Часть 1: Нейрон и нейронная сеть

Романов Михаил, Игорь Слинько

Нервная система человека

Нервная система человека

Мозг
(обработка информации,
принятие решения)

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Сигналы от
органов чувств

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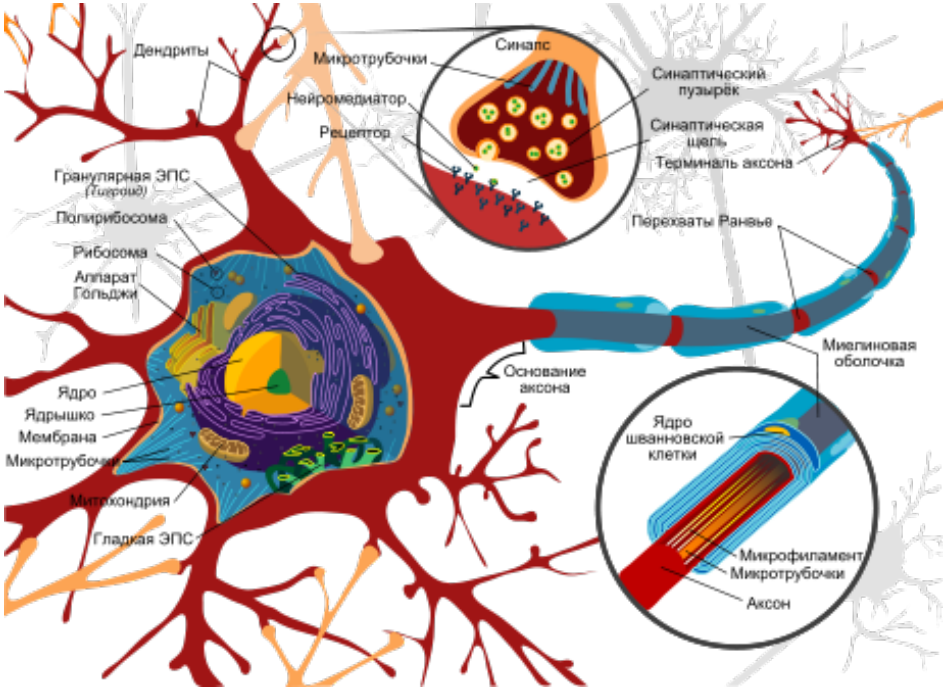
(обработка информации,
принятие решения)

↑ Сигналы от органов чувств

↓ Сигналы к мышцам

Биологический нейрон



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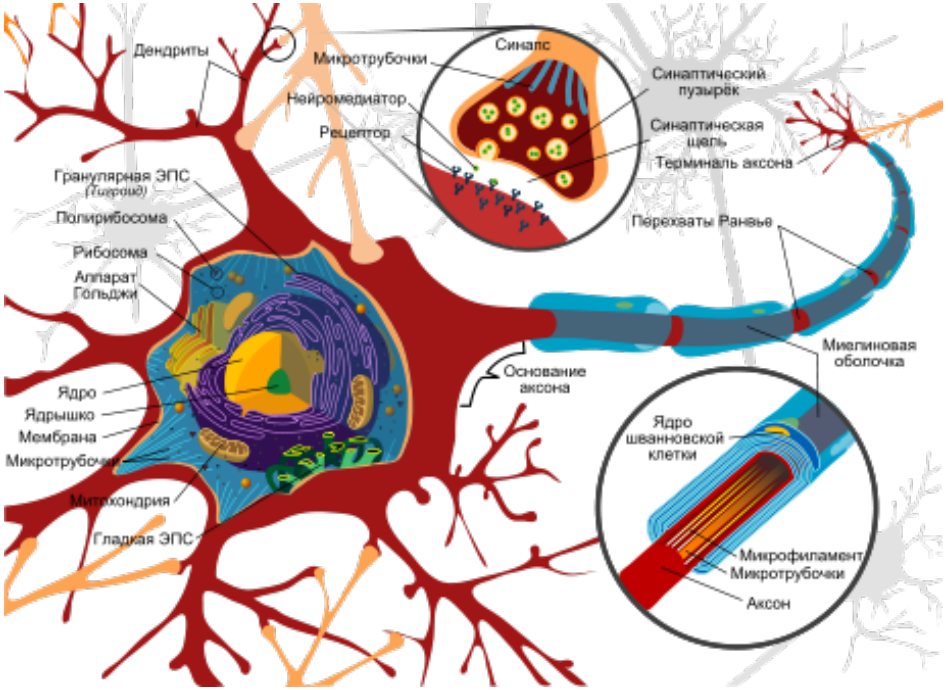
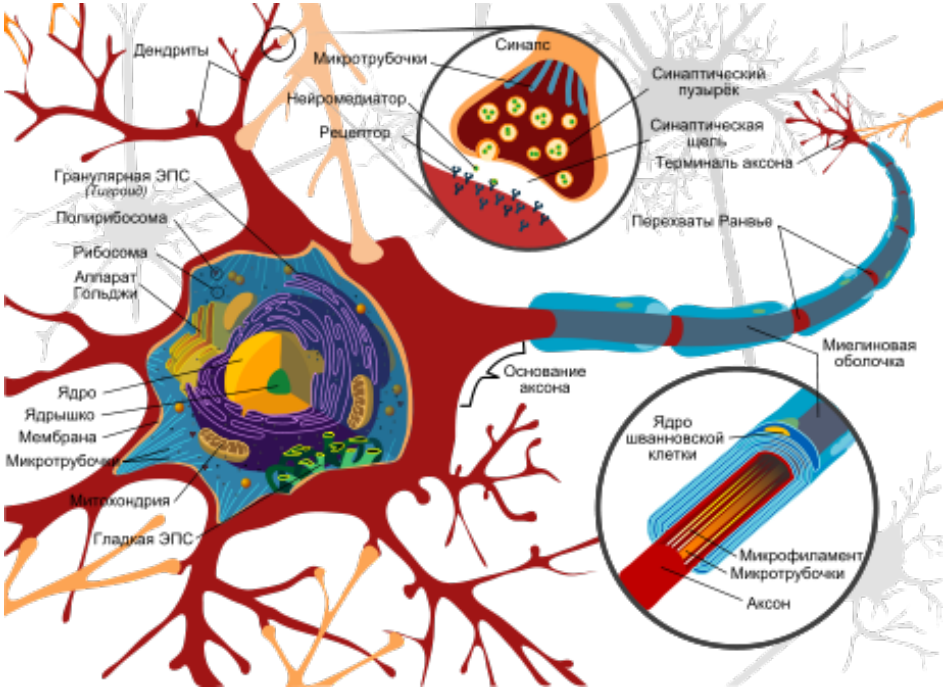
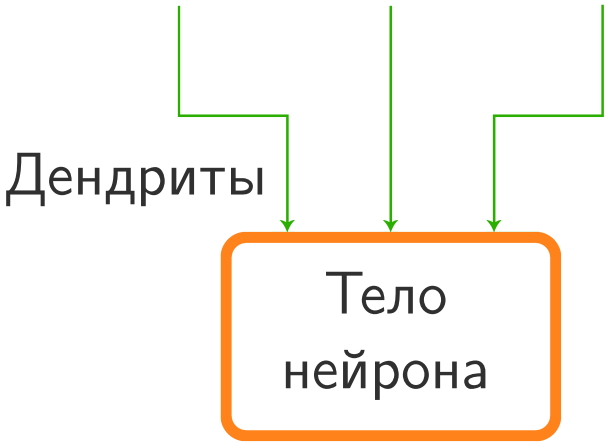


Тело нейрона

Биологический нейрон





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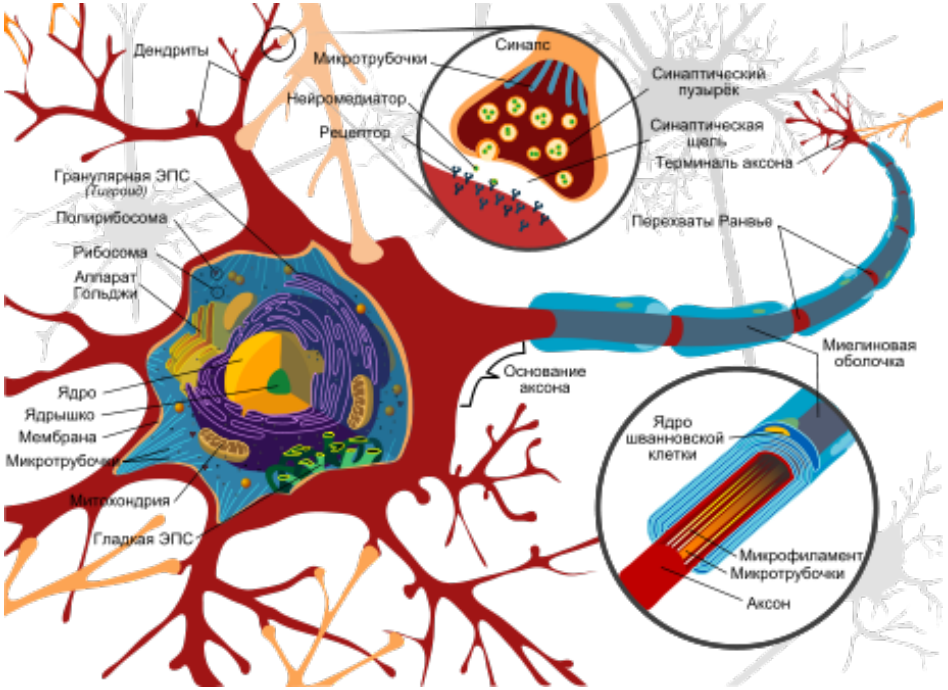

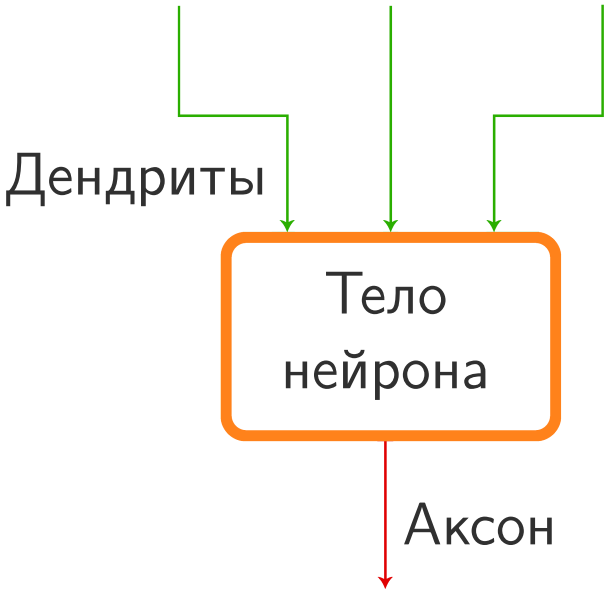

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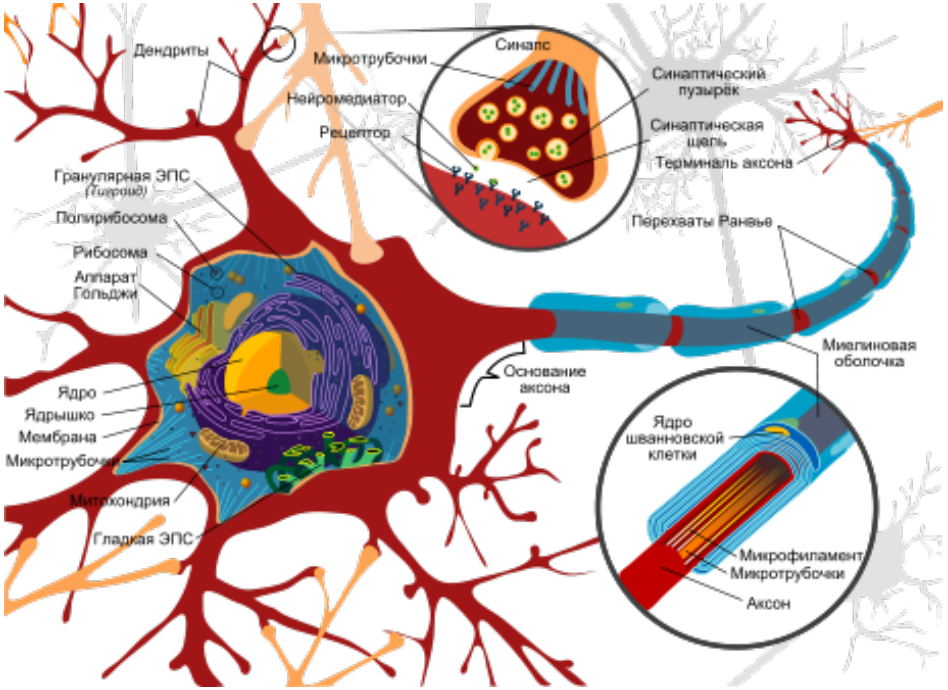
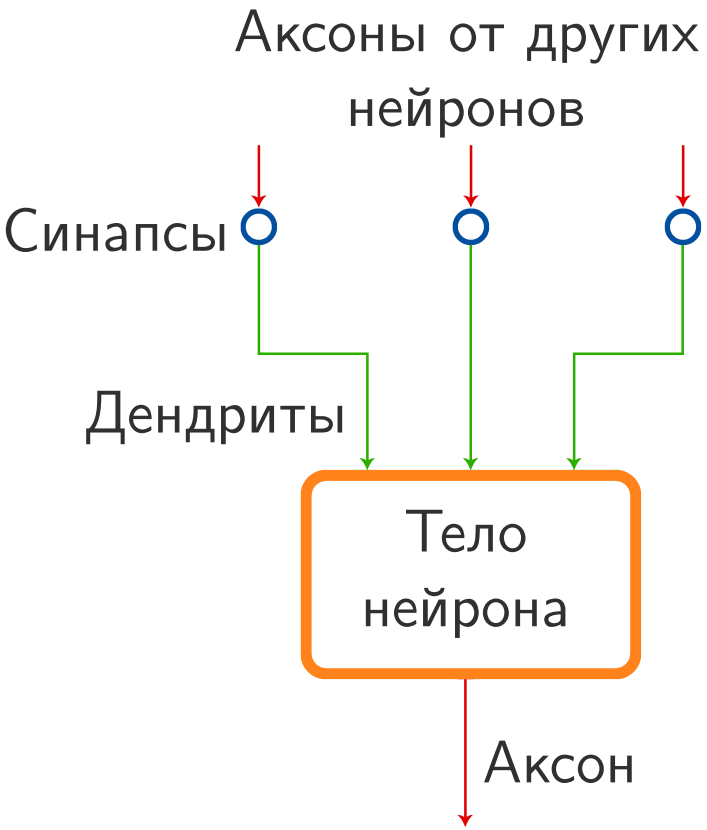
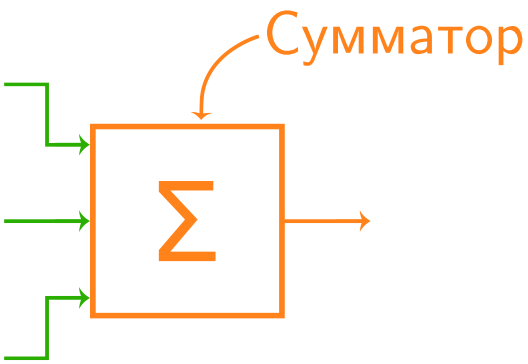




