Gym's interface

We need to install `gym` first. Executing the following in a Jupyter notebook should work:

```
!pip install cmake 'gym[atari]' scipy
```

Once installed, we can load the game environment and render what it looks like:

```
import gym

env = gym.make("Taxi-v2").env

env.render()
```

```
+---------+
|R: | : :G|
| : : : : |
| : : : : |
| | : | : |
|Y| : |B: |
+---------+
```

The core gym interface is `env`, which is the unified environment interface. The following are the `env` methods that would be quite helpful to us:

- `env.reset`: Resets the environment and returns a random initial state.
- `env.step(action)`: Step the environment by one timestep. Returns
  - **observation**: Observations of the environment
  - **reward**: If your action was beneficial or not
  - **done**: Indicates if we have successfully picked up and dropped off a passenger, also called one *episode*
  - **info**: Additional info such as performance and latency for debugging purposes
- `env.render`: Renders one frame of the environment (helpful in visualizing the environment)

Note: We are using the `.env` on the end of `make` to avoid training stopping at 200 iterations, which is the default for the new version of Gym (reference).

Reminder of our problem

Here's our restructured problem statement (from Gym docs):

"There are 4 locations (labeled by different letters), and our job is to pick up the passenger at one location and drop him off at another. We receive +20 points for a successful drop-off and lose 1 point for every time-step it takes. There is also a 10 point penalty for illegal pick-up and drop-off actions."

Let's dive more into the environment.

```python
env.reset() # reset environment to a new, random state
env.render()

print("Action Space {}".format(env.action_space))
print("State Space {}".format(env.observation_space))
```

```
+---------+
|R: | : :G|
| : : : : |
| : : : : |
| | : | : |
|Y| : |B: |
+---------+

Action Space Discrete(6)
State Space Discrete(500)
```

- The **filled square** represents the taxi, which is yellow without a passenger and green with a passenger.
- The **pipe** ("|") represents a wall which the taxi cannot cross.
- **R, G, Y, B** are the possible pickup and destination locations. The **blue letter** represents the current passenger pick-up location, and the **purple letter** is the current destination.

As verified by the prints, we have an **Action Space** of size 6 and a **State Space** of size 500. As you'll see, our RL algorithm won't need any more information than these two things. All we need is a way to identify a state uniquely by assigning a unique number to every possible state, and RL learns to choose an action number from 0-5 where:

- 0 = south
- 1 = north
- 2 = east
- 3 = west
- 4 = pickup
- 5 = dropoff

Recall that the 500 states correspond to an encoding of the taxi's location, the passenger's location, and the destination location.
Reinforcement Learning will learn a mapping of **states** to the optimal **action** to perform in that state by *exploration*, i.e. the agent explores the environment and takes actions based off rewards defined in the environment.

The optimal action for each state is the action that has the **highest cumulative long-term reward**.

Back to our illustration

We can actually take our illustration above, encode its state, and give it to the environment to render in Gym. Recall that we have the taxi at row 3, column 1, our passenger is at location 2, and our destination is location 0. Using the Taxi-v2 state encoding method, we can do the following:

```
state = env.encode(3, 1, 2, 0) # (taxi row, taxi column, passenger index, destination index)
print("State:", state)

env.s = state
env.render()
```

```
State: 328
+---------+
|R: | : :G|
| : : : : |
| : : : : |
| | : | : |
|Y| : |B: |
+---------+
```

We are using our illustration's coordinates to generate a number corresponding to a state between 0 and 499, which turns out to be **328** for our illustration's state.

Then we can set the environment's state manually with `env.env.s` using that encoded number. You can play around with the numbers and you'll see the taxi, passenger, and destination move around.

The Reward Table

When the Taxi environment is created, there is an initial Reward table that's also created, called `P`. We can think of it like a matrix that has the number of states as rows and number of actions as columns: States x actions matrix

Since every state is in this matrix, we can see the default reward values assigned to our illustration's state:

```
env.P[328]
```

OUT:

```
{0: [(1.0, 428, -1, False)],
 1: [(1.0, 228, -1, False)],
 2: [(1.0, 348, -1, False)],
 3: [(1.0, 328, -1, False)],
 4: [(1.0, 328, -10, False)],
 5: [(1.0, 328, -10, False)]}
```

This dictionary has the structure `{action: [(probability, nextstate, reward, done)]}`.

