

KD5081 Theory, Computation & Experiment

Laboratory Scripts

BSc (Hons) / MPhys Physics

BSc (Hons) / MPhys Physics with Astrophysics

Department of Mathematics, Physics and Electrical Engineering

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Laboratory Rules

1-Strictly no food or drink is to be consumed in the labs.

2-Coats and bags tucked away under bench or in pigeon holes.

3-Come to lab prepared to perform the experiment. Follow all written and verbal instructions. When in doubt, ask.

4-Report all accidents, injuries or breakage to a member of staff immediately. Also, report any equipment that you suspect is malfunctioning.

5-Be on time!

6-Bring your labbook or go and buy a new one.

7-Read the script in advance.

8-Every lab report must include:

8.1 The names of all the people in your lab group

8.2 The date

8.3 The experiment name and number

8.4 Objectives of the lab

8.5 Any relevant background information

8.6 The data recorded during the lab in pen with units and error estimates, arranged neatly on the page (use tables when appropriate).

8.7 Error analysis

8.8 Discussion of the data

8.9 A conclusion

9-Use equipment with care for the purpose for which it is intended.

I have read and agree to abide by the Physics Laboratory Rules

.....

Print name and sign



**No mobile phones
in this laboratory**



**No food or drinks
in this laboratory**



**No personal audio
devices in this laboratory**

Values of Physical Constants

Electron-volt	1 eV	=	1.602x10 ⁻¹⁹ J	
Unified atomic mass unit	1 u	=	1.660x10 ⁻²⁷ kg	=931MeV
Rest mass of electron	m_e	=	9.109x10 ⁻³¹ kg	=0.000549 u
Rest mass of proton	m_p	=	1.673x10 ⁻²⁷ kg	= 1.007276 u
Rest mass of neutron	m_n	=	1.675x10 ⁻²⁷ kg	= 1.008665 u
Charge of electron	e	=	1.602x10 ⁻¹⁹ C	
Electron charge/mass ratio	e/m_e	=	1.759x10 ¹¹ C kg ⁻¹	
Rydberg constant	R_∞	=	1.097x10 ⁷ m ⁻¹	
Radioactivity (Curie)	1 Ci	=	3.7x10 ¹⁰ s ⁻¹	
Avogadro constant	N_A	=	6.022x10 ²³ mol ⁻¹	
Planck constant	h	=	6.626x10 ⁻³⁴ J s	
Boltzmann constant	k	=	1.381x10 ⁻²³ J K ⁻¹	
Speed of light in vacuum	c	=	2.998x10 ⁸ m s ⁻¹	
Permittivity of vacuum	ϵ_0	=	8.854x10 ⁻¹² F m ⁻¹	
Permeability of vacuum	μ_0	=	1.257x10 ⁻⁶ H m ⁻¹	= 4 π x10 ⁻⁷ H m ⁻¹
Ideal gas constant	R	=	8.314 J K ⁻¹ mol ⁻¹	
Standard volume of ideal gas at s.t.p	V_0	=	2.242x10 ⁻² m ³ mol ⁻¹	
Stefan-Boltzmann constant	σ	=	5.670x10 ⁻⁸ W m ⁻² K ⁻⁴	
Standard gravitational acceleration	g	=	9.81 m s ⁻²	
Gravitational constant	G	=	6.673x10 ⁻¹¹ N m ² kg ⁻²	
Standard atmospheric pressure		=	1.013x10 ⁵ N m ⁻²	= 760 mm Hg
Density of mercury at 20°C		=	1.36x10 ⁴ kg m ⁻³	
Specific heat capacity of water at 20°C		=	4182 J kg ⁻¹ K ⁻¹	

Greek Alphabet

Letter	Name	Equivalent
A α	Alpha	A
B β	Beta	B
Γ γ	Gamma	G
Δ δ	Delta	D
E ε	epsilon	E
Z ζ	Zeta	z
H η	Eta	\bar{e}
Θ θ	Theta	Th
I ι	Iota	I
K κ	Kappa	K
Λ λ	Lambda	L
M μ	Mu	M
N ν	Nu	N
Ξ ξ	Xi	x
O \omicron	Omicron	o
Π π	Pi	P
P ρ	Rho	R
Σ σ	sigma	S
T τ	Tau	T
Y υ	Upsilon	U
Φ ϕ	Phi	F
X χ	Chi	ch
Ψ ψ	Psi	Ps
Ω ω	Omega	\bar{o}

Presenting Numbers with Errors & Significant Figures

- Experimental errors should be rounded to one significant figure:
60±3 cm and not 60.1±3.2 cm
- Round the experimental result to the same number of places as the uncertainty:
1.2±0.2 cm and not 1.234±0.2 cm
- When using scientific notation the following should be adopted:
(9.0±0.3)×10⁻³ kg and not 9.0×10⁻³±0.3×10⁻³ kg
- “104.03” and “0.0010400” have both 5 significant figures (or digits)

Standard errors: General Formula and Combination

- If $Z=Z(a,b,c,\dots)$ then the error in z is given by:

$$\Delta Z = \sqrt{\left(\frac{\partial Z}{\partial a} \Delta a\right)^2 + \left(\frac{\partial Z}{\partial b} \Delta b\right)^2 + \left(\frac{\partial Z}{\partial c} \Delta c\right)^2 + \dots}$$

- Combination of standard errors:

$$\left. \begin{array}{l} Z = A + B \\ Z = A - B \end{array} \right\} \quad (\Delta Z)^2 = (\Delta A)^2 + (\Delta B)^2$$

$$\left. \begin{array}{l} Z = AB \\ Z = \frac{A}{B} \end{array} \right\} \quad \left(\frac{\Delta Z}{Z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$$

$$Z = A^n \quad \left(\frac{\Delta Z}{Z}\right) = n \left(\frac{\Delta A}{A}\right)$$

$$Z = \ln A \quad \Delta Z = \frac{\Delta A}{A}$$

$$Z = \exp A \quad \frac{\Delta Z}{Z} = \Delta A$$

$$Z = \sin A \quad \Delta Z = \Delta A \cos A$$

$$Z = \cos A \quad \Delta Z = \Delta A \sin A$$

Experiment 1 – Optical Properties of Semiconductors

Objectives

The purpose of this laboratory is to explore some fundamentals optical properties of semiconductors via analysis of transmission versus wavelength curves.

Health & safety consideration

This laboratory does not contain any specific health and safety hazards.



When you arrive in the lab... Switch on the PC and the spectrophotometer to initialize and warm up (switch is on the right hand side).



There are high quality and expensive internal mirrors inside the system and you must not touch them. **You must also wear gloves when inserting/removing samples.**

Introduction

When light interacts with a medium some of it will be transmitted, reflected or absorbed. Reflectance R is the amount of flux reflected by a surface, normalized by the amount of flux incident on it. Transmittance T is the amount of flux transmitted by a surface, normalized by the amount of flux incident on it. Any flux not reflected or transmitted is absorbed (A). Conservation of energy requires that:

$$R + T + A = 1 \quad (1)$$

The transmittance T is defined as:

$$T = \frac{I}{I_0} \quad (2)$$

where I_0 is the intensity of the incident radiation entering the medium and I the intensity of the transmitted radiation leaving the medium. T is usually expressed in percentage (%T). I and I_0 are related by the Beer-Lambert law:

$$I = I_0 \exp(-\alpha x) \quad (3)$$

where x the material thickness and α the absorption coefficient and is wavelength dependent.

The relationship between %T and absorbance (A) is given by the following equation:

$$A = 2 - \log(\%T) \quad (4)$$

The optical absorption spectrum of a semiconductor yields information on the energy band structure of the semiconductor. In particular if the frequency of the radiation ν is such that $E_g < h\nu$ where E_g is the energy bandgap of the semiconductor then strong absorption takes place, each photon absorbed creates an electron-hole pair. If however $E_g > h\nu$ then this process cannot take place and there is usually little absorption in this region. This is illustrated in Figure 1 a). Thus by measuring the position of the absorption edge, one can determine the energy bandgap of the material.

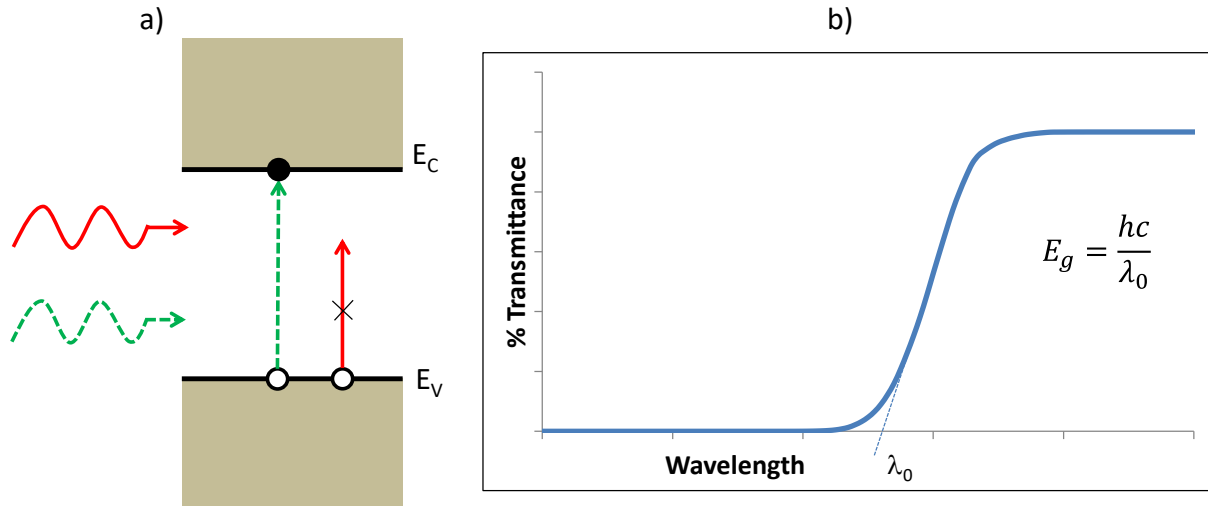


Figure 1: a) Light absorption in semiconductor with an energy gap spanning the top of its valence band E_V and the bottom of its conduction band E_C . Green-dashed line for a photon with energy greater than E_g and red-solid line for a photon whose energy is smaller than E_g . b) Determination of energy bandgap from transmission spectra.

It is, however, often difficult to locate the absorption edge precisely as it may be “smeared out” over a wide wavelength range. A simple criterion that suffices in this case is to take the point which is obtained by extrapolating the transmission curves (see Figure 1 b)) so that we have the relationship between energy bandgap and λ_0 given by:

$$E_g = h\nu = \frac{hc}{\lambda_0} \quad (4)$$

For wavelengths greater than that corresponding to the bandgap it will be observed that the transmission is much less than 100%, this is due to multiple reflections at the air-semiconductor interfaces (light scattering from surface defects). The fraction of light reflected (R) from a single interface of normal incidence is given by (assuming the true absorption is zero):

$$R = \left(\frac{n-1}{n+1} \right)^2 \quad (6)$$

where n is the refractive index of the material. The total transmission of a parallel sided sample is then given by:

$$T = \frac{(1-R)^2}{1-R^2} \quad (7)$$

This thickness of a thin layer may be measured by investigating the transmission (or reflection) spectrum of the thin layer over a suitable wavelength range. Interference occurs between light beams reflected at the upper and lower surfaces of the film and as the wavelength is varied a series of maxima and minima are obtained in the transmitted (reflected) intensity (see Figure 2).

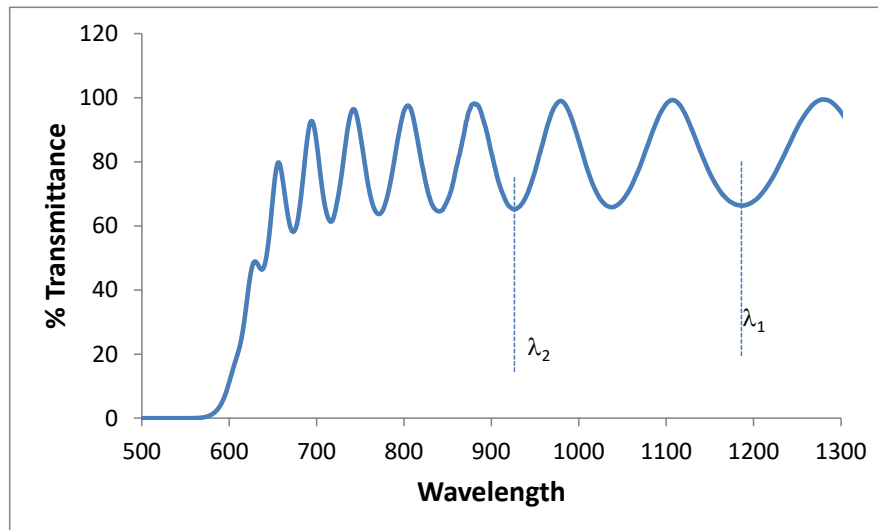


Figure 2: Transmission spectra of a selenium thin film on glass.

The thickness of the thin film is given by:

$$t = \frac{N\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)(n^2 - \sin^2 \theta)^{1/2}} \quad (8)$$

where λ_1 and λ_2 are either maxima or minima ($\lambda_1 > \lambda_2$), N is the number of cycles between these two wavelength, n is the refractive index of the thin film and θ is the angle of incidence of the light.

Equipment

- Shimadzu UV2600plus spectrophotometer
- Samples: Si, GaAs, CdS and ITO (indium doped tin oxide)

Procedure and analysis

Following the procedure to operate the spectrophotometer (at the end of this script) measure the transmission spectra of the four samples provided.

👉 **Take care when handling the samples as those are fragile. Be careful when inserting sample in the sample holder.**

👉 **Do not touch the mirrors! They are very expensive and get damaged easily.**

The operating instructions of the spectrophotometer are provided at the end of the script.

- (1) Using the instruction below measure the transmission spectra of the samples provided in the range **300-1400** nm. Make sure to save all your spectra.
- (2) Plot all data in Excel. You can copy the data from UV Probe directly (click the data icon to show the data).
- (3) Give an estimation for the bandgap values (in eV). Comments on graphs and the bandgap values with regards to the samples colour and its potential at absorbing light of certain wavelengths.
- (4) For those samples for which it is possible to make measurements above the absorption edge, calculate the refractive index. Hint: from the transmittance curve, calculate a value for R around the absorption edge using equation (7) and then use equation (6) to get n .
- (5) Calculate the absorption coefficients (in cm^{-1}) around the absorption edge for Si and GaAs, given a film thickness of 500 μm and 250 μm , respectively.
- (6) Calculate the thickness of the CdS and ITO thin films.
- (7) What would happen to the interferences fringes if the film was thicker (or thinner)?
- (8) A more precise technique to extrapolate the energy bandgap is to use a Tauc plot using the relation:

$$\alpha h\nu = C(h\nu - E_g)^n \quad (9)$$

where C is a constant and n a number characterising the transition process in the semiconductor responsible for the absorption. n can take values of 1/2, 1, 3/2 or 2. For GaAs, ITO and CdS samples $n=1/2$ characteristics of a direct transition (see module EN0583) while for Si, $n=2$ characteristics of an indirect transition. Thus a plot of $(\alpha h\nu)^2$ as a function of energy will yield the energy bandgap of the semiconductor at the intercept on the x-axis by extrapolating the linear part of the graph.

