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We will find the current response of a single Dirac cone, with a temperature gradient $\nabla_y T$ and a magnetic field B_z. The current response of interest in the given geometry is thus in the x-direction,

$$J^x = \chi^{xy}\frac{-\nabla_y T}{T}, \tag{0.1}$$

with χ^{xy} being the response[1]. In the derivation of Chernodub, Cortijo, and Vozmediano [3] the response

$$\chi^{xy} = \frac{e^2 v_F B}{18\pi^2\hbar} \tag{0.2}$$

was found, while the derivation of Arjona, Chernodub, and Vozmediano [1] found [2]

$$\chi^{xy} = \frac{e^2 v_F B}{4\pi^2\hbar}. \tag{0.3}$$

Recall the linear response from the Kubo formalism in Eq. (**??**), found through Luttinger's approach.

$$\langle J^i\rangle(t,\boldsymbol{r}) = \int_{-\infty}^{\infty} \mathrm{d}t'\mathrm{d}\boldsymbol{r}' \int_{-\infty}^{t'} \mathrm{d}t'' \left\{ \frac{-iv_F}{\hbar}\Theta(t-t')\left\langle [J^i(t,\boldsymbol{r}), T^{0j}(t'',\boldsymbol{r}')]\right\rangle \right\} \partial'_j\psi(t',\boldsymbol{r}'). \tag{0.4}$$

Fourier transforming now to the frequency and momentum domain, will be beneficial in our calculations. As before, the non-perturbed system will be taken to be time and position invariant, such that the correlator in Eq. (0.4) can be taken to depend only on the differences $t-t''$ and $\boldsymbol{r}-\boldsymbol{r}'$. Starting with Fourier transforming the position part, notice that the structure of Eq. (0.4) is

$$\langle J^i\rangle(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}'\chi(\boldsymbol{r}-\boldsymbol{r}')\partial'_j\psi(\boldsymbol{r}'),$$

where the temporal parts were dropped for clarity. This is a convolution, and the Fourier transform is thus simply given by the product of the two factors [8].

$$\langle J^i\rangle(\boldsymbol{q}) = \chi(\boldsymbol{q})(iq_j)\psi(\boldsymbol{q}), \tag{0.5}$$

where it was also used that the Fourier transform of a derivative gives the component of the variable. Showing explicitly how to find the form of the response χ in momentum space is often overlooked in much literature, and as it does involve some finesse, we want to show it here. This trick is courtesy of Chang [2]. By definition, the Fourier transform

[1] The sign in Eq. (0.1) depends on the choice of the response function being the response of the gravitational potential or the temperature gradient. Thus, the sign may differ in the literature.

[2] The paper is somewhat unclear on what is their final result, as there is some possible confusion related to the number of Landau levels included and whether one is including both or only one Dirac cone. The above result is what is meant, to the best of our understanding.

of the response is, where the variable of integration has been chosen to be $\boldsymbol{r}-\boldsymbol{r}'$ for later convenience,

$$\chi(\boldsymbol{q}) = \int \mathrm{d}(\boldsymbol{r}-\boldsymbol{r}') e^{-i\boldsymbol{q}(\boldsymbol{r}-\boldsymbol{r}')} \chi(\boldsymbol{r}-\boldsymbol{r}') \tag{0.6}$$

$$= \int \mathrm{d}(\boldsymbol{r}-\boldsymbol{r}') e^{-i\boldsymbol{q}(\boldsymbol{r}-\boldsymbol{r}')} C \left\langle \left[J^i(\boldsymbol{r}), T^{0j}(\boldsymbol{r}') \right] \right\rangle, \tag{0.7}$$

$$\tag{0.8}$$

where C denotes t-dependent prefactors and integrals over time are omitted, again for clarity of notation. Note that

$$\int \mathrm{d}(\boldsymbol{r}-\boldsymbol{r}') = \frac{1}{\mathcal{V}} \int \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r}', \tag{0.9}$$

where $\mathcal{V}$ is the volume of the system. Thus,

$$\begin{aligned} \chi(\boldsymbol{q}) &= \frac{1}{\mathcal{V}} \int \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r}' e^{-i\boldsymbol{q}(\boldsymbol{r}-\boldsymbol{r}')} C \left\langle \left[J^i(\boldsymbol{r}), T^{0j}(\boldsymbol{r}') \right] \right\rangle \\ &= \frac{C}{\mathcal{V}} \left\langle \left[J^i(\boldsymbol{q}), T^{0j}(-\boldsymbol{q}) \right] \right\rangle. \end{aligned} \tag{0.10}$$

Considering now the temporal part, the procedure is simpler. The linear response still has the form of a convolution, as the response function is only dependent on the difference $t-t'$ by

$$\chi(t-t') = \int_{-\infty}^{0} \mathrm{d}t'' \Theta(t-t') \left\langle \left[J(t-t'), T(t'') \right] \right\rangle, \tag{0.11}$$

where t'' was shifted by t', and then the translational invariance of the correlator was used. In frequency space

$$\chi(\omega) = \int \mathrm{d}t e^{i\omega t} \chi(t) \tag{0.12}$$

$$= \int \mathrm{d}t e^{i\omega t} \int_{-\infty}^{0} \mathrm{d}t'' \Theta(t) \left\langle \left[J(t), T(t'') \right] \right\rangle. \tag{0.13}$$

In frequency and momentum space the response function is thus

$$\chi^{ij}(w, \boldsymbol{q}) = \frac{-iv_F}{\mathcal{V}\hbar} \int \mathrm{d}t e^{i\omega t} \int_{-\infty}^{0} \mathrm{d}t' \Theta(t) \left\langle \left[J^i(t, \boldsymbol{q}), T^{0j}(t', -\boldsymbol{q}) \right] \right\rangle. \tag{0.14}$$

0.1 Eigenvalue problem of the Landau levels of a Weyl Hamiltonian

To evaluate the correlator of the response function, the matrix elements of the current and stress-energy tensor must be found. In order to do this, we find eigenstates in the Landau basis of the system. The Weyl Hamiltonian

$$H_s = s v_F \sigma^i \left(p_i + eA_i\right), \tag{0.15}$$

with s being the chirality, p_i the momentum operator, and $e = |e|$ the coupling constant to the electromagnetic field $\boldsymbol{A}$. Choose coordinates such that $\boldsymbol{B} = B_z \hat{\boldsymbol{z}}$, which in the Landau gauge gives $\boldsymbol{A} = -B_z y\, \hat{\boldsymbol{x}}$. As the Hamiltonian is invariant in x and z, take the plane wave ansatz $\phi(\boldsymbol{r}) = e^{ik_x x + ik_z z}\phi(y)$. It then follows

$$H_s\phi(\boldsymbol{r}) = E\phi(\boldsymbol{r}) \implies \tilde{H}_s\phi(y) = E\phi(y), \tag{0.16}$$

where $\tilde{H}$ is the result of replacing $p_z \to \hbar k_z, p_x \to \hbar k_x$ in H_s, as the plane wave part of ϕ have these eigenvalues. Absorb the chirality s as a sign in the velocity v_F, for more concise notation. Thus, writing everything explicitly, the spectrum is given by

$$-\hbar v_F \begin{pmatrix} -k_z & \partial_y + eyB_z/\hbar - k_x \\ -\partial_y + eyB_z/\hbar - k_x & k_z \end{pmatrix} \phi(y) = E\phi(y). \tag{0.17}$$

We will now find the spectrum E of the Hamiltonian.

Inspired by the derivation for the spectrum of the 2D Dirac Hamiltonian in [14], we introduce the length scale $l_B = \sqrt{\hbar/eB}$, and the dimensionless quantity $\chi = y/l_B - k_x l_B$. In dimensionless quantities Eq. (0.17) becomes

$$-\frac{\hbar v_F}{l_B} \begin{pmatrix} -k_z l_B & \partial_\chi + \chi \\ -\partial_\chi + \chi & k_z l_B \end{pmatrix} \phi(y) = E\phi(y). \tag{0.18}$$

Let the operators $a = \left(\chi + \partial_\chi\right)/\sqrt{2},\ a^\dagger = \left(\chi - \partial_\chi\right)/\sqrt{2}$. One may easily verify the commutation relation $[a, a^\dagger] = 1$; they are ladder operators of the harmonic oscillators, whose eigenstates are $|n\rangle$, and where $a\,|n\rangle = \sqrt{n}\,|n-1\rangle\,, a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$. In terms of these operators, the system is

$$-\frac{\sqrt{2}\hbar v_F}{l_B} \begin{pmatrix} -\frac{k_z l_B}{\sqrt{2}} & a \\ a^\dagger & \frac{k_z l_B}{\sqrt{2}} \end{pmatrix} |\phi\rangle = E\,|\phi\rangle\,. \tag{0.19}$$

Take the ansatz

$$|\phi\rangle = \begin{pmatrix} \beta\,|n-1\rangle \\ \alpha\,|n\rangle \end{pmatrix}, \tag{0.20}$$

which is the most general form of $|\phi\rangle$ with any hope of being an eigenstate. This leads to

$$-\frac{\sqrt{2}\hbar v_F}{l_B} \begin{pmatrix} (-\gamma\beta + \alpha\sqrt{n})\,|n-1\rangle \\ (\beta\sqrt{n} + \gamma\alpha)\,|n\rangle \end{pmatrix} = E\,|\phi\rangle\,, \tag{0.21}$$

with $\gamma = k_z l_B/\sqrt{2}$. For $n > 0$ this leads to the equation for ϕ to be an energy eigenfunction

$$-\gamma + \frac{\alpha}{\beta}\sqrt{n} = \frac{\beta}{\alpha}\sqrt{n} + \gamma. \tag{0.22}$$

Solving for α/β this gives

$$\frac{\alpha}{\beta} = \frac{\gamma}{\sqrt{n}} \pm \sqrt{1 + \frac{\gamma^2}{n}}, \tag{0.23}$$

and thus

$$E = \pm v_F \sqrt{\frac{2n\hbar^2}{l_B^2} + k_z^2\hbar^2} = \pm s v_F \sqrt{2neB\hbar + k_z^2\hbar^2}, \tag{0.24}$$

where we reintroduced the explicit s. For $n = 0$ the annihilation operator a destroys the vacuum state $|0\rangle$, and the energy is instead $E_0 = -\hbar s k_z v_F$. The excited energy states are doubly degenerate; we choose to denote the energy levels by $m \in \mathbb{Z}$, where the sign from $\pm s$ is taken care of by the sign of this quantum number, and the harmonic oscillator levels n are given by its absolute value $|m|$. The energy levels are

$$E_{k_z m s} = \operatorname{sign}(m) v_F \sqrt{2|m|eB\hbar + k_z^2\hbar^2} \qquad \text{for } m \neq 0, \tag{0.25}$$

$$E_{k_z 0 s} = -s\hbar k_z v_F \qquad \text{for } m = 0. \tag{0.26}$$

We now find the corresponding eigenvectors of the system. The solution to the one dimensional harmonic oscillator in position space is, in dimensionless coordinates ξ, [7, Eq. 18.39.5]

$$\langle \xi | n \rangle = \phi_n(\xi) = \frac{1}{\sqrt{2^n n!}} \pi^{-\frac{1}{4}} e^{-\frac{\xi^2}{2}} H_n(\xi), \tag{0.27}$$

where H_n are the Hermite polynomials. Thus,

$$\langle \chi | \phi \rangle = \begin{pmatrix} \beta \langle \chi | n-1 \rangle \\ \alpha \langle \chi | n \rangle \end{pmatrix} = e^{-\frac{\chi^2}{2}} \begin{pmatrix} \frac{\beta}{\sqrt{2^{n-1}(n-1)!\sqrt{\pi}}} H_{n-1}(\chi) \\ \frac{\alpha}{\sqrt{2^n n! \sqrt{\pi}}} H_n(\chi) \end{pmatrix} \tag{0.28}$$

Choosing

$$\alpha = \sqrt{\frac{\gamma^2}{n}} \implies \beta = \frac{1}{1 \pm \sqrt{1 + \frac{n}{\gamma^2}}} = \pm\frac{\gamma^2}{n}\left(\sqrt{1 + \frac{n}{\gamma^2}} - 1\right), \tag{0.29}$$

gives

$$\phi(\chi) = e^{-\frac{\chi^2}{2}} \sqrt{\frac{\gamma^2}{n}} \begin{pmatrix} \frac{\pm\sqrt{\frac{\gamma^2}{n}}\left(\sqrt{1+\frac{n}{\gamma^2}}-1\right)}{\sqrt{2^{n-1}(n-1)!\sqrt{\pi}}} H_{n-1}(\chi) \\ \frac{1}{\sqrt{2^n n!\sqrt{\pi}}} H_n(\chi) \end{pmatrix}. \tag{0.30}$$

There are thus four quantum numbers related to the eigenvectors, k_x, k_z, m, s. Reintroducing $\chi = (y - k_x l_B^2)/l_B$ and normalizing

$$\phi_{\boldsymbol{k}ms}(\boldsymbol{r}) = \frac{1}{\sqrt{L_x L_z}} \frac{e^{ik_x x} e^{ik_z z}}{\sqrt{\alpha_{k_z ms}^2 + 1}} e^{-\frac{\left(y - k_x l^2\right)^2}{2l_B^2}} \begin{pmatrix} \frac{\alpha_{k_z ms}}{\sqrt{2^{M-1}(M-1)!\sqrt{\pi} l_B}} H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right) \\ \frac{1}{\sqrt{2^M M! \sqrt{\pi} l_B}} H_M\left(\frac{y-k_x l_B^2}{l_B}\right) \end{pmatrix}, \tag{0.31}$$

where capital letters indicate absolute value of corresponding quantity, $M = |m|, \boldsymbol{k} = (k_x, k_z)$, and with the normalization factor

$$\alpha_{k_z ms} = \frac{-\sqrt{2eB\hbar M}}{\frac{E_{k_z ms}}{sv_F} - \hbar k_z}. \tag{0.32}$$

0.2 Analytical expressions for the operators

We will here find analytical expressions for the current operator $J^i(\omega, \boldsymbol{q})$ and stress-energy tensor $T^{0j}(\omega, \boldsymbol{q})$, needed to calculate the correlation function. The fields are given, in the position basis, by

$$\psi = \sum_{\boldsymbol{k}n} \langle \boldsymbol{r} | \boldsymbol{k}ns \rangle \, a_{\boldsymbol{k}ns}(t) = \sum_{\boldsymbol{k}n} \phi_{\boldsymbol{k}ns}(\boldsymbol{r}) a_{\boldsymbol{k}ns}(t), \tag{0.33}$$

$$\psi^\dagger = \sum_{\boldsymbol{k}n} \langle \boldsymbol{k}ns | \boldsymbol{r} \rangle \, a^\dagger_{\boldsymbol{k}ns}(t) = \sum_{\boldsymbol{k}n} \phi^*_{\boldsymbol{k}ns}(\boldsymbol{r}) a^\dagger_{\boldsymbol{k}ns}(t). \tag{0.34}$$

Here $a^\dagger_\lambda(t) = \exp(iE_\lambda t/\hbar) a^\dagger_\lambda$ and $a^\dagger_\lambda, a_\lambda$ are the creation and annihilation operators of the state with quantum numbers λ. The current operator $\hat{\boldsymbol{J}} = e\hat{\boldsymbol{v}}$, where $\hat{\boldsymbol{v}}$ is the velocity operator. Using the relation of Heisenberg operators $\dot{A} = [A, H]/i\hbar$ [9], for the operator A and Hamiltonian H, the operator

$$\begin{aligned} \boldsymbol{v} = \dot{\boldsymbol{r}} &= \frac{1}{i\hbar} [\boldsymbol{r}, H] && (0.35) \\ &= \frac{sv_F \sigma^i}{i\hbar} [\boldsymbol{r}, p_i + eA_i] && (0.36) \\ &= \frac{sv_F \sigma^i}{i\hbar} (i\hbar + e[\boldsymbol{r}, A_i]) && (0.37) \\ &= sv_F \sigma^i, && (0.38) \end{aligned}$$

and thus

$$J^x = \psi^\dagger \hat{J}^x \psi = sv_F e \sum_{\boldsymbol{k}m, \boldsymbol{l}n} \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) \sigma^x \phi_{\boldsymbol{l}ns}(\boldsymbol{r}) a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{l}ns}(t). \tag{0.39}$$

Similarly, the T^{0y} component of the stress-energy tensor of the theory is given by [1]

$$\begin{aligned} T^{0y}(t, \boldsymbol{r}) = \sum_{\boldsymbol{k}m, \boldsymbol{l}n} \frac{1}{4} \Big\{ & \left[v_F \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) p_y \phi_{\boldsymbol{l}ns}(\boldsymbol{r}) - v_F \left(p_y \phi^*_{\boldsymbol{k}ms} \right) \phi_{\boldsymbol{l}ns} \right] a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{l}ns}(t) \\ & + \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) s\sigma^y \phi_{\boldsymbol{l}ns}(\boldsymbol{r}) \left[a^\dagger_{\boldsymbol{k}ms}(t) i\hbar \partial_0 a_{\boldsymbol{l}ns}(t) - i\hbar \left(\partial_0 a^\dagger_{\boldsymbol{k}ms}(t) \right) a_{\boldsymbol{l}ns}(t) \right] \\ & + \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) s\sigma^y (2\mu) \phi_{\boldsymbol{l}ns}(\boldsymbol{r}) a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{l}ns}(t) \Big\}. \end{aligned} \tag{0.40}$$

Here, also a non-zero potential μ is included. Our final result will be given at zero potential, however it is included in the calculations as it might be of interest to consider finite potential in later work. Recalling the time dependence of $a(t), a^\dagger(t)$ we have that

$$i\hbar\partial_0 a_\lambda(t) = E_\lambda a_\lambda, \quad i\hbar\partial_0 a_\lambda^\dagger(t) = -E_\lambda a_\lambda^\dagger,$$

which further simplifies the expression.

If time, derive T

Fourier transforming the position gives

$$J^x(t, \boldsymbol{q}) = \sum_{\boldsymbol{k}m,\boldsymbol{l}n} J^x_{\boldsymbol{k}ms,\boldsymbol{l}ns}(\boldsymbol{q}) a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{l}ns}(t), \tag{0.41}$$

$$T^{0y}(t, -\boldsymbol{q}) = \sum_{\boldsymbol{k}m,\boldsymbol{l}n} T^{0y}_{\boldsymbol{k}ms,\boldsymbol{l}ns}(\boldsymbol{q}) a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{l}ns}(t), \tag{0.42}$$

where the matrix elements in momentum space are given by

$$J^x_{\boldsymbol{k}ms,\boldsymbol{l}ns}(\boldsymbol{q}) = \int \mathrm{d}\boldsymbol{r} e^{-i\boldsymbol{q}\boldsymbol{r}} s v_F e \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) \sigma^x \phi_{\boldsymbol{l}ns}(\boldsymbol{r}), \tag{0.43}$$

$$\begin{aligned} T^{0y}_{\boldsymbol{k}ms,\boldsymbol{l}ns}(\boldsymbol{q}) &= \frac{1}{4}\int \mathrm{d}\boldsymbol{r} e^{i\boldsymbol{q}\boldsymbol{r}} \left[v_F \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) p_y \phi_{\boldsymbol{l}ns}(\boldsymbol{r}) - v_F (p_y \phi^*_{\boldsymbol{k}ms}) \phi_{\boldsymbol{l}ns}(\boldsymbol{r}) \right] \\ &+ \frac{1}{4}\int \mathrm{d}\boldsymbol{r} e^{i\boldsymbol{q}\boldsymbol{r}} \phi^*_{\boldsymbol{k}ms}(\boldsymbol{r}) s \sigma^y (E_{\boldsymbol{k}_z ms} + E_{\boldsymbol{l}_z ns} - 2\mu) \phi_{\boldsymbol{l}ns}(\boldsymbol{r}). \end{aligned} \tag{0.44}$$

Note that as $T^{0y}(t, -\boldsymbol{q})$ will be used later, we here for convenience included the sign into the definition of the matrix element $T^{0y}_{\boldsymbol{k}ms,\boldsymbol{l}ns}$, as is reflected in the sign of the exponent of Eq. (0.44).

