ELEC 2100 Review

L1

CT: $x(t)$

DT: $x[n]$

$\alpha_1, \alpha_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$e^{j\theta} = \cos\theta + j\sin\theta$

Complex Plane

Complex Numbers

$z = \underbrace{a}_{\text{real}} + \underbrace{b}_{\text{imag}}j$ rectangular form

$a = Re\{z\} = r\cos\theta$

$b = Im\{z\} = r\sin\theta$

$z = \underbrace{r}_{\text{real}}e^{j\underbrace{\theta}_{\text{radian}}}$ polar / exponential form

$r = |z| = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\frac{b}{a}$ $\theta = \theta + 2\pi$

Addition / Multiplication / Division ..

Conjugate: $z^* = a - bj = re^{-j\theta}$

Properties:

Complex Conjugate - Properties

1. Adding a complex number with its conjugate produces 2 times the real part. In other words, the real part is the ***half sum*** of the conjugate pair.

$$\boxed{z + z^* = 2a = 2\mathrm{Re}\{z\} \text{ or } \mathrm{Re}\{z\} = (z+z^*)/2}$$

2. Subtracting a complex number by its conjugate produces $2j$ times the imaginary part

$$\boxed{z - z^* = 2bj = 2j\,\mathrm{Im}\{z\} \text{ or } \mathrm{Im}\{z\} = (z-z^*)/2j}$$

3. Multiplying a complex number with its conjugate produces its magnitude square

$$\boxed{zz^* = (a+bj)(a-bj) = a^2 - (bj)^2 = a^2 + b^2 = re^{j\theta}re^{-j\theta} = r^2}$$

or $\boxed{|z| = \left(zz^*\right)^{1/2}}$

Complex Conjugate - Properties

4. Conjugate of a sum is the sum of the conjugates and conjugate of a product is the product of the conjugates. That is, for a set of complex numbers z_i's:

$$\boxed{\left(\sum z_i\right)^* = \sum z_i^* \ ; \ \left(\prod z_i\right)^* = \prod z_i^*}$$

Sigma notation for a sum

Pi notation for a product

5. To conjugate an exponential, we conjugate both the base and the exponent :

$$(z_1{}^{z_2})^* = (z_1{}^*)^{z_2^*}$$

In ELEC2100, the base z_1 is typically real; hence we only need to worry about conjugating the exponent:

$$(r_1{}^{z_2})^* = r_1{}^{z_2^*} \quad \text{where } r_1 \text{ is real}$$

Try proving properties 4 and 5 yourself?

L2

Multiply signal by constant

Adding signals: Straight + Straight = Straight

3a. ***Time Shifting*** $[n-n_0]$ delay $[n+n_0]$ lead $n_0>0$

3b. ***Time Reflection/Reversal***

3c. ***Time Scaling*** $|\alpha|>1$ compression $|\alpha|<1$ expand $\alpha<0$ reversal.

Shift first then do other

Even: $x(-t) = x(t)$

Odd: $x(-t) = -x(t)$ equal to zero at $t=0$

Decomposition of Signal into Even and Odd Parts

- Any signal can be viewed as the sum of an even part and an odd part:

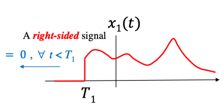
$$x(t) = x_{even}(t) + x_{odd}(t) = \mathcal{E}v\{x(t)\} + \mathcal{O}d\{x(t)\}$$

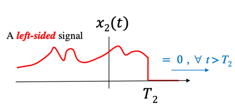
- We can find the even and odd parts by the *half-sum* and *half difference* of $x(t)$ and its time-reversed signal $x(-t)$

$$\underbrace{x_{even}(t) = \frac{x(t)+x(-t)}{2}}_{\text{half-sum}} \qquad \underbrace{x_{odd}(t) = \frac{x(t)-x(-t)}{2}}_{\text{half-difference}}$$

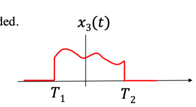
3. Right-Sided and Left-Sided Signals

- A signal is ***right-sided*** if there is a time T_1 for which $x(t) = 0$, $\underset{\text{for all}}{\forall}\, t < T_1$: signal with *initial rest*.
- A signal is ***left-sided*** if there is a time T_2 for which $x(t) = 0$, $\forall\, t > T_2$: signal with *final rest*.





