

# FMCW Radar.- Northe, Velez, Woerner

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## Experimental task 1: VCO Characteristic curve

### 4.1.- VCO Characteristic Curve

4.1.1.- Connect the "User 1" port, whose output frequency is  $1/(16 \times 4096) = 1/65536$  of the output frequency

of VCO fVCO, to an oscilloscope, change the control voltage V from 1.0V to 7.0V with a step of 0.4 V using "T1\_Control\_Voltage.exe" and record the frequency for for each step in a Table.

4.1.2.- Taking into account that a frequency divider circuit with a factor of 4096 is used here, calculate

the output frequency fVCO at VCO's RFOUT port and record them in the third row of

Define control voltage values

```
clc, clear;
V = 1.0:0.4:7.0; % Voltage steps from 1.0V to 7.0V in 0.4V steps
```

Preallocate arrays for measured frequency (fr) and VCO frequency (f\_VCO)

```
fr = zeros(size(V)); % Frequency measured at User 1 port
fVCO = zeros(size(V)); % VCO output frequency (calculated)
```

Measured Frequency Values (in kHz):

```
fr = [358.7, 361.62, 364.04, 366.17, 368.19, 370.16, 371.88, 373.55, 375.09,
376.65, 378.12, 379.58, 380.86, 382.14, 383.3, 384.9];
```

Calculate VCO frequency

According to the instruction:  $f_r = f_{VCO} / 65536 \rightarrow f_{VCO} = f_r * 65536$

```
fVCO = (fr * 1e3 * 65536)/1e9; %result in GHz
```

Display Results in Table

```
T = table(V', fr', fVCO', 'VariableNames', {'V', 'f_r_kHz', 'f_VCO_GHz'});
disp('Table 4.1: Measured results of the VCO's characteristic');
```

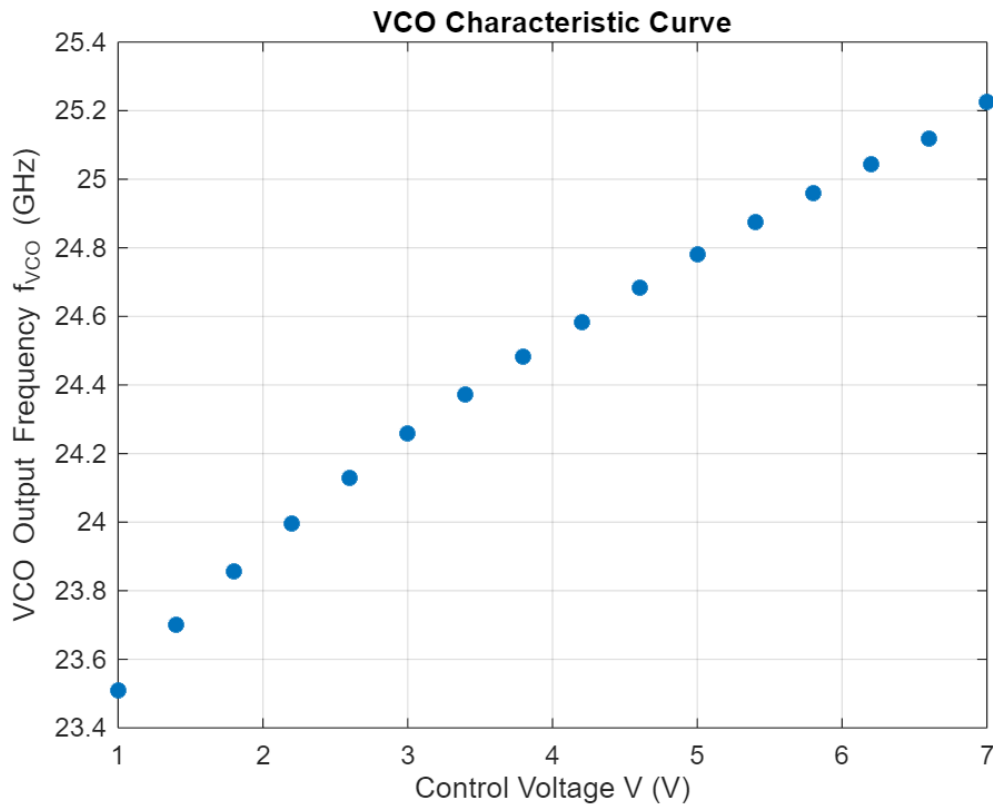
Table 4.1: Measured results of the VCO's characteristic

```
disp(T);
```

V	f_r_kHz	f_VCO_GHz
1	358.7	23.508
1.4	361.62	23.699
1.8	364.04	23.858
2.2	366.17	23.997
2.6	368.19	24.13
3	370.16	24.259
3.4	371.88	24.372
3.8	373.55	24.481
4.2	375.09	24.582
4.6	376.65	24.684
5	378.12	24.78
5.4	379.58	24.876
5.8	380.86	24.96
6.2	382.14	25.044
6.6	383.3	25.12
7	384.9	25.225

## Plot of the calculated VCO Characteristic Curve

```
figure;
plot(V, fVCO, '*', 'LineWidth', 2, 'LineStyle', 'none');
xlabel('Control Voltage V (V)');
ylabel('VCO Output Frequency f_{VCO} (GHz)');
title('VCO Characteristic Curve');
grid on;
```



**4.1.3.- Depict the VCO's characteristic curve  $f_{VCO}$  (V ) using MATLAB (use `interp1()` for interpolation. and choose `spline` for interpolation method, attach the source code to your final report).**

Calculate Interpolated VCO frequency

```
stepsize = 0.01;
V_fine = 1.0:stepsize:7.0;
f_VCO_interpol = interp1(V, fVCO, V_fine, 'spline');
```