- (9) Plot the absorption coefficients as a function of wavelength and comment.
- (10) Make sure all your data is saved and retrievable.
- (11) When you have finished please close the software, turn the instrument and PC off.

Procedure to acquire a transmission spectrum using the Shimadzu UV-2600 Spectrophotometer

The instrument is a double beam spectrophotometer which means that two beams of light are generated, one being used as a reference and the other as the sample beam. The resulting data is the difference between the two (see Figure 3 showing the light paths and the integrating sphere).

- (1) Turn PC on.

- (2) Turn instrument on (on/off switch on right hand side). Front LEDs will be flashing for 3-4min. Wait for green LED to stop flashing after a few successive beeps.
- (3) Open UVProbe software.
- (4) Click “connect” at the bottom of screen.
- (5) The checklist window should be all green; click OK.
- (6) Click the “measurement/method” button (green M in yellow dot icon, or menu “edit” then “method”).
- (7) In the measurement tab set wavelength start =1400nm; end=300nm; scan speed to “fast” and scan mode to single.
- (8) In “instrument parameters” tab, the measuring mode should be transmittance and the slit width 5nm. Also set detector unit to external (2 detectors) and S/R exchange set to normal. Click OK.
- (9) You now need to do a baseline correction of the system. Make sure no sample has been inserted then click the “baseline” button. The baseline is used to remove the background noise and to make sure that both reference and sample beams are identical. In the present case the instrument makes sure that at each wavelength the recorded signal corresponds to 100% T for both beam.
- (10) Load your sample on the right hand holder and place a plain glass slide in the reference position (see Figure 3 b) and close the top cover.
- (11) Click the “START” button (you will be asked for a filename and location) and save your data when finish.

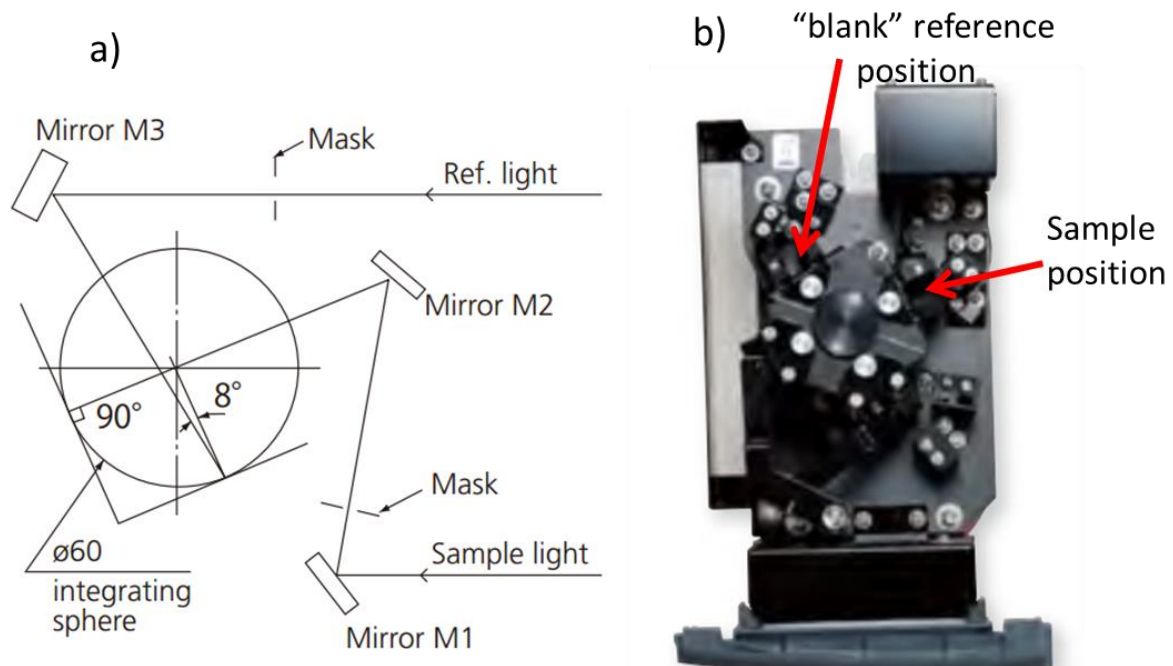


Figure 3: Simple diagram of a double beam spectrophotometer.

Experiment 2 – Thermal Exchanges

Objectives

The purpose of this laboratory is to study thermal exchanges by radiation and convection of a heat exchanger. In the first part of the experiment you will study the system working as a solar panel when the radiator is placed under a halogen lamp. In the second part of the laboratory the system will be used as a radiator with hot water pumped through the radiator.

Health & safety consideration

This laboratory uses a high power halogen lamp as a heating source or hot water as a heating fluid. Care must be taken when operating the apparatus as pipes and surfaces will be hot. **Especially, do not touch the lamp once on.**

When you arrive you should turn the water bath on (set at 25°C) and let the system stabilise for some time.

Introduction

Heat transfers can occur simultaneously by **convection, conduction** and **radiation**.

- If h is the convection coefficient at the external surface of the panel then the power loss P_c by the panel due to convection is:

$$P_c = hA(\theta_p - \theta_{amb}) \quad (1)$$

with A the panel area, θ_p the temperature of the panel and θ_{amb} the ambient temperature. As a first approximation $h \sim 10 \text{ W m}^{-2} \text{ K}^{-1}$ in air without wind and at room temperature.

- In terms of radiated power the panel is assumed to behave as a grey body with absorbing power α and emissive power ε such that:

$$\alpha = \varepsilon < 1 \quad (2)$$

If the panel receives an energetic flux Φ from an external source, it will absorb:

$$P_a = \alpha A \Phi \quad (3)$$

If the panel surface temperature is θ_p (in K) then the radiated power is

$$P_r = MS = \varepsilon A \sigma \theta_p^4 \quad (4)$$

where σ is Stefan's constant and equal to $5.668 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$.

- For the panel in air at ambient temperature it will absorb:

$$P = \alpha A \sigma \theta_{amb}^4 \quad (5)$$

- In both type of experiments the power emitted or received by the flowing water is given by:

$$P_w = mc|\theta_o - \theta_i| \quad (6)$$

where m is the mass flow (g/s) and c the water heat capacity ($c=4.18 \text{ Jg}^{-1}\text{K}^{-1}$).

Equipment

Radiator (frontal area of $0.40 \times 0.45 = 0.18 \text{ m}^2$)

Halogen Lamp

Water recirculator

Fan

Thermometers, flow meter and thermocouples

Glass cover

Preliminary question

(1) At equilibrium the balance of power (ΔP) must be equal to zero. This means that the amount of power received must be equal to the amount of power emitted. **Write this equation for both cases (solar panel and radiator mode).**



It is strongly advised that you check with a member of staff that your derivation is correct.

Procedure and analysis

You will need to record temperature of the water flowing through the radiator and the inlet (θ_i) and outlet (θ_o) positions. The temperature of the panel itself (θ_p) will be measured with the infrared sensor. Use a thermometer to measure the ambient temperature (θ_{amb}).

In all cases you must wait until temperatures are stabilised for the system to be assumed to operate in a quasi-stationary state. It will take between 10 and 20 min for each experiment giving you plenty of time to perform the calculations.

The water flow should be kept constant (flow of around 0.2 L min^{-1}) and when simulating wind place the fan at approximately 30 cm from the panel with an incidence of 45° , speed selector position 6.

(1) Turn on the thermocouple display to show the inlet and outlet temperature. You will notice a difference between the two readings due to a difference in calibration of the thermocouple. **Record both temperature and keep that in mind for the rest of the experiments.**

(2) This should have already been done but just in case, set the recirculator to 25°C (heating and cooling mode) and let the system stabilise.

(3) Check the flow meter (Digiflow 6710M) is power on and that the flow is around 0.2 L/min .

Part A: Operation as a solar panel

In the experiment the panel is illuminated with a halogen lamp. The absorbed energy is transferred to the water circulating through the panel. You will also investigate the effect of wind. At the end of this experiment you should be able to understand the different types of heat exchanges at the panel surface and to quantify them. You will be able to calculate the efficiency η of the system in the various configurations tested defined as:

$$\eta = \frac{\text{Power received by water}}{\text{Power received by panel}} \quad (7)$$

The panel ($A = 0.18 \text{ m}^2$) is placed $\sim 15 \text{ cm}$ from the halogen lamp which irradiate the panel with a flux of $\Phi = 1250 \text{ Wm}^{-2}$ and you can assume that all the power received comes from the light.

(1) Without front glass cover.

Let the system stabilise (around 15-20 min) and record all the temperatures. Calculate:

- (i) The power absorbed by the water;
- (ii) The panel efficiency;
- (iii) The power released by the panel by convection using the value of h given previously;
- (iv) The absorption coefficient α of the panel;
- (v) The power radiated by the panel.

(2) Without front glass cover but with wind.

Let the system stabilise and record all the temperatures. Calculate:

- (i) The power absorber by the water;
- (ii) The panel efficiency;
- (iii) The power radiated by the panel;
- (iv) The thermal convection coefficient h of the panel.

(3) With front glass cover (no wind).

In this case you can assume no convection losses. Explain why.

Take the measurements to calculate the power absorbed by the water and the efficiency of the panel.

Note: to save time it is suggested that once you have collected the data then switch the lamp off and set the recirculator to 60°C.

Finally calculate the transmission coefficient T of the glass defined by:

$$T = \frac{P_w \text{ with glass}}{P_w \text{ without glass}} \quad (8)$$

Conclude on the three experiments.

Part B: Operation as a radiator

In this experiment the system is used as a radiator. The water flowing through the heat exchanger is heated by the water recirculator and the panel radiate heat to the room. This experiment will study heat exchanges by convection in particular.

(1) Switch off the lamp and set the recirculator the heat the water up to 60°C. Remove the glass cover from the front of the panel.

(2) Record the temperatures and calculate:

- (i) The power released by the water;
- (ii) The power radiated by the panel;
- (iii) The power given by convection;
- (iv) The thermal convection coefficient h of the panel.

(3) Add wind and repeat for the above.

Compare your results and conclude on your work.

Experiment 3 – Hall Effect and Conductivity in Semiconductors

Objectives

The purpose of this laboratory is to understand the physical principles of the Hall Effect. This will be used to determine some fundamental properties of semiconductors.

Health & safety consideration

This laboratory does not contain any specific health and safety hazards.

Introduction

The Hall Effect was discovered by Hall in 1879 when he investigated the nature of the force on a conductor carrying a current in a magnetic field. In particular he measured the transdirection voltage on gold foils. The Hall Effect measurement technique has found wide application in the characterisation of semiconductor materials because it gives the conductivity, the carrier density and the mobility, all three being fundamental properties of the investigated material.

- **Hall effect**

When a current I flows in a metal or semiconductor in a direction perpendicular to a magnetic field \vec{B} , the electrons, travelling at a velocity \vec{v} in opposite direction to the current, are deflected towards the edge of the metal by the Lorentz force \vec{F}_B :

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad (1)$$

where q is the elementary charge. Note that the above equation is a vector product, not scalar.

After some time, a net negative charge is created on one edge of the metal and a net positive charge on the opposite edge (see figure 1). This charge distribution creates an electric field \vec{E} , which exerts a force on the electrons in a direction opposite to that exerted by the magnetic field. When the magnitude of the force ($|\vec{F}_E| = q|\vec{E}|$) exerted on the electrons by \vec{E} is equal to the magnitude of the force ($|\vec{F}_B|$) exerted on them by \vec{B} , the net number of electrons on the edges of the metal no longer changes with time. This result is called the **Hall Effect**. The potential difference V_H between the edges of the metal when equilibrium is reached is called the **Hall voltage** and is given by:

$$V_H = wE \quad (2)$$

with w the width of the semiconductor.

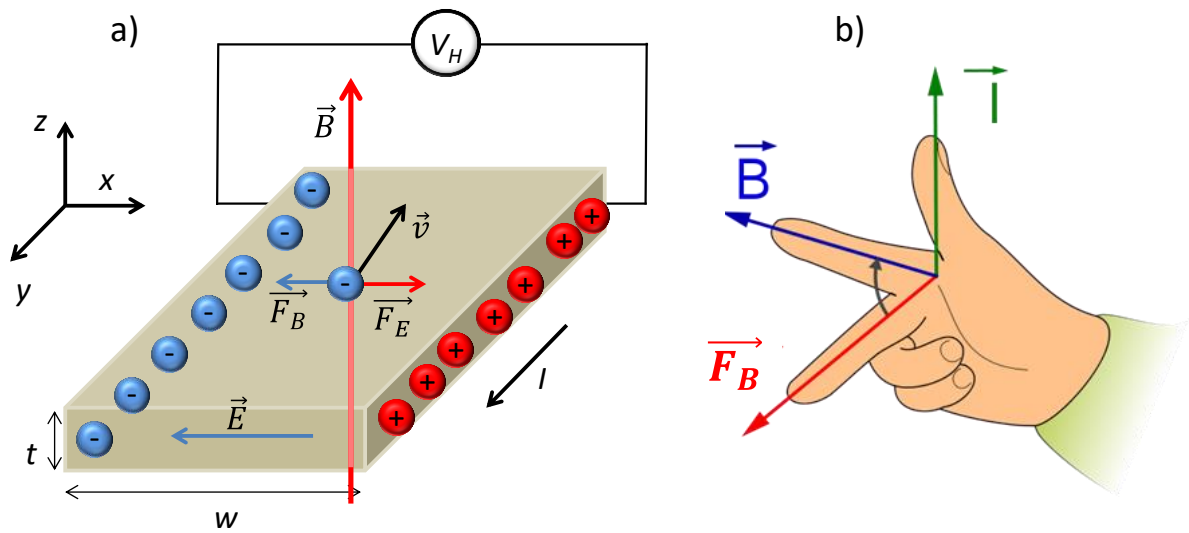


Figure 1: a) Schematic description of the Hall effect at equilibrium in a conductor. b) right hand rule used for positive charges. The electrons have drifted to one side of the conductor leaving a net positive charge on the opposite side. This distribution of charge creates an electric field that can be measured as Hall voltage.

The Hall effect can be used to demonstrate that the charge carriers in metal are negatively charged. If the charge carriers in a metal were positive and moving in the direction of the current shown in figure 1, those positive charges would collect on the same edge of the conductor as the electrons shown in figure 1a), giving an electric field with the opposite sign. Thus the charge carriers in conductors are negatively charged and must be electrons.

The Hall effect can be used to determine the magnitude of the magnetic field as used in the Hall probe. At equilibrium $|\vec{F}_E| = |\vec{F}_B|$ thus:

$$qE = qvB \quad (3)$$

$$\Leftrightarrow B = \frac{E}{v} = \frac{V_H}{vw} \quad (4)$$

The drift velocity v of an electron in a conductor can be related to the magnitude of the current density J in the conductor using:

$$J = \frac{I}{A} = nqv \quad (5)$$

where $A=wt$ is the cross section area of the metal and n is the number of electron per unit volume also known as carrier density. Combining equation (4) and (5) yields the Hall voltage:

$$V_H = \frac{IB}{nqt} \quad (6)$$

The **Hall coefficient** is defined as the ratio of induced electric field E to the product JB :

$$R_H = \frac{E}{JB} = \frac{V_H \times t}{IB} = \frac{3}{8} \pi \frac{1}{nq} \quad (7)$$

For n-type semiconductors the majority carriers are electrons and the concentration is noted n (negative), while for p-type semiconductors the majority carriers are holes and the concentration is noted p (positive).

