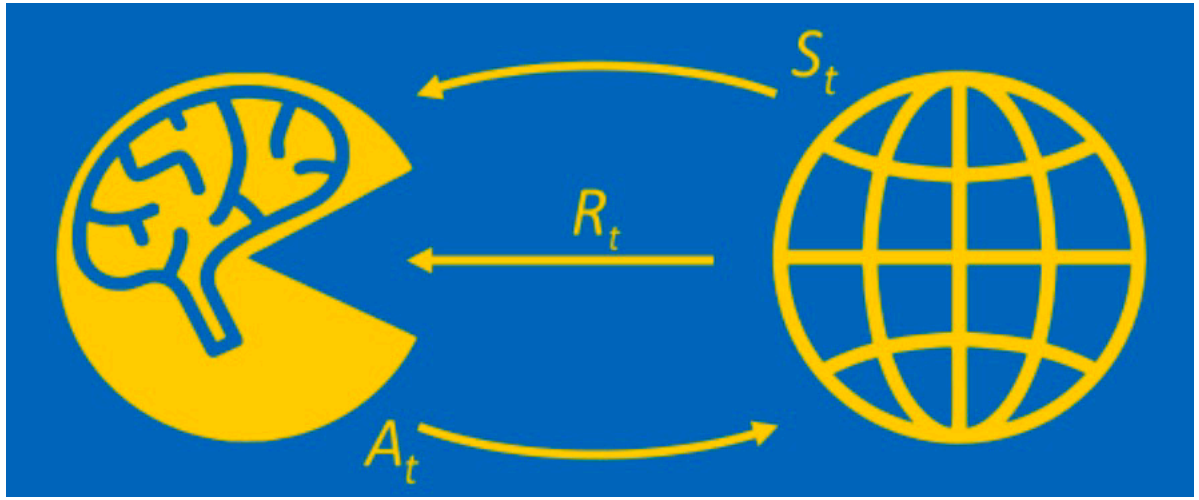
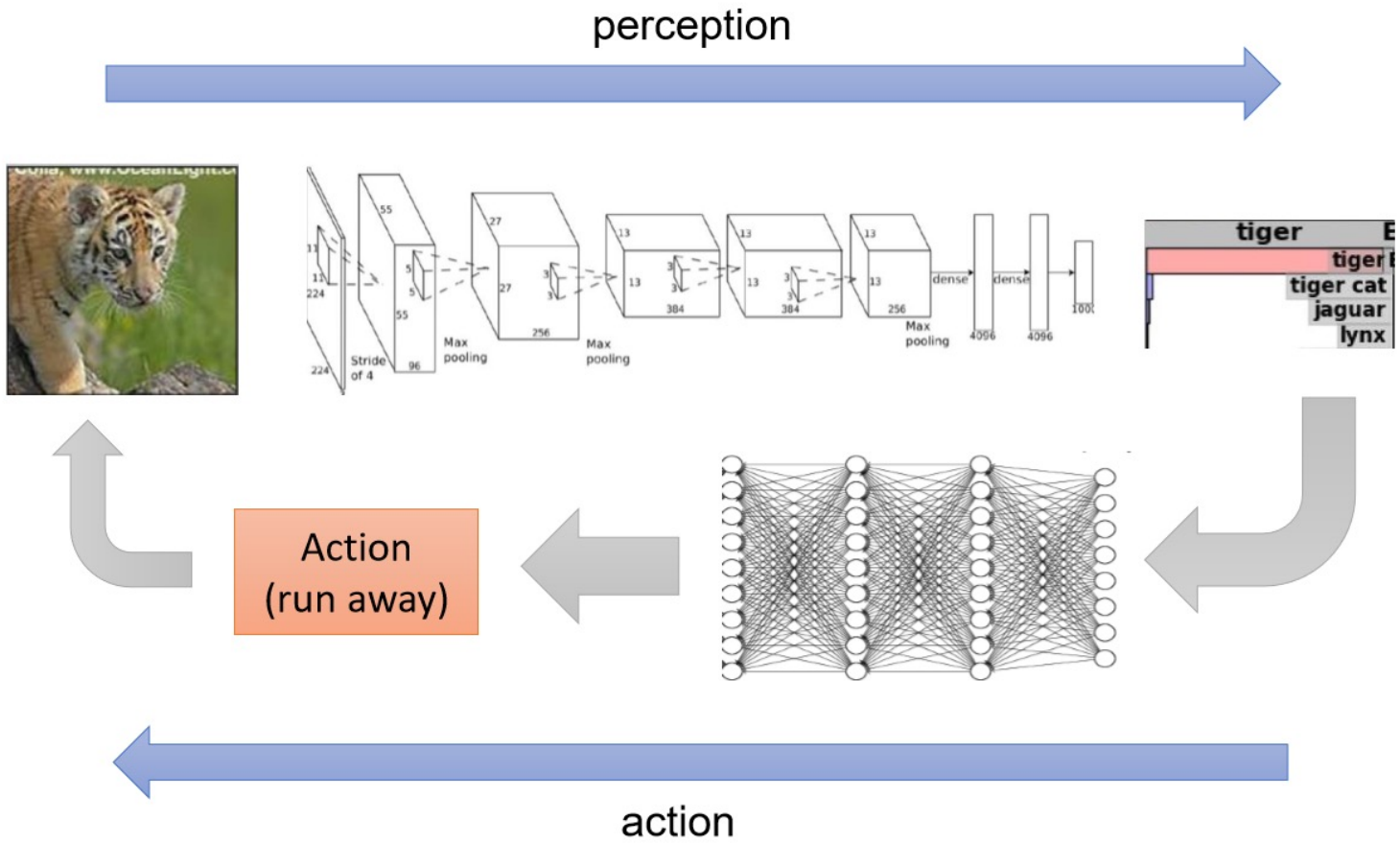
Intro to Reinforcement Learning