A few things to note:

- The 0-5 corresponds to the actions (south, north, east, west, pickup, dropoff) the taxi can perform at our current state in the illustration.
- In this env, `probability` is always 1.0.
- The `nextstate` is the state we would be in if we take the action at this index of the dict
- All the movement actions have a -1 reward and the pickup/dropoff actions have -10 reward in this particular state. If we are in a state where the taxi has a passenger and is on top of the right destination, we would see a reward of 20 at the dropoff action (5)
- `done` is used to tell us when we have successfully dropped off a passenger in the right location. Each successfull dropoff is the end of an **episode**

Note that if our agent chose to explore action two (2) in this state it would be going East into a wall. The source code has made it impossible to actually move the taxi across a wall, so if the taxi chooses that action, it will just keep accruing -1 penalties, which affects the **long-term reward**.

Solving the environment without Reinforcement Learning

Let's see what would happen if we try to brute-force our way to solving the problem without RL.

Since we have our `P` table for default rewards in each state, we can try to have our taxi navigate just using that.

We'll create an infinite loop which runs until one passenger reaches one destination (one **episode**), or in other words, when the received reward is 20.

The `env.action_space.sample()`method automatically selects one random action from set of all possible actions.

Let's see what happens:

```
env.s = 328  # set environment to illustration's state


epochs = 0
penalties, reward = 0, 0


frames = [] # for animation


done = False


while not done:
    action = env.action_space.sample()
    state, reward, done, info = env.step(action)


    if reward == -10:
        penalties += 1


    # Put each rendered frame into dict for animation
    frames.append({
        'frame': env.render(mode='ansi'),
        'state': state,
        'action': action,
        'reward': reward
        }
    )


    epochs += 1


print("Timesteps taken: {}".format(epochs))
print("Penalties incurred: {}".format(penalties))
```

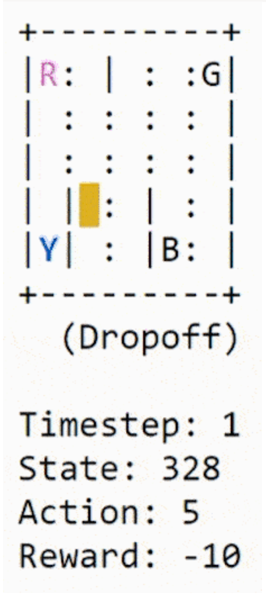
OUT:
```
Time steps taken: 1117

Penalties incurred: 363
```

```python
from IPython.display import clear_output
from time import sleep

def print_frames(frames):
    for i, frame in enumerate(frames):
        clear_output(wait=True)
        print(frame['frame'].getvalue())
        print(f"Timestep: {i + 1}")
        print(f"State: {frame['state']}")
        print(f"Action: {frame['action']}")
        print(f"Reward: {frame['reward']}")
        sleep(.1)

print_frames(frames)
```



```
+---------+
|R: | : :G|
| : : : : |
| : : : : |
| | : | : |
|Y| : |B: |
+---------+
  (Dropoff)

Timestep: 1
State: 328
Action: 5
Reward: -10
```

Not good. Our agent takes thousands of timesteps and makes lots of wrong drop offs to deliver just one passenger to the right destination.

This is because we aren't *learning* from past experience. We can run this over and over, and it will never optimize. The agent has no memory of which action was best for each state, which is exactly what Reinforcement Learning will do for us.

Enter Reinforcement Learning

We are going to use a simple RL algorithm called *Q-learning* which will give our agent some memory.

Intro to Q-learning

Essentially, Q-learning lets the agent use the environment's rewards to learn, over time, the best action to take in a given state.

In our Taxi environment, we have the reward table, `P`, that the agent will learn from. It does thing by looking receiving a reward for taking an action in the current state, then updating a *Q-value* to remember if that action was beneficial.

The values store in the Q-table are called a *Q-values*, and they map to a `(state, action)`combination.

A Q-value for a particular state-action combination is representative of the "quality" of an action taken from that state. Better Q-values imply better chances of getting greater rewards.

For example, if the taxi is faced with a state that includes a passenger at its current location, it is highly likely that the Q-value for `pickup` is higher when compared to other actions, like `dropoff` or `north`.

