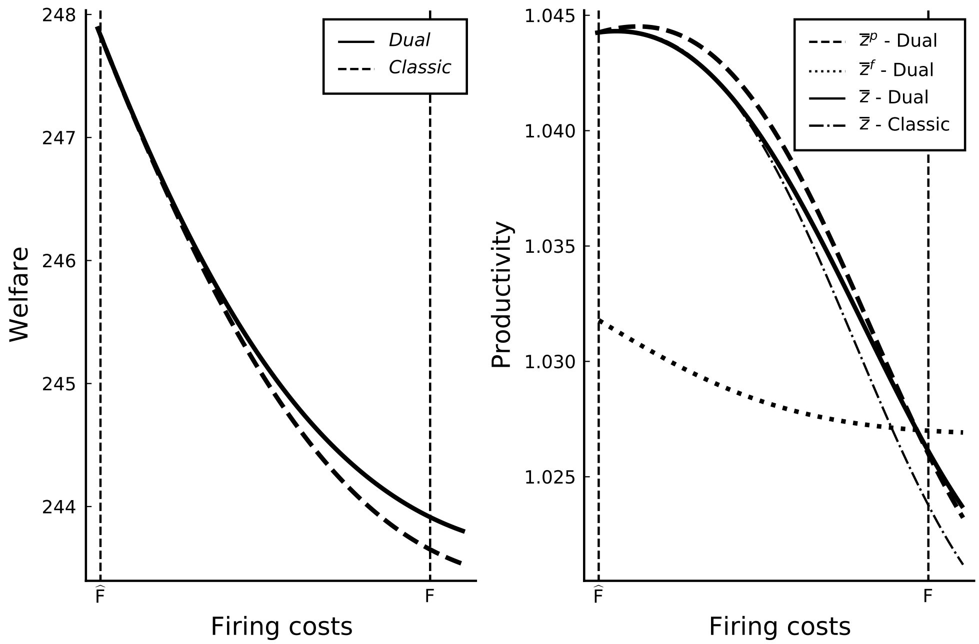

Figure A.5: Social welfare and firing costs

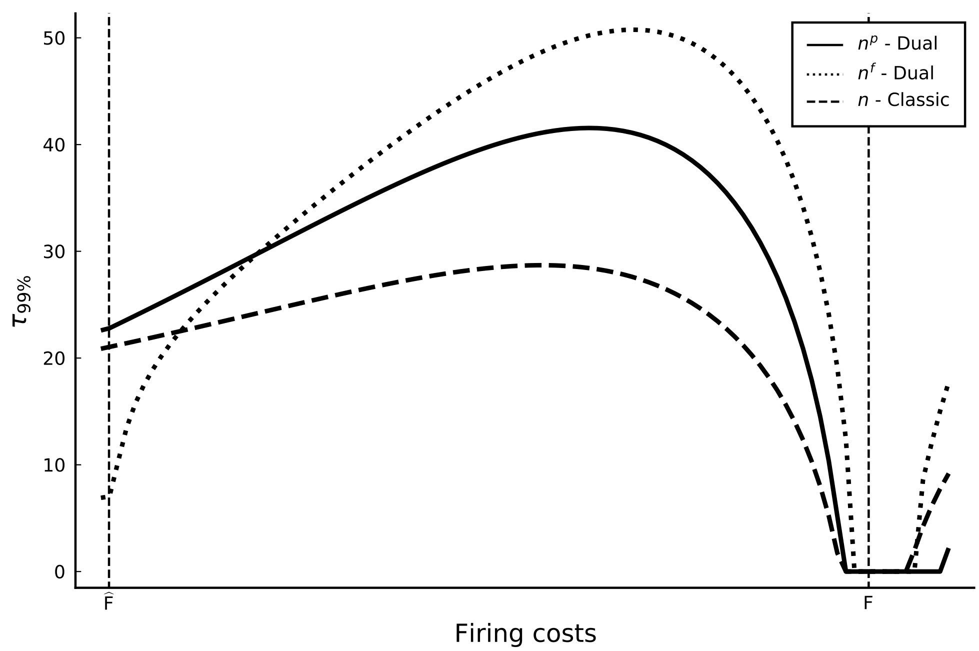

Figure A.6: $\tau_{99\%}$ of different reforms with respect to the post-reform firing costs in months

A.3 $s/\xi = 0$

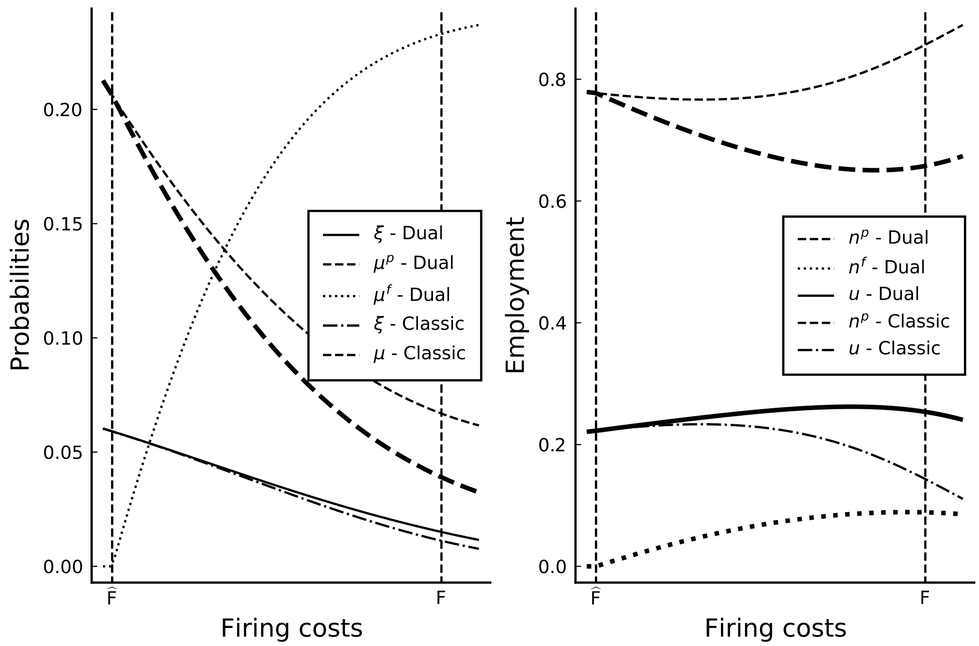

Figure A.7: Steady-state employment values and firing costs

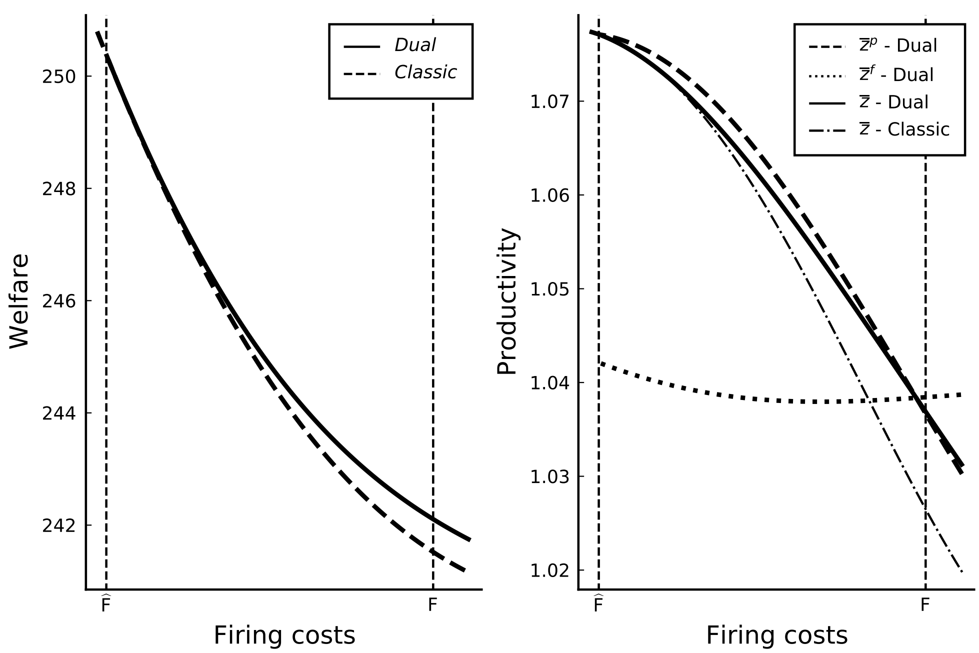

Figure A.8: Social welfare and firing costs

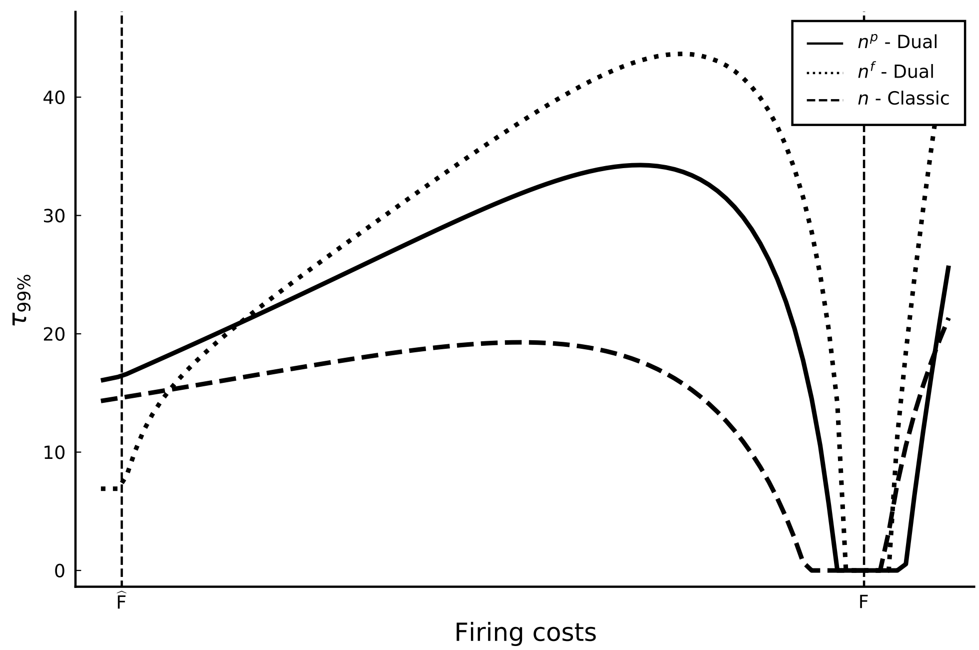

Figure A.9: $\tau_{99\%}$ of different reforms with respect to the post-reform firing costs in months