As was noted earlier, the eigenvectors are plane waves in the x, z-directions, and the non-trivial part is the y-dependent $\phi(y)$. Thus, we want to express these matrix elements in terms of $\phi(y)$. The sum over $\boldsymbol{l}$ in Eq. (0.41) can be replaced by an integration, as it is a good quantum number. As usual, the measure in the integration is given by the density of states in momentum space, the well known $L_i/2\pi$, with L_i being the length of the system in the i-direction.

$$\begin{aligned} J^x(t, \boldsymbol{q}) &= \sum_{\boldsymbol{k}m,n} \int \mathrm{d}l_x \mathrm{d}l_z \frac{L_x L_z}{4\pi^2} J^x_{\boldsymbol{k}ms,\boldsymbol{l}ns}(\boldsymbol{q}) a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{l}ns}(t) \\ &= \int \mathrm{d}l_x \mathrm{d}l_z \int \mathrm{d}y e^{-iq_y y} \delta(l_x - k_x - q_x) \delta(l_z - k_z - q_z) s v_F e \phi^*_{\boldsymbol{k}ms}(y) \sigma^x \phi_{\boldsymbol{l}ns}(y). \end{aligned} \tag{0.45}$$

The Dirac delta functions appeared from taking the integrals from the matrix element over x and z, as the integrand in these variables was only plane waves. The exact same procedure may be done for the stress-energy tensor in Eq. (0.42). Eliminating l by doing the integrals yields

$$J^x(t, \boldsymbol{q}) = \sum_{\boldsymbol{k},mn} J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathsf{q}ns}(\boldsymbol{q}) a^\dagger_{\boldsymbol{k}ms}(t) a_{\boldsymbol{k}+\mathsf{q}ns}(t), \tag{0.46}$$

$$T^{0y}(t, -\boldsymbol{q}) = \sum_{\boldsymbol{\kappa},\mu\nu} T^{0y}_{\boldsymbol{\kappa}\mu s,\boldsymbol{\kappa}-\mathsf{q},\nu s}(\boldsymbol{q}) a^\dagger_{\boldsymbol{\kappa}\mu s}(t) a_{\boldsymbol{\kappa}-\mathsf{q}\nu s}(t), \tag{0.47}$$

where $\mathfrak{q} = (q_x, q_z)$. Keeping in mind that $a^\dagger_\lambda(t) = e^{iE_\lambda t/\hbar} a^\dagger_\lambda$, and that

$$\left\langle \left[a^\dagger_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathfrak{q}ns}, a^\dagger_{\boldsymbol{\kappa}\mu s} a_{\boldsymbol{\kappa}-\mathfrak{q}\nu s} \right] \right\rangle = \delta_{\boldsymbol{k},\boldsymbol{\kappa}-\mathfrak{q}} \delta_{m,\nu} \delta_{\boldsymbol{k}+\mathfrak{q},\boldsymbol{\kappa}} \delta_{n,\mu} \left[n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathfrak{q}ns} \right], \tag{0.48}$$

the correlation function is given by

$$\begin{aligned} \left\langle \left[J^x(t,\boldsymbol{q}), T^{0y}(t',-\boldsymbol{q}) \right] \right\rangle = \sum_{\boldsymbol{k}mn} e^{\frac{i}{\hbar}(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathfrak{q}ns})t} e^{\frac{i}{\hbar}(E_{\boldsymbol{k}+\mathfrak{q}ns} - E_{\boldsymbol{k}ms})t'} \\ \times J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) T^{0y}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) \left[n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathfrak{q}ns} \right]. \end{aligned} \tag{0.49}$$

We are now ready to find the correlation function χ^{xy} given in Eq. (0.14)

$$\chi^{xy}(\omega,\boldsymbol{q}) = \frac{-iv_F}{\mathcal{V}\hbar} \int \mathrm{d}t e^{i\omega t} \int_{-\infty}^{0} \mathrm{d}t' \Theta(t) \left\langle \left[J^x(t,\boldsymbol{q}), T^{0y}(t',-\boldsymbol{q}) \right] \right\rangle. \tag{0.50}$$

Introduce as usual a decay factor $e^{-\eta(t-t')}$ to ensure convergence in the time integrals, and make a change of variables $t' \to -t'$. The integral part of Eq. (0.50), ignoring everything without time dependence for clarity, is then

$$\begin{aligned} \lim_{\eta\to 0} \int_0^\infty \mathrm{d}t \mathrm{d}t' \exp\left[\frac{i}{\hbar} \left(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathfrak{q}ns} + \omega\hbar + i\eta\hbar \right) t \right] \exp\left[\frac{i}{\hbar} \left(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathfrak{q}ns} + i\eta\hbar \right) t' \right] \\ = \lim_{\eta\to 0} \frac{\hbar}{i} \left[E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathfrak{q}ns} + \omega\hbar + i\eta\hbar \right]^{-1} \frac{\hbar}{i} \left[E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathfrak{q}ns} + i\eta\hbar \right]^{-1}. \end{aligned} \tag{0.51}$$

The response function then reads

$$\begin{aligned} \chi^{xy}(\omega,\boldsymbol{q}) = \frac{iv_F\hbar}{\mathcal{V}} \lim_{\eta\to 0} \sum_{\boldsymbol{k}mn} J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) T^{0y}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) \left[n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathfrak{q}ns} \right] \\ \left[E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}-\mathfrak{q}ns} + \omega\hbar + i\eta\hbar \right]^{-1} \left[E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathfrak{q}ns} + i\eta\hbar \right]^{-1}, \end{aligned} \tag{0.52}$$

where the matrix elements are

$$J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) = \int \mathrm{d}y e^{-iq_y y} s v_F e \phi^*_{\boldsymbol{k}ms}(y) \sigma^x \phi_{\boldsymbol{k}+\mathfrak{q}ns}(y), \tag{0.53}$$

$$\begin{aligned} T^{0y}_{\boldsymbol{k}ms,\boldsymbol{k}-\mathfrak{q}ns}(\boldsymbol{q}) = \frac{1}{4} \int \mathrm{d}y e^{iq_y y} \left[v_F \phi^*_{\boldsymbol{k}ms}(y) p_y \phi_{\boldsymbol{k}-\mathfrak{q}ns}(y) - v_F p_y \phi^*_{\boldsymbol{k}ms}(y) \phi_{\boldsymbol{k}-\mathfrak{q}ns}(y) \right] \\ + \frac{1}{4} \int \mathrm{d}y e^{iq_y y} \phi^*_{\boldsymbol{k}ms}(y) s \sigma^y \left(E_{\boldsymbol{k}ms} + E_{\boldsymbol{k}-\mathfrak{q}ns} - 2\mu \right) \phi_{\boldsymbol{k}-\mathfrak{q}ns}(y). \end{aligned} \tag{0.54}$$

We will consider the response function in the static limit $\lim_{\omega\to 0} \lim_{\boldsymbol{q}\to 0}$. We may use the property of the limit of a product of functions $\lim A \cdot B = \lim A \cdot \lim B$ to write

$$\lim_{\omega\to 0} \lim_{\boldsymbol{q}\to 0} \chi^{xy}(\omega,\boldsymbol{q}) = \frac{iv_F\hbar}{\mathcal{V}} \sum_{\boldsymbol{k}mn} \frac{J^x_{\boldsymbol{k}ms,\boldsymbol{k}ns} T^{0y}_{\boldsymbol{k}ns,\boldsymbol{k}ms} [n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}ns}]}{(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}ns})(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}ns})}, \tag{0.55}$$

where the current and energy-momentum tensor matrix elements are the expression given in Eqs. (0.53) and (0.54) taken in the limit.

0.2.1 Finding numerical values for the matrix elements

Compared to the procedure used by arjonaFingerprintsConformalAnomaly2019[1], taking the limit of each matrix element by itself greatly simplifies the calculation.

Let

$$\phi_{\boldsymbol{k}ms}(y) = e^{-\frac{(y-k_x l_B^2)^2}{2l_B^2}} \begin{pmatrix} a_{\boldsymbol{k}ms} H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right) \\ b_{\boldsymbol{k}ms} H_M\left(\frac{y-k_x l_B^2}{l_B}\right) \end{pmatrix}, \tag{0.56}$$

thus implicitly defining the prefactors $a_{\boldsymbol{k}ms}, b_{\boldsymbol{k}ms}$.

The current operator

The matrix element

$$\begin{aligned}
&J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) && (0.57)\\
&= \int \mathrm{d}y e^{-iq_y y} s v_F e \phi^*_{\boldsymbol{k}ms}(y)\sigma^x \phi_{\boldsymbol{k}+\mathfrak{q}ns}(y) \\
&= s v_F e \int \mathrm{d}y \exp\left\{-iq_y y - \frac{(y-k_x l_B^2)^2 + (y-(k_x+q_x)l_B^2)^2}{2l_B^2}\right\} && (0.58)\\
&\quad\left[a_{\boldsymbol{k}ms} b_{\boldsymbol{k}+\mathfrak{q}ns} H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right) H_N\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right.\\
&\left.+ b_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathfrak{q}ns} H_M\left(\frac{y-k_x l_B^2}{l_B}\right) H_{N-1}\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right]\\
&= s v_F e \int \mathrm{d}y \exp\left[-\left\{y + \frac{l_B^2}{2}(iq_y - 2k_x - q_x)\right\}^2 / l_B^2\right] && (0.59)\\
&\quad \exp\left[-\frac{1}{4} l_B^2 \left\{\mathfrak{q}_y^2 + 2i(2k_x+q_x)q_y\right\}\right]\\
&\quad\left[a_{\boldsymbol{k}ms} b_{\boldsymbol{k}+\mathfrak{q}ns} H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right) H_N\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right.\\
&\left.+ b_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathfrak{q}ns} H_M\left(\frac{y-k_x l_B^2}{l_B}\right) H_{N-1}\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right],
\end{aligned}$$

where we completed the square in the exponent, to get the form $e^{-a(y+b)^2}$. Also, $\mathfrak{q}_y = (q_x, q_y)$, was introduced, not to be confused with $\mathfrak{q} = (q_x, q_z)$. By introducing $\tilde{y} = \frac{y}{l_B} + l_B(iq_y - q_x - 2k_x)/2$ the matrix element may be rewritten

$$\begin{aligned}
J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) &= s v_F e \int \mathrm{d}\tilde{y}\, l_B \exp\left[-\frac{1}{4} l_B^2 \left\{\mathfrak{q}_y^2 + 2i(2k_x+q_x)q_y\right\}\right]\\
&\quad e^{-\tilde{y}^2}\left[a_{\boldsymbol{k}ms} b_{\boldsymbol{k}+\mathfrak{q}ns} H_{M-1}\left(\tilde{y} + \frac{l_B}{2}(q_x - iq_y)\right) H_N\left(\tilde{y} + \frac{l_B}{2}(-q_x - iq_y)\right)\right.\\
&\quad\left.+ b_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathfrak{q}ns} H_M\left(\tilde{y} + \frac{l_B}{2}(q_x - iq_y)\right) H_{N-1}\left(\tilde{y} + \frac{l_B}{2}(-q_x - iq_y)\right)\right].
\end{aligned} \tag{0.60}$$

Taking the limit we find the simple form

$$J_{\boldsymbol{k}ms;\boldsymbol{k}ns} = sv_F e \int \mathrm{d}\tilde{y} e^{-\tilde{y}} \left[a_{\boldsymbol{k}ms} b_{\boldsymbol{k}ns} H_{M-1}(\tilde{y}) H_N(\tilde{y}) + b_{\boldsymbol{k}ms} a_{\boldsymbol{k}ns} H_M(\tilde{y}) H_{N-1}(\tilde{y}) \right]. \tag{0.61}$$

We employ now the orthogonality relation of the Hermite polynomials [7, Table 18.3.1]

$$\int\limits_{-\infty}^{\infty} \mathrm{d}x e^{-x^2} H_n(x) H_m(x) = \sqrt{\pi} 2^n n! \delta_{n,m} \tag{0.62}$$

to write

$$J_{\boldsymbol{k}ms,\boldsymbol{k}ns} = sv_F e \sqrt{\pi} (a_{\boldsymbol{k}ms} b_{\boldsymbol{k}ns} \delta_{M-1,N} 2^N N! + a_{\boldsymbol{k}ms} b_{\boldsymbol{k}ns} \delta_{M,N-1} 2^M M!). \tag{0.63}$$

With

$$a_{\boldsymbol{k}ms} b_{\boldsymbol{k}ns} = \frac{\alpha_{\boldsymbol{k}ms}}{\sqrt{\alpha_{\boldsymbol{k}ms}^2 + 1} \sqrt{\alpha_{\boldsymbol{k}ns}^2 + 1}} \left[2^{N+M-1} (M-1)! N! \pi l_B^2 \right]^{-\frac{1}{2}}, \tag{0.64}$$

$$b_{\boldsymbol{k}ms} a_{\boldsymbol{k}ns} = \frac{\alpha_{\boldsymbol{k}ns}}{\sqrt{\alpha_{\boldsymbol{k}ms}^2 + 1} \sqrt{\alpha_{\boldsymbol{k}ns}^2 + 1}} \left[2^{N+M-1} (N-1)! M! \pi l_B^2 \right]^{-\frac{1}{2}}. \tag{0.65}$$

we find explicitly

$$\begin{aligned} J_{\boldsymbol{k}ms,\boldsymbol{k}ns} = &\frac{sv_F e}{\sqrt{\alpha_{\boldsymbol{k}ms}^2 + 1} \sqrt{\alpha_{\boldsymbol{k}ns}^2 + 1}} \\ &\left(\alpha_{\boldsymbol{k}ms} \sqrt{\frac{2^N N!}{2^{M-1}(M-1)!}} \delta_{M-1,N} + \alpha_{\boldsymbol{k}ns} \sqrt{\frac{2^M M!}{2^{N-1}(N-1)!}} \delta_{M,N-1} \right). \end{aligned} \tag{0.66}$$

The stress-energy tensor operator

Consider the first part of the stress-energy matrix element

$$T^{0y\,(1)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{1}{4} \int \mathrm{d}y e^{iq_y y} \phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y) s\sigma^y (E_{k\mu s} + E_{\lambda\nu s} - 2\mu) \phi_{\boldsymbol{k}ms}(y). \tag{0.67}$$

Recall that

$$\phi_{\boldsymbol{k}ms}(y) = e^{-\frac{(y-k_x l_B^2)^2}{2l_B^2}} \begin{pmatrix} a_{\boldsymbol{k}ms} H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right) \\ b_{\boldsymbol{k}ms} H_M\left(\frac{y-k_x l_B^2}{l_B}\right) \end{pmatrix}. \tag{0.68}$$

The form of the integrand is very similar to the current matrix case, with the exchange of the Pauli matrix $\sigma^x \to \sigma^y$, thus giving an additional i and a negative sign to the first term.

$$\begin{aligned} &T^{0y\,(1)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) \\ &= \frac{is}{4} (E_{k\mu s} + E_{\lambda\nu s} - 2\mu) \int \mathrm{d}y e^{iq_y y} e^{-\frac{(y-k_x l_B^2)^2 + (y-(k_x+q_x) l_B^2)^2}{2l_B^2}} \\ &\quad \left[-a_{\boldsymbol{k}+\mathsf{q}ns} b_{\boldsymbol{k}ms} H_{N-1}(\dots) H_M(\dots) + b_{\boldsymbol{k}+\mathsf{q}ns} a_{\boldsymbol{k}ms} H_N(\dots) H_{M-1}(\dots) \right]. \end{aligned} \tag{0.69}$$

Taking care to note that the factor from the Fourier transform, that was $e^{-iq_y y}$ in the current matrix element is here $e^{+iq_y y}$, a similar completion of the square is done

$$
\begin{aligned}
&T^{0y\,(1)}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) \\
&= \frac{is}{4}(E_{k\mu s} + E_{\lambda\nu s} - 2\mu)\exp\left[-\frac{l_B^2}{4}\left\{\mathfrak{q}_y^2 - 2iq_y(2k_x + q_x)\right\}\right] \\
&\int \mathrm{d}y \exp\left[-\left\{y + \frac{l_B^2}{2}(-iq_y - 2k_x - q_x)\right\}^2 / l_B^2\right] \\
&[-a_{\boldsymbol{k}+\mathfrak{q}ns} b_{\boldsymbol{k}ms} H_{N-1}(\dots)H_M(\dots) + b_{\boldsymbol{k}+\mathfrak{q}ns} a_{\boldsymbol{k}ms} H_N(\dots)H_{M-1}(\dots)]\,.
\end{aligned}
\tag{0.70}
$$

The arguments of the Hermite polynomials have been dropped for brevity of notation. As before make a change of variables to get the integral on the form of the shifted orthogonality relation for the Hermite polynomials Eq. (**??**). Upon introducing $\tilde{y} = \frac{y}{l_B} + l_B(-iq_y - q_x - 2k_x)/2$ the shifted orthogonality relation is used on the expression

$$
\begin{aligned}
T^{0y\,(1)}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{is}{4}(E_{k\mu s} + E_{\lambda\nu s} - 2\mu)\exp\left[-\frac{l_B^2}{4}\left\{\mathfrak{q}_y^2 - 2iq_y(2k_x + q_x)\right\}\right] \int \mathrm{d}\tilde{y}\, l_B e^{-\tilde{y}^2} \\
\left[-a_{\boldsymbol{k}+\mathfrak{q}ns} b_{\boldsymbol{k}ms} H_{N-1}\left(\tilde{y} + \frac{l_B}{2}(iq_y - q_x)\right) H_M\left(\tilde{y} + \frac{l_B}{2}(iq_y + q_x)\right)\right. \\
\left. + b_{\boldsymbol{k}+\mathfrak{q}ns} a_{\boldsymbol{k}ms} H_N\left(\tilde{y} + \frac{l_B}{2}(iq_y - q_x)\right) H_{M-1}\left(\tilde{y} + \frac{l_B}{2}(iq_y + q_x)\right)\right].
\end{aligned}
\tag{0.71}
$$

The terms in the integrand are exactly the same as in the current matrix element case, just in the reverse order and with $q_y \to -q_y$. By Eqs. (**??**) and (**??**)

$$
\begin{aligned}
T^{0y\,(1)}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{is}{4}\frac{(E_{k\mu s} + E_{\lambda\nu s} - 2\mu)}{\sqrt{\alpha_{\boldsymbol{k}ms}^2 + 1}\sqrt{\alpha_{\boldsymbol{k}ns}^2 + 1}} \\
\left(\alpha_{\boldsymbol{k}ms}\sqrt{\frac{2^N N!}{2^{M-1}(M-1)!}}\delta_{M-1,N} - \alpha_{\boldsymbol{k}ns}\sqrt{\frac{2^M M!}{2^{N-1}(N-1)!}}\delta_{M,N-1}\right).
\end{aligned}
\tag{0.72}
$$

where $\bar{\boldsymbol{q}} = (q_x, -q_y, q_z)$.

Consider now the latter part of the stress-energy tensor, which is split into two parts

$$
T^{0y\,(2)}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = +\frac{1}{4}\int \mathrm{d}y e^{iq_y y} v_F \phi^*_{\boldsymbol{k}+\mathfrak{q}ns}(y) p_y \phi_{\boldsymbol{k}ms}(y), \tag{0.73}
$$

$$
T^{0y\,(3)}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = -\frac{1}{4}\int \mathrm{d}y e^{iq_y y} v_F \left(p_y \phi^*_{\boldsymbol{k}+\mathfrak{q}ns}(y)\right) \phi_{\boldsymbol{k}ms}(y). \tag{0.74}
$$

Recall that $\phi_{\boldsymbol{k}ms}(y)$, defined in Eq. (0.56), consists of two y-dependent factors: $\exp\left[-\frac{(y-k_x l_B^2)^2}{2l_B^2}\right]$ and the Hermite polynomials. The operator p_y thus produces two terms when operating on ϕ. The first term, coming from the exponent, is proportional to $y - k_x l_B^2$. The operator in Eqs. (0.73) and (0.74) acts on ϕ with the quantum number $\boldsymbol{k}$ and $\boldsymbol{k}+\mathfrak{q}$, respectively; when summing the two contributions, everything thus cancels except for a term proportional to q_x, which vanishes in the local limit.