Математическая модель нейрона



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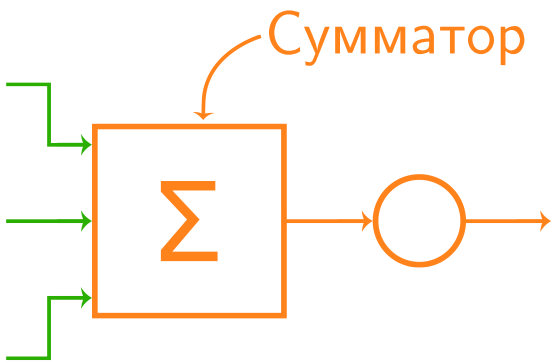

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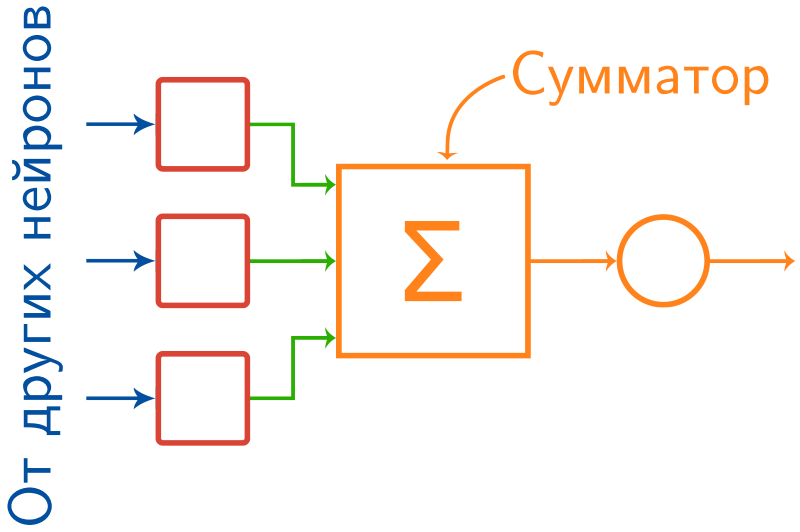
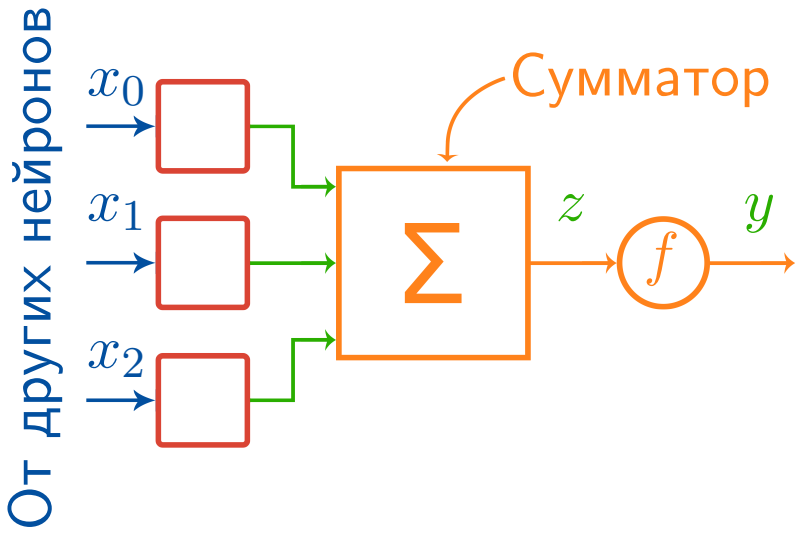


Математическая модель нейрона

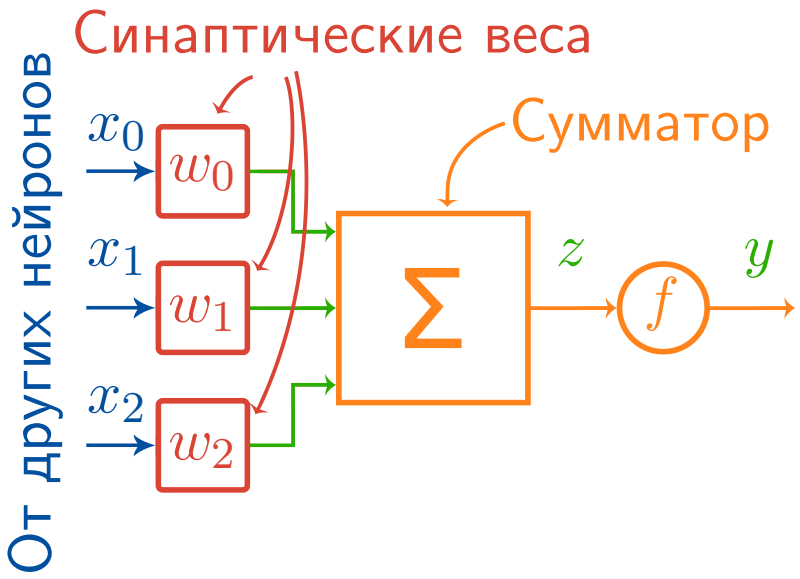


$y = f(z) = f(x_0 + x_1 + x_2)$

$f(z)$ – функция активации

x_0, x_1, x_2 – входные значения

Математическая модель нейрона



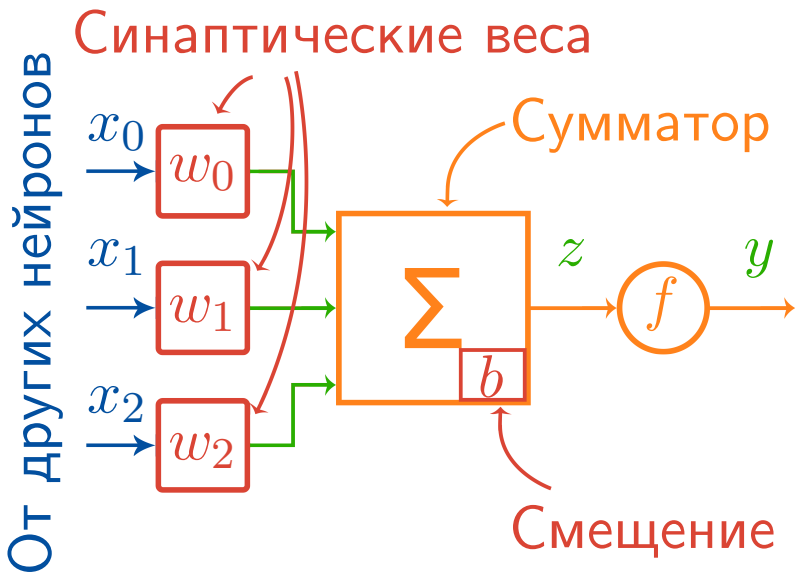
$y = f(z) = f(w_0 x_0 + w_1 x_1 + w_2 x_2)$

$f(z)$ – функция активации

x_0, x_1, x_2 – входные значения

w_0, w_1, w_2 – веса

Математическая модель нейрона



$y = f(z) = f(w_0 x_0 + w_1 x_1 + w_2 x_2 + b)$

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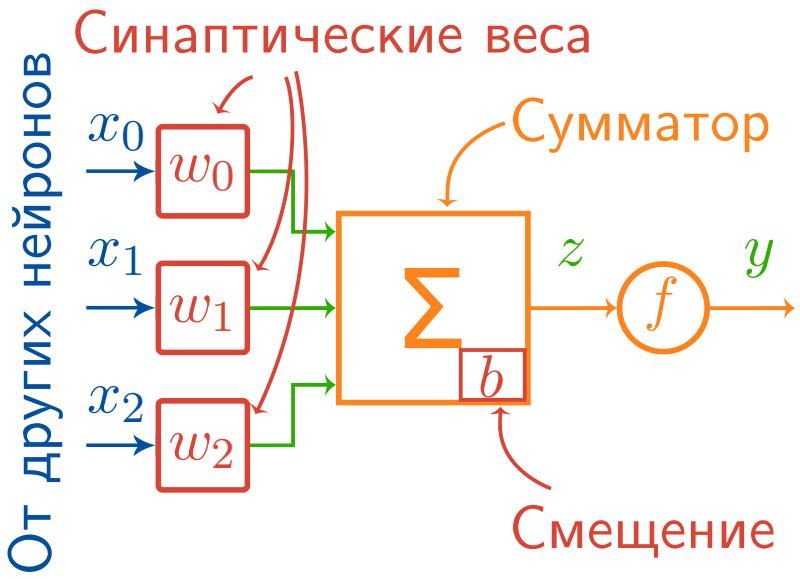
x_0, x_1, x_2 – входные значения

w_0, w_1, w_2 – веса

b – смещение

Математическая модель нейрона



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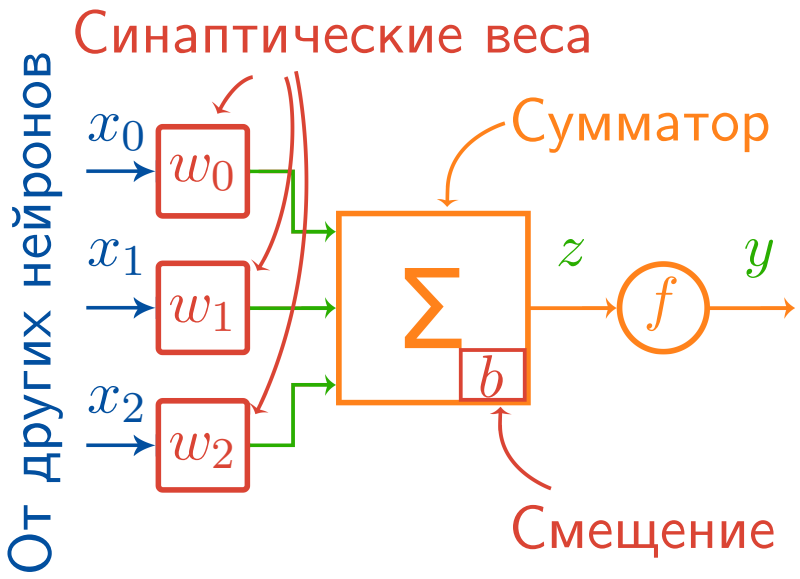
$$y = f(z) = f\left(\sum_{i=1}^{n} w_i x_i + b\right)$$

w_i, b – настраиваемые параметры

Математическая модель нейрона



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$y = f(z) = f(\langle \mathbf{w}, \mathbf{x} \rangle + b)$

$\langle \mathbf{w}, \mathbf{x} \rangle$ – скалярное произведение

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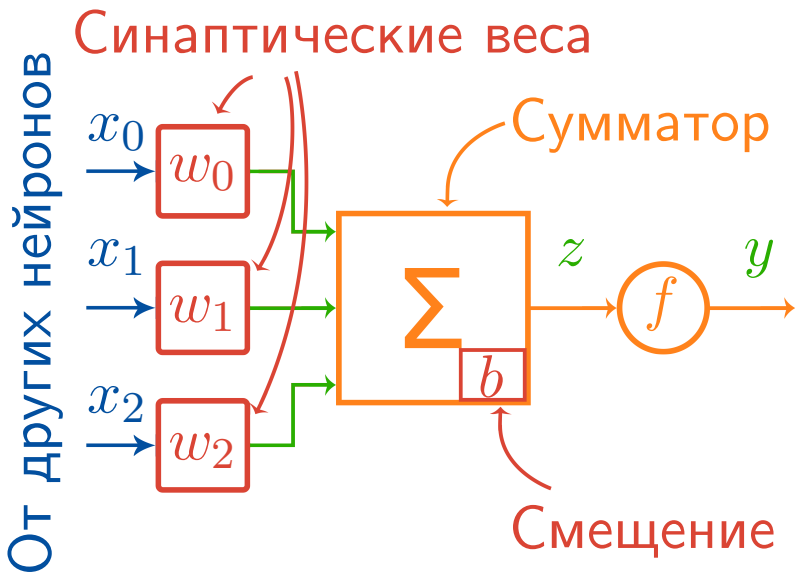

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Пороговая функция активации:

$$f(z) = \begin{cases} 0, & \text{if } z \leqslant 0 \\ 1, & \text{if } z > 0 \end{cases}$$

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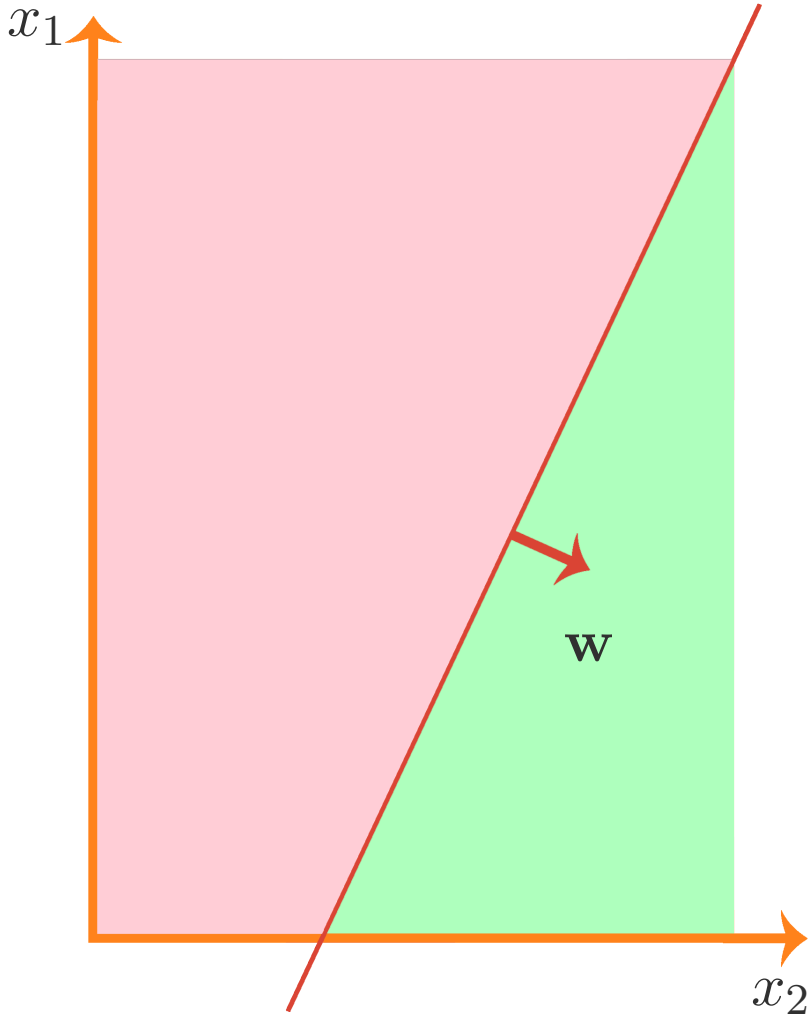
Разделяющая поверхность