A.4 $u = 0.1$

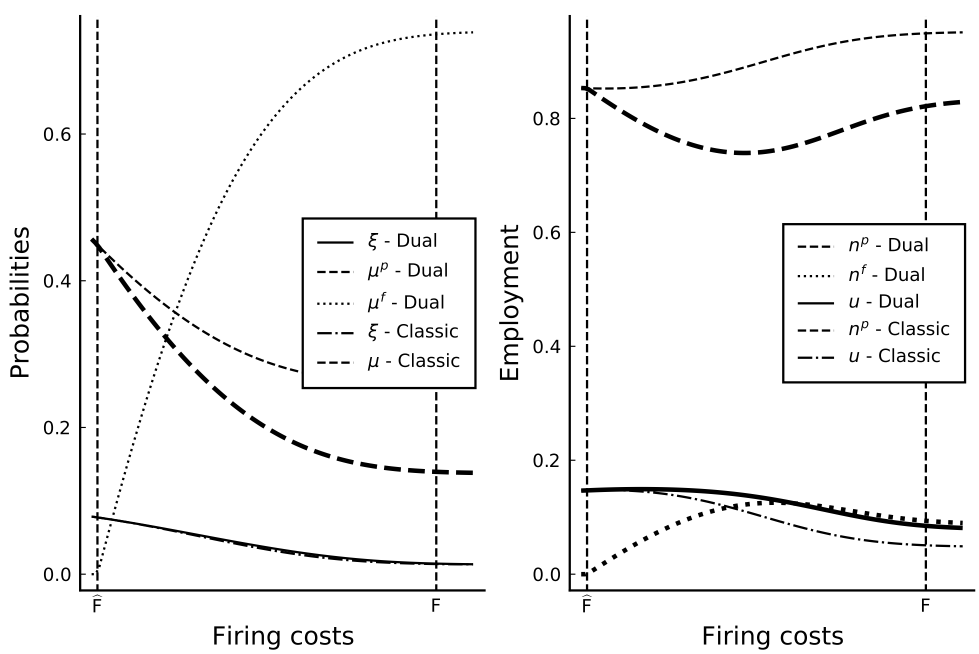

Figure A.10: Steady-state employment values and firing costs

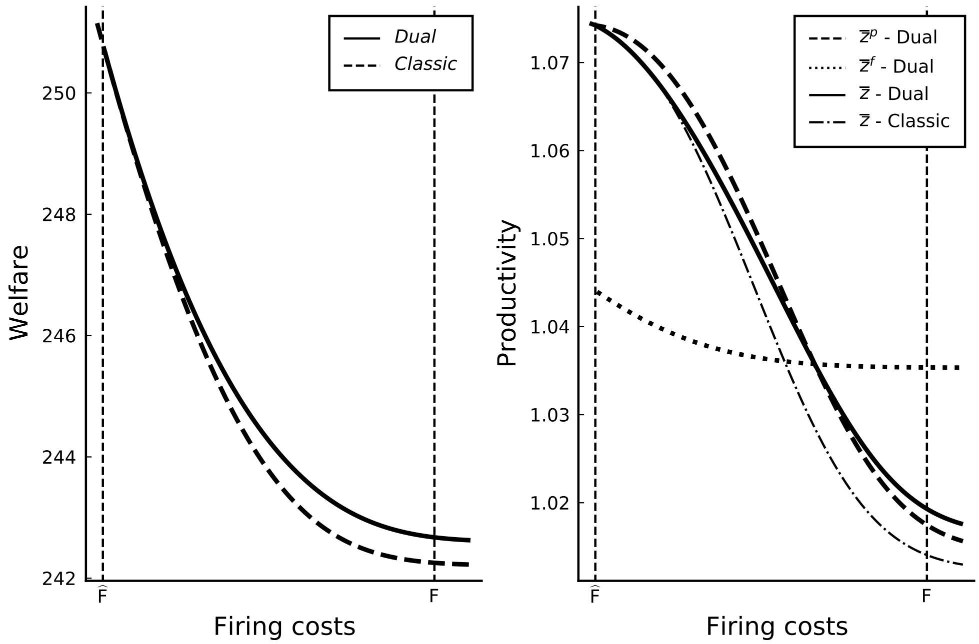

Figure A.11: Social welfare and firing costs

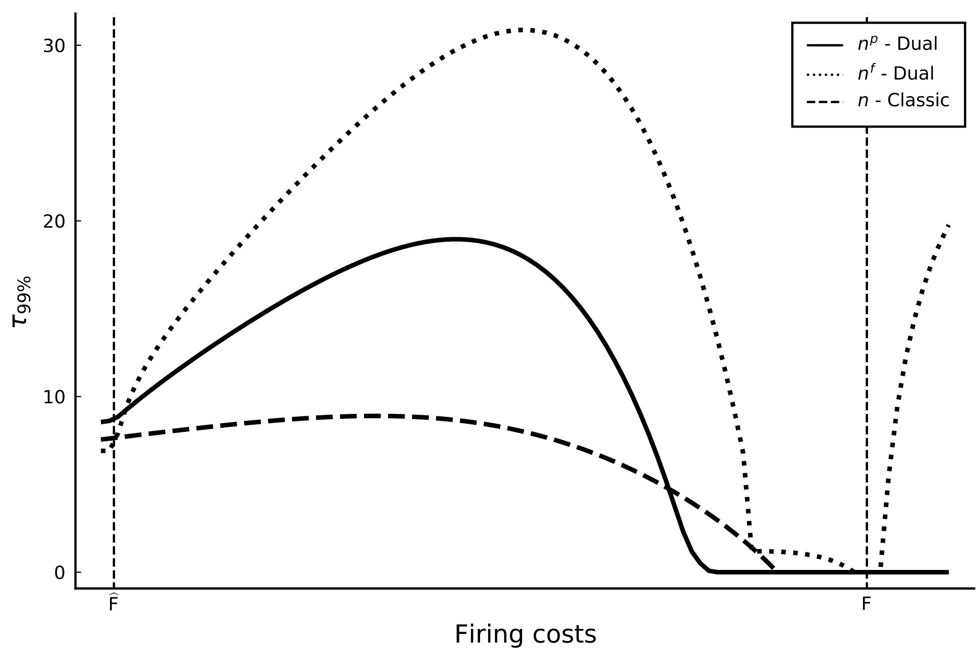

Figure A.12: $\tau_{99\%}$ of different reforms with respect to the post-reform firing costs in months

B Proofs

Proposition 1 I denote $\rho^p = 1/(r+s+\lambda)$ and $\rho^f = 1/(r+\delta+\lambda)$. As mentioned above, $\rho^p > \rho^f$.

- Assume that $z^* > z^f$. (2.3.11) implies that $\rho^p z^* = \left(\rho^p - \rho^f\right) z^* + \rho^f z^* = \rho^p z^c + \rho^f \left(z^* - z^f\right)$. Since $z^* - z^f > 0$, the latter equality implies $z^* > z^c$.

- Assume that $z^* > z^c$. Again, jointly with algebraic manipulations, (2.3.11) implies that $\rho^f z^c = -\left(\rho^p - \rho^f\right) z^c + (\rho^p - \rho^f) z^* + \rho^f z^f > -\left(\rho^p - \rho^f\right) z^c + (\rho^p - \rho^f) z^c + \rho^f z^f > \rho^f z^f$, which entails that $z^c > z^f$.

- Assume that $z^c > z^f$. Algebraic manipulations and (2.3.11) imply that $\left(\rho^p - \rho^f\right) z^* = \rho^p \left(z^c - z^f\right) + \left(\rho^p - \rho^f\right) z^f > \left(\rho^p - \rho^f\right) z^f$, which implies $z^* > z^f$. □

Proposition 2 Referring to (2.3.13),

- If open-ended workers are the only ones hired, then $max\left[z^f, z^*\right] \leq z^f$, implying that $z^* \leq z^f$. Referring to proposition 2, the latter inequality entails $z^f \leq z^c$. As a result, $z^* \leq z^f \leq z^c$.

- If job creation is dual, then

$$\begin{cases} 0 < \max\left[z^f, z^*\right] \\ z^f < z^* \end{cases}$$

  Using proposition 2, the latter system of inequalities boils down to $\max\left[0, z^f\right] < z^*$.