It remains to consider the result of p_y operating on the Hermite polynomials. Let $\tilde{p}_y$ indicate the p_y operator acting only on the Hermite polynomial part of ϕ, and use the property of Hermite polynomials $\partial_x H_n(x) = 2nH_{n-1}(x)$ [7, Eq. 18.9.25].

$$\phi^*_{\boldsymbol{k}+\mathfrak{q}ns}(y)\tilde{p}_y\phi_{\boldsymbol{k}ms} = -i\hbar \exp\left\{-\frac{(y-k_x l_B^2)^2 + (y-(k_x+q_x)l_B^2)^2}{2l_B^2}\right\}$$
$$\frac{2}{l_B}\left\{(M-1)a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathfrak{q}ns}H_{M-2}\left(\frac{y-k_x l_B^2}{l_B}\right)H_{N-1}\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right.$$
$$\left. + Mb_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathfrak{q}ns}H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right)H_N\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right\}, \quad (0.75)$$

With the now well-known completion of the square

$$\int \mathrm{d}y e^{iq_y y}\phi^*_{\boldsymbol{k}+\mathfrak{q}ns}(y)\tilde{p}_y\phi_{\boldsymbol{k}ms}(y) = -i\hbar \exp\left[-\frac{l_B^2}{4}\left\{\mathfrak{q}_y^2 - 2iq_y(2k_x+q_x)\right\}\right]$$
$$\int \mathrm{d}y \exp\left[-\left\{y+\frac{l_B^2}{2}(-iq_y-2k_x-q_x)\right\}^2/l_B^2\right]$$
$$\frac{2}{l_B}\left\{(M-1)a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathfrak{q}ns}H_{M-2}\left(\frac{y-k_x l_B^2}{l_B}\right)H_{N-1}\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right.$$
$$\left. + Mb_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathfrak{q}ns}H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right)H_N\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right\}. \quad (0.76)$$

Upon introducing $\tilde{y} = \frac{y}{l_B} + l_B(-iq_y - q_x - 2k_x)/2$, as before, the expression reduces to

$$\int \mathrm{d}y e^{iq_y y}\phi^*_{\boldsymbol{k}+\mathfrak{q}ns}(y)\tilde{p}_y\phi_{\boldsymbol{k}ms}(y) = -i\hbar \exp\left[-\frac{l_B^2}{4}\left\{q_x^2 + q_y^2 - 2iq_y(2k_x+q_x)\right\}\right]$$
$$\int \mathrm{d}\tilde{y} l_B \exp\left[-\tilde{y}^2\right]$$
$$\frac{2}{l_B}\left\{(M-1)a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathfrak{q}ns}H_{M-2}\left(\tilde{y}+\frac{l_B}{2}(iq_y+q_x)\right)H_{N-1}\left(\tilde{y}+\frac{l_B}{2}(iq_y-q_x)\right)\right.$$
$$\left. + Mb_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathfrak{q}ns}H_{M-1}\left(\tilde{y}+\frac{l_B}{2}(iq_y+q_x)\right)H_N\left(\tilde{y}+\frac{l_B}{2}(iq_y-q_x)\right)\right\}, \quad (0.77)$$

where we may use equation (**??**) to evaluate the integral.

Comparing with Eqs. (**??**) and (**??**), it is apparent that

$$
\begin{aligned}
&\int \mathrm{d}\tilde{y}e^{-\tilde{y}^2} a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathsf{q}ns}H_{M-2}(\dots)H_{N-1}(\dots) \\
&\qquad = \int \mathrm{d}\tilde{y}e^{-\tilde{y}^2}\frac{a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathsf{q}ns}}{a_{\boldsymbol{k}m\mp 1s}b_{\boldsymbol{k}+\mathsf{q}n\mp 1s}} a_{\boldsymbol{k}m\mp 1s}b_{\boldsymbol{k}+\mathsf{q}n\mp 1s}H_{M-2}(\dots)H_{N-1}(\dots) \\
&\qquad = \frac{a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathsf{q}ns}}{l_B a_{\boldsymbol{k}m\mp 1s}b_{\boldsymbol{k}+\mathsf{q}n\mp 1s}} \frac{\Xi_1(\bar{\boldsymbol{q}}, m\mp 1, n\mp 1, s)}{\sqrt{\alpha^2_{\boldsymbol{k}m\mp 1s}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}n\mp 1s}+1}} \\
&\qquad = \frac{\alpha_{\boldsymbol{k},m,s}\alpha_{\boldsymbol{k}+\mathsf{q},n,s}}{l_B\sqrt{2(M-1)}\alpha_{\boldsymbol{k},m\mp 1,s}} \frac{\Xi_1(\bar{\boldsymbol{q}}, m\mp 1, n\mp 1, s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1}},
\end{aligned}
\tag{0.78}
$$

where $m \mp 1$ and $n \mp 1$ should be read with the upper sign for $m, n > 0$, and the lower sign otherwise. For the second term

$$
\begin{aligned}
&\int \mathrm{d}\tilde{y}e^{-\tilde{y}^2} b_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathsf{q}ns}H_{M-1}(\dots)H_{N}(\dots) \\
&\qquad = \frac{b_{\boldsymbol{k}ms}}{a_{\boldsymbol{k}ms}} a_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathsf{q}ns}H_{M-1}(\dots)H_{N}(\dots) \\
&\qquad = \frac{b_{\boldsymbol{k}ms}}{l_B a_{\boldsymbol{k}ms}} \frac{\Xi_1(\bar{\boldsymbol{q}}, m, n, s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1}} \\
&\qquad = \frac{1}{\alpha_{\boldsymbol{k}ms}l_B\sqrt{2M}} \frac{\Xi_1(\bar{\boldsymbol{q}}, m, n, s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1}}.
\end{aligned}
\tag{0.79}
$$

Thus,

$$
\begin{aligned}
\int \mathrm{d}y e^{iq_y y}\phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y)\tilde{p}_y\phi_{\boldsymbol{k}ms}(y) &= -i\hbar \exp\left[-\frac{l_B^2}{4}\left\{q_x^2+q_y^2-2iq_y(2k_x+q_x)\right\}\right] \\
&\left\{\sqrt{2(M-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathsf{q}ns}}{l_B\alpha_{\boldsymbol{k}m\mp 1s}} \frac{\Xi_1(\bar{\boldsymbol{q}}, m\mp 1, n\mp 1, s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1}}\right. \\
&\qquad \left. +\frac{\sqrt{2M}}{l_B\alpha_{\boldsymbol{k}ms}} \frac{\Xi_1(\bar{\boldsymbol{q}}, m, n, s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1}}\right\}.
\end{aligned}
\tag{0.80}
$$

Similarly, for $T^{0y\,(3)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q})$, one has

$$
\begin{aligned}
\left(\tilde{p}_y\phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y)\right)\phi_{\boldsymbol{k}ms}(y) &= -i\hbar\exp\left\{-\frac{(y-k_x l_B^2)^2+(y-(k_x+q_x)l_B^2)^2}{2l_B^2}\right\} \\
&\frac{2}{l_B}\left\{(N-1)a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathsf{q}ns}H_{M-1}\left(\frac{y-k_x l_B^2}{l_B}\right)H_{N-2}\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right. \\
&\qquad \left. +Nb_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathsf{q}ns}H_{M}\left(\frac{y-k_x l_B^2}{l_B}\right)H_{N-1}\left(\frac{y-(k_x+q_x)l_B^2}{l_B}\right)\right\}
\end{aligned}
\tag{0.81}
$$

which with the same procedure as above gives

$$\int \mathrm{d}y e^{iq_y y}\left(\tilde{p}_y \phi^*_{\boldsymbol{k}+\mathrm{q}ns}(y)\right)\phi_{\boldsymbol{k}ms}(y) = -i\hbar \exp\left[-\frac{l_B^2}{4}\left\{q_x^2+q_y^2-2iq_y(2k_x+q_x)\right\}\right]$$
$$\left\{\sqrt{2(N-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{l_B\alpha_{\boldsymbol{k}+\mathrm{q}n\mp 1s}}\frac{\Xi_2(\bar{\boldsymbol{q}},m\mp 1,n\mp 1,s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1}}\right.$$
$$\left.+\frac{\sqrt{2N}}{l_B\alpha_{\boldsymbol{k}+\mathrm{q}ns}}\frac{\Xi_2(\bar{\boldsymbol{q}},m,n,s)}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1}}\right\}. \tag{0.82}$$

In summary then, we have

$$J_{\boldsymbol{k}ms;\boldsymbol{k}ns} = \Gamma_{\boldsymbol{k}\mathrm{q}mns} s v_F e\left(\alpha_{\boldsymbol{k}ms}\sqrt{\frac{2^N N!}{2^{M-1}(M-1)!}}\delta_{M-1,N} + m\leftrightarrow n\right), \tag{0.83}$$

$$T^{0y\,(1)}_{\boldsymbol{k}ns,\boldsymbol{k}ms} = \frac{is\Gamma_{\boldsymbol{k}\mathrm{q}mns}}{4}\left(E_{\boldsymbol{k}ms}+E_{\boldsymbol{k}ns}-2\mu\right) \tag{0.84}$$
$$\left(\alpha_{\boldsymbol{k}ms}\sqrt{\frac{2^N N!}{2^{M-1}(M-1)!}}\delta_{M-1,N} - m\leftrightarrow n\right)$$

$$T^{0y\,(2)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = -\frac{i\hbar v_F\Gamma^+_{\boldsymbol{k}\mathrm{q}mns}}{4l_B}\left\{\frac{\sqrt{2M}}{\alpha_{\boldsymbol{k}ms}}\Xi_1(\bar{\boldsymbol{q}},m,n,s)\right. \tag{0.85}$$
$$\left.+\sqrt{2(M-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\alpha_{\boldsymbol{k}m\mp 1s}}\Xi_1(\bar{\boldsymbol{q}},m\mp 1,n\mp 1,s)\right\}$$

$$T^{0y\,(3)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{i\hbar v_F\Gamma^+_{\boldsymbol{k}\mathrm{q}mns}}{4l_B}\left\{\frac{\sqrt{2N}}{\alpha_{\boldsymbol{k}+\boldsymbol{q}ns}}\Xi_2(\bar{\boldsymbol{q}},m,n,s)\right. \tag{0.86}$$
$$\left.+\sqrt{2(N-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\alpha_{\boldsymbol{k}+\mathrm{q}n\mp 1s}}\Xi_2(\bar{\boldsymbol{q}},m\mp 1,n\mp 1,s)\right\}$$

where $m\leftrightarrow n$ represent the preceding term under the interchange of m,n and where we have defined $\Gamma_{\boldsymbol{k},\mathrm{q},m,n,s} = \left[(\alpha^2_{\boldsymbol{k}ms}+1)(\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1)\right]^{-\frac{1}{2}}$.

0.2.2 Comment on the energy-momentum tensor

The symmetric form of the energy-momentum tensor, used by arjonaFingerprintsConformalAnomaly2019, gives additional contributions to the energy-momentum matrix element. We will here show that in the case of no tilt, these contributions are identical to those of the non-symmetric tensor. In the tilted case, however, the contributions differ.

The first other contribution is **take care of prefactors**

$$\left(\frac{\sqrt{M}}{\alpha_{\boldsymbol{k}ms}}+\sqrt{(M-1)}\alpha_{\boldsymbol{k}ns}\right)\alpha_{\boldsymbol{k}ms}\delta_{M-1,N}. \tag{0.87}$$

The normalization factor, given in dimensionless quantities is,

$$\alpha_{k_z ms} = -\frac{s\sqrt{M}}{\epsilon_m - s\kappa}.$$

Inserting this, and using the explicit form of the energy for $m \neq 0$

$$\epsilon_n = \mathrm{sign}(m)\sqrt{M + \kappa^2},$$

for the case $N > 0$ the contribution can be shown to be

$$-s(\epsilon_m + \epsilon_n)\alpha_{\boldsymbol{k}ms}\delta_{M-1,N}. \tag{0.88}$$

For $n = 0$, the second term of Eq. (0.87) is zero, and we have

$$-(\epsilon_m - s\kappa)\alpha_{kms}\delta_{M-1,N}, \tag{0.89}$$

and by identifying $\epsilon_0 = -s\kappa$ this has the same form as Eq. (0.88).

0.2.3 Numerical value of response function

Final form of response function

It is now finally possible to write out the entire response function. The response function will be split into three parts, $\chi^{xy\,(i)}$, $i = 1, 2, 3$ corresponding to the three parts of the stress-energy tensor. Also, the sum over the $\boldsymbol{k}$ values will be replaced by an integral. Firstly, we will show that the sum over k_x is restricted; recall that the eigenfunctions are exponentially centered around $y_0 = k_x l_B^2$, which for a finite sample we expect to be restricted to $0 \leq y_0 \leq L_y$. This restricts the k_x sum to $0 \leq k_x \leq L_y/l_B^2 = L_y eB/\hbar$, resulting in the k_x summation giving a finite degeneracy contribution [12, Ch. 1.4.1, 5].

$$\sum_{\boldsymbol{k}} = \sum_{k_x=0}^{L_y eB/\hbar} \sum_{k_z} \to \frac{L_x L_z}{(2\pi)^2} \int_0^{L_y eB/\hbar} \mathrm{d}k_x \int \mathrm{d}k_z \tag{0.90}$$

$$= \frac{\mathcal{V}eB}{(2\pi)^2\hbar} \int \mathrm{d}k_z. \tag{0.91}$$

Recall the response function

$$\chi^{xy}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \sum_{\boldsymbol{k},mn} \frac{1}{\mathcal{V}} \frac{iv_F\hbar J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) T^{0y}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) \left[n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathrm{q}ns}\right]}{(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathrm{q}ns} + i\hbar\eta)(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathrm{q}ns} + \hbar\omega + i\hbar\eta)}. \tag{0.92}$$

Splitting up the stress-energy tensor, and considering each part of the response function by itself, as explained above, gives

$$
\begin{aligned}
\chi^{xy\,(i)}(\omega,\boldsymbol{q}) = &\lim_{\eta\to 0}\frac{eBiv_F\hbar}{\hbar(2\pi)^2}\sum_{mn}\int \mathrm{d}k_z[n_{\boldsymbol{k}ms}-n_{\boldsymbol{k}+\mathrm{q}ns}] \\
&\frac{J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q})T^{0y\,(i)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q})}{(E_{\boldsymbol{k}ms}-E_{\boldsymbol{k}+\mathrm{q}ns}+i\hbar\eta)(E_{\boldsymbol{k}ms}-E_{\boldsymbol{k}+\mathrm{q}ns}+\hbar\omega+i\hbar\eta)} \\
= &\lim_{\eta\to 0}\frac{-e^2Bv_F^2s\Gamma^-_{\boldsymbol{k}\mathrm{q}mns}\Gamma^+_{\boldsymbol{k}\mathrm{q}mns}}{4(2\pi)^2}\sum_{mn}\int \mathrm{d}k_z \\
&\frac{(\Xi_1+\Xi_2)\,(\dots)^i\,[n_{\boldsymbol{k}ms}-n_{\boldsymbol{k}+\mathrm{q}ns}]}{(E_{\boldsymbol{k}ms}-E_{\boldsymbol{k}+\mathrm{q}ns}+i\hbar\eta)(E_{\boldsymbol{k}ms}-E_{\boldsymbol{k}+\mathrm{q}ns}+\hbar\omega+i\hbar\eta)},
\end{aligned}
\tag{0.93}
$$

where

$$
(\dots)^1 = s(E_{\boldsymbol{k}ms}+E_{\boldsymbol{k}+\mathrm{q}ns}-2\mu)(\Xi_1(\bar{\boldsymbol{q}},m,n,s)-\Xi_2(\bar{\boldsymbol{q}},m,n,s)), \tag{0.94}
$$

$$
(\dots)^2 = \frac{\hbar v_F}{l_B}\left\{\frac{\sqrt{2M}}{\alpha_{\boldsymbol{k}ms}}\Xi_1(\bar{\boldsymbol{q}},m,n,s)\ +\sqrt{2(M-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\alpha_{\boldsymbol{k}m-1s}}\Xi_1(\bar{\boldsymbol{q}},m-1,n-1,s)\right\}, \tag{0.95}
$$

$$
(\dots)^3 = \frac{\hbar v_F}{l_B}\left\{\frac{\sqrt{2N}}{\alpha_{\boldsymbol{k}+\mathrm{q}ns}}\Xi_2(\bar{\boldsymbol{q}},m,n,s)\ +\sqrt{2(N-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\alpha_{\boldsymbol{k}+\mathrm{q}n-1s}}\Xi_2(\bar{\boldsymbol{q}},m-1,n-1,s)\right\}. \tag{0.96}
$$

In the interest of facilitating for the integration over k_z, make a change of variables in the integration, by introducing the dimensionless quantity $\kappa_z = \hbar k_z/\sqrt{2eB\hbar}$, which also normalizes the energy to $E_{\boldsymbol{k}ms} = v_F\sqrt{2eB\hbar}\epsilon_{\kappa_z ms}$, which is seen directly from the expression of the energy eigenvalues. After some cleaning up, among other taking the product $\Gamma^-_{\boldsymbol{k}\mathrm{q}mns}\Gamma^+_{\boldsymbol{k}\mathrm{q}mns}$ and using $l_B = \sqrt{\hbar/(eB)}$,

$$
\chi^{xy\,(i)}(\omega,\boldsymbol{q}) = \lim_{\eta\to 0}\frac{-1}{4(2\pi)^2}\frac{e^2Bsv_F}{\hbar}e^{-\frac{l_B^2}{2}q^2}\sum_{mn}\int \mathrm{d}\kappa_z\xi(\kappa_z)\,(\Xi_1+\Xi_2)\,\Delta^i, \tag{0.97}
$$

where we defined

$$
\xi(\kappa_z) = \frac{[n_{\kappa ms}-n_{\kappa+\mathrm{q}ns}]\left[(\alpha^2_{\kappa ms}+1)(\alpha^2_{\kappa+\mathrm{q}ns}+1)\right]^{-1}}{(\epsilon_{\kappa ms}-\epsilon_{\kappa+\mathrm{q}ns}+i\frac{\hbar\eta}{v_F\sqrt{2eB\hbar}})(\epsilon_{\kappa ms}-\epsilon_{\kappa+\mathrm{q}ns}+\frac{\hbar\omega}{v_F\sqrt{2eB\hbar}}+i\frac{\hbar\eta}{v_F\sqrt{2eB\hbar}})} \tag{0.98}
$$

and

$$
\Delta^1 = s(\epsilon_{\boldsymbol{\kappa}+m+s}+\epsilon_{\boldsymbol{\kappa}+\mathrm{q}ns}-2\frac{\hbar\mu}{v_F\sqrt{2eB\hbar}})(\Xi_1(\bar{\boldsymbol{q}},m,n,s)-\Xi_2(\bar{\boldsymbol{q}},m,n,s)), \tag{0.99}
$$

$$
\Delta^2 = \frac{\sqrt{M}}{\alpha_{\boldsymbol{k}ms}}\Xi_1(\bar{\boldsymbol{q}},m,n,s)+\sqrt{(M-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\alpha_{\boldsymbol{k}m-1s}}\Xi_1(\bar{\boldsymbol{q}},m-1,n-1,s), \tag{0.100}
$$

$$
\Delta^3 = \frac{\sqrt{N}}{\alpha_{\boldsymbol{k}+\mathrm{q}ns}}\Xi_2(\bar{\boldsymbol{q}},m,n,s)+\sqrt{(N-1)}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\alpha_{\boldsymbol{k}+\mathrm{q}n-1s}}\Xi_2(\bar{\boldsymbol{q}},m-1,n-1,s). \tag{0.101}
$$

Considering now the local limit $q \to 0$, and using the previously found selection rules for products of the Ξ_i-functions. Using that the Laguerre polynomials are unity at the origin and writing out the Ξ-functions with the selection rules

$$\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (1)}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{1}{4(2\pi)^2} \frac{e^2 B v_F}{\hbar} \sum_{mn} \int \mathrm{d}\kappa_z \xi(\kappa_z)(\epsilon_{\boldsymbol{\kappa}ms} + \epsilon_{\boldsymbol{\kappa}ns} - 2\frac{\hbar\mu}{v_F\sqrt{2eB\hbar}})(\alpha^2_{\kappa ns}\delta_{N,M+1} - a^2_{\kappa ms}\delta_{N,M-} \tag{0.102}$$

$$\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (2)}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{1}{4(2\pi)^2} \frac{e^2 B s v_F}{\hbar} \sum_{\substack{mn \\ N=M-1}} \int \mathrm{d}\kappa_z \xi(\kappa_z) \left\{ \sqrt{M}\alpha_{\boldsymbol{\kappa}ms} + \sqrt{M-1}\alpha_{\boldsymbol{\kappa}ns}\alpha^2_{\boldsymbol{\kappa}ms} \right\}, \tag{0.103}$$

$$\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (3)}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{-1}{4(2\pi)^2} \frac{e^2 B s v_F}{\hbar} \sum_{\substack{mn \\ N=M+1}} \int \mathrm{d}\kappa_z \xi(\kappa_z) \left\{ \sqrt{N}\alpha_{\boldsymbol{\kappa}ns} + \sqrt{N-1}\alpha_{\boldsymbol{\kappa}ms}\alpha^2_{\boldsymbol{\kappa}ns} \right\}. \tag{0.104}$$

The function ξ is odd under interchange of m, n, i.e. $\xi_{m,n} = -\xi_{n,m}$. Using this, we may simplify our expressions some. Firstly, in the second term of Eq. (0.102), we may relabel $m \leftrightarrow n$, and using that ξ is odd to get twice the first term. Doing a similar trick for Eq. (0.103) renders it identical to Eq. (0.104). In total, we get

$$\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (1)}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{1}{4(2\pi)^2} \frac{e^2 B v_F}{\hbar} \sum_{N=M+1} \int \mathrm{d}\kappa_z 2\xi(\kappa_z)(\epsilon_{\boldsymbol{\kappa}ms} + \epsilon_{\boldsymbol{\kappa}ns} - 2\frac{\hbar\mu}{v_F\sqrt{2eB\hbar}})\alpha^2_{\kappa ns}, \tag{0.105}$$

$$\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (2,3)}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{-1}{4(2\pi)^2} \frac{e^2 B s v_F}{\hbar} \sum_{\substack{mn \\ N=M+1}} \int \mathrm{d}\kappa_z 2\xi(\kappa_z) \left\{ \sqrt{N}\alpha_{\boldsymbol{\kappa}ns} + \sqrt{N-1}\alpha_{\boldsymbol{\kappa}ms}\alpha^2_{\boldsymbol{\kappa}ns} \right\}. \tag{0.106}$$

Here, $\chi^{xy\ (2,3)} = \chi^{xy\ (2)} + \chi^{xy\ (3)}$.

