

7

LOGICAL AGENTS

In which we design agents that can form representations of a complex world, use a process of inference to derive new representations about the world, and use these new representations to deduce what to do.

REASONING
REPRESENTATION
KNOWLEDGE-BASED
AGENTS

Humans, it seems, know things; and what they know helps them do things. These are not empty statements. They make strong claims about how the intelligence of humans is achieved—not by purely reflex mechanisms but by processes of **reasoning** that operate on internal **representations** of knowledge. In AI, this approach to intelligence is embodied in **knowledge-based agents**.

The problem-solving agents of Chapters 3 and 4 know things, but only in a very limited, inflexible sense. For example, the transition model for the 8-puzzle—knowledge of what the actions do—is hidden inside the domain-specific code of the `RESULT` function. It can be used to predict the outcome of actions but not to deduce that two tiles cannot occupy the same space or that states with odd parity cannot be reached from states with even parity. The atomic representations used by problem-solving agents are also very limiting. In a partially observable environment, an agent’s only choice for representing what it knows about the current state is to list all possible concrete states—a hopeless prospect in large environments.

LOGIC

Chapter 6 introduced the idea of representing states as assignments of values to variables; this is a step in the right direction, enabling some parts of the agent to work in a domain-independent way and allowing for more efficient algorithms. In this chapter and those that follow, we take this step to its logical conclusion, so to speak—we develop **logic** as a general class of representations to support knowledge-based agents. Such agents can combine and recombine information to suit myriad purposes. Often, this process can be quite far removed from the needs of the moment—as when a mathematician proves a theorem or an astronomer calculates the earth’s life expectancy. Knowledge-based agents can accept new tasks in the form of explicitly described goals; they can achieve competence quickly by being told or learning new knowledge about the environment; and they can adapt to changes in the environment by updating the relevant knowledge.

We begin in Section 7.1 with the overall agent design. Section 7.2 introduces a simple new environment, the wumpus world, and illustrates the operation of a knowledge-based agent without going into any technical detail. Then we explain the general principles of **logic**

in Section 7.3 and the specifics of **propositional logic** in Section 7.4. While less expressive than **first-order logic** (Chapter 8), propositional logic illustrates all the basic concepts of logic; it also comes with well-developed inference technologies, which we describe in sections 7.5 and 7.6. Finally, Section 7.7 combines the concept of knowledge-based agents with the technology of propositional logic to build some simple agents for the wumpus world.

7.1 KNOWLEDGE-BASED AGENTS

KNOWLEDGE BASE
SENTENCE

KNOWLEDGE
REPRESENTATION
LANGUAGE
AXIOM

The central component of a knowledge-based agent is its **knowledge base**, or KB. A knowledge base is a set of **sentences**. (Here “sentence” is used as a technical term. It is related but not identical to the sentences of English and other natural languages.) Each sentence is expressed in a language called a **knowledge representation language** and represents some assertion about the world. Sometimes we dignify a sentence with the name **axiom**, when the sentence is taken as given without being derived from other sentences.

INFERENCE

There must be a way to add new sentences to the knowledge base and a way to query what is known. The standard names for these operations are TELL and ASK, respectively. Both operations may involve **inference**—that is, deriving new sentences from old. Inference must obey the requirement that when one ASKS a question of the knowledge base, the answer should follow from what has been told (or TELLED) to the knowledge base previously. Later in this chapter, we will be more precise about the crucial word “follow.” For now, take it to mean that the inference process should not make things up as it goes along.

BACKGROUND
KNOWLEDGE

Figure 7.1 shows the outline of a knowledge-based agent program. Like all our agents, it takes a percept as input and returns an action. The agent maintains a knowledge base, *KB*, which may initially contain some **background knowledge**.

Each time the agent program is called, it does three things. First, it TELLS the knowledge base what it perceives. Second, it ASKS the knowledge base what action it should perform. In the process of answering this query, extensive reasoning may be done about the current state of the world, about the outcomes of possible action sequences, and so on. Third, the agent program TELLS the knowledge base which action was chosen, and the agent executes the action.