For each case, the converse propositions are straightforward using (2.3.13). □

Proposition 3

- Considering the case where job creation is dual and there is endogenous destruction of open-ended jobs, We differentiate the different equations.

$$\begin{aligned}
&\frac{r+\xi}{r+s+\lambda} dz^p = \frac{\eta\gamma}{1-\eta} d\theta - (r+s) dF \\
&\frac{r+\delta}{r+\delta+\lambda} dz^f = \frac{\eta\gamma}{1-\eta} d\theta \\
&dz^c = dz^p + (r+s+\lambda) dF \\
&\left(\frac{1}{r+s+\lambda} - \frac{1}{r+\delta+\lambda}\right) dz^* = \frac{dz^c}{r+s+\lambda} - \frac{dz^f}{r+\delta+\lambda} \\
&-\frac{\gamma q'}{(1-\eta) q^2(\theta)} d\theta = -\left(1 - G\left(z^*\right)\right)\left(\frac{1}{r+s+\lambda} - \frac{1}{r+\delta+\lambda}\right) dz^* - \left(1 - G\left(z^f\right)\right) \frac{dz^f}{r+\delta+\lambda}
\end{aligned}$$

  Substituting the expression of dz^p into the definition of dz^c, we get

$$\frac{dz^c}{r+s+\lambda} = \frac{1}{r+\xi}\left(\frac{\eta\gamma}{1-\eta} d\theta + \lambda G\left(z^p\right) dF\right)$$

  In turn, this expression for dz^c can be substituted into the definition of dz^*.

$$\left(\frac{1}{r+s+\lambda} - \frac{1}{r+\delta+\lambda}\right) dz^* = \frac{1}{r+\xi}\left(\frac{\eta\gamma}{1-\eta} d\theta + \lambda G\left(z^p\right) dF\right) - \frac{1}{r+\delta+\lambda} dz^f$$

  Reintroducing the expression of dz^f, the differentiated job creation condition becomes,

$$-\frac{\gamma q'}{(1-\eta)q^2(\theta)}d\theta = -\frac{1-G\left(z^*\right)}{r+\xi}\left[\frac{\eta\gamma}{1-\eta}d\theta + \lambda G\left(z^p\right)dF\right] - \frac{G\left(z^*\right)-G\left(z^f\right)}{r+\delta}\frac{\eta\gamma}{1-\eta}d\theta$$

As a result,

$$\frac{\partial\theta}{\partial F} = -\frac{\frac{1-G(z^*)}{r+\xi}\lambda G\left(z^p\right)}{-\frac{\gamma q'}{(1-\eta)q^2(\theta)} + \frac{\eta\gamma}{1-\eta}\left[\frac{1-G(z^*)}{r+\xi} + \frac{(G(z^*)-G(z^f))}{r+\delta}\right]} < 0$$

This entails $\frac{\partial z^f}{\partial F} < 0$ and $\frac{\partial z^p}{\partial F} < 0$.

In addition,

$$\begin{aligned}
\frac{\partial z^c}{\partial F} &\propto \frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F} + \lambda G\left(z^p\right) \\
\frac{\partial z^c}{\partial F} &\propto -\frac{\gamma q'}{(1-\eta)q^2(\theta)} + \frac{G\left(z^*\right)-G\left(z^f\right)}{r+\delta}\frac{\eta\gamma}{1-\eta} > 0
\end{aligned}$$

Jointly with the fact that $\frac{\partial z^f}{\partial F} < 0$, We get that $\frac{\partial z^*}{\partial F} > 0$.

- When $z^p \leq 0$, we have that $\frac{\partial\theta}{\partial F} = 0$ and, consequently, $\frac{\partial z^f}{\partial F} = 0$ and $\frac{\partial z^c}{\partial F} = 0$. This leads to $\frac{\partial z^*}{\partial F} = 0$.
- Similar computations can be carried out for the other case, where job creation occurs through open-ended contracts only. The only change lies in the job creation condition.

$$-\frac{\gamma q'}{(1-\eta)q^2(\theta)}d\theta = -\frac{1-G\left(z^c\right)}{r+s+\lambda}dz^c$$

Introducing the expression of dz^c, the differentiated job creation condition becomes,

$$-\frac{\gamma q'(\theta)}{(1-\eta)q^2(\theta)}d\theta = -\frac{1-G\left(z^c\right)}{r+\xi}\left(\frac{\eta\gamma}{1-\eta}d\theta + \lambda G\left(z^p\right)dF\right)$$

As a result,

$$\frac{\partial\theta}{\partial F} = -\frac{\frac{1-G(z^c)}{r+\xi}\lambda G\left(z^p\right)}{-\frac{\gamma q'}{(1-\eta)q^2(\theta)} + \frac{\eta\gamma}{1-\eta}\frac{1-G(z^c)}{r+\xi}} < 0$$

and

$$\begin{aligned}
\frac{\partial z^c}{\partial F} &\propto \frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F} + \lambda G\left(z^p\right) \\
\frac{\partial z^c}{\partial F} &\propto -\frac{\gamma q'}{(1-\eta)q^2(\theta)} > 0
\end{aligned}$$

$\frac{\partial z^p}{\partial F} < 0$ is still true. $\square$

Proposition 4 To prove this result, we will rely on the two corner equilibria that are implicitly present in proposition 2.

- In the first case, consider the case where $z^f = z^c = z^* = \widehat{z}$, which is the knife-edge case associated with proposition 1. In this case, the equilibrium is summed up by equations (2.3.13) and (2.3.4), where θ and z^f are respectively replaced by $\widehat{\theta}$ and $\widehat{z}$. Taking into account the fact that $z^f = z^c = z^*$ in the job creation equation, the two latter equations boil down to

$$\widehat{z} = \left(1 + \frac{\lambda}{r+\delta}\right)\left(b + \frac{\eta\gamma\widehat{\theta}}{1-\eta}\right) - \frac{\lambda}{r+\delta}E_z$$
$$\frac{\gamma}{(1-\eta)q\left(\widehat{\theta}\right)} = \int_{\widehat{z}}^{+\infty}(1-G(z))\,dz$$

  Given the equilibrium values $\left(\widehat{\theta}, \widehat{z}\right)$, $z^c = \widehat{z}$ along with (2.3.2) and (2.3.7) entail that F_1 verifies

$$\widehat{z} - b - \lambda\widehat{F} + \frac{\lambda}{r+s+\lambda}\int_{\widehat{z}-(r+s+\lambda)F_1}^{+\infty}(1-G(z))dz = \frac{\eta\gamma\widehat{\theta}}{1-\eta}$$

  Proposition 2 ensures that job creation only occurs through open-ended contracts. Replacing μ^f by zero in (2.4.4) demonstrates that $n^f = 0$ at the equilibrium.

  A marginal increase in firing costs immediately entails an increase in z^c while z^f diminishes according to proposition 3. The initial situation being $z^f = z^c = \widehat{z}$, we now face $z^f < \widehat{z} < z^c$, which is equivalent to $z^f < z^*$ as Proposition 1 states. Job creation is now dual, as asserts Proposition 2. Conversely, a marginal decrease in firing costs implies a cut in z^* and z^c as well as an increase in z^f, which results in $z^c < z^f$. The latter is equivalent to $z^* < z^c$ under proposition 2. The resulting equilibrium only involves open-ended contracts.

- In the second case, consider the case where $z^p = 0$. There is no endogenous destruction of open-ended matches. The equilibrium can be summed up by $\left(\widetilde{F}, \theta, z^p, z^c, z^f, z^*\right)$ verifying equations (2.3.2), (2.3.4), (2.3.7), (2.3.11) and (2.3.13) under the additional constraint that $z^p = 0$.

C Dynamics

In this section, I describe the employment dynamics after a shock to firing costs.

C.1 Dual-to-dual and classic-to-dual transitions

I shall distinguish the continuing open-ended employment $n_t^{c,p}$ and the new open-ended employment $n_t^{0,p}$. After a shock in firing costs, I assume that new open-ended matches do not renegotiate wages and their outside option does not include the payment of firing costs in case of disagreement. I do so to avoid overlooking the impact of firing costs.

I consider the model starting from the equilibrium defined by $\left(\theta_0, z_0^p, z_0^c, z_0^f, z_0^*\right)$ with firing costs F_0 at time $t = 0^-$ and the associated open-ended and fixed-term employment values $n_0^{c,p}$, $n_0^{0,p}$ and

n_0^f. The former tuple solely consists in forward-looking variables. Therefore, assuming that firing costs jump from F_0 to F on time $t = 0$, the equilibrium tuple defined in definition 1 immediately jumps to its new value $\left(\theta, z^p, z^c, z^f, z^*\right)$. As for employment, notice that if $z^p > z_0^p$ — which occurs if $F < F_0$ — continuing open-ended matches such that $z_0^p < z < z^p$ split right away. Similarly, new open-ended matches such that $\max\left[z_0^*, z_0^c\right] < z < z^c$ also separate immediately. Denoting δ_0 Dirac function in 0, open-ended and fixed-term contracts, equations (2.4.1)-(2.4.2) are amended as follows

$$\left\{\begin{array}{ll} \dot{n}_t^{c,p} & = -\xi n_t^{c,p} + \lambda\left(1 - G\left(z^p\right)\right) n_t^{0,p} - \delta_0 \alpha_0 n_t^{c,p} \\ \dot{n}_t^{0,p} & = -(s+\lambda) n_t^{0,p} + \mu^p \left(1 - n_t^{c,p} - n_t^{0,p} - n_t^f\right) - \delta_0 \beta_0 n_t^{0,p} \\ \dot{n}_t^f & = -\delta n_t^f + \mu^f \left(1 - n_t^{c,p} - n_t^{0,p} - n_t^f\right) \end{array}\right.$$

where

$$\alpha_0 = \frac{G(\max\{z_0^p, z^p\}) - G\left(z_0^p\right)}{1 - G\left(z_0^p\right)}$$
$$\beta_0 = \frac{G(\max\left[z_0^*, z_0^c, z^c\right]) - G\left(\max\left[z_0^*, z_0^c\right]\right)}{1 - G\left(\max\left[z_0^*, z_0^c\right]\right)}$$

and ξ, μ^p and μ^f are computed as described in section 2.4 using the equilibrium tuple $\left(\theta, z^p, z^c, z^f, z^*\right)$.