Calculate Ideal Interpolation

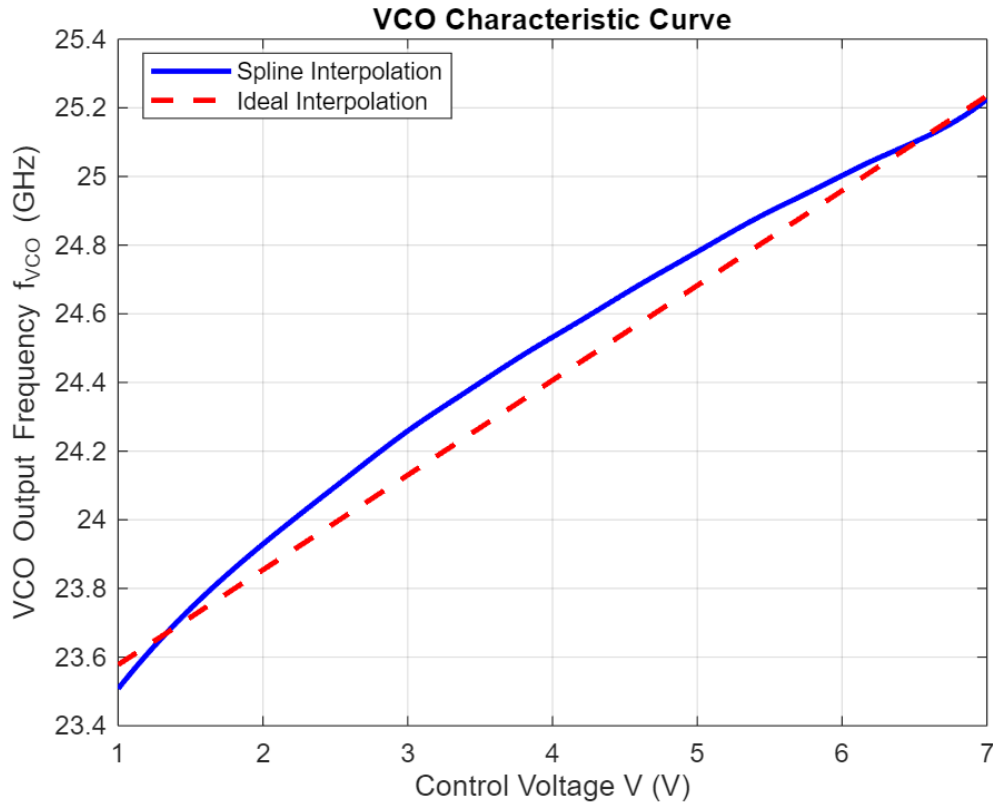
```
Offset_min = 0.07;
Offset_max = 0.01;
Vmin = min(f_VCO_interpol)+ Offset_min;
Vmax = max(f_VCO_interpol)+ Offset_max;
dist = Vmax-Vmin;
dim = size(V_fine);
steps = dist / dim(2);
f_VCO_ideal = Vmin:steps:Vmax;
stepsize2 = 6 / 601;
V_fine_2 = 1:stepsize2:7;
```

Plot interpolated VCO Characteristic Curve

```
figure;
plot(V_fine, f_VCO_interpol, 'b-', 'LineWidth', 2);
hold on;
plot(V_fine_2, f_VCO_ideal, 'r--', 'LineWidth', 2);
hold off;
xlabel('Control Voltage V (V)');
```

```
ylabel('VCO Output Frequency f_{VCO} (GHz)');
title('VCO Characteristic Curve');
legend('Spline Interpolation', 'Ideal Interpolation','Location', 'best');
grid on;

legend(["Spline Interpolation", "Ideal Interpolation"], "Position", [0.1530
0.8305 0.2750, 0.0774])
```



#### 4.1.4.- Specify the control voltage $V_C$ for a triangular FMCW RADAR using measured VCO Curve

Given:

$f_0 = 24$  GHz

$B = 500$  MHz

$f_{DA} = 100$  kHz  $\rightarrow 500\text{MHz} / 100 \text{ kHz} = 5000$

$T_{\text{sweep}} = 10$  ms

Get relevant voltage range

```
% Frequency range in GHz
f_min = 23.75;
f_max = 24.25;
T_sweep = 10; %ms

% mask frequencies of the relevant region
%mask = f_VCO_ideal((f_VCO_ideal >= f_min) & (f_VCO_ideal <= f_max))
%mask = (f_VCO_ideal >= f_min) & (f_VCO_ideal <= f_max)

% Choose the related voltages to the frequencies
```

```
%V_selected = V_fine_2(mask)
```

```
df = 0.0001;          % 100 kHz = 0.0001 GHz

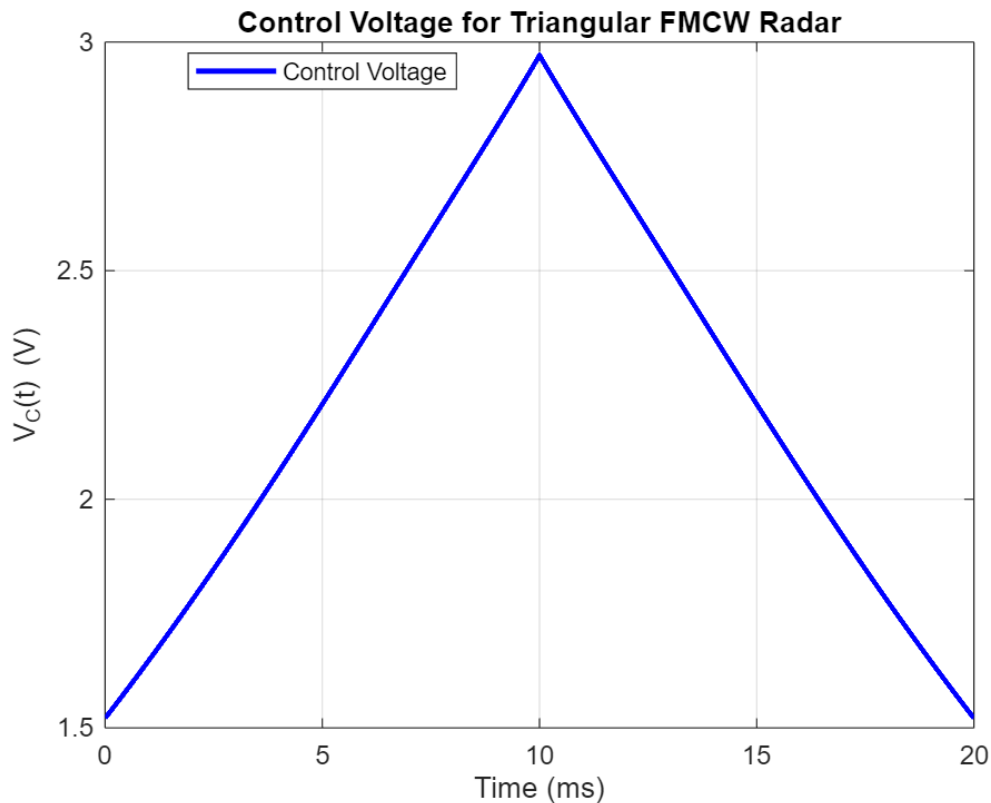
% Step 2: Create desired frequency array with 100kHz steps
f_uniform = f_min:df:f_max;

% Step 3: Invert the interpolation: find V values for desired f values
% Use interp1 with frequency as x and voltage as y
%V_interp = interp1(f_VCO_ideal, V_fine_2, f_uniform, 'spline');
V_interp = interp1(f_VCO_interpol, V_fine, f_uniform, 'spline');

% step 4: remove any NaNs due to extrapolation
valid_idx = ~isnan(V_interp);
V_interp = V_interp(valid_idx);
V_full_sweep = [V_interp flip(V_interp(1:end-1))];
f_uniform = f_uniform(valid_idx);
timestep = T_sweep / 5000;
time = 0:timestep:2*T_sweep;
```

Plot the  $V_c(t)$  curve

```
figure;
plot(time, V_full_sweep, 'b-', 'LineWidth', 2);
xlabel('Time (ms)');
ylabel('V_C(t) (V)');
title('Control Voltage for Triangular FMCW Radar');
legend('Control Voltage', 'Location', 'best');
grid on;
```



#### 4.1.5.- Why is the VCO's characteristic curve needed before the radar measurement?

Because the VCO output frequency is not perfectly linear to its input voltage. Therefore we have to measure its characteristic curve and map the input voltage values to the actually generated frequencies to ensure, that the frequency sweep we generate behaves the way we want.

## Experimental task 2: Distance Measurement

A corner reflector is placed in front of the radar ( $1.5 \text{ m} < r < 5 \text{ m}$ ). The parameters of a triangular FMCW radar are given as follows:  $f_0 = 24 \text{ GHz}$ ,  $B = 1 \text{ GHz}$ ,  $T_{\text{sweep}} = 10 \text{ ms}$ .

**4.2.1.- Estimate the minimal AD sampling frequency  $f_s$  for this experiment according to the Nyquist sampling criterion (the real sampling frequency should be 1.3 times of the theoretical one). Examine whether all sampling points can be used or whether there is an overlap between up- and down-sweep. Choose the sampling frequency in a way that the Nyquist criterion is fulfilled and a Fast Fourier Transform can be used.**

The following snippet computes the required  $f_{s\_min}$  and  $f_{s\_min\_fft}$  to satisfy these conditions:

```
clear; clc;

c0 = physconst('LightSpeed');
fc = 24e9;           % [Hz]
r = 5;              % [m]
B = 1e9;             % [Hz]
T_sweep = 10e-3;    % [s]

gamma = B / T_sweep;
f_b_max = (2 * gamma * r) / c0;
```

```
f_s_min = 2 * f_b_max * 1.3
```

```
f_s_min =  
8.6727e+03
```

```
f_s_fft_min = f_s_min * 2
```

```
f_s_fft_min =  
1.7345e+04
```

#### 4.2.2 Range Estimation $r$ from $S_{if}$

```
load("ws_task2.mat");
```

```
c0 = physconst('LightSpeed');
```

```
f_c      = 24e9;      % [Hz]
```

```
T_sweep = 10e-3;     % [s]
```

```
BW = fmax - fmin;
```

```
gamma = BW / (T_sweep);
```

```
% n samples which are needed for one sweep
```

```
N = round(T_sweep * fs);
```