Область положительных значений:
$f(\langle \mathbf{w}, \mathbf{x} \rangle + b) > \frac{1}{2}$
Если функия пороговая, то:
$\langle \mathbf{w}, \mathbf{x} \rangle + b > 0$
Разделябщая поверхность:
$\langle \mathbf{w}, \mathbf{x} \rangle + b = 0$

Функции активации (передаточные функции)

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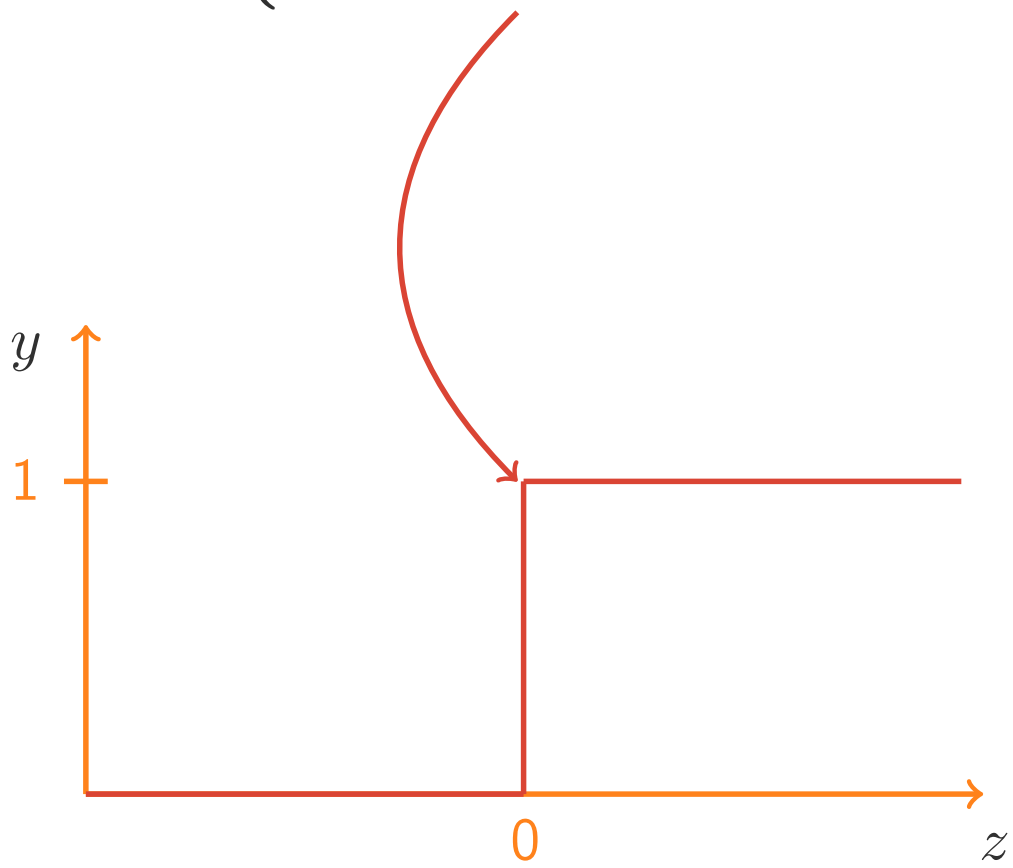

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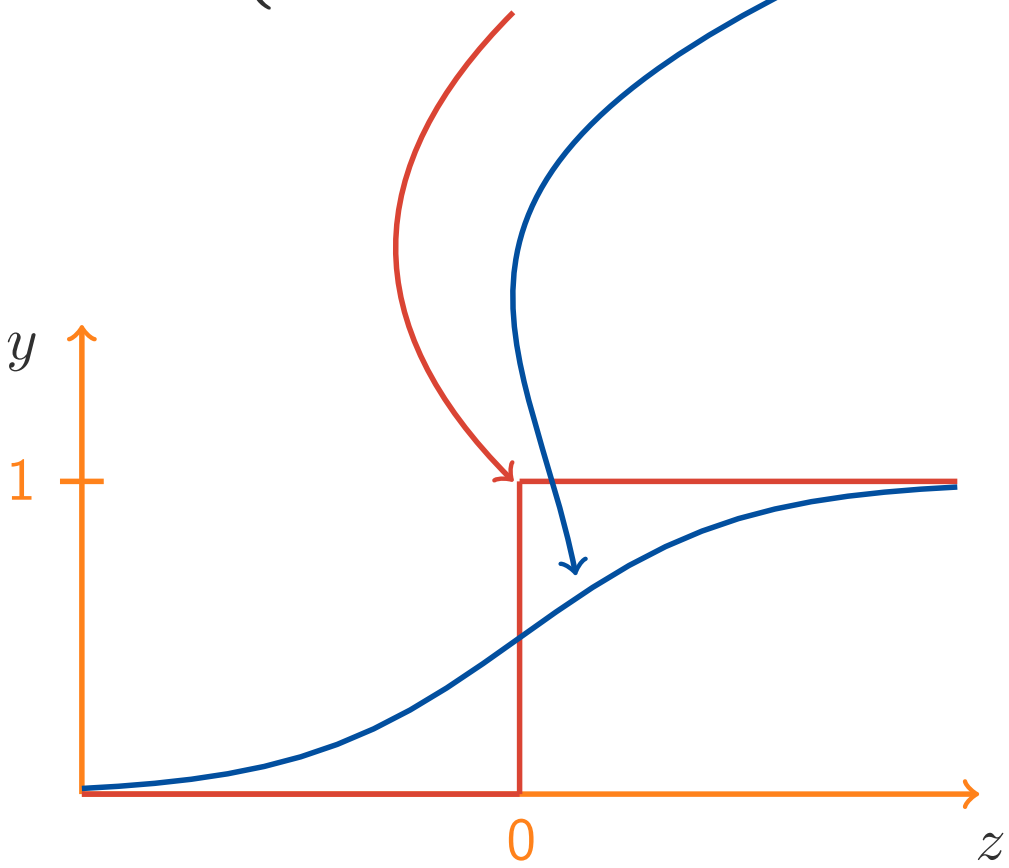
Сигмоида:

$$\sigma(z) = \frac{1}{1+e^{-z}}$$



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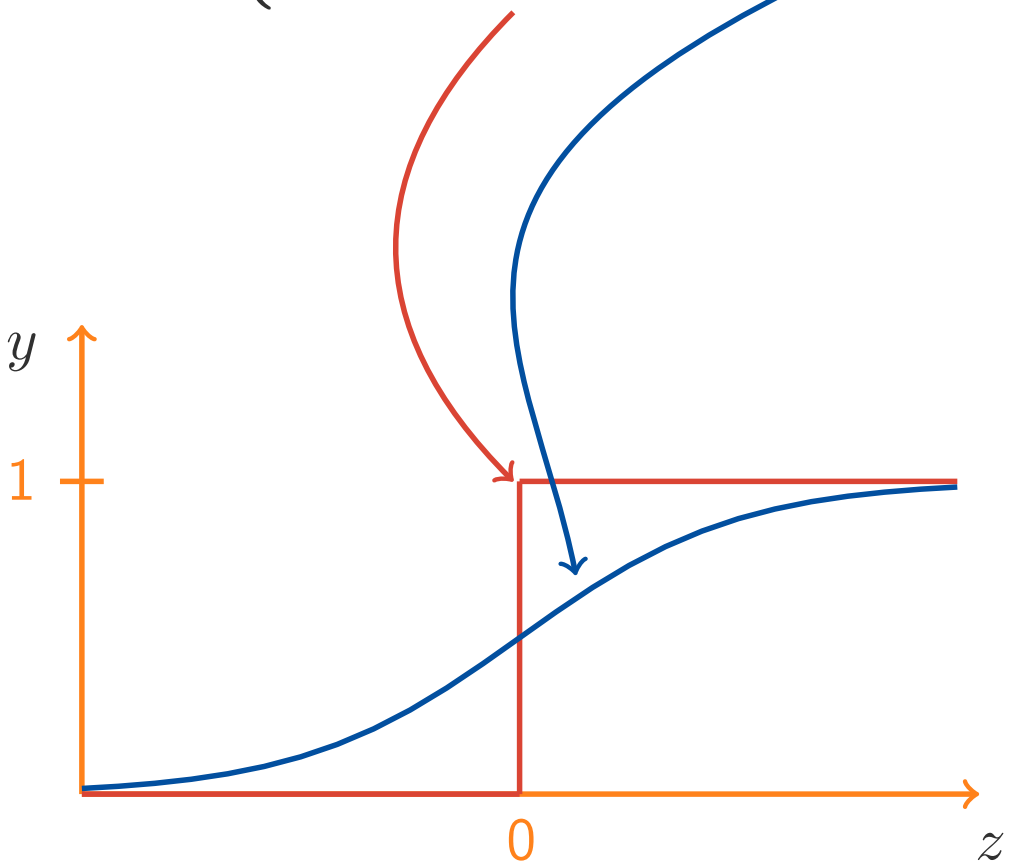

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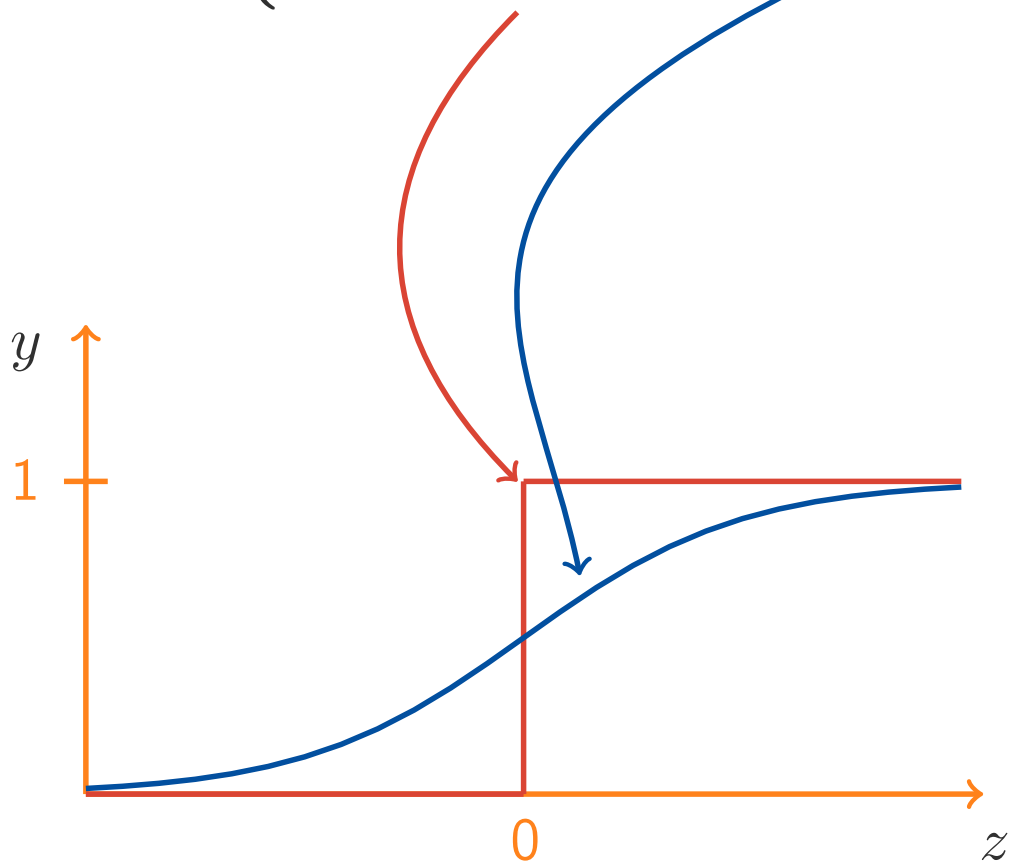

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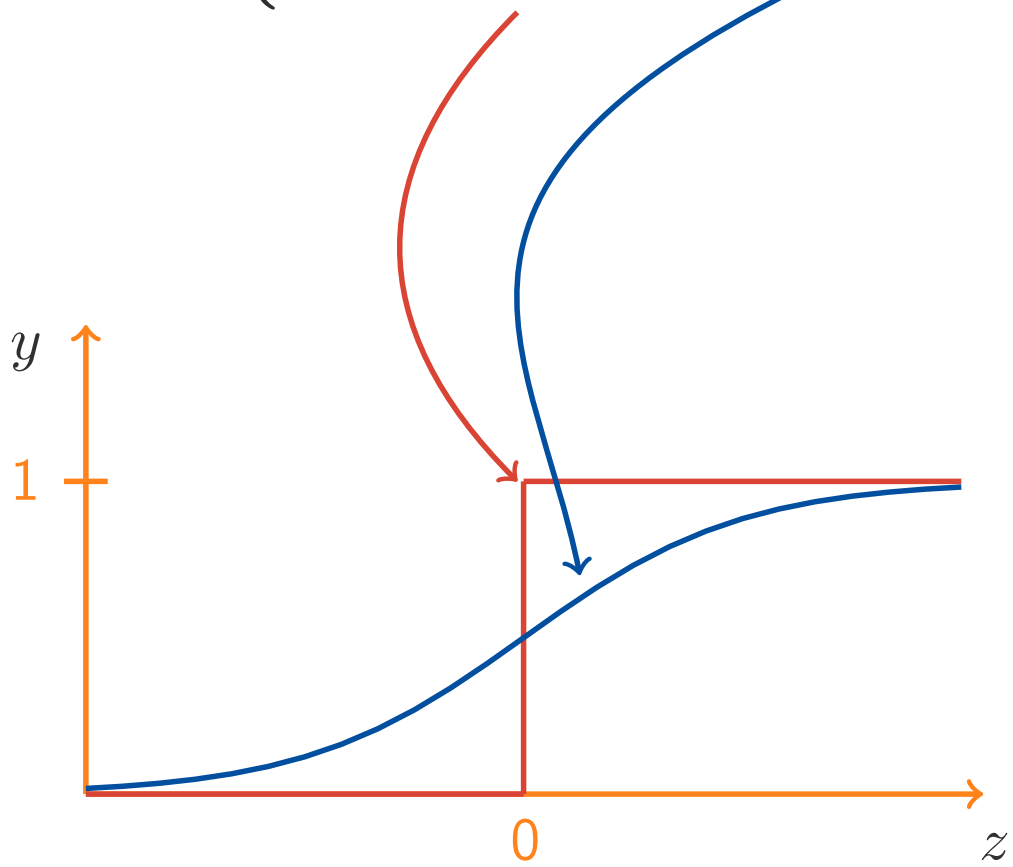

