Priors

$\pi = [\, p(\text{knows before}) \quad 1-p(\text{knows before}) \,]$

$= [\, p(L_0) \quad 1-p(L_0) \,]$

Transition Probability (Übergangswahr.)

	Mastered	not Mastered
Mastered	1	0 (forget)
not Mastered	p(Transit)	1 - p(Transit)

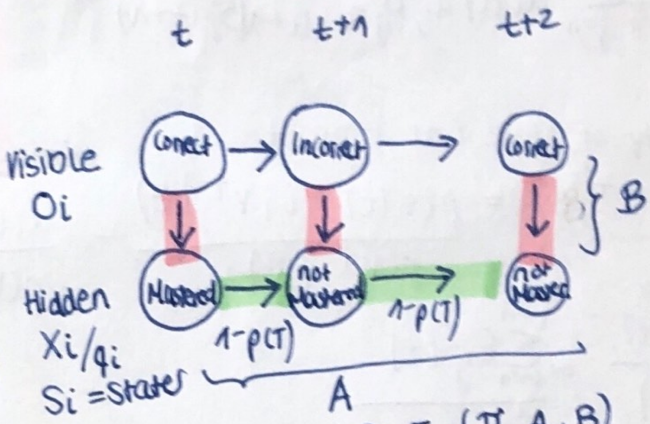
$A = \begin{bmatrix} 1 & 0 \\ p(T) & 1-p(T) \end{bmatrix}$ $\quad a_{ij} = P(q_{t+1} = S_j \mid q_t = S_i)$

Emission Probability (Ausgabewahrscheinl.)

visibles v

states S_i	Correct	Incorrect
Mastered	1-p(slip)	p(slip)
not Mastered	p(Guess)	1-p(Guess)

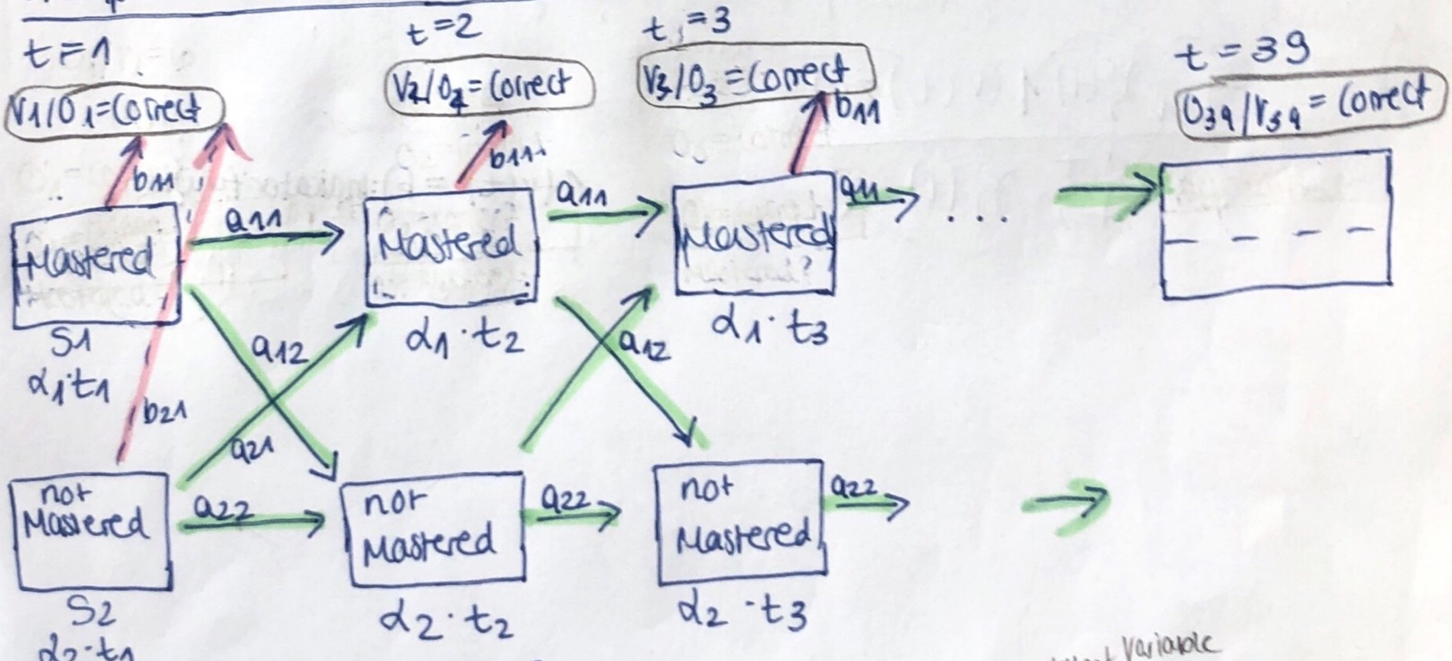
$B = \begin{bmatrix} 1-p(S) & p(S) \\ p(G) & 1-p(G) \end{bmatrix}$



