

A Simple Two-stage detector for Massive MIMO Systems with one-bit ADCs

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Abstract—In massive multiple-input multiple-output (m-MIMO) systems, the expensive analog to digital converters (ADCs) with high power consumption, connected to the base station (BS) antennas have become a bottleneck for practical implementation of this technology. Motivated by this, a pair of one-bit ADCs connected to each antenna has been proposed recently to provide more energy efficiency (EE) in such m-MIMO systems. In this paper, a two-stage detector with one-bit ADCs based on the zero-forcing (ZF) and maximum likelihood (ML) detectors is proposed for the uplink of a single cell multiuser m-MIMO system. The proposed detector outperforms the ZF detector and its performance is similar to the ML detector with much lower complexity. In addition, we study the case where we have some hardware impairments and develop a proper detector based on the ZF and ML detectors. Simulation results show that this detector also performs close to the ML detector in the same scenario.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is one of the key technologies proposed to implement 5G cellular networks, and it is known as a promising technology for meeting the increasing demand of wireless throughput by some advantages such as high spectral efficiency, reduced radiated power, greater simplicity in signal processing, array gain, and diversity gain [1]. Despite these benefits, there exist some obstacles for practical implementation of m-MIMO such as expensive radio frequency (RF) chains with high power consumption, connected to a large number of antennas at the base station (BS). If we only consider the transmit power, m-MIMO is energy efficient, but considering the power consumption of the equipment connected to each antenna, m-MIMO is not an energy efficient technique [2]. In this paper, we consider the uplink of m-MIMO with one-bit analog to digital converters (ADCs). Therefore, the receiver consists of the RF chain and baseband signal processing components which are connected to each other by one-bit ADCs [3]. The power consumption of ADCs such as $\Sigma\Delta$ scales linearly with the sampling rate [4]. As the result, using low-resolution ADCs is a way to reduce the total power consumption.

Low-resolution ADCs have been used in different scenarios in the other studies. In [5], the channel capacity is obtained in an m-MIMO system with one-bit ADCs. The spectral efficiency of m-MIMO systems with one-bit ADCs is analysed in [6] and a closed-form achievable rate for m-MIMO wideband system is

derived. It is proved in [6] that the wideband m-MIMO works well with one-bit ADCs.

In addition some recent papers have presented new studies based on the m-MIMO systems with one-bit ADCs. The channel estimation problem is studied in [7] as one of the crucial issues to implement the m-MIMO systems with one-bit ADCs. The paper presented in [7] uses an iterative algorithm to increase the estimation accuracy without using extra training symbols. The paper in [8] uses faster than symbol rate (FTSR) sampling in the uplink scenario and it is shown that the system has better performance at the same SNRs than the case which does not use the FSTR. One of the most important issues related to the m-MIMO systems with one-bit ADCs is the use of a low complexity detector which compensates the operational degradation caused by using one-bit ADCs. the paper in [9] presents a low complexity maximum likelihood detection (MLD) algorithm which reduces the search space. So in this paper we propose a low complexity detection algorithm as in the following.

There exist different hardware impairments at the transceiver side such as, phase noise, peak to average power ratio (PAPR), IQ imbalance, quantization errors and amplifier non-linearities [10]. In [11], hardware impairments at both BSs and user equipments (UEs) are incorporated in the study of m-MIMO systems. It is proved that the hardware impairments limit the channel estimation accuracy and make an upper bound on the maximum uplink/downlink capacity of each UE. The impact of impairments in the large scale arrays of antennas vanishes asymptotically [11], therefore it enables us to use non-ideal hardware in m-MIMO to reduce the price while the total power consumption does not increase. Some compensation algorithms are used to mitigate these hardware impairments, but there remain some residual hardware impairments.

In this paper, We present a two-stage detector with one-bit ADCs which utilizes the ZF detector at the first stage and the ML detector with reduced search space at the second stage. Our two-stage detector outperforms the ZF detector and has approximately the same performance as the ML detector while the complexity is much lower. We consider aforementioned residual impairments in this paper to show their impact on the performance of the detectors.

The rest of the paper is organized as follows. In section II, we explain our system model. In section III, our proposed two-stage detector is presented and the effect of hardware impairments is explained. In section V, we evaluate the performance of the proposed detector, ZF and ML detector by simulation. The conclusion is presented in section VI.

