UTGAVE 25. nov. 2021

LABORATORIUM I EMNENE

TFY4155/FY1003 ELEKTRISITET OG MAGNETISME

for studenter ved studieprogrammene

MTFYMA/MLREAL/BFY/BKJ

NTNU

Våren 2022

Forord

Dette heftet inneholder tekster til laboratoriekurset til emnene TFY4155/FY1003 Elektrisitet og magnetisme.

Denne versjonen markerer restrukturing som følge av den nye XFys ferdighetstreng programmet. Laboppgavene ble fornyet for å muliggjøre mer egenstendig eksperimentering, sette fokus på dokumentasjon med labjournal og introdusere mer feilanalyse.

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Contents

1 The Lorentz force

Goal

In this lab assignment you will

- document experiments in the lab journal,
- perform error analysis of the experiment,
- use numerical methods to model the experiment,
- study forces on electrons in homogeneous and constant electric and magnetic fields,
- determine the ratio e/m between the charge of the electron e and the mass m (the Thomson experiment).

1.1 Theoretical background

The theory is presented for the presentation of the calculation tasks below. Reference is also made to the textbook and the lectures in electromagnetism.

1.2 Calculation tasks

1.2.1 The Lorentz force

The force from electric and magnetic fields on charged particles is often called the Lorentz force. The force on a charged particle with charge q moving at velocity $\vec{v}$ in an electrostatic field $\vec{E}$ and a magnetostatic field $\vec{B}$ is given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}. \tag{1-1}$$

To create a fast electron beam, electric fields are always used to accelerate the electrons. However, the expression for Lorentz force shows that both electric and magnetic fields give rise to a force on charged particles.

1. Explain from the expression of the Lorentz force why magnetic fields (as opposed to electric fields) can not give a charged particle increased kinetic energy.

1.2.2 Charges in an E-field

A homogeneous electric field is most easily set up between two parallel plate electrodes. This geometry provides an electric field $E = U_\mathrm{a}/\ell$ where U_a is the voltage between the plates and ℓ the distance between them. A charged particle with charge q and mass m experiences an acceleration

$$a = \frac{qE}{m} = \frac{qU_\mathrm{a}}{m\ell}, \tag{1-2}$$

when travelling between the two plates. This is used in sources of charged particle beams.

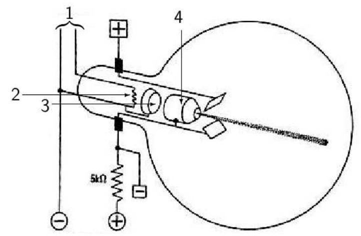

Figure 1.1: Sketch of a electron beam source. **(1)** AC source (5-10 V). **(2)** Incandescent filament heats up the **(3)** oxide layer. The acceleration voltage between the filament and **(4)** anode accelerated the electrons.

Figure 1.1 shows how an electron beam source can be made. A tungsten filament is heated by electric current passing through it. When it is red hot, it emits electrons from the surface. The filament is placed just behind a plate electrode with holes in it. Voltage is applied between the filament and the plate electrode. The electrons are accelerated towards the plate electrode and the electrons passing through the hole go out in a beam on the other side of the plate. Another way to make electrons in electron sources is to use an oxide cathode that is heated indirectly by a filament. This construction provides better field conditions during acceleration, and it is this version that is shown in Figure 1.1.

Assume that an electron is accelerated from a standstill to a voltage of 30 V in a static electric field.

2. What is the speed of the electron after the acceleration?

In particle accelerator physics, the energy unit "electron volt" is abbreviated eV. A particle with the charge 1 e gets the energy 1 eV when it is accelerated through a voltage drop of 1 V.

3A. How many joules is equivalent to 1 eV?

3B. What is the energy of an electron that is accelerated to a voltage of 30 V expressed in the unit eV?