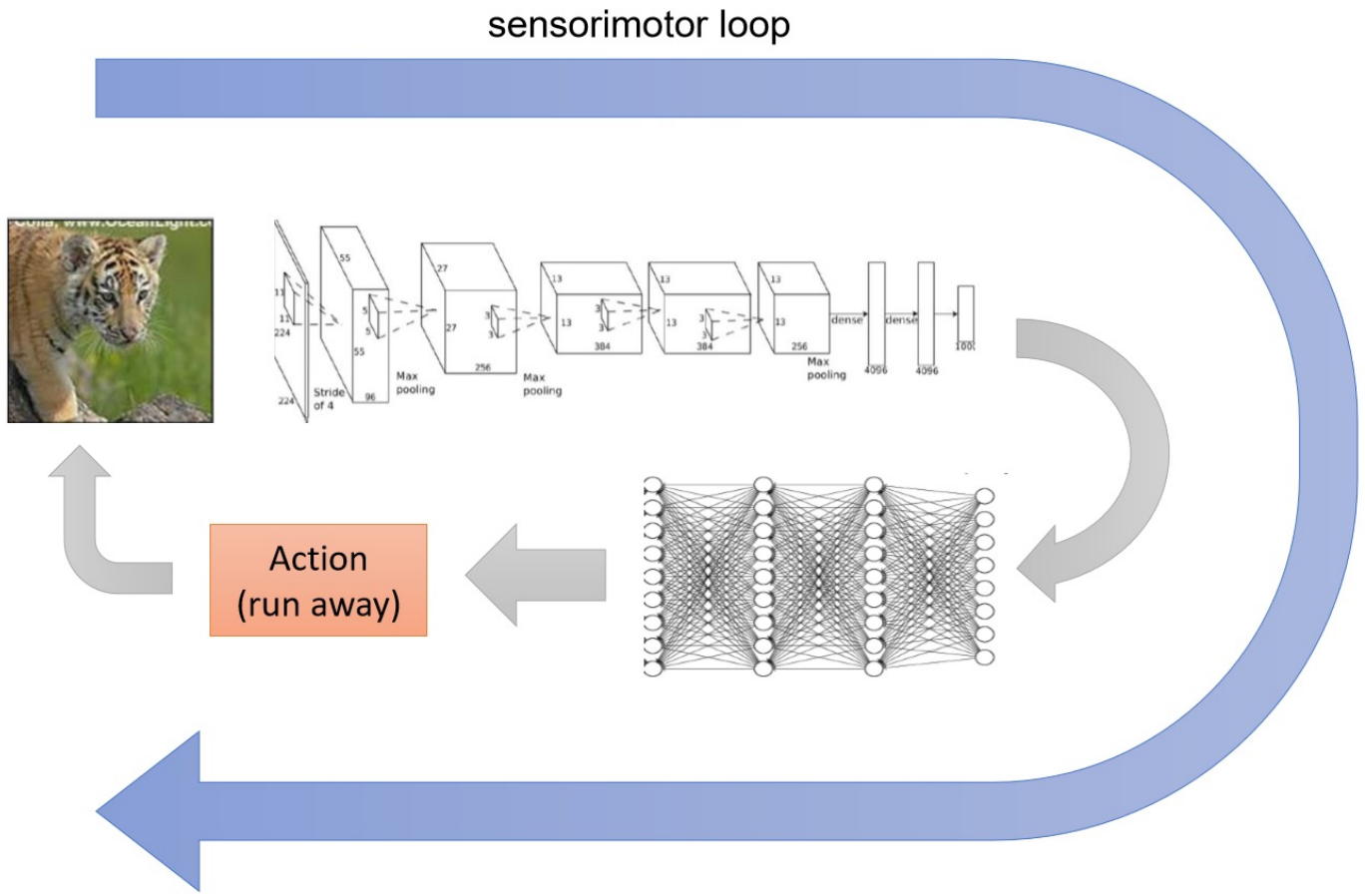


Sergey Sviridov

От обучения с учителем к обучению с подкреплением



От обучения с учителем к обучению с подкреплением



RL vs Supervised Learning

Instructive

vs

Evaluative

feedback

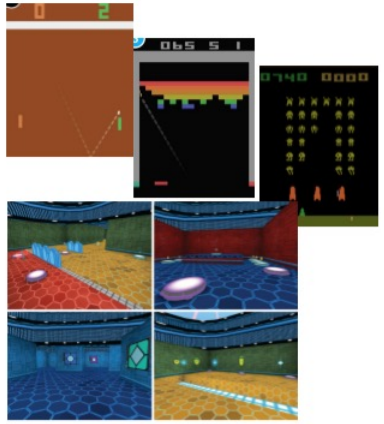
Примеры обучения с подкреплением

Kohl and Stone, 2004

Ng et al, 2004

Tedrake et al, 2005

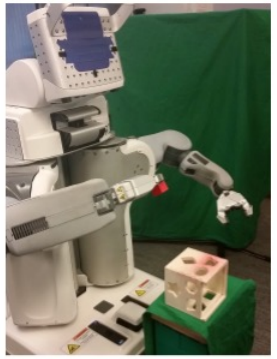
Kober and Peters, 2009



Mnih et al 2013 (DQN)
Mnih et al, 2015 (A3C)

Silver et al, 2014 (DPG)
Lillicrap et al, 2015 (DDPG)

Schulman et al, 2016 (TRPO + GAE)



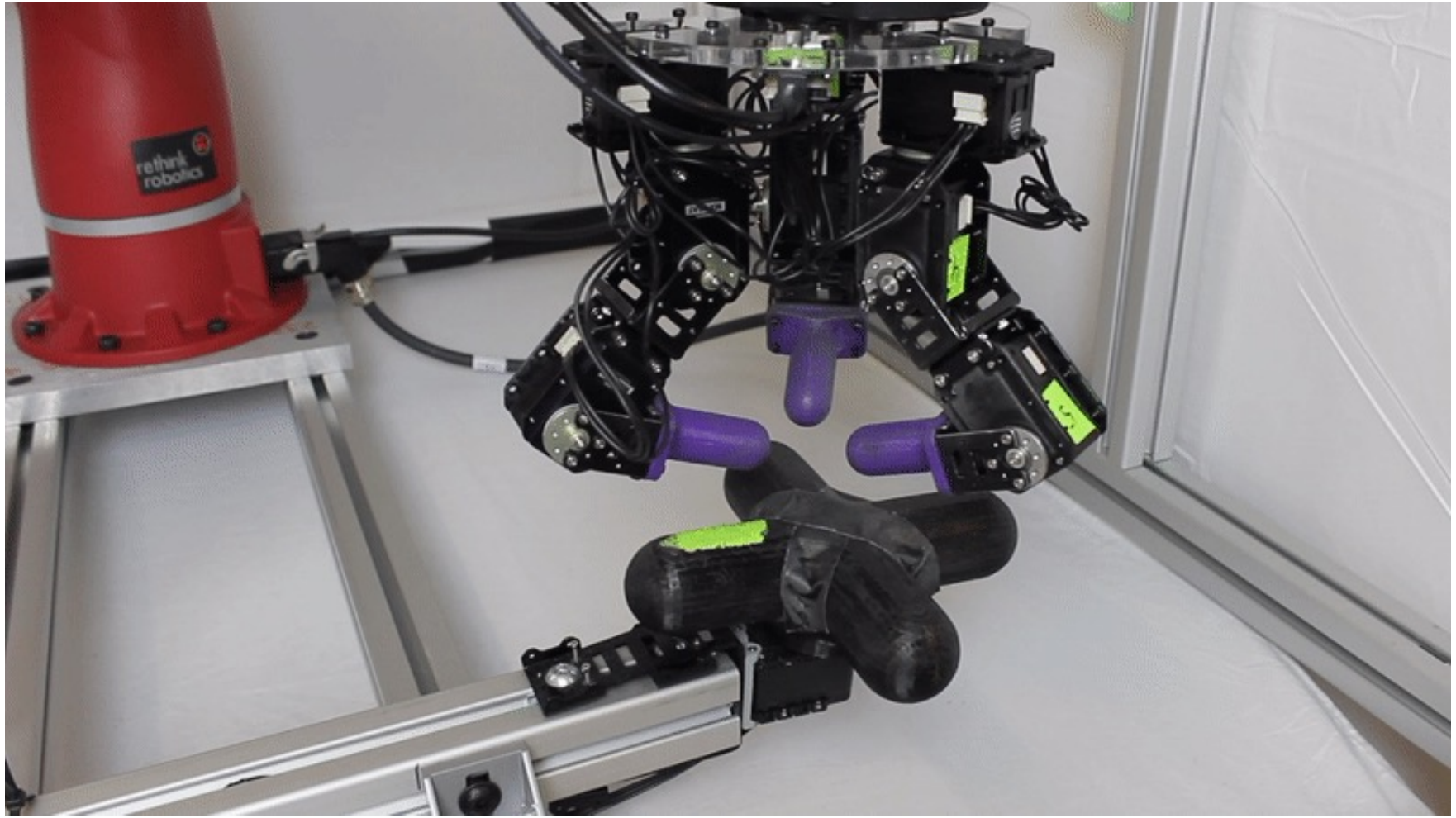
Levine*, Finn*, et al, 2016 (GPS)

Silver*, Huang*, et al, 2016 (AlphaGo)

