UTGAVE 25. nov. 2021

LABORATORIUM I EMNENE

TFY4155/FY1003 ELEKTRISITET OG MAGNETISME

for studenter ved studieprogrammene

MTFYMA/MLREAL/BFY/BKJ

NTNU

Våren 2022

Forord

Dette heftet inneholder tekster til laboratoriekurset til emnene TFY4155/FY1003 Elektrisitet og magnetisme.

Denne versjonen markerer restrukturing som følge av den nye XFys ferdighetstreng programmet. Laboppgavene ble fornyet for å muliggjøre mer egenstendig eksperimentering, sette fokus på dokumentasjon med labjournal og introdusere mer feilanalyse.

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25. november 2019

Contents

1 LABORATORY SAFETY

The experiments in this laboratory course are safe and do not pose any danger if sensible and obvious precautions are followed and the students follow the safety instructions given below. There is always a potential danger of electric shock or fire wherever there are wall outlets, sockets, wiring or connections, which are found in all laboratories and homes. The purpose of this chapter is to provide you with general information that will prepare you to perform laboratory tests without putting yourself or others at unnecessary risk, and to describe the procedure to follow if you or anyone else nearby is in an emergency.

1.1 The electrical equipment

As stated in Ohm's law, $V = RI$, and the law of power development in a resistor, $P = VI = RI^2$, there will always be a voltage drop V over a live conductor with current I and resistance R, and there will always be generated heat in the conductor (provided that $R > 0$).

To protect the consumer against electric shock, the outer casing of all equipment with a conductive outer casing is permanently connected to earth. This is done through the ground connector in the plug. If a fault occurs which causes the supply voltage of the equipment to come into contact with the conductive outer casing, the safety earthing will ensure that the voltage is connected to earth and not applied to the outer casing, with the dangers this entails. The current will thus be short-circuited to earth and normally the fuse will blow, causing the current to be interrupted. [1]

1: It is important in this regard that the safety earth line is dimensioned so that it can withstand the current in the main supply line, i.e., it must have at least the same cross-section as the main supply line.

To avoid personal safety being dependent on the power fuses, electrical energy networks are often equipped with a so-called *nullstrømbryter*. The neutral circuit breaker can detect earth leakage currents that are much less than the hedging value of the course. The switch does this by comparing the current in the two wires of a course, ie the current into a consumer with the current returning from the consumer. Loss of electricity at the consumer is interpreted as leakage current to earth. If the zero current switch detects a difference between the phase currents that is greater than a certain preset value (eg 30 mA), then the current is interrupted.

A third method of protecting the consumer against electric shock is *double insulated* equipment (Class II equipment, marked with a double square: ⧈). Do not attempt to ground double insulated equipment as this will reduce contact safety.

Experience has shown that the vast majority of faults in electrical systems are earthing faults. Careful handling of earth and correct earthing of electrical equipment is a prerequisite for safe use of electrical energy and for electrical equipment to work correctly. Never plug electrical equipment that requires a grounded plug into a non-grounded electrical outlet.

1.2 Connecting electrical circuits

According to the Norwegian Electricity Regulatory Authority's regulations, all AC voltages above 25 V rms value and all ripple-free DC voltages above 60 V must be properly insulated. When connecting circuits in the laboratory, make sure that these regulations are followed. [2]

2: Is this still the law?

You must make sure that the wires you use are sufficiently large, so that they do not heat up. Although the resistance in electrical wires is small, the value will not always be negligible. For twisted copper cables used for fixed installations in houses (with 230 V), the regulations state that a cable with a cross-section of 2,5 mm^2 can carry a maximum current of 16 A. As a rule of thumb in the laboratory, we can assume that 5 A/mm^2 at voltages below 300 V will be safe. The flexible laces we use for connections all have cross-sections that are larger than 1 mm^2. Currents up to 5 A will therefore not present any problem for these laces. Another factor you need to take care of is choosing a large enough cross-section on your wires so that the voltage drop across the wires does not contribute to measurement errors.

Be careful not to send current through coils / drums with electrical wiring. The requirements described above apply to wires in the open with natural air cooling. If the wire is coiled tightly together, the heat development will lead to a significantly greater temperature increase which can lead to the insulation melting.

When using electrical connection equipment, it is important that you make sure that all equipment is in order. Look for damage to wires and especially connectors. It is also important that you treat the coupling material so that it is not damaged. Be sure to place the wires during use so that there is no danger of them getting caught in moving parts or getting pinched so that the insulation is damaged. The cables should also be left free so that they have sufficient cooling. When the wiring material is not in use, it should not be twisted up as heavy twisting can damage the insulation and the connection between the conductor and the socket.

1.3 The effect of electrical currents on the body

The danger of electric shock is given by the magnitude of the current flowing through the body and the current path it follows. How much voltage the body can withstand will depend on the total resistance in the circuit where the body is included. Figure 1.1 shows the human body as part of a circuit.

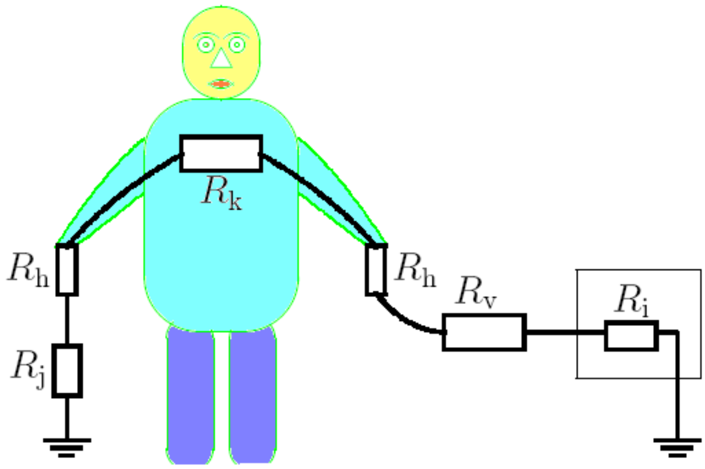

Figure 1.1: The human body as part of a circuit, where R_j is the insulation resistance between the body and earth, R_h is the transition resistance in the skin, R_k is the body's internal resistance, R_i is the internal resistance of the voltage source that drives the current in the circuit and R_v is the insulation resistance between the voltage terminal of the voltage source and the body.

Current can damage the body, either by damaging body tissue when heated, or by the current interfering with the electrical signals in the nervous system. The heating is given by the current I and the resistance R in the tissue being heated. A sustained flow through the tissue will cause an increasing amount of heat energy and risk of injury. Disruption of the electrical signals in the nervous system can, among other things lead to heart fibrillation and paralysis. In particular, alternating current in the frequency range 15 to 60 Hz is dangerous for the nervous system. Our network supply of 50 Hz is therefore in the dangerous category. When the frequency of the alternating current increases, the influence on the nervous system will decrease, and for frequencies above 10 kHz the influence is almost gone. However, the heating effect is still present.