Notation: the lower and upper boldface letters are used for column vectors and matrices, respectively. $\mathbb{E}\{x\}$ represents the expectation of random variable x . $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^{-1}$ are used to denote the transpose, hermitian and pseudo inverse operators respectively. $\|\cdot\|$ denotes l_2 -norm. $\text{Re}\{s\}$ and $\text{Im}\{s\}$ represent real and image part of s . $\mathbb{C}^{m \times n}$ denotes the set of all complex $m \times n$ matrices and $\mathbb{R}^{m \times n}$ denotes the set of all real $m \times n$ matrices.

II. SYSTEM MODEL

We consider the uplink of a single-cell multi-user m-MIMO system with K single-antenna users and one N -antenna BS. All the users transmit independent data symbols with the power P to the BS, simultaneously. The received signal at the BS is

$$\mathbf{r} = \sqrt{P} \sum_{k=1}^K \mathbf{h}_k s_k + \mathbf{n} = \sqrt{P} \mathbf{H} \mathbf{s} + \mathbf{n}, \quad (1)$$

where \mathbf{h}_k is the channel vector between the k -th user and the BS, s_k is the transmitted symbol from the k -th user which is selected from an M -ary PSK constellation set $\mathcal{A} = \{a_1, \dots, a_M\}$ with $\mathbb{E}\{s_k\} = 0$ and $\mathbb{E}\{|s_k|^2\} = 1$, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$ is the additive white Gaussian noise (AWGN) and $\mathbf{H} \in \mathbb{C}^{N \times K}$ is the channel matrix whose elements are distributed independently and identically as $\mathcal{CN}(0, 1)$; we assume that the BS has perfect channel state information (CSI).

Our proposed detector is based on the output of one-bit ADCs as in Fig.1; so the quantized received signal is modeled based on the one-bit ADCs as

$$\hat{\mathbf{r}} = \mathbf{Q}(\mathbf{r}) = \mathbf{Q}(\sqrt{P} \mathbf{H} \mathbf{s} + \mathbf{n}), \quad (2)$$

where $\mathbf{Q}(\cdot)$ represents the quantization function that operates on the real and imaginary parts separately as

$$\mathbf{Q}(\mathbf{r}) = \text{sgn}(\text{Re}(\mathbf{r})) + j \text{sgn}(\text{Im}(\mathbf{r})). \quad (3)$$

The $\text{sgn}(\cdot)$ represents for the sign function that maps the positive values to 1 and the negative values to -1. Therefore we have

$$\hat{\mathbf{r}} = [\hat{r}_1, \dots, \hat{r}_N]^T, \quad \hat{r}_i \in \{1 + j, 1 - j, -1 + j, -1 - j\}, \quad (4)$$

which $\hat{\mathbf{r}}$ will be used in section III to implement detectors with one-bit ADCs.

III. PROPOSED EFFICIENT DETECTOR

The ZF detector is a linear low complexity detector commonly used in m-MIMO systems with full-resolution ADCs. It can support more users than the ML detector because its complexity does not depend on the number of the users, but the ZF detector using one-bit ADCs achieves error floor rate at high SNR regions. In this Section, we propose a two-stage

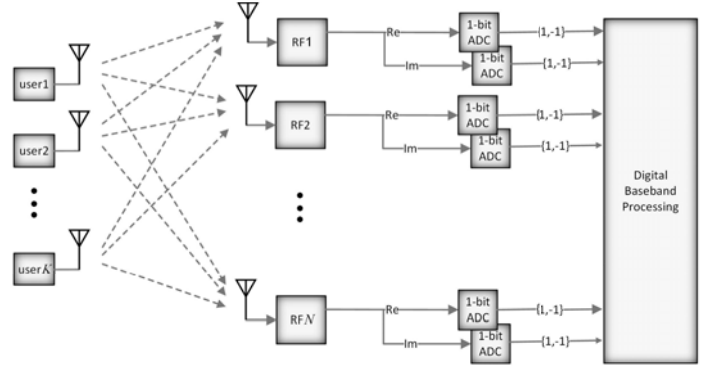


Fig. 1. Receiver model with one bit ADCs

one-bit detector for the problem in [12] in Subsection III. A whose performance is close to the ML detector in [12], but the complexity is much lower. In addition, we model the hardware impairment similar to [13] in Subsection III. B and evaluate the performance of the system with impairments.