Q-values are initialized to an arbitrary value, and as the agent exposes itself to the environment and receives different rewards by executing different actions, the Q-values are updated using the equation:

What is this saying?

$$Q(state, action) \leftarrow (1-\alpha)Q(state, action) + \alpha\Big(reward + \gamma \max_{a} Q(next\ state, all\ actions)\Big)$$

Where:

- α (alpha) is the learning rate ($0 < \alpha \leq 1$) - Just like in supervised learning settings, α is the extent to which our Q-values are being updated in every iteration.

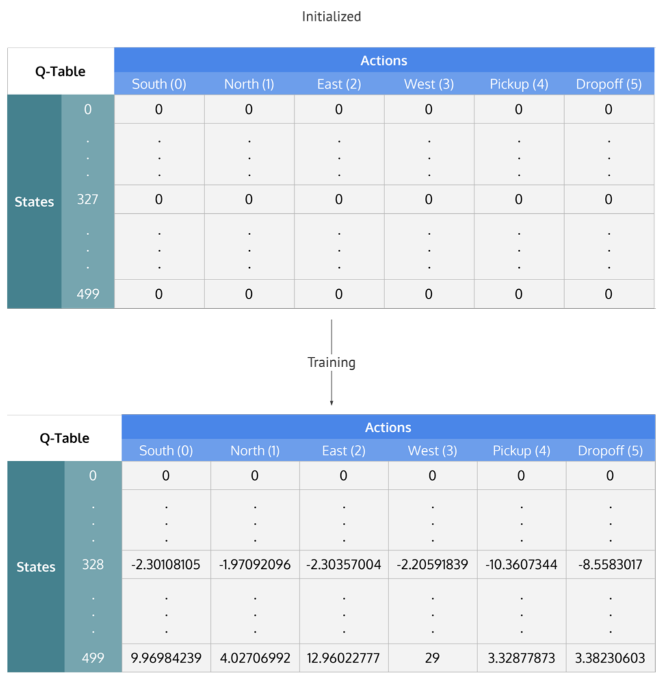
- γ (gamma) is the discount factor ($0 \leq \gamma \leq 1$) - determines how much importance we want to give to future rewards. A high value for the discount factor (close to **1**) captures the long-term effective award, whereas, a discount factor of **0** makes our agent consider only immediate reward, hence making it greedy.

We are assigning (←), or updating, the Q-value of the agent's current *state* and *action* by first taking a weight (1−*α*) of the old Q-value, then adding the learned value. The learned value is a combination of the reward for taking the current action in the current state, and the discounted maximum reward from the next state we will be in once we take the current action.

Basically, we are learning the proper action to take in the current state by looking at the reward for the current state/action combo, and the max rewards for the next state. This will eventually cause our taxi to consider the route with the best rewards strung together.

The Q-value of a state-action pair is the sum of the instant reward and the discounted future reward (of the resulting state). The way we store the Q-values for each state and action is through a **Q-table**

The Q-table is a matrix where we have a row for every state (500) and a column for every action (6). It's first initialized to 0, and then values are updated after training. Note that the Q-table has the same dimensions as the reward table, but it has a completely different purpose.


Initialized
Training

Q-Table		Actions					
		South (0)	North (1)	East (2)	West (3)	Pickup (4)	Dropoff (5)
States	0	0	0	0	0	0	0
	. . .	. . .	. . .	. . .	. . .	. . .	. . .
	327	0	0	0	0	0	0
	. . .	. . .	. . .	. . .	. . .	. . .	. . .
	499	0	0	0	0	0	0

Q-Table		Actions					
		South (0)	North (1)	East (2)	West (3)	Pickup (4)	Dropoff (5)
States	0	0	0	0	0	0	0
	. . .	. . .	. . .	. . .	. . .	. . .	. . .
	328	-2.30108105	-1.97092096	-2.30357004	-2.20591839	-10.3607344	-8.5583017
	. . .	. . .	. . .	. . .	. . .	. . .	. . .
	499	9.96984239	4.02706992	12.96022777	29	3.32877873	3.38230603

Q-Table values are initialized to zero and then updated during training to values that optimize the agent's traversal through the environment for maximum rewards

Summing up the Q-Learning Process

Breaking it down into steps, we get

- Initialize the Q-table by all zeros.
- Start exploring actions: For each state, select any one among all possible actions for the current state (S).
- Travel to the next state (S') as a result of that action (a).
- For all possible actions from the state (S') select the one with the highest Q-value.
- Update Q-table values using the equation.
- Set the next state as the current state.
- If goal state is reached, then end and repeat the process.