Sequence $V^T = [\text{Correct}_1, \text{Correct}_2 \ldots \text{Correct}_{39}]$

$\Theta = (\pi, A, B)$

Example of 1 student & 1 skill



E-Step: Update emission prob. B

latent Variable

$$\boxed{\varepsilon_{ij}(t)} = \frac{\alpha_i(t)\, a_{ij}\, b_{jkv(t+1)}\, \beta_j(t+1)}{\sum_{i=1}^{M}\sum_{j=1}^{M} \alpha_i(t)\, a_{ij}\, b_{jkv(t+1)}\, \beta_j(t+1)}$$

latent Variable

$$\gamma_i(t) = \sum_{j=1}^{M} \varepsilon_{ij}(t)$$

M-Step: update

$$\hat{a}_{ij} = \frac{\boxed{\sum_{t=1}^{T-1} \varepsilon_{ij}(t)}}{\underbrace{\sum_{t=1}^{T-1}\sum_{j=1}^{M} \varepsilon_{ij}(t)}_{\gamma_i(t)}} \qquad \hat{b}_{jk} = \frac{\sum_{t=1}^{T} \gamma_j(t)\, 1(v(t)=k)}{\sum_{t=1}^{T} \gamma_j(t)}$$

Re-estimate probabilities

① $\hat{a}_{ij}$ = $\dfrac{\text{expected number of transitions from hidden state } i \text{ to state } j}{\text{expected number of transition from hidden state } i}$

② $\hat{b}_{jk}$ = $\dfrac{\text{expected number of times in hidden state } j \text{ and observing } v(k)}{\text{expected number of times in hidden state } j}$

Nominator:

① $p(s(t)=i, s(t+1)=j \mid V^T, \theta) = \dfrac{p(s(t)=i, s(t+1)j, V^T \mid \theta)}{p(V^T \mid \theta)}$

$$\varepsilon_{ij}(t) = \frac{\alpha_i(t) a_{ij} b_{jk} v(t+1) \beta_j(t+1)}{\sum_{i=1}^{M}\sum_{j=1}^{M} \alpha_i(t) a_{ij} b_{jk} v(t+1) \beta_j(t+1)}$$

Prob of observation sequence V^T by any path given the model θ

Denominator:

probability of state i at time t

$$p(s(t)=i \mid V^T, \theta) = \frac{p(s(t)=i, V^T \mid \theta)}{p(V^T \mid \theta)} = \frac{\alpha_i(t)\beta_i(t)}{p(V^T \mid \theta)} = \frac{\alpha_i(t)\beta_i(t)}{\sum_{i=1}^{M} \alpha_i(t)\beta_i(t)} = \gamma_i(t)$$

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \varepsilon_{ij}(t)}{\sum_{t=1}^{T-1} \gamma_i(t)} = \frac{\sum_{t=1}^{T-1} \varepsilon_{ij}(t)}{\sum_{j=1}^{M} \varepsilon_{ij}(t)}$$

Identical

② $$\hat{b}_{jk} = \frac{\sum_{t=1}^{T} \gamma_j(t) \underbrace{1(v(t)=k)}_{\text{observing } v_k}}{\sum_{t=1}^{T} \gamma_j(t)}$$

Probability of being in state j at time t

$1(v(t)=k)$: Indicator function

indicates whether this belongs to 1? So whether it is true?

Generated Data

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
(Mastered) Hidden:	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0
Visible: (correct/ Incorrect)	0	0	0	1	0	0	0	0	1	1	0	1	0	1	1	0	0	0	1	0

$\hookrightarrow \pi = [0, 1]$

- 0: knows before L_0
- 1: doesn't know before $1-L_0$

Transition Probability

$$\hat{A} = \begin{bmatrix} p(\text{Mastered} \mid \text{Mastered}) & p(\text{not Mastered} \mid \text{Mastered}) \\ p(\text{Mastered} \mid \text{not Mast.}) & p(\text{not Mast.} \mid \text{not Mast.}) \end{bmatrix} = \begin{bmatrix} \overset{(0.1)}{2/19} & \overset{(0.16)}{3/19} \\ \underset{(0.16)}{3/19} & \underset{(0.6)}{11/19} \end{bmatrix}$$

Emission Probability

$$\hat{B} = \begin{bmatrix} p(\text{Correct} \mid \text{Mastered}) & p(\text{Incorrect} \mid \text{Mastered}) \\ p(\text{Correct} \mid \text{not Mast.}) & p(\text{Incorrect} \mid \text{not Mastered}) \end{bmatrix} = \begin{bmatrix} 3/19 & 2/19 \\ 4/19 & 11/19 \end{bmatrix}$$