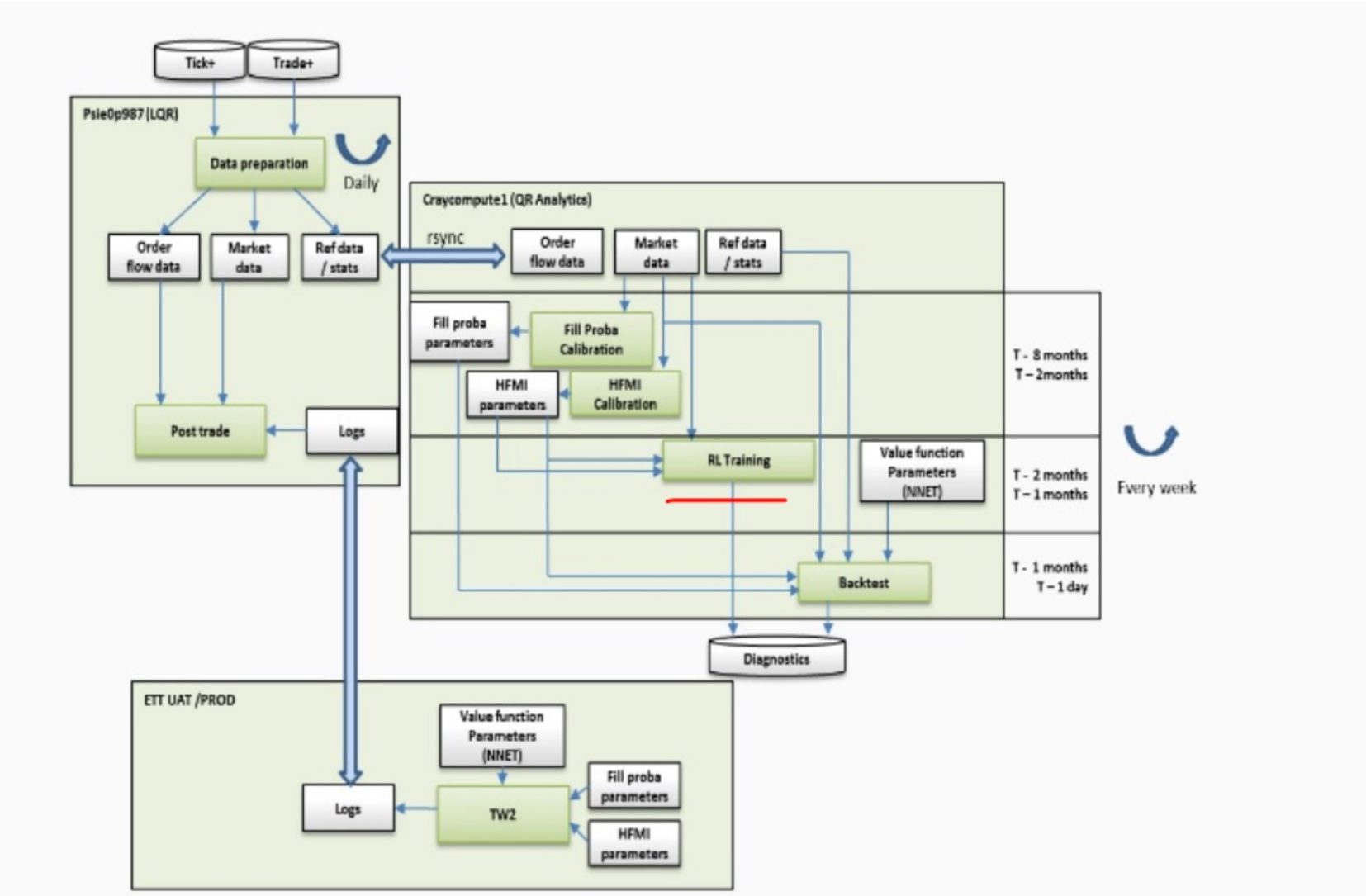
Примеры. AlphaGo и AlphaGo Zero

Примеры. Open AI Five

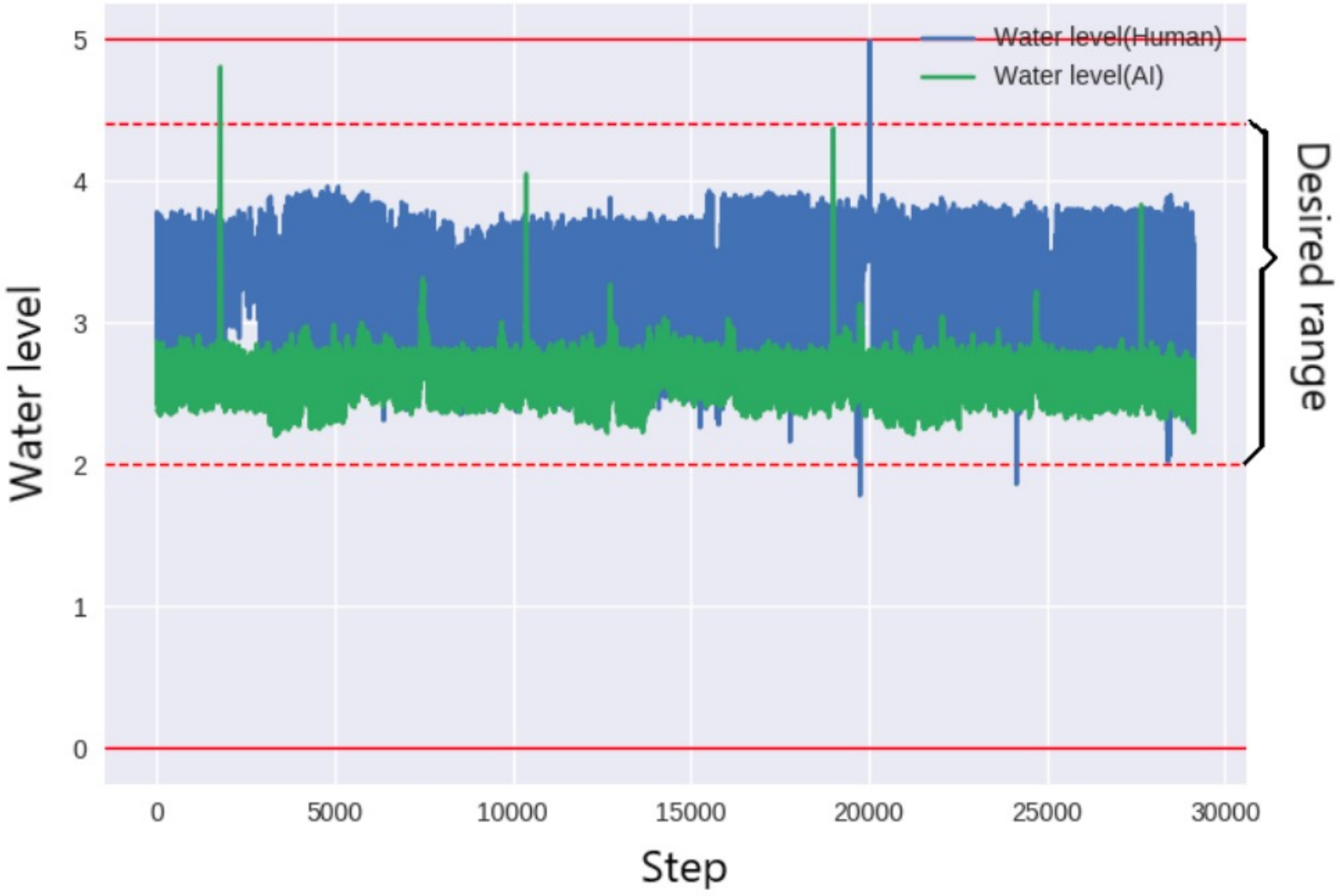
Примеры. Dexterous Manipulation with Reinforcement Learning



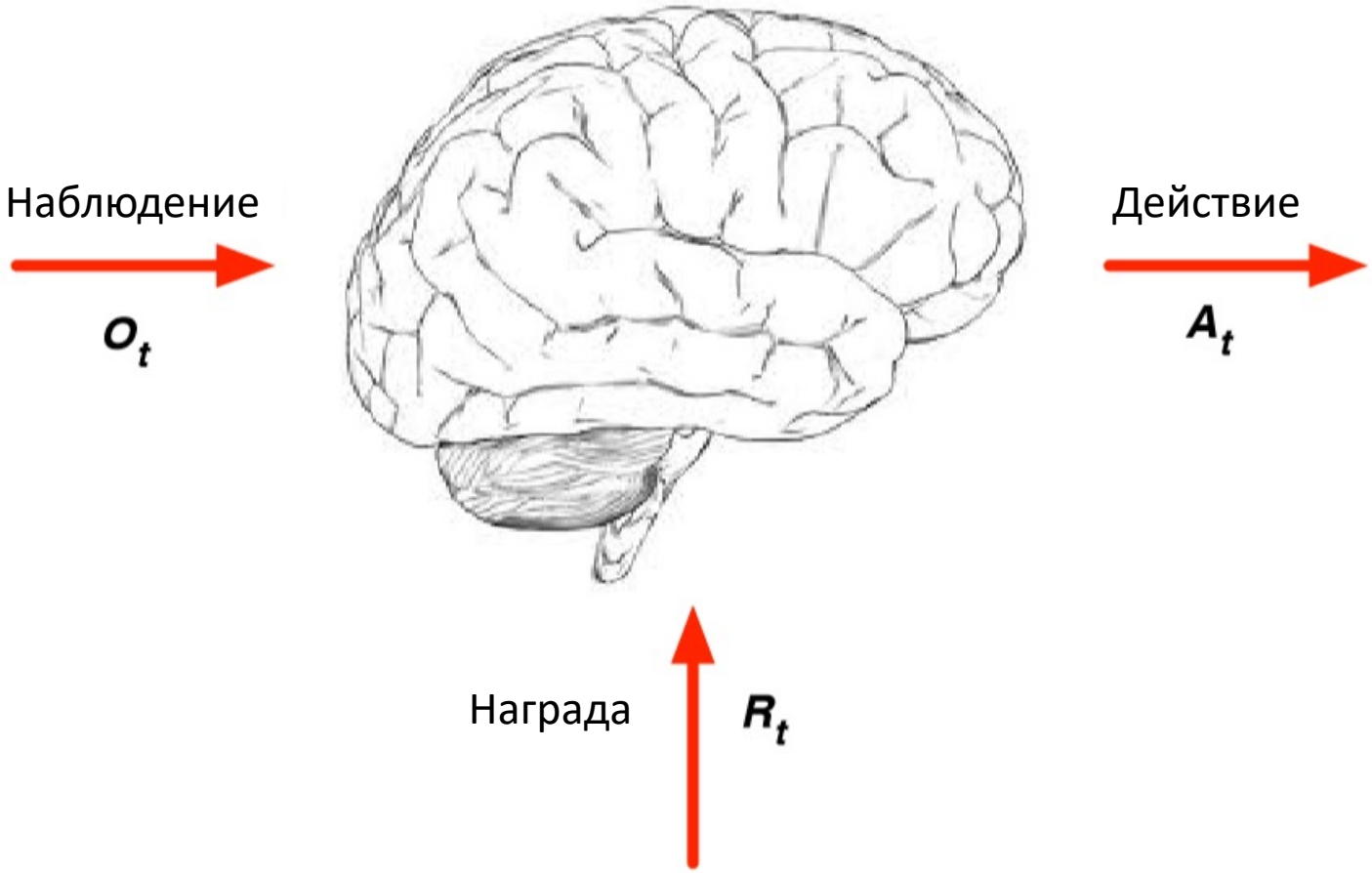
Примеры. Trading Application at JP Morgan Chase



Примеры. Automating Water Purification Plant Operations



Агент



Внутри RL агента

Основные компоненты любого RL агента:

- Политика (Policy): функция поведения агента
- Функция стоимости (Value function): функция оценки состояния и/или действия
- Модель (Model): представление агента о среде

Политика (Policy)

- Политика определяет поведение агента
- Представляет собой отображение состояний в действия
- Детерминированная политика: $a = \pi(s)$
- Стохастическая политика: $\pi(a|s) = \mathbb{P}[A_t = a|S_t = s]$

Функция стоимости (Value function)

- Функция стоимости является оценкой будущих наград
- Используется для оценки насколько хорошим или плохим является состояние
- Используется для выбора лучшего действия

