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Praktikum Mikrowellentechnik

Investigation of Analog and Digital Demodulation Methods

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1 Introduction

With modulation, information content is impressed in some form on a high-frequency, sinusoidal carrier signal. This can be done by changing the amplitude, phase or frequency of the carrier signal and is done by a so-called modulator. Modulation is used to bring a low-frequency information signal into a form suitable for the transmission channel for the purpose of information transmission. An example of a transmission channel is the radio transmission link shown in simplified form in Fig. 1.1.

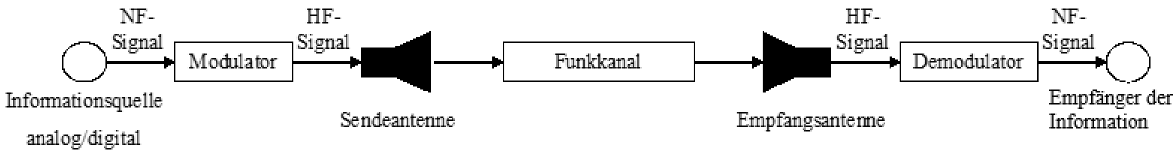


Fig. 1.1: Radio transmission link.

The high-frequency carrier signal is much more suitable for transmission on a radio channel than the low-frequency signal. By using different carrier frequencies, the entire available frequency range of the radio channel can be divided into several communication channels. As a result, many different communication signals (for example from different radio stations) can be transmitted at the same time without influencing one another. In addition, antennas can only radiate electromagnetic waves efficiently if the wavelength is in the range of the antenna dimensions. A sound signal with the frequency range $30\,\mathrm{Hz} \ldots 15\,\mathrm{kHz}$, for example, has a free space wavelength range of $20\,\mathrm{km} \ldots 10\,000\,\mathrm{km}$, which would require huge antennas with a very large relative bandwidth. To avoid large antenna dimensions, the signal must have a sufficiently high frequency, which is achieved by the high-frequency carrier signal.

Demodulation is the recovery of the low-frequency signal after receiving the high-frequency signal (modulated carrier). To do this, the received signal must be sent to a demodulator, which recovers the low-frequency information signal from the modulated carrier signal.

A distinction is made between analog and digital modulation schemes. With analog modulation, a continuous information signal is modulated onto the carrier, with digital modulation, on the other hand, a digital (time and value discrete) signal is used. Due to the advancing digitization (digital television, digital radio, mobile radio), analog modulation schemes are being replaced by digital modulation schemes in many areas. However, there will still be applications

Table 1.1: Overview of modulation schemes

Analog modulation Continuous information signal	Digital modulation Time- and value-discrete information signal
Amplitude modulation (AM)	Amplitude shift keying (ASK)
Frequency modulation (FM)	Frequency shift keying (FSK)
Phase modulation (PM)	Phase shift keying (PSK)

in communication and sensor technology in which analog technology will be preferred for economic and technical reasons (simplicity, low space requirements, low production costs). The cost factor and the dimensions of an RF module play an important role in modern technology. In this situation, good physical ideas and knowledge of analog technology are often much more important than the mathematical formalism of the digital world.

There are a number of different modulation schemes that differ in how the amplitude, frequency, or phase of the carrier signal is changed. Table 1.1 gives an overview. The oldest modulation scheme is amplitude modulation. In the case of amplitude modulation, the amplitude of the carrier signal is continuously changed in proportion to the information signal. It is still used today in medium wave and long wave broadcasting. Frequency modulation followed later, which has several advantages over amplitude modulation, including greater noise reduction, which leads to improved sound quality in the case of radio, for example. It is mainly used for VHF. The digital modulation schemes are used for wireless digital data traffic (e.g., WLAN, WiMAX, Bluetooth, HiperLAN), wireless telephony (DECT, GSM, UMTS) and navigation (GPS). A detailed description of digital modulation schemes can be found in [Jon01] and [Str90]. The advantages of digital modulation schemes include greater flexibility in signal coding, higher spectral efficiency and higher resistance to mutual interference thanks to built-in error correction. It also makes sense to use digital modulation for the transmission of digital data.

As with modulation, a distinction is made between analog and digital demodulation methods. Analog components are mainly used for analog demodulation, while with digital demodulation the received signal is first sampled (digitized) and then converted into an information signal with the aid of signal processors or other digital circuits. In this way, both an analog and a digital signal can be digitally demodulated.

2 Representation of Modulated Signals

If a high-frequency carrier signal

$$s_{\mathrm{T}}(t) = a_{\mathrm{T}} \cos(\omega_0 t) \tag{2.1}$$

is modulated with a low-frequency information signal $s_{\mathrm{N}}(t)$, the result at the output of the modulator in Fig. 2.1 is the modulated carrier signal

$$s(t) = a(t) \cos(\omega_0 t + \varphi(t)) \tag{2.2}$$

Depending on the modulation scheme, the information in the signal $s_{\mathrm{N}}(t)$ is either in the amplitude $a(t)$, in the phase $\varphi(t)$ or in both parameters. $\omega_0 = 2\pi f_0$ is called the angular frequency of the carrier.
The instantaneous angular frequency $\omega(t)$ of the modulated carrier signal results from the derivation of the phase of $s(t)$ to

$$\omega(t) = \frac{d}{dt}(\omega_0 t + \varphi(t)) = \omega_0 + \frac{d\varphi(t)}{dt} \tag{2.3}$$

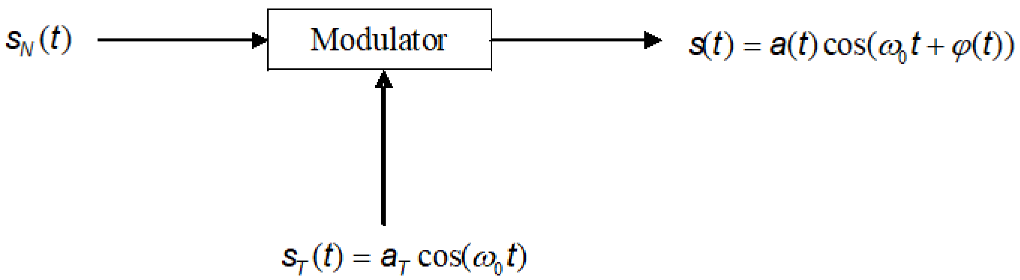


Fig. 2.1: Modulation principle.

Accordingly, the instantaneous frequency $f(t)$ is given by

$$f(t) = \frac{\omega(t)}{2\pi} = f_0 + \frac{1}{2\pi}\frac{d\varphi(t)}{dt} \tag{2.4}$$

This shows that the instantaneous frequency $f(t)$ and the instantaneous phase $\varphi(t)$ are closely related, which makes simultaneous and independent frequency and phase modulation impossible.

The modulated carrier frequency can also be written complex-valued in the form

$$\underline{s}(t) = \underline{u}(t) \cdot \mathrm{e}^{\mathrm{j}\omega_0 t} \tag{2.5}$$

The complex envelope and the equivalent lowpass signal of $s(t)$ are given by

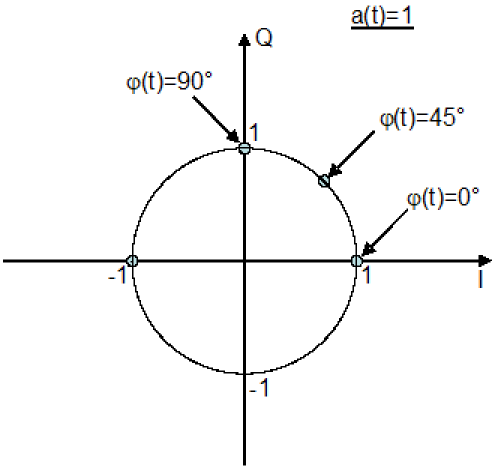
$$\underline{u}(t) = a(t) \cdot \mathrm{e}^{\mathrm{j}\varphi(t)} = a(t)\cos(\varphi(t)) + \mathrm{j}a(t)\sin(\varphi(t)) = s_{\mathrm{I}}(t) + \mathrm{j}s_{\mathrm{Q}}(t) \tag{2.6}$$

with the inphase and quadrature components

$$s_{\mathrm{I}}(t) = a(t)\cos(\varphi(t)) \tag{2.7}$$

$$s_{\mathrm{Q}}(t) = a(t)\sin(\varphi(t)) \tag{2.8}$$

The inphase and quadrature components represent the real and imaginary part of the complex envelope, respectively. Because of (2.7) and (2.8), $s_{\mathrm{I}}(t)$ and $s_{\mathrm{Q}}(t)$ can also be used instead of the amplitude $a(t)$ and the phase $\varphi(t)$ to characterize modulation schemes. Especially with digital modulation schemes, $s_{\mathrm{I}}(t)$ and $s_{\mathrm{Q}}(t)$ only accept discrete values. They can then be drawn as points in an IQ or constellation diagram as in Fig. 2.2.