Laplace transforms are particularly useful when it comes to solving systems of differential equations embedding Dirac functions. Denoting $N^{c,p}$, $N^{0,p}$ and N^f the Laplace transforms of $n^{c,p}$, $n^{0,p}$ and n^f, the latter system becomes

$$\left\{\begin{array}{rl} pN^{c,p}(p) - n_0^{c,p} & = -\xi N^{c,p} + \lambda\left(1 - G\left(z^p\right)\right) N^{0,p} - \alpha_0 n_0^{c,p} \\ pN^{0,p}(p) - n_0^{0,p} & = -(s+\lambda) N^{0,p} + \mu^p \left(\frac{1}{p} - N^{c,p}(p) - N^{0,p}(p) - N^f(p)\right) - \beta_0 n_0^{0,p} \\ pN^f(p) - n_0^f & = -\delta N^f(p) + \mu^f \left(\frac{1}{p} - N^{c,p}(p) - N^{0,p}(p) - N^f(p)\right) \end{array}\right.$$

The system above being linear in $N^{c,p}(p)$, $N^{0,p}(p)$ and $N^f(p)$, one may isolate the latter to obtain

$$\begin{aligned} N^{c,p}(p) = & \Big\{ c_0^{c,p} p^3 + (s + \lambda + \delta + \mu^f + \mu^p) c_0^{c,p} p^2 \\ & + \left(\left[(s+\lambda)(\delta + \mu^f) + \delta\mu^p\right] c_0^{c,p} + \lambda\left(1 - G\left(z^p\right)\right)\left[\mu^p\left(1 - n_0^f\right) + \left(\delta + \mu^f\right) c_0^{0,p}\right]\right) p \\ & + \lambda\left(1 - G\left(z^p\right)\right)\mu^p\delta \Big\} \\ & \Big\{ p(p + s + \lambda)(p^2 + (\xi + \delta + \mu^p + \mu^f)p + \mu^f\xi + \delta\xi + \delta\mu^p) \Big\}^{-1} \end{aligned}$$

$$\begin{aligned} N^{0,p}(p) = & \Big\{ c_0^{0,p} p^3 + \left[\left(\xi + \delta + \mu^f\right) c_0^{0,p} + \mu^p\left(1 - c_0^{c,p} - n_0^f\right)\right] p^2 \\ & + \left[\mu^p\delta\left(1 - c_0^{c,p}\right) + \xi\mu^p\left(1 - n_0^f\right) + \xi\left(\delta + \mu^f\right) c_0^{0,p}\right] p + \xi\delta\mu^p \Big\} \\ & \Big\{ p(p + s + \lambda)(p^2 + (\xi + \delta + \mu^p + \mu^f)p + \mu^f\xi + \delta\xi + \delta\mu^p) \Big\}^{-1} \end{aligned}$$

$$N^f(p) = \frac{n_0^f p^2 + \left[\mu^f \left(1 - c_0^{c,p} - c_0^{0,p}\right) + (\xi + \mu^p)\, n_0^f\right] p + \xi \mu^f}{p(p^2 + (\xi + \delta + \mu^p + \mu^f)p + \mu^f \xi + \delta\xi + \delta\mu^p)}$$

where $c_0^{c,p} = (1 - \alpha_0)\, n_0^{c,p}$ and $c_0^{0,p} = (1 - \beta_0)\, n_0^{0,p}$.

The denominators of fractions above have roots 0, α_1, α_2 and $\omega = -(s + \lambda)$ where

$$\begin{aligned}
\alpha_1 &= \frac{1}{2}\left(-\left(\xi + \delta + \mu^p + \mu^f\right) + \sqrt{\left(\xi + \delta + \mu^p + \mu^f\right)^2 - 4\left(\xi\delta + \xi\mu^f + \mu^p\delta\right)}\right)\\
\alpha_2 &= \frac{1}{2}\left(-\left(\xi + \delta + \mu^p + \mu^f\right) - \sqrt{\left(\xi + \delta + \mu^p + \mu^f\right)^2 - 4\left(\xi\delta + \xi\mu^f + \mu^p\delta\right)}\right)
\end{aligned}$$

Using a partial fraction decomposition, one may write for $i \in \{0, c\}$

$$\begin{aligned}
N^{i,p}(p) &= \frac{n_\infty^{i,p}}{p} + \frac{\rho_0^{i,p}}{p - \omega} + \frac{\rho_1^{i,p}}{p - \alpha_1} + \frac{\rho_2^{i,p}}{p - \alpha_2} \quad i \in \{0, c\}\\
N^f(p) &= \frac{n_\infty^f}{p} + \frac{\rho_1^f}{p - \alpha_1} + \frac{\rho_2^f}{p - \alpha_2}
\end{aligned}$$

where $n_\infty^{i,p}$, $\rho_0^{i,p}$, $\rho_1^{i,p}$, ρ_1^f, $\rho_2^{i,p}$ and ρ_2^f verify

$$\begin{aligned}
n_\infty^{c,p} &= \frac{\lambda\left(1 - G\left(z^p\right)\right)\mu^p\delta}{(s + \lambda)(\mu^f\xi + \delta\xi + \delta\mu^p)}\\
n_\infty^{0,p} &= \frac{\xi\mu^p\delta}{(s + \lambda)(\mu^f\xi + \delta\xi + \delta\mu^p)}\\
n_\infty^f &= \frac{\mu^f\xi}{\mu^p\delta + \xi\delta + \mu^f\xi}\\
\rho_0^{c,p} &= \frac{\left[(\xi + \delta + \mu^f + \omega)c_0^{0,p} + \mu^p\left(1 - c_0^{c,p} - n_0^f\right)\right]\omega + \mu^p\delta\left(1 - c_0^{c,p}\right) + \xi\mu^p\left(1 - n_0^f\right) + (\delta + \mu^f)\xi c_0^{0,p} - \alpha_1\alpha_2 n_\infty^{0,p}}{\left(\omega - \alpha_1\right)\left(\omega - \alpha_2\right)}\\
\rho_0^{0,p} &= -\rho_0^{c,p}\\
\rho_1^{i,p} &= \frac{\alpha_1 A_{i,p} + B_{i,p}}{\alpha_1 - \alpha_2}\\
\rho_1^f &= \frac{\alpha_1 A_f + B_f}{\alpha_1 - \alpha_2}\\
\rho_2^{i,p} &= -\frac{\alpha_2 A_{i,p} + B_{i,p}}{\alpha_1 - \alpha_2}\\
\rho_2^f &= -\frac{\alpha_2 A_f + B_f}{\alpha_1 - \alpha_2}
\end{aligned}$$

and where

$$\begin{aligned}
n_0^p &= n_0^{c,p} + n_0^{0,p}\\
A_{c,p} &= c_0^{c,p} - n_\infty^{c,p} - \rho_0^{c,p}\\
A_{0,p} &= c_0^{0,p} - n_\infty^{0,p} - \rho_0^{0,p}
\end{aligned}$$

$$
\begin{aligned}
A_f &= n_0^f - n_\infty^f \\
B_{c,p} &= \left(\delta + \mu^p + \mu^f\right) c_0^{c,p} - (\omega+\xi) c_0^{0,p} + (\alpha_1+\alpha_2)\, n_\infty^{c,p} + (\alpha_1+\alpha_2-\omega)\, \rho_0^{c,p} \\
B_{0,p} &= \left(\xi + \delta + \mu^f\right) c_0^{0,p} + \mu^p \left(1 - c_0^{c,p} - n_0^f\right) + (\alpha_1+\alpha_2)\, n_\infty^{0,p} + (\alpha_1+\alpha_2-\omega)\, \rho_0^{0,p} \\
B_f &= (\xi + \mu^p)\, n_0^f + \mu^f \left(1 - c_0^{c,p} - c_0^{0,p}\right) + (\alpha_1+\alpha_2)\, n_\infty^f
\end{aligned}
$$

The inverse Laplace transforms lead to

$$
\begin{aligned}
n_t^{i,p} &= n_\infty^{i,p} + \rho_0^{i,p} exp\{\omega t\} + \rho_1^{i,p} exp\{\alpha_1 t\} + \rho_2^{i,p} exp\{\alpha_2 t\} \quad i \in \{0, c\} \\
n_t^f &= n_\infty^f + \rho_1^f exp\{\alpha_1 t\} + \rho_2^f exp\{\alpha_2 t\}
\end{aligned}
$$

Total open-ended employment n_t^p verifies

$$
n_t^p = n_\infty^p + \rho_1^p exp\{\alpha_1 t\} + \rho_2^p exp\{\alpha_2 t\}
$$

where

$$
\begin{aligned}
n_\infty^p &= \frac{\mu^p \delta}{\mu^f \xi + \delta\xi + \delta\mu^p} \\
\rho_1^p &= \frac{\alpha_1 A_p + B_p}{\alpha_1 - \alpha_2} \\
\rho_2^p &= -\frac{\alpha_2 A_p + B_p}{\alpha_1 - \alpha_2} \\
A_p &= c_0^{c,p} + c_0^{0,p} - n_\infty^p \\
B_p &= \left(\delta + \mu^f\right) c_0^{c,p} + \mu^p \left(1 - n_0^f\right) + (\alpha_1+\alpha_2)\, n_\infty^p
\end{aligned}
$$

The classic-to-dual transition obeys the same equations with the only additional condition $n_0^f = 0$ and $\beta_0 = \frac{G(\max\{z_0^c, z^c\}) - G(z_0^c)}{1 - G(z_0^c)}$.