- **Conductivity and mobility**

The conductivity σ of the sample is given by:

$$\sigma = nq\mu \quad (8)$$

where μ is the carrier mobility. Note that the conductivity is the reciprocal of the resistivity ρ ($\sigma = 1/\rho = L/(A \times R)$) where L is the length of the material and R its resistance.

Therefore both the carrier concentration and the mobility can be found if the Hall coefficient and conductivity (or resistivity) are measured.

Equipment

- Electromagnets and power supply (EA-PS-2084-05 B)
- Teslameter with Hall probe (unilab)
- Multimeter (Tema 72-8720)
- Current source (RS Microcal 1030)
- N and p-type Ge samples

Procedure and analysis

Part A: Electromagnet calibration

The electromagnet is used to provide a uniform and large magnetic field compare to a simple coil of wire. In an electromagnet the current flow in the coils is enhanced by filling the coils with ferromagnetic core material (usually iron) to increase the field obtained in the gap between the pole pieces.

- (1) Connect the coils in series.
- (2) Zero the Hall probe. Make sure the probe is well away from the electromagnet.
- (3) Position the Hall probe between the pole pieces. Remember about the orientation of the probe (See figure 2).
- (4) Make a drawing of the setup showing the direction of current flow and magnetic field.
- (5) Record the value of the magnetic field created when you set a coil current to **0.10 A**. Compare this value with the calibration curve of the coils given in the next figure.

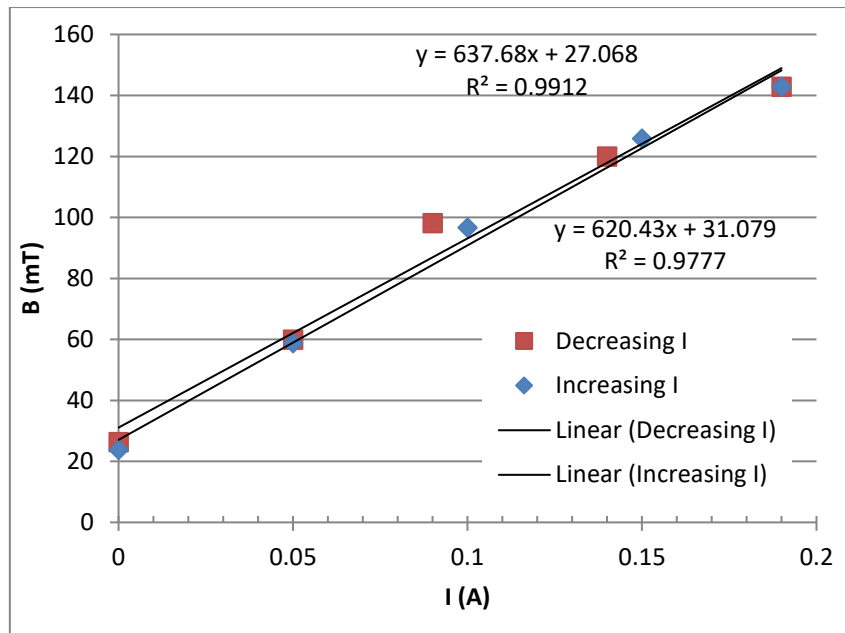


Figure 2: Magnet calibration curves.

(6) Looking at figure 2, does the magnetic field vary reversibly with current when you sweep from low to high and high to low? If not what is observed? What could account for this behaviour?

Part B: Conductivity measurement

In this section you will attempt to measure the conductivity of the semiconductor samples provided. They are doped germanium, mounted on a circuit board (as shown in figure 3). The dimensions of the sample are given on the printed circuit board (error is ± 0.1 mm).

Please take care as the samples are fragile!

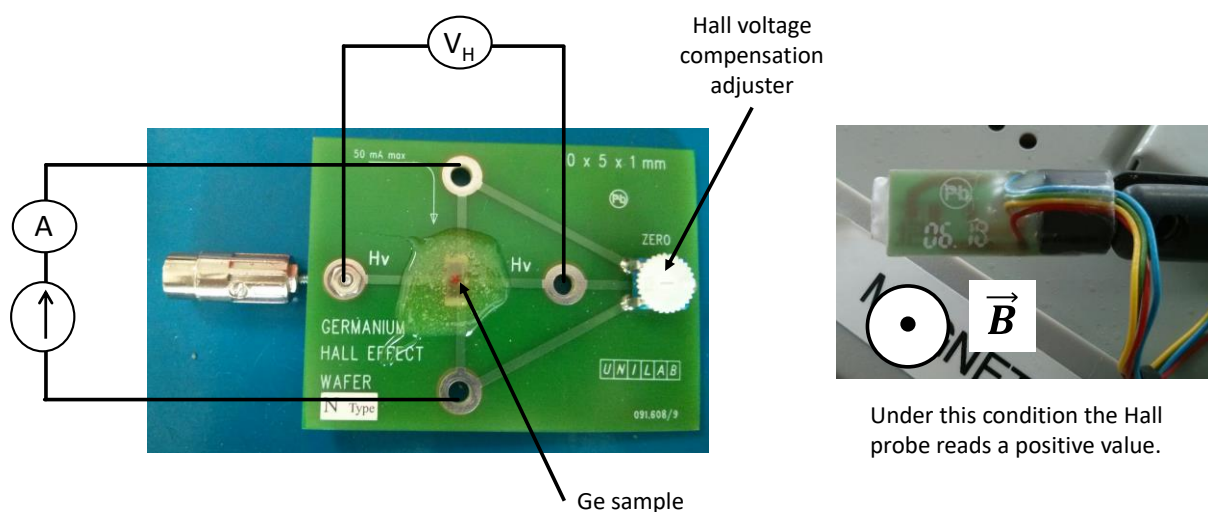


Figure 3: Test sample (left) and orientation of Hall probe to determine the magnetic field direction.

(1) Measure the electrical resistance of the sample (along the long axis of the sample). **Make sure the arrow on the potentiometer is pointing directly away from the sample for the resistance measurement.**

(2) Calculate the conductivity and resistivity of the sample.

Part C: Hall Effect measurement

(1) Connect the circuit according to figure 4, the connectors coming out from the brass arm are labelled and correspond to the electrodes also labelled on the circuit board. Draw a diagram of the connections taking in account the polarity and orientation of the sample when inserted through the magnets (see figure 5). Also take in account the direction of the magnetic field. The red connectors are positive and the black connectors are negative.

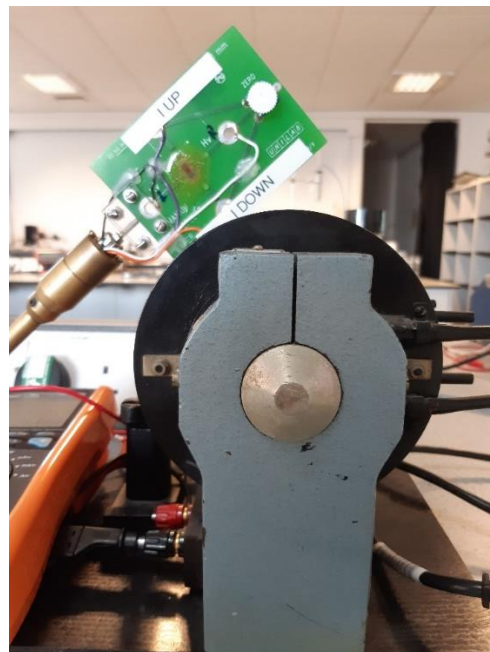
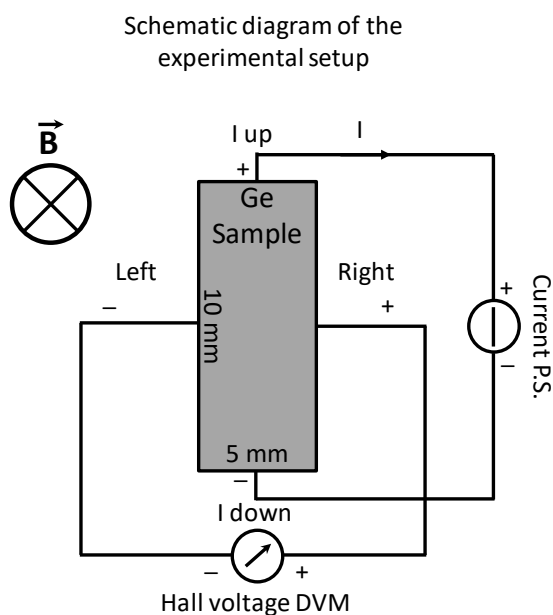


Figure 4 Left: Example of diagram to check the conductivity of the Ge sample (the small rectangle mounted on the circuit board). **Under those conditions a positive Hall voltage will indicate an n-type sample.** Right: Insertion of the Ge sample using the movable arm holding the circuit board.

(2) Using the RS Microcal 1030 (100 mA scale) current source set the current going through the sample to 40 mA (use the dial knob on the unit). Set the current in the “normal” direction on the Microcal using the toggle switch.

Maximum current must not exceed 50 mA, which could result in burning the sample.

(3) With the sample away from the electromagnets check the measured Hall voltage (set the multimeter to DC millivolt range using the appropriate controls). If it is not nil adjust the potentiometer (the small wheel on the circuit board see figure 3) to compensate the voltage value of the Hall voltage so it becomes as close as possible to zero volts.

- (4) Set a magnetic field of **around 100 mT**. Place the sample between the magnets (see figure 4) and record the Hall voltage keeping the Microcal power supply at 40 mA. Make sure to record the position of the sample in your lab book according to the magnetic field direction.
- (5) Keeping the magnetic field constant, record V_H while decreasing the current to zero in 5 mA steps. Make sure that for each measurement you take the circuit board away from the magnet and set $V_H=0$ V using the compensator before reinserting the sample through the magnets.
- (6) Now measure the Hall voltage V_H as a function of magnetic field (**Maximum voltage for magnet power supply: 70 V**) for a sample current of 25 mA.
- (7) Plot graphs of V_H versus I for constant B and V_H versus B for constant I .
- (8) Derive the unit for R_H knowing that 1 volt-second is equal to 1 tesla square meter and that 1 ampere is equal 1 coulomb per second.
- (9) Calculate the Hall coefficient R_H .
- (10) From R_H and σ , calculate the carrier concentration n (or p) and the Hall mobility μ .
- (11) Using the slope of the graphs comments on the polarity of R_H and the sample conductivity.

When finished make sure to turn the current source OFF (battery powered).

Experiment 4 – The Photovoltaic Effect and Solar Cells

Objectives

The purpose of this laboratory is to investigate the current voltage characteristics of a silicon solar cell under dark and controlled illumination and to derive its fundamental parameters. This laboratory is also an opportunity to understand the principle operations of a source-measure unit.

Health & safety consideration

After long period of utilisation the housing and other part of the simulator system will get hot. Take care.

Do not touch the hot plate when in use.

The solar simulator emits UV radiations. Wear the goggles provided when doing measurement under illumination.

Introduction

The photovoltaic (PV) effect in a semiconductor p-n junction is the combined result of two processes. The first one is the generation of carriers (electron hole pairs) by absorption of incident photons in the depletion region of a semiconductor (provided that $h\nu > E_g$). The second process consists of the separation of the carriers due to the high built in electric field collection of those generated carriers in order to create an electrical current. This process is shown in Figure 1.

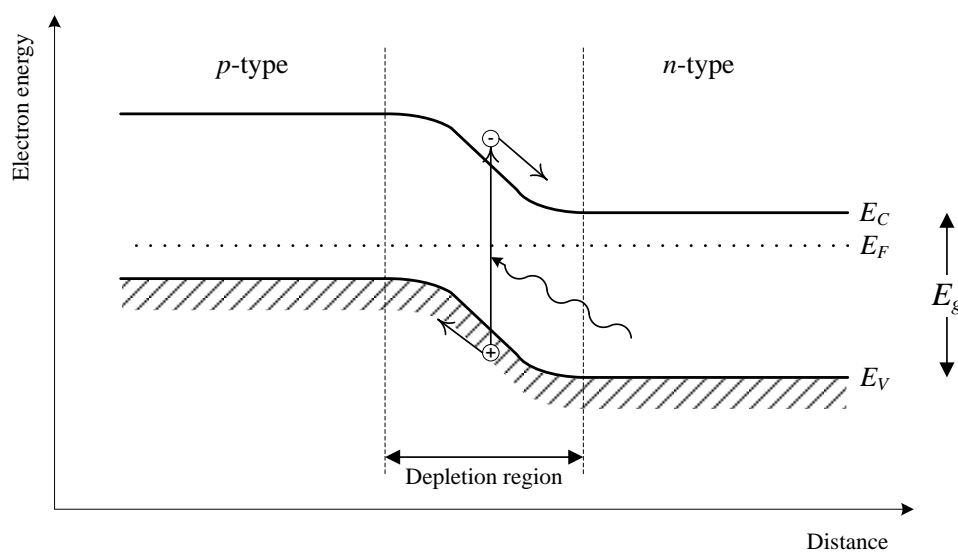


Figure 1: Illustration of the photovoltaic effect.

If the device is left effectively an open circuit an externally measurable potential will appear between the p and n regions, the size of which is dependent on the number of electron-hole pairs generated; that is, on the intensity of the incident radiation. In this mode of operation the device is known as a **photovoltaic cell**. If, on the other hand, the p and n regions are short-circuited externally (or a reverse bias voltage is applied across them), then a current will flow which again is dependent on the intensity of radiation. In this mode the device is called a **photodiode**. The connection of an external resistor will give rise to external power generation, which enables the conversion of light energy into electrical energy as, for example, in the solar cell.

The current voltage (I - V) behaviour of a solar cell under illumination is taken to be:

$$I = I_0 \left[\exp\left(\frac{qV}{nkT}\right) - 1 \right] - I_L \quad (1)$$

where I_0 is the dark reverse saturation current, I_L is the light generated current, k is Boltzmann constant, q the electronic charge, T the temperature and n is the ideality factor. The ideality factor is used to measure how closely the diode of the solar cell follows the ideal diode equation and should be between 1 and 2.

So basically the I - V curve of a solar cell is the superposition of the I - V curve of the solar cell diode in the dark with the light-generated current (equation 1).

The I - V curve is used to derive fundamentals parameters about the solar cell and especially its efficiency:

- **Short circuit current, I_{SC} :** I_{SC} is the current through the solar cell when the voltage across the solar cell is zero (i.e., when the solar cell is short circuited). I_{SC} is due to the generation and collection of light-generated carriers. For an ideal solar cell at most moderate resistive loss mechanisms, the I_{SC} and the light-generated current I_L are identical. I_{SC} depends on the area of the solar cell, the number of photons, the spectrum of the incident light, the optical properties of the solar cell and its collection probability.

$$I_{SC} = I_L \quad (2)$$

- **Open circuit voltage V_{OC} :** V_{OC} is the maximum voltage available from a solar cell, and this occurs at zero current. V_{OC} corresponds to the amount of forward bias on the solar cell due to the bias of the solar cell junction with the light-generated current.

$$V_{OC} = \frac{nkT}{q} \ln\left(\frac{I_L}{I_0} + 1\right) \quad (3)$$

- **Fill factor, FF :** I_{SC} and V_{OC} are the maximum current and voltage respectively from a solar cell. However, at both of these operating points, the power from the solar cell is zero. The FF is defined as the ratio of the maximum power from the solar cell to the product of V_{OC} and I_{SC} . Graphically, the FF is a measure of the "squareness" of the curve and is also the area of the largest rectangle which will fit in the I - V curve.

$$FF = \frac{I_{mpp} \times V_{mpp}}{I_{SC} \times V_{OC}} \quad (4)$$

where I_{mpp} and V_{mpp} are the coordinate of the maximum power point.