$a(t)$	$\varphi(t)$	$s_I(t) + js_Q(t)$
1	0	1
1	45	$\frac{1}{\sqrt{2}}(1+j)$
1	90	j
1	180	-1

Fig. 2.2: Constellation diagram.

3 Analog Modulation

In analog modulation, a continuous, low-frequency signal $s_\mathrm{N}(t)$ is modulated onto a high-frequency carrier. In the following only the simplest case of a harmonic information signal $s_\mathrm{N}(t)$ with a constant amplitude a_N, the angular frequency $\omega_\mathrm{N} = 2\pi f_\mathrm{N}$ and the period $T_\mathrm{N} = 1/f_\mathrm{N}$ resulting in

$$s_\mathrm{N}(t) = a_\mathrm{N}\cos(\omega_\mathrm{N} t) \tag{3.1}$$

is regarded. However, other waveforms are used later in the experiments.

3.1 Amplitude Modulation (AM)

In the case of amplitude modulation, only the amplitude of the carrier is changed, so the information is only contained in the amplitude. The advantage of amplitude modulation is that it enables the simplest of transmitters and receivers to be used.

3.1.1 Representation of AM Signals in the Time Domain

If the low-frequency, harmonic signal from (3.1) is fed to an amplitude modulator, the amplitude-modulated carrier signal according to (2.2) results at its output. Its amplitude has the form

$$a(t) = A_\mathrm{T} + s_\mathrm{N}(t) = A_\mathrm{T} + a_\mathrm{N}\cos(\omega_\mathrm{N} t) = A_\mathrm{T}(1 + m\cos(\omega_\mathrm{N} t)) \tag{3.2}$$

where A_T is the constant mean amplitude of $s(t)$ and $m = a_\mathrm{N}/A_\mathrm{T}$ is called modulation depth. To achieve that $a(t)$ is always positive, the modulation depth m has to be smaller than 1 ($A_\mathrm{T} > a_\mathrm{N}$), because this is the only way to guarantee distortion-free modulation. The amplitude $a(t)$ varies with frequency ω_N between the values $A_\mathrm{T}(1-m)$ and $A_\mathrm{T}(1+m)$.
If it is taken into account that, due to the amplitude modulation, $\varphi(t) = \mathrm{const}$ applies to the phase and therefore $\varphi(t) = 0$ can be set without loss of generality, the AM signal results to

$$s(t) = a(t)\cos(\omega_0 t) = A_\mathrm{T}(1 + m\cos(\omega_\mathrm{N} t))\cos(\omega_0 t) \tag{3.3}$$

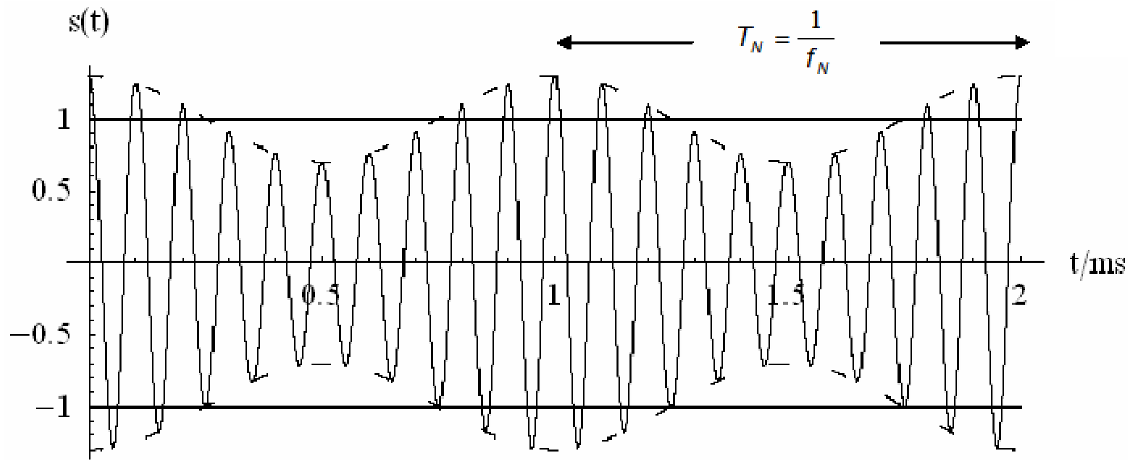


Fig. 3.1: AM signal with $f_0 = 10\,\text{kHz}$, $f_\text{N} = 1\,\text{kHz}$, $A_\text{T} = 1$ and $m = 0.3$.

Fig. 3.1 shows an AM signal in the time domain. The amplitude or envelope is depicted as a dashed line. Because of the modulation depth $m = 0.3$, it fluctuates between 0.7 and 1.3 around the mean carrier amplitude $A_\text{T} = 1$.

3.1.2 Spectrum of AM Signals

By using the trigonometric relation

$$\cos\alpha \cdot \cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta) \tag{3.4}$$

(3.3) can be reformulated to

$$\begin{aligned} s(t) &= A_\text{T}(1 + m\cos(\omega_\text{N}t))\cos(\omega_0 t) \\ &= A_\text{T}(\cos(\omega_0 t) + m\cos(\omega_\text{N}t)\cos(\omega_0 t)) \\ &= A_\text{T}\cos(\omega_0 t) + \frac{mA_\text{T}}{2}\cos((\omega_0+\omega_\text{N})t) + \frac{mA_\text{T}}{2}\cos((\omega_0-\omega_\text{N})t) \end{aligned} \tag{3.5}$$

The spectrum of the AM signal is thus made up of three components. First the angular carrier frequency ω_0 with the amplitude A_T and the two side frequencies $\omega_0 + \omega_\text{N}$ and $\omega_0 - \omega_\text{N}$ both with the amplitude $mA_\text{T}/2$. The left part of Fig. 3.2 shows the spectrum of $s(t)$.

Usually, however, not just a single harmonic signal is modulated, but an entire spectrum, e.g., when transmitting music. In addition to the carrier frequency, there are not two discrete side frequencies anymore, but two complete sidebands that extend from $f_0 \pm f_\text{min}$ to $f_0 \pm f_\text{max}$ (see right part of Fig. 3.2). Thus, another name for this type of modulation is double sideband mod-

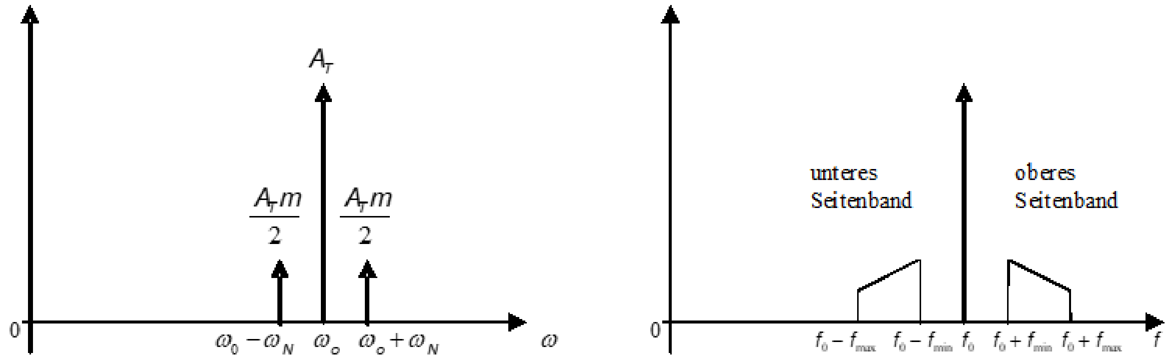


Fig. 3.2: Spectrum of a simple AM signal (left) and sidebands of a complexer AM signal (right).

ulation. In the case of music, for example, the two limiting frequencies are about $f_{\min} \approx 30\,\mathrm{Hz}$ and $f_{\max} \approx 15\,\mathrm{kHz}$.

3.1.3 Demodulation Principle

To recover the signal $s_{\mathrm{N}}(t)$ from the received AM signal $s(t)$ after the transmission, an amplitude demodulator is needed. In the following, only the simplest amplitude demodulator, the so-called envelope detector, is presented. More advanced and complexer amplitude demodulators can be found, for example, in [ZB99]. The envelope detector consists of a diode D followed by a lowpass filter consisting of R and C, as shown in Fig. 3.3.

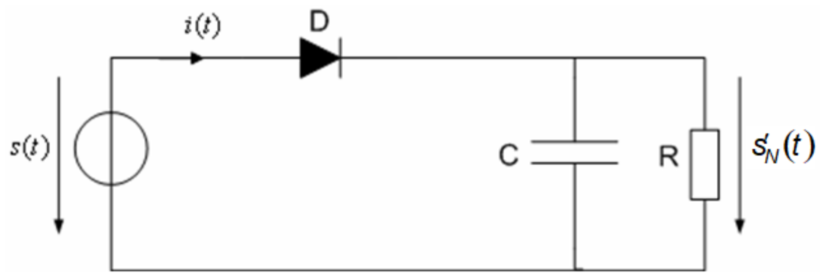


Fig. 3.3: Envelope detector.