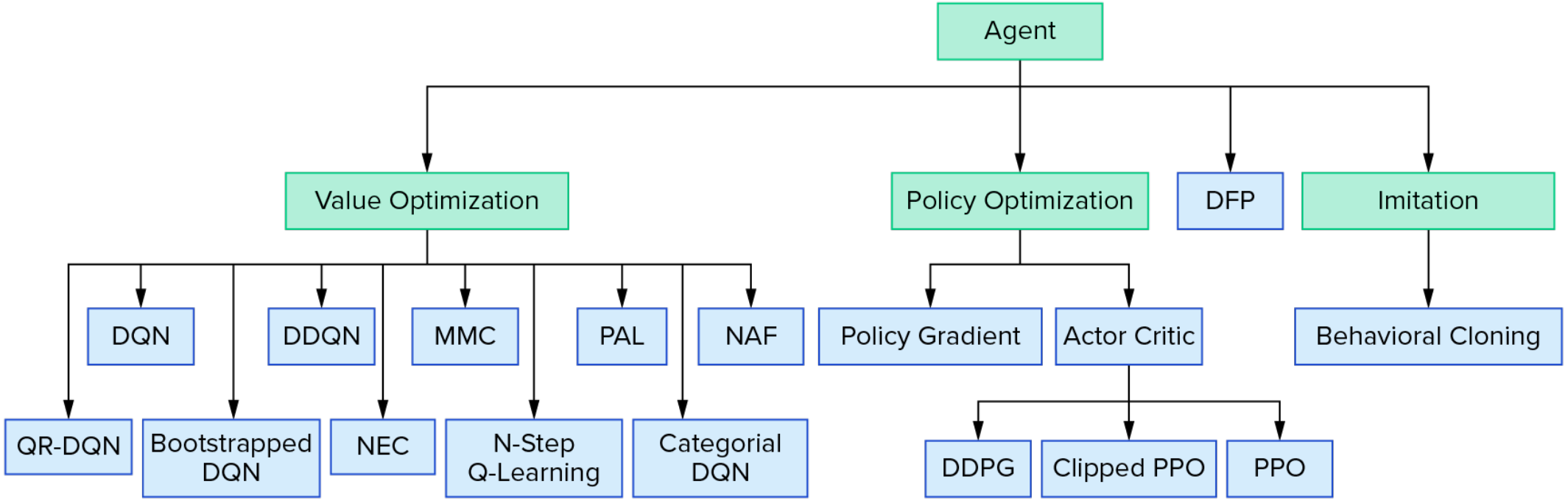
$$v_\pi(s) = \mathbb{E}_\pi \left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s \right]$$

Модель

- Модель предсказывает, что произойдет с окружением в следующий момент времени
- *P* предсказывает следующее состояние
- *R* предсказывает следующую награду

$$\mathcal{P}_{ss'}^{a} = \mathbb{P}[S_{t+1} = s' \mid S_t = s, A_t = a]$$
$$\mathcal{R}_{s}^{a} = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a\right]$$

Методы обучения с подкреплением (1)



Методы обучения с подкреплением (2)

- Model-free (без модели)
  - Есть политика и/или функция стоимости
  - Нет модели
- Model-base (основанные на модели)
  - Есть политика и/или функция стоимости
  - Есть модель

Методы обучения с подкреплением (3)

- On-Policy (используется одна политика)
  - Не можем использовать старый опыт после обновления политики
- Off-Policy (используется behavioral и target политики)
  - Можем использовать старый опыт после обновления политики

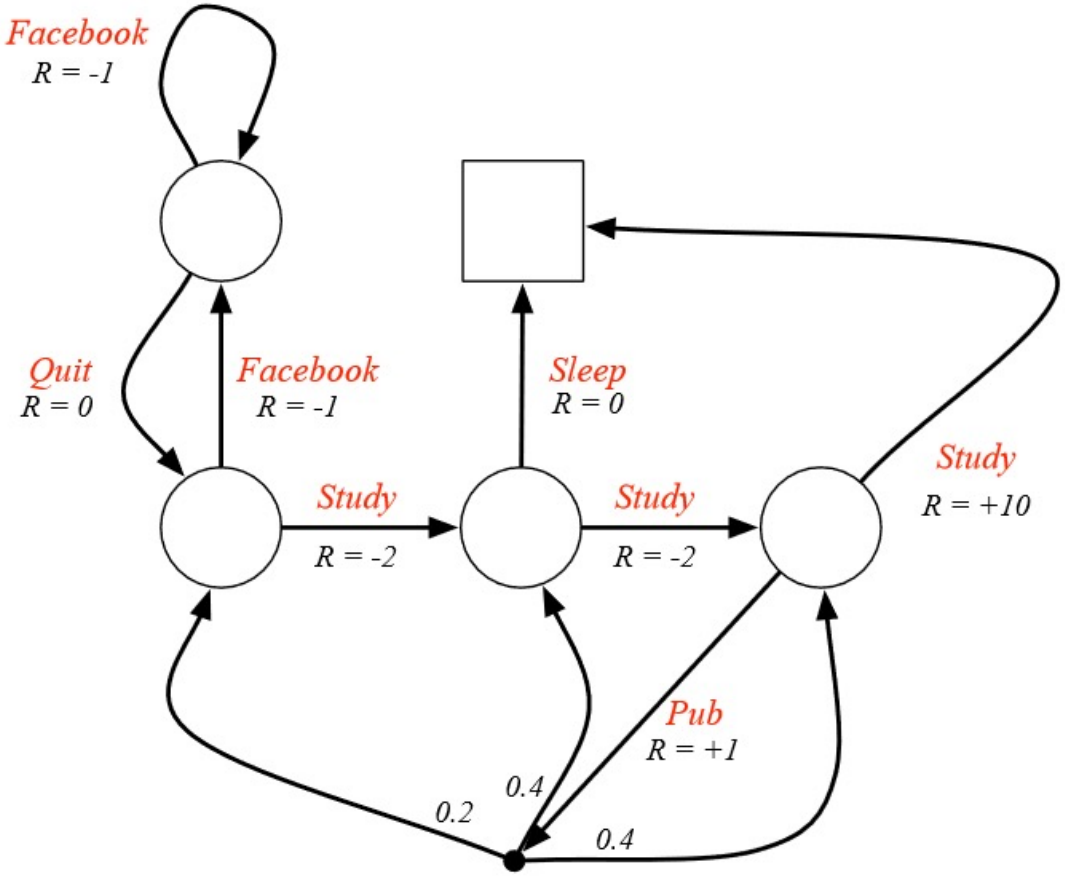
Markov Decision Process

Definition

A *Markov Decision Process* is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states (discrete or continuous)
- $\mathcal{A}$ is a finite set of actions (discrete or continuous)
- $\mathcal{P}$ is a state transition probability matrix,
  $\mathcal{P}_{ss'}^{a} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s, A_t = a\right]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}_s^a = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a\right]$
- γ is a discount factor $\gamma \in [0, 1]$.

Markov Decision Process: Example



Source: D. Silver, DeepMind, Lectures

Policy

Definition

A *policy* π is a distribution over actions given states,

$$\pi(a|s) = \mathbb{P}\left[A_t = a \mid S_t = s\right]$$

- Policy completely describes agent's behavior
- MDP policies depend on the current state
- Policies are stationary (time-independent)

Value Function

Definition

The *state-value function* $v_\pi(s)$ of an MDP is the expected return starting from state s, and then following policy π

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s \right]$$

Definition

The *action-value function* $q_\pi(s, a)$ is the expected return starting from state s, taking action a, and then following policy π

$$q_\pi(s, a) = \mathbb{E}_\pi \left[G_t \mid S_t = s, A_t = a \right]$$

Bellman Expectation Equation

The state-value function can again be decomposed into immediate reward plus discounted value of successor state,

$$v_\pi(s) = \mathbb{E}_\pi \left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

The action-value function can similarly be decomposed,

$$q_\pi(s, a) = \mathbb{E}_\pi \left[R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a \right]$$

Optimal Value Function

Definition

The *optimal state-value function* $v_*(s)$ is the maximum value function over all policies

$$v_*(s) = \max_{\pi} v_\pi(s)$$

The *optimal action-value function* $q_*(s, a)$ is the maximum action-value function over all policies

$$q_*(s, a) = \max_{\pi} q_\pi(s, a)$$

Optimal Policy

Define a partial ordering over policies

$$\pi \geq \pi' \text{ if } v_\pi(s) \geq v_{\pi'}(s), \forall s$$

Theorem

For any Markov Decision Process

- *There exists an optimal policy π_* that is better than or equal to all other policies, $\pi_* \geq \pi, \forall \pi$*
- *All optimal policies achieve the optimal value function, $v_{\pi_*}(s) = v_*(s)$*
- *All optimal policies achieve the optimal action-value function, $q_{\pi_*}(s, a) = q_*(s, a)$*

Finding Optimal Policy

An optimal policy can be found by maximising over $q_*(s, a)$,

$$\pi_*(a|s) = \begin{cases} 1 & \text{if } a = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q_*(s, a) \\ 0 & \textit{otherwise} \end{cases}$$