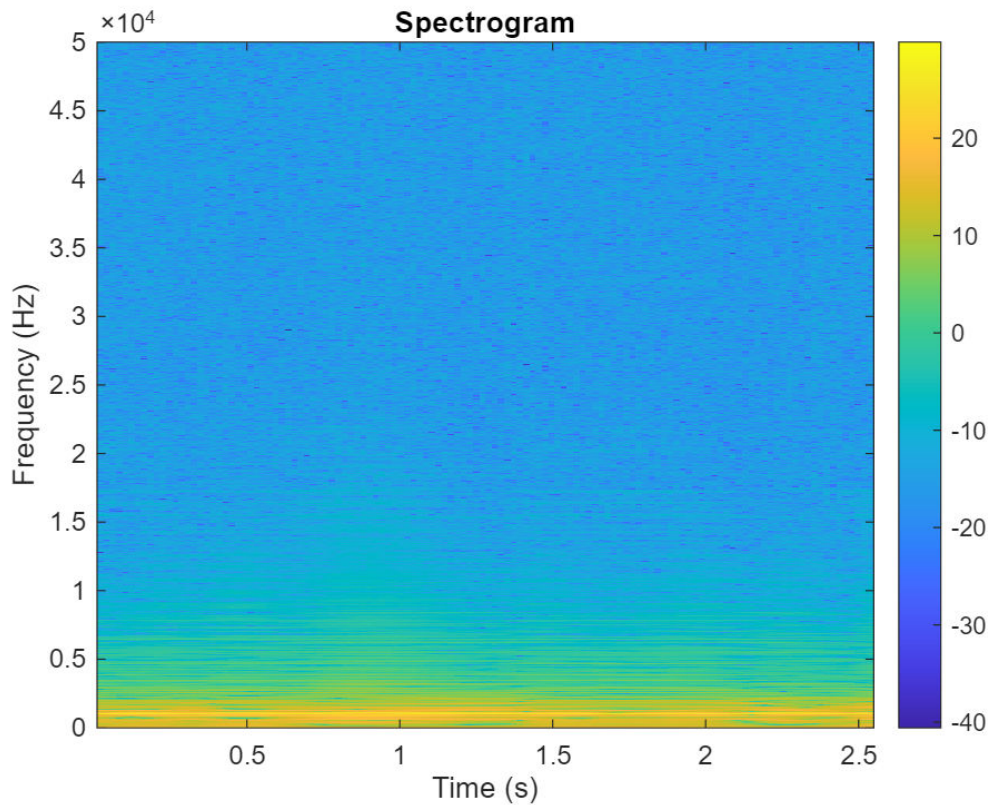
```
fft_length = 2^nextpow2(N);
```

```
% 4 * for smooth vel
```

```
fft_length = 4 * fft_length
```

```
fft_length =  
4096
```

```
[s, f, t] = make_spectrogram(sif, fs, fft_length);
```



```
[~, time_slices] = size(s);

v_r = zeros(1, time_slices);
range = zeros(1, time_slices);

for t = 1:time_slices
    % for each spectrum of the spectrogram
    % find the two local max (try to avoid finding two adjacent bins)
    % then use f_b + f_d formulas

    spec = s(:, t);
    spec = abs(spec);

    max_mask = islocalmax(spec);
    peak_vals = spec(max_mask);

    % for debugging the peaks
    %plot_spectrum_with_peaks(spec, max_mask, fs);

    if numel(peak_vals) >= 2
        [~, idx_sorted] = maxk(peak_vals, 2);
        peak_indices_in_spec = find(max_mask);
        top2_indices = peak_indices_in_spec(idx_sorted);
        top2_values = spec(top2_indices);
    elseif numel(peak_vals) == 1
        top2_indices = find(max_mask);
        top2_values = spec(top2_indices);
    else
        top2_indices = [];
    end
end
```



```

        top2_values = [];
    end

    top2_indices = top2_indices * fs / fft_length;

    f_b = 0.5 * (top2_indices(1) + top2_indices(2));
    f_d = 0.5 * (top2_indices(1) - top2_indices(2));

    range(t) = c0 / (2 * gamma) * f_b;
    v_r(t)    = c0 / (2 * f_c)    * f_d;
end

r_min = min(range);
r_max = max(range);
v_min = min(v_r);
v_max = max(v_r);

fprintf('Raw:\n');

```

Raw:

```
fprintf('Range:   Min = %.3f m, Max = %.3f m\n', r_min, r_max);
```

Range: Min = 1.446 m, Max = 1.665 m

```
fprintf('Velocity: Min = %.3f m/s, Max = %.3f m/s\n', v_min, v_max);
```

Velocity: Min = -0.305 m/s, Max = 0.610 m/s

```

range_filtered = sgolayfilt(range, 3, 11);
v_r_filtered   = sgolayfilt(v_r, 3, 11);

r_min_f = min(range_filtered);
r_max_f = max(range_filtered);

v_r_min_f = min(v_r_filtered);
v_r_max_f = max(v_r_filtered);

fprintf('\nFiltered:\n');

```

Filtered:

```
fprintf('Range:   Min = %.3f m, Max = %.3f m\n', r_min_f, r_max_f);
```

Range: Min = 1.447 m, Max = 1.669 m

```
fprintf('Velocity: Min = %.3f m/s, Max = %.3f m/s\n', v_r_min_f, v_r_max_f);
```

Velocity: Min = -0.371 m/s, Max = 0.688 m/s

```
fprintf('For Q. 4.2.2: %.3f m\n', mean(range_filtered)); %Computation for
estimating the average range
```

For Q. 4.2.2: 1.528 m

```

% Time or index axis
t = 1:length(range);