In order to understand the principle of the envelope detector, it is initially assumed that the capacitor C has not yet been installed. Consequently, a pure series connection of the diode D and the resistor R is considered.
If the received AM signal $s(t)$ is now applied to the series circuit of diode and resistor, the AM signal is rectified. The characteristic of the diode, that is used in Fig. 3.4, was assumed to be ideal with a sharp kink. The diode acts like a switch where only the positive half-waves of the AM signal can pass. The result is the current curve $i(t)$ shown in the upper right in Fig. 3.4.

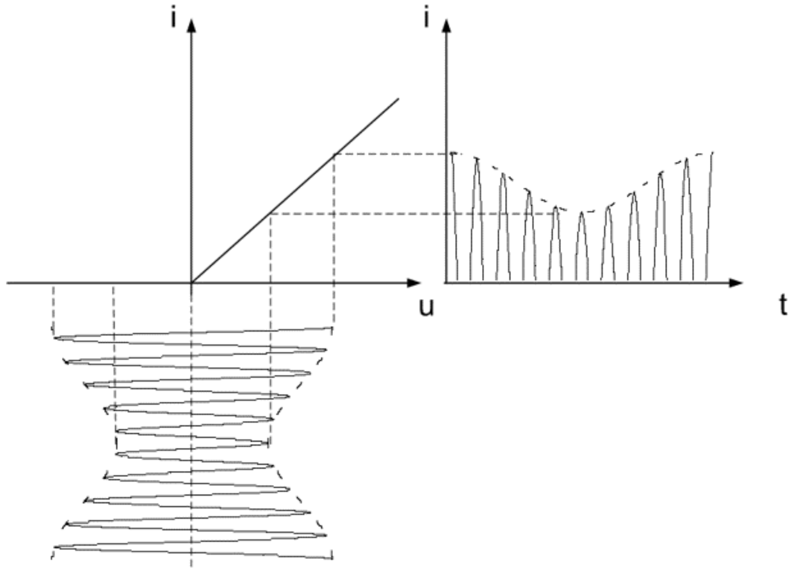


Fig. 3.4: Rectification of an AM signal.

If now a capacitor C is connected in parallel to R, the actual envelope detector in Fig. 3.3 results. The capacitor and the resistor form a lowpass filter. The capacitor has the task of filling the currentless pauses between the individual half-waves. It charges during the positive half-wave via the diode and discharges during the negative half-wave of the AM signal via the resistor, as shown in Fig. 3.5.

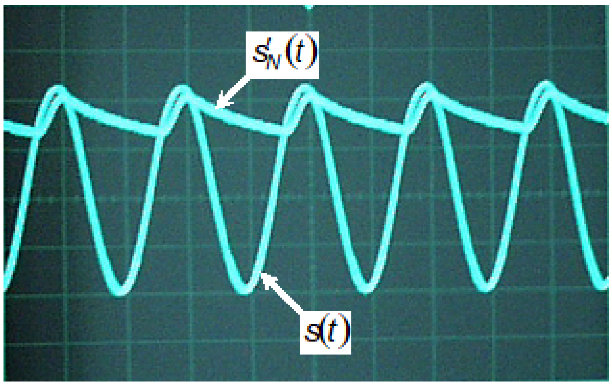


Fig. 3.5: Charging and discharging of the capacitor.

If the time constant $\tau = R \cdot C$ is chosen correctly, the voltage $s'_{\mathrm{N}}(t)$ at the output of the envelope detector follows the envelope curve $a(t)$ of the modulated carrier signal $s(t)$ which is the input of the envelope detector (see Fig. 3.6). The demodulated signal $s'_{\mathrm{N}}(t)$ then only differs by an additive constant and a scaling factor from $s_{\mathrm{N}}(t)$.
If the time constant $\tau = R \cdot C$ is too small, $s'_{\mathrm{N}}(t)$ decreases during the negative half-wave, so that the demodulated signal $s'_{\mathrm{N}}(t)$ has high-frequency distortions that appear on the oscilloscope

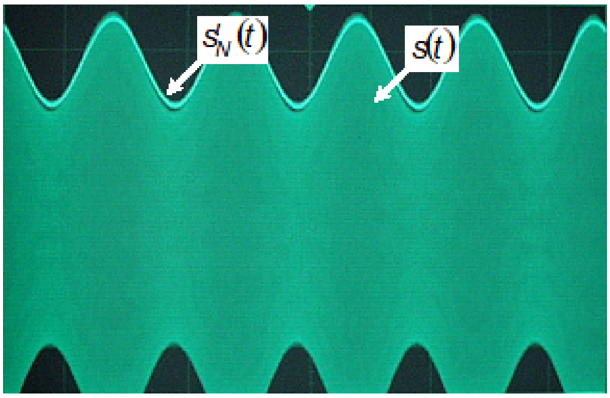


Fig. 3.6: Modulated signal $s(t)$ and the successfully demodulated signal $s'_{\mathrm{N}}(t)$ ($\tau = R \cdot C \rightarrow$ optimum).

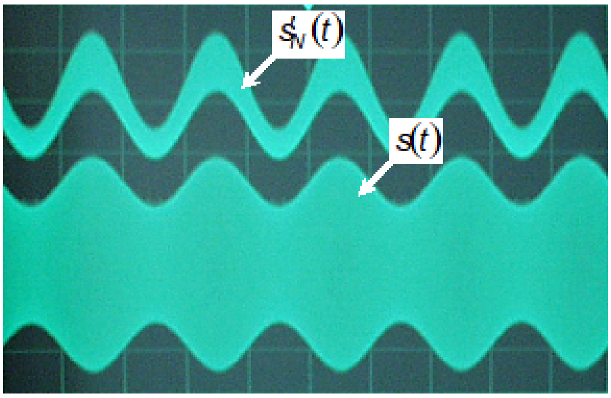


Fig. 3.7: High-frequency distortion ($\tau = R \cdot C \rightarrow$ too small).

as a broadening of the signal curve (see Fig. 3.7).
If τ is too big, $s'_{\mathrm{N}}(t)$ can't follow rapid changes in the envelope curve no longer, so that low-frequency distortions occur (see Fig. 3.8). Good results can be obtained with the dimensioning specification

$$\frac{1}{\omega_0} \ll \tau < \frac{1}{2\pi m f_{\max}} \tag{3.6}$$

where $f_{\max}$ represents the maximum frequency of the modulating or the baseband signal.

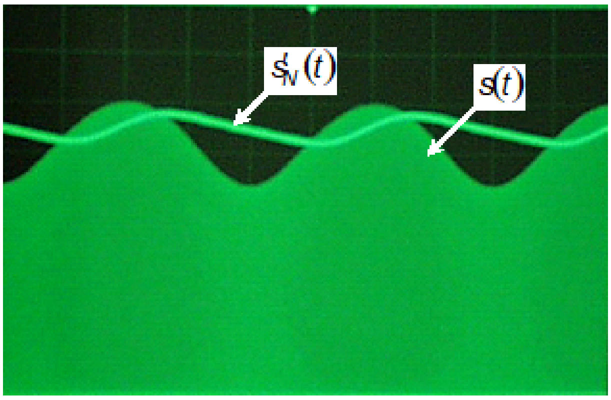


Fig. 3.8: Low-frequency distortion ($\tau = R \cdot C \rightarrow$ too big).

3.2 Angle Modulation

In contrast to amplitude modulation, angle modulation changes the frequency or phase of the carrier signal. A distinction is therefore made between frequency modulation (FM) and phase modulation (PM). One advantage of angle modulation compared to amplitude modulation is the improved signal-to-noise ratio, which, however, has to be bought at the expense of an increased bandwidth.