Note the $\alpha_{\kappa ns}$ factors coming from the Ξ functions; it is also important to note that the latter term involves the chirality s, which is of concern as this could make pairs of nodes cancel, however, we will see that the term ends up not being dependent on s. In the static limit $\lim_{\omega\to 0}$ and with the potential $\mu = 0$ the integral may be solved. Including only the lowest Landau level contribution, that is, the sum restricted to $m = 0, n = \pm 1$, the contributions were found to be

$$\lim_{\omega\to 0}\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (1)} = \frac{1}{2}\frac{e^2 B v_F}{4(2\pi)^2\hbar}, \tag{0.107}$$

$$\lim_{\omega\to 0}\lim_{\boldsymbol{q}\to 0} \chi^{xy\ (3)} = \frac{1}{2}\frac{e^2 B s^2 v_F}{4(2\pi)^2\hbar}. \tag{0.108}$$

For clarity s^2 was included explicitly, showing that the aforementioned s-dependence is not an issue. The total transverse response function with only first level contributions is therefore

$$\lim_{\omega \to 0} \lim_{\boldsymbol{q} \to 0} \chi^{xy} = \frac{e^2 B v_F}{4(2\pi)^2 \hbar}. \tag{0.109}$$

Including higher order contributions, only gives additional numerical prefactors,

$$\lim_{\omega \to 0} \lim_{\boldsymbol{q} \to 0} \chi^{xy} = \gamma_M \frac{e^2 B v_F}{4(2\pi)^2 \hbar}, \tag{0.110}$$

with $\gamma_0 = 1, \gamma_{20} \approx 2$, and where γ_M goes as $\log M$. The first 300 contributions were calculated, shown in Figure **??**.

0.3 Discussion of results

In the static and local limit $\lim_{\omega \to 0} \lim_{\boldsymbol{q} \to 0}$ the transverse response function χ^{xy} of the charge current to a temperature perturbation

$$J^x = \chi^{xy} \frac{-\nabla^y T}{T} \tag{0.111}$$

from a single Dirac point was found to be

$$\lim_{\omega \to 0} \lim_{\boldsymbol{q} \to 0} \chi^{xy} = \gamma_N \frac{e^2 B v_F}{4(2\pi)^2 \hbar}, \tag{0.112}$$

with γ_N a prefactor dependent on how many landau levels are included in the final evaluation of the response function. The response function is independent of the chirality s of the Dirac point. It was found that $\gamma_0 = 1, \gamma_{20} \approx 2$ and that the prefactor goes like $\log N$.

Firstly, the result differ slightly from that found by Arjona, Chernodub, and Vozmediano [1]

$$\lim_{\omega \to 0} \lim_{\boldsymbol{q} \to 0} \chi^{xy} = 2\gamma_N \frac{e^2 B v_F}{4(2\pi)^2 \hbar}, \tag{0.113}$$

which differ by a factor of two.

Secondly, the sum will diverge as $N \to \infty$. However, not all Landau levels are filled, and thus the sum should not be taken to all levels. Similarly to a Quantum Hall effect, the number of filled bands, the filling factor ν, is inverse proportional to the B-field strength

$$\nu \propto \frac{1}{B}. \tag{0.114}$$

Thus, we expect that the N-sum should be truncated at a Landau level, given by the filling factor ν. A detailed derivation of the exact truncation of the N-sum has not been done. If a precise result for the numerical prefactor is found to be of importance, this should be straightforward.

The divergence is not discussed by Arjona, Chernodub, and Vozmediano, where only the values of $N = 0$ and $N = 20$ are given, and the final result is that of $N = 20$. Furthermore, they state that the contributions from higher values of N decrease very rapidly.

However, we found the contributions to go like $1/x$, which is not decreasing rapidly enough to give a finite total contribution, thus giving the total contribution diverging logarithmically. **Say that we are communicating with them to better understand their chioce of**
Comparing our result with the different procedure done by Chernodub, Cortijo, and Vozmediano [3], the numerical prefactor found in our calculation including only the first term ($M = 0$) coincides very well with the numerical prefactor found there, with a ratio of 16/18.

0.4 Type II Weyl semimetals

The conic section problem with the intersecting plane restricted to pass through the node of the cone is trivially seen to have two solutions: a point and two intersecting lines. Despite this, the possibility of a Weyl cone tilted beyond the Fermi level was never considered before Soluyanov et al. described this new class of Weyl semimetals in 2015. This now seemingly obvious possibility made an already rich field even more exciting, opening up for a wider range of novel and interesting effects. **add some concrete examples or cites**

In the case of massless fermions, the particle physics equivalent of the Weyl semimetal, such a tilt is not possible, due to the requirement of Lorentz invariance **add cite or explain**. In condensed matter physics, however, this is not an issue, and it is indeed a real class of materials **cite examples**. We denote these types of materials Type-II Weyl semimetals, as opposed to Type-I. The transition between Type-I and Type-II is abrubt – the Fermi surface goes form a single point to two intersecting lines, in other words going from a zero dimensional to a one dimensional surface. **Make sure this is indeed a one dimensional surface**

Make sure it is one dim also for the 3D case, quadric surface, not conic intersection

Type-II also has electron and particle pockets at the Fermi level. While the density of states for a Type-I semimetal goes to zero as one approaches the Fermi level, this causes Type-II to have a finite density of states at the Fermi level. **End with something like: all in all this gi**

0.4.1 Hamiltonian

We will firstly consider a slightly more realistic toy model for a Weyl semimetal, with a parameter taking the system from a Type-I to a Type-II. This is instructive both in order to more intuitively see the origin of the terms causing the tilting of the Dirac cone, and also to see how two Dirac cones in the same Brillouin zone tilt in relation to each other. We will then continue by linearizing the model around the Weyl points, regaining the familiar form of a Dirac cone, with an additional anisotropy term causing the tilt.

Using the general time-reversal breaking model described by McCormick, Kimchi, and Trivedi we have

$$\begin{aligned} H(\boldsymbol{k}) = {} & [(\cos k_x + \cos k_z - 2)m + 2t(\cos k_x - \cos k_0)]\,\sigma_1 \\ & - 2t \sin k_y \sigma_2 - 2t \sin k_z \sigma_3 + \gamma(\cos k_x - \cos k_0). \end{aligned} \tag{0.115}$$

The model has Weyl nodes at $\boldsymbol{K'} = (\pm k_0, 0, 0)$, and the parameter γ controls the tilting of the emerging cones. A value of $\gamma = 0$ gives no tilt, while for $\gamma > |2t|$ the Type-II system emerges. Figure 2 shows the cross section $k_y = 0$ of the eigenvalues of this system, as γ is gradually increased from 0 to 0.15 **verify numbers**. The γ-term "warps" the bands, and in the limit of Type-II the hole band crosses the Fermi level into positive energy, while the particle band crosses the Fermi level into negative energies. We call these hole and electron pockets, respectively.

Linearizing around the Weyl nodes reduces to the familiar expression of a Dirac cone

$$H(\boldsymbol{K}'^{\pm} + \boldsymbol{k}) \approx \mp 2t k_x \sin k_0 \sigma_1 - 2t(k_y \sigma_2 + k_z \sigma_3) \mp \gamma k_x \sin k_0 \sigma_0, \quad k_x, k_y, k_z \ll 1. \tag{0.116}$$

When the separation between the two nodes is π, i.e. $k_0 = \pi/2$, the linearized Hamiltonian of around the cone, is

$$H'(\boldsymbol{k}) = \mp 2tk_x\sigma_x - 2tk_y\sigma_y - 2tk_z\sigma_z \mp \gamma k_x. \tag{0.117}$$

However, as the two noes are brought closer together, the effective Fermi velocity in the x-direction is rescaled, and the system is anisotropic even for no tilt ($\gamma = 0$). The expression may be made even more clear by moving the sign $\pm$-sign into the tilt parameter γ. The Hamiltonian is invariant under a sign change of the first term, as the isotropic Dirac Hamiltonian is invariant under inversion. In the tilt-term, we move the sign dependence into γ, and the linearized model is

$$H'(\boldsymbol{k}) = -2t\boldsymbol{k}\boldsymbol{\sigma} - \gamma^{\pm}k_x, \tag{0.118}$$

where $\gamma^{\pm} = \pm\gamma$ with the upper sign corresponding to the node at $k_x = +k_0$ and the lower sign corresponds to the node at $k_x = -k_0$. As expected, we get two Dirac cones, tilting in opposite direction, but with the same amount. **How does this affect the Berry curvature and chern**

Maybe prettier/more correct to invert ky and kz, as that would also give the opposite chira

The linearized model are accurate in describing low energy interactions around the Fermi level. For higher energies their validity falls apart, and more complex models are warranted. In our calculations the linear models is sufficient, and much easier to work with, and we will thus mainly consider the linear model from here on.

- gives rise to cones tilting opposite direction
- Linearized model valid for low energy interaction. For higher energy, the perfect cone model is not valid, as the cones does in fact touch.
- In this model, the hole pocket is "shared" between the two cones. There are also models with individual pockets (see [6])

0.4.2 Eigenstates and Landau levels

The eigenvalues of Type-II Weyl semimetal simple to find, and are not qualitatively different from those of Type-I, other than the appearence of particle and hole pockets at the Fermi level. We will also consider the Landau levels of these materials, which importantly are very different from Type-I. In fact, erroneous treatment of the Landau spectrum of Type-II semimetals caused the original paper describing Type-II materials to mistakenly assert that the chiral anomaly would not be present for certain directions of a background magnetic field [11][10].

Eigenstates, spin, berry, etc

The issue with the Landau level description is that for certain directions of the B-field, the levels break down and become imaginary. For Type-I materials, the description is valid for all dierections of the B-field, but as the cone tip into a Type-II material, the description breaks down when the B-field and tilt direction are perpendicular [10], and figures/conicSection.pdf

Figure 1:

as the magnitude of the tilt is increased, the Landau levels are only valid up to a certain angle between the tilt direction and magnetic field.

Consider the Hamiltonian

$$H = v_F \boldsymbol{t}\boldsymbol{k} + s(\boldsymbol{v} \odot \boldsymbol{k})\boldsymbol{\sigma}, \tag{0.119}$$

where $\boldsymbol{t}$ is the *tilt vector* and $\boldsymbol{v}$ is the Fermi velocity, which in general is anisotropic. To find the Landau levels in a magnetic field $\boldsymbol{B} = B_z\hat{z}$, we will "Lorentz boost" the system to a frame where the cone is not tilted, where we may use the usual approach for finding the Landau levels. Firstly, assume that the tilt vector $\boldsymbol{t}$ is in the x, z-plane, $\boldsymbol{t} = (t_\perp, 0, t_\parallel)$, which we can always achieve by a rotation around z. **Proof by figure** Introduce the $\boldsymbol{B}$-field by the minimal coupling $\boldsymbol{k} \to \boldsymbol{k}^B = \boldsymbol{k} + e\boldsymbol{A}$. We take the field to be in the z-direction, and use the Landau gague $\boldsymbol{A} = -B_z y\hat{x}$. The Hamiltonian can thus be written

$$H_B = v_F t_\perp \left(k_x - eB_z y\right) + v_F t_\parallel k_z + s v_y k_y \sigma_y + s v_z k_z \sigma_z + s v_x \left(k_x - eB_z y\right)\sigma_x, \tag{0.120}$$

and the Landau level equation is

$$\left(H_B - E\right)|\psi\rangle = 0. \tag{0.121}$$

figures/typeIIridgeline.png

Figure 2: **Write this** The values of the parameters were chosen to be $m = 0.15, t = -0.05$, and $2k_0 = \pi$.

In order to use the ladder operator method used for the untilted cone, we must get rid of the k_x^B on the diagonal of the Hamiltonian. [3] To achieve this, we will use a "Lorentz transformation", which as we will show only leave k_z and E in the diagonal. Act with the hyperbolic rotation operator $\exp[\Theta/2\sigma_x]$ on Eq. (0.121), and insert identity on the form $\exp[\Theta/2\sigma_x]\exp[-\Theta/2\sigma_x]$ before the state vector. By introducing the state in the rotated frame $|\tilde{\psi}\rangle = \exp[-\Theta/2\sigma_x]\mathcal{N}|\psi\rangle$, with $\mathcal{N}$ a normalization factor compensating for the non-unitarity of the transformation, we get the eigenvalue equation

$$\left(\exp[\Theta/2\sigma_x]H_B\exp[\Theta/2\sigma_x] - E\exp[\Theta\sigma_x]\right)|\tilde{\psi}\rangle = 0. \tag{0.122}$$

We rewrite the Hamiltonian (Eq. (0.120)) in a more compact and explicit way

$$H_B = v_F\left(t_\perp k_x^B + t_\| k_z^B\right)\mathcal{I}_2 + \sum_i s v_i k_i^B \sigma_i, \tag{0.123}$$

where $\mathcal{I}_2$ is the identity matrix of size 2. We now make the fortunate observation that, with the hyperbolic rotation operator denoted $R = \exp[\Theta/2\sigma_x]$, the diagonal elements of

$$R\sigma_i R$$

[3] It would also be possible to choose the frame such that the tilt was both in x and y direction, in which case we would get ladder operators also on the diagonal. This system, albeight tedious, could also have been solved directly. **Verify this**

figures/movetypeiinode.png

Figure 3: A Type-II Weyl semimetal with separation between the nodes $2k_0 = 0, \pi/2, \pi$. See main text for details about the model.

are zero for $i = y$ and non-zero for $i = x, z$. We may thus rotate the x and z in and out of the diagonal elements, without accidentaly rotating the y components into the diagonal.

The problematic part of the Hamiltonian with regards to finding the Landau levels, are the terms containing k_x^B on the diagonal, i.e.

$$v_F t_\perp k_x^B \mathcal{I}_2 + s v_x k_x^B \sigma_x.$$

We will now find the boost parameter that eliminates k_x from the diagonal. We have

$$R^2 = e^{\Theta\sigma_x} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \tag{0.124}$$

and as $[R, \sigma_x] = 0$,

$$R\sigma_x R = R^2\sigma_x = \begin{pmatrix} \sinh\theta & \cosh\theta \\ \cosh\theta & \sinh\theta \end{pmatrix}, \tag{0.125}$$

as the effect of σ_x is to transpose the rows. The requirement for k_x^B to be rotated out of the diagonal is thus

$$t_\perp \cosh\theta + s\sinh\theta = 0. \tag{0.126}$$

Solving for θ we get

$$\theta = \log(\pm\frac{\sqrt{s - t_\perp}}{\sqrt{s + t_\perp}}). \tag{0.127}$$

NB: depending of choice of sign in log, we get different signs in answer Alternatively, written in a slightly suggestive form,

$$\tanh\theta = -st_{\perp}. \tag{0.128}$$

For pedagogic reasons, include arctanh, which is only valid for -1 ¡ x ¡ 1, explicitly showing

Before we proceed any further, we will put the above into a more solid context, defining some useful quantities and more carefully investigte what is going on, which will be of help later when considering for the regions of validity, and the physical reason behind it.

nice plot of the landau levels acutally being squeezed

Introduce the dimensionless *tilt parameter*

$$\boldsymbol{t} = \left(\frac{\omega_{0x}}{v_x}, \frac{\omega_{0y}}{v_y}, \frac{\omega_{0z}}{v_z}\right).$$

Let also $v_i = v_0 a_i$, where $\boldsymbol{a}$ is a vector describing the anisotropy of the system. In these parameters, the eigenvalues of the system are

$$E(\boldsymbol{k}) = \boldsymbol{\omega_0 k} \pm \sqrt{(v_i k_i)^2} = \sqrt{(t_i v_i k_i)^2} \pm \sqrt{(v_i k_i)^2}. \tag{0.129}$$

The system is Type-II if the first term dominates for any $\boldsymbol{k}$, and Type-I if the last term dominates [11]. The $\boldsymbol{t}$-vector is thus a convenient tool for categorization – if $t > 1$ we have a Type-II, else we have a Type-I.

Proof Proof: We may always rotate our coordinate system such that, without loss of generality, $\boldsymbol{t} = t\hat{x}$. In that case, the first term obviously dominates in the x-direction, when $t > 1$.

Expressed in the parameter t, the result in Eq. (0.128) has an intuitive, and quite visual, interpretation. As described above, we have rotated our frame such that the tilt is confined to the x, z-plane, i.e. no tilt in the y-direction. The required hyperbolic tilt angle to eliminate the k_x^B in the diagonal elements of the Hamiltonian, originating from the tilt, was

$$\theta = -s \tanh^{-1}\frac{\omega_{\perp}}{v_x} = -s\tanh^{-1} t_x. \tag{0.130}$$

The inverse of tan, of course, diverges as the argument approaches ± 1, as shown in Figure 4. For $t_x < 1$ we are able to find an angle θ which transforms our Hamiltonian into a form which we may solve. For $t_x \geq 1$, however, no (real) solution of θ exits, and the Landau level description collapses. **Is this argumentation sufficient? What if we just have to use a diffe**

Discuss magnetic vs electric regime It is also interesting that there are no restrictions on the tilt in z, t_z. Visually, this can be visualized by plotting the t-vector inside a unit sphere, shown in Figure 5. If the vector is outside the unit sphere, it is a Type-II, if it is inside, it is a Type-I. Also, if the projection of the vector onto the x, y-plane is on the unit disk, the Landau level description is valid, if not, the Landau levels collapse. All Type-I materials may thus be described by Landau levels, while it for Type-II is only valid for certain directions of the t-vector. As the t-vector gets larger, the region of valid directions is reduced to an ever smaller cone around z.