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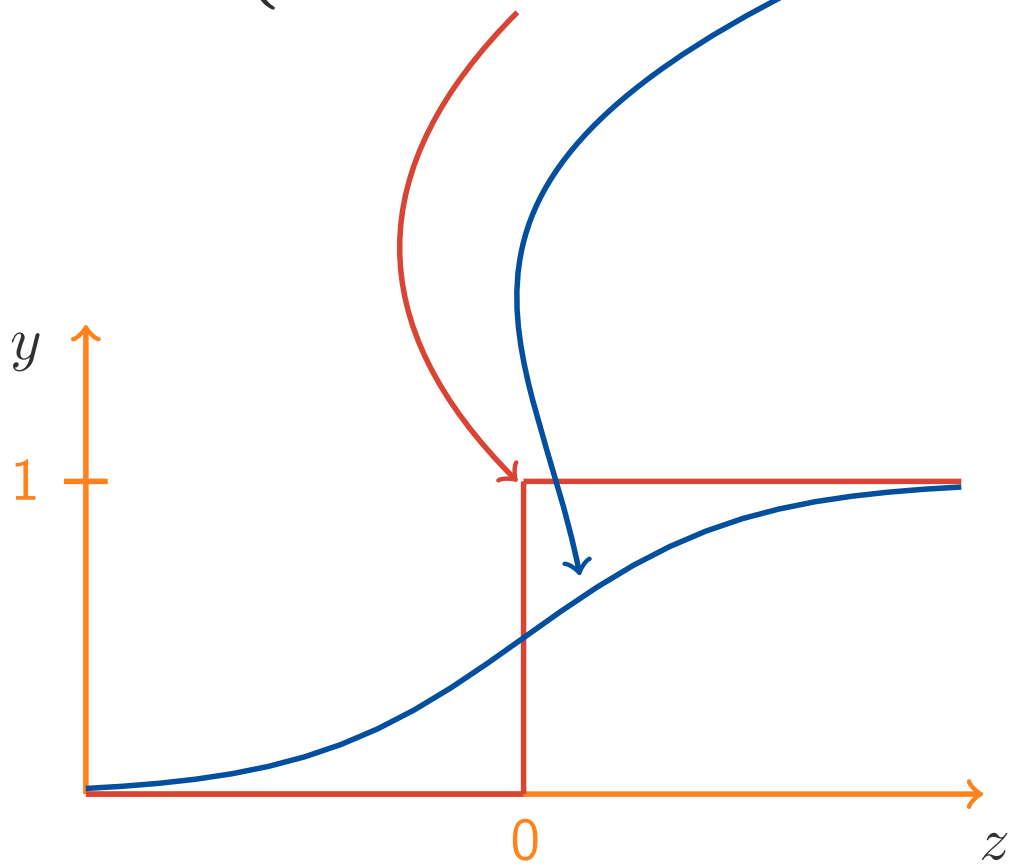

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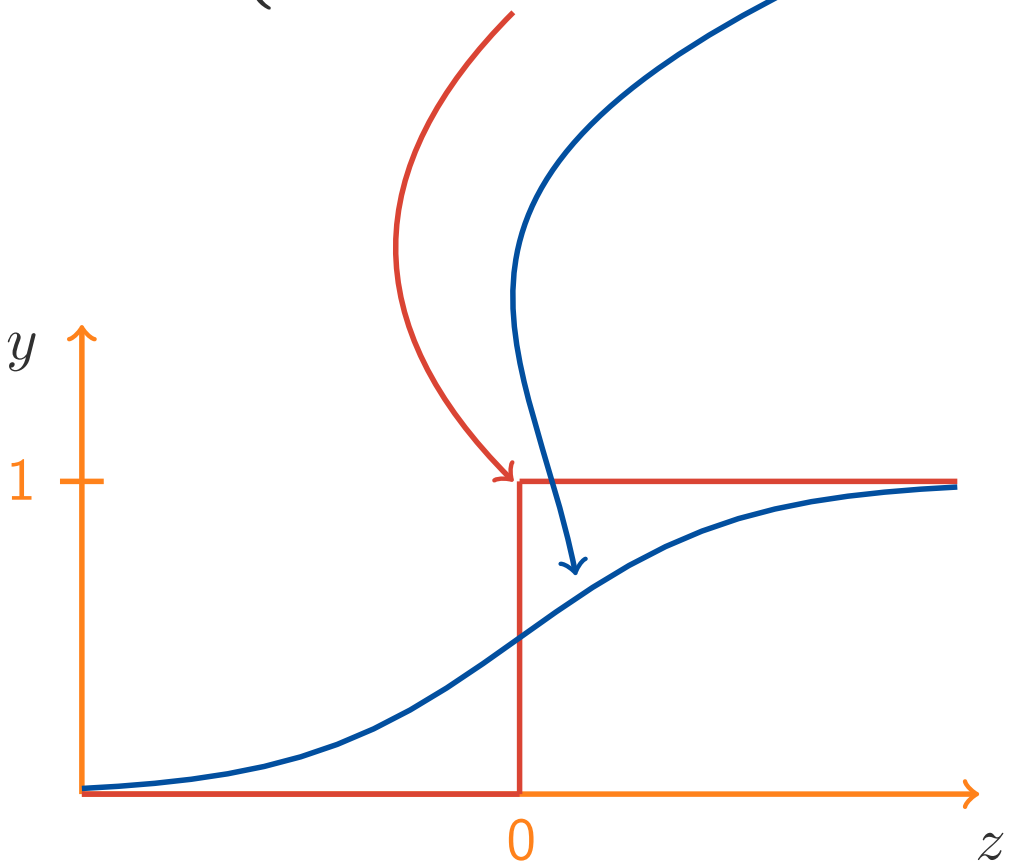

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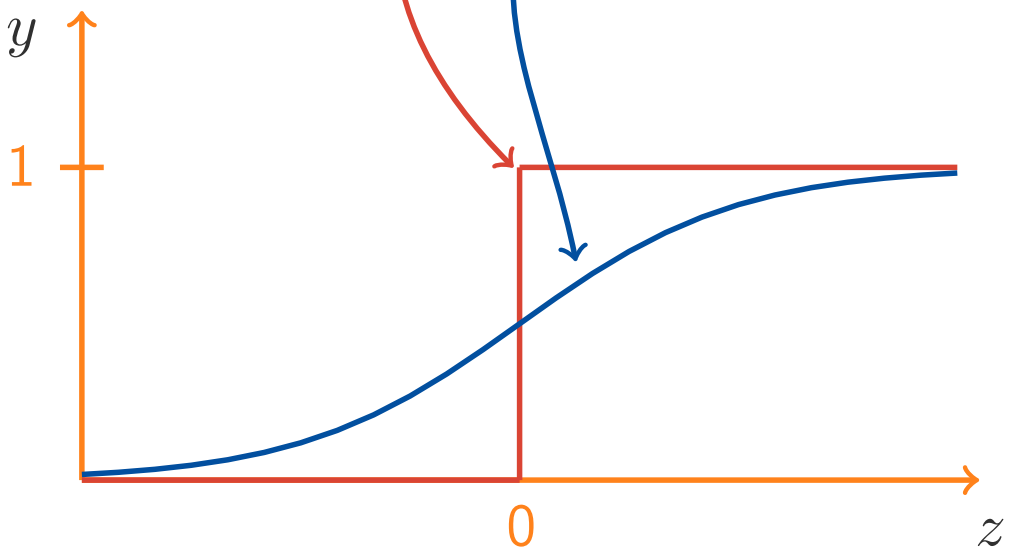

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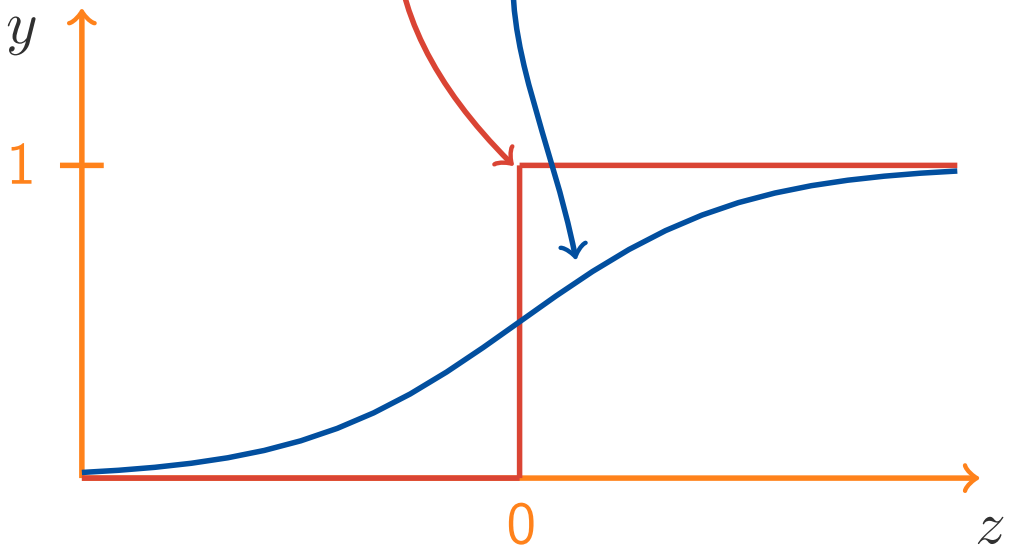

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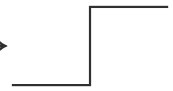
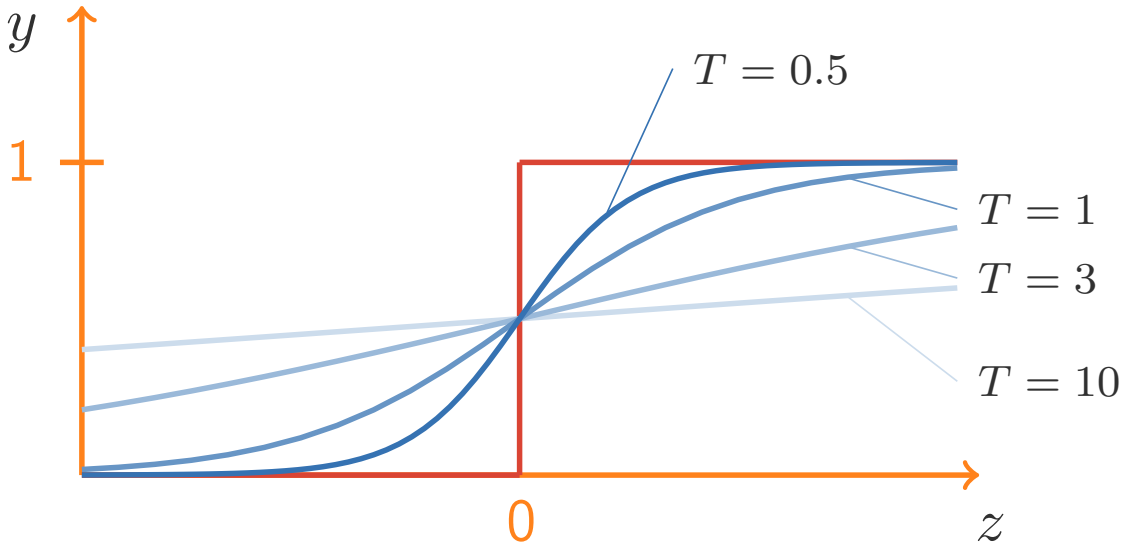
при $T \to 0$ $\sigma(z) \to$ 



Оператор “НЕ”

$\overline{x}$

x (input)	$\overline{x}$ (output)
0	1
1	0

Задача: сделать нейрон, который выполняет операцию отрицания

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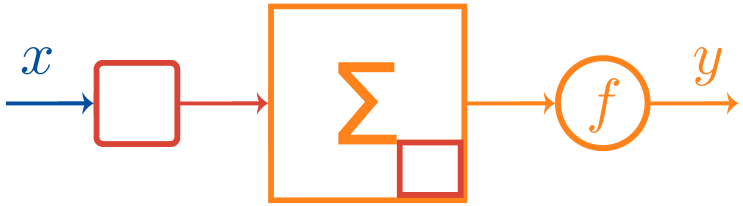

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$\overline{x}$

x (input)	$\overline{x}$ (output)
0	1
1	0

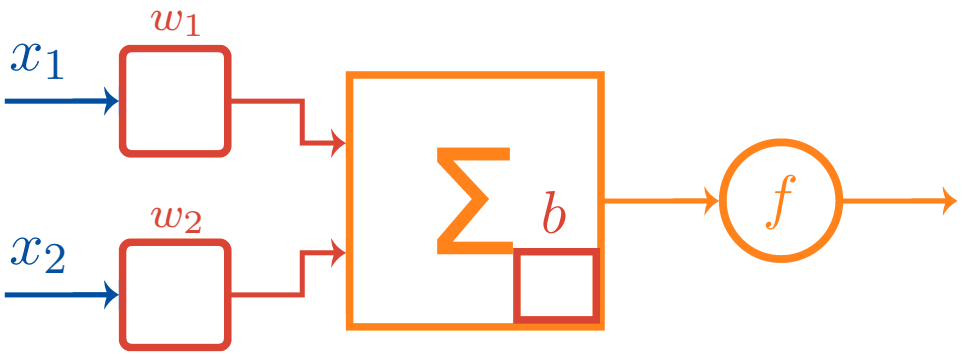
Задача: сделать нейрон, который выполняет операцию отрицания

Оператор “И”

$x_1 \ \& \ x_2$

x_1 (input)	x_2 (input)	$x_1 \ \& \ x_2$ (output)
1	1	1
1	0	0
0	1	0
0	0	0

Оператор “И”