3.2.1 Representation of FM and PM Signals in the Time Domain

Here, too, a harmonic low-frequency signal is used as in (3.1). Because of the angle modulation, $a(t) = a_\mathrm{T} = \mathrm{const}$ applies to the amplitude. With phase modulation, the phase $\varphi(t)$ in (2.2) is directly proportional to the modulating information signal $s_\mathrm{N}(t)$, meaning

$$\varphi(t) = K_\varphi s_\mathrm{N}(t) = K_\varphi a_\mathrm{N} \cos(\omega_\mathrm{N} t) = \Delta\varphi \cos(\omega_\mathrm{N} t) \tag{3.7}$$

$\Delta\varphi = K_\varphi \cdot a_\mathrm{N}$ is the phase deviation. Hence, the PM signal results to

$$s(t) = a_\mathrm{T} \cos(\omega_0 t + \Delta\varphi \cos(\omega_\mathrm{N} t)) \tag{3.8}$$

With frequency modulation not the phase but the frequency $d\varphi/dt$ in (2.2) is directly proportional to $s_\mathrm{N}(t)$, meaning

$$\frac{d\varphi(t)}{dt} = K_\omega s_\mathrm{N}(t) = K_\omega a_\mathrm{N} \cos(\omega_\mathrm{N} t) = \Delta\omega \cos(\omega_\mathrm{N} t) \tag{3.9}$$

$\Delta\omega = K_\omega \cdot a_\mathrm{N}$ is the frequency deviation. The phase

$$\varphi(t) = \frac{\Delta\omega}{\omega_\mathrm{N}} \sin(\omega_\mathrm{N} t) = \eta \sin(\omega_\mathrm{N} t) \tag{3.10}$$

can be calculated by integrating (3.9). $\eta = \Delta\omega/\omega_\mathrm{N}$ is called modulation index. Hence, the FM signal is given by

$$s(t) = a_\mathrm{T} \cos(\omega_0 t + \eta \sin(\omega_\mathrm{N} t)) \tag{3.11}$$

According to (3.7), the phase deviation $\Delta\varphi$ is the maximum value of the phase change. And according to (3.9), the frequency deviation $\Delta\omega$ is the maximum deviation of the instantaneous angular frequency $\omega(t)$ from the carrier frequency ω_0. η is the phase deviation of the frequency modulation. In the following illustration only frequency modulation is used. An FM signal in the time domain is shown in Fig. 3.9. Here, also the instantaneous frequency

$$f(t) = \frac{\omega(t)}{2\pi} = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} = f_0 + \frac{1}{2\pi}\Delta\omega \cos(\omega_\mathrm{N} t) \tag{3.12}$$

is depicted. Because of the modulation index $\eta = 4$, $f(t)$ oscillates between 6 kHz and 14 kHz around the carrier frequency $f_0 = 10\,\mathrm{kHz}$.

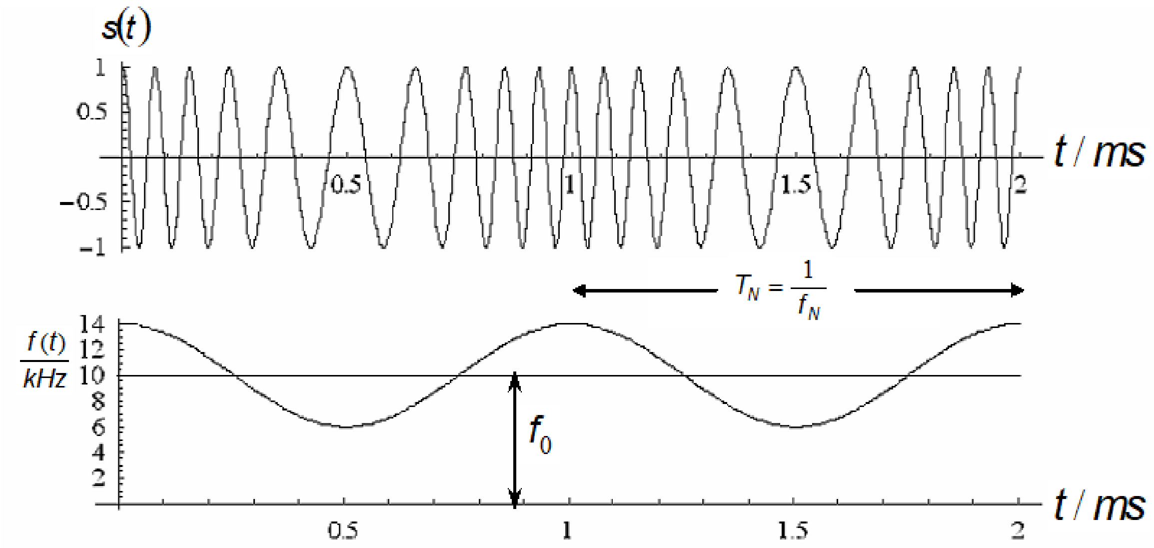


Fig. 3.9: FM signal and instantaneous frequency for $f_0 = 10\,\mathrm{kHz}$, $f_\mathrm{N} = 1\,\mathrm{kHz}$, $a_\mathrm{T} = 1$ and $\eta = 4$.

3.2.2 Spectrum of FM Signals

To calculate the spectrum of an FM signal the Fourier series

$$\mathrm{e}^{\mathrm{j}\eta\sin(\omega_\mathrm{N}t)} = \sum_{k=-\infty}^{\infty} J_k(\eta)\mathrm{e}^{\mathrm{j}k\omega_\mathrm{N}t} \tag{3.13}$$

is required. J_k is the Bessel function of the first kind with order k. The following derivation also uses the relationship $J_{-k}(\eta) = (-1)^k J_k(\eta)$. Hence, (3.11) can be rewritten to

$$\begin{aligned} s(t) &= a_\mathrm{T}\cdot\cos(\omega_0 t + \eta\sin(\omega_\mathrm{N}t)) \\ &= a_\mathrm{T}\cdot\mathrm{Re}\big\{\mathrm{e}^{\mathrm{j}(\omega_0 t+\eta\sin(\omega_\mathrm{N}t))}\big\} \\ &= a_\mathrm{T}\sum_{k=-\infty}^{\infty} J_k(\eta)\cdot\mathrm{Re}\big\{\mathrm{e}^{\mathrm{j}(\omega_0+k\omega_\mathrm{N})t}\big\} \\ &= a_\mathrm{T}\sum_{k=-\infty}^{\infty} J_k(\eta)\cos((\omega_0+k\omega_\mathrm{N})t) \\ &= a_\mathrm{T}\cdot J_0(\eta)\cdot\cos(\omega_0 t) \\ &\quad + a_\mathrm{T}\sum_{k=1}^{\infty} J_k(\eta)\big[\cos((\omega_0+k\omega_\mathrm{N})t) + (-1)^k\cos((\omega_0-k\omega_\mathrm{N})t)\big] \end{aligned} \tag{3.14}$$

The spectral lines are thus at the distance $\pm k\omega_\mathrm{N}$ from the angular carrier frequency ω_0 and their height relative to a_T is given by the Bessel function of the first kind with order k. Fig. 3.10 shows the spectrum of the FM signal for $\eta = 5$.

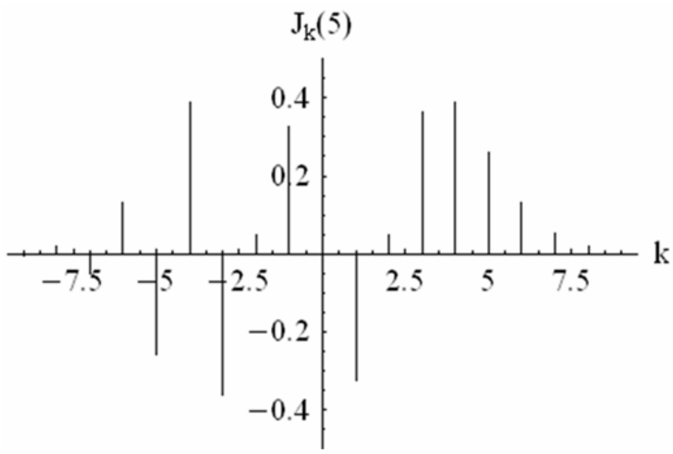


Fig. 3.10: Spectrum of an FM signal with $\eta = 5$.

The Bessel function $J_k(\eta)$ tends towards zero for a constant value of η with increasing order k.

For this reason, the spectral lines at a greater distance from the angular carrier frequency ω_0 are very small and thus have very little influence on the signal properties. Correspondingly, a real channel with a band-limited transfer function can also transmit an FM signal with virtually no distortion.

3.2.3 Demodulation Principle

In order to recover the low-frequency information signal $s_\mathrm{N}(t)$ from an FM signal, a frequency demodulator is required. In the following, only the so-called slope detector is covered. Further frequency demodulators can be found in [ZB99]. The slope detector consists of a parallel resonant circuit wit L_p and C_p followed by an envelope detector, as shown in Fig. 3.11.

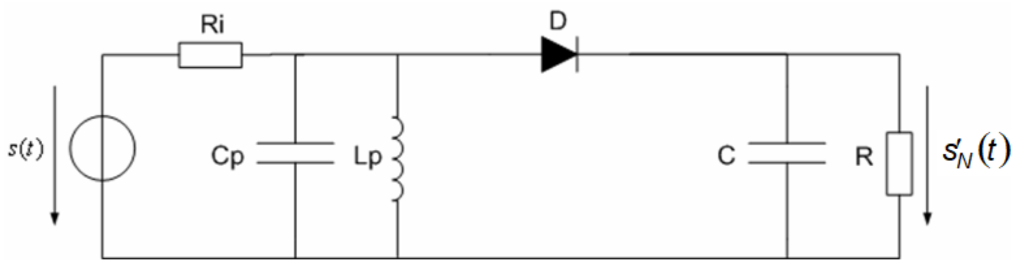


Fig. 3.11: Slope detector.