The details of the representation language are hidden inside three functions that implement the interface between the sensors and actuators on one side and the core representation and reasoning system on the other. MAKE-PERCEPT-SENTENCE constructs a sentence asserting that the agent perceived the given percept at the given time. MAKE-ACTION-QUERY constructs a sentence that asks what action should be done at the current time. Finally, MAKE-ACTION-SENTENCE constructs a sentence asserting that the chosen action was executed. The details of the inference mechanisms are hidden inside TELL and ASK. Later sections will reveal these details.

The agent in Figure 7.1 appears quite similar to the agents with internal state described in Chapter 2. Because of the definitions of TELL and ASK, however, the knowledge-based agent is not an arbitrary program for calculating actions. It is amenable to a description at

```

function KB-AGENT(percept) returns an action
  persistent: KB, a knowledge base
               t, a counter, initially 0, indicating time

  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action

```

Figure 7.1 A generic knowledge-based agent. Given a percept, the agent adds the percept to its knowledge base, asks the knowledge base for the best action, and tells the knowledge base that it has in fact taken that action.

KNOWLEDGE LEVEL

the **knowledge level**, where we need specify only what the agent knows and what its goals are, in order to fix its behavior. For example, an automated taxi might have the goal of taking a passenger from San Francisco to Marin County and might know that the Golden Gate Bridge is the only link between the two locations. Then we can expect it to cross the Golden Gate Bridge *because it knows that that will achieve its goal*. Notice that this analysis is independent of how the taxi works at the **implementation level**. It doesn't matter whether its geographical knowledge is implemented as linked lists or pixel maps, or whether it reasons by manipulating strings of symbols stored in registers or by propagating noisy signals in a network of neurons.

IMPLEMENTATION LEVEL

DECLARATIVE

A knowledge-based agent can be built simply by TELLing it what it needs to know. Starting with an empty knowledge base, the agent designer can TELL sentences one by one until the agent knows how to operate in its environment. This is called the **declarative** approach to system building. In contrast, the **procedural** approach encodes desired behaviors directly as program code. In the 1970s and 1980s, advocates of the two approaches engaged in heated debates. We now understand that a successful agent often combines both declarative and procedural elements in its design, and that declarative knowledge can often be compiled into more efficient procedural code.

We can also provide a knowledge-based agent with mechanisms that allow it to learn for itself. These mechanisms, which are discussed in Chapter 18, create general knowledge about the environment from a series of percepts. A learning agent can be fully autonomous.

7.2 THE WUMPUS WORLD

WUMPUS WORLD

In this section we describe an environment in which knowledge-based agents can show their worth. The **wumpus world** is a cave consisting of rooms connected by passageways. Lurking somewhere in the cave is the terrible wumpus, a beast that eats anyone who enters its room. The wumpus can be shot by an agent, but the agent has only one arrow. Some rooms contain