An electron beam from an electron source with zero potential accelerated to a potential U_a can be deflected in a transverse electric field $E = U_\mathrm{d}/d$ over a distance L as shown in Figure 1.2. The deflection angle α is given by

$$\tan\alpha = \frac{1}{2}\frac{U_\mathrm{d}}{U_\mathrm{a}}\frac{L}{d}. \tag{1-3}$$

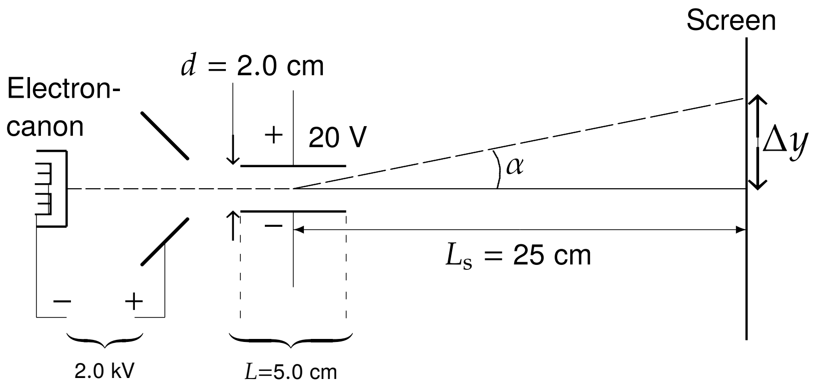

Figure 1.2: Deflection of an electron beam in a transverse electric field. The numerical values in the figure are used in the exercise.

Equation (1-3) describes the physical laws that form the basis of the oscilloscope, which is an instrument for measuring electrical voltage. The voltage to be measured is placed between the transverse deflection plates and the deflection of the electron beam observed on a phosphorescent screen is a measure of the voltage.

Assume the electrons are accelerated by a 2 kV acceleration potential and the other parameters are as shown in Figure 1.2.

4. How much deflection Δy does the beam on the screen get if the deflection voltage is 20 V?

1.2.3 Charges in a B-field

When an electron moves in a space free of electric field but with a magnetic field B, the Lorentz force at all times acts normal to the path (unless the path is parallel to the magnetic field). Let us assume the electron is shot into the B-field at velocity perpendicular to the field, the Lorentz force then becomes $F_\mathrm{B} = evB$ and constitutes the centripetal force. The electron orbit becomes a circle of radius r with centripetal acceleration $a_\mathrm{s} = v^2/r = F_\mathrm{B}/m$. The speed v can be determined from the kinetic energy of the electron given by $\frac{1}{2}mv^2 = eU_\mathrm{a}$. By putting these equations together, we get the following expression for the radius

$$r = \frac{1}{B}\sqrt{\frac{2U_\mathrm{a}}{e/m}}. \tag{1-4}$$

5. Set up a curve diagram curve intersections over r sfa. B in the range $0 < B < 15\,\text{gauss}$ for $U_a = 20$, 40 and 60 V. Use, for example, Python or Matlab.

5. Set up a set of curves for the radius r as a function of B. Do this for the range $B \in (0\,\text{gauss}, 15\,\text{gauss})$ with $U_a = 20, 40,$ and 60V. Use for example Python or Matlab.

1.2.4 Determination of *e*/*m* (The Thomson experiment)

In 1897, Thomson discovered [1] extremely light, negatively charged particles – which later turned out to be electrons. That same year, he used deflection in static electric and magnetic fields to find the relationship between their charge and mass. The expression (1-4) can be rewritten to

$$\frac{e}{m} = \frac{2U_a}{B^2 r^2}. \tag{1-5}$$

1: Sir Joseph John Thomson (1856–1940) was a British physicist and Nobel Laureate in Physics, credited with the discovery of the electron, the first subatomic particle to be discovered.

The ratio e/m can thus be determined by measuring the radius r of the electron path of electrons accelerated by a voltage U_a in a magnetic field B perpendicular to the path.

You will repeat Thomson's experiment in the laboratory. The size of the equipment limits the diameter of the largest electron orbit to approx. 8 cm.

6. Use the graph of the curves that you made in the problem above to estimate the appropriate magnetic field B for three different acceleration voltages $U_a = 20$, 40 and 60 V.

When rewriting (1-5) you get

$$m = \frac{qB^2 r^2}{2U_a}, \tag{1-6}$$

showing that the mass m of a charged particle with a known charge q that has been accelerated to a known potential U_a can be determined by measuring the magnetic flux density B and the orbital radius r of the particle in a magnetic field. This is the idea behind the magnetostatic mass spectrometer which is one of the most important instruments for determining atomic and molecular particle masses. With his work, Thomson laid the foundation for this technique.

