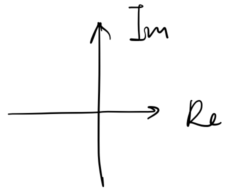
ELEC 2100 Review

Euler's Formula

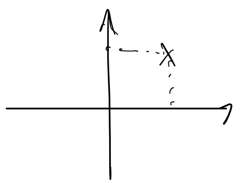
$$e^{j\theta} = \cos\theta + j\sin\theta$$

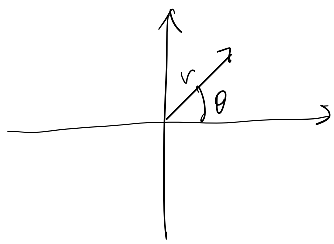
$j = \sqrt{-1}$ and θ is real

Complex Plane



Recantagular Form / Polar Form





(a,b) (r,θ)

$r = \sqrt{a^2+b^2}$ $\theta = \tan^{-1}\frac{b}{a}$

$Z = a + bi$

real imaginary

$Z = re^{j\theta}$ $(r\angle\theta)$

$e^{j\theta}$: unit circle

$|e^{j\theta}| = 1$

$\theta = \theta + 2\pi$

$$z_1 z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

$$z = a + bj$$

$|z|$ magnitude $= \sqrt{a^2+b^2}$

$\measuredangle z = \theta = \tan^{-1}\frac{b}{a}$ phase Beware of quadrants!

$a = Re\{z\} = r\cos\theta$

$b = Im\{z\} = r\sin\theta$

$z = \underline{r e^{j\theta}} = r\cos\theta + jr\sin\theta$

$\uparrow$

Euler's Formula

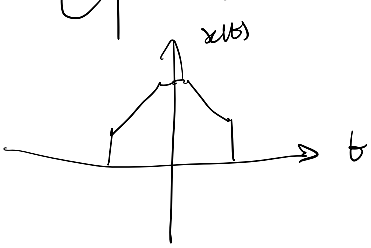
Even and Odd Signal

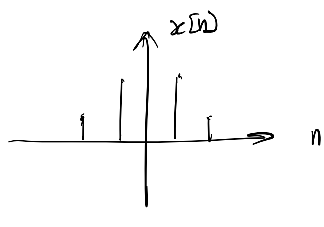
$Ev(x_1(t)) = \frac{x_1(t) + x_1(-t)}{2}$ $\quad$ $Odd(x_1(t)) = \frac{x_1(t) - x_1(-t)}{2}$

even part $\quad$ odd part

Even: Unchanged under time reversal.

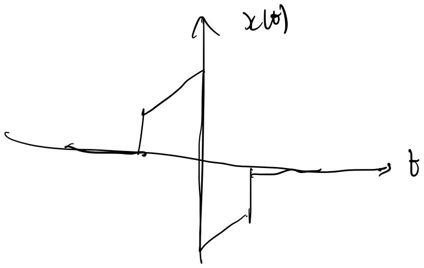
CT: $x(-t) = x(t)$ $\quad$ DT: $x[-n] = x[n]$

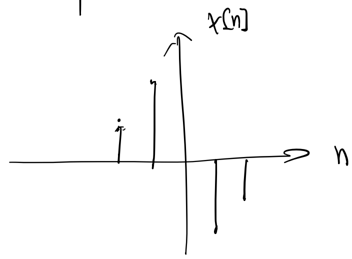




Odd: negated under time reversal. antisymmetric

CT: $x(-t) = -x(t)$ DT: $x[-n] = -x[n]$





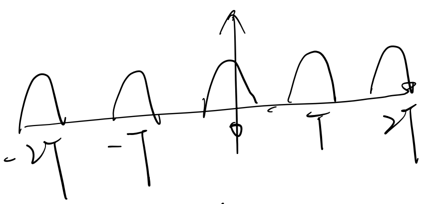
$x(0) = 0$!!