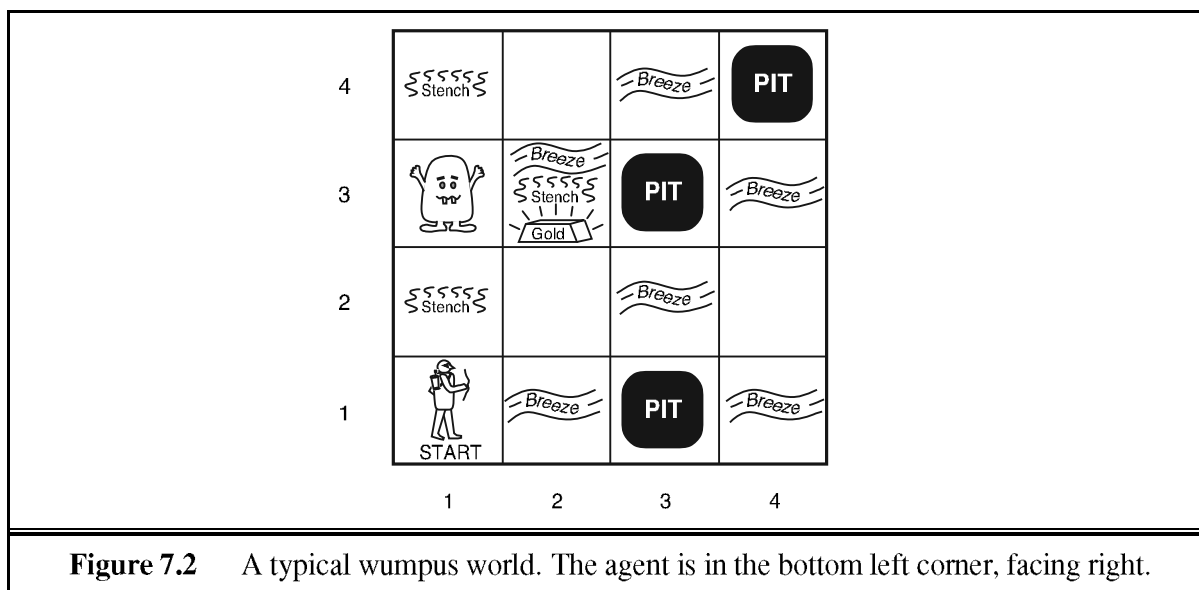
bottomless pits that will trap anyone who wanders into these rooms (except for the wumpus, which is too big to fall in). The only mitigating feature of this bleak environment is the possibility of finding a heap of gold. Although the wumpus world is rather tame by modern computer game standards, it illustrates some important points about intelligence.

A sample wumpus world is shown in Figure 7.2. The precise definition of the task environment is given, as suggested in Section 2.3, by the PEAS description:

- **Performance measure:** +1000 for climbing out of the cave with the gold, −1000 for falling into a pit or being eaten by the wumpus, −1 for each action taken and −10 for using up the arrow. The game ends either when the agent dies or when the agent climbs out of the cave.
- **Environment:** A 4×4 grid of rooms. The agent always starts in the square labeled [1,1], facing to the right. The locations of the gold and the wumpus are chosen randomly, with a uniform distribution, from the squares other than the start square. In addition, each square other than the start can be a pit, with probability 0.2.
- **Actuators:** The agent can move *Forward*, *TurnLeft* by 90° , or *TurnRight* by 90° . The agent dies a miserable death if it enters a square containing a pit or a live wumpus. (It is safe, albeit smelly, to enter a square with a dead wumpus.) If an agent tries to move forward and bumps into a wall, then the agent does not move. The action *Grab* can be used to pick up the gold if it is in the same square as the agent. The action *Shoot* can be used to fire an arrow in a straight line in the direction the agent is facing. The arrow continues until it either hits (and hence kills) the wumpus or hits a wall. The agent has only one arrow, so only the first *Shoot* action has any effect. Finally, the action *Climb* can be used to climb out of the cave, but only from square [1,1].
- **Sensors:** The agent has five sensors, each of which gives a single bit of information:
 - In the square containing the wumpus and in the directly (not diagonally) adjacent squares, the agent will perceive a *Stench*.
 - In the squares directly adjacent to a pit, the agent will perceive a *Breeze*.
 - In the square where the gold is, the agent will perceive a *Glitter*.
 - When an agent walks into a wall, it will perceive a *Bump*.
 - When the wumpus is killed, it emits a woeful *Scream* that can be perceived anywhere in the cave.

The percepts will be given to the agent program in the form of a list of five symbols; for example, if there is a stench and a breeze, but no glitter, bump, or scream, the agent program will get [*Stench*, *Breeze*, *None*, *None*, *None*].

We can characterize the wumpus environment along the various dimensions given in Chapter 2. Clearly, it is discrete, static, and single-agent. (The wumpus doesn't move, fortunately.) It is sequential, because rewards may come only after many actions are taken. It is partially observable, because some aspects of the state are not directly perceivable: the agent's location, the wumpus's state of health, and the availability of an arrow. As for the locations of the pits and the wumpus: we could treat them as unobserved parts of the state that happen to be immutable—in which case, the transition model for the environment is completely



known; or we could say that the transition model itself is unknown because the agent doesn't know which *Forward* actions are fatal—in which case, discovering the locations of pits and wumpus completes the agent's knowledge of the transition model.