1.2.5 Voltage components

The so-called voltage divider is among the most common electrical connections. You must use it in the experiment. In its simplest form, the voltage divider looks like the top circuit in Figure 1.3.

Through the coupling, the voltage V_{in} can be reduced to V_{out}. To find the size of V_{out} as a function of V_{in} you can look at the current I through the resistors R_1 and R_2 which according to Ohm's is

$$I = \frac{V_{\text{in}}}{R_1 + R_2}. \tag{1-7}$$

Use Ohm's law again on R_2:

$$V_{\text{out}} = IR_2 = \frac{R_2}{R_1 + R_2} V_{\text{in}}. \tag{1-8}$$

The voltage divider can be made variable by replacing R_1 and R_2 with a potentiometer, a resistor with variable resistance, as shown in the middle circuite in Figure 1.3. Together with a fixed voltage source, this provides an opportunity to create a cheap variable voltage source.

However, the voltage divider has a disadvantage.

7. Can you find what disadvantage?
TIP: What happens to V_{out} when you start drawing power from the output? See the bottom circuit in Figure 1.3. The resulting resistance across the parallel connection of R_2 and R_{L} is denoted $R_2||R_{\text{L}}$ and is equal to $(R_2R_{\text{L}})/(R_2 + R_{\text{L}})$.

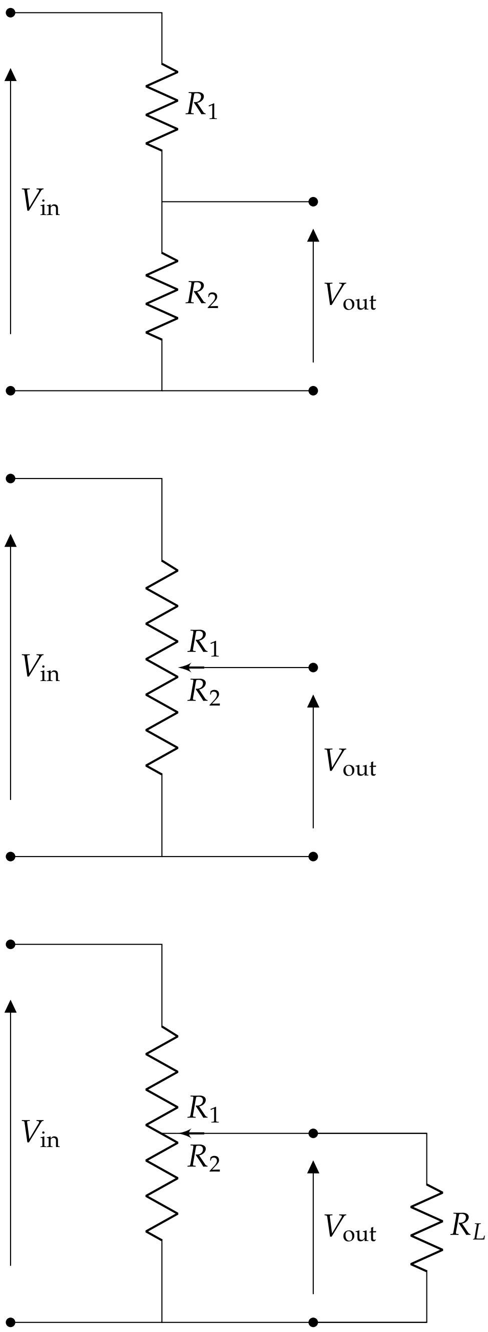

Figure 1.3: Voltage dividers. **(Top)** With fixed resistors. **(Middle)** With variable resistance (potentiometer). **(Bottom)** With variable resistance and a load R_L.

1.3 Experimental

1.3.1 Equipment

- **Electron fine beam tube.** Leybold Hereaus 555 571. [2]
- **Power supply for accelerator and focusing electrode.** Leybold Didactics 521 651 Tube power supply 0 ... 500 V.
- **Power supply for the Helmholtz coils.** Mascot, type 719. Used as a power source. Range: 0- 15 V, 30 mA - 2 A.
- **Multimeter for coild current.** Escort EDM168A, 3 digits,

2: http://www.hep.fsu.edu/~wahl/phy3802/expinfo/em/555571e.pdf or equivalent.

- **Multimeter for acceleration voltage.** Escort EDM168A, 3 digits, or equivalent.
- **Junction box with voltage divider.**
- Various wires of different lengths.