C.2 Dual-to-classic transition

Keeping the same notations as above, with the exception that the possibility to hire through fixed-term contracts is shut down after the reform. The resulting equilibrium is defined by the tuple (z^p, z^c, θ) which verifies equations (2.3.2), (2.3.7) and the modified job creation condition

$$
\frac{\gamma}{(1-\eta)q(\theta)} = \frac{1}{r+s+\lambda} \int_{z^c}^{+\infty} (1 - G(z))\, dz \tag{C.2.1}
$$

The only remaining transition probabilities on the labor market are the job-finding probability $\mu = p(\theta)\,(1 - G(z^c))$ and the job-destruction probability ξ, which is left unchanged. Thus, the system of equation describing dynamics after the reform on time $t = 0$ is

$$
\left\{
\begin{aligned}
\dot{n}_t^{c,p} &= -\xi n_t^{c,p} + \lambda \left(1 - G(z^p)\right) n_t^{0,p} - \delta_0 \alpha_0 n_t^{c,p} \\
\dot{n}_t^{0,p} &= -(s+\lambda) n_t^{0,p} + \mu \left(1 - n_t^{c,p} - n_t^{0,p} - n_t^f\right) - \delta_0 \beta_0 n_t^{0,p} \\
\dot{n}_t^f &= -\delta n_t^f
\end{aligned}
\right. \tag{C.2.2}
$$

where α_0 and β_0 are defined as in the dual-to-dual-reform paragraph.
Laplace transforms lead to

$$\begin{aligned}
N^{c,p}(p) = &\Big\{c_0^{c,p}p^3 + \left[(\mu+\delta-\omega)c_0^{c,p} - (\omega+\xi)c_0^{0,p}\right]p^2 + \left[\delta(\mu-\omega)c_0^{c,p} - (\omega+\xi)\left(\delta c_0^{0,p} + \mu\left(1-n_0^f\right)\right)\right]p \\
&-(\omega+\xi)\mu\delta\Big\}\left\{p(p-\omega)(p+\delta)(p+\xi+\mu)\right\}^{-1} \\
N^{0,p}(p) = &\frac{c_0^{0,p}p^3 + \left[\mu\left(1-c_0^{c,p}-n_0^f\right) + (\xi+\delta)c_0^{0,p}\right]p^2 + \left[\mu\left(\delta\left(1-c_0^{c,p}\right) + \xi\left(1-n_0^f\right)\right) + \xi\delta c_0^{0,p}\right]p + \xi\delta\mu}{p(p-\omega)(p+\delta)(p+\xi+\mu)} \\
N^f(p) = &\frac{n_0^f}{p+\delta}
\end{aligned}$$

Enforcing a partial fraction decomposition and the inverse Laplace transform yields

$$\begin{aligned}
n_t^{i,p} &= n_\infty^{i,p} + \rho_0^{i,p} exp\{\omega t\} + \rho_1^{i,p} exp\{-\delta t\} + \rho_2^{i,p} exp\{-(\xi+\mu)t\} \\
n_t^f &= n_0^f exp\{-\delta t\}
\end{aligned}$$

where

$$\begin{aligned}
n_\infty^{c,p} &= \frac{(\omega+\xi)\mu}{(\xi+\mu)\omega} \\
n_\infty^{0,p} &= -\frac{\xi\mu}{(\xi+\mu)\omega} \\
\rho_0^{c,p} &= \frac{-\left[\mu\left(1-n_0^f-c_0^{c,p}\right) + (\omega+\xi+\delta)c_0^{0,p}\right]\omega + \delta\mu c_0^{c,p} - \xi\left(\delta c_0^{0,p} + \mu\left(1-n_0^f\right)\right) - \delta(\xi+\mu)n_\infty^{c,p}}{(\omega+\delta)(\omega+\xi+\mu)} \\
\rho_0^{0,p} &= -\rho_0^{c,p} \\
\rho_1^{c,p} &= \frac{\mu c_0^{c,p} - (\omega+\xi)c_0^{0,p} - (\xi+\mu)n_\infty^{c,p} - (\xi+\mu+\omega)\rho_0^{c,p}}{\xi+\mu-\delta} \\
\rho_1^{0,p} &= \frac{\mu\left(1-c_0^{c,p}-n_0^f\right) + (\omega+\xi)c_0^{0,p} - (\xi+\mu)n_\infty^{0,p} - (\xi+\mu+\omega)\rho_0^{0,p}}{\xi+\mu-\delta} \\
\rho_2^{c,p} &= c_0^{c,p} - n_\infty^{c,p} - \rho_0^{c,p} - \rho_1^{c,p} \\
\rho_2^{0,p} &= c_0^{0,p} - n_\infty^{0,p} - \rho_0^{0,p} - \rho_1^{0,p}
\end{aligned}$$

C.3 Classic-to-classic transitions

In a classic framework, open-ended employments verify

$$\left\{\begin{array}{ll}
\dot{n}_t^{c,p} & = -\xi n_t^{c,p} + \lambda\left(1-G\left(z^p\right)\right)n_t^{0,p} - \delta_0\alpha_0 n_t^{c,p} \\
\dot{n}_t^{0,p} & = -(s+\lambda)n_t^{0,p} + \mu^p\left(1-n_t^{c,p}-n_t^{0,p}\right) - \delta_0\beta_0 n_t^{0,p}
\end{array}\right.$$

Solutions verify

$$n_t^{i,p} = n_\infty^{i,p} + \rho_0^{i,p} exp\left\{\omega t\right\} + \rho_1^{i,p} exp\left\{-(\xi+\mu)t\right\}, \quad i \in \{0,c\}$$

where

$$\begin{aligned}
n_\infty^{c,p} &= \frac{\mu(\omega+\xi)}{\omega(\xi+\mu)} \\
n_\infty^{0,p} &= -\frac{\mu\xi}{\omega(\xi+\mu)} \\
\rho_0^{c,p} &= \frac{\mu c_0^{c,p} - (\omega+\xi)c_0^{0,p} - (\xi+\mu)n_\infty^{c,p}}{\omega+\xi+\mu} \\
\rho_0^{0,p} &= -\rho_0^{c,p} \\
\rho_1^{i,p} &= c_0^{i,p} - n_\infty^{i,p} - \rho_0^{i,p}, \quad i \in \{0,c\}
\end{aligned}$$

D The model with firing-cost uncertainty

D.1 The model

Whenever possible, I simplify notations and substitute dependence in F_i with the corresponding subscripts $i \in \{1,2\}$.

$$\begin{aligned}
rV(F) &= -\gamma + q\left(\theta(F)\right) \int \max\left[J_0^p\left(z,F\right) - V(F), J^f(F) - V(F), 0\right] dG(z) \\
&\quad + \epsilon\left(\frac{V_1+V_2}{2} - V(F)\right) \\
rU(F) &= b + p(\theta(F)) \int \max\left(W_0^p\left(z',F\right) - U(F), W^f\left(z',F\right) - U(F), 0\right] dG\left(z'\right) \\
&\quad + \epsilon\left(\frac{U_1+U_2}{2} - U(F)\right) \\
rJ^p(z,F) &= z - w^p(z,F) + s\left(V - J^p(z,F)\right) + \lambda \int \left(\max\left[J^p\left(z',F\right), V(F) - F\right] - J^p(z,F)\right) dG\left(z'\right) \\
&\quad + \epsilon\left(\frac{1}{2}\max\left[J^p\left(z,F_1\right), V_1 - F_1\right] + \frac{1}{2}\max\left[J^p\left(z,F_2\right), V_2 - F_2\right] - J^p(z,F)\right) \\
rW^p(z,F) &= w^p(z,F) + \lambda \int \left(\max\left[W^p\left(z',F\right), U(F)\right] - W^p(z,F)\right) dG\left(z'\right) + s\left(U(F) - W^p(z,F)\right) \\
&\quad + \epsilon\left(\frac{1}{2}\max\left[W^p\left(z,F_1\right), U_1\right] + \frac{1}{2}\max\left[W^p\left(z,F_2\right), U_2\right] - W^p(z,F)\right) \\
rJ_0^p(z,F) &= z - w_0^p(z,F) + s\left(V(F) - J_0^p(z)\right) + \lambda \int \left(\max\left[J^p\left(z',F\right), V(F) - F\right] - J_0^p(z,F)\right) dG\left(z'\right) \\
&\quad + \epsilon\left(\frac{1}{2}\max\left[J_0^p\left(z,F_1\right), V_1\right] + \frac{1}{2}\max\left[J_0^p\left(z,F_2\right), V_2\right] - J_0^p(z,F)\right) \\
rW_0^p(z,F) &= w_0^p(z,F) + \lambda \int \left(\max\left[W^p\left(z',F\right), U(F)\right] - W_0^p(z,F)\right) dG\left(z'\right) + s\left(U(F) - W_0^p(z,F)\right) \\
&\quad + \epsilon\left(\frac{1}{2}\max\left[W_0^p\left(z,F_1\right), U_1\right] + \frac{1}{2}\max\left[W_0^p\left(z,F_2\right), U_2\right] - W_0^p(z,F)\right)
\end{aligned}$$

$$
\begin{aligned}
rJ^f(z,F) &= z - w^f(z,F) + \lambda \int \Big(J^f(z',F) - J^f(z,F)\Big)\, dG(z') + \delta\Big(V(F) - J^f(z,F)\Big) \\
&\quad + \epsilon\left(\frac{1}{2}J^f(z,F_1) + \frac{1}{2}J^f(z,F_2) - J^f(z,F)\right) \\
rW^f(z,F) &= w^f(z,F) + \lambda \int \Big(W^f(z',F) - W^f(z,F)\Big)\, dG(z') + \delta\Big(U(F) - W^f(z,F)\Big) \\
&\quad + \epsilon\left(\frac{1}{2}W^f(z,F_1) + \frac{1}{2}W^f(z,F_2) - W^f(z,F)\right)
\end{aligned}
$$

Free entry implies $V_1 = V_2 = 0$. Taking into account the Nash-bargaining rules (2.0.1)–(2.0.3), the job creation condition is

$$
\frac{\gamma}{(1-\eta)q(\theta(F))} = \int \max\Big[S_0^p(z',F), S^f(z',F), 0\Big]\, dG(z')
$$

Considering the unemployed's value function, the previous equation and Nash-bargaining rules (2.0.1)–(2.0.3) imply

$$
(r+\epsilon)U_i = \left(1 + \frac{\epsilon}{r}\right) b + \frac{\eta\gamma}{1-\eta}\left(\theta_i + \frac{\epsilon}{r}\frac{\theta_1+\theta_2}{2}\right)
$$