For an agent in the environment, the main challenge is its initial ignorance of the configuration of the environment; overcoming this ignorance seems to require logical reasoning. In most instances of the wumpus world, it is possible for the agent to retrieve the gold safely. Occasionally, the agent must choose between going home empty-handed and risking death to find the gold. About 21% of the environments are utterly unfair, because the gold is in a pit or surrounded by pits.

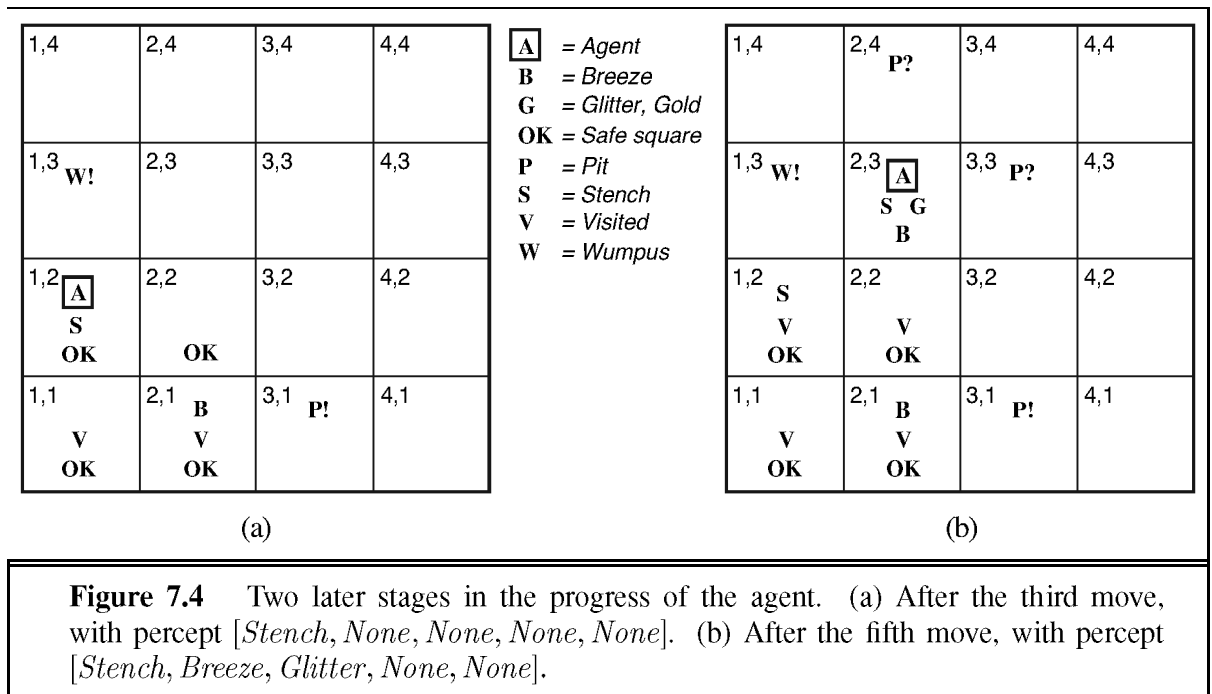
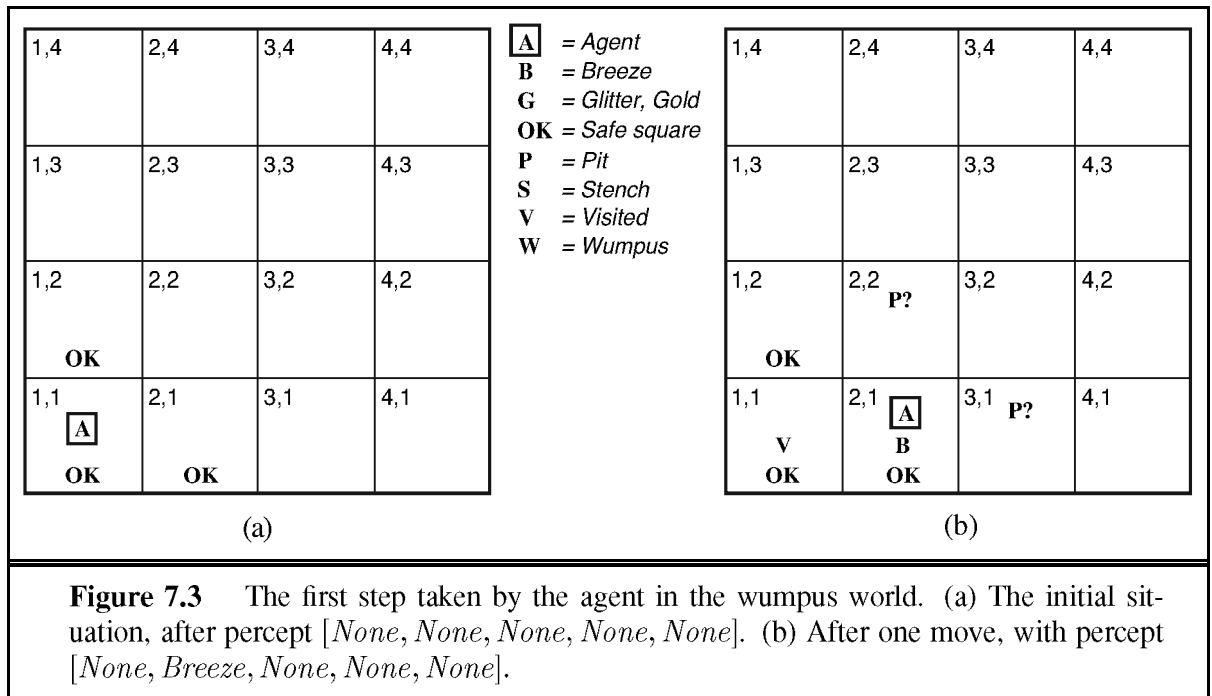
Let us watch a knowledge-based wumpus agent exploring the environment shown in Figure 7.2. We use an informal knowledge representation language consisting of writing down symbols in a grid (as in Figures 7.3 and 7.4).