The apparatus is shown schematically in Figures 1.4 and 1.5. The electron gun (K) emits electrons that can be deflected in a static electric field between the parallel deflection plates (P) and/or in a static magnetic field from the Helmholtz coils. The glass flask around the electron gun is filled with thin hydrogen gas to a pressure of approximately 10^{-5} atmospheres, 1 Pa. Shock between the electrons and the gas will excite the electrons in the gas molecules which then relaxe and emit light (same phenomenon as in the northern lights). The trace of the electrons then becomes visible as a blue-green stripe.

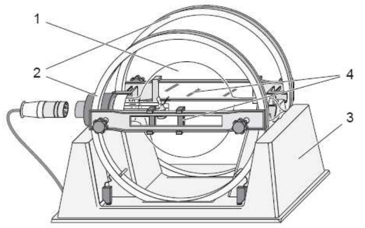

Figure 1.4: The Leybold apparatus with the electron canon inside a glass vessel (1). Helmholtz coils (2), holder (3), and radius measuring device (4).

Study the electron gun through the glass flask. Also study the disassembled cannon from an old beam generator laid out in the laboratory.

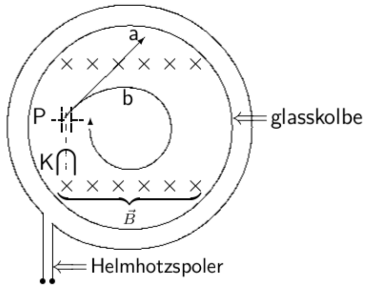

Figure 1.5: Schematic drawing. The electron canon (K), deflection of the electron beam between the deflection plates (P). The magnetic field is indicated with crosses, ×, into the paper and the electron beam is shown without (a) and with (b) the B-field.

1.3.2 Connection of voltages to the electron gun

Figures 1.7 and 1.6 show schematically the electrical connections for the electron gun. Note the common ground point (output 2 on Leybold Power supply)

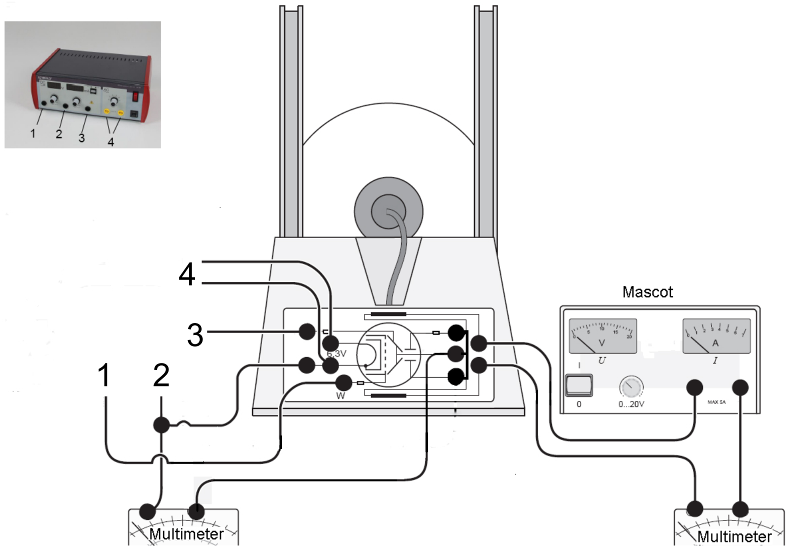

Figure 1.6: Wiring diagram for the electron canon. The power supply Leybold and multimeter for measuring the anode voltage. See Figure 1.7 for details of the wiring diagram on the beam apparatus.

There are several alternative ways to connect the electron gun, but it is important to connect correctly. However, it is not difficult to break a couple of fuses if you connect incorrectly, so ask the supervisor if you are in doubt.

Connect the circuit according to figure 1.6.

Notes and tips:

- You must connect socket 4 to the unit. Which one you use is irrelevant, but one is grounded via contacts b and c_1 in figure 1.7.
- The unit must be switched off during connection.
- Ask the lab supervisor to approve the connection.
- Switch on the unit and adjust the glow voltage to "6" (V) and observe that the wire glows. This assumes that the ceiling lighting is switched off and the table lighting is dimmed. Note that the glow voltage can be lowered to 5 V during the experiments.
- Switch off the unit again.