There is always a deterministic optimal policy for any MDP

If we know $q_*(s, a)$, we immediately have the optimal policy

From Q-value iteration to Q-learning

$Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$$

From Q-value iteration to Q-learning

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$
- (Tabular) Q-Learning: replace expectation by samples
  - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$
  - Consider your old estimate: $Q_k(s,a)$
  - Consider your new sample estimate:

    $$\text{target}(s') = R(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$
  - Incorporate the new estimate into a running average:

    $$Q_{k+1}(s,a) \leftarrow (1-\alpha) Q_k(s,a) + \alpha \left[\text{target}(s') \right]$$

Q-learning

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal

Approximate Q-Learning

- Instead of a table, we have a parametrized Q function: $Q_\theta(s,a)$
  - Can be a linear function in features:

$$Q_\theta(s,a) = \theta_0 f_0(s,a) + \theta_1 f_1(s,a) + \cdots + \theta_n f_n(s,a)$$

  - Or a complicated neural net
- Learning rule:
  - Remember: $\text{target}(s') = R(s,a,s') + \gamma \max_{a'} Q_{\theta_k}(s',a')$
  - Update:

$$\theta_{k+1} \leftarrow \theta_k - \alpha \nabla_\theta \left[\frac{1}{2} (Q_\theta(s,a) - \text{target}(s'))^2 \right] \Big|_{\theta=\theta_k}$$

Fitted Q-iteration

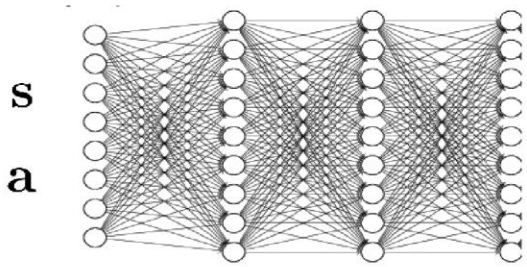
full fitted Q-iteration algorithm:

parameters

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy — dataset size N, collection policy

$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ — iterations K
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$ — gradient steps S



$Q_\phi(\mathbf{s}, \mathbf{a})$
parameters ϕ

Recap: Q-learning

full fitted Q-iteration algorithm:

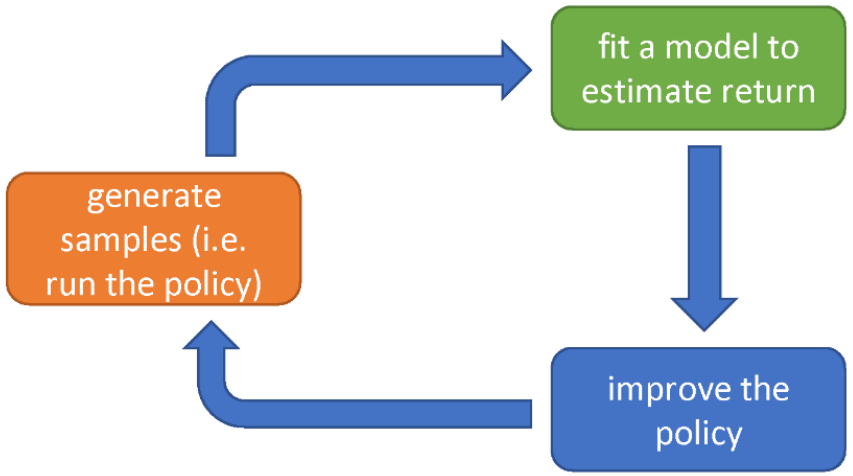
1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

$K\times$

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

$$Q_\phi(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}')$$



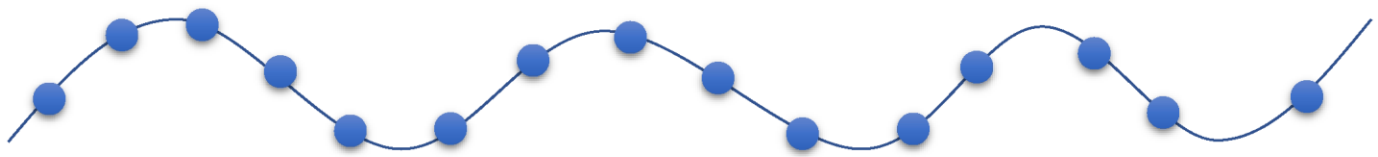
$$\mathbf{a} = \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a})$$

Correlated samples in online Q-learning

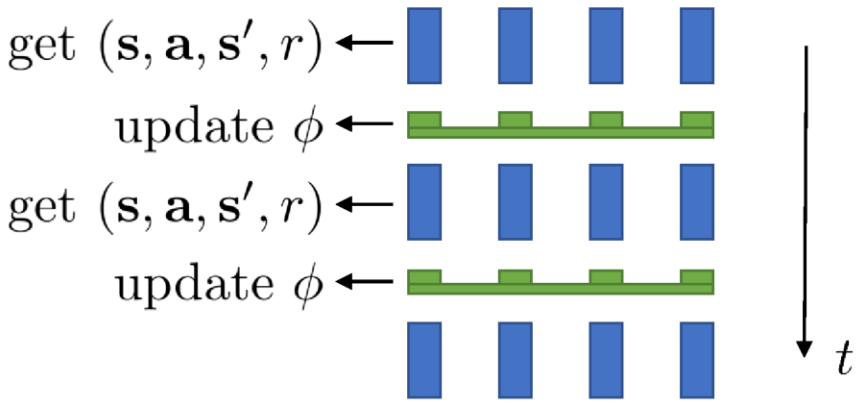
online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

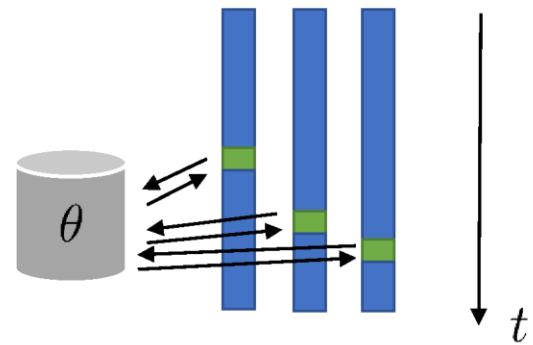
- sequential states are strongly correlated
- target value is always changing



synchronized parallel Q-learning



asynchronous parallel Q-learning



Another solution: replay buffers

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

special case with K = 1, and one gradient step

full fitted Q-iteration algorithm:

~~1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy~~

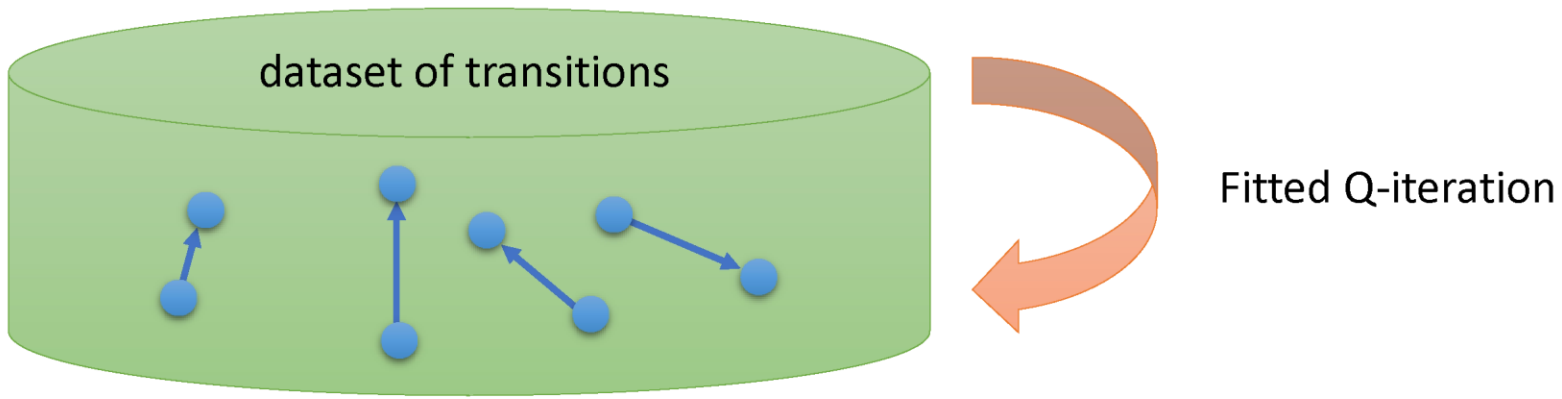
$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

any policy will work! (with broad support)

just load data from a buffer here

still use one gradient step



Another solution: replay buffers

Q-learning with a replay buffer:

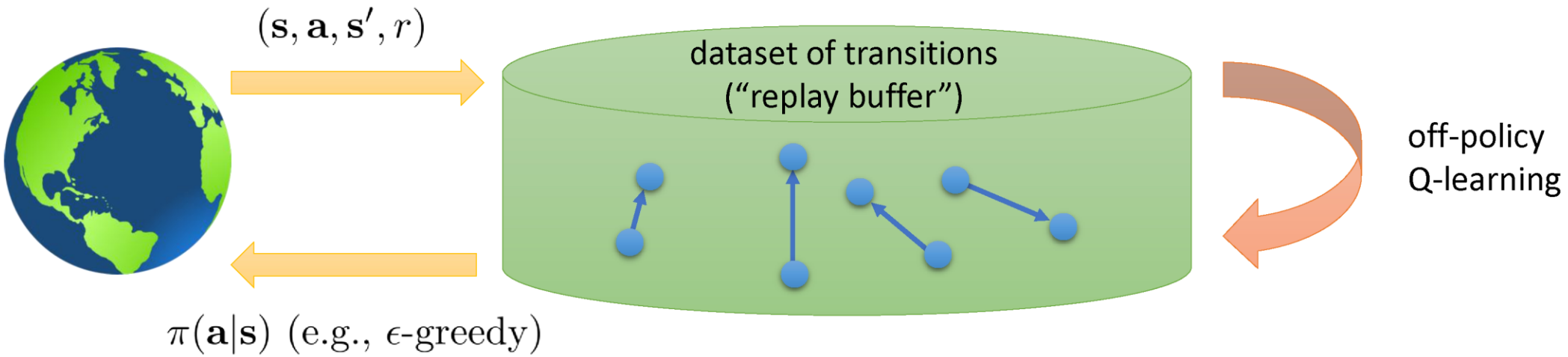
1. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
2. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

+ samples are no longer correlated

+ multiple samples in the batch (low-variance gradient)

but where does the data come from?

need to periodically feed the replay buffer…



Putting it together

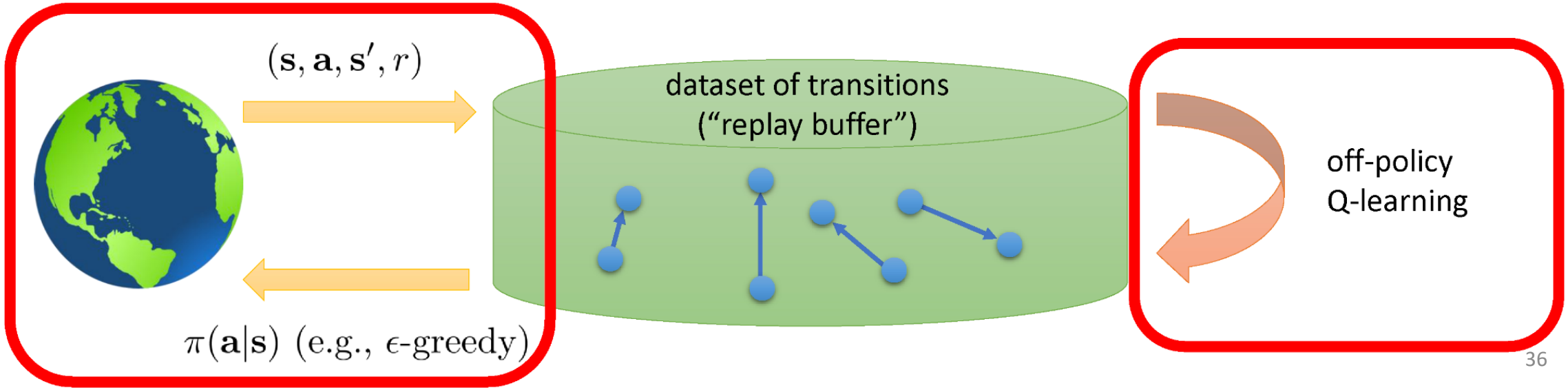
full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$K\times$

2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

K = 1 is common, though larger K more efficient



What's wrong?

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

~~these are correlated!~~

use replay buffer

Q-learning is *not* gradient descent!

$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

This is still a problem!

Q-Learning and Regression

full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$
2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

$K\times$

one gradient step, moving target

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

$K\times$

perfectly well-defined, stable regression

Q-Learning with target networks

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$
3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \underline{[r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)]})$

$N\times$

$K\times$

targets don't change in inner loop!

supervised regression

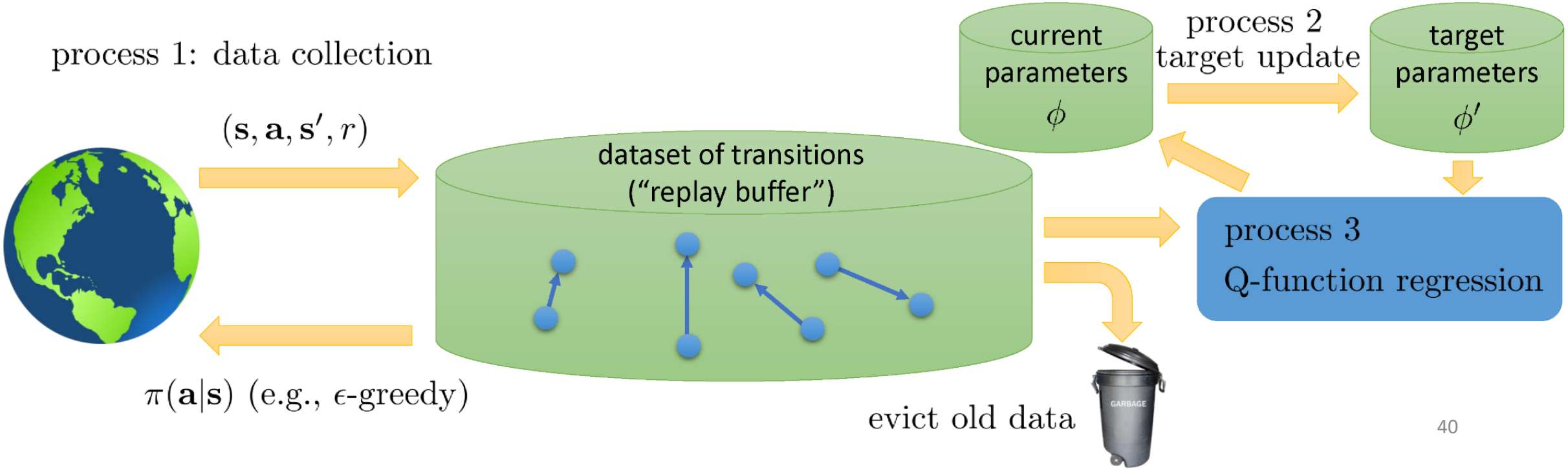
A more general view

Q-learning with replay buffer and target network:

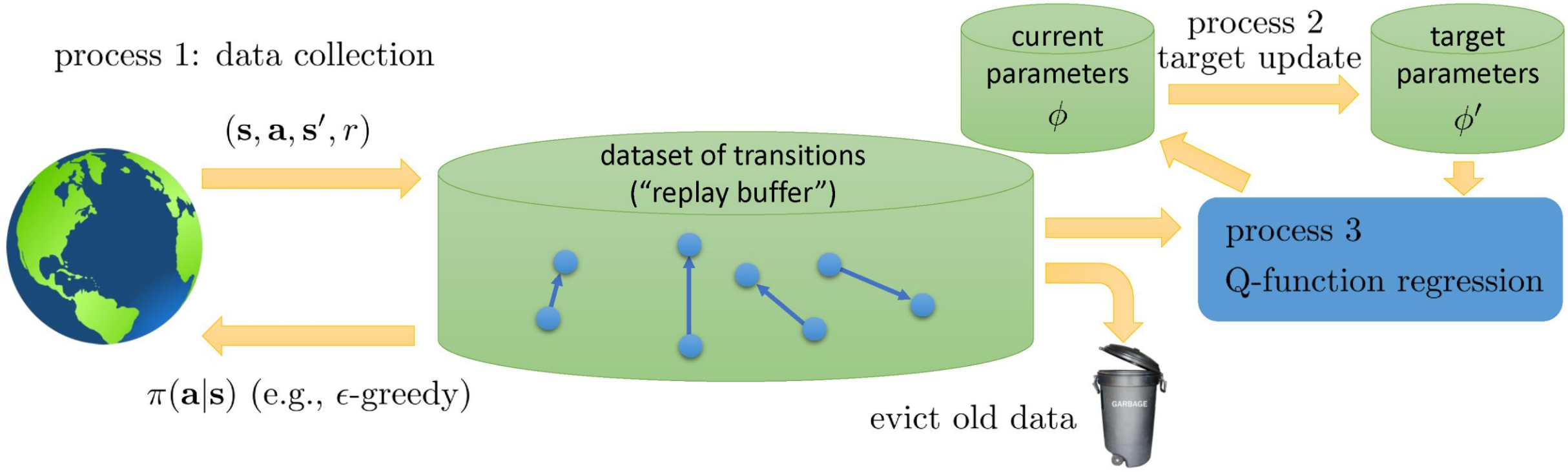
1. save target network parameters: $\phi' \leftarrow \phi$
2. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