- A finite duration signal is both right-sided and left-sided.



AKW

Periodic Signal

Poisson Sum $\quad \tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t-kT)$

e^{at}

$x(t) = \cos(\omega t + \phi)$

angular freq $\quad$ offset/phase

$f = \frac{\omega}{2\pi} \quad \rightarrow \quad \cos(2\pi f t + \phi)$

$T = \frac{1}{f} = \frac{2\pi}{\omega}$

Sine, Cosine, and Derivative

- The sine function is the same as cosine except for a 90° phase lag $\left(\phi = -\frac{\pi}{2}\right)$.

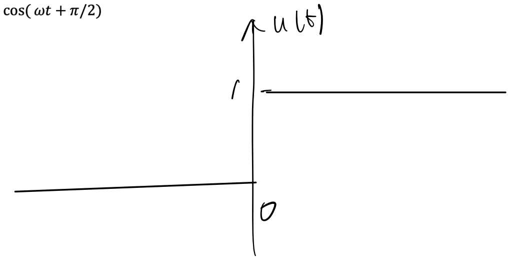
$$\sin(\omega t) = \cos(\omega t - \pi/2)$$

- The *derivative of a sinusoid is also a sinusoid at the same frequency*, but there is a multiplication by the angular frequency and a phase advance of 90°:

$$\frac{d\cos(\omega t)}{dt} = -\omega\sin(\omega t) = -\omega\cos(\omega t - \pi/2) = \omega\cos(\omega t + \pi/2)$$

$$\frac{d\sin(\omega t)}{dt} = \omega\cos(\omega t) = \omega\sin(\omega t + \pi/2)$$

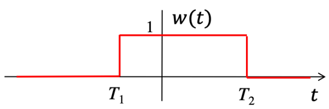
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



Window/Rectangular Signal as Difference of Unit Steps

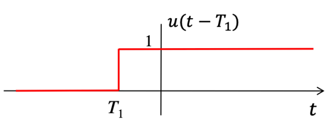
A window/rectangular signal is equal to 1 in the time interval (T_1, T_2) and is equal to 0 otherwise.

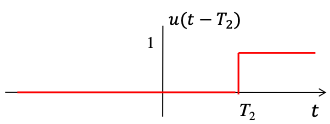
$$\boxed{w(t) = \begin{cases} 1, & T_1 < t < T_2 \\ 0 & otherwise \end{cases}}$$



We can represent the above window as the difference of two shifted unit steps:

$$\boxed{w(t) = u(t - T_1) - u(t - T_2)}$$





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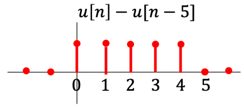
DT $u[n]$ & $\delta[n]$

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

DT window $\quad u[n] - u[n-N] = \begin{cases} 0 & n < 0;\ n \geq N \\ 1 & 0 \leq n \leq N-1 \end{cases}$

Example:

$u[n] - u[n-5]$



Note that while in the CT case $u(t) - u(t-T)$ is a window from 0 to T, $u[n] - u[n-N]$ is a DT window from 0 to $N-1$.

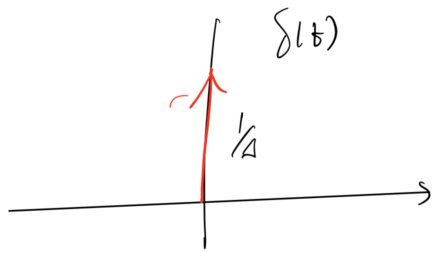
$\delta[n]$ unit impulse/ delta function

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

CT $\delta(t)$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$\delta(t) = \frac{du(t)}{dt}$$



Mathematical Properties of the Impulse Signal

1. Zero everywhere else, infinite at $t = 0$:

$$\delta(t) = 0 \ \ \forall t \neq 0$$

$$\lim_{t \to 0} \delta(t) = +\infty$$

2. Unit total area:

The total area under the $\delta(t)$ function is 1:

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

3. Integral is the unit step: $\quad u(t)$ is the first integral of $\delta(t)$

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

Alternate notations for first integral:

$$y(t) = \int_{-\infty}^{t} x(\tau) d\tau$$

$$y(t) = \int x(t)$$

$$y(t) = x^{(-1)}(t)$$

4. **Scaled impulse** $k\delta(t)$ has total area k.