A. Proposed detector with the assumption of ideal hardware

Our two-stage detector uses ZF detector at the first stage to mitigate the effect of fading coefficients and then it uses ML detector with reduced search space. So we present the ZF detector and the ML detectors respectively as the following.

$$\hat{\mathbf{s}}_{\text{ZF}} = (\mathbf{H} \mathbf{H}^H)^{-1} \mathbf{H}^H \hat{\mathbf{r}}, \quad (5)$$

because the norm square of $\hat{\mathbf{s}}_{\text{ZF}}$ may not equal to K , the BS normalizes $\hat{\mathbf{s}}_{\text{ZF}}$ as

$$\bar{\mathbf{s}}_{\text{ZF}} = \sqrt{K} \frac{\hat{\mathbf{s}}_{\text{ZF}}}{\|\hat{\mathbf{s}}_{\text{ZF}}\|}. \quad (6)$$

Then the user-by-user detection is used,

$$\hat{s}_{\text{ZF},k} = \underset{s' \in \mathcal{A}}{\text{argmin}} \|\bar{s}_{\text{ZF},k} - s'\|^2, \quad (7)$$

where $\bar{s}_{\text{ZF},k}$ is the k -th element of $\bar{\mathbf{s}}_{\text{ZF}}$. Then, in order to obtain a closed form of the ML detector presented in [12], we convert the complex baseband signal into the real domain similarly, as in the following

$$\mathbf{H} = [\mathbf{g}_1 \quad \mathbf{g}_2 \quad \dots \quad \mathbf{g}_N]^T \quad (8)$$

where $\mathbf{g}_n^T \in \mathbb{C}^{1 \times K}$ is the n -th row of the channel matrix \mathbf{H} , so the received signal is rewritten in the real form as

$$\mathbf{G}_{\text{R},n} = \begin{bmatrix} \text{Re}(\mathbf{g}_n^T) & -\text{Im}(\mathbf{g}_n^T) \\ \text{Im}(\mathbf{g}_n^T) & \text{Re}(\mathbf{g}_n^T) \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{\text{R},n,1}^T \\ \mathbf{g}_{\text{R},n,2}^T \end{bmatrix} \in \mathbb{R}^{2 \times 2K} \quad (9)$$

$$\mathbf{g}_{\text{R},n,1} = \begin{bmatrix} \text{Re}(\mathbf{g}_n) \\ -\text{Im}(\mathbf{g}_n) \end{bmatrix}, \quad \mathbf{g}_{\text{R},n,2} = \begin{bmatrix} \text{Im}(\mathbf{g}_n) \\ \text{Re}(\mathbf{g}_n) \end{bmatrix} \quad (10)$$

$$\mathbf{n}_{\text{R},n} = \begin{bmatrix} \text{Re}(\mathbf{n}_n) \\ \text{Im}(\mathbf{n}_n) \end{bmatrix} = \begin{bmatrix} n_{\text{R},n,1} \\ n_{\text{R},n,2} \end{bmatrix} \in \mathbb{R}^{2 \times 1} \quad (11)$$

$$\mathbf{s}_R = \begin{bmatrix} \text{Re}(\mathbf{s}) \\ \text{Im}(\mathbf{s}) \end{bmatrix} \quad (12)$$

$$n_{R,n,i} \sim \mathcal{N}(0, \frac{\sigma^2}{2}),$$

the real and imaginary components of the Gaussian noise are iid. The received signal representation is

$$\mathbf{r}_{R,n} = \begin{bmatrix} \text{Re}(r_n) \\ \text{Im}(r_n) \end{bmatrix} = \begin{bmatrix} r_{R,n,1} \\ r_{R,n,2} \end{bmatrix} \quad (13)$$

$$= \sqrt{P}\mathbf{G}_{R,n}\mathbf{s}_R + \mathbf{n}_{R,n}$$

and the quantized form is

$$\hat{\mathbf{r}}_{R,n} = \begin{bmatrix} \text{sgn}(\text{Re}(r_n)) \\ \text{sgn}(\text{Im}(r_n)) \end{bmatrix} = \begin{bmatrix} \hat{r}_{R,n,1} \\ \hat{r}_{R,n,2} \end{bmatrix}. \quad (14)$$