To extract an electron beam from the cathode, the electrons must be accelerated as explained above in the theory section. This is done by applying an acceleration voltage between the cathode and the anode.

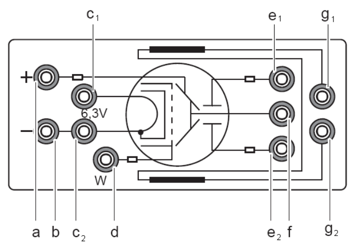

Figure 1.7: Wiring diagram for the apparauts' side panel.
a: Anode
b: Cathode
c: Cathode heating
d: Wehnelt cylinder
e: Deflection plates
f: Anode, for symmetrical adjustment of the deflection voltage
g: Helmholtz coils

Connect the acceleration voltage according to figure 1.6.

Notes and tips:

- Make sure that the power supply is switched off and that the power and voltage adjustment knobs are turned all the way down when connecting.
- Connect Leybold output 3 to "a" and output 2 to "b".
- Starting potential is $U_d = 0$, connect therefore the two deflection plates (e $_1$ and e $_2$) and possibly ground or connect them to the anode, to prevent charge from building up on them.
- To avoid measurement errors due to voltage drop across the series resistor, you can connect the voltmeter directly to the anode instead of "f". Use measuring range 1000 V DC. Check the value of the Leybold unit. How big a measurement error do you get and what determines the size of the error?
- The series resistors mounted internally in the equipment are inserted for safety reasons to avoid excessive current in the circuit in the event of an internal short circuit (e.g. a gas discharge).

Now it is ready to turn on all voltages and see that the electron beam is visible:

- Ask the lab supervisor to approve the connection.
- Switch on the glow voltage.
- Check that the filament starts to glow.
- Increase the acceleration voltage slowly until you (at 20–30 V or slightly higher if your eyes have not adapted to the dark) see the trace of the electron beam in the hydrogen gas as a blue-violet stripe out of the electron gun.
- Observe how the beam changes when you change the acceleration voltage.
- Why does the beam not reach the glass wall at low acceleration? At higher voltages, quite abruptly, the beam will meet the wall, then lower the voltage slightly. This happens at different voltages at the different setups, follow closely from around 100 V, and possibly upwards to approx. 150 V.
- NOTE: If the anode starts to glow, turn off the appliance and let it cool down. Never use higher acceleration voltages than necessary to avoid glowing.

Make a sketch of the beam path for different acceleration voltages.

Procedure to turn the apparatus off:

- Switch off the glow voltage.
- Reduce the acceleration voltage to zero and switch off the power supply.

NOTE: It is not dangerous to have on acceleration voltage and magnetic field without having a glow voltage, but it is not good to have on glow voltage without acceleration voltage. Therefore, always switch off the glow voltage when you have short or long breaks.

In an electron beam, all the particles have the same sign of charge. The beam will therefore have a tendency to diverge due to internal Coulomb power. If the focusing ring (Wehnelt cylinder) in front of the cathode is made slightly negative, this will have a focusing effect on the beam.

Connect the Wehnelt cylinder according to Figure 1.6. Output 1 to "W". Notes and tips:

- Check that the power supply is switched off and that the power and voltage adjustment knobs are turned down completely before connecting it.
- After connecting the Wehnelt cylinder and switching on the power supply, adjust the voltage on the focusing electrode (Wehnelt cylinder) if you think the beam is poorly focused. Use maximum 10 V.

1.3.3 Deflection in an E-field

Task: *You will study the deflection of an electron beam in an electrostatic field.*

In the setup so far, you have put both deflection plates at the same voltage, namely the anode voltage U_a. To obtain deflection, one plate is placed on a somewhat lower voltage. This is done by adding a voltage divider to the anode voltage; the principle of voltage division is shown in Figure 1.3. Here you have to make changes to the connection. The anode voltage (3) should now be connected with a junction box according to the diagram in Figure 1.8 shows changes you must make from the connection in Figure 1.6 where the deflection potential was zero. The connection between e_1 and e_2 is removed.

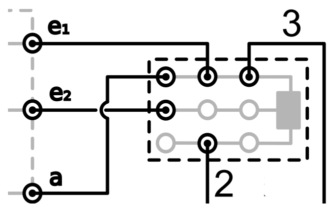

Figure 1.8: Circuit diagram for connecting the voltage divider.