The parallel resonant circuit is used for an FM-AM conversion of the FM signal $s(t)$ at the input of the demodulator. The resulting AM signal, which remains frequency-modulated, is then demodulated as described in Subsection 3.1.3 by means of a subsequent envelope detector. The transfer function of the parallel resonant circuit with a series-connected resistor R_i, which causes the FM-AM conversion, is given by

$$\underline{G}(\omega) = \frac{\mathrm{j}\omega L_\mathrm{p}}{\mathrm{j}\omega L_\mathrm{p} + R_\mathrm{i}(1 - L_\mathrm{p} C_\mathrm{p} \omega^2)} \tag{3.15}$$

It can be assumed to be linear around the inflection point of a slope (see left part of Fig. 3.12). When properly tuned, the parallel resonant circuit converts the FM signal into an AM/FM signal, which is then demodulated as described before by means of an envelope detector. Due to the non-linear characteristic, the dynamic range is limited to a relatively small area and can only be placed in the area around the inflection point.

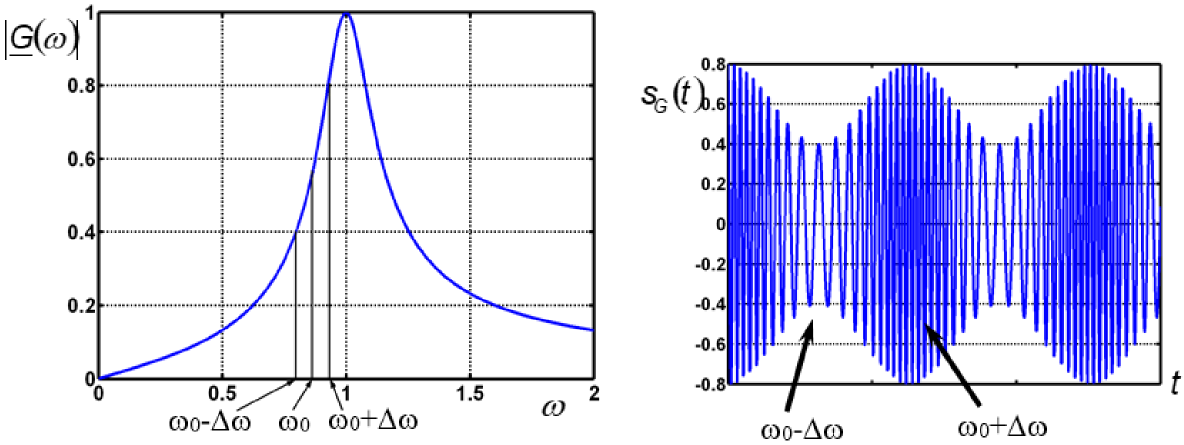


Fig. 3.12: Transfer function (left) and output signal (right) of the resonant circuit.

4 Digital Modulation

The task of digital modulation is to assign bits or bit groups to wave trains (real signals). To represent the individual bits, a discrete-valued, low-frequency, periodic square-wave signal with the Fourier series

$$s_{\mathrm{N}}(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_{\mathrm{N}}t) - \frac{1}{3}\cos(3\omega_{\mathrm{N}}t) + \frac{1}{5}\cos(5\omega_{\mathrm{N}}t) - \frac{1}{7}\cos(7\omega_{\mathrm{N}}t) \pm \dots\right) \tag{4.1}$$

is used. Fig. 4.1 shows $s_{\mathrm{N}}(t)$ for the frequency $f_{\mathrm{N}} = 1\,\mathrm{kHz}$ and the cycle duration T_{N}.

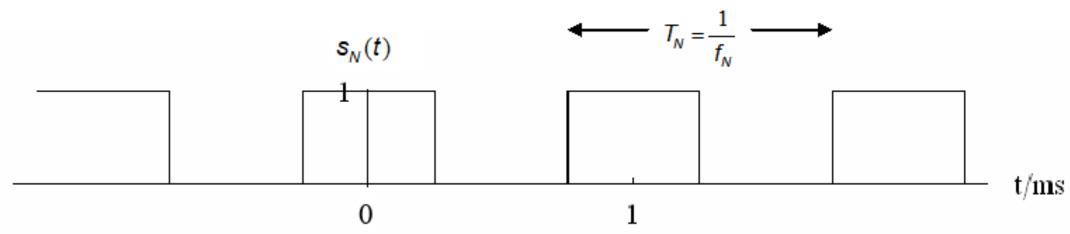


Fig. 4.1: Discrete-valued, low-frequency information signal.

4.1 ASK – Amplitude Shift Keying

With the ASK scheme, the information is modulated onto the amplitude. According to Fig. 4.1, the amplitude can only have the values 0 and 1. So the signal is switched on and off in the bit cycle.

Fig. 4.2 shows an ASK signal with the carrier frequency $f_0 = 10\,\mathrm{kHz}$. In addition, Fig. 4.3 depicts the associated constellation diagram.

ASK signals can be demodulated either with an envelope detector or with digital demodulation methods.

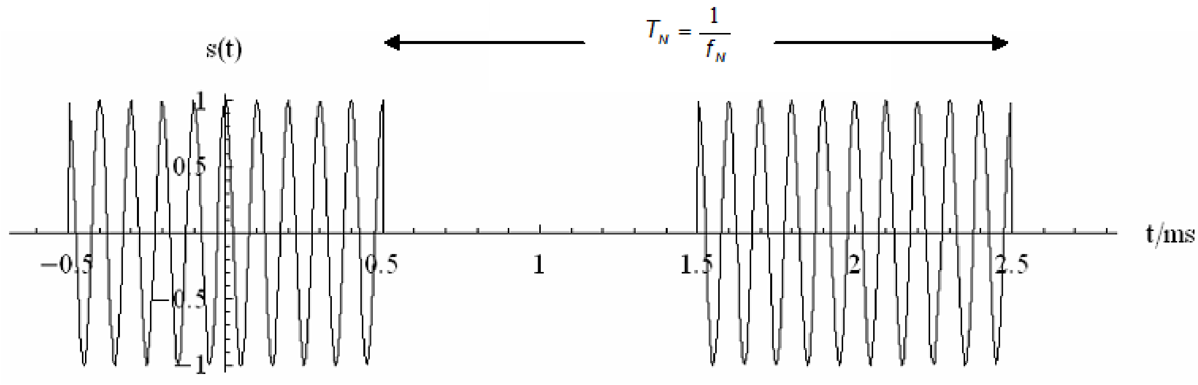


Fig. 4.2: ASK signal with $f_0 = 10\,\mathrm{kHz}$, $f_\mathrm{N} = 0.5\,\mathrm{kHz}$ and $a_\mathrm{T} = 1$.

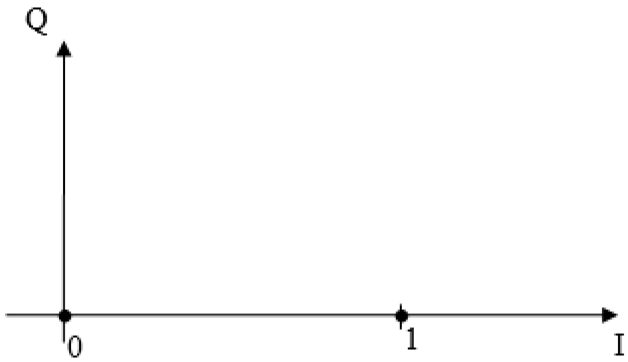


Fig. 4.3: Constellation diagram of the ASK signal.

4.2 PSK – Phase Shift Keying

With PSK, the phase is changed in the bit cycle and, depending on the PSK variant, can assume two or more discrete values. In general, an M-PSK signal according to (2.2) is represented by

$$s(t) = a_\mathrm{T} \cdot \cos(\omega_0 t + \varphi(t)), \quad \varphi(t) = \frac{2\pi m}{M}, \quad m = 0, 1, \ldots, M-1 \tag{4.2}$$

With 2-PSK (two-phase shift keying, $M = 2$), also called BPSK (binary phase shift keying), the phase can only take one of the values "0" and "π". Fig. 4.4 shows a 2-PSK signal with the carrier frequency $f_0 = 5\,\mathrm{kHz}$. Additionally, Fig. 4.5 shows the constellation diagram of a 2-PSK (left) and a 4-PSK signal (right).
For the demodulation of PSK signals, digital demodulation methods are normally used. They require a recovery of the reference phase or other measures to remove the phase ambiguity (e.g., differential phase shift keying). Chapter 5 explains the principle of PM demodulation using an IQ demodulator.