Algebraic manipulations along with the Nash-bargaining assumption deliver the following expressions for surpluses

$$
\begin{aligned}
(r+s+\lambda+\epsilon)S^p(z,F) &= z - (r+\epsilon)U(F) + (r+s+\epsilon)F + \lambda \int \max\Big[S^p(z',F), 0\Big]\, dG(z') \\
&\quad + \frac{\epsilon}{2}\left(\max\left[S^p(z,F_1), 0\right] + \max\left[S^p(z,F_2), 0\right]\right) \\
&\quad + \epsilon\left(\frac{U_1+U_2}{2} - \frac{F_1+F_2}{2}\right)
\end{aligned}
\tag{D.1.1}
$$

$$
\begin{aligned}
(r+s+\lambda+\epsilon)S_0^p(z,F) &= z - \lambda F - (r+\epsilon)U(F) + \epsilon\frac{U_1+U_2}{2} + \lambda \int \max\Big[S^p(z',F), 0\Big]\, dG(z') \\
&\quad + \frac{\epsilon}{2}\left(\max\left[S_0^p(z,F_1), 0\right] + \max\left[S_0^p(z,F_2), 0\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
(r+\delta+\lambda+\epsilon)S^f(z,F) &= z - (r+\epsilon)U(F) + \lambda \int S^f(z',F)\, dG(z') \\
&\quad + \epsilon\left(\frac{S^f(z,F_1)+S^f(z,F_2)}{2} + \frac{U_1+U_2}{2}\right)
\end{aligned}
$$

Proposition 5. *S^p, S_0^p and S^f are increasing in z and there exists a unique collection of thresholds $(z^p(F), z^c(F), z^f(F), z^*(F))$ such that*

$$
\begin{cases}
S^p(z^p(F), F) = 0 \\
S_0^p(z^c(F), F) = 0 \\
S^f\left(z^f(F), F\right) = 0 \\
S_0^p(z^*(F), F) = S^f(z^*(F), F)
\end{cases}
$$

Proof. I first demonstrate that S^p is increasing in z.

- If z is such that $S^p\left(z, F_i\right) > 0,\ i \in \{1,2\}$, differentiating (D.1.1) with respect to z yields

$$(r+s+\lambda+\epsilon)\frac{\partial S^p}{\partial z} = 1 + \frac{\epsilon}{2}\left(\left.\frac{\partial S^p}{\partial z}\right|_{(z,F_1)} + \left.\frac{\partial S^p}{\partial z}\right|_{(z,F_2)}\right) \tag{D.1.2}$$

  Considering the latter equation at (z, F_1) and (z, F_2) leads to the following linear system

$$\begin{cases} (r+s+\lambda+\epsilon)\left.\frac{\partial S^p}{\partial z}\right|_{(z,F_1)} = 1 + \frac{\epsilon}{2}\left(\left.\frac{\partial S^p}{\partial z}\right|_{(z,F_1)} + \left.\frac{\partial S^p}{\partial z}\right|_{(z,F_2)}\right) \\ (r+s+\lambda+\epsilon)\left.\frac{\partial S^p}{\partial z}\right|_{(z,F_2)} = 1 + \frac{\epsilon}{2}\left(\left.\frac{\partial S^p}{\partial z}\right|_{(z,F_1)} + \left.\frac{\partial S^p}{\partial z}\right|_{(z,F_2)}\right) \end{cases}$$

  Solving this system yields

$$\left.\frac{\partial S^p}{\partial z}\right|_{(z,F_i)} = \frac{1}{r+s+\lambda},\ i \in \{1,2\}$$

  As a result, reinjecting these partial derivatives in the equation (D.1.2)

$$\frac{\partial S^p}{\partial z} = \frac{1}{r+s+\lambda}$$

- If z is such that $S^p\left(z, F_i\right) \leq 0,\ i \in \{1,2\}$, a similar procedure leads to

$$\frac{\partial S^p}{\partial z} = \frac{1}{r+s+\lambda+\epsilon}$$

- If z is such that $S^p\left(z, F_1\right) > 0$ and $S^p(z, F_2) \leq 0$ or such that $S^p\left(z, F_1\right) \leq 0$ and $S^p(z, F_2) > 0$, a similar procedure leads to

$$\frac{\partial S^p}{\partial z} = \frac{1}{r+s+\lambda+\frac{\epsilon}{2}}$$

S^p being continuous and piece-wise linear in z given firing costs F, there exists a unique $z^p(F)$ such that $S^p\left(z^p(F), F\right) = 0$.

An analogous calculation for $\partial S_0^p/\partial z$ leads to

$$\frac{\partial S_0^p}{\partial z} = \begin{cases} \frac{1}{r+s+\lambda} & \text{if } S_0^p\left(z, F_i\right) > 0,\ i \in \{1,2\} \\ \frac{1}{r+s+\lambda+\epsilon} & \text{if } S_0^p\left(z, F_i\right) \leq 0,\ i \in \{1,2\} \\ \frac{1}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } S_0^p\left(z, F_1\right) > 0 \text{ and } S_0^p\left(z, F_2\right) \leq 0 \\ & \text{or } S_0^p\left(z, F_1\right) \leq 0 \text{ and } S_0^p\left(z, F_2\right) > 0 \end{cases}$$

Again, S_0^p is continuous and piece-wise linear in z given firing costs F. Thus, there exists a unique $z^c(F)$ such that $S_0^p\left(z^c(F), F\right) = 0$.

Finally, I find that

$$\frac{\partial S^f}{\partial z} = \frac{1}{r+\delta+\lambda}$$

S^f is linear. Therefore, there exists a unique $z^f(F)$ such that $S^f\left(z^f(F),F\right)=0$.

Assuming that $s+\epsilon<\delta$, $\partial S_0^p/\partial z>\partial S^f/\partial z$. Thus, S_0^p and S^f being continuous and respectively piece-wise linear and linear, there exists a unique $z^*(F)$ such that $S_0^p\left(z,F\right)\geq S^f\left(z,F\right)$ for $z\geq z^*(F)$.

z^f verifies

$$\begin{aligned}
&z^f-\left(1+\frac{\epsilon}{r}\right)b-\frac{\eta\gamma}{1-\eta}\left(\theta+\frac{\epsilon}{r}\overline{\theta}\right)+\lambda\int S^f\left(z',F\right)dG\left(z'\right)\\
&+\epsilon\left(\frac{S^f(z^f,F_1)+S^f(z^f,F_2)}{2}+\overline{U}\right)=0
\end{aligned}$$

S^f being linear in z, S^f verifies

$$S^f(z,F)=\frac{z-z^f}{r+\delta+\lambda}$$

As a result, z^f verifies

$$\frac{r+\delta+\epsilon}{r+\delta+\lambda}z^f-\left(1+\frac{\epsilon}{r}\right)b-\frac{\eta\gamma}{1-\eta}\left(\theta+\frac{\epsilon}{r}\overline{\theta}\right)+\frac{\lambda Ez}{r+\delta+\lambda}-\frac{\epsilon}{2}\frac{z_1^f+z_2^f}{r+\delta+\lambda}+\epsilon\overline{U}=0$$

Differentiating the latter equation with respect to F yields

$$\frac{r+\delta+\epsilon}{r+\delta+\lambda}\frac{\partial z^f}{\partial F}=\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}\tag{D.1.3}$$

z^p verifies

$$\begin{aligned}
&z^p-\left(1+\frac{\epsilon}{r}\right)b-\frac{\eta\gamma}{1-\eta}\left(\theta+\frac{\epsilon}{r}\overline{\theta}\right)+(r+s+\epsilon)F+\lambda\int\max\left[S^p\left(z',F\right),0\right]dG\left(z'\right)\\
&+\frac{\epsilon}{2}\left(\max\left[S^p\left(z^p,F_1\right),0\right]+\max\left[S^p\left(z^p,F_2\right),0\right]\right)\\
&+\epsilon\left(\overline{U}-\overline{F}\right)=0
\end{aligned}$$

Differentiating the latter equation with respect to F yields

$$\begin{aligned}
\frac{\partial z^p}{\partial F}=&\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}-(r+s+\epsilon)-\lambda\int_{z^p(F)}^{+\infty}\left.\frac{\partial S^p}{\partial F}\right|_{(z',F)}dG(z')-\epsilon\frac{\mathbb{1}\{z^p(F)>\max(z_1^p,z_2^p)\}}{r+s+\lambda}\frac{\partial z^p}{\partial F}\\
&-\frac{\epsilon}{2}\frac{\mathbb{1}\{\min[z_1^p,z_2^p]<z^p(F)\leq\max[z_1^p,z_2^p]\}}{r+s+\lambda+\frac{\epsilon}{2}}\frac{\partial z^p}{\partial F}
\end{aligned}$$

where I denote $z_i^j\equiv z^j(F_i)$ for $j\in\{*,c,p,f\}$ and $i\in\{1,2\}$.