The sign-refined channel matrix related to the n -th antenna is

$$\tilde{\mathbf{G}}_{R,n} = \begin{bmatrix} \tilde{\mathbf{g}}_{R,n,1}^T \\ \tilde{\mathbf{g}}_{R,n,2}^T \end{bmatrix}, \quad (15)$$

where, $\tilde{\mathbf{g}}_{R,n,i}$ is defined as

$$\tilde{\mathbf{g}}_{R,n,i} = \hat{r}_{R,n,i}\mathbf{g}_{R,n,i}. \quad (16)$$

According to these definitions, the ML detector would be [12]

$$\hat{\mathbf{s}}_{R,ML} = \underset{\mathbf{s}'_R \in \mathcal{A}_R^K}{\text{argmax}} \prod_{i=1}^2 \prod_{n=1}^N \Phi(\sqrt{2P}\tilde{\mathbf{g}}_{R,n,i}^T \mathbf{s}'_R) \quad (17)$$

$$\Phi(t) = \int_{-\infty}^t \frac{1}{\sqrt{2\pi}} e^{-\frac{\tau^2}{2}} d\tau,$$

where \mathcal{A}_R^K is the K -ary Cartesian product set of $\mathcal{A}_R = \left\{ \begin{bmatrix} \text{Re}(a_1) \\ \text{Im}(a_1) \end{bmatrix}, \dots, \begin{bmatrix} \text{Re}(a_M) \\ \text{Im}(a_M) \end{bmatrix} \right\}$, which is ordered first according to the real parts of the constellations and then according to the imaginary parts.

Then, our proposed two-stage detector would be the following algorithm by the use of aforementioned detectors.

Algorithm 1

input: quantized baseband vector $\hat{\mathbf{r}}$, channel matrix \mathbf{H} and \mathbf{G}

step1

$$1: \hat{\mathbf{s}}_{ZF} = (\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{H}^H\hat{\mathbf{r}}$$

2: normalize the norm of $\hat{\mathbf{s}}_{ZF}$ to \sqrt{K}

$$\bar{\mathbf{s}}_{ZF} = \sqrt{K} \frac{\hat{\mathbf{s}}_{ZF}}{\|\hat{\mathbf{s}}_{ZF}\|}$$

$$3: \hat{s}_{ZF,k,1} = \underset{s' \in \mathcal{A}}{\text{argmin}} |\bar{s}_{ZF,k} - s'|^2, \quad k=1,\dots,K$$

4: find the second nearest point with the minimum Euclidean distance for each user, separately

$$\hat{s}_{ZF,k,2} = \underset{\substack{s' \in \mathcal{A} \\ s' \neq \hat{s}_{ZF,k,1}}}{\text{argmin}} |\bar{s}_{ZF,k} - s'|^2, \quad k=1,\dots,K$$

step2

1: find new search space based on the two nearest points.

The set of \mathcal{B}_R^K consists of all of the K -ary vectors that each element of these vectors can be chosen from just two probable points (the two nearest points). In addition, each vector is ordered first according to the real parts of its elements and then according to the imaginary parts.

2: detect the transmitted vector by the ML detector, using the new search space

$$\hat{\mathbf{s}}_{R,ML} = \underset{\mathbf{s}'_R \in \mathcal{B}_R^K}{\text{argmax}} \prod_{i=1}^2 \prod_{n=1}^N \Phi(\sqrt{2P}\tilde{\mathbf{g}}_{R,n,i}^T \mathbf{s}'_R)$$

output: $\hat{\mathbf{s}}_{R,ML}$

B. Proposed detector with the assumption of hardware impairments

Hardware impairments at the transceiver side have different effects on the performance of the system due to the transmission technology. Compensation algorithms are used to mitigate transceiver hardware impairments, but there remain residual impairments [11]. The impairment model does not depend on the specific front-end design [3].

In this paper, the effect of these impairments on the detectors are investigated. This paper uses the impairment model proposed in [13] with aggregate impact of different hardware impairments assumption.

assumption: UEs are equipped with ideal hardwares and just the impairments of the hardwares connected to the BS antennas at receiver side are considered.