The agent's initial knowledge base contains the rules of the environment, as described previously; in particular, it knows that it is in [1,1] and that [1,1] is a safe square; we denote that with an "A" and "OK," respectively, in square [1,1].

The first percept is $[None, None, None, None, None]$, from which the agent can conclude that its neighboring squares, [1,2] and [2,1], are free of dangers—they are OK. Figure 7.3(a) shows the agent's state of knowledge at this point.

A cautious agent will move only into a square that it knows to be OK. Let us suppose the agent decides to move forward to [2,1]. The agent perceives a breeze (denoted by "B") in [2,1], so there must be a pit in a neighboring square. The pit cannot be in [1,1], by the rules of the game, so there must be a pit in [2,2] or [3,1] or both. The notation "P?" in Figure 7.3(b) indicates a possible pit in those squares. At this point, there is only one known square that is OK and that has not yet been visited. So the prudent agent will turn around, go back to [1,1], and then proceed to [1,2].

The agent perceives a stench in [1,2], resulting in the state of knowledge shown in Figure 7.4(a). The stench in [1,2] means that there must be a wumpus nearby. But the



wumpus cannot be in [1,1], by the rules of the game, and it cannot be in [2,2] (or the agent would have detected a stench when it was in [2,1]). Therefore, the agent can infer that the wumpus is in [1,3]. The notation **W!** indicates this inference. Moreover, the lack of a breeze in [1,2] implies that there is no pit in [2,2]. Yet the agent has already inferred that there must be a pit in either [2,2] or [3,1], so this means it must be in [3,1]. This is a fairly difficult inference, because it combines knowledge gained at different times in different places and relies on the lack of a percept to make one crucial step.

The agent has now proved to itself that there is neither a pit nor a wumpus in [2,2], so it is OK to move there. We do not show the agent's state of knowledge at [2,2]; we just assume that the agent turns and moves to [2,3], giving us Figure 7.4(b). In [2,3], the agent detects a glitter, so it should grab the gold and then return home.