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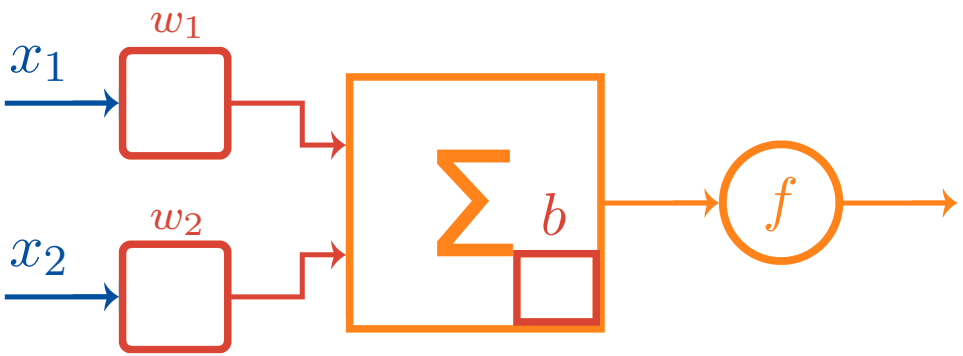
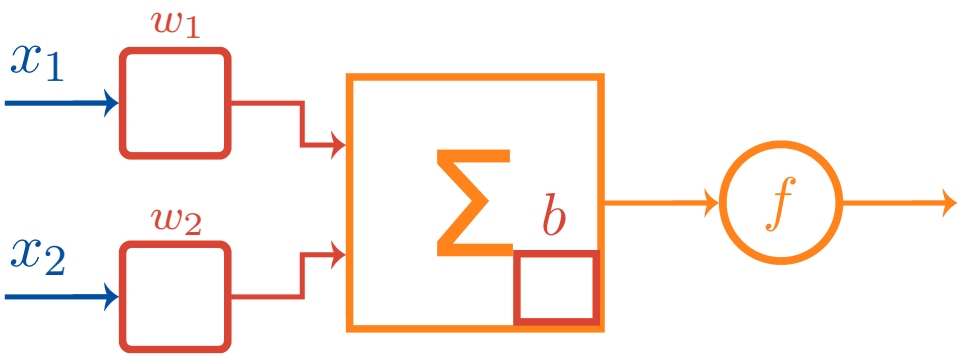


$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

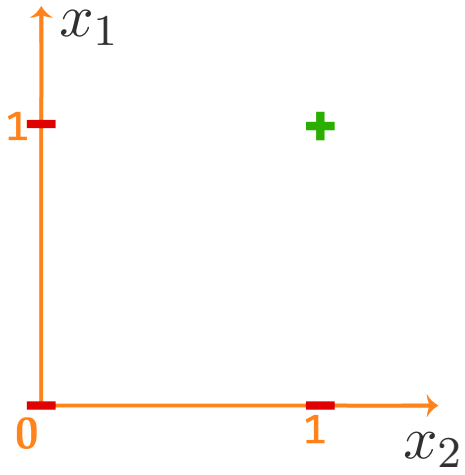
Оператор “И”

$x_1 \ \& \ x_2$

x_1 (input)	x_2 (input)	$x_1 \ \& \ x_2$ (output)
1	1	1
1	0	0
0	1	0
0	0	0



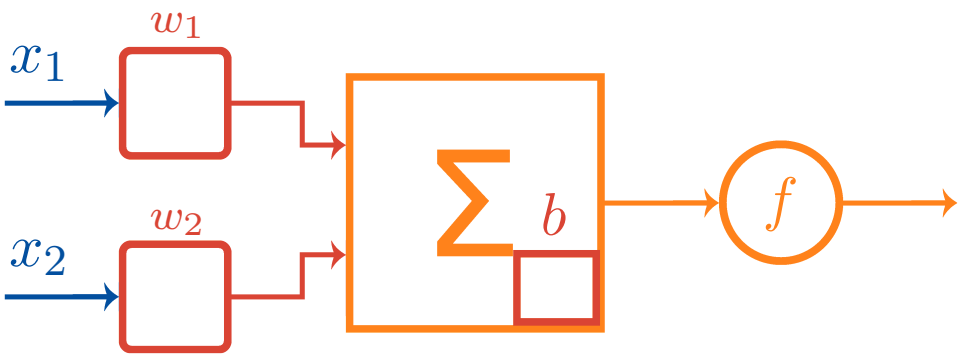
$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



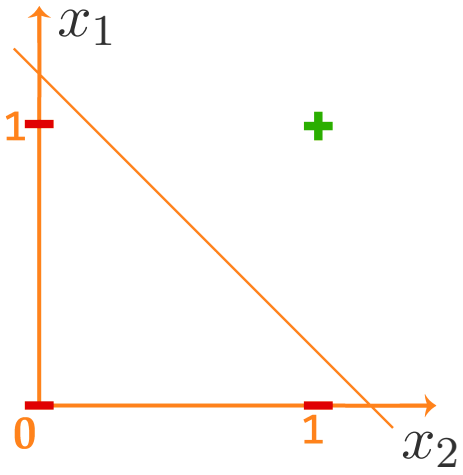
Оператор “И”

$x_1 \ \& \ x_2$

x_1 (input)	x_2 (input)	$x_1 \ \& \ x_2$ (output)
1	1	1
1	0	0
0	1	0
0	0	0



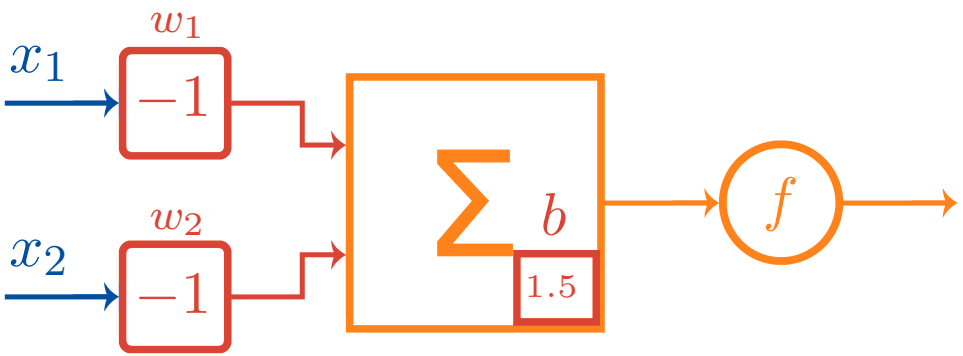
$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



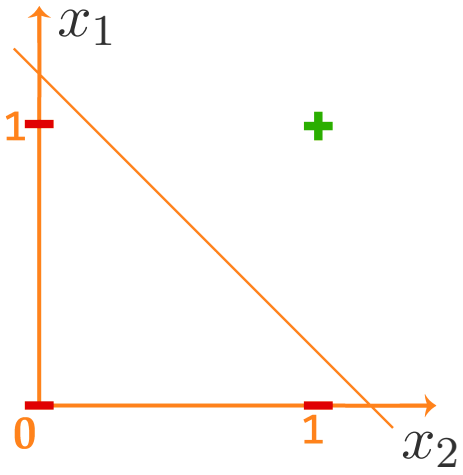
Оператор “И”

$x_1 \ \& \ x_2$

x_1 (input)	x_2 (input)	$x_1 \ \& \ x_2$ (output)
1	1	1
1	0	0
0	1	0
0	0	0



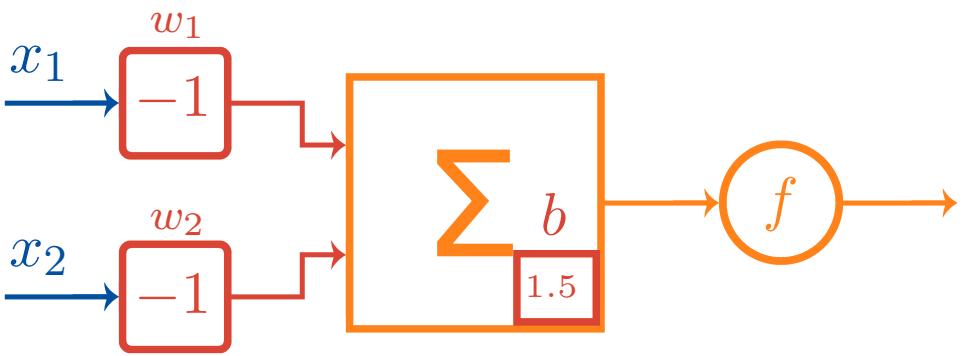
$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



Оператор “И”

$x_1 \ \& \ x_2$

x_1 (input)	x_2 (input)	$x_1 \ \& \ x_2$ (output)
1	1	1
1	0	0
0	1	0
0	0	0



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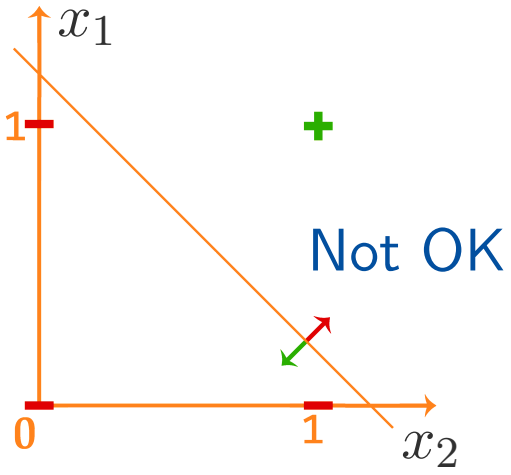
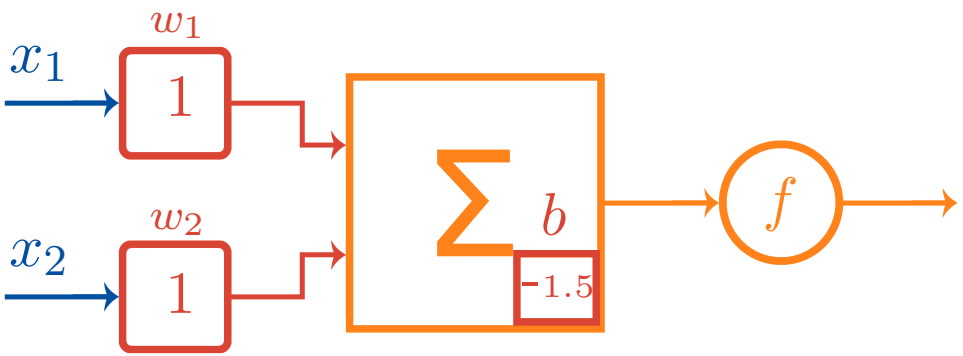


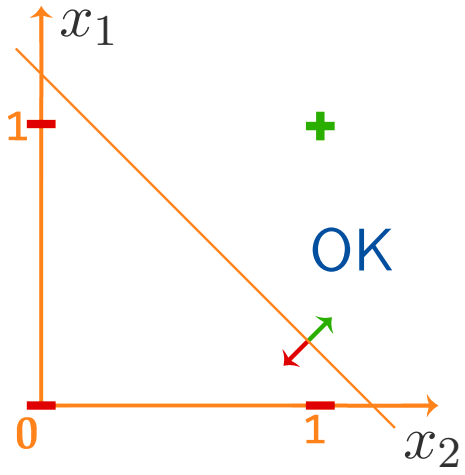
Оператор “И”

$x_1 \ \& \ x_2$

x_1 (input)	x_2 (input)	$x_1 \ \& \ x_2$ (output)
1	1	1
1	0	0
0	1	0
0	0	0



$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

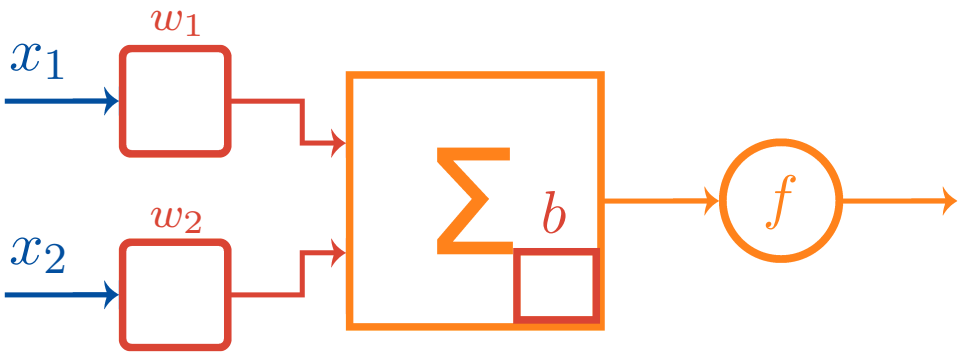


Оператор “ИЛИ”

$$x_1 \mid\mid x_2$$

x_1 (input)	x_2 (input)	$x_1 \mid\mid x_2$ (output)
1	1	1
1	0	1
0	1	1
0	0	0

Оператор “ИЛИ”



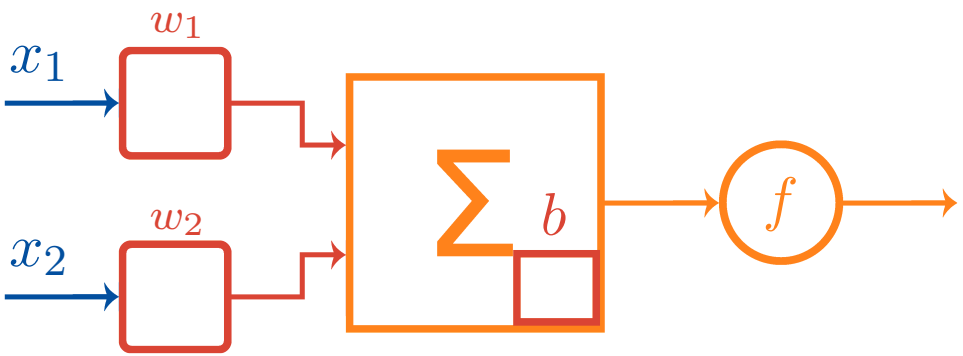
$$x_1 \mid\mid x_2$$

x_1 (input)	x_2 (input)	$x_1 \mid\mid x_2$ (output)
1	1	1
1	0	1
0	1	1
0	0	0

Оператор “ИЛИ”

$$x_1 \mid\mid x_2$$

x_1 (input)	x_2 (input)	$x_1 \mid\mid x_2$ (output)
1	1	1
1	0	1
0	1	1
0	0	0



$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b \geqslant 0 \\ 0w_1 + 1w_2 + b \geqslant 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