Clearly $$\int_{-\infty}^{\infty} k\delta(t)dt = k\int_{-\infty}^{\infty}\delta(t)dt = k$$

We label a scaled impulse by its area as shown previously:

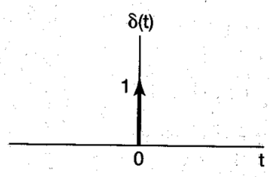


Figure 1.35 Continuous-time unit impulse.

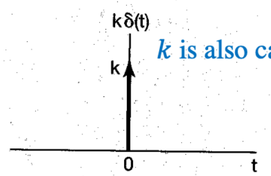


Figure 1.36 Scaled impulse.

k is also called the magnitude of the impulse

Integrating a scaled impulse $k\delta(t)$ gives $ku(t)$:

$$\int_{-\infty}^{t} k\delta(\tau)d\tau = ku(t)$$

$$x(t)\delta(t) = x(0)\delta(t)$$

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$\int_{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$$

$$\int_{\infty} |\delta(t)|^2 dt = \lim_{\Delta\to 0}\int_0^{\Delta}\left|\frac{1}{\Delta}\right|^2 dt = \lim_{\Delta\to 0}\frac{1}{\Delta} \to \infty$$ infinite energy

Summary – Key Properties of $\delta(t)$ and $\delta[n]$

Integral/first sum is unit step: $\boxed{u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau}$ $\boxed{u[n] = \sum_{m=-\infty}^{n}\delta[m]}$

Sampling: $\boxed{x(t)\delta(t) = x(0)\delta(t)}$ $\boxed{x[n]\delta[n] = x[0]\delta[n]}$

$\boxed{x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)}$ $\boxed{x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]}$

Sifting: $\boxed{\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)}$ $\boxed{\sum_{-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]}$

Complex Exponential: e^{st}

Complex Frequency: $s = \sigma + j\omega$

Complex Sinusoid: $e^{j\omega t}$

$$e^{st} = e^{\sigma t} e^{j\omega t}$$

$e^{\sigma t}$: decay $\sigma < 0$, grows $\sigma > 0$

$e^{j\omega t}$: oscillation

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$e^{\sigma t}\cos\omega t = \mathrm{Re}\{e^{st}\} = \frac{1}{2}e^{st} + \frac{1}{2}(e^{st})^*$$

$$= \frac{1}{2}e^{\sigma t}e^{j\omega t} + \frac{1}{2}e^{\sigma t}e^{-j\omega t}$$

↳ Conjugate = negate ω

$$\cos\omega t = \mathrm{Re}\{e^{j\omega t}\} = \frac{1}{2}e^{j\omega t} + \frac{1}{2}e^{-j\omega t}$$

One sided

$e^{st}u(t)$ right

$e^{st}u(-t)$ left

DT

e^{st} (CT) $\longrightarrow$ e^{sn} (DT)

$e^{sn} \rightarrow z^n \qquad z = e^s = e^{\sigma} e^{j\omega}$

$\hookrightarrow z^n = |z| e^{j\omega n}$

DT complex sinusoid: $e^{j\omega n}$

CT and DT Complex Frequency

- s and z are both called ***complex frequency***, for CT and DT case respectively. $z = e^s$
- s and z concisely describe the rate at which the signal grows or decays and oscillates.

CT: $s = \text{Re}\{s\} + j\,\text{Im}\{s\}$

rate of growth/decay ($Re\{s\} = 0$ means no growth/decay)

frequency of oscillation ($Im\{s\} = 0$ *means no oscillation*)

DT: $z = |z|e^{j\measuredangle z}$

rate of growth/decay ($|z| = 1$ *means* no growth/decay)

frequency of oscillation ($\measuredangle z = 0$ means no oscillation)

DT Sinusoids – Fact 1: Periodicity

1. DT sinusoid is periodic only if its ordinary frequency f is a rational number, or its angular frequency ω is a rational number times 2π.)

- If a DT sinusoid $\cos(\omega n)$ is periodic, it is unchanged after time shift by some integer N:

$$\cos(\omega(n+N)) = \cos(\omega n + \omega N) = \cos(\omega n) \;\; \forall n$$

This means the phase change ωN must be an integer multiple of 2π;

i.e., $\omega N = m2\pi$ for some integer m, or $\omega = m2\pi/N$

Hence $\omega = \frac{m}{N} \times 2\pi$, a rational number $\frac{m}{N}$ multiplied by 2π .