Note that in each case for which the agent draws a conclusion from the available information, that conclusion is *guaranteed* to be correct if the available information is correct. This is a fundamental property of logical reasoning. In the rest of this chapter, we describe how to build logical agents that can represent information and draw conclusions such as those described in the preceding paragraphs.

7.3 LOGIC

This section summarizes the fundamental concepts of logical representation and reasoning. These beautiful ideas are independent of any of logic's particular forms. We therefore postpone the technical details of those forms until the next section, using instead the familiar example of ordinary arithmetic.

SYNTAX

In Section 7.1, we said that knowledge bases consist of sentences. These sentences are expressed according to the **syntax** of the representation language, which specifies all the sentences that are well formed. The notion of syntax is clear enough in ordinary arithmetic: “ $x + y = 4$ ” is a well-formed sentence, whereas “ $x4y+ =$ ” is not.

SEMANTICS

TRUTH

POSSIBLE WORLD

A logic must also define the **semantics** or meaning of sentences. The semantics defines the **truth** of each sentence with respect to each **possible world**. For example, the semantics for arithmetic specifies that the sentence “ $x + y = 4$ ” is true in a world where x is 2 and y is 2, but false in a world where x is 1 and y is 1. In standard logics, every sentence must be either true or false in each possible world—there is no “in between.”¹

MODEL

When we need to be precise, we use the term **model** in place of “possible world.” Whereas possible worlds might be thought of as (potentially) real environments that the agent might or might not be in, models are mathematical abstractions, each of which simply fixes the truth or falsehood of every relevant sentence. Informally, we may think of a possible world as, for example, having x men and y women sitting at a table playing bridge, and the sentence $x + y = 4$ is true when there are four people in total. Formally, the possible models are just all possible assignments of real numbers to the variables x and y . Each such assignment fixes the truth of any sentence of arithmetic whose variables are x and y . If a sentence α is true in model m , we say that m **satisfies** α or sometimes m **is a model of** α . We use the notation $M(\alpha)$ to mean the set of all models of α .

SATISFACTION

ENTAILMENT

Now that we have a notion of truth, we are ready to talk about logical reasoning. This involves the relation of logical **entailment** between sentences—the idea that a sentence *follows logically* from another sentence. In mathematical notation, we write

$$\alpha \models \beta$$

¹ **Fuzzy logic**, discussed in Chapter 14, allows for degrees of truth.