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$$x_1 \mid\mid x_2$$

x_1 (input)	x_2 (input)	$x_1 \mid\mid x_2$ (output)
1	1	1
1	0	1
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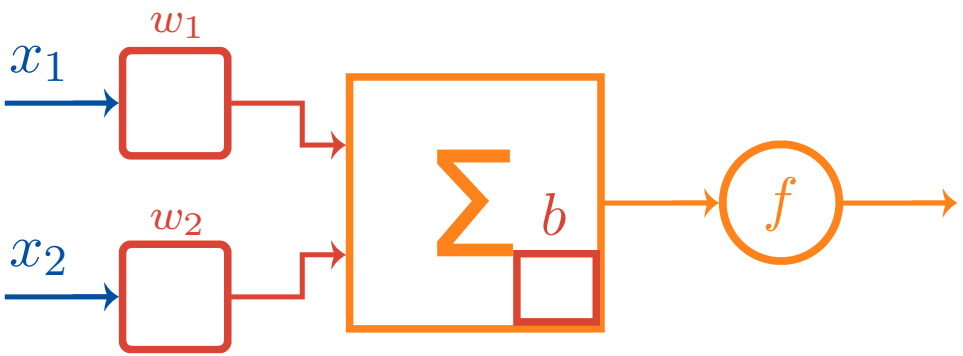
0	0	0



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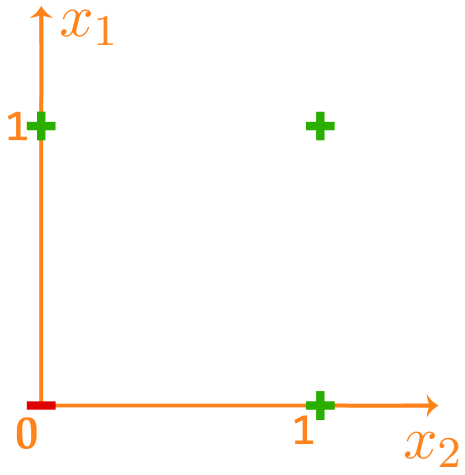
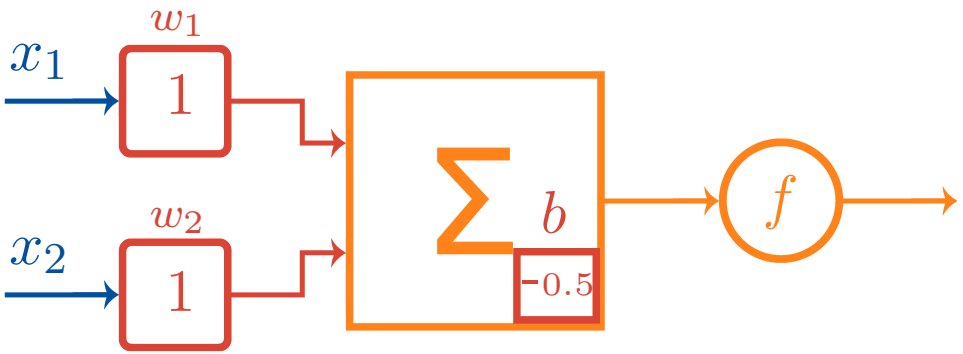


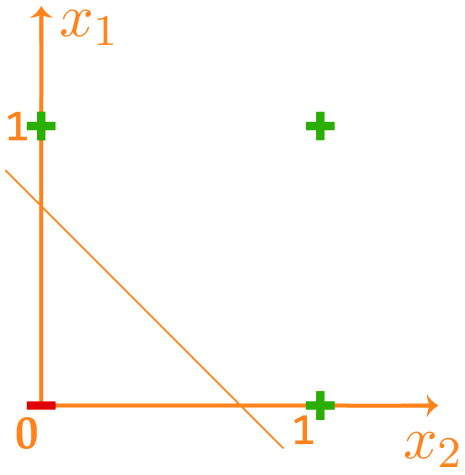
Оператор “ИЛИ”

$$x_1 \mid\mid x_2$$

x_1 (input)	x_2 (input)	$x_1 \mid\mid x_2$ (output)
1	1	1
1	0	1
0	1	1
0	0	0



$$\begin{cases} 1w_1 + 1w_2 + b \geqslant 0 \\ 1w_1 + 0w_2 + b \geqslant 0 \\ 0w_1 + 1w_2 + b \geqslant 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



Оператор “Исключающее ИЛИ”

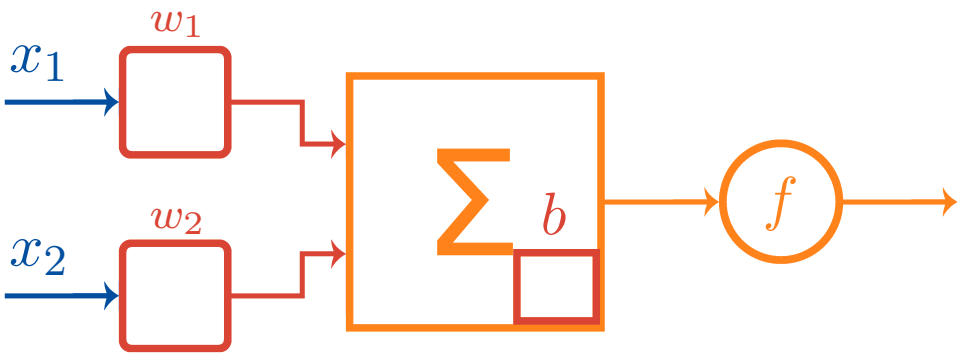
$x_1 \oplus x_2$

x_1 (input)	x_2 (input)	$x_1 \oplus x_2$ (output)
1	1	0
1	0	1
0	1	1
0	0	0

Оператор “Исключающее ИЛИ”

$$x_1 \oplus x_2$$

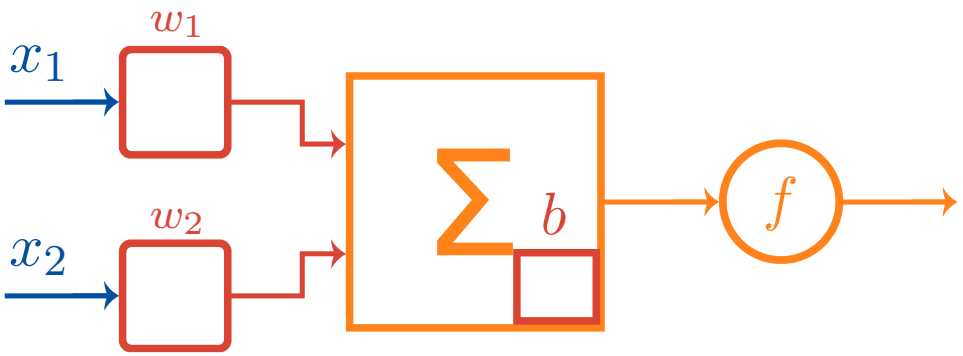
x_1 (input)	x_2 (input)	$x_1 \oplus x_2$ (output)
1	1	0
1	0	1
0	1	1
0	0	0



Оператор “Исключающее ИЛИ”

$x_1 \oplus x_2$

x_1 (input)	x_2 (input)	$x_1 \oplus x_2$ (output)
1	1	0
1	0	1
0	1	1
0	0	0

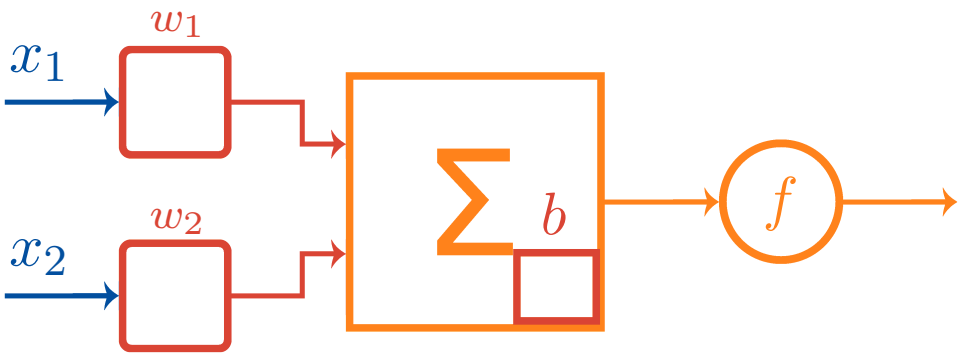


$$\begin{cases} 1w_1 + 1w_2 + b < 0 \\ 1w_1 + 0w_2 + b \geqslant 0 \\ 0w_1 + 1w_2 + b \geqslant 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

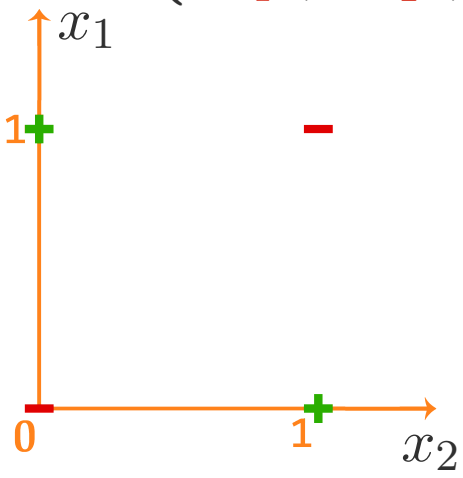
Оператор “Исключающее ИЛИ”

$x_1 \oplus x_2$

x_1 (input)	x_2 (input)	$x_1 \oplus x_2$ (output)
1	1	0
1	0	1
0	1	1
0	0	0



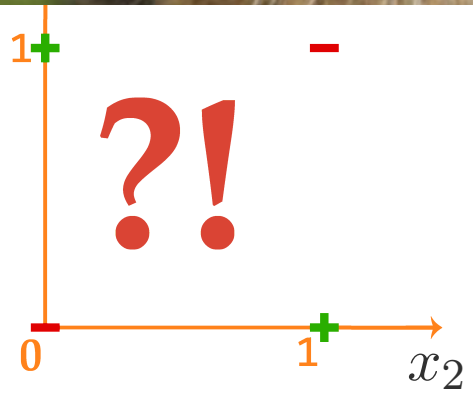
$$\begin{cases} 1w_1 + 1w_2 + b < 0 \\ 1w_1 + 0w_2 + b \geqslant 0 \\ 0w_1 + 1w_2 + b \geqslant 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



Оператор “Исключающее ИЛИ”

$$x_1 \oplus x_2$$

x_1 (input)	x_2 (input)	$x_1 \oplus x_2$ (output)
1	1	0
1	0	1
0	1	1
0	0	0

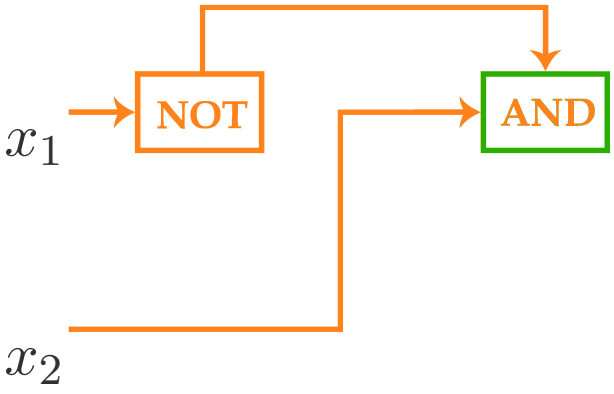


Решение

$$x_1 \oplus x_2 = (\overline{x}_1 \ \& \ x_2) \ || \ (x_1 \ \& \ \overline{x}_2)$$

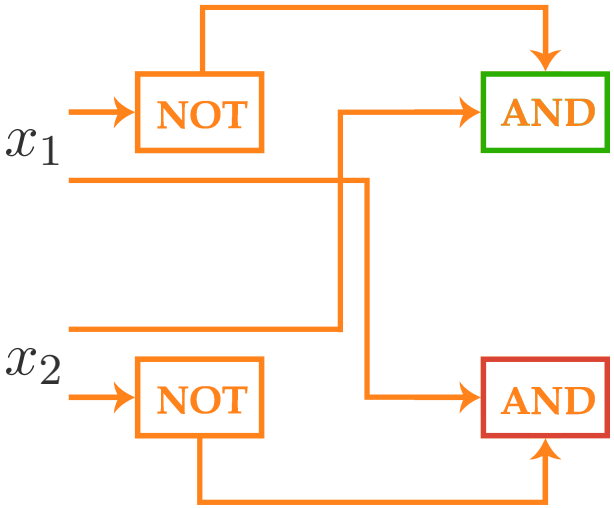
Решение

$$x_1 \oplus x_2 = \underbrace{(\overline{x}_1 \ \& \ x_2)} \ || \ (x_1 \ \& \ \overline{x}_2)$$



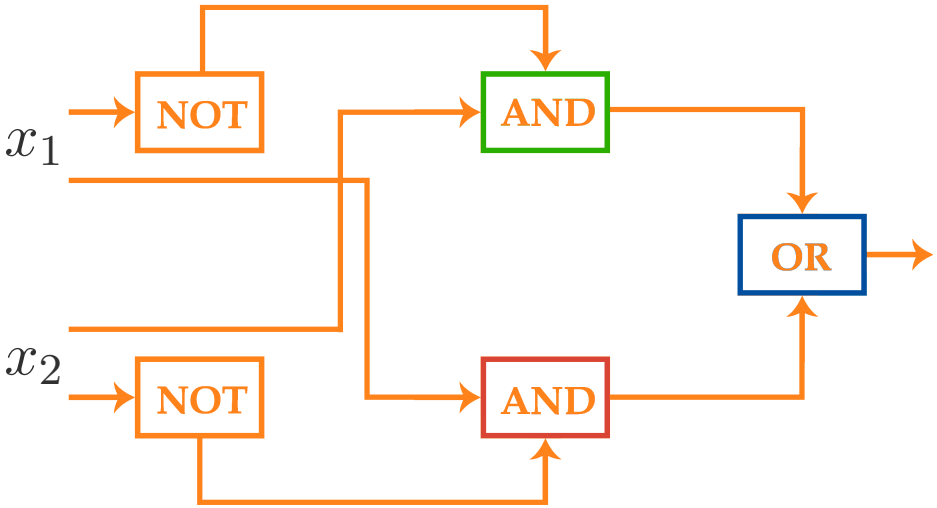
Решение

$$x_1 \oplus x_2 = \underbrace{(\overline{x}_1 \ \& \ x_2)} \ || \ \underbrace{(x_1 \ \& \ \overline{x}_2)}$$