Meanwhile,

$$(r+s+\lambda+\epsilon)\frac{\partial S^p}{\partial F}=-\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}+r+s+\epsilon+\lambda\int_{z^p(F)}^{+\infty}\left.\frac{\partial S^p}{\partial F}\right|_{(z',F)}dG(z')$$

Integrating the latter equation between z^p and $+\infty$ leads to

$$(r+\xi+\epsilon)\int_{z^p}^{+\infty}\left.\frac{\partial S^p}{\partial F}\right|_{(z',F)} dG(z') = (1-G(z^p))\left(-\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}+r+s+\epsilon\right)$$

As a result, $\partial z^p/\partial F$ is

$$\begin{aligned}\frac{\partial z^p}{\partial F} =& \frac{r+s+\lambda+\epsilon}{r+\xi+\epsilon}\left(\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}-(r+s+\epsilon)\right)-\epsilon\frac{\mathbb{1}\{z^p>\max(z_1^p,z_2^p)\}}{r+s+\lambda}\frac{\partial z^p}{\partial F}\\ &-\frac{\epsilon}{2}\frac{\mathbb{1}\{\min[z_1^p,z_2^p]<z^p\leq\max[z_1^p,z_2^p]\}}{r+s+\lambda+\frac{\epsilon}{2}}\frac{\partial z^p}{\partial F}\end{aligned}$$

which is equivalent to

$$\begin{aligned}&(r+\xi+\epsilon)\left(\frac{\mathbb{1}\{z^p>\max(z_1^p,z_2^p)\}}{r+s+\lambda}+\frac{\mathbb{1}\{\min(z_1^p,z_2^p)<z^p\leq\max(z_1^p,z_2^p)\}}{r+s+\lambda+\frac{\epsilon}{2}}+\frac{\mathbb{1}\{z^p\leq\min(z_1^p,z_2^p)\}}{r+s+\lambda+\epsilon}\right)\frac{\partial z^p}{\partial F}\\ &=\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}-(r+s+\epsilon)\end{aligned} \tag{D.1.4}$$

z^c verifies

$$\begin{aligned}&z^c-\lambda F-\left(1+\frac{\epsilon}{r}\right)b-\frac{\eta\gamma}{1-\eta}\left(\theta+\frac{\epsilon}{r}\overline{\theta}\right)+\epsilon\overline{U}+\lambda\int\max\left[S^p\left(z',F\right),0\right]dG\left(z'\right)\\ &+\frac{\epsilon}{2}\left(\max\left[S_0^p\left(z^c,F_1\right),0\right]+\max\left[S_0^p\left(z^c,F_2\right),0\right]\right)=0\end{aligned}$$

Using similar steps to derive $\partial z^p/\partial F$, $\partial z^c/\partial F$ is

$$\begin{aligned}&(r+\xi+\epsilon)\left(\frac{\mathbb{1}\{z^c>\max(z_1^c,z_2^c)\}}{r+s+\lambda}+\frac{\mathbb{1}\{\min(z_1^c,z_2^c)<z^c\leq\max(z_1^c,z_2^c)\}}{r+s+\lambda+\frac{\epsilon}{2}}+\frac{\mathbb{1}\{z^c\leq\min(z_1^c,z_2^c)\}}{r+s+\lambda+\epsilon}\right)\frac{\partial z^c}{\partial F}\\ &=\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}+\lambda G\left(z^p\right)\end{aligned}$$

As for z^*, it verifies

$$S_0^p\left(z^*,F\right)=S^f\left(z^*,F\right)$$

Differentiating the latter equation with respect to F leads to

$$\left(\left.\frac{\partial S_0^p}{\partial z}\right|_{(z^*,F)}-\left.\frac{\partial S^f}{\partial z}\right|_{(z^*,F)}\right)\frac{\partial z^*}{\partial F}=\left.\frac{\partial S^f}{\partial F}\right|_{(z^*,F)}-\left.\frac{\partial S_0^p}{\partial F}\right|_{(z^*,F)}$$

id est

$$
\begin{aligned}
&\left(\frac{\mathbb{1}\{z^* > \max[z_1^c, z_2^c]\}}{r+s+\lambda} + \frac{\mathbb{1}\{\min[z_1^c, z_2^c] < z^* \leq \max[z_1^c, z_2^c]\}}{r+s+\lambda+\frac{\epsilon}{2}} + \frac{\mathbb{1}\{z^* \leq \min[z_1^c, z_2^c]\}}{r+s+\lambda+\epsilon}\right)\frac{\partial z^*}{\partial F} \\
= &\left(\frac{\mathbb{1}\{z^c > \max[z_1^c, z_2^c]\}}{r+s+\lambda} + \frac{\mathbb{1}\{\min[z_1^c, z_2^c] < z^c \leq \max[z_1^c, z_2^c]\}}{r+s+\lambda+\frac{\epsilon}{2}} + \frac{\mathbb{1}\{z^c \leq \min[z_1^c, z_2^c]\}}{r+s+\lambda+\epsilon}\right)\frac{\partial z^c}{\partial F} \qquad \text{(D.1.5)} \\
&- \frac{1}{r+\delta+\lambda}\frac{\partial z^f}{\partial F}
\end{aligned}
$$

□

Lemma 1. *These assertions are equivalent*

1. $z^* > z^f$
2. $z^* > z^c$
3. $z^c > z^f$

Proof. For the sake of brevity, I write z^i instead of z^i for $i \in \{z^f, z^c, z^*\}$.

- If $z^* > z^f$, then $S_0^p(z^*, F) = S^f(z^*, F) > S^f\left(z^f, F\right) = 0$ because S^f is increasing in z. Since S_0^p is increasing in z, it implies that $z^* > z^c$.
- If $z^* > z^c$, then $S^f(z^c, F) > S_0^p(z^c, F) = 0 = S^f(z^f, F)$, where the inequality stems from the definition of z^*. As S^f is increasing in z, $z^c > z^f$.
- If $z^c > z^f$, then $S^f\left(z^f, F\right) = 0 = S_0^c(z^c, F) > S_0^c\left(z^f, F\right)$, where the inequality stems from the fact that S_0^p is increasing in z. Using the later inequality and the definition of z^*, one gets $z^f < z^*$.

□

Proposition 6. *Considering an equilibrium with dual job creation, the following inequalities are verified*

$$
\begin{cases}
z_1^p > z_2^p \\
z_1^c < z_2^c \\
z_1^f > z_2^f \\
z_1^* < z_2^* \\
z_i^f < z_i^c < z_i^*
\end{cases}
$$

Proof. I assume that job creation is dual at the considered equilibrium. According to the lemma above, the thresholds verify $z^f < z^c < z^*$ and the job creation condition boils down to

$$
\frac{\gamma}{(1-\eta)q(\theta)} = \int_{z^*}^{+\infty} S_0^p\left(z', F\right) dG(z') + \int_{z^f}^{z^*} S^f\left(z', F\right) dG\left(z'\right)
$$

Differentiating the latter with respect to F leads to

$$
-\frac{\gamma q'(\theta)}{(1-\eta)q^2(\theta)}\frac{\partial \theta}{\partial F} = \int_{z^*}^{+\infty} \left.\frac{\partial S_0^p}{\partial F}\right|_{(z',F)} dG(z') + \int_{z^f}^{z^*} \left.\frac{\partial S^f}{\partial F}\right|_{(z',F)} dG(z')
$$

$$= -\frac{G(z^*) - G(z^f)}{r+\delta+\lambda}\frac{\partial z^f}{\partial F} - (1-G(z^*))\left(\frac{\mathbb{1}\{z^c > \max[z_1^c, z_2^c]\}}{r+s+\lambda} + \right.$$
$$\left. + \frac{\mathbb{1}\{\min[z_1^c, z_2^c] < z^c \leq \max[z_1^c, z_2^c]\}}{r+s+\lambda+\frac{\epsilon}{2}} + \frac{\mathbb{1}\{z^c \leq \min[z_1^c, z_2^c]\}}{r+s+\lambda+\epsilon}\right)\frac{\partial z^c}{\partial F}$$
$$= -\frac{G(z^*) - G(z^f)}{r+\delta+\epsilon}\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F} - \frac{1-G(z^*)}{r+\xi+\epsilon}\left(\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F} + \lambda G(z^p)\right)$$

As a result,

$$\frac{\partial\theta}{\partial F} = -\frac{\frac{1-G(z^*)}{r+\xi+\epsilon}\lambda G(z^p)}{-\frac{\gamma q'(\theta)}{(1-\eta)q^2(\theta)} + \frac{\eta\gamma}{1-\eta}\left(\frac{G(z^*)-G(z^f)}{r+\delta+\epsilon} + \frac{1-G(z^*)}{r+\xi+\epsilon}\right)} < 0$$

It immediately follows from (D.1.3) and (D.1.4) that $\partial z^f/\partial F$ and $\partial z^p/\partial F$

$$\frac{\partial z^f}{\partial F} < 0$$
$$\frac{\partial z^p}{\partial F} < 0$$

As for $\partial z^c/\partial F$, it verifies

$$\frac{\partial z^c}{\partial F} \propto -\frac{\gamma q'}{(1-\eta)q^2(\theta)} + \frac{G(z^*) - G(z^f)}{r+\delta+\epsilon}\frac{\eta\gamma}{1-\eta} > 0$$

Using (D.1.5) and the signs of $\partial z^c/\partial F$ and $\partial z^f/\partial F$, it follows that

$$\frac{\partial z^*}{\partial F} > 0$$

As a result, since $F_1 < F_2$, I get

$$\begin{cases} z_1^p > z_2^p \\ z_1^c < z_2^c \\ z_1^f > z_2^f \\ z_1^* < z_2^* \end{cases}$$

□

Using the last proposition, $\partial S^p/\partial z$ verifies

$$\frac{\partial S^p}{\partial z} = \begin{cases} \frac{1}{r+s+\lambda+\epsilon} & \text{if } z \leq z_2^p \\ \frac{1}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z_2^p \leq z \leq z_1^p \\ \frac{1}{r+s+\lambda} & \text{if } z \geq z_1^p \end{cases}$$

The present discounted value of unemployment U depends on the firing cost through the labor market tightness, which accounts for the i subscript, with $i \in \{1,2\}$. Their expressions still verify (2.2.6).

