1 Non-flat metric

In the main text, we have used the Lagrangian

$$L_s = i\phi^\dagger \tilde{\sigma}^\mu \partial_\mu \phi, \tag{1}$$

where we have used *modified* Pauli matrices $\tilde{\sigma}^\mu = \sigma^\mu + t^\mu,\ t^\mu = (0, \boldsymbol{t})$. We here present an alternative, where we instead consider moving the tilting into the metric, i.e. considering a non-tilted cone in curved spacetime. In essence, we want

$$g^{\mu\nu}\sigma_\nu p_\mu$$

to give $(\sigma^\mu + t^\mu)p_\mu$. We see that this involves putting t^μ on the 0-components of the metric.

Consider the metric

$$g^{\mu\nu} = \eta^{\mu\nu} + t^\mu(\delta_0^\mu + \delta_0^\nu) = \begin{pmatrix} 1 & t^x & t^y & t^z \\ t^x & -1 & 0 & 0 \\ t^y & 0 & -1 & 0 \\ t^z & 0 & 0 & -1 \end{pmatrix}. \tag{2}$$

The top row is however problematic, as it gives an unwanted

$$g^{0\nu}\sigma_\nu = \sigma^0 - \boldsymbol{t}\boldsymbol{\sigma}.$$

Interestingly, the metric thus *cannot* be symmetric! We conclude that the appropriate choice is

$$g^{\mu\nu} = \eta^{\mu\nu} + t^\mu\delta_0^\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ t^x & -1 & 0 & 0 \\ t^y & 0 & -1 & 0 \\ t^z & 0 & 0 & -1 \end{pmatrix}. \tag{3}$$

Consider therefore the Lagrangian

$$\mathcal{L} = \phi^\dagger g^{\mu\sigma}\sigma_\sigma \partial_\mu \phi. \tag{4}$$

Using once again the canoncial defintion

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial^\nu \phi \tag{5}$$

$$= \phi^\dagger g^{\mu\sigma}\sigma_\sigma \partial^\nu \phi, \tag{6}$$

we find the component of the energy-momentum tensor

$$T^{y0} = \phi^\dagger g^{y\sigma}\sigma_\sigma \partial^0 \phi = \phi^\dagger(t^y\sigma_0 + \sigma_y)\partial^0\phi. \tag{7}$$

Remark 1. The four-Pauli matrices are defined as

$$\sigma^\mu = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) = (\sigma^0, \sigma_x, \sigma_y, \sigma_z), \tag{8}$$

where the matrices with lower roman indices are the well-known Pauli matrices. Thus, the four-matrices with lowered inddex are

$$\sigma_\mu = g_{\mu\nu}\sigma^\mu = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3). \tag{9}$$