Problem with the energy-momentum tensor

We have a response function

$$\chi(t, \boldsymbol{r}) = \int \mathrm{d}t' \mathrm{d}\boldsymbol{r}' \left\{ \left\langle \left[\boldsymbol{J}(t, \boldsymbol{r}, T^{00}(t', \boldsymbol{r}') \right] \right\rangle \psi(t', \boldsymbol{r}') \right\}, \tag{1}$$

where $\boldsymbol{J}$ is a charge current, $T^{\mu\nu}$ is the energy-momentum tensor, and ψ is a gravitational potential [1]. Using the continuity relation of the energy-momentum tensor

$$\partial_0 T^{00} + \partial_i T^{0i} = 0 \implies T^{00}(t, \boldsymbol{r}) = -\int_{-\infty}^{t} \mathrm{d}t' \partial_i T^{0i}(t', \boldsymbol{r}),$$

and integration by parts

$$\int uv' = uv - \int u'v$$

we get

$$\chi = \int \mathrm{d}t' \mathrm{d}\boldsymbol{r}' \int \mathrm{d}t'' \left\{ \left\langle \left[\boldsymbol{J}(t, \boldsymbol{r}, T^{0j}(t'', \boldsymbol{r}') \right] \right\rangle \partial_j' \psi(t', \boldsymbol{r}') \right\}. \tag{2}$$

Now, our response function, an observable, depends on the value of T^{0j} itself, not the divergence.

[1] Yes, condensed matter physisists have a hack where they use gravitational potential to "simulate" temperature.