Current paths through the heart and lung region are particularly dangerous due to the vital organs that exist there. The kidneys are also particularly sensitive to current damage.

From the circuit in figure 1.1 we see that the current through the body is given by

$$I = \frac{V}{R_\mathrm{j} + 2R_\mathrm{h} + R_\mathrm{k} + R_\mathrm{i} + R_\mathrm{v}}. \tag{1-1}$$

When referring to the voltage on a supply network, we usually refer to the root mean square voltage, defined as

$$V_\mathrm{eff} = \sqrt{\langle V(t)^2 \rangle} = \sqrt{\frac{1}{T}\int_0^T V_0^2 \sin^2\left(\frac{2\pi t}{T}\right) \mathrm{d}t} = \frac{V_0}{\sqrt{2}}. \tag{1-2}$$

This means that for a 230 volt system, the peak voltage (amplitude) is $V_0 = \sqrt{2} \times 230\,\mathrm{V} = 325\,\mathrm{V}$.

Safety precautions against electrical damage involve reducing the value of the current through the body to harmless values. This can be done for a given voltage V by making sure that the sum of the resistances below the fractional line in eq. (1-1) is sufficiently large:

$\mathbf{R_v}$ Ordinary electrical insulation of electrical equipment involves making R_v sufficiently large.

$\mathbf{R_k}$ The internal resistance of the body R_k is constant and of the order of 500 Ω. Note that the limit for mandatory isolation of AC voltage of 25 V effective voltage corresponds to 50 mA when we assume that the resistance in the circuit consists only of the body's internal resistance, ie worst case.

R_h The transition resistance R_h in the skin can vary from 0 Ω for moist skin to well above 10 000 Ω for dry skin. R_h also decreases with increasing voltage and it also varies with the frequency so that it is lowest in the frequency range around 50 Hz. This helps to make AC voltage more dangerous than DC voltage. Another factor that makes alternating voltage more dangerous than direct voltage is that the magnitude of the voltage referred to when alternating voltage is mentioned is the effective voltage. However, the peak voltage is $\sqrt{2}\times$ higher, as described in equation (1-2).

R_j If the body is not in contact with the ground, R_j will be large. Why do the birds not die when they are on power lines (see figure 1.2)? Answer this question by modifying Figure 1.1. Now look at figure 1.3, and then answer the question asked there.

Figure 1.2: En fugl som sitter på en kraftlinje.

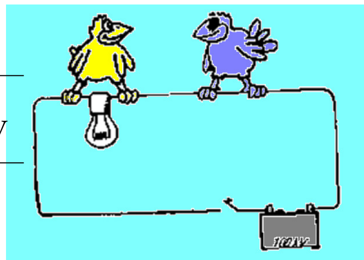

Figure 1.3: Hva skjer når den slås på?

In addition to the above, it is important to be aware that high-frequency electromagnetic radiation in the MHz - GHz range (microwaves - radar) can be dangerous for the body in that the electric alternating field leads to heating of the body tissue, the same effect which provides heating in a microwave. Such high-frequency heating is extra dangerous because it often takes place without normal pain functions taking effect. People should be shielded if the power per unit area against the body is > 0,01 W/cm^2.

Exposure characteristics	Magnetic flux density
Occupational[b]	
Exposure of head and trunk	2 T
Exposure of limbs[c]	8 T
General public[d]	
Exposure of any part of the body	400 mT

[a] ICNIRP recommends that these limits should be viewed operationally as spatial peak exposure limits.
[b] For specific work applications, exposure up to 8 T can be justified, if the environment is controlled and appropriate work practices are implemented to control movement-induced effects.
[c] Not enough information is available on which to base exposure limits beyond 8 T.
[d] Because of potential indirect adverse effects, ICNIRP recognizes that practical policies need to be implemented to prevent inadvertent harmful exposure of persons with implanted electronic medical devices and implants containing ferromagnetic material, and dangers from flying objects, which can lead to much lower restriction levels such as 0.5 mT.

1.4 Effect of static magnetic field

The International Commission on Non-Ionizing Radiation Protection (ICNIRP) provides in its guidelines recommended limit values for exposure to electromagnetic fields (see Table 1.1). In Norway, the Radiation Protection Regulations stipulate that ICNIRP's guidelines must be followed, and that all exposure must be kept as low as practicable. For static magnetic fields, the recommended limit is 2 T for workers and 400 mT for the general population. Individuals with pacemakers, ferromagnetic implants or implanted electronic components should not be exposed to magnetic fields above 0,5 mT. [3] In the experiment using the Helmholtz coils the magnetic flux density B in the immediate vicinity of the coils has a value slightly exceeding 0,5 mT for $I = 1$ A and $R = 0{,}07$ m, where I is the current flowing in the coils and R is the average radius of the coils . But the field decreases rapidly outside the coils; the magnitude of B at a point perpendicular to the coil at a distance 0,15 m (or greater) from the center of the coil is less than 50 µT. If a current of 0,5 A is used, the field everywhere will be lower than the recommended limit for people with implants. Students with an implant that is sensitive to magnetic fields should inform the person in charge, who will ensure that this student has a partner who does not have an implant and that the partner makes the measurements.

3: http://www.icnirp.de/documents/emfgdl.pdf [*Health Physics* **96** (2009) 504–519.]

Important: Students with an implant that is sensitive to magnetic fields should inform the person in charge.

1.5 Common sense in handling electrical equipment

In addition to the usual risks associated with electricity, some laboratories have high voltage equipment [4] which poses an even greater potential hazard. Students should be extra careful with such equipment, and should learn how to disconnect the power source in an emergency. Here are some rules that must be followed when working with and near electricity.

4: For our purposes, it will be sufficient to consider any voltage higher than 50 V as high voltage.

1. Do not work with electricity if your hands, feet, or other parts of your body are wet, or if you are standing on a wet floor.
2. Inspect electrical equipment (with power off and unplugged) for broken wires and damaged connections. If such a thing is detected, do not use the equipment. Then report it to the person in charge so that the equipment can be repaired.
3. Never attempt to repair electrical equipment yourself — this must be done by qualified personnel.
4. If you are subjected to even a mild shock from any equipment, report it immediately so that it may be repaired.
5. Do not use or store extremely flammable liquids near electrical equipment. Some substances, such as ether, can be ignited by sparks from electrical equipment.
6. Always use earthed plugs in earthed wall sockets. Never attempt to insert a grounded electrical plug into an unearthed wall outlet.
7. Extension cords should not be used instead of permanent wiring; they should only be used temporarily and they should not be pulled under doors, across hallways, through windows or holes in walls, around pipes or near sinks.
8. Do not overload circuits by using branch outlets on a standard outlet.
9. Do not remove or change safety measures on high voltage equipment. Remember they are there to protect you.
10. Make sure that the power is switched off during connection or when making changes to the circuit.
11. Make sure that all equipment is set up with the correct

settings according to the measurement to be performed before the circuit is made live.