Or, the ordinary frequency $f = m/N$ is rational.

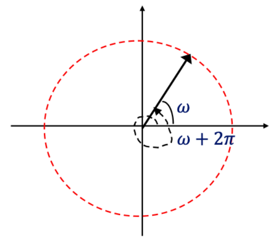
DT Sinusoid – Fact 2: Periodicity of DT Frequency

2. DT frequency is periodic (Very Important!)

In DT:

$\boxed{\begin{array}{l}\omega \text{ and } \omega + k2\pi \text{ refer to the same angular frequency!} \\ f \text{ and } f + k \text{ refer to the same ordinary frequency!}\end{array}}$

Angular frequency ω is the phase change per unit time. For a DT signal we observe the phase change only at discrete time instants, so there is no difference between a phase change of ω and phase change of $\omega + k2\pi$.



$$\cos((\omega + k2\pi)n) = \cos(\omega n + \cancelto{0}{kn2\pi}) = \cos(\omega n)$$

E.g.: 0.9 Hz, 1.9 Hz, 2.9 Hz, -0.1 Hz , -1.1 Hz are all the same frequency!

(In addition, because cosine is an even function, -0.9 Hz, 0.1 Hz, 1.1 Hz, 2.1 Hz , etc., are also the same frequency! We will come back to this later.)

E.g. $\cos(0.9 \times 2\pi n) \equiv \cos(-1.1 \times 2\pi n)$

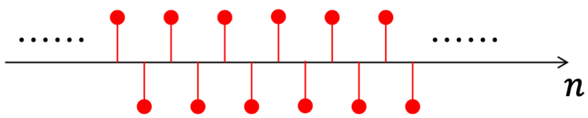
E.g. $\cos(0.9 \times 2\pi n) \equiv \cos(1.1 \times 2\pi n)$

Maximum Frequency of DT Signals

$\boxed{\text{The fastest rate at which a DT signal can oscillate is at an ordinary frequency of } f = ½ \text{, or angular frequency of } \omega = \pi.}$

The following DT oscillation, with period N=2, represents the DT oscillation at the highest frequency: (What if N=1?)

$$\cos \pi n = (-1)^n$$



But note that we have not specified what the "time unit" is. If the time unit is 1 ns (DT signal generated at 1 GHz), then an $f = ½$ actually represents a frequency of 0.5 GHz.

Examples:

- Traditional digital telephone networks sample at 8 KHz $\Longrightarrow$ maximum frequency in speech signal transmitted is 4 KHz.
- Compact Disc music sample at 44.1 KHz $\Longrightarrow$ maximum frequency in CD music is 22.05 KHz.

L4

Series : convolve
parallel : add ?

Characterization of Systems

Memoryless depends only on input at t/n

Causality don't depend on future input

Stability BIBO

Invertible one-to-one
can deduce $x(t)$ from $y(t)$

时不变

Time Invariance Shifted input - shifted output

$x(t) \to y(t)$

$x(t-t_0) \to y(t-t_0)$

If multiplied by function of time then not TI $y(t) = t\,x(t)$

Raising input to n^{th} power $x^t(t)$

Time Reversal, Taken Even, Odd part

$x(-t)$ $Ev\{x(t)\}$ $Od\{x(t)\}$

Differentiation : $\checkmark$

Integration : TI when window moves with time

Linearity : Superposition

CT: if $x_1(t) \rightarrow y_1(t)$ where "$x \rightarrow y$" means x produces the output y

and $x_2(t) \rightarrow y_2(t)$

then $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$

Input that is a weighted sum will produce an output that is the same weighted sum of the individual outputs.

DT: if $x_1[n] \rightarrow y_1[n]$ and $x_2[n] \rightarrow y_2[n]$

then $ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$

additive & Scaling

Zero-input $\rightarrow$ zero - output

If $x_k(t) \rightarrow y_k(t)$ $k = 1, \dots K$

$$\sum_{k=1}^{K} a_k x_k(t) \rightarrow \sum_{k=1}^{K} a_k y_k(t)$$

Differentiation & Integration is Linear

LTI system : Linear + TI

No Yes No Yes ~~No~~

No No Yes ~~Yes~~ Yes

No No ~~NO~~ No NO

e^x bounded if x bounded

Time Reversal.