$x(-0) = -x(0) \Rightarrow x(0) = -x(0) \Rightarrow x(0) = 0$

Periodic Signal & Poisson Sum

$x(t+T) = x(t) \quad \forall t$ $x[n+T] = x[n] \quad \forall n$

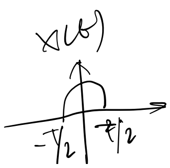
Periodic: unchanged after given time-shifts



time shift by T, no change

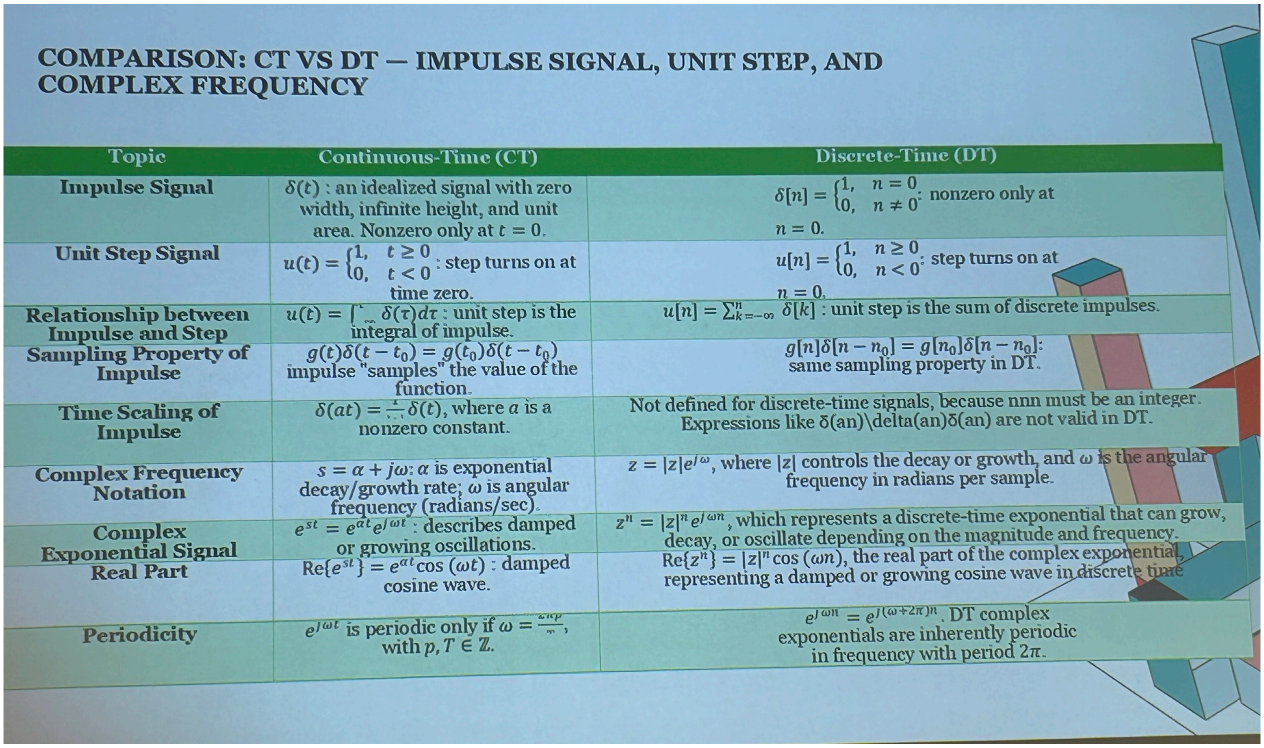
Poisson Sum

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t-kT)$$



$$x(t) = \begin{cases} \tilde{x}(t) & -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases}$$

(Shifted copy of original signal)