• **Efficiency, η :** The efficiency is the most commonly used parameter to compare the performance of one solar cell to another. Efficiency is defined as the ratio of power output (P_{mpp}) from the solar cell to input power (P_{in}) from the sun. In addition to reflecting the performance of the solar cell itself, the efficiency depends on the spectrum and intensity of the incident sunlight and the temperature of the solar cell. Therefore, conditions under which efficiency is measured must be carefully controlled in order to compare the performance of one device to another.

$$P_{mpp} = I_{mpp} \times V_{mpp} \quad (5)$$

$$\eta = \frac{P_{mpp}}{P_{in}} = \frac{I_{SC} \times V_{OC} \times FF}{P_{in}} \quad (6)$$

Equipment

Solar simulator mounted on base

Safety goggles

Si solar cell

Source measure unit (Keithley 2401)

Hot plate

Procedure and analysis

In this laboratory you will use a source measure unit (SMU) to perform the data acquisition. A SMU integrates a voltage source, a current source, a voltage meter, a current meter, and an ohmmeter combined into a single instrument. This is a high end instrument!

Part A: Current voltage characteristic in the dark

(1) Connect the setup as shown in Figure 3. Dark conditions can be created by placing the cover on top of the solar cell. Make sure to connect the circuit in **4 wire configuration**.

(2) Remind yourself what is the purpose of 4 wire measurements and write an explanation in your lab book.

(3) Configure the SMU to perform 4 wire measurements.

-Power up the SMU

-Press Config

-Press "V" button (left of "Meas" function)

-“Sense mode” should now be flashing, press enter. If not flashing press the left or right key.

-Press right arrow key to make “4-wire” display flash

-Press “enter” followed by “exit”

A “4W” should now appear on the top line of the display screen.

(4) To acquire the I - V curve you will want to source voltage and measure the current. Set the instrument to do this by pressing “V” under source and “I” under meas.

(5) One of the key features of the SMU is the compliance (Cmpl). It can be regarded as safety function to limit the current (or voltage) allowed to flow in the circuit. The compliance must always be greater than the value you are trying to measure. When sourcing voltage you must set a maximum allowed current. In the case of the solar cell being investigated I recommend to **set the compliance to 0.2 A**.

To adjust the compliance use the edit buttons (edit, left/right, up down arrows and range up/down).

(6) Measure the current as a function of voltage flowing through the solar cell from **-0.20 V to 0.60 V in 0.05 V steps**. Use the edit buttons to set the desired sourcing voltage. The output of the unit can be turn on by pressing the output ON/OFF button.

(7) When finish turn the SMU output OFF.

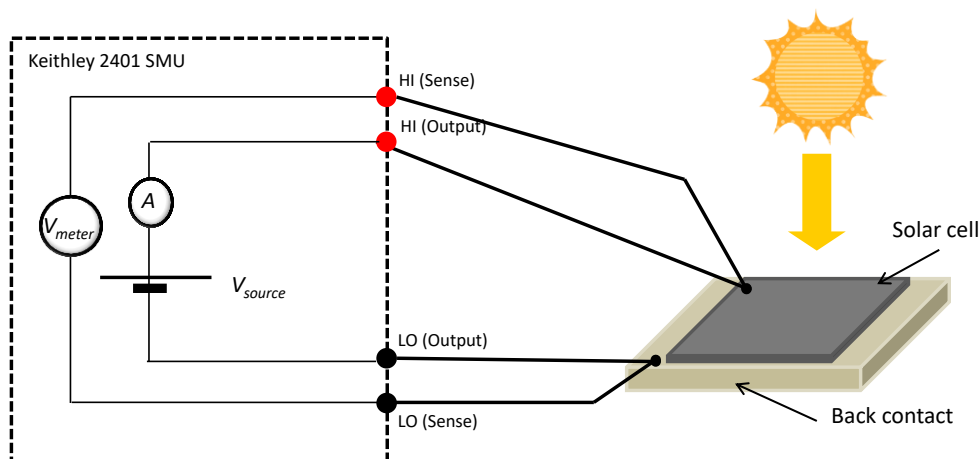


Figure 3: General experimental setup.

Part B: Current voltage characteristic under illumination

In order to turn the solar simulator on you must follow the procedure below (use the photograph of Figure 4 if needed). **The solar simulator must not be turn off until you have finished all the experiments (end of part C).**

(1) Close the shutter of the simulator (downward position).

(2) Turn on the lamp power supply. The current preset reading should be below 7A. If it is not call a member of staff. **Do not adjust the lamp current knob.**

(3) Press the lamp start button.

● HOT SURFACES!

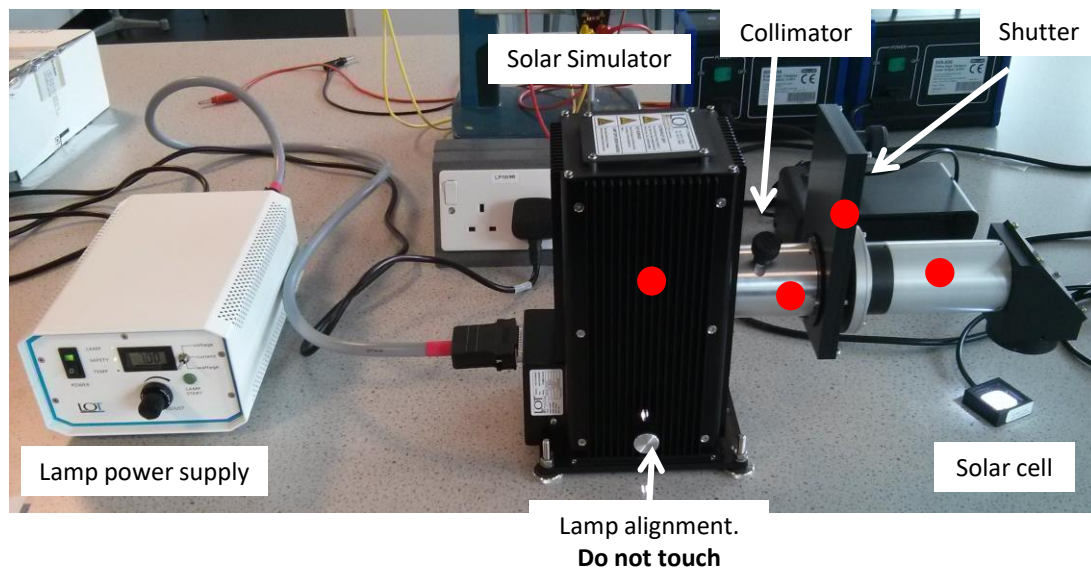


Figure 4: Experimental setup



From this point on please wear the protective eyewear.

(4) Now place the solar cell under the solar simulator and on top of the hot plate. Make sure it is fairly centred within the illumination area when you open the shutter.

(5) Measure the current as a function of voltage flowing through the solar cell from **-0.20 V to 0.60 V**. Keep the compliance set at 200 mA and close the shutter and turn SMU output off when finished.

(6) Plot a graph of current (I) as a function of voltage (V).

(7) On the same graph plot the I - V curve for the dark measurements.

(8) Give values for I_{sc} , V_{oc} , P_{mpp} , FF and the solar cell efficiency, given the irradiance from the solar simulator is 1000 Wm^{-2} and the cell area is 4 cm^2 . Note that P_{mpp} can be found in the fourth quadrant of the I - V curve, taking the absolute value of power.

Part C: Influence of temperature on V_{oc} and I_{sc}

You will now be using the hot plate to investigate the properties of V_{oc} and I_{sc} as a function of temperature.

(1) Set the hot plate at 283 K using the increase/decrease keys (press and hold to initiate change) only and record the V_{oc} and I_{sc} . Remember about the SMU settings when measuring voltage or current (i.e. to measure V_{oc} you must source a current).

(2) Only open the shutter to make the measurements and close it afterwards.

(3) Repeat the procedure as the temperature increases in 10K step up to 343K.

(4) Plot a graph of V_{oc} as a function of temperature and comment on the variations of J_{sc} .

The reverse saturation current I_0 is strongly dependent on temperature and is the cause of the variations observed:

$$I_0 \approx CT^\gamma \exp\left(-\frac{E_g(T=0K)}{kT}\right) \quad (7)$$

where C is a temperature independent constant and γ a number close to 3.

The temperature dependence of the energy bandgap E_g of the semiconductor is also responsible for the observed trends. As temperature increases E_g decreases and thus the solar cell is capable of absorbing photons with longer wavelength, increasing the value of short circuit current. As you should have observed the impact is still minimal on the I_{sc} value since $I_0 \ll I_L$. For V_{oc} , the relationship with temperature can be simplified to:

$$V_{oc} = \frac{E_g(T=0K)}{q} - AT \quad (8)$$

where A is the temperature coefficient which reflect the dark current characteristics in the solar cells.

(5) Calculate the temperature coefficient and energy bandgap for the solar cell investigated here.

(6) Calculate I_0 at 25°C, considering $C=1$.

(7) Using Microsoft Excel fit your experimental light I-V curve using equation 1. You should also be able to fit the dark curve by using equation 1 and $I_L=0$.

(8) Show the demonstration to obtain equation (8). Starting with equation (3), you can assume that $n=1$ (ideal case) and $I_L/I_0 \gg 1$.

Experiment 5 – Heat Engine Cycles

Objectives

The purpose of this laboratory is to observe the practical demonstration of a heat engine cycle and to derive fundamental quantities of heat, work and efficiency at various point of the cycle.

Health & safety consideration

This laboratory uses hot water. Please do not move the hot water container and follow staff instructions.

Introduction

A heat engine is a device that does work by extracting thermal energy from a hot reservoir and exhausting thermal energy to a cold reservoir. In this experiment, the heat engine consists of air inside a cylinder which expands when the attached can is immersed in hot water. The expanding air pushes on a piston and does work by lifting a weight. The heat engine cycle is completed by immersing the can in cold water, which returns the air pressure and volume to the starting values.

The theoretical maximum efficiency η of a heat engine depends only on the temperature of the hot reservoir, T_H , and the temperature of the cold reservoir, T_C . The maximum efficiency is given by

$$\eta = 1 - \frac{T_C}{T_H} \quad (1)$$

The actual efficiency is defined as

$$\eta = \frac{W}{Q_H} \quad (2)$$

where W is the work done by the heat engine on its environment and Q_H is the heat extracted from the hot reservoir.

At the beginning of the cycle, the air is held at a constant temperature while a weight is placed on top of the piston. Work is done on the gas and heat is exhausted to the cold reservoir. The internal energy of the gas ($\Delta U = nC_V\Delta T$) does not change since the temperature does not change. According to the First Law of Thermodynamics, $\Delta U = Q - W$, where Q is the heat added to the gas and W is the work done by the gas.

In the second part of the cycle, heat is added to the gas, causing the gas to expand, pushing the piston up, doing work by lifting the weight. This process takes place at constant pressure (atmospheric pressure) because the piston is free to move. For an isobaric process, the heat added to the gas is $Q_P = nC_P\Delta T$, where n is the number of moles of gas in the container, C_P is the molar heat capacity for constant pressure, and ΔT is the change in temperature. The work done by the gas is

found using the First Law of Thermodynamics, $W=Q-\Delta U$, where Q is the heat added to the gas and ΔU is the change in internal energy of the gas, given by $\Delta U=nC_V \Delta T$, where C_V is the molar heat capacity for constant volume.

Since air consists mostly of diatomic molecules, $C_V=5/2 R$, and $C_P=7/2 R$.

In the third part of the cycle, the weight is lifted off the piston while the gas is held at the hotter temperature. Heat is added to the gas and the gas expands, doing work. During this isothermal process, the work done is given by

$$W = nRT \ln \left(\frac{V_f}{V_i} \right) \quad (3)$$

where V_i is the initial volume at the beginning of the isothermal process and V_f is the final volume at the end of the isothermal process. Since the change in internal energy is zero for an isothermal process, the First Law of Thermodynamics shows that the heat added to the gas is equal to the work done by the gas:

$$\Delta U = Q - W = 0 \quad (4)$$

In the final part of the cycle, heat is exhausted from the gas to the cold reservoir, returning the piston to its original position. This process is isobaric and the same equations apply as in the second part of the cycle.

Equipment

- Heat engine / gas law apparatus
- Temperature, pressure and motion sensors
- Plastic containers
- Ice cube and hot water
- Laptop

Procedure and analysis

Part A: Data Acquisition

The experimental setup is shown in Figure 1. The heat engine is attached to the rod by sliding the heat engine's rod clamp onto the rod. The heat engine should be oriented with the piston end up and the heat engine should be positioned close to the bottom of the rod stand.

The rotary motion sensor is attached to the top of the rod stand and with the medium groove of the pulley of the sensor aligned so that a string coming from the centre of the heat engine's piston platform will pass over the pulley.

One end of a piece of string is threaded through the hole in the top of the piston platform and tied to the shaft of the piston under the piston platform. See Figure 2. The other end of the string is passed over the medium step of rotary motion sensor pulley and attached to the mass hanger and masses totalling 35 grams. This mass acts as a counterweight for the piston.

The piston is positioned about 2 or 3 cm from the bottom of the cylinder and the tube from the can is attached to one port on the heat engine and the tube from the pressure sensor is attached to the other port on the heat engine.



Figure 1: Experimental setup

- (1) Connect the Pressure Sensor to Channel A, the two Temperature Sensors to Channels B and C, and the Rotary Motion Sensor to Channels 1 and 2 on the computer interface.
- (2) Put hot water (about 80°C, ask staff member to provide) into one of the plastic containers (about half full). Put ice water in the other plastic container. The large (about 3 litre) containers keep the hot and cold temperatures constant during the heat engine cycle.
- (3) Place one temperature sensor in the hot water and place the other temperature sensor in the cold water. Note that the temperature sensors are labelled hot and cold in the software program so you will have to pay attention to which sensor you put in the hot water and which is in the cold water.
- (4) Run Data Studio software and load the file “HeatEngineCycle” located on the desktop.

(5) Perform the following cycle without hesitating between steps. You may want to practice a few times before recording a data run. Start with the can in the cold water. This starting point will be called point A. Record the height of the bottom of the piston. Start recording data on the computer.

A→B	Place the 200g mass on the platform.
B→C	Move the can from the cold bath to the hot bath.
C→D	Remove the 200g mass from the platform.
D→A	Move the can from the hot bath to the cold bath.

(6) Export the graph of the cycle. Label the four corners of your graph as A, B, C, and D. Identify the temperatures at points A, B, C, and D. Put arrows on the cycle to show the direction of the process.

(7) Identify the types of processes (i.e., isothermal, etc.) and the actual physical performance (put mass on, put in hot bath, etc.) for A to B, B to C, C to D, and D to A.

(8) Identify and label the two processes in which heat is added to the gas.

Part B: Analysis

(1) Calculate the ideal (maximum) efficiency for a heat engine operating between the two temperatures using Equation 1.

(2) Calculate Q_H , the heat added to the gas by the hot reservoir during the isobaric expansion from B to C and the isothermal expansion from C to D. You will need to calculate the following:

(a) We do not know the initial volume, V_A , but we can calculate it by measuring the volume of the can and adding the initial volume of air in the cylinder. We will ignore the volume in the tubes.

$$V = (\pi r^2 h)_{can} + (Ah_o)_{cylinder} \quad (5)$$

where A is the cross-sectional area of the piston.