12. Make sure that the components used are of a rating that can withstand the applied current and voltage.
13. Replace broken fuses with new ones of the correct type and with the correct specifications.
14. *If you think someone has been subjected to a severe electric shock:*
    - do not remove the injured person from the electrical source until the power has been switched off.
    - if you do not turn off the power, use an insulator such as a dry rope, cloth or broom handle to pull the person away from the power.
    - check if you are able to make contact with the person.
    - check if there is breathing.
    - secure open airways.
    - if the injured does not breathe or is experiencing cardiac arrest, call the emergency services at 113.
    - provide cardiopulmonary resuscitation (CPR) using 30 compressions and 2 breaths. Continue until the emergency services arrive.
    - if the injured person breathes by themselves, put the person in the recovery position (on their side, with open airways).
    - look for burns and cool burns with water and treat them as third-degree burns.
    - after first aid has been given, the injured must be supervised at all times. Conscious people should be talked to. All injuries should be treated further by medical personnel.
    - people sent to the Emergency Room by taxi / private car must be followed all the way in and out. Bring any HSE data sheets!
    - all incidents / near incidents are to be regarded as deviations in relation to normal operation, and must be reported on the deviation form.

In the experiment with Coulomb's law, the high-voltage cannon used to charge the bullets provide 25 kV when activated. However, the current is limited to 75 µA. Such currents usually have no physiological effect on the body, but still: *the cannon is to be handled as if it was lethal*.

2 STATIC MAGNETIC FIELD

Goal

In this lab assignment you will

- develop your skills in documenting laboratory work with the lab journal,
- set up and carry out physical measurements,
- perform basic error analysis,
- learn to measure magnetic fields with Hall effect gaussmeter,
- study the magnetic field around a short coil, Helmholtz coil and solenoid,
- use Python or Excel spreadsheets to compare calculated and measured results.

2.1 Theoretical background

Biot-Savart's law is presented here. Some theory is presented in the text for the calculation tasks in section 2.2. Otherwise, reference is made to the textbook and the lectures in electromagnetism.

2.1.1 Biot-Savart's Law

Electric fields are generated around any electric charge while magnetic fields are generated around electric charges that move. In 1820, Jean-Baptiste Biot and Félix Savart performed magnetic field experiments on live conductors and set up the following expression for the magnetic field contribution $\mathrm{d}\vec{B}$ at a point P in space from the current I in a wire element $\mathrm{d}\vec{s}$,

$$\mathrm{d}\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\vec{s} \times \mathbf{r}}{r^2}, \tag{2-1}$$

where $\vec{r}$ is the position vector of the point P measured from the current element $I\,\mathrm{d}\vec{s}$, $\mathbf{r} = \vec{r}/r$ is the unit vector along $\vec{r}$ and μ_0 is the magnetic permeability in empty space. Note the r^{-2} dependence, the same as for the electric field strength in Coulomb's law. The superposition principle applies to magnetic fields, and consequently the total field strength $\vec{B}$ at point P from the whole conductor can be found by integrating over the length s of the conductor:

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int_s \frac{\mathrm{d}\vec{s}\times\mathbf{r}}{r^2}. \tag{2-2}$$

In the experiment, you will examine the magnetic field along the axis of circular coils. The simplest circular spool you can make consists of a simple wire loop as shown in Figure 2.1.

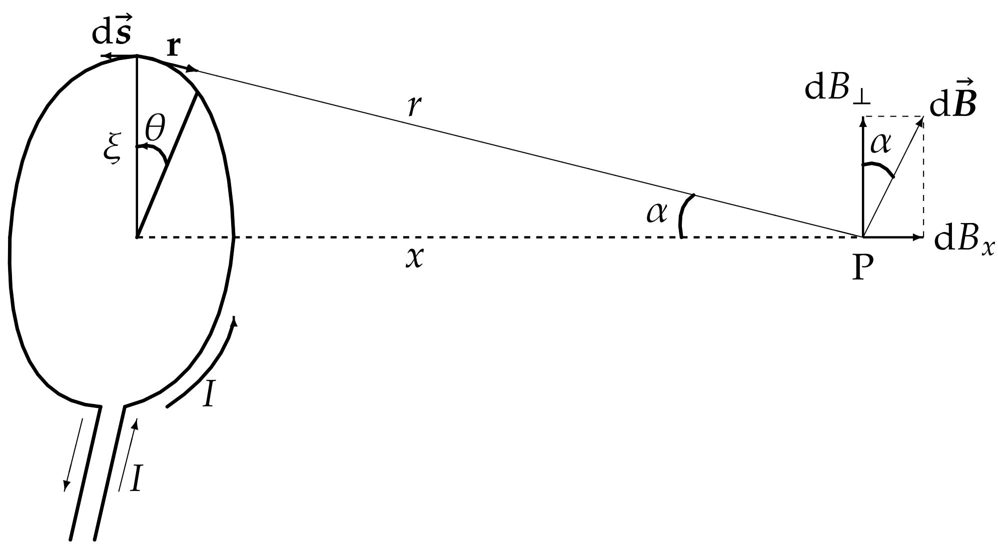

Figure 2.1: The magnetic field on the normal axis of a circular coil.

The magnetic field on the axis of the coil will have a component in the axis direction x and a component normal to this. According to Biot-Savart's law, the normal component will be zeroed out by contributions (integration) over the entire loop, as the symmetry of the system gives equal strength in all directions. For the x component we get by inserting $r^2 = x^2 + \xi^2$ and using $\sin\alpha = \xi/r$,

$$\begin{aligned}\mathrm{d}B_x &= \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}s}{x^2+\xi^2}\sin\alpha \\ &= \frac{\mu_0 I}{4\pi}\frac{\xi\,\mathrm{d}\theta}{x^2+\xi^2}\frac{\xi}{\sqrt{x^2+\xi^2}},\end{aligned} \tag{2-3}$$

where θ is the integration angle around the loop circle.

Integrated over the loop with $\int \mathrm{d}\theta = 2\pi$ gives this result

$$B_x = \frac{\mu_0 I}{4\pi} \frac{2\pi\xi^2}{(x^2+\xi^2)^{3/2}} = \frac{\mu_0 I}{2\xi}\left(1+\frac{x^2}{\xi^2}\right)^{-3/2}. \qquad \text{(2-4)}$$

For $x = 0$ (middle of the loop) this gives $B_x = \mu_0 I/2\xi$ and for $x \gg \xi$ (far away from the loop), $B_x = \mu_0 I\xi^2/2x^3$.