COMPARISON: CT VS DT — IMPULSE SIGNAL, UNIT STEP, AND COMPLEX FREQUENCY

Topic	Continuous-Time (CT)	Discrete-Time (DT)
Impulse Signal	$\delta(t)$: an idealized signal with zero width, infinite height, and unit area. Nonzero only at $t = 0$.	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$: nonzero only at $n = 0$.
Unit Step Signal	$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$: step turns on at time zero.	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$: step turns on at $n = 0$.
Relationship between Impulse and Step	$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$: unit step is the integral of impulse.	$u[n] = \sum_{k=-\infty}^{n} \delta[k]$: unit step is the sum of discrete impulses.
Sampling Property of Impulse	$g(t)\delta(t - t_0) = g(t_0)\delta(t - t_0)$ impulse "samples" the value of the function.	$g[n]\delta[n - n_0] = g[n_0]\delta[n - n_0]$: same sampling property in DT.
Time Scaling of Impulse	$\delta(at) = \frac{1}{\lvert a \rvert}\delta(t)$, where a is a nonzero constant.	Not defined for discrete-time signals, because nnn must be an integer. Expressions like δ(an)\delta(an)δ(an) are not valid in DT.
Complex Frequency Notation	$s = \alpha + j\omega$: α is exponential decay/growth rate; ω is angular frequency (radians/sec).	$z = \lvert z \rvert e^{j\omega}$, where $\lvert z \rvert$ controls the decay or growth, and ω is the angular frequency in radians per sample.
Complex Exponential Signal	$e^{st} = e^{\alpha t}e^{j\omega t}$: describes damped or growing oscillations.	$z^n = \lvert z \rvert^n e^{j\omega n}$, which represents a discrete-time exponential that can grow, decay, or oscillate depending on the magnitude and frequency.
Real Part	$\mathrm{Re}\{e^{st}\} = e^{\alpha t}\cos(\omega t)$: damped cosine wave.	$\mathrm{Re}\{z^n\} = \lvert z \rvert^n \cos(\omega n)$, the real part of the complex exponential, representing a damped or growing cosine wave in discrete time
Periodicity	$e^{j\omega t}$ is periodic only if $\omega = \frac{2\pi p}{T}$, with $p, T \in \mathbb{Z}$.	$e^{j\omega n} = e^{j(\omega + 2\pi)n}$. DT complex exponentials are inherently periodic in frequency with period 2π.

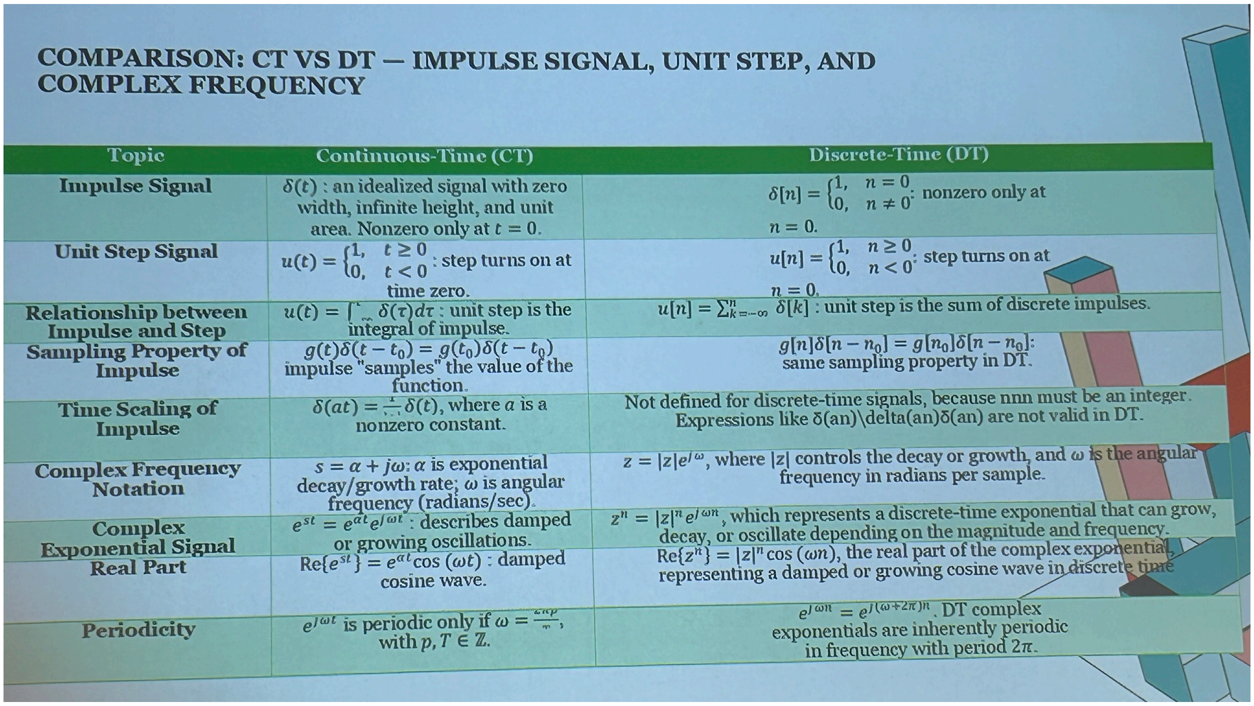
$\underline{u(t) \Leftrightarrow \delta(t)}$