The derivative above and the definition of z^p yield another expression for S^p.

$$S^p(z,F_2) = \begin{cases} \frac{z-z_2^p}{r+s+\lambda+\epsilon} & \text{if } z \leq z_2^p \\ \frac{z-z_2^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z_2^p < z < z_1^p \\ \frac{z-z_1^p}{r+s+\lambda} + \frac{z_1^p - z_2^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z \geq z_1^p \end{cases}$$

$$S^p(z,F_1) = \begin{cases} \frac{z-z_2^p}{r+s+\lambda+\epsilon} + \frac{z_2^p - z_1^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z \leq z_2^p \\ \frac{z-z_1^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z_2^p < z < z_1^p \\ \frac{z-z_1^p}{r+s+\lambda} & \text{if } z \geq z_1^p \end{cases}$$

Using an integration by parts in (D.1.1), z_1^p and z_2^p verify

$$\begin{aligned} & z_2^p - (r+\epsilon)U_2 + (r+s+\epsilon)F_2 + \epsilon\left(\frac{U_1+U_2}{2} - \frac{F_1+F_2}{2}\right) \\ & + \frac{\lambda}{r+s+\lambda+\frac{\epsilon}{2}} \int_{z_2^p}^{z_1^p} (1-G(x))\,dx + \frac{\lambda}{r+s+\lambda} \int_{z_1^p}^{+\infty} (1-G(x))\,dx = 0 \\ & z_1^p - (r+\epsilon)U_1 + (r+s+\epsilon)F_1 + \epsilon\left(\frac{U_1+U_2}{2} - \frac{F_1+F_2}{2}\right) \\ & + \frac{\lambda}{r+s+\lambda} \int_{z_1^p}^{+\infty} (1-G(x))\,dx + \frac{\epsilon}{2}\frac{z_1^p - z_2^p}{r+s+\lambda+\frac{\epsilon}{2}} = 0 \end{aligned}$$

In the same manner, z_1^c and z_2^c verify

$$\begin{aligned} & z_2^c - (r+\epsilon)U_2 - \lambda F_2 + \epsilon\frac{U_1+U_2}{2} + \frac{\lambda}{r+s+\lambda+\frac{\epsilon}{2}} \int_{z_2^p}^{z_1^p} (1-G(x))\,dx + \frac{\lambda}{r+s+\lambda} \int_{z_1^p}^{+\infty} (1-G(x))\,dx \\ & + \frac{\epsilon}{2}\frac{z_2^c - z_1^c}{r+s+\lambda+\frac{\epsilon}{2}} = 0 \\ & z_1^c - (r+\epsilon)U_1 - \lambda F_1 + \epsilon\frac{U_1+U_2}{2} + \frac{\lambda}{r+s+\lambda} \int_{z_1^p}^{+\infty} (1-G(x))\,dx = 0 \end{aligned}$$

Similarly, thresholds z_i^f are such that

$$z_i^f = \frac{r+\delta+\lambda}{r+\delta+\epsilon}\left((r+\epsilon)U_i - \frac{\delta}{r+\delta}\epsilon\frac{U_1+U_2}{2}\right) - \frac{\lambda}{r+\delta}Ez$$

I assume that ϵ and the distance between F_1 and F_2 is small enough so that $z_i^* > z_2^c$ for all $i \in \{1,2\}$. In this case, z_i^* verifies

$$\left(\frac{1}{r+s+\lambda} - \frac{1}{r+\lambda+\delta}\right) z_i^* = \frac{z_i^c}{r+s+\lambda} - \frac{z_i^f}{r+\lambda+\delta}$$

and job creation verifies

$$\frac{\gamma}{(1-\eta)q\left(\theta_i\right)}=\frac{1}{r+s+\lambda}\int_{z_i^*}^{+\infty}(1-G(z))\,dz+\frac{1}{r+\delta+\lambda}\int_{z_i^f}^{z_i^*}(1-G(z))\,dz$$

Consequently, the equilibrium can be summed up by a continuous time Markov process over space $\left\{\left(\theta_i, z_i^p, z_i^c, z_i^f, z_i^*, F_i\right)\right\}_{i=1,2}$ with switching probability ϵ and 2x2 transition matrix filled with $1/2$. State variables are $\left(n^{c,p}, n^{0,p}, n^f\right)$.

Taking into account the Nash-bargaining rules (2.0.1)–(2.0.3), wages of continuing open-ended workers and new open-ended workers with productivity z and firing costs F_i write

$$\begin{aligned}
w^p\left(z, F_i\right) &= \eta\left(z+(r+s+\epsilon)F_i-\epsilon\frac{F_1+F_2}{2}+\gamma\theta_i\right)+(1-\eta)b \\
w_0^p\left(z, F_i\right) &= \eta\left(z-\lambda F_i+\gamma\theta_i\right)+(1-\eta)b
\end{aligned}$$

D.2 Wage dynamics

In the subsequent simulated method of moments, the average open-ended wage needs to represent a given proportion of the average firing costs. Consequently, I derive the dynamics of the average open-ended wage to meet this requirement.

I denote $W_t^{c,p}$ and $W_t^{0,p}$ aggregate wages continuing open-ended workers and new open-ended workers earn at time t. As in Appendix C, I consider the model starting from the equilibrium defined by $\left(\theta_0, z_0^p, z_0^c, z_0^f, z_0^*\right)$ with firing costs F_0 at time $t=0^-$. I assume that firing costs jump from F_0 to F on time $t=0$, the equilibrium tuple defined in definition 1 immediately jumps to its new value $\left(\theta, z^p, z^c, z^f, z^*\right)$. The initial open-ended and fixed-term employment values are $n_0^{c,p}$, $n_0^{0,p}$ and n_0^f. Similarly, I denote $W_0^{c,p}$ and $W_0^{0,p}$ the initial values of aggregate wages earned by open-ended workers. Aggregate open-ended wages verify the following system of differential equations.

$$\begin{aligned}
\dot{W}_t^{c,p} =& -(s+\lambda)W_t^{c,p}+\lambda\left(1-G\left(z^p\right)\right)\mathbb{E}\left[w^p(z,F)\mid z\geq z^p\right]n_t^p \\
&-\delta_0\alpha_0\mathbb{E}\left[w^p(z,F)\mid z_0^p\leq z\leq\max\left[z^p, z_0^p\right]\right]n_t^{c,p} \\
\dot{W}_t^{0,p} =& -(s+\lambda)W_t^{0,p}+\mu^p\mathbb{E}\left[w_0^p(z,F)\mid z\geq\max\left[z^*, z^c\right]\right]\left(1-n_t^p-n_t^f\right) \\
&-\delta_0\beta_0\mathbb{E}\left[w_0^p(z,F)\mid\max\left[z_0^c, z_0^*\right]\leq z\leq\max\left[z^c, z_0^c, z_0^*\right]\right]n_t^{0,p}
\end{aligned}$$

The solutions write

$$W_t^{i,p}=W_\infty^{i,p}+\nu_0^{i,p}exp\{\omega t\}+\nu_1^{i,p}exp\{\alpha_1 t\}+\nu_2^{i,p}exp\{\alpha_2 t\},\quad i\in\{c,0\}$$

where

$$\begin{aligned}
W_\infty^{c,p} &= \frac{\lambda\left(1-G\left(z^p\right)\right)}{s+\lambda}E^{c,p}n_\infty^p \\
W_\infty^{0,p} &= \frac{\mu^p}{s+\lambda}E^{0,p}\left(1-n_\infty^p-n_\infty^f\right)
\end{aligned}$$

$$
\begin{aligned}
\nu_0^{c,p} &= W_0^{c,p} - \alpha_0 E_0^{c,p} n_0^{c,p} - \lambda \left(1 - G\left(z^p\right)\right) E^{c,p} \left(\frac{n_\infty^p}{s+\lambda} + \frac{\rho_1^p}{\alpha_1 - \omega} + \frac{\rho_2^p}{\alpha_2 - \omega} \right) \\
\nu_0^{0,p} &= W_0^{0,p} - \beta_0 E_0^{0,p} n_0^{0,p} + \left(\frac{\rho_1^p + \rho_1^f}{\alpha_1 - \omega} + \frac{\rho_2^p + \rho_2^f}{\alpha_2 - \omega} - \frac{1 - n_\infty^p - n_\infty^f}{s+\lambda} \right) \mu^p E^{0,p} \\
\nu_i^{c,p} &= \frac{\lambda \left(1 - G\left(z^p\right)\right)}{\alpha_i - \omega} E^{c,p} \rho_i^p \quad i \in \{1,2\} \\
\nu_i^{0,p} &= -\mu^p \frac{\rho_i^p + \rho_i^f}{\alpha_i - \omega} E^{0,p} \quad i \in \{1,2\} \\
E^{c,p} &= \mathbb{E}\left[w^p(z,F) \mid z \geq z^p\right] \\
E_0^{c,p} &= \mathbb{E}\left[w^p(z,F) \mid z_0^p \leq z \leq \max\left[z^p, z_0^p\right]\right] \\
E^{0,p} &= \mathbb{E}\left[w_0^p(z,F) \mid z \geq \max\left[z^*, z^c\right]\right] \\
E_0^{0,p} &= \mathbb{E}\left[w_0^p(z,F) \mid \max\left[z_0^*, z_0^c\right] \leq z \leq \max\left[z_0^*, z_0^c, z^c\right]\right]
\end{aligned}
$$

D.3 The simulated method of moments

1. Set $N = 1,000$ and $D = 1,000$
2. Simulate N trajectories on time interval $[0, D]$ for shocks on firing costs, choosing the initial state of firing costs at random.
3. Set a n_Δ-point grid for the firing cost wedge $\Delta = \mid \frac{F_1 - F}{F} \mid = \mid \frac{F_2 - F}{F} \mid$. I set n_Δ to 20.
4. For each i in $\{1, ..., n_\Delta\}$, for each j in $\{1, ..., N\}$, minimize the wedge between simulated moments and moments in Table 3.2 optimizing over parameters $(F, b, s, \sigma_z, m, \gamma)$.

Parameters $(r, \sigma, \eta, \lambda, \delta)$ remain at their baseline values. The results are pretty robust; changing the set of parameters the simulated method of moments optimizes over does not significantly change the results.