```

```

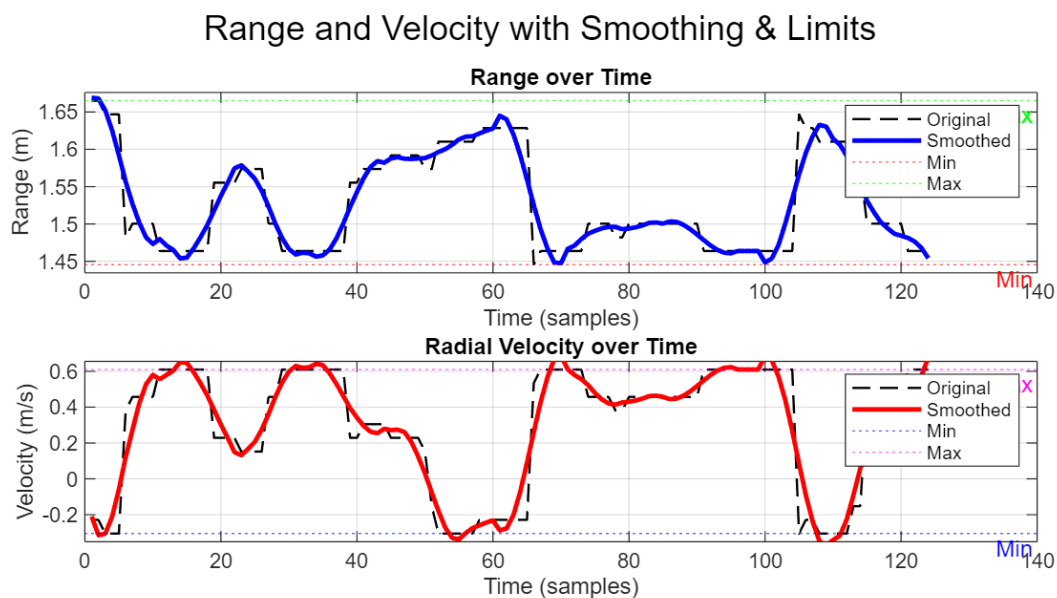
% Create figure
figure('Color', 'w', 'Position', [100, 100, 1000, 500]);

% === Plot 1: Range ===
subplot(2,1,1);
plot(t, range, '--k', 'LineWidth', 1); hold on;
plot(t, range_filtered, 'b', 'LineWidth', 2);
yline(r_min, ':r', 'Min', 'LabelVerticalAlignment','bottom');
yline(r_max, ':g', 'Max', 'LabelVerticalAlignment','bottom');
grid on;
title('Range over Time');
xlabel('Time (samples)');
ylabel('Range (m)');
legend('Original', 'Smoothed', 'Min', 'Max');
ylim([r_min - 0.05*(r_max - r_min), r_max + 0.05*(r_max - r_min)]);

% === Plot 2: Velocity ===
subplot(2,1,2);
plot(t, v_r, '--k', 'LineWidth', 1); hold on;
plot(t, v_r_filtered, 'r', 'LineWidth', 2);
yline(v_min, ':b', 'Min', 'LabelVerticalAlignment','bottom');
yline(v_max, ':m', 'Max', 'LabelVerticalAlignment','bottom');
grid on;
title('Radial Velocity over Time');
xlabel('Time (samples)');
ylabel('Velocity (m/s)');
legend('Original', 'Smoothed', 'Min', 'Max');
ylim([v_min - 0.05*(v_max - v_min), v_max + 0.05*(v_max - v_min)]);

sgtitle('Range and Velocity with Smoothing & Limits');

```



```
fprintf('Range Resolution: %.3f m\n', c0 / (2 * 1e9));
```

Range Resolution: 0.150 m

$r = 1.528m$  computed as the average of the range.

### Explanation:

Check the answer for question 4.3 where the whole procedure for generating the range and velocity plots is written. As the code generating both of them is the same, with the only difference being the data set that is loaded at the beginning.

We observe that the range is correctly obtained from the measurements performed with the FMCW radar. However, the measurements indicate that the position of the target was not constant, but rather there was a small movement range during the measurements of  $\pm 10$  cm w.r.t. the mean position of 1.5 meters. This can be explained by slight movements from the person holding the target, this small fluctuation could cause, after the whole procedure of recovering the range that the estimation falls into the wrong bin, and therefore the error causes the estimation to be off by  $\pm 15$ cm, which is what we observe on our plots.

#### 4.2.3 Indicate the false target

The false target can be seen by the strong peak in the  $s_{if}$  signal in the frequency domain.

Here the strongest peak is in the DC/0-Hz Bin. This has to be ignored in the signal processing as this signal is the result from TX-RX coupling. The coupled signal has the strongest signal power and almost no phase shift and no doppler shift. Therefore the result from the mixing process is  $f_{TX} - f_{RX} = 0$  and  $f_{TX} + f_{RX}$  and the both frequencies.

But only the 0Hz results is visible due to image filtering after the mixing stage.

#### 4.2.4 Theoretical vs. Measured SNR

```
clear; clc;

c0 = physconst('LightSpeed');
k_b = physconst('Boltzmann');

db_2_lin    = @(g) 10^(g / 10);
dbm_2_lin   = @(g) 10.^((g - 30) / 10);

lin_2_db    = @(g) 10 * log10(g);
lin_2_dBm   = @(g) lin_2_db(g) + 30;

f_2_lambda = @(f) c0 / f;

T          = 300;           % [K]
fc         = 24e9;          % [Hz]
fs         = 100e3;         % [Hz]
r          = 1.56;          % [m]
a          = 0.15;          % [m]
P_vco      = 12;            % [dBm]
L_pd       = 4;             % [dB]
G_ant      = 15;            % [dBi]
G_lna      = 18;            % [dB]
L_mix      = 8;             % [dB]
G_ifa      = 34;            % [dB]
F_lna      = 2.6;           % [dB]
F_mix      = 8;             % [dB]
```

```

F_ifa    = 19.1;           % [dB]
F_ad     = 59;             % [dB]

lambda = f_2_lambda(fc);
rcs = 4 * pi * a^4 / (3 * lambda^2);

D_f = lin_2_db((4 * pi)^3 * r^4 / (rcs * lambda^2));

P_in  = P_vco - L_pd + G_ant - D_f + G_ant;
N_t   = lin_2_dBm(k_b * T * fs);
SNR_i = P_in - N_t;

f_lna = db_2_lin(F_lna);
l_mix = db_2_lin(-F_mix);
g_lna = db_2_lin(G_lna);
g_ifa = db_2_lin(G_ifa);
f_ad  = db_2_lin(F_ad);
f_mix = db_2_lin(-F_mix);
f_ifa = db_2_lin(F_ifa);

F =      f_lna + ...
      (f_mix - 1) / g_lna + ...
      (f_ifa - 1) / (g_lna * l_mix) + ...
      (f_ad - 1) / (g_lna * l_mix * g_ifa);
NF = 10 * log10(F);

SNR_o = SNR_i - NF