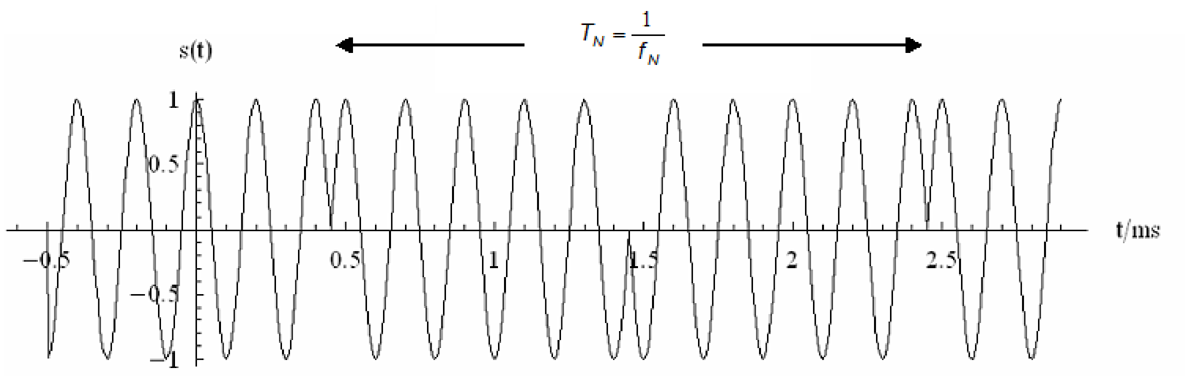


Fig. 4.4: 2-PSK signal with $f_0 = 10\,\mathrm{kHz}$, $f_\mathrm{N} = 0.5\,\mathrm{kHz}$ and $a_\mathrm{T} = 1$.

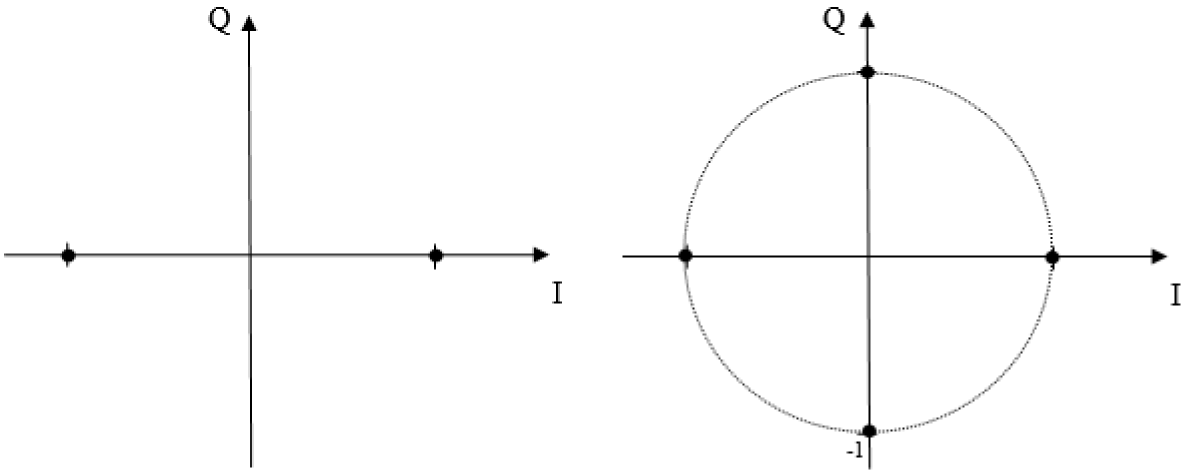


Fig. 4.5: Constellation diagram of a 2-PSK signal (left) and a 4-PSK signal (right) with $a_\mathrm{T} = 1$.

4.3 FSK – Frequency Shift Keying

With an FSK signal, the frequency $d\varphi/dt$ is changed in the bit cycle. With 2-FSK the keying takes place between the angular frequencies ω_1 and ω_2. The carrier frequency is $\omega_0 = (\omega_1 + \omega_2)/2$, so it is exactly in the middle between ω_1 and ω_2. It should be noted that the carrier frequency ω_0 is not transmitted, only ω_1 and ω_2. The frequency deviation is thus $\Delta\omega = (\omega_2 - \omega_1)/2$. Figure 4.6 shows an example of a 2-FSK signal with the frequencies $f_1 = 5\,\mathrm{kHz}$ and $f_2 = 10\,\mathrm{kHz}$.

The demodulation of FSK signals can be done either with an slope detector or with digital demodulation methods, e.g., with an IQ demodulator.

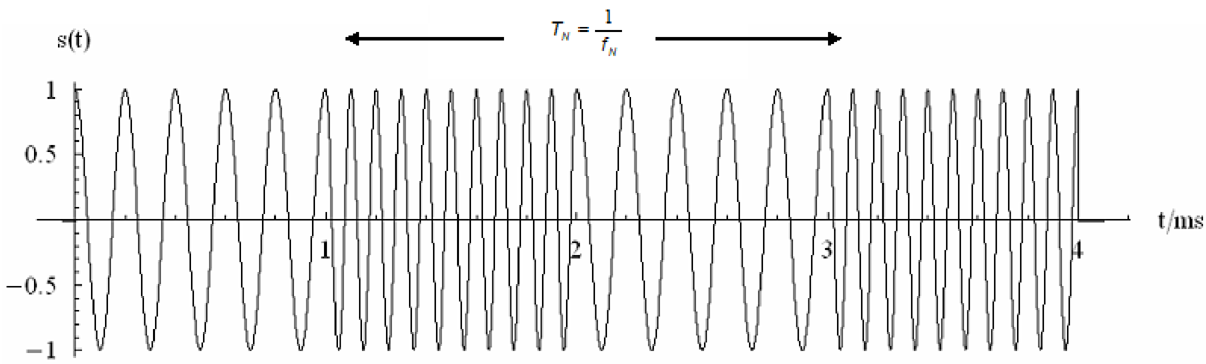


Fig. 4.6: 2-FSK signal with $f_1 = 10\,\mathrm{kHz}$, $f_2 = 5\,\mathrm{kHz}$, $f_\mathrm{N} = 0.5\,\mathrm{kHz}$ and $a_\mathrm{T} = 1$.

5 IQ Modulation

After changing the sign before $\varphi(t)$ in (2.2), all modulation schemes can be described by

$$\begin{aligned} s(t) &= a(t) \cdot \cos(\omega_0 t - \varphi(t)) \\ &= a(t) \cdot \cos(\varphi(t)) \cdot \cos(\omega_0 t) + a(t) \cdot \sin(\varphi(t)) \cdot \sin(\omega_0 t) \\ &= s_\mathrm{I}(t) \cdot \cos(\omega_0 t) + s_Q(t) \cdot \sin(\omega_0 t) \end{aligned} \tag{5.1}$$

$s_\mathrm{I}(t)$ and $s_Q(t)$ represent the inphase and the quadrature component, respectively. For $\varphi(t) = 0$ a pure amplitude modulation results, but all other modulation schemes can also be represented with (5.1). If $s_\mathrm{I}(t) = f_1(t)$ and $s_\mathrm{Q}(t) = f_2(t)$ are set, where $f_1(t)$ and $f_2(t)$ are arbitrary functions, the modulation scheme is called quadrature modulation (IQ modulation). The IQ signal results from

$$s(t) = f_1(t) \cdot \cos(\omega_0 t) + f_2(t) \cdot \sin(\omega_0 t) \tag{5.2}$$

With IQ modulation, two information signals are transmitted simultaneously on one carrier. Fig. 5.1 shows the principle of a transmission with IQ modulation. For demodulation, the IQ signal is multiplied by $\cos(\omega_0 t)$ and $\sin(\omega_0 t)$ and then filtered with a lowpass. By using trigonometric relations, for the upper branch in Fig. 5.1

$$f_1(t) \cdot \cos^2(\omega_0 t) + f_2(t) \cdot \sin(\omega_0 t) \cdot \cos(\omega_0 t) = \frac{1}{2}(f_1(t) + f_1(t) \cdot \cos(2\omega_0 t) + f_2(t) \cdot \sin(2\omega_0 t)) \tag{5.3}$$

holds. Accordingly, for the lower branch

$$f_1(t) \cdot \cos(\omega_0 t) \cdot \sin(\omega_0 t) + f_2(t) \cdot \sin^2(\omega_0 t) = \frac{1}{2}(f_2(t) + f_1(t) \cdot \sin(2\omega_0 t) - f_2(t) \cdot \cos(2\omega_0 t)) \tag{5.4}$$

results. The subsequent lowpass filters then suppresses the high-frequency components $f_1(t) \cdot \cos(2\omega_0 t)$, $f_2(t) \cdot \sin(2\omega_0 t)$, $f_1(t) \cdot \sin(2\omega_0 t)$ and $f_2(t) \cdot \cos(2\omega_0 t)$.