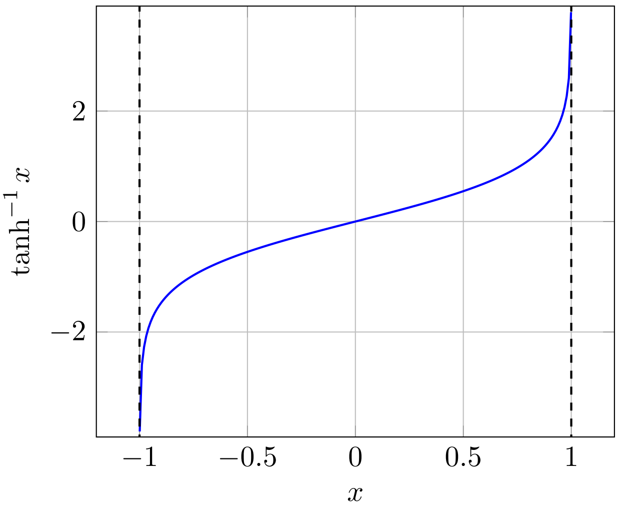

Figure 4: Plot of $\tanh^{-1}$, which diverges as the argument goes to ± 1.

We now return to solving Eq. (0.122), using the solution angle we just found. By insertion, and after some clean up, we get *note to thorvald: we chose $\theta = Log\,(+\dots)$*

$$
\begin{aligned}
&(\exp[\Theta/2\sigma_x] H_B \exp[\Theta/2\sigma_x] - E\exp[\Theta\sigma_x])\,|\tilde{\psi}\rangle = 0 \\
&= v_F \begin{pmatrix} k_z(1+t_\parallel\gamma) - \frac{E\gamma}{v_F} & -ik_y + k_x/\gamma - \frac{E\gamma\beta}{v_F} + k_z t_\parallel \gamma\beta \\ ik_y + k_x/\gamma - \frac{E\gamma\beta}{v_F} + k_z t_\parallel\gamma\beta & -k_z(1-t_\parallel\gamma) - \frac{E\gamma}{v_F} \end{pmatrix} |\psi\rangle\,. \qquad (0.131)
\end{aligned}
$$

In order to simplify this further, absorb $\gamma\beta(k_z t_\parallel - E/v_F)$ into k_x. Thus, let

$$
\begin{aligned}
\tilde{k}_x &= k_x/\gamma + \gamma\beta(k_z t_\parallel - E/v_F), \\
\tilde{k}_y &= k_y, \\
\tilde{k}_z &= k_z.
\end{aligned} \qquad (0.132)
$$

The eigenvalue equation is simply

$$
\left[\gamma\left(t_\parallel \tilde{k}_z - \frac{E}{v_F}\right)\mathcal{I}_2 + \tilde{k}_i\sigma_i\right]|\tilde{\psi}\rangle = 0. \qquad (0.133)
$$

If we now again introduce the magnetic field using minimal coupling, $k_x \to k_x - eyB_z$, this corresponds to an effective field $B_z\frac{v_x v_y}{v_F^2}/\gamma$ in the new quantities. This is because $\tilde{k}_x \to \tilde{k}_x - e\tilde{y}\frac{v_y v_x}{v_F^2}B_z/\gamma$, where the rescaled $\tilde{y} = \frac{v_F}{v_y}y$.

The Landau level equation thus reads

$$
\left[\sum_i v_F\left(\tilde{k}_i + e\tilde{A}_i\right)\sigma_i\right]|\tilde{\psi}\rangle = (E - t_\parallel v_F \tilde{k}_z)\gamma\,|\tilde{\psi}\rangle\,, \qquad (0.134)
$$

figures/tiltSpherewBackground.png

Figure 5: TODO

where $\tilde{\boldsymbol{A}} = -B_z/\gamma\tilde{y}\hat{x}$. We may thus use directly the result for the untilted cone, **eq ref**, giving

$$\left(E - t_{\|} v_F \tilde{k}_z\right)\gamma = \mathrm{sign}(m) v_F \sqrt{2|m|e\frac{B}{\gamma}\hbar + \tilde{k}_z^2\hbar^2}, \qquad m \neq 0, \tag{0.135}$$

$$\left(E - t_{\|} v_F \tilde{k}_z\right)\gamma = -s\hbar\tilde{k}_z v_F, \qquad m = 0. \tag{0.136}$$

Cleaning up, we get

$$E = t_{\|} v_F \tilde{k}_z + \mathrm{sign}(m) v_F \sqrt{2|m|e\frac{B}{\gamma^3}\hbar + \tilde{k}_z^2\hbar^2/\gamma^2}, \qquad m \neq 0, \tag{0.137}$$

$$E = \tilde{k}_z v_F \left(t_{\|} - s\hbar/\gamma\right), \qquad m = 0. \tag{0.138}$$

As the tilt is increased, $\gamma = 1/\sqrt{1-\beta^2}$ diverges to infinity. With the trivial substitution $\alpha = \frac{1}{\gamma}$, which goes to zero, this gets an intuitive interpretation.

$$E = t_{\|} v_F \tilde{k}_z + \mathrm{sign}(m) v_F \alpha \sqrt{2|m|eB\alpha\hbar + \tilde{k}_z^2\hbar^2} \tag{0.139}$$

As the tilt increases, the Landau levels converge towards $t_{\parallel} v_F \tilde{k}_z$. **Note that $\tilde{k} \to 0$ as we overtilt as wel**
In particular, the separation between Landau levels m **maybe use the word cyclotron frequency**
is reduced by a factor $\alpha^{\frac{3}{2}}$. The effect of the tilt on the Landau levels is to squeeze the Landau levels together, and we will call the $\alpha^{\frac{3}{2}}$ the *squeezing factor*. We note that when approaching the degree of tilt where we are no longer able to find a boost which enables us to solve for the Landau levels, i.e. when $\beta \to 1$, the squeezing factor goes to zero. As the tilt exceeds this limit, the squeezing factor is imaginary.

Consider now isotropic velocities $v_i = v_F$. Even for anisotripic systems, we may rescale the momenta to arrive at such a description. We may rewrite the energy as

$$E = \begin{cases} t_{\parallel} v_F k_z + \operatorname{sign}(m) v_F \alpha \sqrt{2|m|eB\alpha + k_z^2} & m \neq 0 \\ t_{\parallel} v_F k_z - s\alpha v_F k_z & m = 0, \end{cases}$$

where $\hbar = 1$. Notice that this is exactly

$$E = t_{\parallel} v_F k_z + \alpha E^0_{m,\alpha B},$$

where $E^0_{m,B}$ is the energy in the untilted case, with magnetic field αB.

The eigenstate of

$$H = v_F \sigma^i (p_i + eA_i),$$

with $A_i = -B_z y \delta_{ix}$, given in the position basis, is

$$\phi_{\boldsymbol{k}ms}(\boldsymbol{r}) = \frac{1}{\sqrt{L_x L_z}} \frac{e^{ik_x x} e^{ik_z z}}{\sqrt{\alpha^2_{k_z ms} + 1}} e^{-\frac{y - k_x l^2}{2 l_B^2}} \begin{pmatrix} \frac{\alpha_{k_z ms}}{\sqrt{2^{M-1}(M-1)!\sqrt{\pi} l_B}} H_{M-1}\left(\frac{y - k_x l_B^2}{l_B}\right) \\ \frac{1}{\sqrt{2^M M! \sqrt{\pi} l_B}} H_M \left(\frac{y - k_x l_B^2}{l_B}\right) \end{pmatrix}, \tag{0.140}$$

where capital letters indicate absolute value of corresponding quantity, $M = |m|, \boldsymbol{k} = (k_x, k_z)$, and with the normalization factor

$$\alpha_{k_z ms} = \frac{-\sqrt{2eB\hbar M}}{\frac{E_{k_z ms}}{s v_F} - \hbar k_z}. \tag{0.141}$$

Taking care to keep track of boosted and rescaled quantites, the eigenstate in the boosted frame is

$$\tilde{\psi}(\tilde{\boldsymbol{r}}) = \frac{1}{\sqrt{L_x L_z}} \frac{e^{i\tilde{k}_x \tilde{x}} e^{ik_z z}}{\sqrt{\alpha^2_{\tilde{k}_z ms} + 1}} e^{-\frac{\left(\tilde{y} - \tilde{k}_x l_{B'}^2\right)^2}{2 l_{B'}^2}} \begin{pmatrix} \frac{\alpha_{\tilde{k}_z ms}}{\sqrt{2^{M-1}(M-1)!\sqrt{\pi} l_{B'}}} H_{M-1}\left(\frac{\tilde{y} - \tilde{k}_x l_{B'}^2}{l_{B'}}\right) \\ \frac{1}{\sqrt{2^M M! \sqrt{\pi} l_{B'}}} H_M \left(\frac{\tilde{y} - \tilde{k}_x l_{B'}^2}{l_{B'}}\right) \end{pmatrix}, \tag{0.142}$$

with

$$\alpha_{\tilde{k}_z ms} = \frac{-\sqrt{2eB'\hbar M}}{\gamma \frac{E_{\tilde{k}_z ms} - t_{\parallel} v_F \tilde{k}_z}{s v_F} - \hbar \tilde{k}_z}, \tag{0.143}$$

where

$$B' = B\alpha.$$

We note that $\alpha_{k_z 0 s} = 0$, so using the explicit form of the energy we may simplify the expression some. For $m \neq 0$

$$\frac{E_{k_z ms} - t_{\parallel} v_F k_z}{s v_F} = \mathrm{sign}(m) s\alpha\sqrt{2MeB\alpha + k_z^2}$$

and thus

$$\alpha_{k_z ms} = \frac{-\sqrt{\alpha M}}{\mathrm{sign}(m)s\sqrt{\alpha M + \kappa^2} - \kappa} \tag{0.144}$$

where we defined the dimensionless $\kappa_z = \sqrt{2eB}k_z$.

Note that

And thus the original eigenstate $|\psi\rangle = 1/\mathcal{N} e^{\theta/2\sigma_x} |\tilde{\psi}\rangle$ of the tilted system is easily found. The normalization factor $\mathcal{N}$ is needed as the boost transformation is not unitary.

Write down the full form of $\phi_{\boldsymbol{k}ms}(\boldsymbol{r})$, taking care to use the original momenta, and not th

Reinserting explicitly, in the boosted frame, that

$$\tilde{q}_x = \alpha q_x - \frac{\beta}{\alpha}\frac{E_{k_z ms} - k_z v_F t_{\parallel}}{v_x} = \alpha q_x - \beta\frac{E^0_{m,\alpha B}}{v_F}$$

and $l_{B'} = \frac{l_B}{\sqrt{\alpha}}$

$$\chi = \frac{y - \tilde{q}_x l_{B'}^2}{l_{B'}} = \sqrt{\alpha}(y - q_x l_B^2)/l_B + \frac{\beta l_B}{\sqrt{\alpha} v_F} E^0_{m,\alpha B}. \tag{0.145}$$

. **Clean up in $\hbar$**

$$\tilde{\phi}_{\boldsymbol{k}ms}(\tilde{\boldsymbol{r}}) = \frac{1}{\sqrt{L_x L_z}} \frac{e^{i\tilde{k}_x\tilde{x}} e^{ik_z z}}{\sqrt{\alpha^2_{\tilde{k}_z ms} + 1}} e^{-\frac{1}{2}\chi^2} \sqrt[4]{\alpha} \begin{pmatrix} \frac{\alpha_{\tilde{k}_z ms}}{\sqrt{2^{M-1}(M-1)!\sqrt{\pi} l_B}} H_{M-1}(\chi) \\ \frac{1}{\sqrt{2^M M! \sqrt{\pi} l_B}} H_M(\chi) \end{pmatrix}. \tag{0.146}$$

For later convenience, let us explicitly define

$$\tilde{\phi}_{\boldsymbol{k}ms}(\tilde{\boldsymbol{r}}) = \frac{e^{i\tilde{k}_x\tilde{x} + ik_z z}}{\sqrt{L_x L_z}} \underbrace{\frac{e^{-\frac{1}{2}\chi^2}\sqrt[4]{\alpha}}{\sqrt{\alpha^2_{\tilde{k}_z ms} + 1}} \begin{pmatrix} \frac{\alpha_{\tilde{k}_z ms}}{\sqrt{2^{M-1}(M-1)!\sqrt{\pi} l_B}} H_{M-1}(\chi) \\ \frac{1}{\sqrt{2^M M! \sqrt{\pi} l_B}} H_M(\chi) \end{pmatrix}}_{\tilde{\phi}_{\boldsymbol{k}ms}(y)}, \tag{0.147}$$

and thus

$$\tilde{\phi}_{\boldsymbol{k}ms}(y) = e^{-\frac{1}{2}\chi^2} \begin{pmatrix} a_{\boldsymbol{k}ms} H_{M-1}(\chi) \\ b_{\boldsymbol{k}ms} H_M(\chi) \end{pmatrix}, \tag{0.148}$$

with

$$a_{\boldsymbol{k}ms} = \frac{\alpha_{\tilde{k}_z ms}\sqrt[4]{\alpha}}{\sqrt{\alpha^2_{\tilde{k}_z ms}+1}\sqrt{2^{M-1}(M-1)!\sqrt{\pi}l_B}}, \tag{0.149}$$

$$b_{\boldsymbol{k}ms} = \frac{\sqrt[4]{\alpha}}{\sqrt{\alpha^2_{\tilde{k}_z ms}+1}\sqrt{2^{M}M!\sqrt{\pi}l_B}}. \tag{0.150}$$

$$\tag{0.151}$$

We proceed now to find the normalization factor $\mathcal{N}$, as it will become necessary in later steps. Recall that

$$|\psi\rangle = \frac{1}{\mathcal{N}}e^{\theta/2\sigma_x}|\tilde{\psi}\rangle,$$

and

$$e^{\theta\sigma_x} = \frac{1}{\alpha}\begin{pmatrix} 1 & -st_x \\ -st_x & 1 \end{pmatrix}.$$

The upper and lower part of the spinor are orthogonal, thus we have

$$\langle\psi|\psi\rangle = \frac{1}{\mathcal{N}^*\mathcal{N}}\frac{1}{\alpha}\langle\tilde{\psi}|\tilde{\psi}\rangle = 1 \implies \mathcal{N}^*\mathcal{N} = \frac{1}{\alpha}. \tag{0.152}$$

We choose $\mathcal{N} = \alpha^{-\frac{1}{2}}$.

Proposition 1

The tilted Hamiltonian

$$H = v_F\boldsymbol{t}\boldsymbol{k} + sv_F\boldsymbol{k}\boldsymbol{\sigma}$$

in a magnetic field $\boldsymbol{B}$ has the Landau levels

$$E = \begin{cases} t_\parallel v_F k_z + \operatorname{sign}(m)v_F\alpha\sqrt{2eB\alpha M + k_z^2} & m \neq 0 \\ t_\parallel v_F k_z - s\alpha v_F k_z & m = 0 \end{cases}.$$

The associated eigenstates in the position basis are

$$\tilde{\psi}(\boldsymbol{r}) = \frac{1}{\mathcal{N}}e^{\theta/2\sigma_x}\frac{e^{ik_x x + ik_z z}}{\sqrt{L_x L_z}}\psi(y),$$

where

$$\psi(y) = e^{-\frac{1}{2}\chi^2}\begin{pmatrix} a_{k_z ms}H_{M-1}(\chi) \\ b_{k_z ms}H_M(\chi) \end{pmatrix},$$

where we have defined $\chi = \sqrt{\alpha}\frac{y-k_x l_B^2}{l_B} + \frac{\beta l_B}{\sqrt{\alpha}v_F}E^0_{m,\alpha B}$ and $a_{k_z ms}, b_{k_z ms}$ are given in Eqs. (0.149).

0.5 The response of a tilted cone

Repeating the calculation of the response function is now straightforward, but rather tedious. Due to the boost transformation, the elements of the spinor in the untilted system, Eq. (0.142), mix. We thus have twice as many terms to keep track of.

Consider the expression for the current operator, Eq. 4.39, which we derived form the time evolution relation

$$\dot{A} = [A, H]/i\hbar,$$

by considering $\boldsymbol{v} = \dot{r}$, as $\boldsymbol{J} = e\boldsymbol{v}$. In the tilted Hamiltonian, the tilt term thus causes another term in the current operator.

$$\begin{aligned} \boldsymbol{v} = \dot{r} &= \frac{1}{i\hbar}[\boldsymbol{r}, H] \\ &= \frac{sv_F\sigma^i}{i\hbar}[\boldsymbol{r}, p_i + eA_i] + \frac{1}{i\hbar}[\boldsymbol{r}, \omega_0\boldsymbol{k}] \\ &= \frac{sv_F\sigma^i}{i\hbar}(i\hbar + e[\boldsymbol{r}, A_i]) + \boldsymbol{\omega}_0 \\ &= sv_F\sigma^i + \boldsymbol{\omega}_0. \end{aligned} \tag{0.153}$$

Take for example the matrix element of the current operator

$$J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) = \int \mathrm{d}y e^{-iq_y y} sv_F e\phi^*_{\boldsymbol{k}ms}(y)\sigma^x\phi_{\boldsymbol{k}+\mathrm{q}ns}(y).$$

We must find the matrix product $\phi\sigma_x\phi$. Recall that $\phi = \frac{1}{\mathcal{N}}e^{\theta/2\sigma_x}\tilde{\phi}$, and thus we must find

$$\phi^*\sigma_x\phi = \frac{1}{\mathcal{N}^*\mathcal{N}}\tilde{\phi}^* e^{\theta/2\sigma_x}\sigma_x e^{\theta/2\sigma_x}\tilde{\phi} = \frac{1}{\mathcal{N}^*\mathcal{N}}\tilde{\phi}^*\sigma_x e^{\theta\sigma_x}\tilde{\phi}.$$

With the previously found solution $\theta = -\tanh^{-1} t_x$, we get the rather simple form

$$e^{\theta\sigma_x} = \begin{pmatrix} 1 & -st_x \\ -st_x & 1 \end{pmatrix} \frac{1}{\sqrt{1-t_x^2}}.$$

With

$$\tilde{\phi} = e^{-\frac{1}{2}\chi^2}\begin{pmatrix} a_{\boldsymbol{k}ms}H_{M-1}(\chi) \\ b_{\boldsymbol{k}ms}H_M(\chi) \end{pmatrix} \tag{0.154}$$

we see how the expressions change when t_x become non-zero. Where we previoulsy had

$$\phi^*_{\boldsymbol{k}ms}\sigma_x\phi_{\boldsymbol{k}+\mathrm{q}ns} = a_{\boldsymbol{k}ms}H_{M-1}(\dots)\left[b_{\boldsymbol{k}+\mathrm{q}ns}H_N(\dots)\right] + \dots \tag{0.155}$$

the contents of the square brackets must now include also the other element of the spinor:

$$\phi^*_{\boldsymbol{k}ms}\sigma_x e^{\theta\sigma_x}\phi_{\boldsymbol{k}+\mathrm{q}ns} = a_{\boldsymbol{k}ms}H_{M-1}(\dots)\left[b_{\boldsymbol{k}+\mathrm{q}ns}H_N(\dots) - st_x a_{\boldsymbol{k}+\mathrm{q}ns}H_{N-1}(\dots)\right]\frac{1}{\sqrt{1-t_x^2}} + \dots \tag{0.156}$$

First of all, let us consider the exponent of the product. Due to the extra term in χ, this becomes more elaborate. The exponent is of course

$$\exp\{-iq_y y - \frac{1}{2}\chi_{\boldsymbol{k}}^2 - \frac{1}{2}\chi_{\boldsymbol{k}+\mathsf{q}}^2\} \tag{0.157}$$