Exploiting learned values

After enough random exploration of actions, the Q-values tend to converge serving our agent as an action-value function which it can exploit to pick the most optimal action from a given state.

There's a tradeoff between exploration (choosing a random action) and exploitation (choosing actions based on already learned Q-values). We want to prevent the action from always taking the same route, and possibly overfitting, so we'll be introducing another parameter called ϵ "epsilon" to cater to this during training.

Instead of just selecting the best learned Q-value action, we'll sometimes favor exploring the action space further. Lower epsilon value results in episodes with more penalties (on average) which is obvious because we are exploring and making random decisions.

Implementing Q-learning in python

Training the Agent

First, we'll initialize the Q-table to a 500×6 matrix of zeros:

```
import numpy as np
q_table = np.zeros([env.observation_space.n,
env.action_space.n])
```

We can now create the training algorithm that will update this Q-table as the agent explores the environment over thousands of episodes.

In the first part of `while not done`, we decide whether to pick a random action or to exploit the already computed Q-values. This is done simply by using the `epsilon` value and comparing it to the `random.uniform(0, 1)` function, which returns an arbitrary number between 0 and 1.

We execute the chosen action in the environment to obtain the `next_state` and the `reward`from performing the action. After that, we calculate the maximum Q-value for the actions corresponding to the `next_state`, and with that, we can easily update our Q-value to the `new_q_value`:

```
%%time
"""Training the agent"""

import random
from IPython.display import clear_output

# Hyperparameters
alpha = 0.1
```

```
gamma = 0.6
epsilon = 0.1

# For plotting metrics
all_epochs = []
all_penalties = []

for i in range(1, 100001):
    state = env.reset()

    epochs, penalties, reward, = 0, 0, 0
    done = False

    while not done:
        if random.uniform(0, 1) < epsilon:
            action = env.action_space.sample() # Explore action space
        else:
            action = np.argmax(q_table[state]) # Exploit learned values

        next_state, reward, done, info = env.step(action)

        old_value = q_table[state, action]
        next_max = np.max(q_table[next_state])

        new_value = (1 - alpha) * old_value + alpha * (reward + gamma * 
next_max)
        q_table[state, action] = new_value

        if reward == -10:
            penalties += 1

        state = next_state
        epochs += 1
    if i % 100 == 0:
        clear_output(wait=True)
        print(f"Episode: {i}")

print("Training finished.\n")
```

OUT:

```
Episode: 100000
Training finished.


Wall time: 30.6 s
```

Now that the Q-table has been established over 100,000 episodes, let's see what the Q-values are at our illustration's state:

```
q_table[328]
```

OUT:

```
array([ -2.30108105,  -1.97092096,  -2.30357004,  -2.20591839,
       -10.3607344 ,  -8.5583017 ])
```

The max Q-value is "north" (-1.971), so it looks like Q-learning has effectively learned the best action to take in our illustration's state!

Evaluating the agent

Let's evaluate the performance of our agent. We don't need to explore actions any further, so now the next action is always selected using the best Q-value:

```
"""Evaluate agent's performance after Q-learning"""

total_epochs, total_penalties = 0, 0
episodes = 100

for _ in range(episodes):
    state = env.reset()
    epochs, penalties, reward = 0, 0, 0

    done = False

    while not done:
        action = np.argmax(q_table[state])
        state, reward, done, info = env.step(action)

        if reward == -10:
            penalties += 1

        epochs += 1

    total_penalties += penalties
    total_epochs += epochs

print(f"Results after {episodes} episodes:")
print(f"Average timesteps per episode: {total_epochs / episodes}")
print(f"Average penalties per episode: {total_penalties / episodes}")
```

OUT:

```
Results after 100 episodes:
Average timesteps per episode: 12.3
Average penalties per episode: 0.0
```

We can see from the evaluation, the agent's performance improved significantly and it incurred no penalties, which means it performed the correct pickup/dropoff actions with 100 different passengers.