$$u(t) = \int_{-\infty}^{n} \delta(\tau)d\tau \qquad u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Sample time t_0 from $g(t)$:

$$g(t)\,\delta(t - t_0) = g(t_0)\,\delta(t - t_0)$$

Samples $g(t_0)$ from $g(t)$



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Time Scaling

$$\delta(at) = \frac{1}{|a|}\delta(t) \quad ?$$

检查

Complex Frequency

$$S = \alpha + j\omega$$

$$z = |z| e^{j\omega}$$

Complex Exponential

$$e^{st} = e^{\cancel{\alpha t}} e^{j\omega t}$$

$$z^n = |z|^n e^{j\omega n}$$

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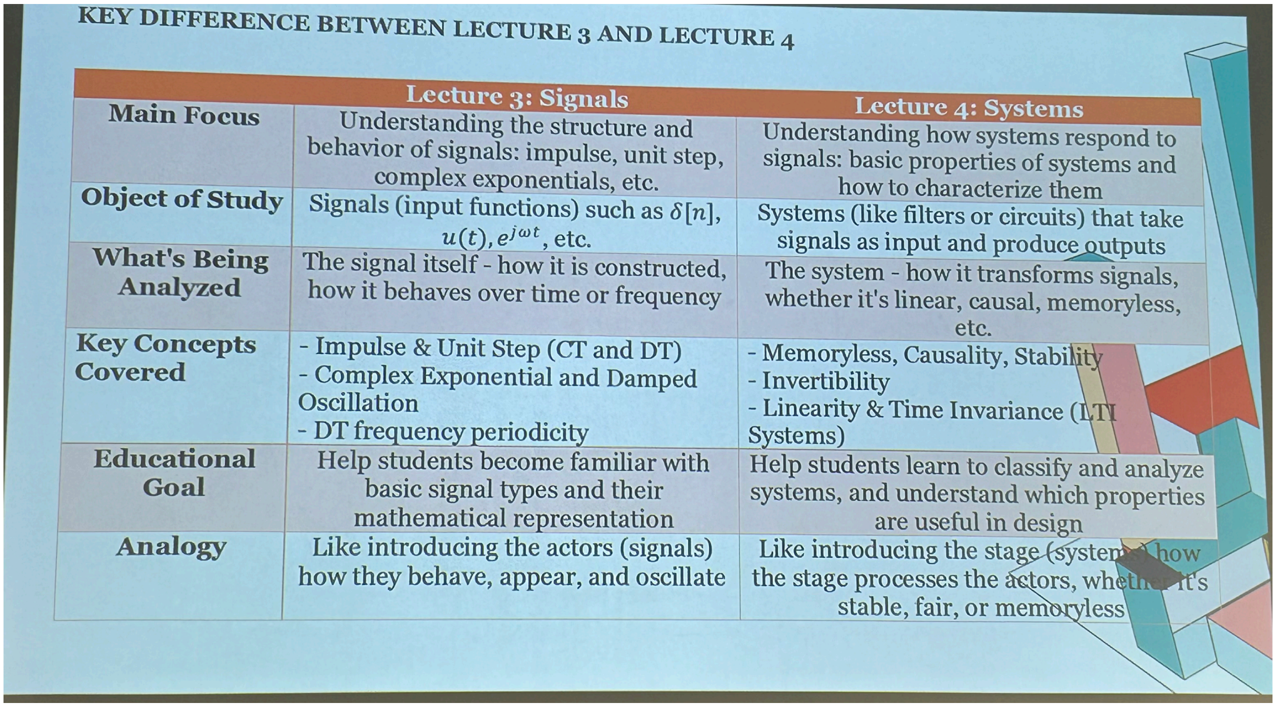
Lecture 4



KEY DIFFERENCE BETWEEN LECTURE 3 AND LECTURE 4

	Lecture 3: Signals	Lecture 4: Systems
Main Focus	Understanding the structure and behavior of signals: impulse, unit step, complex exponentials, etc.	Understanding how systems respond to signals: basic properties of systems and how to characterize them
Object of Study	Signals (input functions) such as $\delta[n]$, $u(t), e^{j\omega t}$, etc.	Systems (like filters or circuits) that take signals as input and produce outputs
What's Being Analyzed	The signal itself - how it is constructed, how it behaves over time or frequency	The system - how it transforms signals, whether it's linear, causal, memoryless, etc.
Key Concepts Covered	- Impulse & Unit Step (CT and DT) - Complex Exponential and Damped Oscillation - DT frequency periodicity	- Memoryless, Causality, Stability - Invertibility - Linearity & Time Invariance (LTI Systems)
Educational Goal	Help students become familiar with basic signal types and their mathematical representation	Help students learn to classify and analyze systems, and understand which properties are useful in design
Analogy	Like introducing the actors (signals) how they behave, appear, and oscillate	Like introducing the stage (system) how the stage processes the actors, whether it's stable, fair, or memoryless