A straightforward but tedious calculation shows that the argument of the exponent can be written as

$$-\frac{\alpha}{l_B^2}\left(y + \frac{l_B^2}{2\alpha}(iq_y - (q_x' + 2k_x'))\right)^2 - \frac{l_B^2}{4\alpha}(q_y^2 + 2i(q_x' + 2k_x')q_y + (q_x')^2), \tag{0.158}$$

where we have defined

$$q_x' = q_x\alpha - \frac{\beta}{v_F}(E^0_{n,\alpha B} - E^0_{m,\alpha B}), \tag{0.159}$$

$$k_x' = k_x\alpha - \frac{\beta}{v_F}E^0_{m,\alpha B}. \tag{0.160}$$

These must not be confused with the transformed momenta $\tilde{k}$, which are similar in form. Eq. (0.158) is on the same for as in the untilted cone case, and we may thus proceed with the same method. Make a change of variable

$$\tilde{y} = \frac{\sqrt{\alpha}}{l_B}\left(y + \frac{l_B^2}{2\alpha}(iq_y - 2k_x' - q_x')\right),$$

Follow up the substitution of the root in the integral. Consider moving the root into Ξ
to get the exponent on the form $e^{-\tilde{y}^2}$. With this substitution,

$$\chi_{\boldsymbol{k}} = \tilde{y} + \frac{l_B}{2\sqrt{\alpha}}\left(q_x' - iq_y\right) \tag{0.161}$$

$$\chi_{\boldsymbol{k}+\mathsf{q}} = \tilde{y} + \frac{l_B}{2\sqrt{\alpha}}\left(-q_x' - iq_y\right) \tag{0.162}$$

Doing this, Eq. (4.59) **fix ref** in the project thesis, becomes

$$\begin{aligned} J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathsf{q}ns}(\boldsymbol{q}) = \frac{sv_Fe}{\sqrt{\alpha}} \int \mathrm{d}\tilde{y}\, l_B \exp\left[-\frac{l_B^2}{4\alpha}\left(q_y^2 + 2i(2k_x' + q_x')q_y + (q_x')^2\right)\right] \\ e^{-\tilde{y}^2}\left[a_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathsf{q}ns}H_{M-1}\left(\chi_{\boldsymbol{k}}\right)H_N\left(\chi_{\boldsymbol{k}+\mathsf{q}}\right)\right. \\ - st_x a_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathsf{q}ns}H_{M-1}\left(\chi_{\boldsymbol{k}}\right)H_{N-1}\left(\chi_{\boldsymbol{k}+\mathsf{q}}\right) \\ + b_{\boldsymbol{k}ms}a_{\boldsymbol{k}+\mathsf{q}ns}H_M\left(\chi_{\boldsymbol{k}}\right)H_{N-1}\left(\chi_{\boldsymbol{k}+\mathsf{q}}\right) \\ \left. - st_x b_{\boldsymbol{k}ms}b_{\boldsymbol{k}+\mathsf{q}ns}H_M\left(\chi_{\boldsymbol{k}}\right)H_N\left(\chi_{\boldsymbol{k}+\mathsf{q}}\right)\right], \end{aligned} \tag{0.163}$$

where we used $\mathcal{N}^*\mathcal{N}\alpha = 1$

$$J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) = sv_F e \frac{\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 + 2i(2k_x' + q_x')q_y + (q_x')^2)\right]}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}}$$
$$\Big[\alpha_{\boldsymbol{k}ms}\Xi_1(\boldsymbol{q},m,n,s) + \alpha_{\boldsymbol{k}+\mathrm{q}ns}\Xi_2(\boldsymbol{q},m,n,s)$$
$$- st_x\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}\Xi_1(\boldsymbol{q},m,n\mp 1,s) - st_x\Xi_2(\boldsymbol{q},m,n\pm 1,s)\Big]. \quad (0.164)$$

We here used definitions of Ξ similar to that in the project thesis, but modified to the tilted case.

$$\frac{\sqrt{\alpha}\alpha_{\boldsymbol{k}ms}\Xi_1(\boldsymbol{q},m,n,s)}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}} = \int \mathrm{d}\tilde{y}\; e^{-\tilde{y}^2} l_B a_{\boldsymbol{k}ms} b_{\boldsymbol{k}+\mathrm{q}ns} H_{M-1}(\chi_{\boldsymbol{k}}) H_N(\chi_{\boldsymbol{k}+\mathrm{q}}) \quad (0.165)$$

$$\frac{\sqrt{\alpha}\alpha_{\boldsymbol{k}+\boldsymbol{q}ns}\Xi_2(\boldsymbol{q},m,n,s)}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}} = \int \mathrm{d}\tilde{y}\; e^{-\tilde{y}^2} l_B b_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathrm{q}ns} H_M(\chi_{\boldsymbol{k}}) H_{N-1}(\chi_{\boldsymbol{k}+\mathrm{q}}) \quad (0.166)$$

Recall the *shifted orthogonality* relation for Hermite polynomials [4, Eq. (7.377)]

$$\int_{-\infty}^{\infty} \mathrm{d}x e^{-x^2} H_m(x+y)H_n(x+z) = 2^n \pi^{\frac{1}{2}} m! y^{n-m} L_m^{n-m}(-2yz), \quad m \le n, \quad (0.167)$$

where L_b^a is the *generalized Laguerre polynomial* of order b and type a. Using that

$$a_{\boldsymbol{k}ms} b_{\boldsymbol{k}+\mathrm{q}ns} = \sqrt{\alpha}\frac{\alpha_{\boldsymbol{k}ms}}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}}\left[2^{N+M-1}(M-1)!N!\pi l_B^2\right]^{-\frac{1}{2}} \quad (0.168)$$

$$b_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathrm{q}ns} = \sqrt{\alpha}\frac{\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}}\left[2^{N+M-1}(N-1)!M!\pi l_B^2\right]^{-\frac{1}{2}} \quad (0.169)$$

$$a_{\boldsymbol{k}ms} a_{\boldsymbol{k}+\mathrm{q}ns} = \sqrt{\alpha}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}}\left[2^{N+M-2}(M-1)!(N-1)!\pi l_B^2\right]^{-\frac{1}{2}} \quad (0.170)$$

$$b_{\boldsymbol{k}ms} b_{\boldsymbol{k}+\mathrm{q}ns} = \sqrt{\alpha}\frac{1}{\sqrt{\alpha_{\boldsymbol{k}ms}^2+1}\sqrt{\alpha_{\boldsymbol{k}+\mathrm{q}ns}^2+1}}\left[2^{N+M}M!N!\pi l_B^2\right]^{-\frac{1}{2}} \quad (0.171)$$

$$(0.172)$$

NB!! definition changed compared to project thesis. Here, we have dropped the α factor

Verify if we need to add some factors due to the tilt.

$$\Xi_1^{(1)}(\boldsymbol{q},m,n,s) = \sqrt{\frac{2^N(M-1)!}{2^{M-1}N!}}\left(\frac{q'_x - iq_y}{2\sqrt{\alpha}}l_B\right)^{N-M+1} L_{M-1}^{N-M+1}\left(\frac{\mathfrak{q}_y^2 l_B^2}{2\alpha}\right), \quad (0.173)$$

$$\Xi_1^{(2)}(\boldsymbol{q},m,n,s) = \sqrt{\frac{2^{M-1}N!}{2^N(M-1)!}}\left(\frac{-q'_x - iq_y}{2\sqrt{\alpha}}l_B\right)^{M-N-1} L_{N}^{M-N-1}\left(\frac{\mathfrak{q}_y^2 l_B^2}{2\alpha}\right), \quad (0.174)$$

$$\Xi_1(\boldsymbol{q},m,n,s) = \begin{cases} \Xi_1^{(1)} & \text{if } N \geq M-1 \\ \Xi_1^{(2)} & \text{if } N \leq M-1 \end{cases}, \quad (0.175)$$

$$\Xi_2^{(1)}(\boldsymbol{q},m,n,s) = \sqrt{\frac{2^{N-1}M!}{2^M(N-1)!}}\left(\frac{q'_x - iq_y}{2\sqrt{\alpha}}l_B\right)^{N-1-M} L_{M}^{N-1-M}\left(\frac{\mathfrak{q}_y^2 l_B^2}{2\alpha}\right), \quad (0.176)$$

$$\Xi_2^{(2)}(\boldsymbol{q},m,n,s) = \sqrt{\frac{2^M(N-1)!}{2^{N-1}M!}}\left(\frac{-q'_x - iq_y}{2\sqrt{\alpha}}l_B\right)^{M-N+1} L_{N-1}^{M-N+1}\left(\frac{\mathfrak{q}_y^2 l_B^2}{2\alpha}\right), \quad (0.177)$$

$$\Xi_2(\boldsymbol{q},m,n,s) = \begin{cases} \Xi_2^{(1)} & \text{if } N-1 \geq M \\ \Xi_2^{(2)} & \text{if } N-1 \leq M \end{cases}, \quad (0.178)$$

Here, $\mathfrak{q}_y = (q'_x, q_y)$. **In the future, might be nice to go over to having only one function, Xi1, a**

Now we will consider the second term of the current operator.

$$J^{(2)}_{\boldsymbol{k}ms;\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) = \int \mathrm{d}y e^{-iq_y y} e\phi^*_{\boldsymbol{k}ms}(y)\omega_{0x}\phi_{\boldsymbol{k}+\mathfrak{q}ns}(y). \quad (0.179)$$

Thus

$$J^{(2)}_{\boldsymbol{k}ms;\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) = \frac{ev_F t_x}{\mathcal{N}^*\mathcal{N}}\int \mathrm{d}y \exp\{-iq_y y - \frac{1}{2}\chi^2_{\boldsymbol{k}} - \frac{1}{2}\chi^2_{\boldsymbol{k}+\mathfrak{q}}\} \tilde{\phi}^*_{\boldsymbol{k}ms}(y)e^{\theta\sigma_x}\tilde{\phi}_{\boldsymbol{k}+\mathfrak{q}ns}(y). \quad (0.180)$$

Using the same substitution and completion of the square as above, this is

$$J^{(2)}_{\boldsymbol{k}ms;\boldsymbol{k}+\mathfrak{q}ns}(\boldsymbol{q}) = \frac{ev_F t_x l_B}{\sqrt{\alpha}}\int \mathrm{d}\tilde{y} \exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 + 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right] e^{-\tilde{y}^2}\big[a_{\boldsymbol{k}ms}H_{M-1}(\chi_{\boldsymbol{k}})\left(a_{\boldsymbol{k}+\mathfrak{q}ns}H_{N-1}(\chi_{\boldsymbol{k}+\mathfrak{q}}) - st_x b_{\boldsymbol{k}+\mathfrak{q}ns}H_N(\chi_{\boldsymbol{k}+\mathfrak{q}})\right) + b_{\boldsymbol{k}ms}H_M(\chi_{\boldsymbol{k}})\left(-st_x a_{\boldsymbol{k}+\mathfrak{q}ns}H_{N-1}(\chi_{\boldsymbol{k}+\mathfrak{q}}) + b_{\boldsymbol{k}+\mathfrak{q}ns}H_N(\chi_{\boldsymbol{k}+\mathfrak{q}})\right)\big]. \quad (0.181)$$

Thus [4]

$$J^{(2)}_{\boldsymbol{k}ms;\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) = ev_F t_x \frac{\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 + 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right]}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1}}$$
$$\Big[- st_x[\alpha_{\boldsymbol{k}ms}\Xi_1(\boldsymbol{q},m,n) + \alpha_{\boldsymbol{k}+\mathrm{q}ns}\Xi_2(\boldsymbol{q},m,n)] + \alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathrm{q}ns}\Xi_1(\boldsymbol{q},m,n\mp 1) + \Xi_2(\boldsymbol{q},m,n\pm 1)\Big]. \quad (0.182)$$

We notice that this part has the same form as $J^{(1)}$, with only a change of the prefactors of the Ξ-functions. *Note to self:* I believe it is here important to consider if we have an inversion symmetric or inversion unsymmetric case. In the former, this makes everything easier, as the nasty terms are dropped.

$$J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) = ev_F s\alpha^2 \frac{\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 + 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right]}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1}}$$
$$[\alpha_{\boldsymbol{k}ms}\Xi_1(\boldsymbol{q},m,n) + \alpha_{\boldsymbol{k}+\mathrm{q}ns}\Xi_2(\boldsymbol{q},m,n)]. \quad (0.183)$$

We used here that $1 - t_x^2 = \alpha^2$.

Stress-energy tensor

Consider now

$$T^{0y(1)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{1}{4}\int \mathrm{d}y e^{iq_y y}\phi^*_{\boldsymbol{k}+\mathrm{q}ns}(y) s\sigma^y (E_{k\mu s} + E_{\lambda\nu s} - 2\mu)\phi_{\boldsymbol{k}ms}(y). \quad (0.184)$$

As

$$\sigma_y e^{\theta/2\sigma_x} = e^{-\theta/2\sigma_x}\sigma_y \quad (0.185)$$

we get the very fortunate result

$$\phi^*\sigma_y\phi = \frac{1}{\mathcal{N}^*\mathcal{N}}\tilde{\phi}^*\sigma_y\tilde{\phi}. \quad (0.186)$$

The first term of the stress-energy tensor thus has the exact same form as the untilted case. Recalling the expression for $\tilde{\phi}$ from Eq. (0.154),

$$\tilde{\phi} = e^{-\frac{1}{2}\chi^2}\begin{pmatrix} a_{\boldsymbol{k}ms}H_{M-1}(\chi) \\ b_{\boldsymbol{k}ms}H_M(\chi)\end{pmatrix},$$

where

$$\chi = \sqrt{\alpha}(y - q_x l_B^2)/l_B - \mathrm{sign}(m)\beta\sqrt{2|m| + \frac{k_z^2 l_B^2}{\alpha}}.$$

[4]Note to self: note that we dropped the $\frac{1}{\sqrt{\alpha}}$ for the $\sqrt{\alpha}$ coming from the Ξ definition.

We thus get

$$T^{0y(1)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{is}{4\mathcal{N}^*\mathcal{N}}(E_{k\mu s} + E_{\lambda\nu s} - 2\mu)\int \mathrm{d}y e^{iq_y y} e^{-\frac{1}{2}(\chi^2_{\boldsymbol{k}+\mathsf{q}}+\chi^2_{\boldsymbol{k}})}$$
$$[-a_{\boldsymbol{k}+\mathsf{q}ns}b_{\boldsymbol{k}ms}H_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}})H_M(\chi_{\boldsymbol{k}}) + b_{\boldsymbol{k}+\mathsf{q}ns}a_{\boldsymbol{k}ms}H_N(\chi_{\boldsymbol{k}+\mathsf{q}})H_{M-1}(\chi_{\boldsymbol{k}})]. \quad (0.187)$$

We will perform once again the completion of the square and substituion of y. The exponent is the same as that which we found for the current operator case, Eq. (0.158), with the change $q_y \to -q_y$. We thus make the change of variables

$$\tilde{y} = \frac{\sqrt{\alpha}}{l_B}\left(y - \frac{l_B^2}{2\alpha}(iq_y + (2k'_x + q'_x))\right), \quad (0.188)$$

giving

$$\chi_{\boldsymbol{k}} = \tilde{y} + \frac{l_B}{2\sqrt{\alpha}}\left(q'_x + iq_y\right), \quad (0.189)$$

$$\chi_{\boldsymbol{k}+\mathsf{q}} = \tilde{y} + \frac{l_B}{2\sqrt{\alpha}}\left(-q'_x + iq_y\right). \quad (0.190)$$

Thus, analogous to Eq. (4.79), we get

$$T^{0y(1)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) =$$
$$\frac{is}{4\mathcal{N}^*\mathcal{N}\sqrt{\alpha}}(E_{k\mu s} + E_{\lambda\nu s} - 2\mu)\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 - 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right]\int \mathrm{d}\tilde{y} l_B e^{-\tilde{y}^2}$$
$$\Big[-a_{\boldsymbol{k}+\mathsf{q}ns}b_{\boldsymbol{k}ms}H_{N-1}(\chi_{\boldsymbol{k}})H_M(\chi_{\boldsymbol{k}+\mathsf{q}}) + b_{\boldsymbol{k}+\mathsf{q}ns}a_{\boldsymbol{k}ms}H_N(\chi_{\boldsymbol{k}})H_{M-1}(\chi_{\boldsymbol{k}+\mathsf{q}})\Big] \quad (0.191)$$

And thus we have

$$T^{0y(1)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{is\alpha}{4}\frac{E_{k\mu s} + E_{\lambda\nu s} - 2\mu}{\sqrt{\alpha^2_{\boldsymbol{k}ms}+1}\sqrt{\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1}} \quad (0.192)$$

$$\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 - 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right] \quad (0.193)$$

$$(-\alpha_{\boldsymbol{k}+\mathsf{q}ns}\Xi_2(\bar{\boldsymbol{q}}, m, n, s) + \alpha_{\boldsymbol{k}ms}\Xi_1(\bar{\boldsymbol{q}}, m, n, s)), \quad (0.194)$$

where $\bar{\boldsymbol{q}} = (q_x, -q_y, q_z)$.

We must now consider the latter parts of the stress energy tensor

$$T^{0y(2)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = +\frac{1}{4}\int \mathrm{d}y e^{iq_y y} v_F \phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y) p_y \phi_{\boldsymbol{k}ms}(y), \quad (0.195)$$

$$T^{0y(3)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = -\frac{1}{4}\int \mathrm{d}y e^{iq_y y} v_F (p_y \phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y))\phi_{\boldsymbol{k}ms}(y). \quad (0.196)$$

Firstly, we note that

$$[p_y, e^{\theta/2\sigma_x}] = 0.$$

Furthermore, exactly as for the untilted case, the momentum operator acting on the exponential prefactor of ϕ gives contributions proportional to q_x. In the local limit $q \to 0$ this term vanishes, and we need only consider the effect of the momentum operator acting on the Hermite polynomials.