```

```

SNR_o =
78.2152

```

$SNR_{Theo} = 78.21dB$

```

clear; clc;

sif_with = load('ws_task2.mat').sif;
sif_without = load('ws_task3.mat').sif;

N = min(length(sif_with), length(sif_without));
sif_with = sif_with(1:N);
sif_without = sif_without(1:N);

spectrum_with = abs(fft(sif_with)).^2 / N;
spectrum_noise = abs(fft(sif_without)).^2 / N;

% real data -> only pos side w/o DC
[~, peak_bin] = max(spectrum_with(2:N/2));
peak_bin = peak_bin + 1;

P_signal_bin = spectrum_with(peak_bin);
P_noise_bin = spectrum_noise(peak_bin);

```

```
% SNR estimate
```

```
SNR_bin = P_signal_bin / max(P_noise_bin, eps);
```

```
SNR_dB = 10 * log10(SNR_bin);
```

```
fprintf('Frequency-domain SNR at peak bin: %.2f dB\n', SNR_dB);
```

Frequency-domain SNR at peak bin: 50.21 dB

```
% some plotting
```

```
f = linspace(0, 1, N);
```

```
figure;
```

```
plot(f(1:N/2), 10*log10(spectrum_with(1:N/2)), 'b', 'DisplayName',  
'Signal+Noise');
```

```
hold on;
```

```
plot(f(1:N/2), 10*log10(spectrum_noise(1:N/2)), 'r', 'DisplayName', 'Noise  
Only');
```

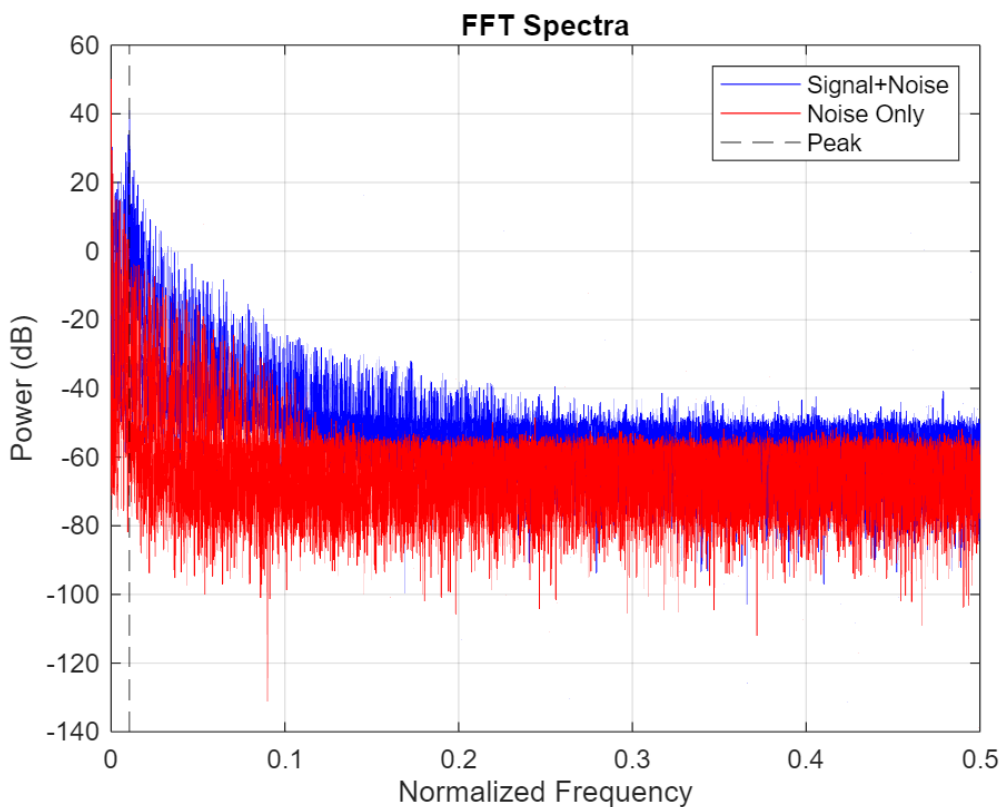
```
xline(f(peak_bin), '--k', 'DisplayName', 'Peak');
```

```
legend; grid on;
```

```
xlabel('Normalized Frequency');
```

```
ylabel('Power (dB)');
```

```
title('FFT Spectra');
```



$$SNR_{Meas} = 50.21dB$$

From the plot, we can observe in red the measurement of the noise floor, while in blue we observe the measurement of the target where we later estimated it's range. Evidently when the Tx and Rx antennas are covered the noise floor has a lower power.

When the transmitter and receiver antennas are covered with a sponge, the transmitted signal is blocked from reaching the receiver, and environmental reflections are suppressed. As a result, the received signal contains only the system's internal noise — primarily thermal noise from the receiver circuitry and quantization noise from the ADC. This is what is shown in the red spectrum, which represents the baseline noise floor of the radar system in the absence of any external signal.