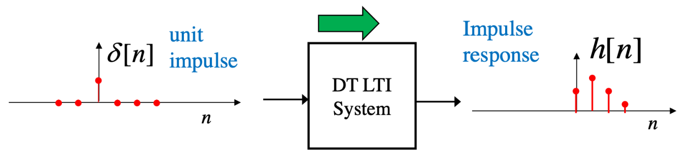
Integration : Linear

LTI

$x[n] \longrightarrow \boxed{\text{system}} \longrightarrow y[n]$

Impulse Response : (DT)

- Let $h[n]$ be the output of a DT LTI system when the input is the unit impulse $\delta[n]$. We call $h[n]$ the **impulse response**.



DT as Weighted Sum

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

Output of an LTI System

- If system is ***linear***, output must be the same weighted sum of the responses to the individual shifted impulses. Let $h_k[n]$ be the system's response to a $\delta[n-k]$. Then

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \longrightarrow \boxed{\text{Linear System}} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h_k[n] \qquad \textbf{Eq. (2.3)}$$

Input is a weighted sum of $\delta[n-k]$ — Output is a weighted sum of responses to $\delta[n-k]$

- If system is ***time-invariant***, $h_k[n]$ must be $h[n-k]$. Hence:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \longrightarrow \boxed{\begin{array}{c}\text{Linear}\\ \text{Time-Invariant}\\ \text{(LTI)}\\ \text{System}\end{array}} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \qquad \textbf{Eq. (2.6)}$$

$h_k[n] = h[n-k]$

- Eq. (2.6) is called the **convolution sum**.
- It ***means*** that $y[n]$, the output of an LTI system at any time n, can be founded by summing up the responses arising from the inputs at all different times k and the response at time n arising from the input at time k is $x[k]h[n-k]$.

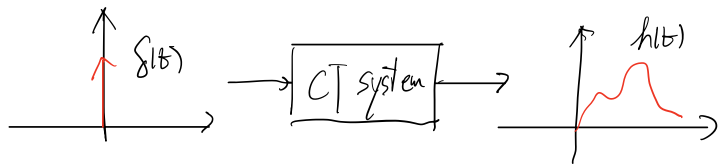
Convolution Sum

Output = Convolution of input & impulse response

$$y[n] = \underset{\text{Convolution}}{x[n] * h[n]} = \underset{\text{Convolution Sum}}{\sum_{k=-\infty}^{\infty} x[n]\,h[n-k]}$$

Convolution is LTI

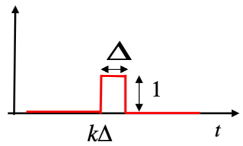
Impulse Response : $h(t)$
(CT)



Approximately: $x(t) \approx \hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\,\delta_\Delta(t-k\Delta)\Delta$

A shifted narrow pulses with unit height

$\delta_\Delta(t-k\Delta)\Delta$



Now take limit $\Delta \to 0$

$$x(t) = \lim_{\Delta\to 0}\hat{x}(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)\,\delta_\Delta(t-k\Delta)\Delta$$

$$= \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

Response of CT LTI System

$\lim_{\Delta\to 0}\delta_\Delta(t) = \delta(t)$

- Let $h_\Delta(t)$ be the response of a CT system to the input $\delta_\Delta(t)$. In limit, $\lim_{\Delta\to 0} h_\Delta(t) = h(t)$
- For a CT LTI system, if the input is a superposition of shifted $\delta_\Delta(t)$, the output is the same superposition of shifted $h_\Delta(t)$:

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta \longrightarrow \boxed{\text{CT LTI system}} \longrightarrow \hat{y}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)h_\Delta(t-k\Delta)\Delta$$

- Now, we take limit $\Delta\to 0$, the input becomes $x(t)$ and the output becomes:

$$x(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \longrightarrow \boxed{\text{CT LTI system}} \longrightarrow$$

$$\boxed{y(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)h_\Delta(t-k\Delta)\Delta = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$$

Eq. 2.33

Convolution of input with impulse response

The Convolution Integral

- In summary, for a CT LTI system, we can determine the output from the input and the impulse response by:

$$\boxed{y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$$

- This is called the ***convolution integral*** and we also use the "*" symbol to denote it:

$$\boxed{y(t)=x(t)*h(t)}$$

- Just like the DT case, for an LTI system, the output is the convolution of the input and the system's impulse response!