2.2 Calculation tasks

2.2.1 Calculation and documentation

To perform calculations and to create tables / diagrams, you can freely choose between suitable programs (for example Python, Matlab, Origin, Excel or similar). Data and results must be documented in the lab journal (for example by pasting printouts of the tables and diagrams in the lab journal or by writing the results by hand). The source code and the original files used for calculation must be available in case of questions. The source code can be pasted into the lab journal.

2.2.2 Short Coil

The magnetic field from a short coil with N windings can be considered far from the coil (in relation to the extent of the coil) as the sum of the magnetic field from N current loops given in equation (2-4). The magnetic field on the axis at a distance x from the mean center [1] of the coil is then according to equation (2-4) given by

1: Recall that the coils have finite thickness, so we use the mean center.

$$B(x) = \frac{N\mu_0 I}{2R}\left(1+\frac{x^2}{R^2}\right)^{-3/2}, \qquad \text{(2-5)}$$

where R is the average radius of the current loops.

Assume that you have a short coil with $N = 330$ windings and an average radius $\langle R\rangle = 70$ mm with a constant current $I = 1{,}00$ A through the windings.

1A. Make a table of the magnetic flux density $B(x)$ on the axis of *the coil in the area* $-20\,\text{cm} < x < 20\,\text{cm}$.

A suggested table is shown in Table 2.1. Constants are entered in rows under the heading.

Magnetic permeability for vacuum, μ_0	1.26E-0.6	H/m
Number of windings, N	330	#
Current, I	1.00	A
Average loop radius, R	0.07	m

Position (cm)	x (m)	Calculated (gauss = $10^{-4}T$)	Measured (gauss)	Difference (%)
24.5	-0.25	0.581		-100.0
29.5	-0.2	1.068		-100.0
34.5	-0.15	2.241		-100.0
39.5	-0.10	5.588		-100.0
44.5	-0.05	15.965		-100.0
47.5	-0.02	26.339		-100.0
48.5	-0.01	28.745		-100.0
49.5	0.00	29.629		-100.0

Table 2.1: Calculated and measured values of the magnetic field B along the axis of a short coil as a function of the distance x from the center of the coil.

1B. Present the result in a graph.

Discuss the calculation of the magnetic field of a short coil when the extent of the coil is taken into account.

The program or spreadsheet in Excel can also be used to calculate expected magnetic field strengths as a result of Biot-Savart's (2-4) law and subsequent comparison with experimentally measured magnetic field strengths in section 2.3.

1C. Include a column for measured value of B and a column for deviation between measured and calculated value given in %, and also prepare to show the deviation graphically. (A suggestion is given in Table 2.1.)

Calculate also one of the values of B by hand, and verify the value of the difference between calculated and measured.

A small challenge for those with extra time:

The actual spool you are going to use in the problem has 22 windings in 15 layers with an effective wire diameter equal to 0,93 mm, wound with inner diameter equal to 126 mm and outer diameter equal to 154 mm.

2. How do you think the real magnetic field near the coil will differ from the magnetic field calculated from equation (2-5)*?* [2]

3. If you were to take into account the extent of the coil during the calculations of the field, how would you proceed? [3]

2.2.3 Helmholtz coils

So-called Helmholtz coils [4] consist of two identical, short coils connected in series set up concentrically at a distance a as shown in figure 2.2. The axial magnetic field on the axis is given by equation (2-5) for each coil. According to the superposition principle, we find the total field at a distance x from the center plane between the coils to be equal

$$B_x(x) = \frac{N\mu_0 I}{2R}\left[\left(1+\frac{(x-a/2)^2}{R^2}\right)^{-3/2} + \left(1+\frac{(x+a/2)^2}{R^2}\right)^{-3/2}\right]. \tag{2-6}$$

Here the average radius of the coils is equal to R, the current through the coils is equal to I and we have neglected the extent of the coils.

If you differentiate equation (2-6) with respect to x you will find that at $x = 0$ the derivatives

$$\frac{\mathrm{d}B}{\mathrm{d}x} = \frac{\mathrm{d}^2B}{\mathrm{d}x^2} = \frac{\mathrm{d}^3B}{\mathrm{d}x^3} = 0$$

when $a = R$. This means that when $a = R$ you have a geometry that gives you a particularly good homogeneous magnetic field in the area between the coils. Such Helmholtz coils are used to set up homogeneous magnetic fields in many situations.

Suppose you have Helmholtz coils with $N = 330$ windings on each coil, $R = 70$ mm and send a constant current $I = 1{,}00$ A through the windings.

4A. Make a table of the magnetic flux density $B(x)$ on the axis of the Helmholtz coil in the range $-20\,\mathrm{cm} < x < 20\,\mathrm{cm}$ *for each of the three different values $a = R/2$, R and $2R$.*

A suggested table is shown in Table 2.2.

4B. Present the result in a curve diagram

2: TIP: Look at the extent of the coil as an error in x and estimate the corresponding error ΔB in $B(x)$.

3: TIP: Divide the coil into e.g. three coils with 110 windings each and calculate the field from each of the coils and superimpose the effect. The best thing you can do is treat each winding separately, and in practice, that means integration over each winding.

4: Named after Hermann Ludwig Ferdinand von Helmholtz (1821-1894), German physicist and physician.

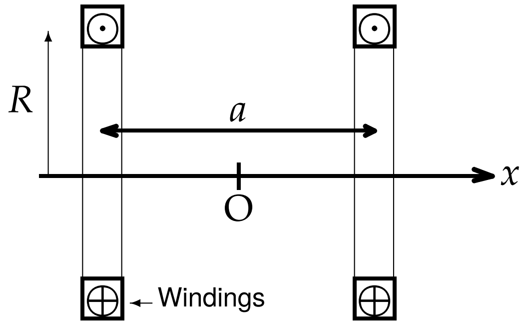

Figure 2.2: Helmholtz coils consist of to thin concentric coils at a distance a from each other.

Table 2.2: Calculated and measured values of the magnetic field B along the axis of a Helmholtz coil as a function of the distance x from the center of the coil, for different values of the separation a.

Magnetic permeability for vaccum, μ_0	1.26E-0.6	H/m
Number of windings, N	330	#
Current, I	1.00	A
Average loop radius, R	0.07	

		$a = 2R$			$a = 2R$			$a = 2R$		
Position (cm)	x (m)	Calculated (gauss)	Measured (gauss)	Difference (%)	Calculated (gauss)	Measured (gauss)	Difference (%)	Calculated (gauss)	Measured (gauss)	Difference (%)
30	-0.20	3.626		-100	2.454		-100	2.213		-100
35	-0.15	9.286		-100	5.478		-100	4.719		-100
40	-0.10	24.643		-100	14.549		-100	11.996		-100
43	-0.07	32.279		-100	26.258		-100	22.393		-100
45	-0.05	30.129		-100	35.312		-100	33.160		-100
47	-0.03	24.981		-100	41.063		-100	45.054		-100
48	-0.02	22.821		-100	42.105		-100	49.866		-100
49	-0.01	21.429		-100	42.382		-100	53.017		-100
50	0.00	20.951		-100	42.402		-100	54.108		-100

The spreadsheet shall also be used for the calculation of expected magnetic field strengths as a result of Biot-Savart's law (2-4) and subsequent comparison with experimentally measured magnetic field strengths in section 2.3.

