Preliminary_Session6_Team1

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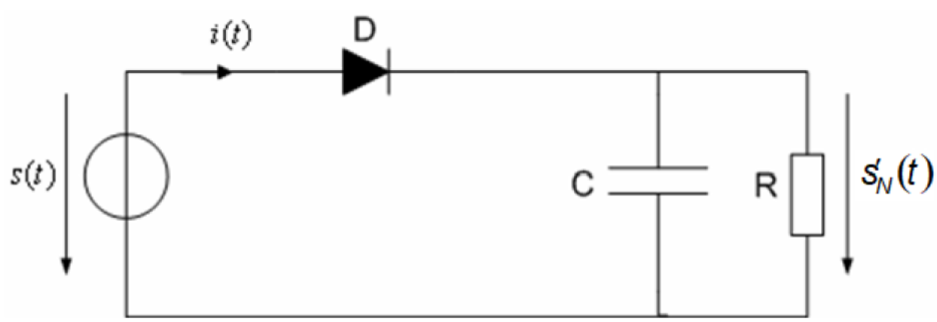
1 Preparation for Experiment.- Northe, Velez, Wörner

1.1 6.1 Exercise 1

An AM modulated signal is to be demodulated with an AM demodulator as shown in Fig. 3.3. The modulating signal represents a harmonic oscillation. For the modulation frequency 20 kHz and the modulation depth m = 33%, calculate the largest possible time constant τ for which distortion-free demodulation is still possible. What is the value of the resistance R at C = 300 pF under these conditions?



1.1.1 Solution: Exercise 1 (AM) — Largest Time Constant for Distortion-Free Demodulation

Given:

- Modulation frequency: $f_{\max} = 20\,\text{kHz}$
- Modulation depth: $m = 0.33$
- Capacitance: $C = 300\,\text{pF} = 300 \times 10^{-12}\,\text{F}$

From the lab manual (Eq. 3.6), the time constant τ of the envelope detector must satisfy:

$$\frac{1}{\omega_0} \ll \tau < \frac{1}{2\pi m f_{\max}}$$

We use the upper limit:

$$\tau_{\max} = \frac{1}{2\pi m f_{\max}}$$

Substituting values:

$$\tau_{\max} = \frac{1}{2\pi \cdot 0.33 \cdot 20 \times 10^3} = \frac{1}{41{,}469.576} \approx 24.1\,\mu\text{s}$$

To find the corresponding resistance R:

$$\tau = R \cdot C \quad \Rightarrow \quad R = \frac{\tau}{C}$$

Substituting:

$$R = \frac{24.1 \times 10^{-6}}{300 \times 10^{-12}} = 80{,}333.33\,\Omega \approx \boxed{80.3\,\text{k}\Omega}$$

The following script computes the value and prints the result:

[2]:
```python
# Given values
f_max = 20e3            # Hz
m = 0.33
C = 300e-12             # Farads

# Compute the maximum time constant tau
import math
tau_max = 1 / (2 * math.pi * m * f_max)

# Compute resistance R
R = tau_max / C

# Display results
print(f"Maximum  : {tau_max * 1e6:.2f} µs")
print(f"Required R: {R / 1e3:.1f} kΩ")
```