The received signal vector is modeled with the assumption of hardware impairments at the receiver side as in [13]

$$\mathbf{r}_{imp} = \sqrt{P} \sum_{k=1}^K \mathbf{h}_k s_k + \boldsymbol{\eta}_r + \mathbf{n} = \sqrt{P}\mathbf{H}\mathbf{s} + \boldsymbol{\eta}_r + \mathbf{n}, \quad (18)$$

$$\boldsymbol{\eta}_r \sim \mathcal{CN}(\mathbf{0}, \delta_r^2 \text{tr}(\mathbf{Q})\mathbf{I}_{N_r}),$$

where $\boldsymbol{\eta}_r$ is an ergodic stochastic vector, \mathbf{Q} is the signal covariance matrix and δ_r is related to the error vector magnitude (EVM) which is in the range $\delta_r \in [0.08, 0.175]$. Considering the new received signal model, we reformulate

the ML detector as

$$\hat{\mathbf{s}}_{R,ML} = \underset{\mathbf{s}'_R \in \mathcal{A}_R^K}{\operatorname{argmax}} \prod_{i=1}^2 \prod_{n=1}^N \Phi\left(\frac{\sqrt{2P}\tilde{\mathbf{g}}_{R,n,i}^T \mathbf{s}'_R}{1 + \delta_r^2 \operatorname{tr}(\mathbf{Q})}\right) \quad (19)$$

$$\Phi(t) = \int_{-\infty}^t \frac{1}{\sqrt{2\pi}} e^{-\frac{\tau^2}{2}} d\tau$$

and the ZF detector based on the impaired received signal is

$$\hat{\mathbf{r}}_{imp} = \mathbf{Q}(\mathbf{r}_{imp}) = \mathbf{Q}(\sqrt{P}\mathbf{H}\mathbf{s} + \boldsymbol{\eta}_r + \mathbf{n}) \quad (20)$$

$$\hat{\mathbf{s}}_{ZF} = (\mathbf{H}\mathbf{H}^H)^{-1} \mathbf{H}^H \hat{\mathbf{r}}_{imp}. \quad (21)$$

Our two-stage detector based on the new ZF and ML detectors will be similar to what we mentioned in Algorithm 1.

IV. NUMERICAL RESULTS

In order to evaluate the performance of the proposed detectors, we provide some simulation examples based on the Monte-Carlo simulation. In order to compare the performance of the detectors we use the symbol error rate (SER) as follows

$$\text{SER} = \frac{1}{K} \sum_{k=1}^K \mathbb{E}\{P_k\}, \quad (22)$$

where P_k is the detection error probability of the k -th user when a symbol is sent. The channel we assume is Rayleigh fading, where the channel coefficients, i.e. the elements of \mathbf{H} are complex Gaussian distributed $\mathcal{CN}(0,1)$. The number of antennas of the BS is $N = 32$ and $N = 128$ and the utilized constellation is Q-PSK ($M = 4$) and 8-PSK ($M = 8$). We set $\delta_r = 0.15$ to evaluate the effect of the hardware impairments. In Fig. 2, we compare the performance of ZF, ML and our proposed two-stage detector. Our two-stage detector performs close to the ML detector, while its complexity is much lower. In order to detect the transmitted vector of the users by the ML detector, we need to search between M^K points; while by the use of our two-stage detector we just requires 2^K searches which is much lower. In Fig. 3 the SER of the ZF and our proposed two-stage detector are depicted and shows that the ZF detector achieves the error floor rate after SNR= 10dB; however the proposed two-stage detector does not have error floor rate. Fig. 4 depicts the effect of the hardware impairments on the performance of the detectors which vanishes by increasing the SNR. Simulation results reveal that the two-stage detector has approximately the same performance as the ML detector and outperforms that of the ZF detector. In Fig. 5 we compare the performance of our proposed detector in ideal and non-ideal hardware situations and observe that although the performance degrades in non-ideal hardware, the performances are close and the two-stage detector is much better than the ZF detector.

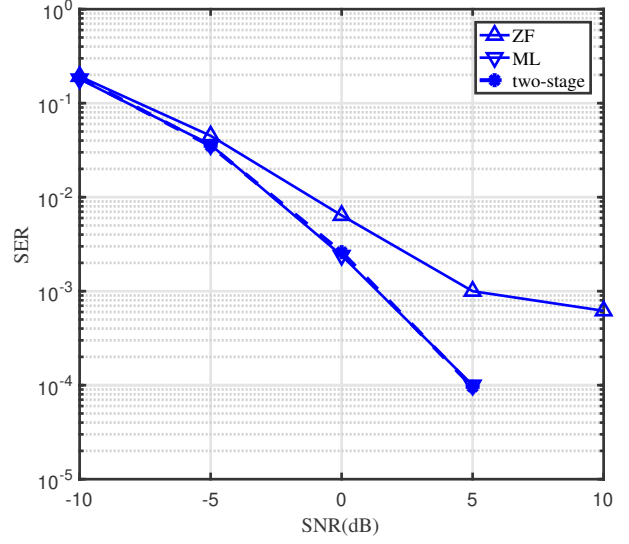


Fig. 2. SER vs. SNR (in dB) of detectors with $K = 4$, $M = 4$, and $N = 32$.