In the analog area, IQ modulation is used, for example, to modulate the color carrier in color television. IQ modulation has become particularly important in the digital sector. If two mutually independent bit signals are used for $s_\mathrm{I}(t) = f_1(t)$ and $s_\mathrm{Q}(t) = f_2(t)$ (see Fig. 5.2), the transmitted bit rate can be doubled by means of IQ modulation.

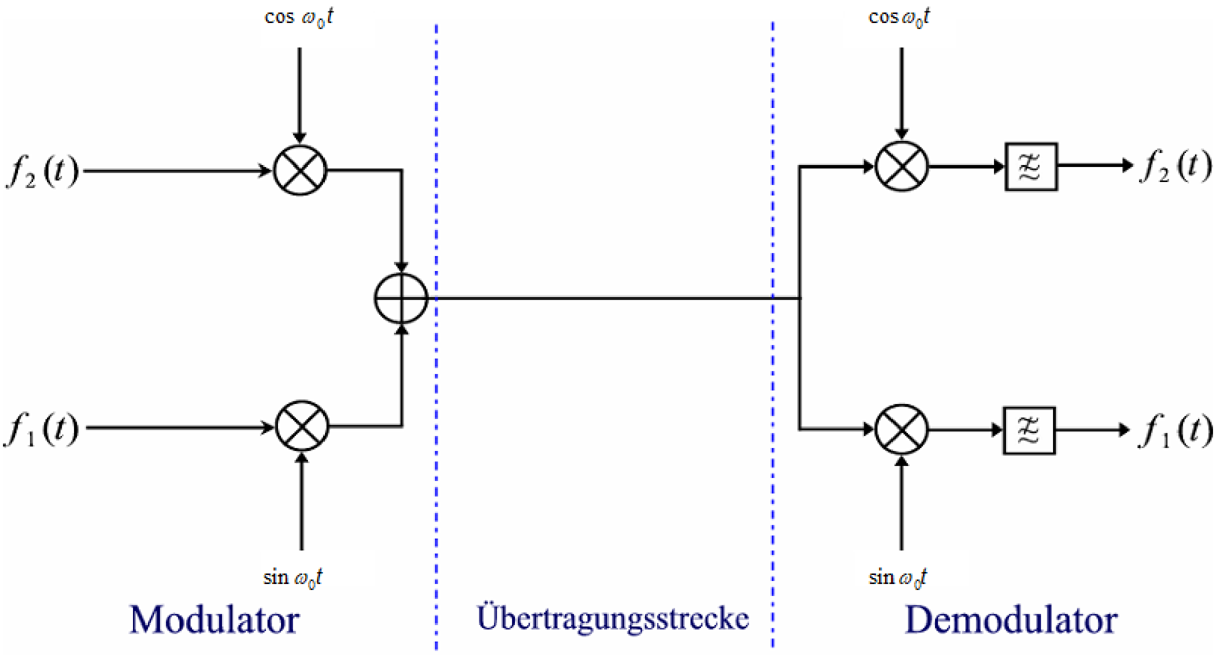


Fig. 5.1: IQ modulation (left) and demodulation (right).

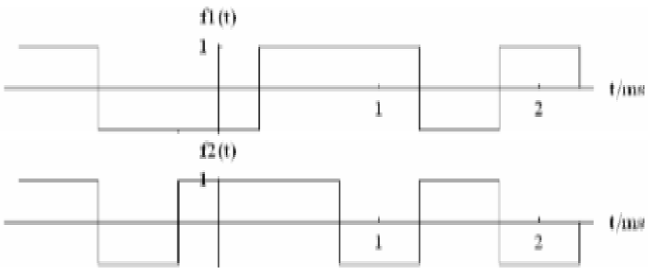


Fig. 5.2: Independent bit streams.

If the inphase and quadrature components in (5.1) only assume the values ± 1, this modulation method is referred to as 4-PSK or 4-QAM (quadrature amplitude modulation) because of the four possible carrier states. Fig. 4.5 (right) shows the associated constellation diagram. If only discrete-value signals are used for $s_{\mathrm{I}}(t) = f_1(t)$ and $s_{\mathrm{Q}}(t) = f_2(t)$, whose amplitudes can take on the values $\pm 1/3$ and ± 1, 16-QAM results (see Fig. 5.3). The gain in bit rate, however, comes at the cost of a loss in the signal-to-noise ratio. This can be recognized qualitatively by the fact that the distances between the individual points in the constellation diagram for higher-order IQ modulation schemes are getting smaller and smaller. The two signals $f_1'(t)$ and $f_2'(t)$ can contain different information, but are dependent on the same initial phase, which was omitted in (5.1). The phase can assume any values due to the transmission path. Phase synchronization may therefore be necessary for distortion-free demodulation.

In addition to digital demodulation of digital signals, digital demodulation of analog signals is

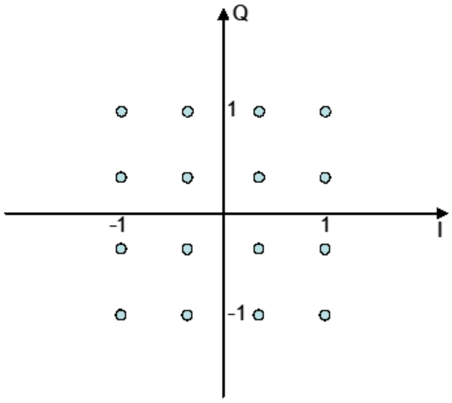


Fig. 5.3: Constellation diagram of a 16-QAM signal.

also possible. The sampled I and Q signals ($f_1'(t)$ and $f_2'(t)$) at the output of the IQ demodulator (right side in Fig. 5.1) can be processed with the help of a microprocessor to demodulate the amplitude, frequency and phase, taking into account (5.1):

AM demodulation
$$a'(t) = \sqrt{(s_{\mathrm{I}}(t))^2 + (s_{\mathrm{Q}}(t))^2} \tag{5.5}$$

PM demodulation
$$\varphi'(t) = \arctan\left(\frac{s_{\mathrm{Q}}(t)}{s_{\mathrm{I}}(t)}\right) \tag{5.6}$$

FM demodulation
$$f'(t) = \frac{1}{2\pi}\frac{d\varphi'(t)}{dt} = \frac{1}{2\pi}\frac{d\left(\arctan\left(\frac{s_{\mathrm{Q}}(t)}{s_{\mathrm{I}}(t)}\right)\right)}{dt} \tag{5.7}$$

In this case, the two I and Q signals have the same amplitude $a(t)$, but are shifted in phase by 90° since $\sin(\varphi) = \cos(\varphi - 90°)$.

6 Exercises

The tasks in this chapter must be completed before the test date and brought along on the day of the test. They are assessed together with the test protocol. Results are only evaluated with a clear calculation method!

6.1 Exercise 1 (AM)

An AM modulated signal is to be demodulated with an AM demodulator as shown in Fig. 3.3. The modulating signal represents a harmonic oscillation. For the modulation frequency 20 kHz and the modulation depth $m = 33\,\%$, calculate the largest possible time constant τ for which distortion-free demodulation is still possible. What is the value of the resistance R at $C = 300\,\mathrm{pF}$ under these conditions?

6.2 Exercise 2 (FM)

Derive equation 3.15.

6.3 Exercise 3 (IQ)

The following signal with a carrier frequency of 1 MHz is present at the input of an IQ demodulator:

$$s(t) = 2\Big[1 + 0.5\cos\Big(4\pi 10^3\,\mathrm{Hz}\cdot t + \frac{\pi}{4}\Big)\Big]\cos\Big(2\pi 10^6\,\mathrm{Hz}\cdot t - \sin\Big(6\pi 10^3\,\mathrm{Hz}\cdot t - \frac{\pi}{4}\Big)\Big) \quad (6.1)$$
$$0 \leq t \leq 1.5\,\mathrm{ms}$$

6.3.1 Use the formulas in Chapter 5 to determine the I and Q signals after the lowpass filters at the output of the IQ demodulator.

6.3.2 Determine for the signal $s(t)$:

- the mean carrier amplitude A_T

- the modulation depth m
- the AM modulation frequency $f_{\mathrm{N,AM}}$
- the peak phase deviation $\Delta\varphi$
- the modulation index η
- the peak frequency deviation Δf
- the FM modulation frequency $f_{\mathrm{N,FM}}$

7 Instructions for the Experiments

7.1 Analog Modulation and Demodulation

Fig. 7.1 shows the basic circuit diagram of a very simple demodulator for AM and FM signals. This modulator has to be parameterized now in a simulation. You will also get to know weak points of this modulator and use measuring devices to determine modulation parameters.