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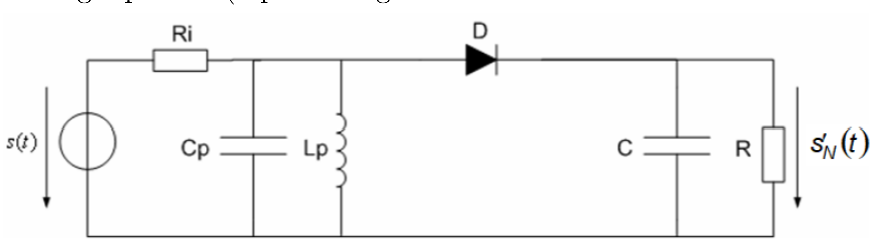
Maximum  : 24.11 µs
Required R: 80.4 kΩ
```

1.2 6.2 Exercise 2

Derive the following equation (representing the transfer function of the circuit used for AM-FM



conversion).

1.2.1 Solution: Exercise 2 (FM) — Derivation of Equation (3.15)

We analyze the FM demodulator circuit **before the diode**, which consists of a series resistor (R_i) and a parallel LC tank ((L_p C_p)).

We aim to compute the transfer function:

$$G(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}}$$

where: - (V_{in}): FM input signal - (V_{out}): voltage across the LC tank

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1.2.2 Step 1: Impedance of the LC Tank

The parallel combination is:

$$Z_{LC} = \left(\frac{1}{j\omega C_p} \parallel j\omega L_p \right) = \frac{\left(\frac{1}{j\omega C_p} \right) \cdot (j\omega L_p)}{\frac{1}{j\omega C_p} + j\omega L_p}$$

Numerator:

$$\frac{1}{j\omega C_p} \cdot j\omega L_p = \frac{L_p}{C_p}$$

Denominator:

$$\frac{1}{j\omega C_p} + j\omega L_p = \frac{1 - \omega^2 L_p C_p}{j\omega C_p}$$

Therefore:

$$Z_{LC} = \frac{L_p / C_p}{(1 - \omega^2 L_p C_p)/(j\omega C_p)} = \frac{j\omega L_p}{1 - \omega^2 L_p C_p}$$

1.2.3 Step 2: Total Series Impedance

The total impedance from input to ground is:

$$Z_{\text{total}} = R_i + Z_{LC} = R_i + \frac{j\omega L_p}{1 - \omega^2 L_p C_p}$$

1.2.4 Step 3: Voltage Divider

Using voltage division:

$$G(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_{LC}}{R_i + Z_{LC}} = \frac{\frac{j\omega L_p}{1-\omega^2 L_p C_p}}{R_i + \frac{j\omega L_p}{1-\omega^2 L_p C_p}}$$

Multiply numerator and denominator by (1 - ˆ2 L_p C_p):

$$G(\omega) = \frac{j\omega L_p}{j\omega L_p + R_i(1-\omega^2 L_p C_p)}$$

1.2.5 Final Result

$$\boxed{G(\omega) = \frac{j\omega L_p}{j\omega L_p + R_i\left(1-\omega^2 L_p C_p\right)}}$$

This transfer function describes how the **slope detector** converts **frequency variations (FM)** into **amplitude variations (AM)**, which can then be demodulated by the envelope detector.

1.3 6.3 Exercise 3 IQ

$$f_c = 1MHz$$
$$s(t) = 2\left[1 + 0.5\cos\left(4\pi\times 10^3\,\text{Hz}\cdot t + \frac{\pi}{4}\right)\right]\cos\left(2\pi\times 10^6\,\text{Hz}\cdot t - \sin\left(6\pi\times 10^3\,\text{Hz}\cdot t - \frac{\pi}{4}\right)\right)$$
$$0 \le t \le 1.5\,\text{ms}$$

1.3.1 6.3.1 Formulas of I and Q signals after LPF

Introducing some signs:

$$\omega_0 = 2\pi\times 10^6 Hz$$
$$\omega_{f1} = 2\pi\times 2\cdot 10^3 Hz$$
$$\omega_{f2} = 2\pi\times 3\cdot 10^3 Hz$$
$$a(t) = 2\left[1 + 0.5\cos\left(4\pi\times 10^3\,\text{Hz}\cdot t + \frac{\pi}{4}\right)\right]$$