(b) Calculate V_D using an isobar and the Ideal Gas Law: $\frac{V_A}{T_A} = \frac{V_D}{T_D}$

(c) Calculate V_C using an Isotherm and the Ideal Gas Law: $P_C V_C = P_D V_D$

(d) Calculate $Q_{C \rightarrow D}$. For an isotherm, $Q = nRT \ln(V_f/V_i)$, and since $PV = nRT$, then

$$Q_{C \rightarrow D} = P_D V_D \ln\left(\frac{V_D}{V_C}\right) \quad (6)$$

(e) Calculate $Q_{B \rightarrow C}$. For an isobar, $Q = nC_p \Delta T$, and since air is a diatomic gas $C_p = 5/2 R$, and $nR = PV/T$,

$$Q_{B \rightarrow C} = \frac{5}{2} \frac{P_D V_D}{T_D} (T_C - T_B) \quad (6)$$

(f) Calculate $Q_H = Q_{B \rightarrow C} + Q_{C \rightarrow D}$

(3) Calculate the work done by the gas by measuring the area inside the curve.

(4) Calculate the efficiency using equation 2 (=work done by gas/ heat extracted from hot reservoir). How does this compare with the ideal efficiency from Step 1?

(5) Calculate the actual work done on the 200 g mass using $W = mgh$. Be careful to use only the change in height of the mass. How does this compare to the work done by the gas from part 3? Does the gas do any work other than lifting the 200 g mass?

(6) Mix some of the ice water with the hot water and vice versa so the two reservoirs are closer to the same temperature. Perform the cycle again. How high is the weight lifted now? What is the theoretical efficiency using the new temperatures?

Experiment 6 – X-ray Diffraction

Objectives

The purpose of this laboratory is to learn the principles of X-ray generation, learn the fundamentals of Bragg diffraction and the application of Bragg's law of crystallography.

Health & safety consideration

This laboratory does not contain any specific health and safety hazards.

Introduction

X-rays were first discovered by Wilhelm Röntgen in 1895. His experiments suggested that X-rays were electromagnetic waves with a wavelength of about 10^{-10} m. Max von Laue proposed in the early 1900s that a crystal could serve as a three-dimensional diffraction grating for X-rays. This was demonstrated experimentally (X-ray diffraction) in 1912 by von Laue, Walter Friedrich and Paul Knipping. Soon after Henry Bragg and his son Lawrence derived Bragg's law.

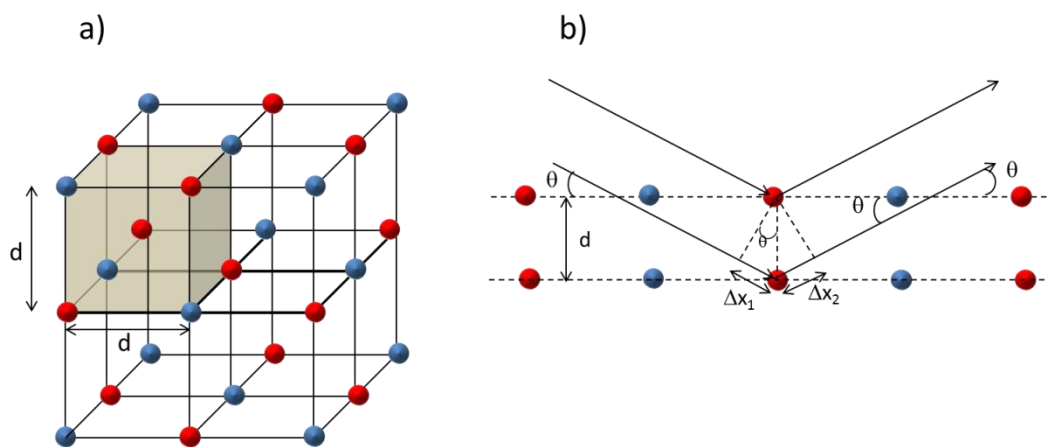


Figure 1: a) A cubic crystal lattice with interatomic spacing d . b) Path length difference for X-rays scattered from two adjacent planes.

Let's assume a cubic crystal such as the one shown in Figure 1 a), with each atom in the lattice a distance d away from its neighbouring atoms in all three directions. We can imagine various lanes of atoms in this crystal. For example, the horizontal planes are composed of atoms spaced a distance d apart, with the planes themselves spaced a distance a from one another. If X-rays are incident on those planes, the row of atoms in the crystalline lattice can act like a diffraction grating.

Interference effects are caused by path length differences. When X-rays scatter off one plane, all the waves remain in phase as long as the incident angle equals the reflected angle. However for two adjacent planes the path length difference for the scattered X-rays from two planes is (see Figure 1 b)):

$$\Delta x = \Delta x_1 + \Delta x_2 = 2d \sin \beta \quad (1)$$

where β is the angle between incoming X-rays and the plane of atoms. Thus the criterion for constructive interference due to Bragg scattering is given by:

$$2d \sin \beta = n\lambda \quad (2)$$

where n is the order of diffraction and λ the X-ray wavelength.

Bremsstrahlung radiation: X-rays are created when fast moving electrons are rapidly decelerated in matter. According to the laws of classical thermodynamics, this deceleration gives rise to electromagnetic radiation which is mainly radiated perpendicular to the direction of the electrons striking the anode. For historical reasons, this X-ray component is referred to as “bremsstrahlung” after the German work for the deceleration process by which it occurs. The bremsstrahlung radiation has a continuous spectrum which extends to a certain maximum frequency (or minimum wavelength).

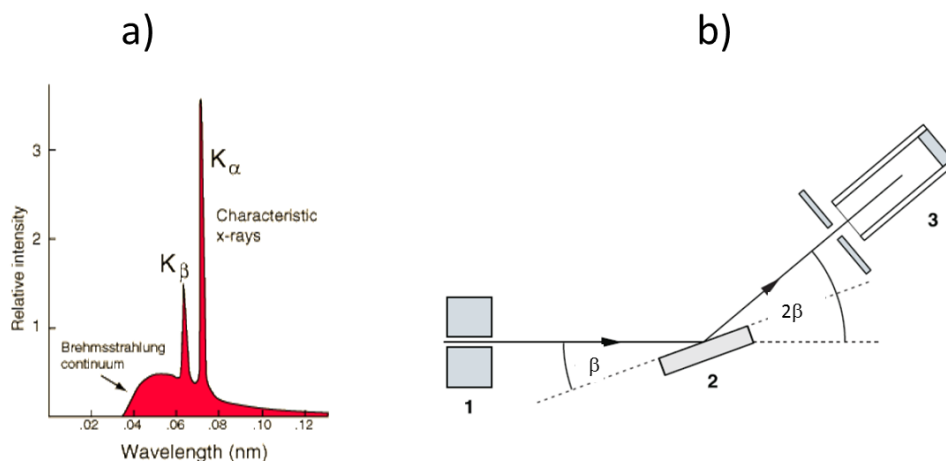


Figure 2: a) Typical emission spectra of an anode. b) Schematic of the experimental setup for β - 2β diffraction.

Atomic emission lines: if the energy of the electrons exceeds a critical value, characteristic X-ray radiation is generated, which appears in the spectrum as individual lines superimposed on the continuous bremsstrahlung spectrum (see Figure 2 a). These lines are generated when high energy electrons penetrate deep into the atomic shells of the anode material and eject electrons from the innermost orbitals. The gaps created in this process are filled by electrons from the outer orbitals. The process results in a photon (X-ray) with a characteristic energy equal to the difference in the energy of the orbitals. As the energy of an orbital is dependent on the material, the resulting X-ray radiation is characteristic of the anode material. The characteristic X-rays emission which is shown as two sharp peaks in Figure 2 a) occurs when vacancies are produced in the $n=1$ or K-shell of the atom

and electrons drop down from above to fill the gap. The X-rays produced by transitions from the $n=2$ to $n=1$ levels are called $K\alpha$ x-rays, and those for the $n=3 \rightarrow 1$ transition are called $K\beta$ X-rays.

Equipment

X-ray diffractometer with Mo anode

NaCl crystal $25 \times 25 \times 4 \text{ mm}^3$ (lattice spacing $a = 282.0 \text{ pm}$)

Zr filter

Nitrile gloves

Why do we use a single crystal of NaCl?

A crystal consists of a series of repeating units of atoms or molecules made up of a lattice (a series of points in space) and a basis (the atoms themselves). The scheme of repetition (the lattice) is defined by three orthogonal vectors, \vec{a} , \vec{b} and \vec{c} . The lattice that the NaCl structure is based upon face centred cubic with a unit cell whose dimension $a = b = c = 5.640 \text{ \AA}$. The structure is shown in Figure 5.

The distance between layers in the NaCl crystal is very regular and is known as the interplanar spacing d (as mentioned earlier) with $d = 2.820 \text{ \AA}$. The wavelength of X-rays is too short to be diffracted by mechanically ruled gratings except at highly grazing angles of incidence. Naturally occurring 3D gratings of appropriate spacing are provided by the regular layers in crystals such as NaCl.

You are using the NaCl crystal with its well defined lattice spacing and known structure as a grating, effectively as part of a spectrometer to study X-rays. However it is important to note that this problem could be inverted. You could analyse diffraction of X-rays from a crystal of unknown material to determine its structure, both its lattice and even its molecular chemical structure. This is the basis of X-ray crystallography, and this is how even complex biological macromolecules such as DNA can be structurally solved.

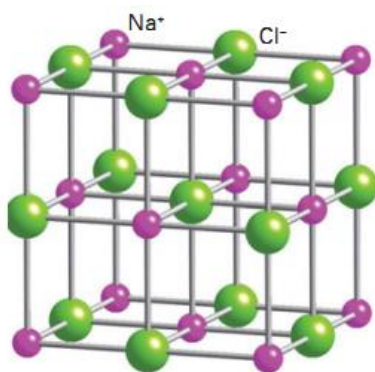


Figure 3: A single unit cell of NaCl.

Procedure and analysis

Note that the θ and β are both used in the script and those describe the same angle. Make sure to save your data throughout.

Part A: Investigation of the energy spectrum of an X-ray tube

In the experiment you will record the **emission spectrum of a Molybdenum anode** by means of Bragg diffraction of the X-rays from a NaCl crystal, as a function of anode voltage and emission current.

The experimental arrangement is shown in Figure 2b). The spectrometer is composed of a goniometer with a NaCl crystal and a Geiger-Müller counter tube in the Bragg configuration. The crystal and counter tube are pivoted with respect to the incident X-ray beam in a β - 2β coupling.

☞ **NaCl crystals are hygroscopic and fragile. Wear the plastic gloves provided to handle the crystal and avoid mechanical stresses on the crystal!**

(1) The crystal will be pre mounted on the system. However if it wasn't you should loosen the knurled screw on the sample holder (item g in Figure 4) in the X-ray machine and lay the NaCl crystal flat on the target stage (**wear the plastic gloves provided to handle the crystal**). Carefully raise the stage as far as it will go ensuring that the crystal is flush against the mounting lip and then tighten the knurled screw with care. **Do not use excessive.** Make sure to close the door when done.

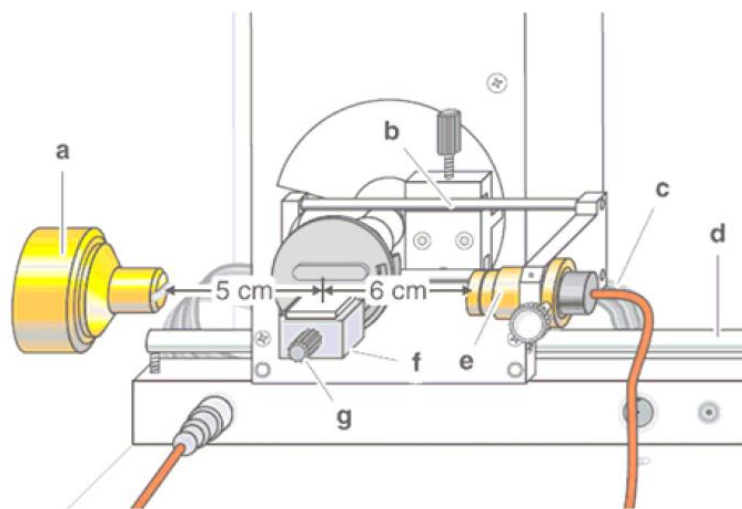


Figure 4: Schematic of the X-ray goniometer.


- (2) Turn on the apparatus and start the "X-ray apparatus" software (in Start/All programs menu). Press the ZERO key on the X-ray system; this will move the sample and detector to the zero position.
- (3) Clear any existing measurement data using the top left hand icon (or F4 key).


(4) Go to the settings window of the software (press F5 if you can't find the icon) and set the tube high voltage to $U=35$ kV. Set the emission current to $I=0.6$ mA, the measuring time per angular step $\Delta t=3$ s and the angular step width $\Delta\beta=0.1^\circ$.

(5) Press the COUPLED key to activate 2θ coupling of target and sensor and set the lower limit of the target angle to $\beta_{\min}=2.5^\circ$ and the upper limit to $\beta_{\max}=12.5^\circ$.

(6) Start measuring by ticking the SCAN tick box. You can close the settings window now.

(7) When the measurement is finished save your data (give it an appropriate filename).

 You can change the unit of the x-axis between angle, wavelength and energy by changing the crystal parameters in the settings window. Do investigate this.

 You can copy the data at any time (right click on graph and select copy table) and paste into Excel.

(8) Repeat the measurement with an emission current of **1 mA**. You can acquire the data to overlay on the previous data.

(9) Whilst holding the left mouse button highlight a peak. Click the right mouse button to access the evaluation functions of the software "X-ray Apparatus". For each peak, select "Calculate Peak Centre" and then mark the full width of the peak by dragging across the peak with the left mouse button. Record the position and width of each peak.

(10) Report the positions and width of $K\alpha$ and $K\beta$ (all in degrees) at the two different currents in your lab book.

(13) What effect does the variation of emission current I have on the shape of the X-ray spectrum? How does the intensity of the bremsstrahlung spectrum and the characteristic lines vary with changes of the emission current?

You will now investigate the effect of varying anode voltage.

(14) Create a new data page. Adjust the voltage to $U=25$ kV and $I=1$ mA.

(15) Collect the data and create a plot with the two spectrum $U=25$ kV and 35 kV ($I=1$ mA). Comment on what you observe.

Part B: The Duane-Hunt relation and determination of Planck's constant.

The bremsstrahlung continuum in the emission spectrum of an X-ray tube is characterised by λ_{\min} , which becomes smaller as the tube high voltage (U) increases (as shown in the previous experiment). W. Duane and F. L. Hunt discovered in 1915 an inverse proportionality between λ_{\min} and U . The Duane-Hunt relationship can be explained by examining some basic quantum mechanical considerations. λ_{\min} corresponds to a maximum frequency (ν_{\max}) and to a maximum energy (E_{\max}).

The most energetic X-ray is generated when all of the kinetic energy (E_{kin}) of the incident electron is converted into radiation:

$$E_{kin} = e \times U \quad (3)$$

where e is the charge of an electron. This yields:

$$\lambda_{min} = \frac{hc}{e \times U} \quad (4)$$

where c is the speed of light and h is Planck's constant.