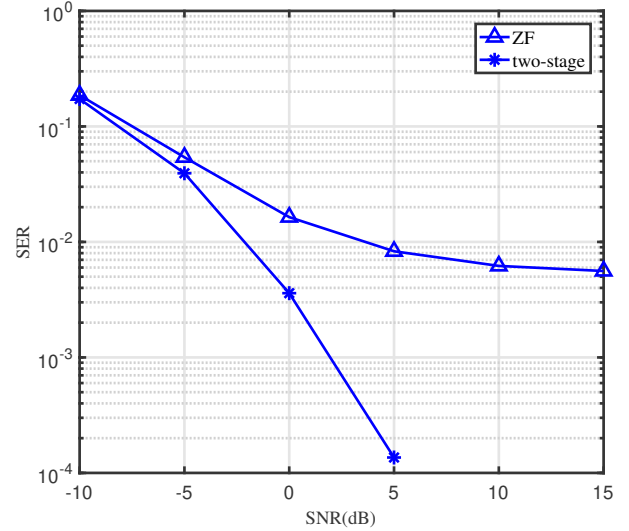


Fig. 3. SER vs. SNR (in dB) of detectors with $K = 8$, $M = 8$, and $N = 128$.

V. CONCLUSION

We proposed a simple two-stage detection method for the uplink multiuser massive MIMO systems with one-bit ADCs. Our proposed detector employs the benefits of ZF and ML detectors at its stages and performs better than that of ZF detector and is much more simpler than the ML detector. In addition, the residual hardware impairment model has been used in our proposed scenario and shows that the effect of the hardware impairments vanishes as SNR increases.

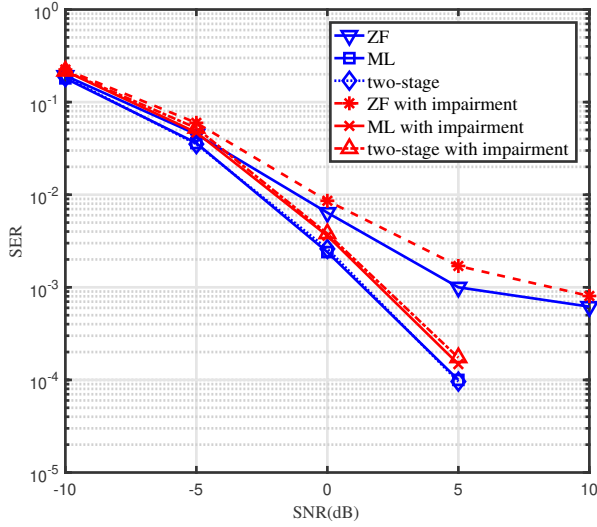


Fig. 4. SER vs. SNR (in dB) of detectors with $K = 4$, $M = 4$, and $N = 32$, $\delta_r = 0.15$.

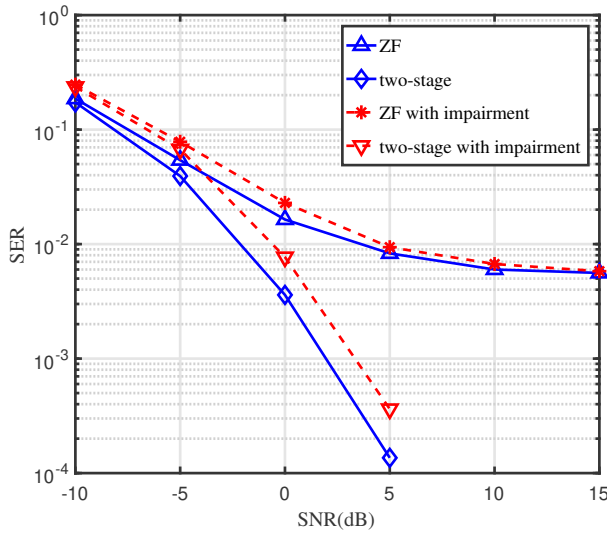


Fig. 5. SER vs. SNR (in dB) of detectors with $K = 8$, $M = 8$, and $N = 128$, $\delta_r = 0.15$.

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