Basic Characteristics

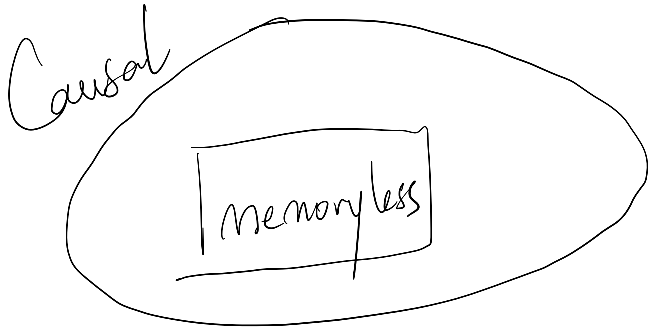
Memoryless

↳ instantaneous

↳ only depend on input signal at time t

Causality

↳ Doesn't depend on future input



Memoryless IS causal

Causal isn't memoryless

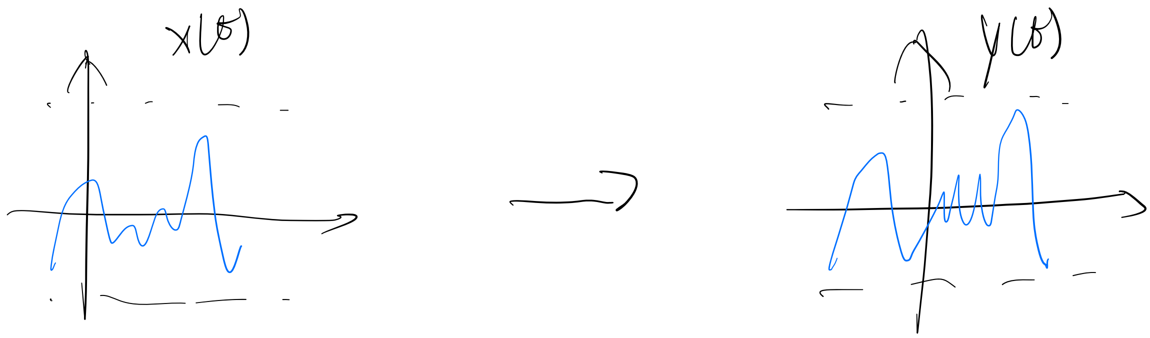
Stability

BIBO Bounded input Bounded output

If $\exists B < \infty$ Such that $|x(t)| \leq B \ \forall(t)$

then $\exists B < \infty$, $|y(t)| \leq B \forall(t)$

Input bounded by B then output too aB



Invertibility

Distinct input → Distinct output

↳ One-to-one

↳ only one corresponding $x(t)$ for a specific output $y(t)$

Time-Invariant (TI)

$x(t) \rightarrow y(t)$, then

$x(t-t_0) \rightarrow y(t-t_0)$

Linearity

↳ Satisfies Superposition

↳ input is weighted sum → output also..

CT : $x_1(t) \to y_1(t)$

$x_2(t) \to y_2(t)$

$ax_1(t) + bx_2(t) \to ay_1(t) + by_2(t)$

Same for DT

Additive and Homogeneity (Scaling)

$x_1(t) + x_2(t) \to y_1(t) + y_2(t)$

$a x_1(t) \to a y_1(t)$

Superposition (linearity)

↳ zero input zero output

$$\sum_{k=1}^{K} a_k x_k(t) \rightarrow \sum_{k=1}^{K} a_k y_k(t)$$

$$e^{x_1[n-1] + x_2[n-1]} \neq e^{x_1[n-1]} + e^{x_2[n-1]}$$

(exponential not linear)

Time Reversal : ~~memoryless~~
~~causal~~
~~time-invariant~~

Integration IS Linear!