(1) Conduct an experiment with the following settings: $U=22$ kV, $I=1$ mA, $\Delta t=30$ s, $\Delta\beta=0.1^\circ$, $\beta_{min}=5.2^\circ$ and $\beta_{max}=6.2^\circ$.

(2) Set the x-axis to wavelength.

(3) Record **at least 3 more** measurements using the parameters from Table 1.

U (kV)	I (mA)	Δt (s)	β_{min} ($^\circ$)	β_{max} ($^\circ$)
24	1.0	30	5.0	6.2
26	1.0	20	4.5	6.2
28	1.0	20	3.8	6.0
30	1.0	10	3.2	6.0
32	1.0	10	2.5	6.0
34	1.0	10	2.5	6.0

Table 1: Parameters for the Duane-Hunt experiment. $\Delta\beta=0.1^\circ$ for all measurements.

(4) Plot of graph of counts as a function of wavelength for each set of experimental parameters. λ_{min} is given by the x intercept of the linear part of the graph.

(5) Plot λ_{min} as a function of the inverse voltage and calculate Planck's constant. Give experimental and relative error.

Part C: Use of an X-ray filter and the measurement of an atomic absorption edge

When X-rays pass through matter they lose energy both by scattering and by absorption. In the absorption process an X-ray photon is absorbed by the atom and the excess energy is transferred to an electron which is expelled from the atom, leaving the atom ionised. This process is called **photoelectron absorption**. Quantitatively, the absorption is given by the linear absorption coefficient μ . The intensity $I(z)$ through the sample decreases exponentially:

$$I(z) = I_0 \exp(-\mu z) \quad (4)$$

where $I_0=I(z=0)$ and I_0 is the incident beam intensity.

The absorption cross-section has an unusual and distinct dependence on the wavelength (and hence energy) of the X-rays. An example is shown in Figure 5. A smooth curve in the plot is followed by very sharp jumps. These discontinuities are called absorption edges and they occur at the

wavelength corresponding to the energy needed to knock out an electron out of an atomic shell in the absorbing material. In particular, the K absorption edge of an element lies very slightly to the short wavelength side of the $K\beta$ lines for that element. Like the characteristic lines, the absorption edge shifts to longer wavelength with decrease in the atomic number.

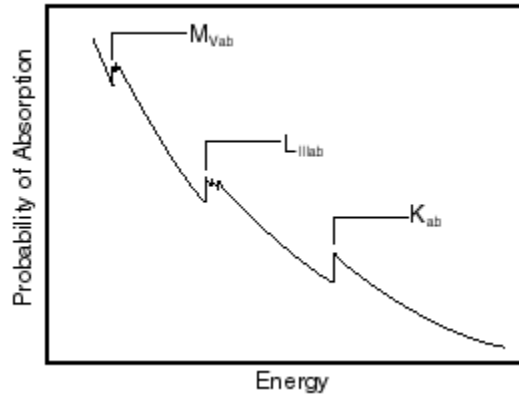


Figure 5: Diagrammatic representation of the variation of the absorption coefficient with energy of the absorbed radiation showing the K (bottom right) and the $L_{I,II,III}$ (centre) absorption edges.

In this experiment you will record the transmission energy spectrum of an X-ray tube with a molybdenum (Mo) anode through a zirconium (Zr) filter. You will investigate how the characteristic spectrum is changed by the addition of an atomic filter, and you will measure the position (in energy) of the Zr K absorption edge. Finally, you will see how the occurrence of atomic absorption edges can be used in X-ray experiments to produce quasi-monochromatic radiation.

- (1) **Gently fit** the zirconium filter onto the front of the collimator. The orientation of the filter does not matter, we are simply using the Zr metal sheet as a beam absorber.
- (2) Acquire data with the following settings: $U=35$ kV, $I=1.0$ mA, $\Delta t=3$ s, $\Delta\beta=0.1^\circ$, $\beta_{min}=2.5^\circ$ and $\beta_{max}=12.5^\circ$.
- (3) Copy and paste your data into a new Excel spreadsheet recorded with the same parameters in part A. Remember to convert angle into energy.
- (4) Plot the obtained spectra with and without (using a dataset from part A) the Zr filter as a function of energy.
- (5) What are the energies of the Mo $K\alpha$ and $K\beta$ fluorescent lines in a spectrum with a Zr filter?
- (6) What effect does the Zr filter have on the shape of the X-ray spectrum?
- (7) Plot the transmission (a ratio of the intensity with the filter to the intensity without the filter) curve as a function of energy (including the y-error bars).
- (8) What is the energy in keV of the Zr absorption edge (evaluate it by inspection of the transmission curve)? Fill the gaps of Table 2.

(9) At research level Cu is often used as the anode. What material would make a good filter to suppress the Cu $K\beta$ line?

Element	K Abs Edge (keV)	$K\alpha$ line (keV)	$K\beta$ line (keV)
Cr	5.987	5.412	5.947
Co	7.712	6.925	7.649
Ni	8.339	7.473	8.265
Cu	8.993	8.041	8.905
Zn	9.673	8.632	9.570
Rb	15.20	13.37	14.96
Y	17.04	14.94	16.73
Zr		15.74	17.66
Nb	18.98	16.60	18.62
Mo	20.00		

Table 2: Energies of K absorption edge and the position of the $K\alpha$ and $K\beta$ for selected elements.

Experiment 7 – Temperature Dependence of PN Junction Characteristics

Objectives

The purpose of this laboratory is to explore the dark characteristics of a diode as a function of temperature.

Health & safety consideration



Never increase the current of the large power supply used for the heating system beyond 0.7A!

Introduction

The base emitter junction of a 1N34A germanium pn diode may be used to investigate the current-voltage (I - V) characteristics of a germanium pn-junction diode. Analysis of pn-junction diode behaviour under small applied DC potential leads to a I - V characteristic of the form

$$I = Ae \left[\frac{N_p D_p}{L_p} + \frac{N_n D_n}{L_n} \right] \exp \left(\frac{-eV_0}{kT} \right) \left[\exp \left(\frac{eV}{kT} \right) - 1 \right] \quad (1)$$

where N_p , N_n are the net densities of holes and electrons in p-type and n-type material, respectively, at the temperature T ; V_0 is the junction barrier potential (of the same order as the energy bandgap), A is the junction area, L_p and L_n are the carrier diffusion lengths. D_p and D_n are the carrier diffusion coefficients, e is the electronic charge, V is the applied potential difference resulting in a current I .

In reverse bias, $V < 0$, and the current saturates to a value:

$$I_s = Ae \left[\frac{N_p D_p}{L_p} + \frac{N_n D_n}{L_n} \right] \exp \left(\frac{-eV_0}{kT} \right) \quad (2)$$

In forward bias, $V > 0$, and the characteristic is described by:

$$I = I_s \left[\exp \left(\frac{eV}{kT} \right) - 1 \right] \quad (3)$$

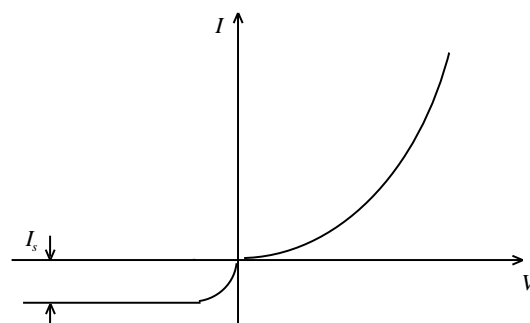


Figure 1: Current voltage characteristic of a pn junction.

Equipment

Germanium diode (1N34A).

Agilent 34450A digital multimeter (used as DC ammeter).

Muirhead decade resistance box

Farnell L30B small power supply

TENMA 728720 digital Mmultimeters (used as DC voltmeter, volt range)

TECA Peltier heating/cooling plate

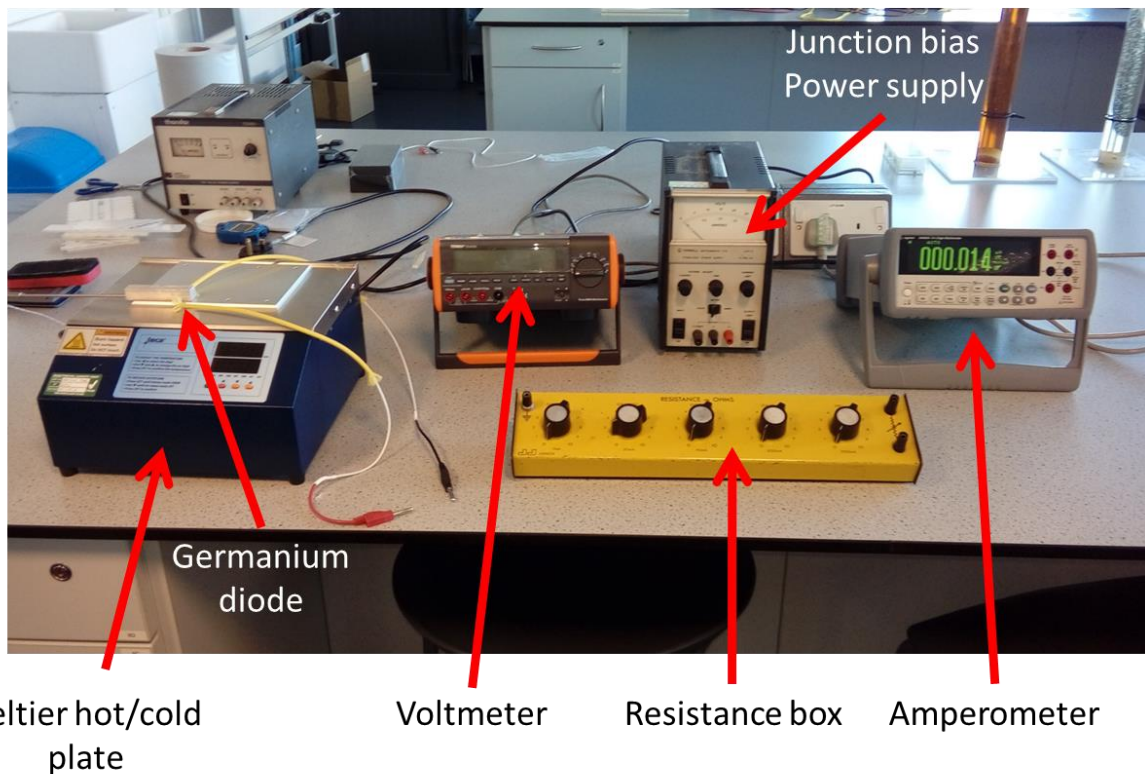


Figure 2: Experimental setup.

Procedure

The characteristics may be obtained with the circuits shown in Figure 3. To obtain the forward characteristic, the applied potential difference is obtained from a low voltage supply in series with a resistance box. **PLEASE ENSURE** that R is set at a **MINIMUM** of $1000\ \Omega$ when first switching on the supply. The applied potential difference when obtaining the reverse characteristic is taken from the voltmeter of the power supply itself.

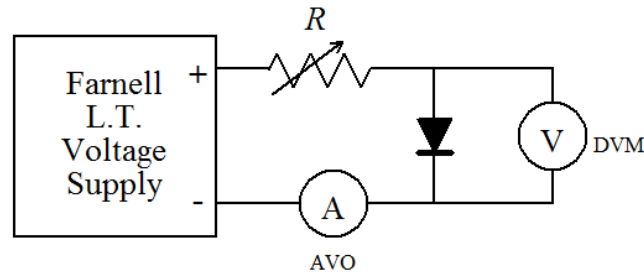


Figure 3: Experimental setup for forward measurements. Note reverse the diode to measure in reverse direction.

(1) At room temperature, obtain the forward characteristic for voltages up to 1 V and also the reverse characteristic for voltages up to 25 V (at least five measurements are required on the forward characteristic between 0 and 0.2 V for better resolution in this range). **(Notice: setup the current range of the multimeter to mA and don't change it during the measurements).**

(2) With the circuit set up to measure the reverse leakage current at 5 V (i.e. the saturation current I_s), switch on the Peltier plate heater/cooler (the switch is located at the back of the unit). Adjust the temperature of the plate to 10 °C. Follow the instruction on the plate itself or below:

- set the plate in auto tune mode press “set” multiple times until the display “PROCESS” (top display) shows *ENAB*.
- Press arrow down until the display called SET (bottom display) shows *ATI* and press “set” to confirm.
- Adjust the temperature to the desired value simply use the left arrow button to select the digit you want to change in the bottom display called SET, and use arrow down or up to change the value, then press “set” to confirm the temperature.

(Notice: the bottom display (in red) is the set temperature and the top display (green) is the actual measured temperature detected by the probe inserted close to the Ge diode).

(3) Record I_s as a function of temperature T for T increasing in step of 5 °C up to 65 °C.

(Notice: setup the current range of the multimeter to 100 μ A and don't change it during the measurements).

(4) Once you reach 65 °C, obtain the forward and reverse characteristics as in (1).

(5) Plot the characteristics for the two steady temperatures on the same graph (room temperature and 65 °C), and summarise the effect of temperature on the junction characteristics.

(6) For the forward characteristic at room temperature, check the applicability of equation (3) by plotting $\log(I + I_s)$ as a function of V , where I_s can be taken as the reverse leakage current at 5 V. Derive a value for e/kT and compare it to the theoretical value. Remember that if $y = \exp(x)$ then $\log(y) = x \log(e)$.

The original equation is only applicable for low values of V (e.g. less than 0.2 V), and it can be shown that the slope of the graph for higher values of V is $\frac{e}{2kT}$. Again, compare with the theoretical value.

(7) In reverse bias, the saturation current can be expressed in the form:

$$I_s = B \exp\left(\frac{-eV_0}{kT}\right) \quad (4)$$

where B is assumed to be constant.

Hence, a plot of $\log(I_s)$ versus $\frac{1}{T}$ should be linear, and the slope allows the determination of the barrier potential V_0 .

Experiment 8 – Radioactive Decay

Objectives

The purpose of this laboratory is to understand the fundamentals of radioactivity via the measurement of α , β and γ particles with a Geiger counter.

Health & safety consideration

The two radioactive sources used in this laboratory (Strontium 90 and Americium 241) have an activity of 5 microcuries (185 kBq). **There is no hazard with a source**, unless it is held close to the body for long periods. The dose rate at a distance of 30 cm from a 5 microcurie Cobalt 60 source is 0.04 millirontgens/hour. In this case an exposure of over **1250 hours** would be needed to exceed the recommended dose which is 50 millirontgens.

The sources are contained within a leaded box with two windows to be opened as required.

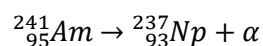
Introduction

Natural elements can be classed in two categories: (i) firstly those elements which nuclei are not subject to any transformation with time; they are said to be stable elements. (ii) Secondly those elements which nuclei emit an ionizing radiation at some time during their lifetime are called radioactive elements or radioelements. Amongst the 92 natural elements known, a dozen are radioactive.

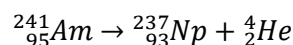
Radioactive decay or **radioactivity** is the spontaneous transformation of a nucleus during which it emits a radiation. The nucleus loses energy by emitting particles such as alpha particles, beta particles or gamma rays.

There are many different types of radioactive decay however the most common spontaneous transformation are summarized below:

- **Alpha (α) decay:** is a type of radioactive decay in which an atomic nucleus (with mass A and atomic number Z) emits an α particle and thereby transforms (or 'decays') into an atom with a mass number A-4 and atomic number Z-2. For example, Americium-241 decaying through α -particle emission to form neptunium-237 can be expressed as;



Because an α particle is the same as the nucleus of a helium-4 atom - consisting of two protons and two neutrons and thus having mass number 4 and atomic number 2 - this can also be written as:



Alpha decay occurs for heavy elements (A large).