The voltage divider consists of a precision potentiometer (Helipot) and is mounted on junction box B. The acceleration voltage U_a is connected across the potentiometer. From the center tapping, we take out a voltage that varies with the position of the potentiometer button, which needs 10 revolutions to turn from top to bottom. The deflection voltage is taken out between the top outlet (which is at potential U_a) and the middle outlet at potential xU_a, where x can vary from 0 to 100 %. This means that one deflection plate is at the anode voltage U_a and the other at xU_a. The deflection voltage then becomes

$$U_d = U_a - xU_a = (1 - x)U_a. \tag{1-9}$$

If you connect incorrectly and take out voltage division to the ground potential, the electron beam will be reduced strongly and easily disappear. You can find out which plate should have lower potential by thinking, or by trial and error.

Connect the deflection potential according to Figure 1.8.

Notes and tips:

- Set the potentiometer to 100 % so that the center tap has the same potential as the anode voltage.
- Switch on glow, focusing voltage and acceleration as described previously.
- Vary the deflection by turning the potentiometer knob.

Explain what you observe with regard to the expression of the Lorentz force in equation (1-1).

Sketch the beam for different deflection stresses, e.g. $U_\mathrm{d} = \frac{1}{2}U_\mathrm{a}$ *and* $\frac{4}{5}U_\mathrm{a}$.

1.3.4 Deflection in a B-field

Task: *You will study the deflection of an electron beam in a static magnetic field.*

- Connect the equipment according to Figure 1.6. Note that the connection of the voltage divider in the previous step is not necessary for the tests to be performed here
- Connect the circuit to the magnets with the Mascot unit in (g).

Notes and tips:

- Verify that the power supply for the Helmholtz coils is switched off and that the control buttons for current and voltage are turned down completely.
- Set the power supply to the power source (METER: A and the control knob for voltage up 1/2 revolution) and set RANGE to 15 V.
- Use the multimeter (6) for current measurement.
- Turn up the current to approx. 1 A, and check that the circuit is working. NOTE: The coils can withstand max. 2 A. Turn the current down again afterwards.
- Switch on the electron beam as described earlier.
- Adjust the acceleration voltage to approx: 100 V and the magnetic current to 0,75 A.

Explain what happens using the expression for the Lorentz force.

Ask the lab supervisor to help you twist the electron beam tube until the beam takes a helical path.

Sketch the beam and explain the reason for the helical path.

1.3.5 The Thomson Experiment: Measurement of e/m

Task: *You will find the relation e/m between the electron charge and mass from equation* (1-5) *by measuring the acceleration voltage U_a, the radius r of the electron orbit and the magnetic field B.*

The magnetic field is pre-calibrated against the current in the coils. The result is given in a calibration curve attached to the apparatus and as an empirical expression of the magnetic field as a function of the measured coil current found from the calibration curve. Alternatively, this expression can be used:

$$B = \mu_0 \cdot \left(\frac{4}{5}\right)^{3/2} \cdot \frac{n}{R} \cdot I, \tag{1-10}$$

with R: Radius of the coils, n: number of turns = 130 per coil.

Notes and tips:

- Align the tube so that the beam becomes a circle and not a helix.
- Set the multimeter for voltage measurement in the range 0-1000 V DC voltage.
- Ask the lab supervisor to show you how to measure the diameter of the electron path using the parallax mirror behind the electron beam tube and a ruler.
- It is possible to photograph the electron path and analyze in TRACKER to get the diameter of the path. Note that you must have a scale (ruler) in the image.

Analysis and Discussion of the Thomson Experiment

- Determine e/m for the electron based on your measurements of U_a, r, I and the calibration curve or formula for $B(I)$.
- If you have time, repeat the experiment for another set of values of U_a and I.
- Estimate the errors in the measured quantities and find the measurement error in e/m. Which sources of error dominate?
- Compare the measured value of e/m with the table value.
- What is the correspondence between the measurement

results from different values of U_a and I and your error estimate?

- Discuss possible applications of the physical effects that have been observed in the experiment.

1.3.6 Closing

Switch off all appliances, unplug and clean all cables. Leave the place in at least as good an order as you found it.