4C. Include a column for measured value of B and a column for deviations between measured and calculated value given in % for $a = 2R$*. Repeat this for the other two values of* a*. Prepare to display the deviation graphically.* (A suggestion is given in the table in Table 2.2.)

Calculate also one of the values of B by hand, and verify the value of the difference between calculated and measured.

2.2.4 Anti-Helmholtz Coil

Anti-Helmholtz coils consist of two identical, short coils connected in series set up concentrically at a distance a as shown in Figure 2.3, with opposite directions of the magnetic field. How do you find the field in this configuration?

5A. Make a table of the magnetic flux density $B(x)$ *on the axis of the Anti-Helmholtz coil in the area* $-20\,\text{cm} < x < 20\,\text{cm}$ *for each of the three different distances* $a = R/2$*,* R *and* $2R$*.*

5B. Present the result in a graph.

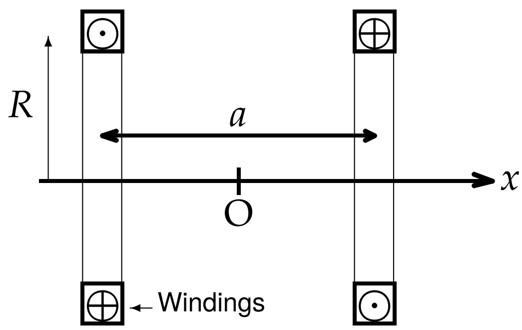

Figure 2.3: Anti-Helmholtz coils consist of two identical, short coils connected in series set up concentrically at a distance a, with the current running opposite of each other.

The spreadsheet shall also be used for the calculation of expected magnetic field strengths as a result of Biot-Savart's law (2-4) and subsequent comparison with experimentally measured magnetic field strengths in section 2.3.

5C. Include a column for measured values of B and a column for discrepancies between measured and calculated values given in %

for $a = 2R$. Repeat this for the other two values of a. Prepare the spreadsheet to display the deviation graphically. (A suggestion is given in Table 2.2.)

Make a control calculation by hand of one of the calculated B. Check the calculated difference by entering the hand-calculated value.

2.2.5 Solenoid

The magnetic field at a point P on the axis of a solenoid can be found from Biot-Savart's law, and the result is

$$B = \frac{N\mu_0 I}{2\ell}(\cos\theta_1 - \cos\theta_2), \tag{2-7}$$

where ℓ is the length of the solenoid and the angles θ_1 and θ_2 are defined in Figure 2.4, and in a larger version in Figure 2.5.

Analytically the angles are given by

$$\cos\theta_1 = \frac{z}{\sqrt{z^2 + R^2}}, \qquad \cos\theta_2 = -\frac{\ell - z}{\sqrt{(\ell - z)^2 + R^2}}, \tag{2-8}$$

where R is the inner radius of the solenoid and z is the distance from the left end of the solenoid to the point P. Inside the solenoid the angles are restricted to $\theta_1 \in \langle 0, \pi/2\rangle$ and $\theta_2 \in \langle \pi/2, \pi\rangle$. The formula also applies outside the solenoid.

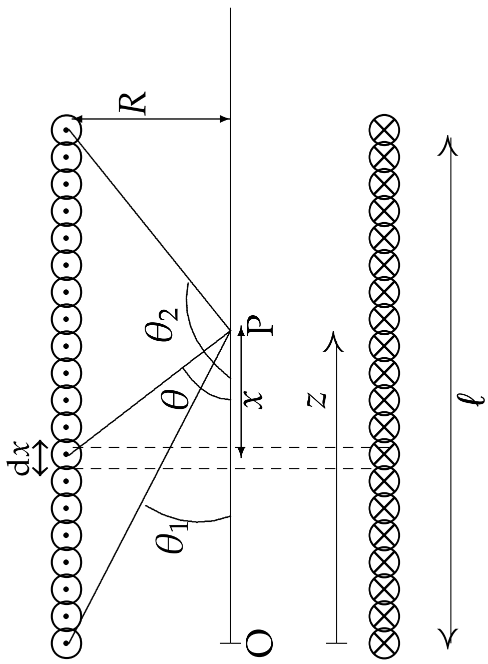

Figure 2.4: A solenoid. The angles are used for calculation of the magnetic field along the axis of the solenoid. See the main text for details. See Figure 2.5 for a larger version.

Proof. We offer here a short proof of Equation (2-7)*:* In equation (2-4) we have found magnetic fields on the axis at a distance x from a single loop. In a solenoid we have many loops close together, we define current per unit length as $IN/\ell = In$, where n is the number of windings per length. Flux density at a point P on the axis then receives a contribution from the thin current loop $\mathrm{d}I = In\,\mathrm{d}x$ in position x, and according to the first part of equation (2-4) the flux density is

$$\mathrm{d}B = \frac{\mu_0\,\mathrm{d}I}{4\pi}\frac{2\pi R^2}{(x^2 + R^2)^{3/2}} = \frac{\mu_0 In}{2}\frac{R^2}{(R^2 + x^2)^{3/2}}\,\mathrm{d}x\,. \tag{2-9}$$

With θ as the angle at point P between the solenoid axis and the periphery at position x (Figure 2.4) we see that

$\tan\theta = R/x$, and differentiation of $x = R/\tan\theta$ gives

$$dx = R\frac{-1}{\tan^2\theta}\frac{1}{\cos^2\theta}\,d\theta = -R\frac{1}{\sin^2\theta}\,d\theta\,. \tag{2-10}$$

We notice that

$$\sin^3\theta = \frac{R^3}{(R^2+x^2)^{3/2}}, \tag{2-11}$$

so that

$$dx = -R\frac{1}{\sin^3\theta}\sin\theta\,d\theta = -R\frac{(R^2+x^2)^{3/2}}{R^3}\sin\theta\,d\theta \tag{2-12}$$

and by substituting $\tan\theta = R/x$ in equation (2-9) and inserting boundaries θ_2 (right end, lowest x) to θ_1 (left end, highest x), we will get

$$B(x) = -\frac{\mu_0 In}{2}\int_{\theta_2}^{\theta_1}\sin\theta\,d\theta = \frac{\mu_0 In}{2}(\cos\theta_1 - \cos\theta_2), \tag{2-13}$$

the result we wanted to show. □

5. What is the expression of the magnetic field deep inside the solenoid, and what is it at the end of the solenoid?
TIP: What are θ_1 and θ_2 at these positions?

Suppose you have a solenoid with $N = 397$ windings, $\ell = 420$ mm, $R = 50$ mm and current $I = 1{,}00$ A.