When the system operates normally with the antennas uncovered, the receiver captures both internal noise and the beat frequency signals resulting from reflections of the transmitted FMCW chirp off nearby objects. These reflected signals add coherent energy at specific frequencies in the FFT, depending on the distance to the target. This results in a significant increase in spectral power, as shown in the blue curve, particularly at lower frequencies corresponding to closer reflectors. The difference in power between the blue and red spectra represents the signal strength and indicates a good signal-to-noise ratio (SNR) in the system.

## 4.3 Velocity measurement

In this task, you will deal with a moving target and try to detect its maximal velocity during the measuring period. The target is supposed to be located 5m away from the radar, the velocity of the target is determined by you, the required  $\Delta r$  is 0.15m and the required  $\Delta v_r$  is 1 m/s.

```
% Constants
c = 3e8;           % Speed of light in m/s
f0 = 24e9;         % Center frequency in Hz (24 GHz)
R_target = 5;      % Target range in meters
range_resolution = 0.15; % Required range resolution (m)
vel_resolution = 1; % Required velocity resolution (m/s)

% --- Parameter Options from Table 4.2 ---
B_options = [200e6, 500e6, 1000e6]; % Bandwidths in Hz
T_options = [2e-3, 5e-3, 10e-3]; % Sweep times in seconds
fs_options = [100e3, 500e3, 1000e3]; % Sampling rates in Hz
```

### 4.3.1.- Determination of system parameters from Table 4.2

```
delta_r = c ./ (2*B_options) % range resolution
```

```
delta_r = 1x3
    0.7500    0.3000    0.1500
```

```
delta_v = c ./ (2 * f0 * T_options) % velocity resolution
```

```
delta_v = 1x3
    3.1250    1.2500    0.6250
```

```
gamma = 1000e6 / (10e-3)
```

```
gamma =
    1.0000e+11
```

```
f_b_max = (2 * gamma * R_target) / (c);
```

```
f_s_min = 2 * f_b_max * 1.3
```

```
f_s_min =  
8.6667e+03
```

```
f_s_fft_min = f_s_min * 2
```

```
f_s_fft_min =  
1.7333e+04
```

Since the required range resolution is 15cm, the Bandwidth should be chosen to  $B = 1000e6$  Hz.

The required velocity resolution is 1m/s. The only  $T_{\text{sweep}}$  which satisfies the condition is 10ms.

After considering the chosen  $B$ ,  $T_{\text{sweep}}$ , the Nyquist criteria and the recommended factor of 1.3, the minimum sampling rate should be around 17.3 kHz. That means the smallest sampling rate from the table (100kHz) is sufficient.

#### 4.3.3 Estimation of $r$ and $v_r$

Explanation:

First we load the dataset with the moving trihedral (ws\_task4.mat) and calculate some constants, which are needed later.

Most importantly is the calculation of the minimum samples needed for one triangle waveform. We have found out that the best spectrogram is, when we use twice this number, otherwise the velocity plot is very distorted.

One more benefit of using more samples is that the fft provides some noncoherent integration gain, which makes it also easier to detect the peaks later on.

For the windowing function we have used the Blackman-Harris function as it has a stronger sidelobe suppression as the Hamming function, therefore helping by pronouncing the peaks.

Further we use some overlap in order to increase the time resolution of the spectrogram.

In the spectrogram we can clearly see the motion of the products from the mixer.

After creating the spectrogram (make\_spectrogram.m) we go through each spectrum/time slice and search there for the prominent peaks, but also ignoring the DC bin.

Then we convert bin to frequency and calculate the  $f_{\text{Beat}}$  and  $f_{\text{Doppler}}$  with the formulas from the PDF and add them to the "global" vector/list/array.

So we have global array for each  $r$  and  $v_r$  estimation from each time slice from the spectrogram.

Finally, we filter each of the arrays and plot them. The coloured line is filtered, the black one the raw one.

In both diagram we can see a sinusoid motion, which makes sense as distance and velocity are related to each other. With some filtering the erratic peaks are mostly removed.

```
clear; clc; close all;  
  
load("ws_task4.mat");  
  
c0 = physconst('LightSpeed');  
f_c = 24e9; % [Hz]  
T_sweep = 10e-3; % [s]
```

```

BW = fmax - fmin;
gamma = BW / (T_sweep);

% n samples which are needed for one sweep
N = round(T_sweep * fs);
fft_length = 2^nextpow2(N);

% 4 * for smooth vel
fft_length = 4 * fft_length