Denote by $\tilde{p}_y$ the momentum operator p_y acting only on the Hermite polynomial part of ϕ. Furthermore, we will use the property of Hermite polynomials $\partial_x H_n(x) = 2nH_{n-1}(x)$ [7, Eq. 18.9.25].

$$\tilde{p}_y \phi_{\boldsymbol{k}ms} = -i\hbar e^{\theta/2\sigma_x} e^{-\frac{1}{2}\chi^2} \partial_y \begin{pmatrix} a_{\boldsymbol{k}ms} H_{M-1}(\chi) \\ b_{\boldsymbol{k}ms} H_M(\chi) \end{pmatrix} \tag{0.197}$$

$$= -i\hbar e^{\theta/2\sigma_x} e^{-\frac{1}{2}\chi^2} 2\frac{\partial \chi}{\partial y} \begin{pmatrix} a_{\boldsymbol{k}ms}(M-1) H_{M-2}(\chi) \\ b_{\boldsymbol{k}ms}(M) H_{M-1}(\chi) \end{pmatrix} \tag{0.198}$$

$$= -i\hbar e^{\theta/2\sigma_x} e^{-\frac{1}{2}\chi^2} \frac{2\sqrt{\alpha}}{l_B} \begin{pmatrix} a_{\boldsymbol{k}ms}(M-1) H_{M-2}(\chi) \\ b_{\boldsymbol{k}ms}(M) H_{M-1}(\chi) \end{pmatrix}. \tag{0.199}$$

And thus, recalling that

$$e^{\theta\sigma_x} = \begin{pmatrix} 1 & -t_x \\ -t_x & 1 \end{pmatrix} \frac{1}{\sqrt{1-t_x^2}},$$

we find the product

$$\begin{aligned} \phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y) \tilde{p}_y \phi_{\boldsymbol{k}ms} = &-\frac{i\hbar 2\sqrt{\alpha}}{l_B\sqrt{1-t_x^2}} e^{-\frac{1}{2}\chi^2_{\boldsymbol{k}} - \frac{1}{2}\chi^2_{\boldsymbol{k}+\mathsf{q}}} \\ &\Big[a_{\boldsymbol{k}+\mathsf{q}ns} H_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}}) \left\{a_{\boldsymbol{k}ms}(M-1)H_{M-2}(\chi_{\boldsymbol{k}}) - t_x b_{\boldsymbol{k}ms} M H_{M-1}(\chi_{\boldsymbol{k}})\right\} \\ &+ b_{\boldsymbol{k}+\mathsf{q}ns} H_N(\chi_{\boldsymbol{k}+\mathsf{q}}) \left\{-t_x a_{\boldsymbol{k}ms}(M-1)H_{M-2}(\chi_{\boldsymbol{k}}) + b_{\boldsymbol{k}ms} M H_{M-1}(\chi_{\boldsymbol{k}})\right\}\Big]. \end{aligned} \tag{0.200}$$

Completing the square and substituting

$$\tilde{y} = \frac{\sqrt{\alpha}}{l_B}\left(y - \frac{l_B^2}{2\alpha}(iq_y + (2k'_x + q'_x))\right)$$

gives

$$\begin{aligned} \int \mathrm{d}y e^{iq_y} \phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y) \tilde{p}_y \phi_{\boldsymbol{k}ms}(y) = &-\frac{i\hbar 2\sqrt{\alpha}}{l_B\sqrt{1-t_x^2}} \exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 - 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right] \\ &\int \mathrm{d}\tilde{y} \frac{l_B}{\sqrt{\alpha}} \\ &\Big[a_{\boldsymbol{k}+\mathsf{q}ns} H_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}}) \left\{a_{\boldsymbol{k}ms}(M-1)H_{M-2}(\chi_{\boldsymbol{k}}) - t_x b_{\boldsymbol{k}ms} M H_{M-1}(\chi_{\boldsymbol{k}})\right\} \\ &+ b_{\boldsymbol{k}+\mathsf{q}ns} H_N(\chi_{\boldsymbol{k}+\mathsf{q}}) \left\{-t_x a_{\boldsymbol{k}ms}(M-1)H_{M-2}(\chi_{\boldsymbol{k}}) + b_{\boldsymbol{k}ms} M H_{M-1}(\chi_{\boldsymbol{k}})\right\}\Big]. \end{aligned} \tag{0.201}$$

We must now evaluate the integral, and express the result in the Ξ-functions.

$$\begin{pmatrix} a_{\boldsymbol{k}+\mathsf{q}ns} H_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}}) \\ b_{\boldsymbol{k}+\mathsf{q}ns} H_N(\chi_{\boldsymbol{k}+\mathsf{q}}) \end{pmatrix}^T \underbrace{\begin{pmatrix} 1 & -t_x \\ -t_x & 1 \end{pmatrix}}_{T} \begin{pmatrix} a_{\boldsymbol{k}ms}(M-1) H_{M-2}(\chi_{\boldsymbol{k}}) \\ b_{\boldsymbol{k}ms} M H_{M-1}(\chi_{\boldsymbol{k}}) \end{pmatrix}$$

For each of the entries in T, we get a product on of Hermite polynomials. Where the untilted cone had two such terms, the tilt parameter t_x now gives two extra products, which we must evaluate. Let $M_{ij}^{(2)}$ be the product corresponding to T_{ij}, i.e.

$$M_{11}^{(2)} = \quad a_{\boldsymbol{k}+\mathfrak{q}ns} a_{\boldsymbol{k}ms} (M-1) H_{N-1}(\chi_{\boldsymbol{k}+\mathfrak{q}}) H_{M-2}(\chi_{\boldsymbol{k}}), \tag{0.202}$$

$$M_{12}^{(2)} = -t_x a_{\boldsymbol{k}+\mathfrak{q}ns} b_{\boldsymbol{k}ms} M H_{N-1}(\chi_{\boldsymbol{k}+\mathfrak{q}}) H_{M-1}(\chi_{\boldsymbol{k}}), \tag{0.203}$$

$$M_{21}^{(2)} = -t_x b_{\boldsymbol{k}+\mathfrak{q}ns} a_{\boldsymbol{k}ms} (M-1) H_{N}(\chi_{\boldsymbol{k}+\mathfrak{q}}) H_{M-2}(\chi_{\boldsymbol{k}}), \tag{0.204}$$

$$M_{22}^{(2)} = \quad b_{\boldsymbol{k}+\mathfrak{q}ns} b_{\boldsymbol{k}ms} M H_{N}(\chi_{\boldsymbol{k}+\mathfrak{q}}) H_{M-1}(\chi_{\boldsymbol{k}}). \tag{0.205}$$

We want to evaluate

$$F_{ij}^{(2)} = \left[(\alpha_{\boldsymbol{k}ms}^2 + 1)(\alpha_{\boldsymbol{k}+\mathfrak{q}ns}^2 + 1)\right]^{\frac{1}{2}} \int \mathrm{d}\tilde{y} e^{-\tilde{y}^2} M_{ij}^{(2)}, \tag{0.206}$$

with the prefactor introduced for later convenience.

Notice that **Verify l_B in this section**

$$F_{12}^{(2)} = -t_x \sqrt{\alpha} \sqrt{\frac{M}{2}} \alpha_{k+q,n} \Xi_2(\bar{\boldsymbol{q}}, m \mp 1, n). \tag{0.207}$$

and

$$F_{21}^{(2)} = -t_x \sqrt{\alpha} \sqrt{\frac{M-1}{2}} \frac{a_{\boldsymbol{k}ms}^2}{l_B a_{\boldsymbol{k}m\mp 1s}} \Xi_1(\bar{\boldsymbol{q}}, m \mp 1, n, s). \tag{0.208}$$

$F_{11}^{(2)}$ and $F_{22}^{(2)}$ are the same as for the untilted case:

$$F_{11}^{(2)} = \sqrt{\alpha} \frac{\alpha_{\boldsymbol{k}ms} \alpha_{\boldsymbol{k}+\mathfrak{q}ns} \sqrt{M-1}}{l_B \sqrt{2}} \Xi_1(\bar{\boldsymbol{q}}, m \mp 1, n \mp 1, s), \tag{0.209}$$

and

$$F_{22}^{(2)} = \sqrt{\alpha} \frac{\sqrt{M}}{l_B \sqrt{2}} \Xi_1(\bar{\boldsymbol{q}}, m, n, s). \tag{0.210}$$

In summary we have

$$T^{0y\ (2)}_{\boldsymbol{k}+\mathfrak{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = +\frac{v_F}{4} \int \mathrm{d}y e^{iq_y q} \phi^*_{\boldsymbol{k}+\mathfrak{q}ns}(y) p_y \phi_{\boldsymbol{k}ms}(y) \tag{0.211}$$

$$= -\frac{i\hbar v_F}{2} \Gamma^+_{\boldsymbol{k}\mathfrak{q}mns} \sum_{i,j} F_{ij}^{(2)}, \tag{0.212}$$

where

$$\Gamma^+_{\boldsymbol{k}\mathfrak{q}mns} = \frac{\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 - 2i(2k'_x + q'_x)q_y + (q'_x)^2)\right]}{\left[(\alpha_{\boldsymbol{k}ms}^2 + 1)(\alpha_{\boldsymbol{k}+\mathfrak{q}ns}^2 + 1)\right]^{\frac{1}{2}} \sqrt{1 - t_x^2}}$$

In a similar procedure, we find $T^{0y\ (2)}_{\boldsymbol{k}+\mathsf{q}ns,\boldsymbol{k}ms}(\boldsymbol{q})$.

$$\tilde{p}_y\phi^*_{\boldsymbol{k}+\mathsf{q}ms} = \frac{-i\hbar\sqrt{\alpha}}{l_B}e^{-\frac{1}{2}\chi^2}\begin{pmatrix} a_{\boldsymbol{k}+\mathsf{q}ms}(M-1)H_{M-2}(\chi) \\ b_{\boldsymbol{k}+\mathsf{q}ms}(M)H_{M-1}(\chi)\end{pmatrix}. \tag{0.213}$$

And thus,

$$\begin{aligned}\left(\tilde{p}_y\phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y)\right)\phi_{\boldsymbol{k}ms} &= -\frac{i\hbar 2\sqrt{\alpha}}{l_B\sqrt{1-t_x^2}}e^{-\frac{1}{2}\chi^2_{\boldsymbol{k}}-\frac{1}{2}\chi^2_{\boldsymbol{k}+\mathsf{q}}} \\ &\Big[a_{\boldsymbol{k}+\mathsf{q}ns}(N-1)H_{N-2}(\chi_{\boldsymbol{k}+\mathsf{q}})\left\{a_{\boldsymbol{k}ms}H_{M-1}(\chi_{\boldsymbol{k}})-t_xb_{\boldsymbol{k}ms}H_M(\chi_{\boldsymbol{k}})\right\} \\ &\quad + b_{\boldsymbol{k}+\mathsf{q}ns}NH_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}})\left\{-t_xa_{\boldsymbol{k}ms}H_{M-1}(\chi_{\boldsymbol{k}})+b_{\boldsymbol{k}ms}H_M(\chi_{\boldsymbol{k}})\right\}\Big].\end{aligned} \tag{0.214}$$

With the now well-known completion of the square and substitution, we have

$$\begin{aligned}\int \mathrm{d}ye^{iq_y}\left[\tilde{p}_y\phi^*_{\boldsymbol{k}+\mathsf{q}ns}(y)\right]\phi_{\boldsymbol{k}ms}(y) &= -\frac{i\hbar 2\sqrt{\alpha}}{l_B\sqrt{1-t_x^2}}\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2-2i(2k'_x+q'_x)q_y+(q'_x)^2)\right] \\ &\int \mathrm{d}\tilde{y}\frac{l_B}{\sqrt{\alpha}} \\ &\Big[a_{\boldsymbol{k}+\mathsf{q}ns}(N-1)H_{N-2}(\chi_{\boldsymbol{k}+\mathsf{q}})\left\{a_{\boldsymbol{k}ms}H_{M-1}(\chi_{\boldsymbol{k}})-t_xb_{\boldsymbol{k}ms}H_M(\chi_{\boldsymbol{k}})\right\} \\ &\quad + b_{\boldsymbol{k}+\mathsf{q}ns}NH_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}})\left\{-t_xa_{\boldsymbol{k}ms}H_{M-1}(\chi_{\boldsymbol{k}})+b_{\boldsymbol{k}ms}H_M(\chi_{\boldsymbol{k}})\right\}\Big].\end{aligned} \tag{0.215}$$

Denote the terms of the integrand by

$$M_{11}^{(3)} = a_{\boldsymbol{k}+\mathsf{q}ns}a_{\boldsymbol{k}ms}(N-1)H_{N-2}(\chi_{\boldsymbol{k}+\mathsf{q}})H_{M-1}(\chi_{\boldsymbol{k}}), \tag{0.216}$$

$$M_{12}^{(3)} = -t_xa_{\boldsymbol{k}+\mathsf{q}ns}b_{\boldsymbol{k}ms}(N-1)H_{N-2}(\chi_{\boldsymbol{k}+\mathsf{q}})H_M(\chi_{\boldsymbol{k}}), \tag{0.217}$$

$$M_{21}^{(3)} = -t_xb_{\boldsymbol{k}+\mathsf{q}ns}a_{\boldsymbol{k}ms}NH_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}})H_{M-1}(\chi_{\boldsymbol{k}}), \tag{0.218}$$

$$M_{22}^{(3)} = b_{\boldsymbol{k}+\mathsf{q}ns}b_{\boldsymbol{k}ms}NH_{N-1}(\chi_{\boldsymbol{k}+\mathsf{q}})H_M(\chi_{\boldsymbol{k}}). \tag{0.219}$$

We must evaluate

$$F_{ij}^{(3)} = \left[(\alpha^2_{\boldsymbol{k}ms}+1)(\alpha^2_{\boldsymbol{k}+\mathsf{q}ns}+1)\right]^{\frac{1}{2}}\int \mathrm{d}\tilde{y}e^{-\tilde{y}^2}M_{ij}^{(3)}. \tag{0.220}$$

From the untilted case we know

$$F_{11}^{(3)} = \sqrt{\frac{N-1}{2}}\frac{\alpha_{\boldsymbol{k}ms}\alpha_{\boldsymbol{k}+\mathsf{q}ns}}{l_B\alpha_{\boldsymbol{k}+\mathsf{q}n\mp 1s}}\Xi_2(\bar{\boldsymbol{q}}, m\mp 1, n\mp 1, s), \tag{0.221}$$

$$F_{22}^{(3)} = \sqrt{\frac{N}{2}}\frac{1}{l_B\alpha_{\boldsymbol{k}+\mathsf{q}ns}}\Xi_2(\bar{\boldsymbol{q}}, m, n, s). \tag{0.222}$$

Furthermore,

$$F_{12}^{(3)} = -t_x \frac{\alpha_{\boldsymbol{k}+\mathrm{q}n}}{\alpha_{\boldsymbol{k}+\mathrm{q}n\mp1} l_B} \sqrt{\frac{N-1}{2}} \Xi_2(\bar{\boldsymbol{q}}, m, n\mp1, s), \tag{0.223}$$

$$F_{21}^{(3)} = -\frac{t_x}{l_B} \sqrt{\frac{N}{2}} \frac{\alpha_{\boldsymbol{k}m}}{\alpha_{\boldsymbol{k}+\mathrm{q}n}} \Xi_2(\bar{\boldsymbol{q}}, m\mp1, n, s). \tag{0.224}$$

We thus have

$$T^{0y\ (3)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = -\frac{v_F}{4} \int \mathrm{d}y e^{iq_y y} \left(p_y \phi^*_{\boldsymbol{k}+\mathrm{q}ns}(y)\right) \phi_{\boldsymbol{k}ms}(y) \tag{0.225}$$

$$= \frac{i\hbar v_F}{2} \Gamma^+_{\boldsymbol{k}\mathrm{q}mns} \sum_{ij} F_{ij}^{(3)}. \tag{0.226}$$

Remember normalization factor $\mathcal{N}$!!! **squeezing figure**
Fix anisotropic directions
Squeezing, etc

We will boost the frame of the tilted cone such that the system is again isotropic, and use the well known solution for the Landau levels in that frame. The issue is that the boost is only valid for certain directions of the field. Add pretty figures describing the situation.

In summary we have

$$J_{\boldsymbol{k}ms;\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) = v_F e s \alpha^2 \Gamma^-_{\boldsymbol{k}\mathrm{q}mns} \left[\alpha_{\boldsymbol{k}ms} \Xi_1(\boldsymbol{q}, m, n, s) + \alpha_{\boldsymbol{k}+\mathrm{q}ns} \Xi_2(\boldsymbol{q}, m, n, s)\right], \tag{0.227}$$

$$T^{0y(1)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{is\alpha}{4}(E_{k\mu s} + E_{\lambda\nu s} - 2\mu)\Gamma^+_{\boldsymbol{k}\mathrm{q}mns} \tag{0.228}$$

$$(-\alpha_{\boldsymbol{k}+\mathrm{q}ns} \Xi_2(\bar{\boldsymbol{q}}, m, n, s) + \alpha_{\boldsymbol{k}ms} \Xi_1(\bar{\boldsymbol{q}}, m, n, s)), \tag{0.229}$$

$$T^{0y\ (2)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = -\frac{i\hbar v_F}{2} \Gamma^+_{\boldsymbol{k}\mathrm{q}mns} \sum_{i,j} F_{ij}^{(2)}, \tag{0.230}$$

$$T^{0y\ (3)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{i\hbar v_F}{2} \Gamma^+_{\boldsymbol{k}\mathrm{q}mns} \sum_{ij} F_{ij}^{(3)}. \tag{0.231}$$

with

$$\Gamma^{\pm}_{\boldsymbol{k}\mathrm{q}mns} = \frac{\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2 + (q_x')^2) \pm i q_y l_B^2 (k_x' + \frac{q_x'}{2}))\right]}{\left[(\alpha^2_{\boldsymbol{k}ms} + 1)(\alpha^2_{\boldsymbol{k}+\mathrm{q}ns} + 1)\right]^{\frac{1}{2}}}$$

0.5.1 Static limit and dimensionless form

We are interested in the response in the static limit $\boldsymbol{q} \to 0$. We may use the property of limits that

$$\lim_{n\to a} A \cdot B = \lim_{n\to a} A \cdot \lim_{n\to a} B.$$

We are thus interested in the current and energy-momentum matrix elements in the static limit. Furthermore, we will write them in dimensionless quantities.

Consider firstly the exponent in the $\Gamma^{\pm}$ factor,

$$\exp\left[-\frac{l_B^2}{4\alpha}(q_y^2+(q_x')^2)\pm iq_y l_B^2(k_x'+\frac{q_x'}{2}))\right].$$

Recall the definition Eq. (0.159),

$$q_x' = q_x\alpha - \frac{\beta}{v_F}(E^0_{n,\alpha B}-E^0_{m,\alpha B}).$$

In the static limit

$$\lim_{\boldsymbol{q}\to 0} q_x' = -\frac{\beta}{v_F}(E^0_{n,\alpha B}-E^0_{m,\alpha B}),$$

and thus the for the exponent one has in the limit

$$\exp\left[-\frac{l_B^2\beta^2}{4\alpha v_F^2}(E^0_{n,\alpha B}-E^0_{m,\alpha B})^2\right].$$

Expressed in the dimensionless energies $\epsilon = \frac{E}{v_F\sqrt{2eB}}$

$$\exp\left[-\frac{\beta^2}{2\alpha}(\epsilon^0_{n,\alpha B}-\epsilon^0_{m,\alpha B})^2\right].$$

The normalization factor $\alpha_{k_z ms}$ is independent on $\boldsymbol{q}$, and already dimensionless. Explicity, it is given in dimensionless quantities as

$$\alpha_{k_z ms} = -\frac{\sqrt{2e\alpha BM}}{\frac{E_{k_z ms}-t_\parallel v_F k_z}{v_F s\alpha}-k_z} = -\frac{\sqrt{\alpha M}}{s\epsilon^0_{m,\alpha B}-\kappa}. \tag{0.232}$$

In the tilted case, the Ξ functions do not have a trivial form in the static limit, as was the case in the untilted case. Define

$$P = \lim_{\boldsymbol{q}\to 0}\frac{l_B q_x'}{\sqrt{2\alpha}} = \frac{\beta}{\sqrt{\alpha}}(\epsilon^0_{n,\alpha B}-\epsilon^0_{m,\alpha B}).$$

In the static limit, the Ξ functions thus take the form

$$\Xi_1^{(1)}(m,n,s) = \sqrt{\frac{2^N(M-1)!}{2^{M-1}N!}}\left(\frac{P}{\sqrt{2}}\right)^{N-M+1} L_{M-1}^{N-M+1}\left(P^2\right), \tag{0.233}$$

$$\Xi_1^{(2)}(\boldsymbol{q},m,n,s) = \sqrt{\frac{2^{M-1}N!}{2^N(M-1)!}}\left(-\frac{P}{\sqrt{2}}\right)^{M-N-1} L_N^{M-N-1}\left(P^2\right), \tag{0.234}$$

$$\Xi_1(\boldsymbol{q},m,n,s) = \begin{cases}\Xi_1^{(1)} & \text{if } N\geq M-1\\ \Xi_1^{(2)} & \text{if } N\leq M-1\end{cases}, \tag{0.235}$$

$$\Xi_2^{(1)}(\boldsymbol{q}, m, n, s) = \sqrt{\frac{2^{N-1}M!}{2^M(N-1)!}} \left(\frac{P}{\sqrt{2}}\right)^{N-1-M} L_M^{N-1-M}\left(P^2\right), \tag{0.236}$$

$$\Xi_2^{(2)}(\boldsymbol{q}, m, n, s) = \sqrt{\frac{2^M(N-1)!}{2^{N-1}M!}} \left(-\frac{P}{\sqrt{2}}\right)^{M-N+1} L_{N-1}^{M-N+1}\left(P^2\right), \tag{0.237}$$

$$\Xi_2(\boldsymbol{q}, m, n, s) = \begin{cases} \Xi_2^{(1)} & \text{if } N-1 \geq M \\ \Xi_2^{(2)} & \text{if } N-1 \leq M \end{cases}. \tag{0.238}$$

0.5.2 Solving the kx integral – draft

We have the response function **comment limits (q-¿0, omega -¿0)** **Make shure there are no alpha**

$$\lim_{\omega\to 0}\lim_{\boldsymbol{q}\to 0} \chi^{xy}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{eBiv_F}{(2\pi)^2} \sum_{mn} \int \mathrm{d}k_z [n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathrm{q}ns}] \frac{J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) T^{0y\ (i)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q})}{(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathrm{q}ns} + i\eta)(E_{\boldsymbol{k}ms} - E_{\boldsymbol{k}+\mathrm{q}ns} + i\eta)}.$$