6A. Make a table of the magnetic flux density $B(z)$ on the axis of the solenoid in the area -10 cm $< z <$ 50 cm where $z = 0$ at the left edge of the solenoid.

A suggested table is shown in Table 2.3.

Position (cm)	z (m)	$\cos\theta_1$	$\cos\theta_2$	Calculated (gauss)	Measured (gauss)	Difference (%)
60	-0.10	-0.894	-0.995	0.600		-100.0
55	-0.05	-0.707	-0.994	1.707		-100.0
53	-0.03	-0.514	-0.994	2.848		-100.0
52	-0.02	-0.371	-0.994	3.696		-100.0
51	-0.01	-0.196	-0.993	4.736		-100.0
50	0.00	0.000	-0.993	5.899		-100.0
49	0.01	0.196	-0.993	7.062		-100.0

Table 2.3: Calculated and measured values of the magnetic field B along the axis of a solenoid as a function of the distance z.

6B. Present the result in a line diagram

The spreadsheet shall also be used for the calculation of expected magnetic field strengths as a result of Biot-Savart's law (2-4) and subsequent comparison with experimentally measured magnetic field strengths in section 2.3.

6C. Include a column for measured value of B and a column for deviations between measured and calculated value given in %. (A suggestion is given in the table in Table 2.3.)

2.2.6 Python Data Analysis

The previous tasks can be done in Python, for example. It is advisable to save the data in text files. Here we use comma separated files as shown below.

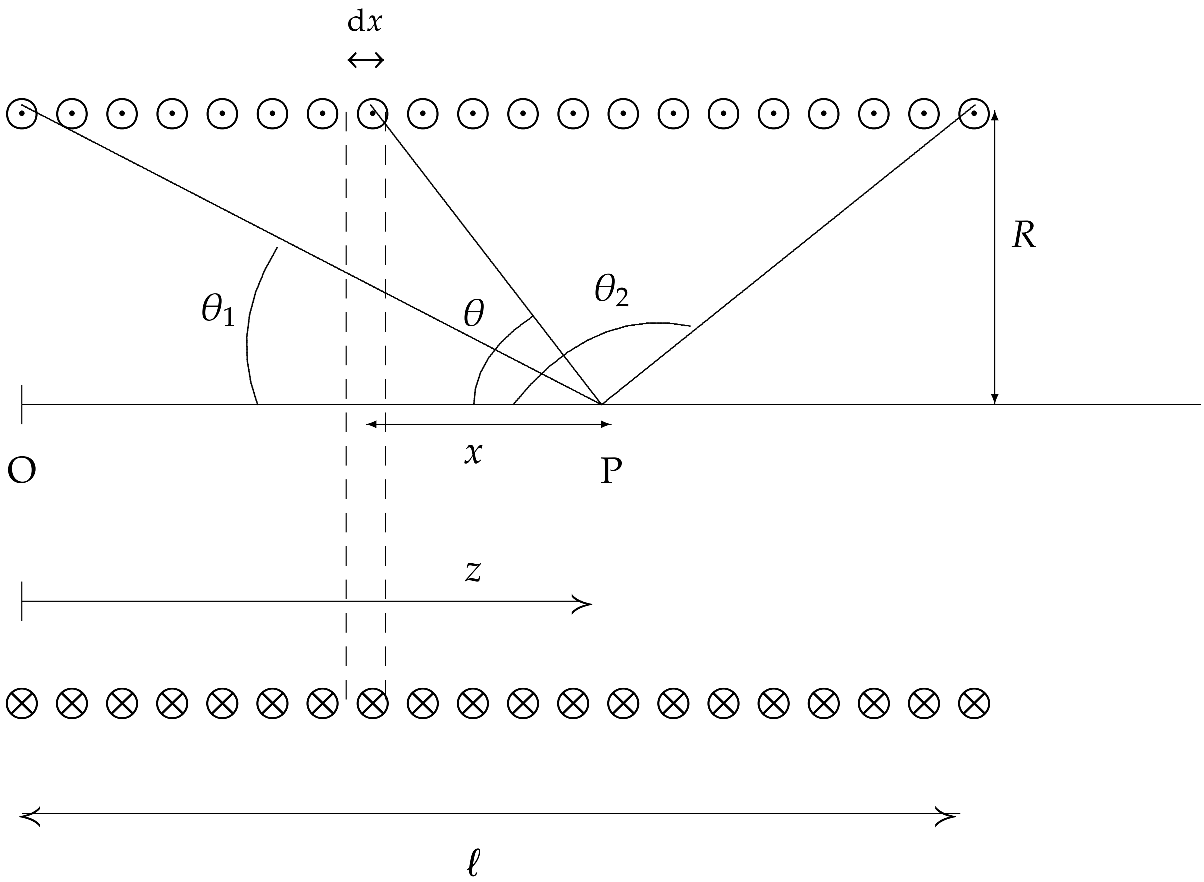

Figure 2.5: A solenoid. The angles are used for calculation of the magnetic field along the axis of the solenoid. See the main text for details.

Tabell 3. Målte og beregnede verdier av magnetfeltet *B* på aksen til en solenoide s.f.a. avstanden *z* fra ene ende

Magnetisk permeabilitet for tomt rom, μ_0	1,26E-06	H/m
Antall viklinger, N	397	#
Spolestrøm, I	1,00	A
Indre radius i solenoiden, R	0,05	m
Lengde, l	0,42	m

Posisjon på målestav (cm)	z (m)	cos θ1	cos θ2	Beregnet B1 gauss	Målt B2 gauss	Differanse B2-B1 (%)
60	-0,10	-0,894	-0,995	0,600		-100,0
55	-0,05	-0,707	-0,994	1,707		-100,0
53	-0,03	-0,514	-0,994	2,848		-100,0
52	-0,02	-0,371	-0,994	3,696		-100,0
51	-0,01	-0,196	-0,993	4,736		-100,0
50	0,00	0,000	-0,993	5,899		-100,0
49	0,01	0,196	-0,993	7,062		-100,0

Figure 2.6: Utdrag fra tabell for magnetfelt på aksen til Helmholtzspoler.

Listing 2.1: Example of CSV file.

```
# Position (m), B-field (Gauss)
0.245,0.0
0.295,0.623
...
```

Here, the first column represents the *position*, and the second column represents *Measured* in Table 2.1. The remaining columns can be calculated in Python. To read the data, we create the following code:

```
# Assumes numpy imported as np.

def read_CSV(filename):
   data = np.loadtxt (filename, delimiter = ',')
   xe = data [:, 0]
   Be = data [:, 1]
   return xe, Be
```

In the function `read_CSV`, the text file with filename *filename* is read. The position data, located in the first column, is stored in the variable `xe` and the data for the measured magnetic field is stored in the variable `Be`.