$$\phi(t) = \sin\left(\omega_{f2}\cdot t - \frac{\pi}{4}\right)$$

Apply cosine subtraction identity:

$$s(t) = a(t)\cos\left(\omega_o\cdot t - \phi(t)\right) = a(t)\left[\cos(\omega_0\cdot t)\cos(\phi(t)) + \sin(\omega_0\cdot t)\sin(\phi(t))\right]$$

Now mix/multiply the bandpass signal with the signal from the LO:

$$s_I(t) = \frac{1}{2}s(t)\cdot\cos\left(\omega_0 t\right)$$

$$s_Q(t) = \frac{1}{2}s(t) \cdot \sin(\omega_0 t)$$

With the following identities:

$$\cos^2(\omega_0 t) = \frac{1+\cos(2\omega_0 t)}{2}$$
$$\sin(\omega_0 t)\cos(\omega_0 t) = \frac{1}{2}\sin(2\omega_0 t)$$

We get this euqation for one branch. We just show the I-part as it is the same for the Q-part:

$$\begin{aligned} s_I(t) &= \frac{1}{2}a(t)\left[\cos(\omega_0 t)\cos(\phi(t)) + \sin(\omega_0 t)\sin(\phi(t))\right] \cdot \cos(\omega_0 t) \\ &= \frac{1}{2}a(t)\left[\cos(\phi(t))\cos^2(\omega_0 t) + \sin(\phi(t))\sin(\omega_0 t)\cos(\omega_0 t)\right] \\ &= \frac{1}{2}a(t)\left[\cos(\phi(t)) \cdot \frac{1+\cos(2\omega_0 t)}{2} + \sin(\phi(t)) \cdot \frac{1}{2}\sin(2\omega_0 t)\right] \end{aligned}$$

Now we want to get rid of the $2\omega_0$ parts, so we have to use a LPF.

$$s_I(t) = \frac{1}{2}a(t)\left[\cos(\phi(t))\right]$$
$$s_Q(t) = \frac{1}{2}a(t)\left[\sin(\phi(t))\right]$$

Resubstitude ϕ and ω terms:

$$s_I(t) = \left[1 + 0.5\cos\left(2\pi \times 2 \cdot 10^3\,\text{Hz} \cdot t + \frac{\pi}{4}\right)\right] \cdot \left[\cos(\sin\left(2\pi \times 3 \cdot 10^3 Hz \cdot t - \frac{\pi}{4}\right))\right]$$
$$s_Q(t) = \left[1 + 0.5\cos\left(2\pi \times 2 \cdot 10^3\,\text{Hz} \cdot t + \frac{\pi}{4}\right)\right] \cdot \left[\sin(\sin\left(2\pi \times 3 \cdot 10^3 Hz \cdot t - \frac{\pi}{4}\right))\right]$$

1.3.2 6.3.2 Determine for the signal $s(t)$:

- Mean carrier amplitude A_T
- Modulation depth m
- AM modulation frequency $f_{N,Am}$
- Peak phase deviation $\Delta\phi$
- Modulation index η
- Peak frequency deviation Δf
- FM modulation frequency $f_{N,FM}$

$$s(t) = \underbrace{2\left[1 + 0.5\cos\left(2\pi \cdot 2 \cdot 10^3 \cdot t + \frac{\pi}{4}\right)\right]}_{\text{AM envelope}} \cdot \cos\left(\underbrace{2\pi \cdot 10^6 \cdot t}_{\text{Carrier}} - \underbrace{\sin\left(2\pi \cdot 3 \cdot 10^3 \cdot t - \frac{\pi}{4}\right)}_{\text{PM phase deviation}}\right)$$

Swing: \$a_{max} = 2 * (1 + 0.5) = 3 \$ and $a_{min} = 2 * (1 - 0.5) = 1$ -> $\Delta a = 2$

$$A_T = 2$$

$$m = \frac{a_n}{A_T} = \frac{1}{2} = 0.5$$

$$f_{N,AM} = 2kHz$$

As an amplitude swing from -1 to 1 equates to 1 rad by converting $A = -1$

$$\Delta\phi = 1rad$$

The modulation index:

$$\eta = \frac{\Delta\omega}{\omega} = \frac{2\pi \cdot 3kHz}{2\pi \cdot 3kHz} = 1$$

Therefore

$$\Delta f = 3kHz$$

$$f_{N,FM} = 3kHz$$