```

```

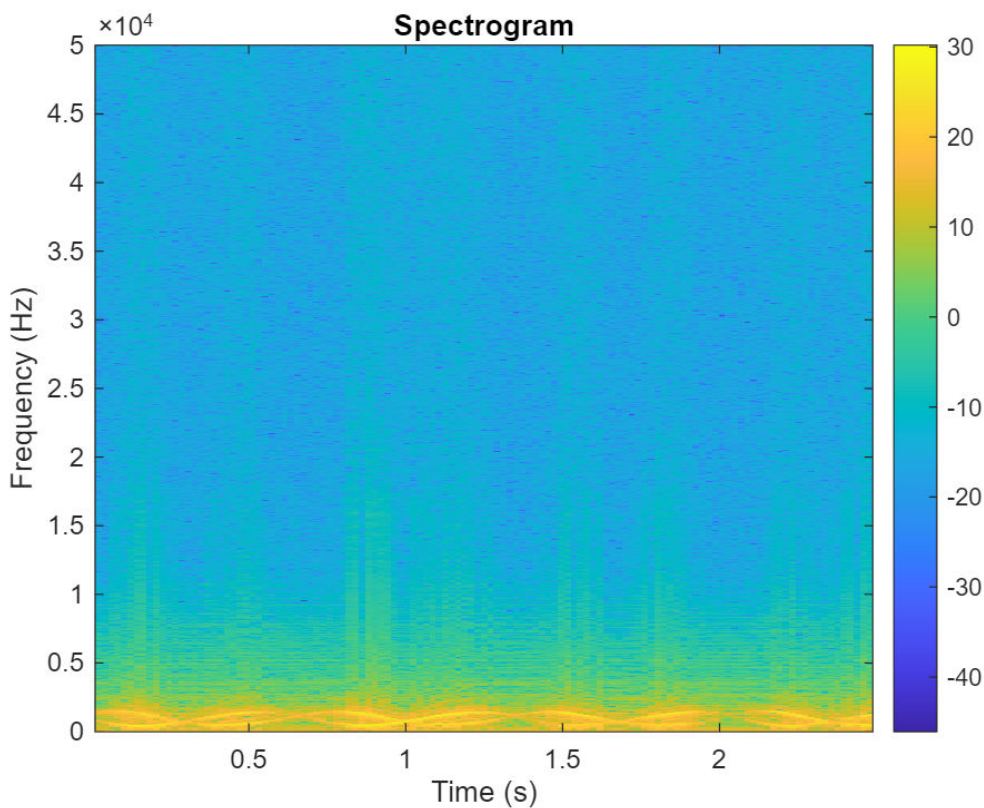
fft_length =
4096

```

```

[s, f, t] = make_spectrogram(sif, fs, fft_length);

```



```

[~, time_slices] = size(s);

v_r = zeros(1, time_slices);
range = zeros(1, time_slices);

for t = 1:time_slices
    % for each spectrum of the spectrogram
    % find the two local max (try to avoid finding two adjacent bins)
    % then use f_b + f_d formulas

    spec = s(:, t);
    spec = abs(spec);

    max_mask = islocalmax(spec);
    peak_vals = spec(max_mask);

```



```

% for debugging the peaks
%plot_spectrum_with_peaks(spec, max_mask, fs);

if numel(peak_vals) >= 2
    [~, idx_sorted] = maxk(peak_vals, 2);
    peak_indices_in_spec = find(max_mask);
    top2_indices = peak_indices_in_spec(idx_sorted);
    top2_values = spec(top2_indices);
elseif numel(peak_vals) == 1
    top2_indices = find(max_mask);
    top2_values = spec(top2_indices);
else
    top2_indices = [];
    top2_values = [];
end

top2_indices = top2_indices * fs / fft_length;

f_b = 0.5 * (top2_indices(1) + top2_indices(2));
f_d = 0.5 * (top2_indices(1) - top2_indices(2));

range(t) = c0 / (2 * gamma) * f_b;
v_r(t)    = c0 / (2 * f_c) * f_d;
end

r_min = min(range);
r_max = max(range);
v_min = min(v_r);
v_max = max(v_r);

fprintf('Raw:\n');

```

Raw:

```
fprintf('Range:   Min = %.3f m, Max = %.3f m\n', r_min, r_max);
```

Range: Min = 0.732 m, Max = 2.086 m

```
fprintf('Velocity: Min = %.3f m/s, Max = %.3f m/s\n', v_min, v_max);
```

Velocity: Min = -3.278 m/s, Max = 3.507 m/s

```

range_filtered = sgolayfilt(range, 3, 11);
v_r_filtered = sgolayfilt(v_r, 3, 11);

r_min_f = min(range_filtered);
r_max_f = max(range_filtered);

v_r_min_f = min(v_r_filtered);
v_r_max_f = max(v_r_filtered);

fprintf('\nFiltered:\n');

```

Filtered:

```
fprintf('Range:   Min = %.3f m, Max = %.3f m\n', r_min_f, r_max_f);
```

Range: Min = 1.013 m, Max = 1.822 m

```
fprintf('Velocity: Min = %.3f m/s, Max = %.3f m/s\n', v_r_min_f, v_r_max_f);
```

Velocity: Min = -3.339 m/s, Max = 3.194 m/s

```
fprintf('For Q. 4.2.2: %.3f m\n', mean(range_filtered));
```

For Q. 4.2.2: 1.403 m

```
% plot
t = 1:length(range);

figure('Color', 'w', 'Position', [100, 100, 1000, 500]);

% Range plots
subplot(2,1,1);
plot(t, range, '--k', 'LineWidth', 1); hold on;
plot(t, range_filtered, 'b', 'LineWidth', 2);
yline(r_min, ':r', 'Min', 'LabelVerticalAlignment','bottom');
yline(r_max, ':g', 'Max', 'LabelVerticalAlignment','bottom');
grid on;
title('Range over Time');
xlabel('Time (samples)');
ylabel('Range (m)');
legend('Original', 'Smoothed', 'Min', 'Max');
ylim([r_min - 0.05*(r_max - r_min), r_max + 0.05*(r_max - r_min)]);

% vel plot
subplot(2,1,2);
plot(t, v_r, '--k', 'LineWidth', 1); hold on;
plot(t, v_r_filtered, 'r', 'LineWidth', 2);
yline(v_min, ':b', 'Min', 'LabelVerticalAlignment','bottom');
yline(v_max, ':m', 'Max', 'LabelVerticalAlignment','bottom');
grid on;
title('Radial Velocity over Time');
xlabel('Time (samples)');
ylabel('Velocity (m/s)');
legend('Original', 'Smoothed', 'Min', 'Max');
ylim([v_min - 0.05*(v_max - v_min), v_max + 0.05*(v_max - v_min)]);

sgtitle('Range and Velocity with Smoothing & Limits');
```

## Range and Velocity with Smoothing & Limits