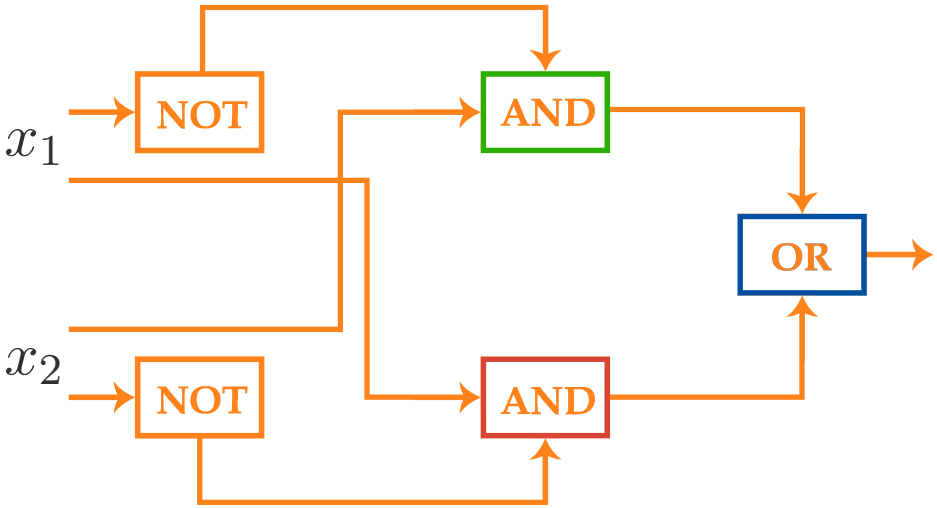
Решение

$$x_1 \oplus x_2 = \overbrace{\underbrace{(\overline{x}_1 \ \& \ x_2)} \ || \ \underbrace{(x_1 \ \& \ \overline{x}_2)}}$$



Решение

$$x_1 \oplus x_2 = \overbrace{\underbrace{(\overline{x}_1 \ \& \ x_2)} \ || \ \underbrace{(x_1 \ \& \ \overline{x}_2)}}$$



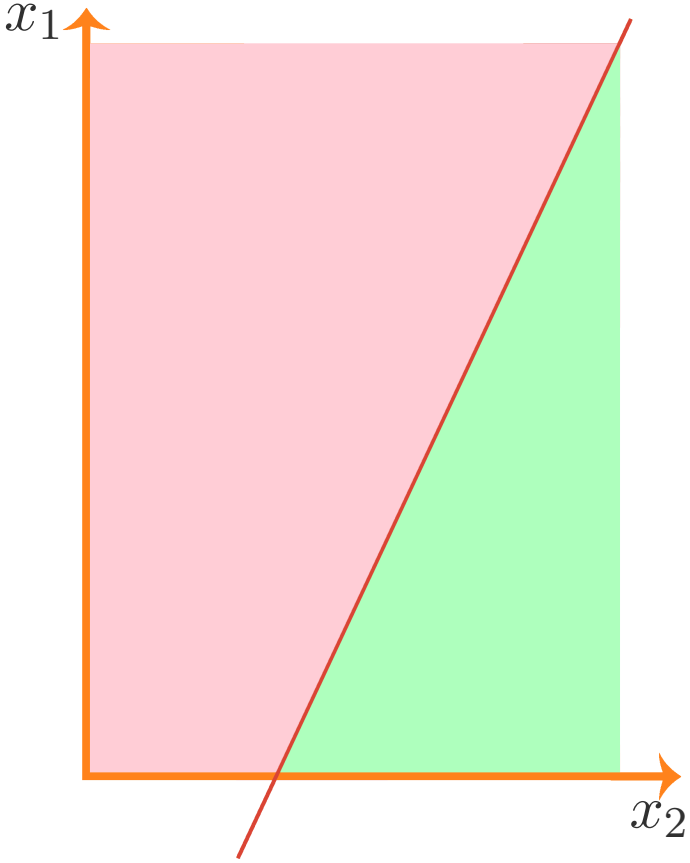
Сеть позволяет сделать нелинейную операцию!!!

Мы использовали 5 нейронов. Но для решения задачи можно использовать всего 3.

Задание: придумать как.

Линейность нейрона и нелинейность сети

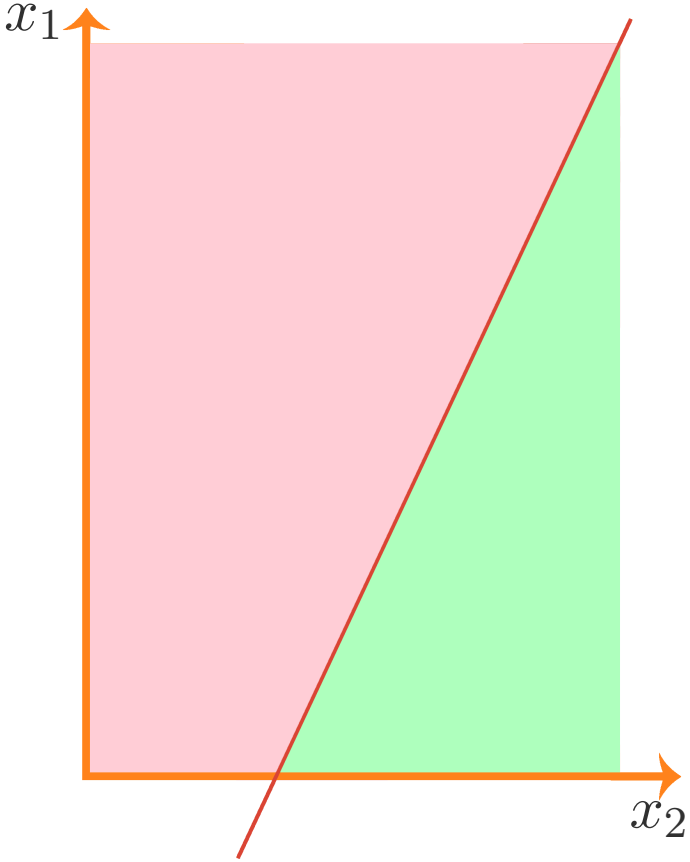
- Разделяющая поверхность нейрона линейна
- Сети – нелинейна



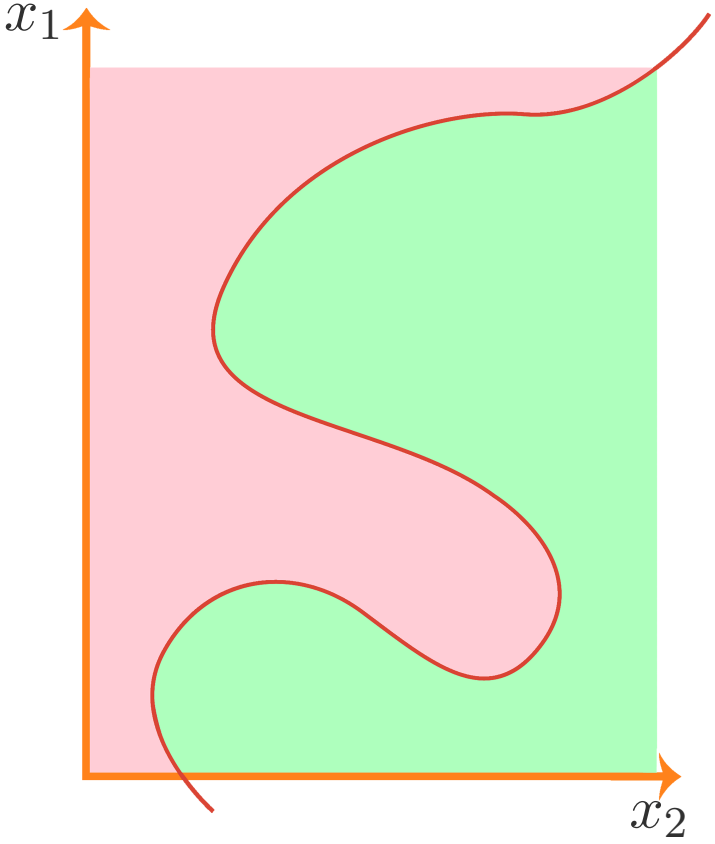
Нейрон

Линейность нейрона и нелинейность сети

- Разделяющая поверхность нейрона линейна
- Сети – нелинейна



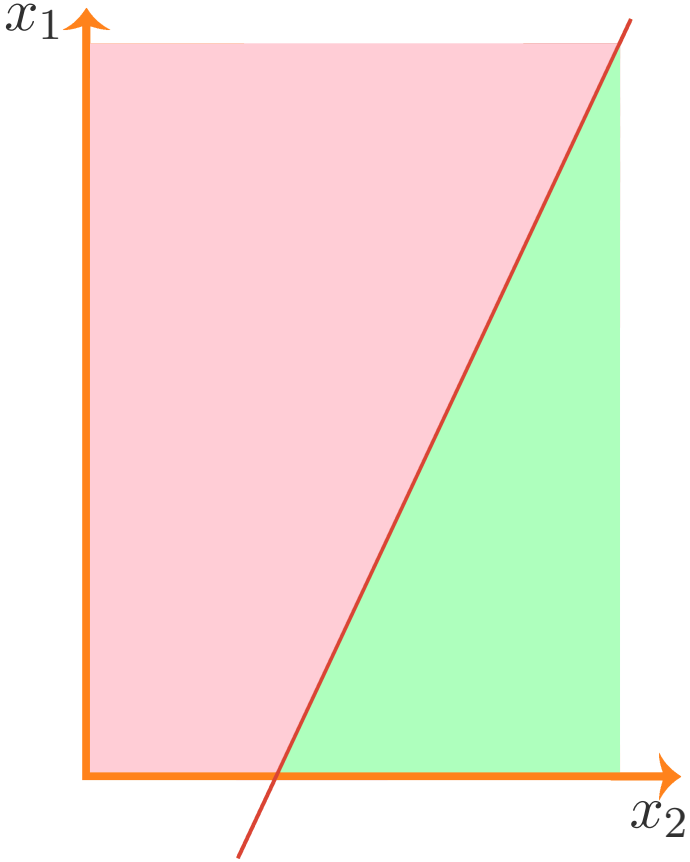
Нейрон



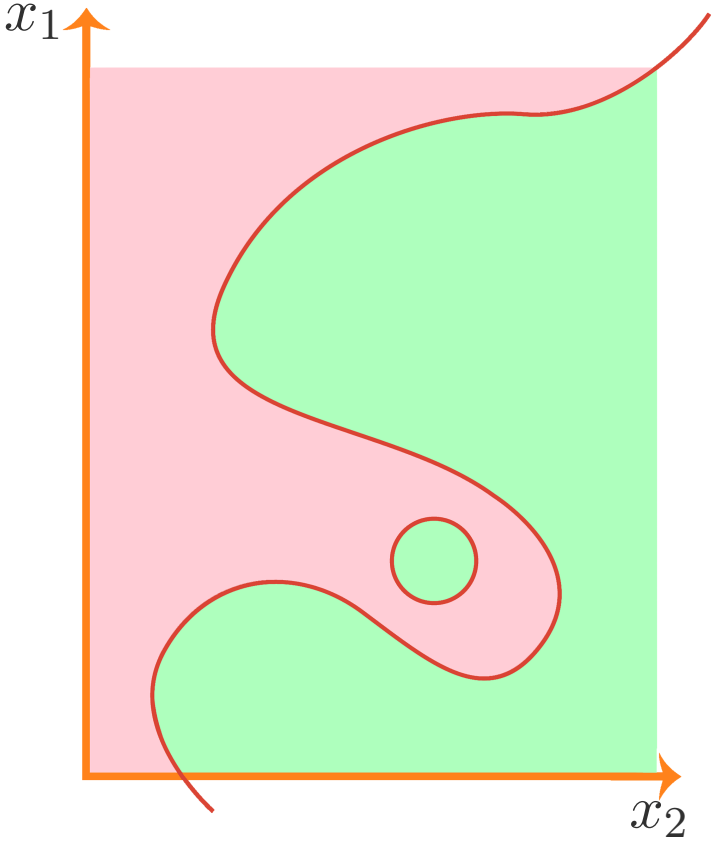
Сеть

Линейность нейрона и нелинейность сети

- Разделяющая поверхность нейрона линейна
- Сети – нелинейна

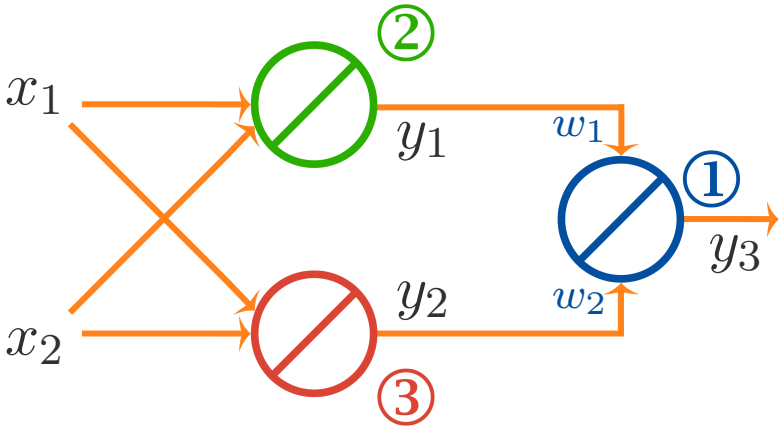


Нейрон

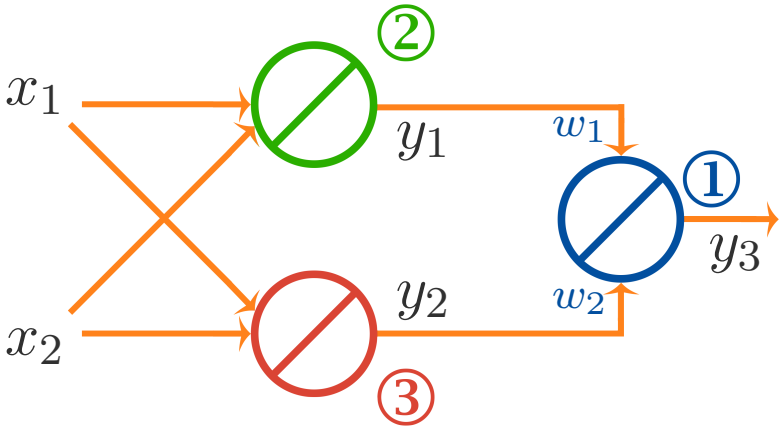


Сеть

Сеть нейронов с линейной активацией

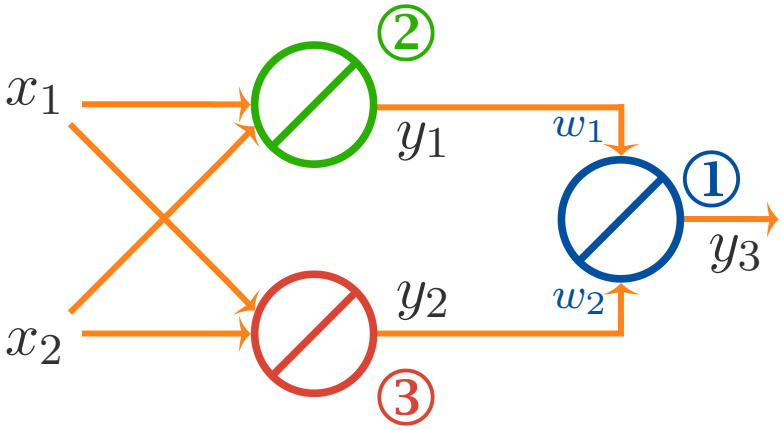


Сеть нейронов с линейной активацией



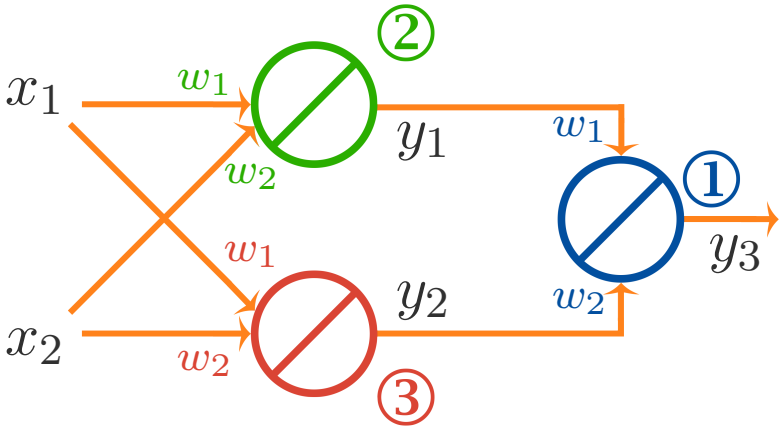
$$y_3 = f\big(w_1^1 y_1 + w_2^1 y_2 + b^1\big) =$$