- **Beta (β) decay:** is a type of radioactive decay in which a beta particle (an electron or a positron) is emitted from an atomic nucleus. β decay is mediated by the weak force: β^- and β^+ . In the case of beta decay that produces an electron emission, it is referred to as β^- , while in the case of a positron emission as β^+ . In both cases the nucleus atomic number Z changing by one without change of mass A .



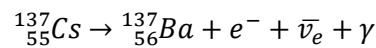
The decay is accompanied by the emission of electron antineutrino ($\bar{\nu}_e$) or electron neutrino (ν_e).

In all cases where β^+ decay of a nucleus is allowed energetically, the electron capture process is also allowed, in which the same nucleus captures an atomic electron with the emission of a neutrino:



- **Spontaneous fission:** This type of decay is found only in very heavy elements (those for which $(Z^2/A) \geq 47$, e.g. Uranium)

- **Gamma (γ) emission:** γ rays ($\lambda < 10$ pm, $f > 10^{19}$ Hz) are produced alongside other forms of radiation such as α or β , and are produced after the other types of decay occur. The mechanism is that when a nucleus emits an α or β particle, the daughter nucleus is usually left in an excited state. It can then move to a lower energy state by emitting a gamma ray photon, in the same way that an atomic electron can jump to a lower energy state by emitting a light ray photon. For example the β^- decay of Caesium-137 into Barium-137 is also emitting a γ ray of energy 0.66 MeV



Similarly the α decay of Americium-241 mentioned earlier also yields the emission of γ ray.

Laws of radioactivity

- Radioactive decay was modelled explicitly for the first time by Rutherford and his student Soddy in 1902: the probability of transformation of a radioactive atom during the time dt is λdt , with λ the decay constant. If N is the number of radioactive atoms at a given time t then the number of atoms dN disintegrating between the time interval t and $t+dt$ is therefore:

$$dN = -\lambda N dt \quad (1)$$

The solution to this first order differential equation is:

$$N(t) = N_0 e^{-\lambda t} \quad (2)$$

where N_0 is the number of atoms at $t=0$. From this can be defined (i) the **half-life** $T_{1/2}$, the time taken for the activity of a given amount of a radioactive substance to decay to half of its initial value:

$$N(T_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad (3)$$

thus
$$T_{1/2} = \frac{\ln 2}{\lambda} \quad (4)$$

and (ii) the **mean lifetime τ** , the average lifetime of a radioactive particle before decay:

$$\tau = \frac{1}{\lambda} \quad (5)$$

The mean lifetime can vary between 10^{-12} s and 10^{17} s. The decay constant can be used to identify a particular atom.

- The law of absorption is given by Beer Lambert relationship (see laboratory 1):

$$I = I_0 \exp(-\mu d) \quad (6)$$

where I_0 is the intensity of the incident radiation entering the medium, I the intensity of the transmitted radiation leaving the medium, d the material thickness and μ the absorption coefficient. The thickness $d_{1/2}$ required to absorb half of the incident radiation is called **half-value layer (HVL)** is:

$$d_{1/2} = \frac{\ln 2}{\mu} \quad (7)$$

Units of radioactivity

You first need to differentiate between the intensity of the emission from a sample of radioactive material, *i.e.* the number of decays per unit time and the effect that a given type of radiation has on the human body.

The SI unit of *radioactivity* is the becquerel (Bq), where 1 Bq = 1 nuclear decay/s. This is an extremely small number. A non-SI but commonly used unit of radioactivity is the curie (Ci), where 1 Ci = 3.7×10^{10} Bq.

The SI unit for the *absorbed dose* is the gray (Gy), defined as 1 joule per kilogram of absorbing material, *i.e.* 1 Gy = 1 J/kg. A commonly used non-SI unit is the rad (rd), where 1 rd = 0.01 Gy.

For X-rays and γ -rays, it is important to measure the amount of ionization (separation of electrons from their previously neutral atoms). This quantity is called *exposure* and the non-SI unit is the röntgen (R), where 1 R = 2.58×10^{-4} C/kg of air.

The extent of damage to biological tissue depends not only on the deposited energy (absorbed dose), but also on the type of particle the nuclear decay emits. Thus a *radiation weight factor w_r* is introduced where w_r ranges from 1 for all photons and β particles to 20 for α particles. The SI unit for the *dose equivalent* is the Sievert (Sv), defined as 1 Sv = $w_r(1 \text{ Gy})$. A non-SI unit is the rem (röntgen equivalent, man), where 1 Sv = 100 rem.

How much radiation are you likely to receive daily?

In the UK the health protection agency has calculated that on average people are exposed to about 2.7 mSv of radiation a year. This comes from natural radon emission seeping through the ground (1.3 mSv), medical x-rays (0.4 mSv), food (0.4 mSv), cosmic rays (0.3 mSv), Earth (0.2 mSv) and

others (including TV, computer, smoke detector etc) (0.1 mSv). In the UK, Cornwall residents are exposed to a higher dose of 7.8 mSv due to radon leaks from the Earth.

A dose of 250 mSv will cause no observable change on the human body. Between 250-1000 mSv slight blood changes may be observed; while a dose of 1000-2000 mSv will cause significant reduction in blood platelets and white blood cells (temporary); between 2000-5000 mSv severe blood damage, nausea, hair loss and death in many cases. A dose of 6000 mSv or more will cause death in less than two months for over 80% of human.

Equipment

Radioactive sources (Americium 241 ($\alpha+\gamma$) and Strontium 90 (β) of both 185 kBq) and holder on breadboard

Lablogic Geiger counter ($\alpha+\beta+\gamma$) and alpha/beta probe ($\alpha+\beta$)

Paper, aluminium and lead sheets

Where possible your graphs will need to include error bars and to give values with its experimental error (even if not asked). Counting events such as radioactive decay or photon counting using a Geiger tube follow a Poisson distribution. If an experiment yields a mean count of N (which is equal to the count rate \times time) then the best estimate of the error in this quantity is $\sqrt{N} = \sigma$. Thus we represent the measurement as $N \pm \sqrt{N}$.

Procedure and analysis

Part A: Natural radiation from the environment

First you will measure radiations from the lab.

(1) Set the Geiger counter as ratemeter: Menu/Config/Mode: ratemeter.

(2) Take 10 readings for the background radiation in the lab and give an average.

(3) The average annual natural dose on humans in the UK is 2700 μSv . Using the meter in dose mode (Menu/Config/Mode: Dose rate) take a few readings of the background value and compare this to the average UK value. Note that an exclusion zone would apply for a dose $> 3000 \mu\text{Sv/hr}$.

Part B: Absorption of α particles using paper

The simplified experimental setup is shown in Figure 1. The radioactive sources are placed in a leaded box with an aperture for each source. The Geiger counter (or paddle counter) is placed directly in front of the source. In most cases, you need to make sure the position of the counter stays constant throughout the experiment as the efficiency of the counter is function of the distance from the source.

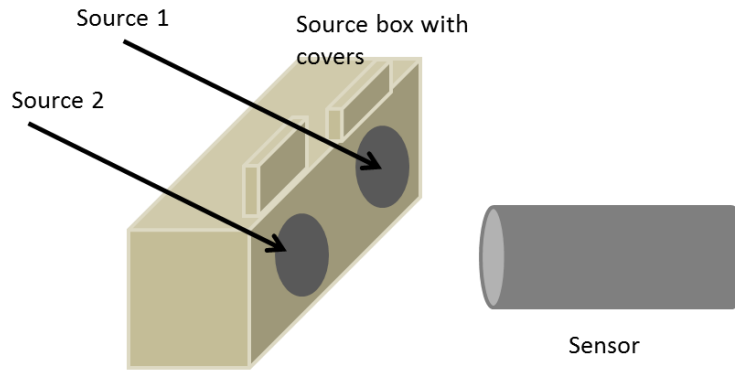


Figure 1: Experimental setup

- (1) Set the paddle detector as close as possible from the Americum-241 shutter. Record the background $Z_{\text{background}}$ radiation (ratemeter mode).
- (2) Open the Americum-241 shutter and record Z (cps).
- (3) Now add a sheet of paper between the two and take a reading.
- (4) Remove the sheet of paper and move the paddle a few cm (2-3 cm) away from the source. Record Z .
- (5) Comment on what you observe.

Part C: Absorption of β particles using aluminium

- (1) Place the Geiger counter in front of Strontium 90 source with the small holder between sensor and source. Record $Z_{\text{background}}$.
- (2) Open the Strontium-90 shutter and record Z .
- (3) Now add a thin sheet of aluminium (0.6 mm thick) on the holder between source and detector and take a new reading. Repeat as you insert more aluminium plates (up to 8).
- (4) Close the source shutter.
- (5) Plot a suitable graph to determine the absorption coefficient μ and half value layer $d_{1/2}$ for aluminium. Using the linear region of the plot give values for both μ and $d_{1/2}$.

(6) The electron (or positron) emitted from β radiation will emit an electromagnetic radiation when decelerated (or accelerated) called Bremsstrahlung radiation (X-ray part of the electromagnetic spectrum). Those X-rays are not absorbed by the Al plates and are detected by the Geiger counter. Looking at your data, where do you observe this? Explain.

Part D: Law of distance applied to β particles

The source emits in all directions. If A is the area of the source (circular aperture of diameter 9.1 mm) then the number of pulses Z is proportional to A/L^2 where L is source to meter distance.

- (1) Bring the Geiger counter as close as possible from strontium 90 source and record $Z_{\text{background}}$.
- (2) Open the Strontium-90 shutter and the number of counts Z .
- (3) Repeat as L increases from 1 to 12 cm.
- (4) Plot $\log Z = f(\log(L))$.
- (5) Using the slope of the graph how can you tell your measurement is correct?

Part E: Determination of half value layer of lead for γ radiations

- (1) Repeat the experiment of Part C for γ radiations (using the Americium-241 source and lead sheets of thickness 0.125mm).
- (2) Repeat (1) using 4 mm thick aluminium plates instead of lead.
- (3) Discuss and conclude on your results.

Experiment 9 – Cosmic-Ray, Muons and Special Relativity

Objectives

The purpose of this experiment is to detect muons originating from cosmic rays coming from deep space and measure their “relativistic” mean speed in order to verify the space-time effects predicted by the special relativity theory.

Health & safety consideration

One of the detectors is mounted on a movable platform, **DO NOT ATTEMPT** to unlock or move the platform. **Any movement of the detector must be performed by a member of staff.**

High voltage cables are attached to the photomultiplier detectors (>2 kV) **DO NOT ATTEMPT** to disconnect these cables.

Introduction

- **Cosmic ray muons**

Muons are fundamental charged particles that were experimentally discovered and theoretically conceived in particle physics in the early 1930s. They belong to the leptons family of elementary particles together with electrons, taus and their associated anti-particles and neutrinos. Their mass is about 207 times that of the electron with the same charge and spin (so called fermions particles). Because of this large mass, neither ordinary radioactive decay or nuclear fission and fusion events are energetic enough to produce muons. It turns out that cosmic rays coming from deep space are a large and reliable source of muons.

Cosmic ray is the term given to very high energy radiation coming from deep space and striking the Earth’s atmosphere every day. The discovery of this radiation is attributed to Victor Hess. In 1912 for the first time in history Hess sent a balloon detector at 5300 m of altitude demonstrating that the atmosphere affected the intensity of the radiation and also that the radiation was not coming from the Sun. The high energy cosmic radiation is thought to come mainly from the energy emitted by supernovae. Cosmic rays striking the Earth’s atmosphere are composed of protons (~90%), α particles (He^{2+} , ~9%) and by other heavier nuclei (~1%). The energy of cosmic rays is very high with a peak distribution at 0.3 GeV. However events with a striking energy up to 1000 TeV have been detected.

The high energy (nuclear) collisions of the primary cosmic radiation with the upper atmosphere generate “showers” of lighter particles, pions π and kaons K as depicted in Figure 1, that quickly decay in muons μ and other particles. Cosmic muons fluxes are dependent on the zenith angle, the Earth’s magnetic field (hence latitude) and the atmospheric conditions. The loss of energy of muons

reaching the Earth's surface is mainly due to the ionisation of atmospheric nitrogen and oxygen and Bremsstrahlung interaction.

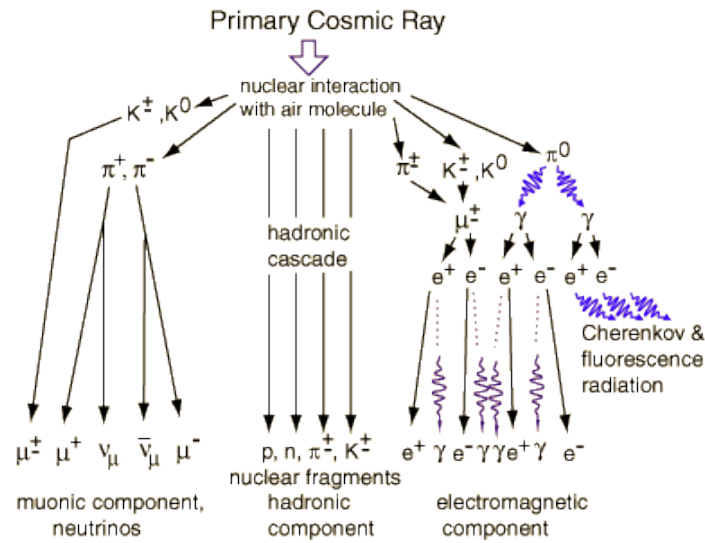


Figure 1: Diagram illustrating “showers” of particles generated at the top of the atmosphere from the nuclear interactions of cosmic rays with the gas present in the atmosphere (source: <http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html>).

Characteristics of muons:		Characteristics of muons generated by cosmic rays:	
Symbol	μ^-	Formation altitude	$\approx 10 \cdot 10^3 \text{ m}$
Mass at rest $m_{\mu 0}$	$105.6583668 \pm 0.0000038 \text{ MeV}/c^2$	Approximate mean energy at the site of production	$\approx 6 \text{ GeV}$ (equivalent speed at $0.9998c$)
Lifetime τ	$2.19703 \pm 0.00004 \text{ } \mu\text{s}$	Range of energy (at the formation)	$0.2 \text{ GeV} - 100 \text{ GeV}$
Electric charge	e^- (electron charge)	Average fluence rate at sea level	$\approx 1.9 \times 10^2 \text{ m}^{-2} \text{ s}^{-1}$
Spin	$1/2$	Energy range at sea level	$0.2 \text{ GeV} - 20 \text{ GeV}$
		Mean energy at sea level	2 GeV

Table 1: Summary of the main properties of the muons and the main characteristics of the highly energetic ones produced by cosmic rays (Cosmic Muons).

Table 1 summarises the properties of muons. The empirical formula for the zenith angle distribution (Figure 2) for the muons angular fluence rate as function of zenith angles $z < 75^\circ$ is:

$$\phi(z) = \phi_0 \cos^n z \quad (1)$$

where ϕ is the fluence rate, ϕ_0 is the angular fluence rate at $z=0$ and n is a function of the energy of the muons.