$h(t)=\delta(t-t_0)$: time delay.

Integrator : $h(t)=u(t)$

L6

Properties of Conv/LTI systems.

Commutative : $x(t)*h(t) = h(t)*x(t)$

Distributive : $x(t)*\{h_1(t)+h_2(t)\} = x(t)*h_1(t) + x(t)*h_2(t)$

Associative :

When we convolve multiple signals in sequence, the order of association does not matter: it does not matter which signal we convolve with which signal first.

$$\{x(t)*h_1(t)\}*h_2(t)=x(t)*\{h_1(t)*h_2(t)\}=x(t)*h_1(t)*h_2(t)$$

$$\underbrace{\{x[n]*h_1[n]\}*h_2[n]}_{\substack{x\text{ first convolving}\\ \text{with } h_1\text{, then } h_2.}}=\underbrace{x[n]*\{h_1[n]*h_2[n]\}}_{\substack{x\text{ convolving with } h_1\\ \text{convolved with } h_2\text{ first}}}=\underbrace{x[n]*h_1[n]*h_2[n]}_{\substack{\text{No need to specify which}\\ \text{convolve with which first}}}$$

This means we can combine two systems in cascade into one by convolving the individual impulse responses:

Characterizing LTI System by Impulse Response

Memoryless: if and only if $h[n]=0$ for all $n \neq 0$

Only memoryless system is $h(t) = A\,\delta(t)$

Causality: Impulse response is zero for all $t<0$

; $h[n]=0$ for $n<0$

Consider CT case $\quad y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

For system to be causal, $y(t)$ cannot depend $x(\tau)$ with $\tau > t$

This means $h(t-\tau)$ must be 0 for all $\tau > t$, or $h(\cdot) = 0$ whenever argument is < 0

Causal Responses $\qquad\qquad$ Non-Causal Responses

$h[n] = 0$ for $n < 0$; $h(t) = 0$ for $t < 0$

Stability An LTI system is BIBO <u>stable</u> iff impulse response is ***absolute integrable*** for CT or ***absolute summable*** for DT, meaning:

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty \; ; \qquad \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

$|\cdot|$ means absolute value or magnitude

Recall that BIBO (Bounded Input Bounded Output stability) means:

if $|x(t)| \leq B < \infty \; \forall t$ then $|y(t)| \leq B' < \infty \; \forall t$

Input $u(t) \rightarrow \int_{-\infty}^{t} h(\tau)d\tau \qquad$ Integral

↙ 微分

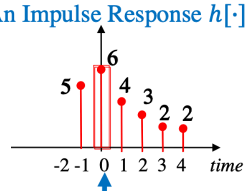
$x^{(k)}(t) * h(t) = x(t) * h^{(k)}(t) = y^{(k)}(t)$

III. Convolution sum & Integral Examples

- The convolution sum for DT is $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
- Computing it conceptually involves the following steps:
  - ✓ Step 1: draw $x[k]$ and $h[k]$ by replacing "n" by "k" k: variable of summation
  - ✓ Step 2: flip $h[k]$ to get $h[-k]$; $h[-k]$ is called the *convolution kernel*   kernel: essence, core
  - ✓ Step 3: Shift $h[-k]$ by n to obtain $h[n-k]$; i.e., recognize $h[0]$ as anchor of kernel and place it at n.
  - ✓ Step 4: Multiple $x[k]$ by $h[n-k]$ and sum over all k.

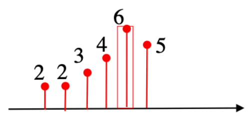
Visualizing the Impulse Response and Convolution kernel

An Impulse Response $h[\cdot]$



$h[0]$: kernel anchor

Corresponding Convolution Kernel



AKW

Kernel as a sequence of values

2	2	3	4	$\boxed{6}$	5

$h[0]$
kernel anchor

L07

$$\langle \vec{x}, \vec{y} \rangle = \sum_n x_n y_n^*$$

$$\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle^*$$