Writing out the matrix products we have

$$J^x_{\boldsymbol{k}ms,\boldsymbol{k}+\mathrm{q}ns}(\boldsymbol{q}) T^{0y\ (i)}_{\boldsymbol{k}+\mathrm{q}ns,\boldsymbol{k}ms}(\boldsymbol{q}) = \frac{v_F e i \alpha^3}{4} e^{-P^2} \frac{(E_{\boldsymbol{k}ms} + E_{\boldsymbol{k}+\mathrm{q}ns})(\alpha^2_{\boldsymbol{k}ms}\Xi_1(m,n)^2 - \alpha^2_{\boldsymbol{k}+\mathrm{q}ns}\Xi_2(m,n)^2)}{(\alpha^2_{\boldsymbol{k}ms}+1)(\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1)} \tag{0.239}$$

$$\lim_{\omega\to 0}\lim_{\boldsymbol{q}\to 0} \chi^{xy}(\omega, \boldsymbol{q}) = \lim_{\eta\to 0} \frac{-e\alpha^3 v_F\sqrt{eB}}{4(2\pi)^2\sqrt{2}} \sum_{mn} \int \mathrm{d}k_z e^{-P^2} \frac{[n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathrm{q}ns}](\epsilon_{\boldsymbol{k}ms} + \epsilon_{\boldsymbol{k}+\mathrm{q}ns})(\alpha^2_{\boldsymbol{k}ms}\Xi_1(m,n)^2 - \alpha^2_{\boldsymbol{k}+\mathrm{q}ns}\Xi_2(m,n)^2)}{(\alpha^2_{\boldsymbol{k}ms}+1)(\alpha^2_{\boldsymbol{k}+\mathrm{q}ns}+1)(\epsilon_{\boldsymbol{k}ms} - \epsilon_{\boldsymbol{k}+\mathrm{q}ns} + i\eta)^2} \tag{0.240}$$

We make the observation that $\Xi_1(M,N) = \Xi_2(N,M)$. The rest of the factors are invariant under the interchange $m \leftrightarrow n$, except for the step functions, which gives an overall sign change. Thus, using $\Xi_1(M,N) = \Xi_2(N,M)$ and relabelling the summation indices we may consider

$$\alpha^2_{\kappa_z ms}\Xi_1^2 - \alpha^2_{\kappa_z ns} \to 2\alpha^2_{\kappa_z ms}\Xi_1^2.$$

We may also simplify the step function expression. Physically, the step function term corresponds to only considering overlap between states with energies of opposite sign. Restricting ourselves to Type-I for now, the energy of the state with quantum number n has the same sign as n itself, excluding of course the zeroth state. For the zeroth state, the sign of the energy is $\mathrm{sign}(-s\kappa_z)$. Using these considerations, we may make certain seletion rules for the sum. In the (m,n)-plane, the first and third quadrant give no contribution, as there $mn > 0$, i.e. they have the same sign. Our sum is thus restricted to the second and fourth quadrant. We have

$$n_{\boldsymbol{k}ms} - n_{\boldsymbol{k}+\mathfrak{q}ns} = \begin{cases} \quad 0 & mn > 0 \text{ or } m, n = 0 \\ -\operatorname{sign}(m) & m, n \neq 0 \\ \quad \operatorname{sign}(n)\theta(\operatorname{sign}(n)s\kappa) & m = 0 \\ -\operatorname{sign}(m)\theta(\operatorname{sign}(m)s\kappa) & n = 0 \end{cases} . \tag{0.241}$$

Furthermore, the contributions from those quadrants are equal, which we will now show.

The mapping $(m, n) \mapsto (-m, -n)$, i.e. a π rotation, transforms points in the second quadrant to the fourth quadrant. We want to consider how the integrand in question transforms under such a mapping. First of all, recall that

$$\alpha_{\kappa_z m} = \frac{-\sqrt{M}}{\epsilon_{\kappa_z m}/s - \kappa_z},$$

and thus $\alpha_{\kappa_z m} = -\alpha_{-\kappa_z -m}$. We consider this factor squared, so no overall sign is introduced. The sign of κ_z is however flipped, but no issue as we integrate over $(-\infty, \infty)$.
Be careful with this argument. It also requires the rest of the integrand to be symmetric u
By Eq. (**??**), Ξ_2 acquires an extra factor $(-1)^{N-M-1}$ when $N - 1 > M$, however this gives no overall change as we consider $(\Xi_2)^2$. Lastly, $(\epsilon_{\kappa_z m} + \epsilon_{\kappa_z n})$ acquires a negative sign change, which cancels with the negative sign from the step function factor. Every term of the sum from the second quadrant may thus be mapped to a term from the fourth quadrant. We may thus restrict the sum to the fourth quadrant, by simply introducing a factor 2.

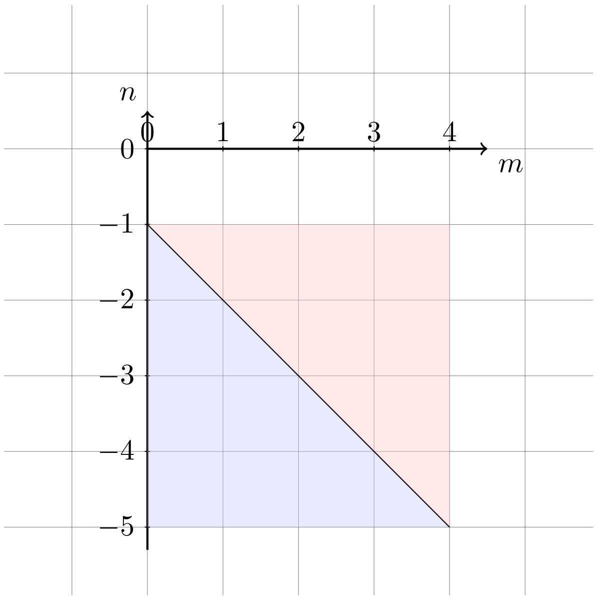

Figure 6: TODO

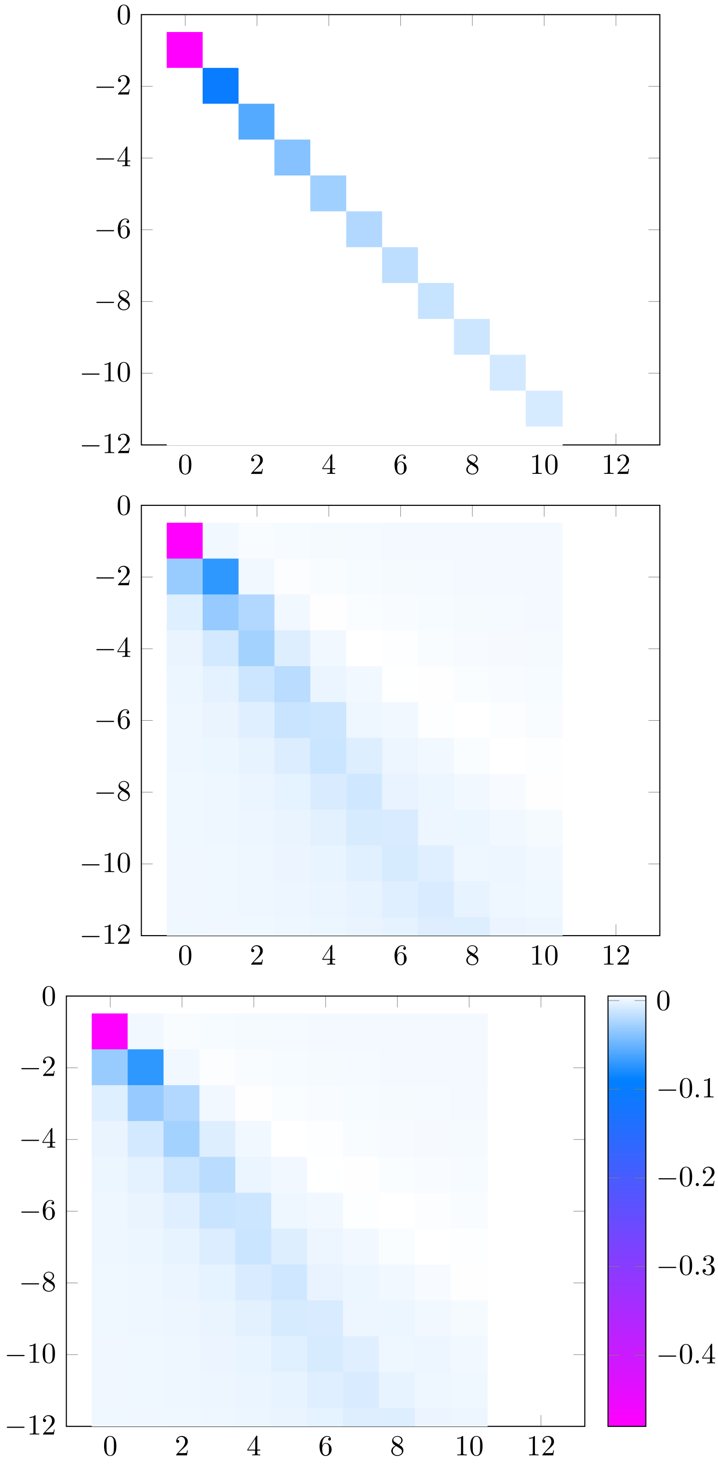

Figure 7: TODO

0.5.3 Tilt parallell to the magnetic field

We consider here a tilt parallel to the magnetic field, $\boldsymbol{t} \parallel \boldsymbol{B}$. We will consider only the canonical part of the energy-momentum tensor, and not the full symmetrized form, as described van der Wurff and Stoof [13].

$$T^{\mu 0} = \frac{i}{2}[\partial_j \bar{\psi} \Gamma^j \gamma^0 \Gamma^\mu \psi - \bar{\psi} \Gamma^\mu \gamma^0 \Gamma^j \partial_j \psi], \tag{0.242}$$

where $\Gamma^\mu = \gamma^\mu + \gamma^0 t^\mu$ with $t^\mu = (0, \boldsymbol{t})$ when inversion symmetry is broken and $\Gamma^\mu = \gamma^\mu = \gamma^0 \gamma^5 t^\mu$ in the inversion symmetric case.

We have, with $\boldsymbol{t}^\chi = \boldsymbol{t}$ in the inversion symmetric case, and $\boldsymbol{t}^\chi = \boldsymbol{t}$ in the symmetry broken case. The expression is exactly the same as for the untilted case, only with different energies. The dimensionless quantities are

$$\epsilon_{\kappa m s} = \begin{cases} t_z^\chi \kappa + \operatorname{sign} m \sqrt{M + \kappa^2} & m \neq 0 \\ (t_z^\chi - s)\kappa & m = 0 \end{cases}. \tag{0.243}$$

The normalization factor $\alpha_{\boldsymbol{k} m s}$ is, expressed in dimensionless quantities,

$$\alpha_{\kappa m s} = -\sqrt{\frac{M}{(\epsilon_{\kappa m s} - t_z^\chi)s - \kappa}}. \tag{0.244}$$

The integrand is

$$\sum_{mn} (\epsilon_{\kappa m s} + \epsilon_{\kappa n s})(\alpha_{\kappa n s}^2 \delta_{N,M+1} - \alpha_{\kappa m s}^2 \delta_{M,N+1}). \tag{0.245}$$

As described above, the contributions from the $\alpha_{\kappa n s}^2$ and $\alpha_{\kappa m s}^2$ terms are identical, and we may simply take

$$\sum_{\substack{mn \\ N=M+1}} (\epsilon_{\kappa m s} + \epsilon_{\kappa n s})\alpha_{\kappa n s}^2. \tag{0.246}$$

For Type-I semimetals, the sign energy of state $m \neq 0$ is given by the sign of m itself. For $m = 0$ the sign of the energy is given by $-s \operatorname{sign} \kappa$. Due to this, the sum is restricted to $n = M+1, m = -M$ and $n = -M-1, m = M$. In the case of Type-II, however, the situation is not so simple. The energy bands cross the Fermi surface, and we must also include in our sum overlap between states of the same sign, i.e. $n = M+1, m = M$ and $n = -M-1, m = -M$, which is non-zero for certain intervals of κ. **Include a figure**

0.6 Notes

0.6.1 Spin states for Dirac cone

See mathematica file.

Consider a simple Dirac cone Hamiltonian $H_D = sv_F \boldsymbol{\sigma p}$, with s denoting the chirality of the cone. The eigenvalues of the system is of course $E = \pm v_F k, \quad k = |\boldsymbol{k}|$. We want to find the eigenstates of this system. Assume plane wave state, and some arbitrary linear combination of spin up and spin down,

$$\psi_\pm = e^{i\boldsymbol{kr}} \alpha \begin{pmatrix} 1 \\ b \end{pmatrix},$$

where α is some normalization. Solving the time independent Schrodinger equation

$$H\psi = E\psi,$$

we may solve for b, which gives

$$b = -\frac{k_z \pm k}{k_x - ik_y}. \tag{0.247}$$

Requiring normalization of the state $\langle\psi|\psi\rangle = 1$ gives the normalization

$$|\alpha|^2 = \frac{1}{1 + |b|^2}.$$

Having found the states, we find the spin expectation value

$$\boldsymbol{S} = \langle\psi|\hat{S}|\psi\rangle, \tag{0.248}$$

where $\boldsymbol{S}$ is the spin expectation value and $\hat{S} = \frac{\boldsymbol{\sigma}}{2}$ is the spin operator, where $\hbar$ was set to 1. Simply evaluating Eq. (0.248), yields

$$\boldsymbol{S} = \pm\frac{\boldsymbol{k}}{2k}. \tag{0.249}$$

The spin structure is that of a hedgehog.

0.6.2 Symmetries

In order to separate weyl cones in momentum, we introduce a pseuod spin degree of freedom, making the system 4x4. We may then get solutions with the cones separated in momentum (or energy). We may also ask what heppens if we try to separate tilted cones?

Firstly, in the most intuitive way to extend the 2x2 tilted cones to 4x4, we get that the cones tilt opposite direction, thus not superimposed even before separating in momentum. They are after that simple to separate in momentum. We might wonder if it makes sense to do it in this way.

The lattice model of the energy dispersion to explain tilted cones gives two cones separated in momentum, and tilting corresponds to "bending" the dispersion curves between them. Maybe we therefore always have cones separated in momentum, and thus tilting superimposed does not make sense? All depends on the origin of the tilt figures/spinStructureWeyl.pdf

Figure 8:

I believe. Also, we must not confuse the global dispersion relation, to the Dirac cones which are expansions around the nodes.

Key to understand how spin behaves in all of this, and also maybe the symmetries.

To properly investigate the symmetry properties of the system, we must consider the 4x4, not 2x2 Hamiltonians. While the 2x2 system does a goood job at describing a single cone, much important phsycis is lost when reducing the 4x4 Hamiltonian. For example, the requirement that the total Berry curvature over the entire Briolluine zone is zero is not met for the 2x2 Hamiltonian, as it describes only one cone of a certain chirality. The 4x4, however, includes two cones, which may in general be superimposed, thus conserving the total zero-divergence of the Berry curvature. As a matter of fact, the inclusion of both cones is important also for symmetry considerations.

Let

$$H = v_F \tau_x \otimes \boldsymbol{\sigma k},$$

where τ is some pseudo spin degree of freedom, transforming like $\boldsymbol{r}$ under parity in time reversal. This system describes two superimposed cones at the origin, with opposite chirality. The effect of parity $\mathcal{P}$ and time reversal $\mathcal{T}$ is

	$\mathcal{P}$	$\mathcal{T}$
τ	-	+
σ	+	-
k	-	-

$$\begin{aligned}\mathcal{P}\tau\mathcal{P}^\dagger &= -\tau, \quad \mathcal{T}\tau\mathcal{T}^\dagger = +\tau \\ \mathcal{P}\sigma\mathcal{P}^\dagger &= +\sigma, \quad \mathcal{T}\sigma\mathcal{T}^\dagger = -\sigma \\ \mathcal{P}k\mathcal{P}^\dagger &= -k, \quad \mathcal{T}k\mathcal{T}^\dagger = -k\end{aligned} \tag{0.250}$$

Obviously then, the Hamiltonian is both time reversal and parity invariant, as $\mathcal{P}\mathcal{P}^\dagger = \mathcal{T}\mathcal{T}^\dagger = 1$.

A tilt term $\tau_x \otimes \mathcal{I}\boldsymbol{\omega}_0\boldsymbol{k}$ breaks time reversal invariance, while maintaining parity invariance. This is due to the two cones of opposite chirality tilting in opposite directions.

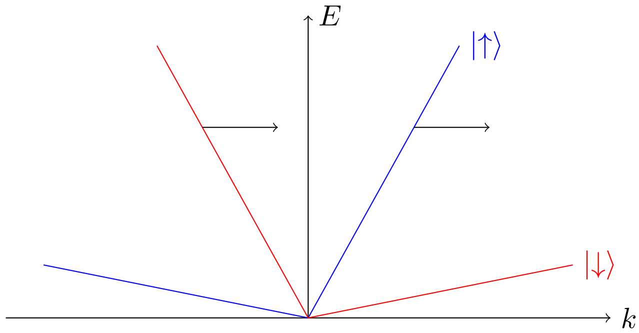

Figure 9: Time reversal breaking in tilted system. Cross section in the tilt direction shown, with blue showing one cone and red the other. Black arrows indicate spin direction, which for $|\uparrow\rangle$ is proportional to k while for $|\downarrow\rangle$ is proportional to $-k$.

The unperturbed Dirac Hamiltonian is Lorentz invariant, given that we consider an "effective speed of light", namely the Fermi velocity, instead of the actual speed of light c. Specifically, Lorentz invariance means invariance under the *Lorentz group*. The Lorentz group is the $O(1,3)$ Lie group that conserves

$$x_\mu x^\mu = t^2 - x^2 - y^2 - z^2,$$

i.e. all isometries of Minkowski space. More specifically, the group consists of all 3D rotations, $O(3)$, and all *boosts*. A boost is a hyperbolic rotation from a spactial dimension to the temporal dimension. If we now direct our focus at the Hamiltonian of the Dirac cone

$$H = \pm v_F \boldsymbol{\sigma}\boldsymbol{p},$$

we may easily show the Lorentz invariance of the system. The time independent Schrodinger equation is

$$H |\psi\rangle = E |\psi\rangle \implies (H^2 - E^2) |\psi\rangle = 0. \tag{0.251}$$

As

$$p^\mu = \left(\frac{E}{c}, \boldsymbol{p} \right),$$

the operator in Eq. (0.251) is nothing more than **Make clear the matrix strucute here. There is an**

$$H^2 - E^2 = v_F^2 \boldsymbol{p}^2 - c^2 \left(p^0\right)^2, \tag{0.252}$$

where we used the anticommutation relation

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij}$$

of the Pauli matrices. Using now the effective speed of light $c = v_F$, Eq. (0.252) is

$$-v_F^2 p_\mu p^\mu. \tag{0.253}$$

Consider now a *tilted* Dirac cone

$$H = \pm v_F \boldsymbol{\sigma} \boldsymbol{p} + \omega_\perp k_x, \tag{0.254}$$

where we, without loss of generality, chose the tilt to be in the x-direction.

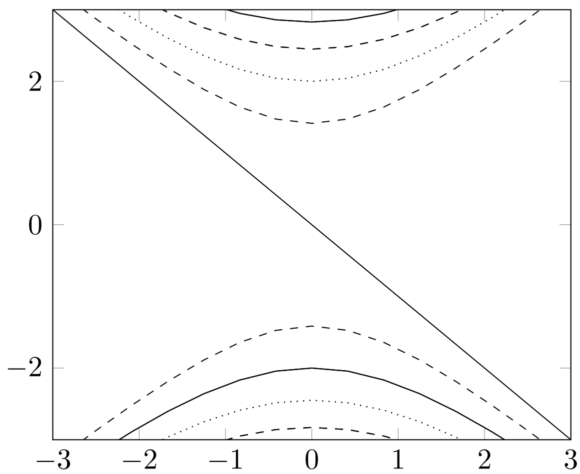

Figure 10:

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