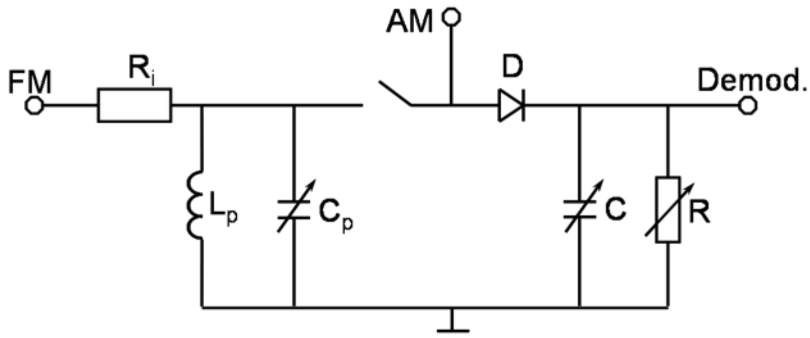


Fig. 7.1: Circuit diagram of an FM/AM demodulator.

In this experiment you will use the “LTSpice” circuit simulation tool from Linear Technology to generate and demodulate amplitude and frequency modulated signals.

Important keyboard shortcuts (are mentioned again in the text):

F1: LTSpice help
F2: Select component symbol
F3: Draw wire
F4: Label net
F5/Del./Entf.: Delete
F6: Copy
F7: Move
F8: Drag
F9: Undo action (very important tool in practice)

7.1.1 Generation of an Amplitude-Modulated Signal

Create a new worksheet via “File/New Schematic”. Press F2 to open the menu for components. Select “voltage” and put two of these voltage sources on the sheet. As in reality, every circuit needs a ground/reference point (or virtual ground). Connect each of the negative poles of the voltage sources to ground. To do this by pressing “g” on the keyboard and pull the appropriate connections (“F3” for pulling cables).

Now create a short open wire at the positive connections of the voltage sources (just draw a line) and assign a “label” to these wires by pressing F4. Name the network of the first voltage source “nf” (this will be your low-frequency signal) and the network of the second voltage source “hf” (this will be your carrier signal).

Now you can calculate the two sources almost as you like: To do this, open the window for selecting the components again and select the “bv” source (arbitrary behaviroal voltage source). Put the source on your sheet. Preferably close to their voltage sources. Also establish a reference to the common (global) ground. Right-click on the “bv” source to open a menu, where you enter the following term in the “Value” line

```
V=V(nf)*V(hf)
```

Now please label the positive pole of the “bv” source in a suitable manner.
Afterwards do the following:

An amplitude-modulated signal results (see also equation 3.3) according to

```
Vout = A_T*[1 + m*cos(w_n*t)]*cos(w_0*t)
```

with w_0 $\gg$ w_n. w_n and w_0 are angular frequencies.

Generate “Vout” with the two voltage sources. Right-click on the voltage sources to open a corresponding menu.

With the LF source, you can model the “1” using the “DC offset”. With “Amplitude” you directly select the modulation depth m ($m < 1$!). The frequency of the LF oscillation is also adjustable. With a phase offset of 90°, you can also convert the sine into a cosine. Proceed in the same way for the "hf" voltage source. Here an amplitude of 1, the phase and the frequency are to be set.

Parameters to be set:

- (Mean) carrier amplitude: 1 V
- Carrier frequency: 1 MHz
- LF frequency: 10 kHz
- Modulation depth: 0.3

You can now start the simulation. Click in the upper status bar on “Simulate”, then on “Run”. When you do this for the first time, a dialog opens. Enter 1e−3 seconds as the stop time. Finish with “ok”. A simulation instruction has now been automatically stored on your worksheet (. tran 1e−3, for transient analysis).

Display the LF, RF and the resulting amplitude-modulated signal (single or double click on the respective networks nf, hf and the network of the bv source). Save the result by selecting the window of the displayed waveform and then activating “Tools/Copy bitmap to Clipboard”. Use, for example, Paint to save your results as a file. Hint: Double click on bv source to get a clean window, then single click on nf and hf signals, so all three signals can be displayed in one window.

Generate the spectrum. To do this, right click on the waveform diagram and there “View/FFT”. All displayed waveforms should already be selected, if not, do so. Then continue with “ok”. In the following window you can select which signals you want to display. Depict all three (without windowing) and take a look at the result. A double-logarithmic representation is selected as standard. You can adjust the axes according to your requirements by right-clicking.

Display the spectrum of the amplitude-modulated signal and use this to determine the modulation depth m. Does it agree with the theoretical? What must be considered when determining the modulation depth in the frequency domain? Save the representation appropriately as before. Save your previous circuit too!

Create an amplitude-modulated signal (0 dBm output power) for the given parameters with the help of the Agilent signal generator and display it with a spectrum analyzer. Use a DC block, otherwise the spectrum analyzer can be damaged! And save the spectrum either by making a screenshot or by taking a photo (e.g., with a mobile phone). Determine the modulation depth m from the values you have read off.

7.1.2 Demodulation of an Amplitude-Modulated Signal

Keep signal settings as before.

An amplitude-modulated signal can be demodulated with a simple lowpass filter and a diode. Place a diode (hotkey d), a resistor (hotkey r) and a capacitor (hotkey c) on your worksheet. Arrange them appropriately and feed them with the output of the bv source. Select $10\,\mathrm{k\Omega}$ for the resistance.

Now find a suitable value for the capacitor to demodulate. Then plot your result and save it as a bitmap as before.
Hint: You can quickly change the value for C yourself and simulate it more often, or you can add a Spice command (select ".op" in the bar at the top). A parameter sweep can then be set here with ".step param C list 100n 1u". 100n would be 100 nF, 1u would be 1 µF. To make this possible, the value of the capacitor "{C}" must be set to make clear that it is a variable.

Demodulate two more signals. To do this, select any type of LF source (e.g., pulse with a defined rise and fall time → sawtooth/triangle/rectangle). Save the carrier, the LF signal and the demodulated signal in the form of a bitmap.

Save your circuit too!

7.1.3 Demodulation of a Frequency-Modulated Signal

Load the file Freq_Demod_raw.asc.

The signals have the following properties:

- 1 MHz carrier frequency (Frequency)
- 200 kHz frequency deviation (FM deviation)
- 10 kHz signal frequency (low-frequency, modulating signal, FM rate)

Start a transient simulation directly and look at the signal in front of the diode.
What frequency do you expect (frequency of the LF signal) and what frequency do you read off? Can you explain this effect? (Hint: Take a look at the transfer function of the pure resonant circuit. To do this, start an AC simulation via the simulation menu. You can also use an FFT at any time to view the spectrum.)
Parameterize the resonant circuit appropriately in order to receive a reasonable signal in front of the diode. Now demodulate the frequency-amplitude-modulated signal with the lowpass.
How and why can the amplifier help you with this? Hint: Look at the signal levels before

and after the diode.
How do you set up the gain appropriately? Save the signal in front of the diode and the demodulated signal as a bitmap.
Save the circuit in the same way.

Now create a frequency-modulated signal with the signal generator.
Look at it with the spectrum analyzer and identify the components of the received signal. Record this with pictures. Compare the characteristics of the measured spectrum to them of the theoretical one.
Before using the spectrum analyzer, make sure that a DC block is connected to its input.

7.2 Digital Demodulation

Generate similar signals on both signal generators (e.g, Agilent and Rohde & Schwarz) with the following settings:
Frequency: 880 MHz (D-net of the GSM communication system)
Amplitude: 10 dBm (Rohde & Schwarz), 0 dBm (Agilent)

IQ Demodulation of ASK-modulated Signals

Connect the {RF $50\,\Omega$} output of the Rohde & Schwarz generator to the {LO} input of the IQ demodulator and the {RF-OUTPUT} of the Agilent signal generator to the {RF} input of the IQ demodulator. Connect the I and Q outputs of the IQ demodulator to the inputs X and Y of the oscilloscope. Modulate the output signal of the Agilent generator with a periodic pulse signal ([Pulse], (Pulse Source): Internal Pulse, (Pulse Period): 120 µs, (Pulse Width): 40 µs, (Pulse): On). Trigger the oscilloscope using the {LF OUTPUT} of the Agilent generator ([LF Out]: On, [Voltage]: 1 Vpp). Determine the amplitudes of the two I and Q signals. Save an image of the signals on the oscilloscope.
Which form of the IQ signals would you expect? Think about how the modulated transmit signal looks like. In addition, think about what an IQ demodulator does or how it is constructed. Why could it be that the displayed IQ signals deviate from theory? What could help?
Hint: You are using two separate devices as transmitters and receivers.
Reconstruct the information signal according to (5.5) from the script (Calculate the amplitude of the received signal. Take a photo or save a picture of the displayed signals.).
Now change the frequency of the Rhode & Schwarz LO generator several times by $\pm 1\,\mathrm{MHz}$ and then set it back to 880 MHz each time to get different I and Q signals at the output of the

IQ demodulator. Save one or two of the corresponding results and perform again the digital amplitude demodulation of the signal at the output according to (5.5). Compare the results with each other.

Explain why the I and Q signals changed after the LO frequency had been changed and restored.

Hint: What is the relationship between frequency and phase?

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