The next thing we need to do is implement a function that calculates the magnetic field according to equation (2-5). We can do this as follows:

```
def B_field_short_coil(x):
    prefactor = N * mu_0 * I0 / (2 * R)
    return prefactor * (1.0 + (x / R) ** 2) ** (-1.5)
```

To plot the result for the calculated field, we add some lines in the `main` function. We also need to define some parameters used in the calculation. The whole program then looks like this:

```
import numpy as np
from matplotlib import pyplot as plt

N = 330 # [] number of windings
I0 = 1.0 # [A] current
mu_0 = 4.0 * np.pi * 1e-7 # [H / m] permeability in
    empty space
R = 0.07 # [m] radius
x0 = 0.400 # [m] center of coil

def B_field_short_coil (x):
    prefactor = N * mu_0 * I0 / (2 * R)
    return prefactor * (1.0 + (x / R) ** 2) ** (- 1.5)

def read_CSV(filename):
```

A quick note about the `main` function. You will quite often see code with a function called `main`, and then a code

```
if(__name__=='__main__'):
    main ()
```

at the end of the file. This is a common convention which is useful for larger projects using several files. It makes it easier for others to see what is the "entry point" of your code, and also prevents accidental execution of code. Feel free to search the web for this if you want to know more.

```
    data = np.loadtxt (filename, delimiter = ',')
    xe = data [:, 0]
    Be = data [:, 1]
    return xe, Be

def main ():
    xe, Be = read_CSV("short_spool.csv")

    # Calculate the B field
    xb = np.linspace (xe [0], xe [-1], 100) # multiple data points
    Bb = B_field_short_coil (xb) * 1e4 # calculated B-field (Gauss)

    # Plot the results
    plt.plot (xb, Bb, label = 'Calculated')
    plt.plot (xe, Be, '.', label = 'Measurement data')
    plt.xlabel ('Distance from center of coil (m)')
    plt.ylabel ('Magnetic field (Gaussian)')
    plt.legend ()
    plt.show ()

if (__name__ == '__main__'):
    main ()
```

You can now create more functions to plot the results for a Helmholtz coil and a solenoid. Remember that it often pays to divide the program into smaller functions. This makes the code easier to read, but also allows you to reuse much of the code for other purposes. More examples of plotting can be found here: https://nbviewer.jupyter.org/urls/www.numfys.net/media/notebooks/basic_plotting.ipynb.

2.2.7 Hall effect probe

When electrons move at speed $\vec{v}$ in a semiconductor located in a magnetic field $\vec{B}$ as shown in Figure 2.7, the electrons will bend to one side. The deflection is given by the Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$, where $q = -e$ for electrons.

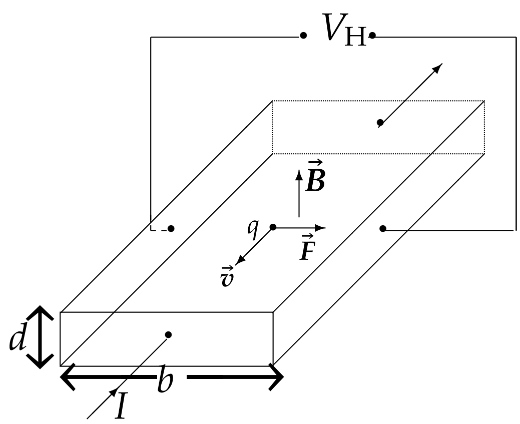

Figure 2.7: Hall effect in a semiconductor probe. A current I is made to flow thorugh the sample, causing the electrons to experience a force $\vec{F}$.

The Lorentz force acts normal to the current direction and normal to B; the deflection due to this causes the electron concentration to become stronger against one wall of the semiconductor. An electric field $\vec{E}$ thus builds up in the conductor with the same direction as the Lorentz force, and this field gives an additional force $\vec{F} = q\vec{E}$ on the electrons. Equilibrium will quickly arise where electrostatic force and Lorentz force are equal so that

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = \vec{0} \quad \Rightarrow \quad \vec{E} = -\vec{v} \times \vec{B}. \tag{2-14}$$

The speed v of the electrons through the semiconductor is determined by the current at the expression $I = nqvA$, where n is the density of electrons and $A = bd$ is the cross section in the conducting direction, see Figure 2.7. This gives

$$v = \frac{I}{A}\frac{1}{nq} = \frac{I}{bd}R_{\mathrm{H}}, \tag{2-15}$$

where we have defined the Hall constant $R_{\mathrm{H}} = 1/nq$.

With b equal to the width of the probe, the voltage $V_{\mathrm{H}} = Eb$ will form across the side walls. If we disregard the sign and use that $\vec{v} \perp \vec{B}$ we get the Hall voltage V_{H}:

$$V_{\mathrm{H}} = Eb = vBb = \frac{R_{\mathrm{H}}I}{d}B, \tag{2-16}$$

where d is the thickness of the Hall probe where the magnetic field B is present. The hall constants R_{H} and d are constants for a given probe and the current I is kept constant. The magnetic flux density B normal to the probe will then be proportional to the Hall voltage V_{H} which is measured with a voltmeter.

The hall effect is used in for example magnetometers [5], instruments for measuring magnetic fields. A Hall effect magnetometer will essentially consist of a current source that provides a constant current I through a probe, and a voltmeter. The probe is placed in the magnetic field to be measured. Through a calibration process, the relationship between the measured Hall voltage and the magnetic field can be established and the voltage reading unit is graded directly in T = Wb/m^2 or gauss = 10^{-4} T.

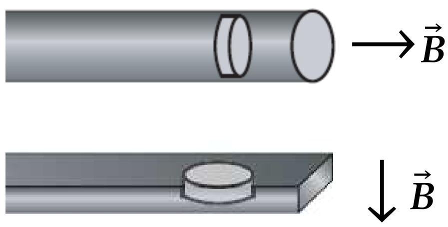

Figure 2.8: Probe geometries: **(Top)** Axial probe, **(Bottom)** Transversal probe.

5: Also known as Gaussmeters and Teslameters.

In practice, the extent of a Hallprobe is of the order of a few mm^2. Two different probe geometries are common: transverse probes and axial probes, as shown in Figure 2.8.

Typical value for the electron density in a doped semiconductor is $n = 10^{20}$ electrons/m^3. By comparison, copper has $n = 10^{28}$ electrons/m^3. In our magnetometers we have $I = 20\,\mathrm{mA}$ and $d = 1{,}0\,\mathrm{mm}$. Assume that the magnetic field varies from 1 to 100 gauss.

7A. What measuring range must the magnetometer's voltmeter have to measure in this magnetic field range? (Assume at $n = 10^{20}$ electrons/m^3.)

7B. Why do you think semiconductors are used in magnetic field probes? Explain why e.g. copper will not be a suitable material.

7C. Also explain why an insulator will not be a suitable material in a magnetic field probe.