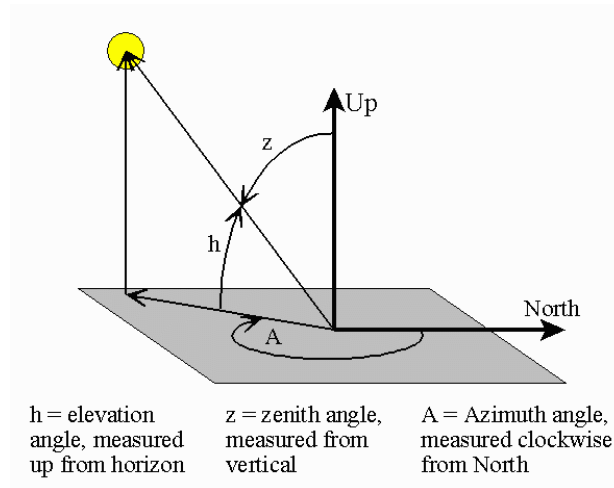


Figure 2: Diagram describing the horizontal coordinate system also called azimuth/elevation system, which uses the observer's local horizon as the fundamental plane.

- **Relativistic speed limit and special relativity**

In classical kinematic the velocity add linearly in Galilean transformations between reference systems. For this reason in the kinematic classical model there is no limit for the relative velocity of two bodies. This was clearly in contradiction with the Maxwell theory of the electromagnetic field which implies that electromagnetic waves travel at the universal speed of light in vacuum regardless of the observer of the wave or the motion of the source. Also other experiments at the end of the 20th century showed the independency of the speed of light upon the motion of the observer or the source. The solution to these contradictions was formulated in 1905 by Albert Einstein in his article on the special relativity. The new relativistic model (based on the constancy of c) which deals with motion approaching the speed of light was able to solve the experimental contradictions and continues to stay valid for motions involving velocities much lower than the speed of light.

One of the most important consequences of the special relativity is the fact that no particle with mass larger than zero can reach the speed limit c . The purpose of this experiment is to verify the existence of this speed limit for high energy particles such as cosmic muons, which travel at “relativistic” speeds close to c , and confirm the inapplicability of the classical Newtonian model to relativistic particles. A clear illustration of the fundamental difference between the Newtonian and relativistic model is shown in Figure 3 using the following equations for the kinetic energy E_k of a muon using:

$$E_k = m_{\mu 0}(\gamma - 1)c^2 = \frac{m_{\mu 0}c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_{\mu 0}c^2 \quad (2)$$

for the special relativity model and

$$E_k = \frac{1}{2}m_{\mu 0}v^2 \quad (3)$$

for the classical model.

Where $m_{\mu 0}$ is the mass at rest of the muon, v is the speed of the muon from an observer on the Earth and $\gamma = 1/(1-v^2/c^2)^{1/2}$ is the Lorentz factor.

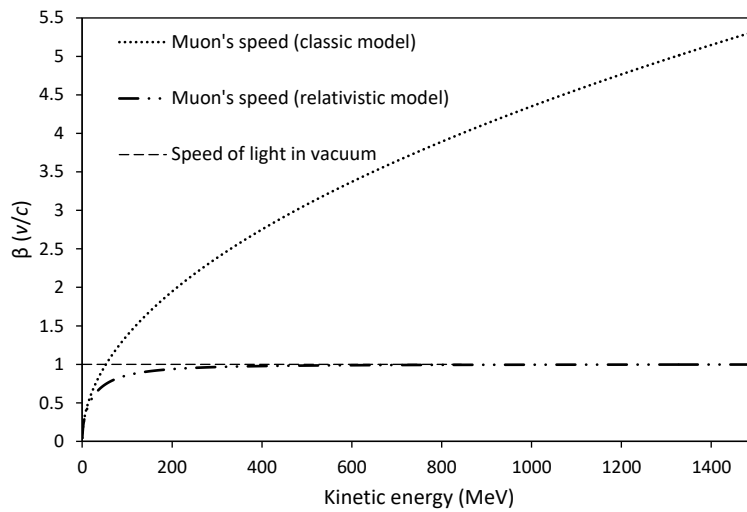


Figure 3: Diagram illustrating the relative speed of cosmic muons as the ratio $\beta = v/c$ at different energies.

The discovery of cosmic muons and their characterisation was also one of the first experimental evidences of the consequences of the special relativity on space and time. If we think in terms of time, from an observer in an inertial frame of reference F (e.g. laboratory) the time interval measured from the laboratory's frame Δt for a cosmic muon travelling at the speed v_x from a position A to B is equal to γ times $\Delta t'$ which is the time interval in the frame of reference of the muon F' , as described by equation (4) and the diagram in Figure 4.

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t' \quad (\gamma > 0) \quad (4)$$

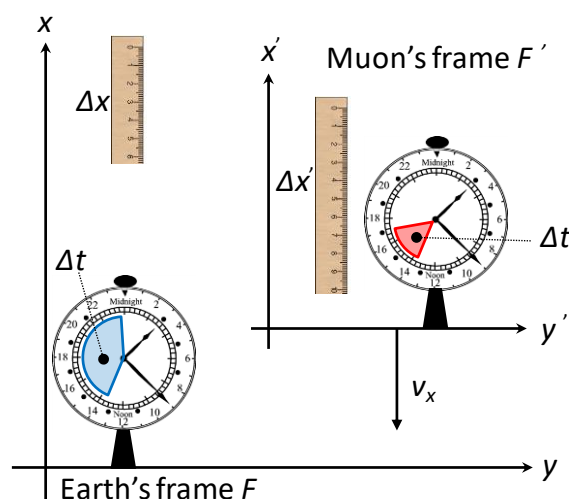


Figure 4: Diagram illustrating the dilation of time phenomenon observed from the Earth's frame and predicted by the special relativity and described by equation (4).

Knowing the lifetime of a muon and the characteristics of the cosmic muons it is easy to demonstrate the dilation of time phenomenon.

Equipment

NIM (Nuclear Instrumentation Module) standard electronics with scintillation detectors.

Measurement principle

The diagram in figure 5 describes the fast timing setup used in this laboratory session to detect the mean speed of cosmic muons. The technique used in this experiment is based on the time of flight (TOF) of the detected particle, which is the time interval for a muon to travel from the top to the bottom scintillator detector. The detectors used in this experiment are made of a special plastic called scintillator, which is a material that is able to emit photons when a charged particle like a muon crosses them.

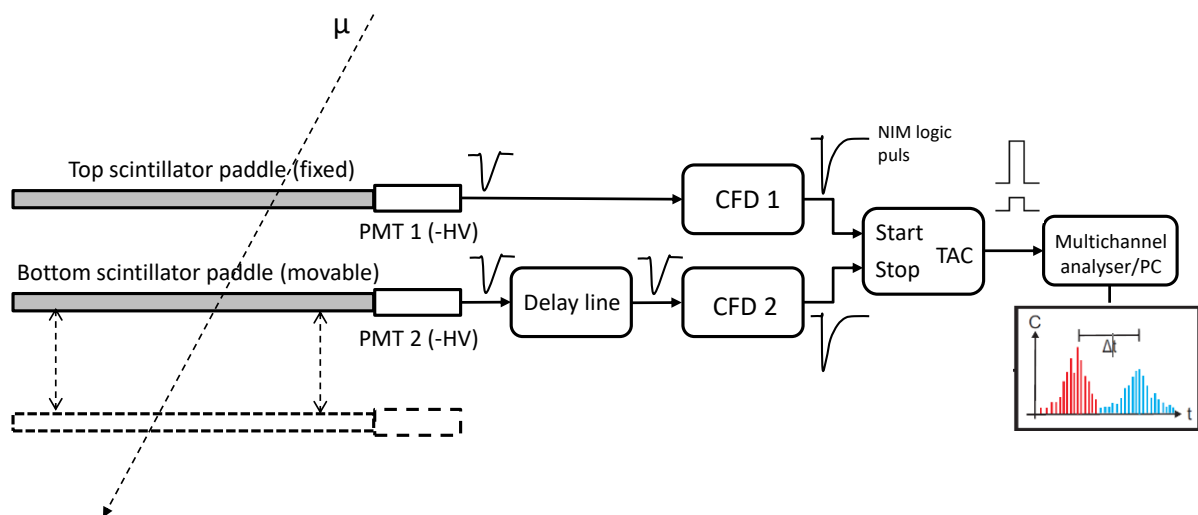


Figure 5: Block diagram of the setup used in the laboratory to detect the mean time of flight (TOF) of cosmic muons.

The detection of one single TOF (also called coincidence) can be summarised in the following steps (see figure 5 as reference):

1. A cosmic muon passes through the top scintillator and consequently thanks to the photomultiplier attached to it and the high energy of the muon, a strong negative voltage peak (order of 1 volt) is generated.
2. The generated signal is modified into a typical NIM pulse by the constant fraction discriminator 1 (CFD1). Another function of this unit is to discriminate (filter) between peaks of different amplitude that might be caused by less energetic particles (e.g. natural α particles background radiation).

3. The NIM pulse coming from the CFD1 is sent to the START input of the TAC unit (Time to Amplitude Converter) which is a device that is able to generate a positive output voltage with amplitude proportional to the START-STOP time interval width of two input signals. In other words it is a slightly more complicated chronometer.
4. In the meantime the same muon detected at the point 1 passes through the bottom detector and generates another pulse that is delayed and sent to the STOP input of the TAC through the discriminator CFD2.
5. The output positive signal of the TAC has now an amplitude that is proportional to the interval of time between the detection of the muon passing from the top to the bottom detector. Different TOFs (coincidence for each muon passing through the top and bottom detectors) will generate a train of pulses at the output of the TAC with different amplitudes.
6. Finally the TAC output is connected to a unit called multichannel analyser (MCA) that essentially recognises signals with different amplitude and record/distribute them in different “channels” called bins (2048 channels in this case), according to their different amplitudes.
7. The software “plots” the MCA records in a bar plot, with the “bin number” as horizontal axis and counts on the vertical axis. The count is the number of times a certain channel (TOF) has been recorded.

With an opportune calibration (already performed before the laboratory session) it is possible to relate the bin number of the horizontal axis of the plot to the TOF and calculate the mean TOF for a large number of coincidences. Different mean TOFs obtained varying the distance between the detectors will allow the calculation of the mean speed of the muons detected in the laboratory session.

The typical distribution of the TOFs recorded by the MCA has a Gaussian bell shape with an associated mean μ (x-axis value corresponding to the peak of the curve) and “width” σ . The σ of the distribution which is a measure of how scattered are the TOFs of the muons is caused by three principal factors:

- Muons reaching the Earth have different speeds.
- Muons have an angular distribution described in equation (1) and for this reason the path length between the two detectors cannot be constant (muons are coming from different angles) hence the recorded TOFs can be different.
- Intrinsic errors caused by the setup of the experiment.

However the mean TOF of the distribution does change with the distance between the detectors and this gives a sufficiently accurate measurement of the mean TOF of the muons and consequently the mean speed of muons can be calculated.

The mean speed of muons $\langle v \rangle$ is given by:

$$\langle v \rangle = \frac{\langle d \rangle}{\langle t \rangle} \quad (9)$$

where $\langle d \rangle$ is the mean distance between top and bottom detector and $\langle t \rangle$ is the associated mean TOF.

Procedure and analysis

Part A: Data acquisition

You will measure the mean TOF for 4 positions of the bottom detector. Acquisition for each position will last 20 minutes. **Do not attempt to change the position of the detector by yourself. Ask a member of staff to do it.**

(1) A member of the staff will switch on the setup for you before the laboratory session. To start the first acquisition go on the Desktop and double click on the icon “Maestro” to open the software interface for the MCA analyser.

(2) Click on the green button icon “Go” (top left), the acquisition should start and you should start to see the first yellow bars appearing on the screen, indicating the detection of the coincidences. Measure the distance between the top and bottom detectors. Take the reading (in mm) at the bottom detector using the yellow tape measure, the reading should be taken at the uppermost edge of the detector.

(3) As soon as the acquisition is complete call a member of the staff to change the position of the detector.

(4) Create your own folder on the Desktop with your name or the name of the group and save the acquired data using the name of the related distance between the detectors (e.g. “526mm”). Repeat steps 2-4 for the other detector positions.

(5) The acquisition of your data is now complete and you can start their analysis. However, before proceeding you need to modify your data and save it as text file. To do this follow this step for each of the 4 saved files:

- a. Double click on the file (e.g. 526mm) this will open the file with Excel. Clear the cells 1 to 12 and 2061 to 2075.
- b. Cut the data from 13 to 2060 and paste it in the cell A1 (top left corner of the spreadsheet).
- c. Save the data in your folder as text, keeping the same name. (Select “Text (Tab delimited)(*.txt)” in the save option of Excel).

Part B: Data analysis

(1) The four set of data saved in your folder, as text file, are distributions with a Gaussian bell shape. You will now use a simple Excel spreadsheet to calculate the mean value (mean bin value) of each set of data, to do this follow these steps:

- a. Open the Gaussian Fit Excel Spread sheet on the Desktop.
- b. Save the Spreadsheet in your folder.
- c. In the Data Ribbon/tab, click the Refresh ALL button.

- d. In the dialogue box find your data file (e.g. 526mm.txt) and click import.
- e. Your data should now be shown as red dots and the theoretical fit as a black dotted line.
- f. Change the numbers in the MU and SIGMA cells until the dotted line is a close match to your data in red (roughly estimate MU and SIGMA looking at the plot. MU is the peak and SIGMA is the width of the bell).
- g. Using the solver, maximise the value for R^2 by varying the values of MU and SIGMA.
- h. Write in your logbook the “fitted” values of MU, SIGMA. Repeat the procedure for each set of data starting from the point c.

(2) It is now time to convert the calculated mean “bin” values MUs and their Standard errors obtained in the previous point into mean TOF (in ns). To do this you simply have to take in account the conversion factor between “bin or channel” and time $\alpha = 39.092 \pm 0.110$ bin/ns.

(3) Using the Excel X Y (Scatter) chart graph type, plot the distance between the detector (in metres) vs TOF points (in seconds) and calculate the mean speed of muons from the slope of the curve with at least 5 significant digits. Using the Excel linear regression function LINEST also calculate the standard error for the angular coefficient. Express your result with the error in m/s and as fraction of c ($\beta = v/c$). List the possible errors in the system that can affect the final result. (Notice: if you don't know how to use Excel, consult the Help file or ask a member of the staff).

(4) Taking in account the lifetime at rest of the muons (muon's frame of reference, see Table 1) calculate the maximum distance they manage to travel within their lifetime, knowing their mean speed (determined in the previous point). Most of muons are generated at $\approx 10,000$ m of altitude; do the muon reach the surface of the Earth before their lifetime is over?

(5) According to the Newtonian mechanics, absolute time exists independently of any frame of reference and progresses at a consistent pace throughout the universe. Newton's theory leads to the results of the previous point since the muon's lifetime is the same regardless the frame of reference. However according to the relativistic theory time is dependent upon the frame of reference. This time calculate the distance travelled by the muons at their mean speed but considering now the lifetime of the muons from an observer in the frame of reference of the laboratory (Earth) according to the special relativity model. (Hint: consider the Lorentz factor γ , the lifetime of muons in the lab reference Δt , see equation 4).

(6) Calculate the mean kinetic energy of the detected muons in GeV considering their experimental mean speed and the relativistic expression for the kinetic energy in the equation(2). Compare this result with the range given in Table (1) and comment.

(7) If even using the special relativity model the speed of the muons differ considerably from the speed of light list the possible explanations for errors introduced by the setup.

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