$N\times$

$K\times$

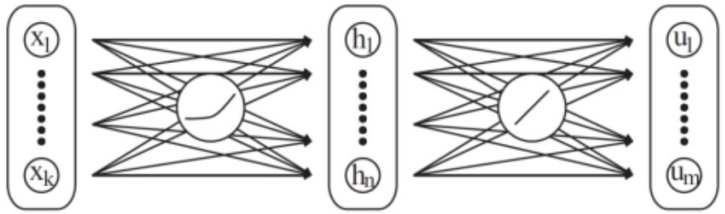


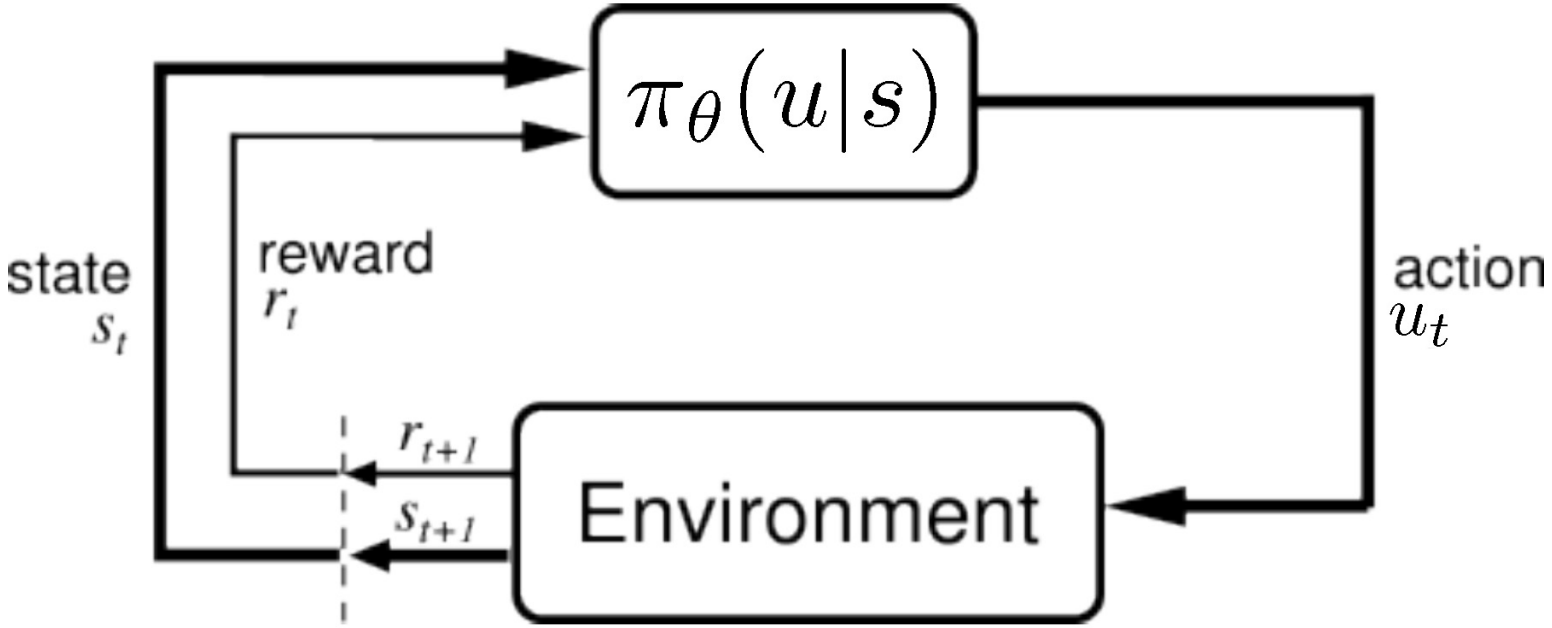
A more general view



- Online Q-learning (last lecture): evict immediately, process 1, process 2, and process 3 all run at the same speed
- DQN: process 1 and process 3 run at the same speed, process 2 is slow
- Fitted Q-iteration: process 3 in the inner loop of process 2, which is in the inner loop of process 1

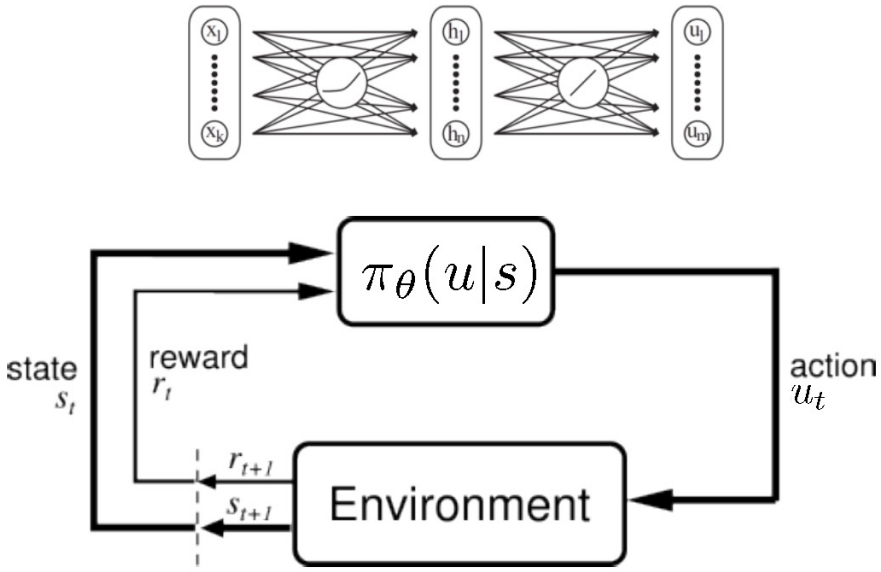
Policy Optimization





[Figure source: Sutton & Barto, 1998]

Policy Optimization



- Consider control policy parameterized by parameter vector θ

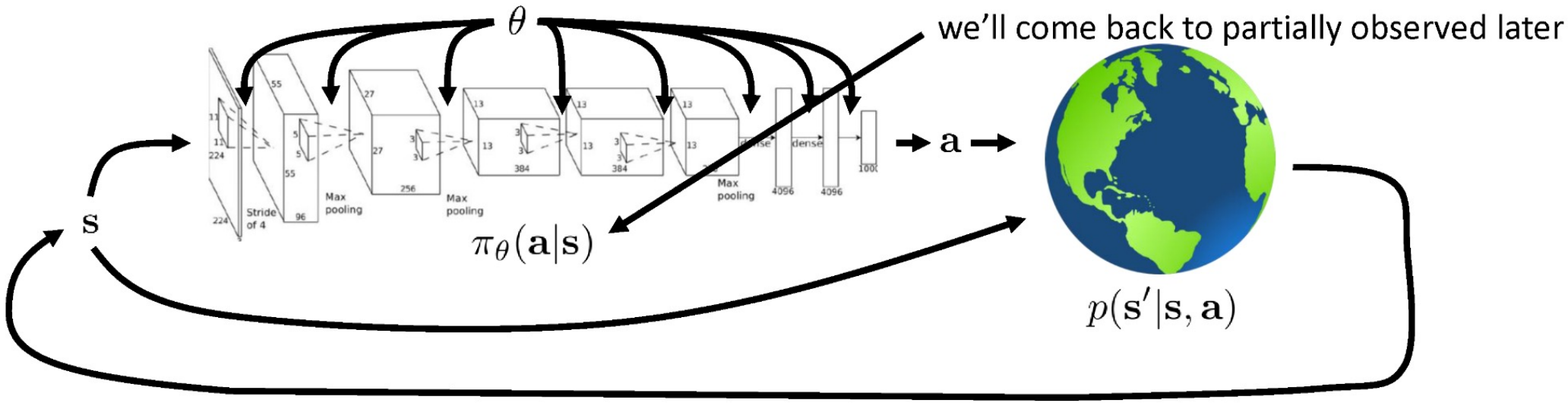
$$\max_\theta \quad \mathrm{E}[\sum_{t=0}^{H} R(s_t)|\pi_\theta]$$

- Stochastic policy class (smooths out the problem):

$\pi_\theta(u|s)$: probability of action u in state s

[Figure source: Sutton & Barto, 1998]

The goal of reinforcement learning

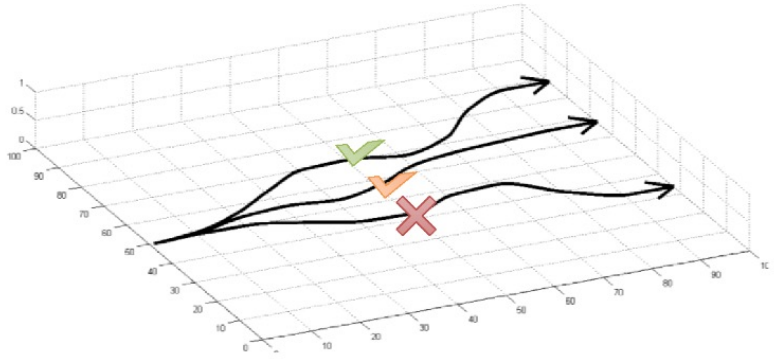


$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

Evaluating the objective

$$\theta^\star = \arg\max_\theta \underbrace{E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right]}_{J(\theta)}$$