Сеть нейронов с линейной активацией



$$y_3 = f\left(w_1^1 y_1 + w_2^1 y_2 + b^1\right) =$$
$$= w_1^1 y_1 + w_2^1 y_2 + b^1 =$$

Сеть нейронов с линейной активацией



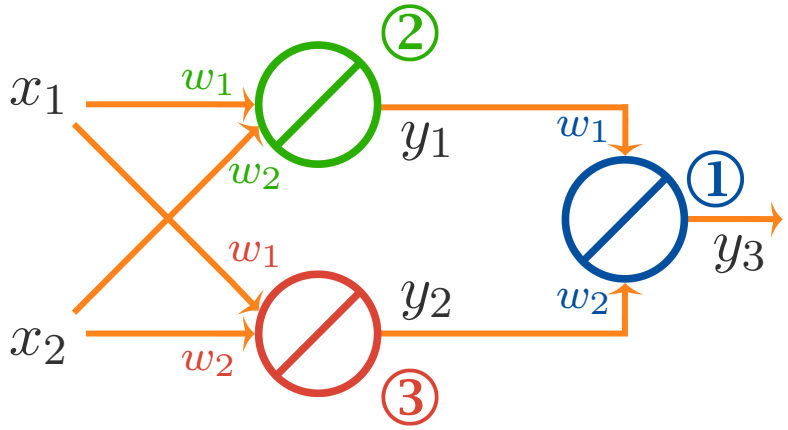
$$y_3 = f\left(w_1^1 y_1 + w_2^1 y_2 + b^1\right) =$$

$$= w_1^1 y_1 + w_2^1 y_2 + b^1 =$$

$$= w_1^1 f\left(w_1^2 x_1 + w_2^2 x_2 + b^2\right) +$$

$$+ w_2^1 f\left(w_1^3 x_1 + w_2^3 x_2 + b^3\right) + b^1 =$$

Сеть нейронов с линейной активацией



$$y_3 = f\left(w_1^1 y_1 + w_2^1 y_2 + b^1\right) =$$

$$= w_1^1 y_1 + w_2^1 y_2 + b^1 =$$

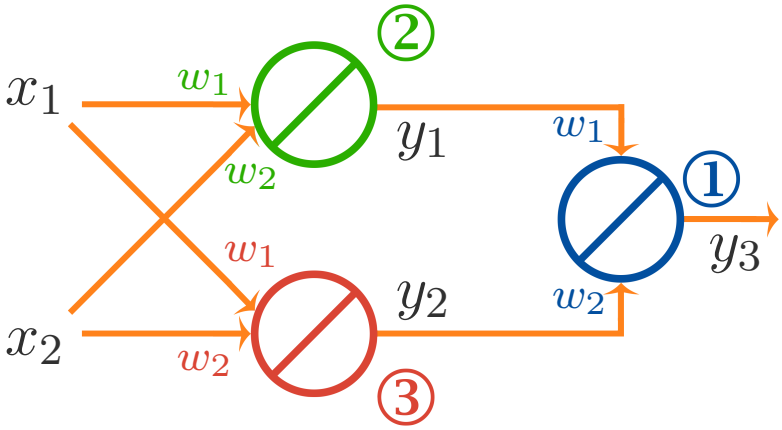
$$= w_1^1 f\left(w_1^2 x_1 + w_2^2 x_2 + b^2\right) +$$

$$+ w_2^1 f\left(w_1^3 x_1 + w_2^3 x_2 + b^3\right) + b^1 =$$

$$= w_1^1 w_1^2 x_1 + w_1^1 w_2^2 x_2 + w_1^1 b^2 +$$

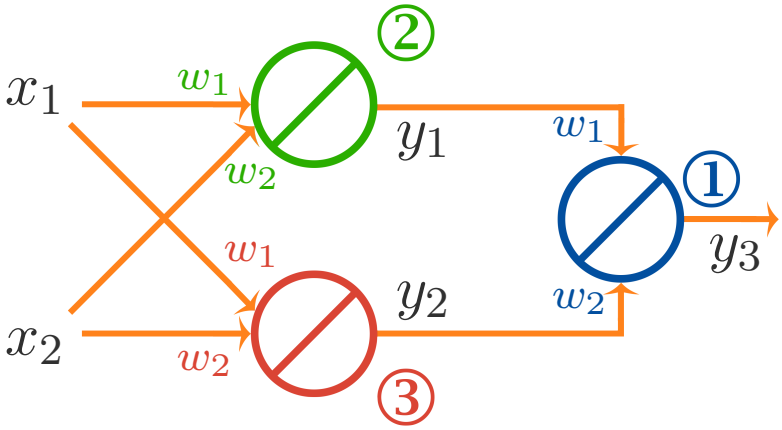
$$+ w_2^1 w_1^3 x_1 + w_2^1 w_2^3 x_2 + w_2^1 b^3 + b^1 =$$

Сеть нейронов с линейной активацией



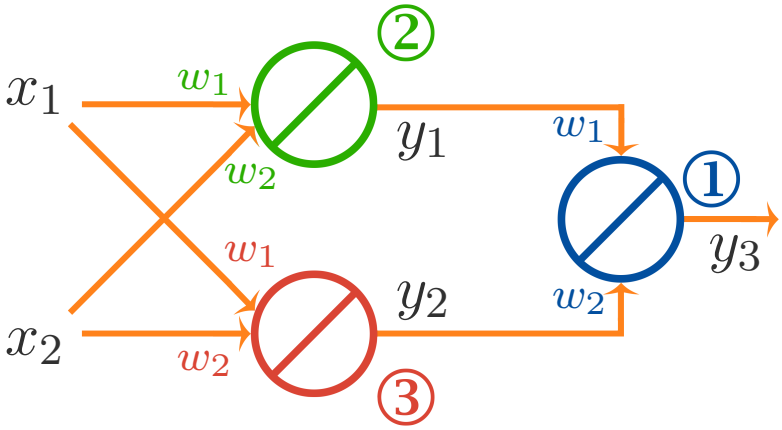
$$
\begin{aligned}
y_3 &= f\left(w_1^1 y_1 + w_2^1 y_2 + b^1\right) = \\
&= w_1^1 y_1 + w_2^1 y_2 + b^1 = \\
&= w_1^1 f\left(w_1^2 x_1 + w_2^2 x_2 + b^2\right) + \\
&+ w_2^1 f\left(w_1^3 x_1 + w_2^3 x_2 + b^3\right) + b^1 = \\
&= w_1^1 w_1^2 x_1 + w_1^1 w_2^2 x_2 + w_1^1 b^2 + \\
&+ w_2^1 w_1^3 x_1 + w_2^1 w_2^3 x_2 + w_2^1 b^3 + b^1 = \\
&= x_1 \left[w_1^1 w_1^2 + w_2^1 w_1^3\right] + x_2 \left[w_1^1 w_2^2 + w_2^1 w_2^3\right] + \\
&+ \left[w_1^1 b^2 + w_2^1 b^3 + b^1\right] =
\end{aligned}
$$

Сеть нейронов с линейной активацией



$$
\begin{aligned}
y_3 &= f\left(w_1^1 y_1 + w_2^1 y_2 + b^1\right) = \\
&= w_1^1 y_1 + w_2^1 y_2 + b^1 = \\
&= w_1^1 f\left(w_1^2 x_1 + w_2^2 x_2 + b^2\right) + \\
&+ w_2^1 f\left(w_1^3 x_1 + w_2^3 x_2 + b^3\right) + b^1 = \\
&= w_1^1 w_1^2 x_1 + w_1^1 w_2^2 x_2 + w_1^1 b^2 + \\
&+ w_2^1 w_1^3 x_1 + w_2^1 w_2^3 x_2 + w_2^1 b^3 + b^1 = \\
&= x_1 \overbrace{\left[w_1^1 w_1^2 + w_2^1 w_1^3\right]}^{\widetilde{w}_1} + x_2 \overbrace{\left[w_1^1 w_2^2 + w_2^1 w_2^3\right]}^{\widetilde{w}_2} + \\
&+ \overbrace{\left[w_1^1 b^2 + w_2^1 b^3 + b^1\right]}^{\widetilde{b}} =
\end{aligned}
$$

Сеть нейронов с линейной активацией



$$
\begin{aligned}
y_3 &= f\left(w_1^1 y_1 + w_2^1 y_2 + b^1\right) = \\
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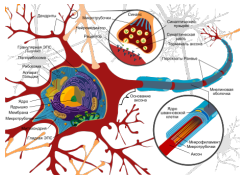
&+ w_2^1 f\left(w_1^3 x_1 + w_2^3 x_2 + b^3\right) + b^1 = \\
&= w_1^1 w_1^2 x_1 + w_1^1 w_2^2 x_2 + w_1^1 b^2 + \\
&+ w_2^1 w_1^3 x_1 + w_2^1 w_2^3 x_2 + w_2^1 b^3 + b^1 = \\
&= x_1 \overbrace{\left[w_1^1 w_1^2 + w_2^1 w_1^3\right]}^{\widetilde{w}_1} + x_2 \overbrace{\left[w_1^1 w_2^2 + w_2^1 w_2^3\right]}^{\widetilde{w}_2} + \\
&+ \overbrace{\left[w_1^1 b^2 + w_2^1 b^3 + b^1\right]}^{\widetilde{b}} = \\
&= x_1 \widetilde{w}_1 + x_2 \widetilde{w}_2 + \widetilde{b}
\end{aligned}
$$

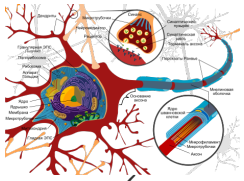
Итог

- Как устроен биологический нейрон



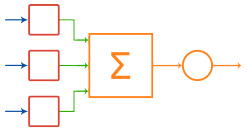
Итог

- Как устроен биологический нейрон



- Как сделать математическую модель нейрона

$$y = \left(\sum_{i=1}^{n} w_i x_i + b \right)$$



Итог

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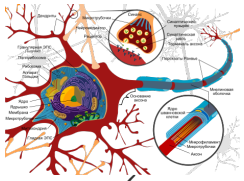

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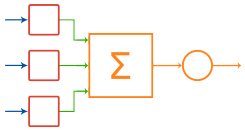
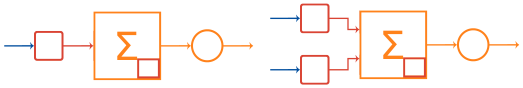
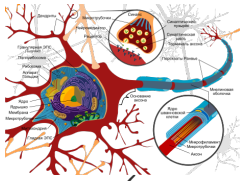


- Как представить некоторые логические операции в виде нейронов



Итог

- Как устроен биологический нейрон



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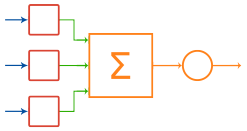
$$y = \left(\sum_{i=1}^{n} w_i x_i + b \right)$$



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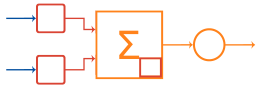
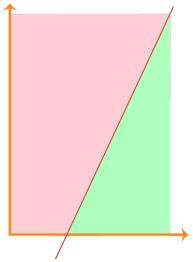
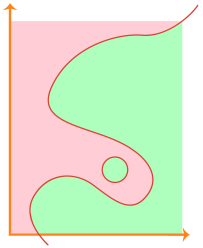


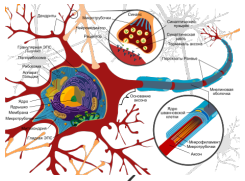
- Нейрон линеен, сеть нелинейна





Итог

- Как устроен биологический нейрон



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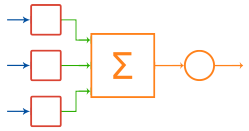
$$y = \left(\sum_{i=1}^{n} w_i x_i + b \right)$$



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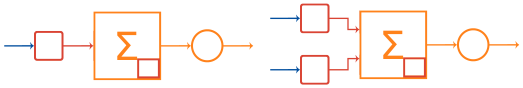
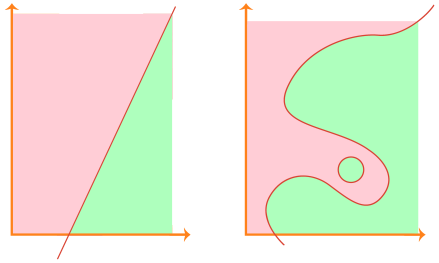
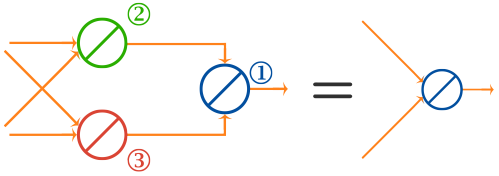


- Нейрон линеен, сеть нелинейна



- Комбинировать полносвязанные линейные нейроны бессмысленно