2.2.8 Magnetic scattering fields

In the laboratory, there are background magnetic fields, or "noise", which can interfere with the magnetic field measurements. These noise fields have three main sources:

- the Earth's magnetic field,
- magnetic fields from magnetic materials in building structures and fixtures,
- induced magnetic fields from for example 230 V power cords.

The Earth's magnetic field, which is constant and of the order of magnitude 1 gauss, is believed to originate from convection currents of electric charges in the Earth's liquid core. Assume that in the middle of the Earth's interior there is a current loop perpendicular to the Earth's axis of rotation. Let the radius of the current loop be $R = R_\mathrm{j}/4$, where $R_\mathrm{j} = 6371$ km is the radius of the Earth. The axial component of the magnetic field in position x on the axis of such a current loop is given by equation (2-4), which reads

$$B_x = \frac{\mu_0 I}{4\pi}\frac{2\pi\xi^2}{(x^2+\xi^2)^{3/2}} = \frac{\mu_0 I}{2\xi}\left(1+\frac{x^2}{\xi^2}\right)^{-3/2}.$$

8. How large must the current in the loop be for the magnetic field on the Earth's surface at the pole ($x = R_j$) to be equal to 1 gauss?

Contributions to the background magnetic field from the building and the inventory will depend on the building and construction materials used. You will evaluate the contributions to this in the laboratory through your own measurements.

Contribution to the background field from the currents in non-shielded wires can be estimated by calculating the field set up by a long straight conductor leading the current I. The azimuthal component of the magnetic field at a distance r from a long conductor can be found from [6] Biot-Savarts Law (2-2)

$$B_\theta(r) = \frac{\mu_0 I}{2\pi r}. \tag{2-17}$$

6: Derivation in e.g. Lillestøl et al. [?, Ch. 23.5].

9. How large is the magnetic flux density at a distance 5 cm from a long 230 V mains lead due to a current 1 A in this wire?

2.2.9 Magnetic field-free room

The permanent magnetic field on the Earth's surface leads to problems with zeroing of magnetometers. However, magnetic fields can be shielded by using materials with high permeability. Pure iron can be used. There are also special alloys that have even higher permeability μ and thus shields even better. An example is mu-metal[7]. Magnetmometers are usually zeroed by inserting the probe into a chamber that shields it from the Earth's magnetic field during zeroing.

7: Mu-metal is an alloy of 77 % nickel, 15 % iron plus some copper and molybdenum. It got its name due to very high value for magnetic permeability, μ.

2.3 Experimental

2.3.1 Equipment

The experiment uses PASCO's data logging system (Capstone) with interfaces and sensors for magnetic fields and position. The following instruments are included in the line-up:

- **Interface.** PASCO 550 Universal Interface.
- **Magnetic field sensor.** PASCO Magnetic Field Sensor CI-6520A. Measuring range: 0,01 G - 100 kG. Resolution: 50 mG. Accuracy: 10 % of reading
- **Zero field chamber.** PASCO Zero Gauss Chamber EM-8652.
- **Position sensor.** PASCO Rotary Motion Sensor PS-2120A. Catalog Resolution: 0,09°. Radii on the three pulleys: 10 mm, 29 mm, 48 mm.

- **Short coils.** 330 Vikings, 22 Vikings / Team × 15 Teams Inner diameter 126 mm, Outer diameter 154 mm. Wire: lacquer-insulated copper, diameter 0,75 mm, maximum coil current: 1,0 A.
- **Solenoid.** 368 windings, length ~ 400 mm, inner diameter 100 mm. Wire: lacquer-insulated copper, diameter 1,0 mm, maximum solenoid current 1,0 A.
- **Multimeter.** Escort Mod. EDM 168A, or equivalent.
- **Miscellaneous:** Meter, screwdriver.
- **Power supply.** Mascot Type 719. Range: 0- 30 V, 30 mA - 2 A. This is a combined power-controlled / voltage-controlled power supply. That is, it can supply a constant current or a constant voltage, up to a certain maximum load. The box has two adjustment buttons, one for voltage marked "0 $\rightarrow$ 30 V" and one for current marked "30 mA $\rightarrow$ 2 A".

  Current controlled power supply (current source) the voltage adjustment is set to maximum. The current is regulated with the current adjustment knob.

  Voltage controlled power supply(voltage source) the power adjustment is set to maximum. The voltage is regulated with the voltage adjustment knob.

  When current controlled, the maximum current that may be supplied is 2 A, given that the resistance is maximum $R = U/I = 30\,\mathrm{V}/2\,\mathrm{A} = 15\,\Omega$. The box also has a button marked "range" which can limit the maximum voltage to 15 V instead of the standard 30 V. If 15 V is selected, the maximum output resistance will be $R = U/I = 15\,\mathrm{V}/2{,}0\,\mathrm{A} = 7{,}5\,\Omega$. At higher output resistance, the output voltage will be kept at maximum value and the current will be lower.

  There is a corresponding maximum regulation for voltage-controlled power supply. 30 V maximum voltage can be given to a resistance of $R = U/I = 30\,\mathrm{V}/2{,}0\,\mathrm{A} = 15\,\Omega$. At lower resistance, the supply will not be able to provide enough current so that the voltage will drop.

2.3.2 Calibration of the Hall probe

It is difficult to produce Hall probes with exactly the same Hall constants. The Hall voltage that occurs at a given magnetic field and current may therefore vary slightly from probe to probe. In order to be able to use several probes for the same Gaussmeter, each probe has often assigned a calibration number which is used to adjust the gain of the Gaussmeter's voltmeter so that the meter shows the correct magnetic field. PASCO's sensors are adjusted and a calibration is not normally needed. Before use, the sensor must be reset.

Resetting:

- Set the area knob on the sensor to position 1x and axial.
- Insert the probe into the zero field chamber and press the zero adjustment button.

Note: When changing the measuring range, make a new reset!

Units and signs of the Hall probe:

- The meter shows the magnetic flux density, B, of the device you select in Capstone.
- The Gaussian meter shows *positive* signs when the field lines have direction *into* the cylindrical tip of the probe.

2.3.3 Magnetic scattering fields

Task: *Investigate the magnetic background fields, noise fields, in the laboratory.*

- How does the measured direction of the earth's magnetism correspond to the assumed direction of our latitude? [8]
- Compare the results with results from the other groups. Is there variation of the measured earth magnetism within the laboratory?
- Will the magnetic field from the iron in the laboratory tables be able to interfere with the measurements?

8: The magnetic north pole is located near the geographical south pole. Magnetic field lines are closed curves and have direction from magnetic north pole to south pole outside of the magnet and from south pole to north pole inside the magnet.

2.3.4 Statistical error in measurements

Reading magnetic field strengths from the Hall probe will give a somewhat varying result. You can use this to find the statistical error.

Task: *Investigate statistical errors in the measurements:*