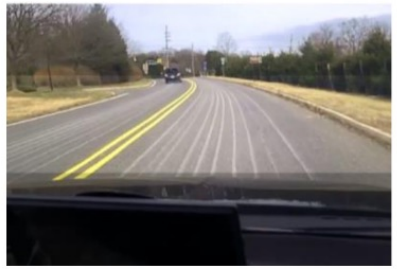
$$J(\theta) = E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right] \approx \frac{1}{N}\sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

sum over samples from π_θ

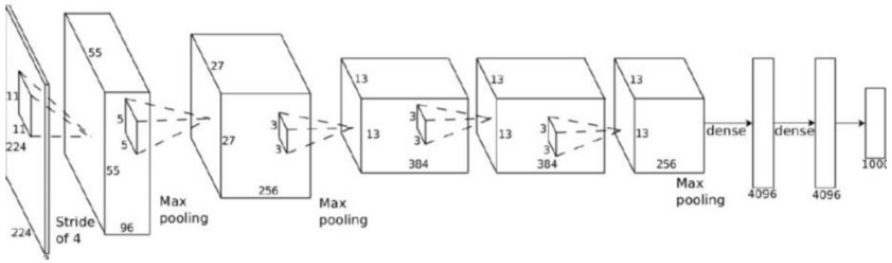
Comparison to maximum likelihood

$$\text{policy gradient:} \quad \nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\right)\left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t},\mathbf{a}_{i,t})\right)$$

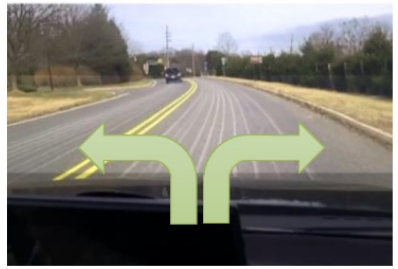
$$\text{maximum likelihood:} \quad \nabla_\theta J_{\mathrm{ML}}(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\right)$$



$\mathbf{s}_t$



$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$



$\mathbf{a}_t$

$\mathbf{s}_t$

$\mathbf{a}_t$

training data

supervised learning

$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

Direct policy differentiation

$$\theta^\star = \arg\max_\theta \underbrace{E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right]}_{J(\theta)}$$

a convenient identity

$$\pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) = \pi_\theta(\tau)\frac{\nabla_\theta \pi_\theta(\tau)}{\pi_\theta(\tau)} = \nabla_\theta \pi_\theta(\tau)$$

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[\underbrace{r(\tau)}_{\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)}] = \int \pi_\theta(\tau) r(\tau) d\tau$$

$$\nabla_\theta J(\theta) = \int \nabla_\theta \pi_\theta(\tau) r(\tau) d\tau = \int \pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) r(\tau) d\tau = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]$$

Direct policy differentiation

$$\theta^\star = \arg\max_\theta J(\theta)$$

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[r(\tau)]$$

$$\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]$$

$$\underbrace{\pi_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

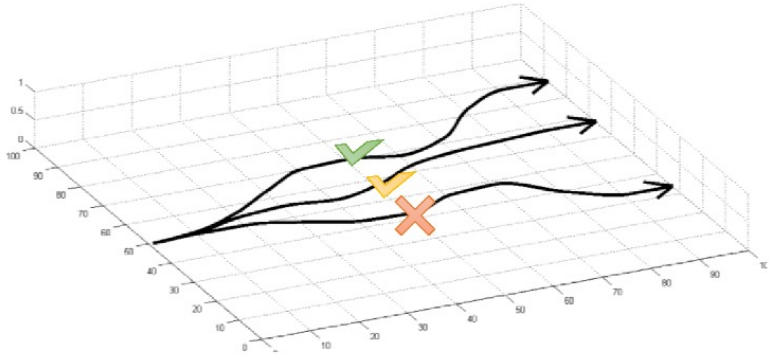
log of both sides

$$\log \pi_\theta(\tau) = \log p(\mathbf{s}_1) + \sum_{t=1}^{T} \log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \log p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\nabla_\theta \left[\cancel{\log p(\mathbf{s}_1)} + \sum_{t=1}^{T} \log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \cancel{\log p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)} \right]$$

$$\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta(\tau)} \left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

Evaluating the policy gradient



$$\text{recall: } J(\theta) = E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

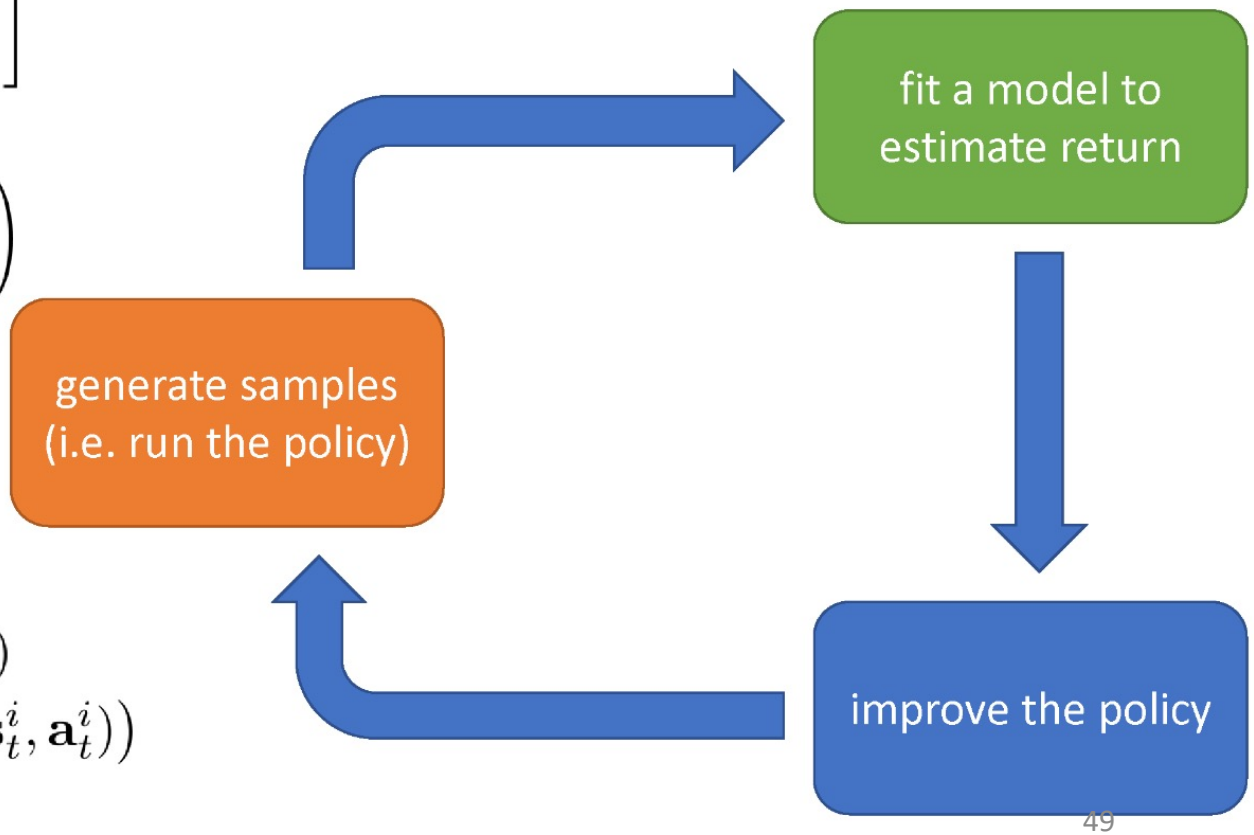
$$\nabla_\theta J(\theta) = E_{\tau \sim \pi_\theta(\tau)} \left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

$$\nabla_\theta J(\theta) \approx \underline{\frac{1}{N} \sum_{i=1}^{N}} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \underline{\left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)}$$

$$\underline{\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)}$$

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t | \mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i | \mathbf{s}_t^i) \right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} R_k$ $\qquad (G_t)$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

What did we just do?

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\right)\left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t},\mathbf{a}_{i,t})\right)$$

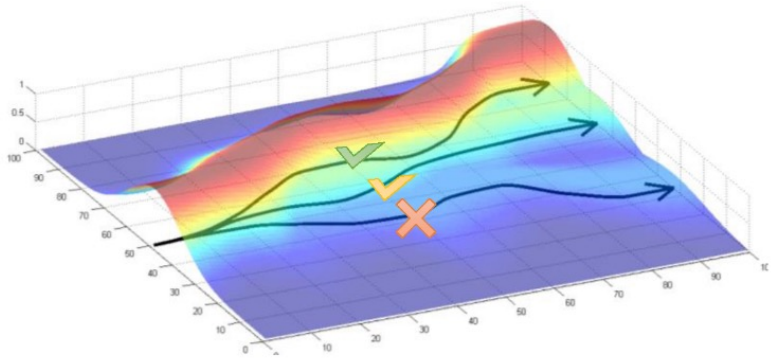
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\underbrace{\nabla_\theta \log \pi_\theta(\tau_i)}_{\sum_{t=1}^{T}\nabla_\theta \log_\theta \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})} r(\tau_i)$$

maximum likelihood: $\nabla_\theta J_{\mathrm{ML}}(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\nabla_\theta \log \pi_\theta(\tau_i)$

good stuff is made more likely

bad stuff is made less likely

simply formalizes the notion of “trial and error”!



REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i,\mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Дополнительные материалы

1. Coursera Reinforcement Learning Specialization
2. UC Berkeley CS 285 Deep Reinforcement Learning
3. MIPT Advanced Topics in Deep Reinforcement learning
4. Introduction to Reinforcement Learning with David Silver
5. Reinforcement Learning: An Introduction

В презентации использованы материалы из источников 2 и 4.