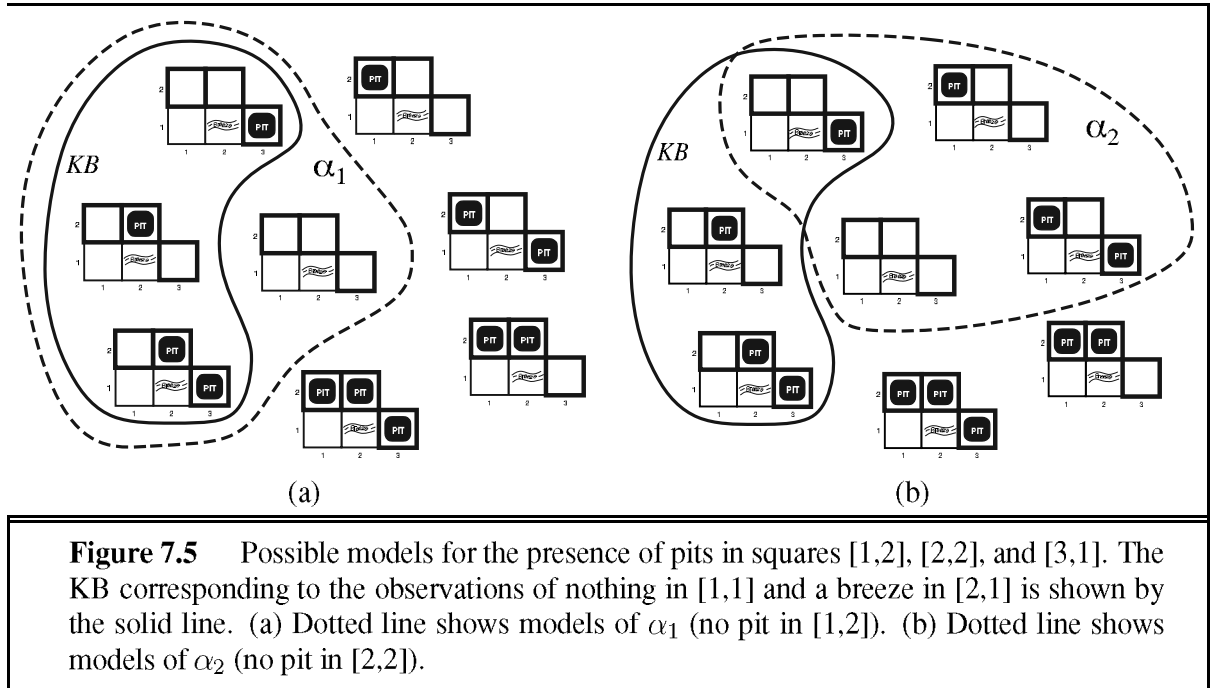


Figure 7.5 Possible models for the presence of pits in squares [1,2], [2,2], and [3,1]. The KB corresponding to the observations of nothing in [1,1] and a breeze in [2,1] is shown by the solid line. (a) Dotted line shows models of α_1 (no pit in [1,2]). (b) Dotted line shows models of α_2 (no pit in [2,2]).

to mean that the sentence α entails the sentence β . The formal definition of entailment is this: $\alpha \models \beta$ if and only if, in every model in which α is true, β is also true. Using the notation just introduced, we can write

$$\alpha \models \beta \text{ if and only if } M(\alpha) \subseteq M(\beta).$$

(Note the direction of the \subseteq here: if $\alpha \models \beta$, then α is a *stronger* assertion than β : it rules out *more* possible worlds.) The relation of entailment is familiar from arithmetic; we are happy with the idea that the sentence $x = 0$ entails the sentence $xy = 0$. Obviously, in any model where x is zero, it is the case that xy is zero (regardless of the value of y).

We can apply the same kind of analysis to the wumpus-world reasoning example given in the preceding section. Consider the situation in Figure 7.3(b): the agent has detected nothing in [1,1] and a breeze in [2,1]. These percepts, combined with the agent's knowledge of the rules of the wumpus world, constitute the KB. The agent is interested (among other things) in whether the adjacent squares [1,2], [2,2], and [3,1] contain pits. Each of the three squares might or might not contain a pit, so (for the purposes of this example) there are $2^3 = 8$ possible models. These eight models are shown in Figure 7.5.²

The KB can be thought of as a set of sentences or as a single sentence that asserts all the individual sentences. The KB is false in models that contradict what the agent knows—for example, the KB is false in any model in which [1,2] contains a pit, because there is no breeze in [1,1]. There are in fact just three models in which the KB is true, and these are

² Although the figure shows the models as partial wumpus worlds, they are really nothing more than assignments of *true* and *false* to the sentences “there is a pit in [1,2]” etc. Models, in the mathematical sense, do not need to have ‘orrible’ airy wumpuses in them.

shown surrounded by a solid line in Figure 7.5. Now let us consider two possible conclusions:

$\alpha_1 = \text{“There is no pit in [1,2].”}$

$\alpha_2 = \text{“There is no pit in [2,2].”}$

We have surrounded the models of α_1 and α_2 with dotted lines in Figures 7.5(a) and 7.5(b), respectively. By inspection, we see the following:

in every model in which KB is true, α_1 is also true.

Hence, $KB \models \alpha_1$: there is no pit in [1,2]. We can also see that

in some models in which KB is true, α_2 is false.

Hence, $KB \not\models \alpha_2$: the agent *cannot* conclude that there is no pit in [2,2]. (Nor can it conclude that there *is* a pit in [2,2].)³

LOGICAL INFERENCE

MODEL CHECKING

The preceding example not only illustrates entailment but also shows how the definition of entailment can be applied to derive conclusions—that is, to carry out **logical inference**. The inference algorithm illustrated in Figure 7.5 is called **model checking**, because it enumerates all possible models to check that α is true in all models in which KB is true, that is, that $M(KB) \subseteq M(\alpha)$.

In understanding entailment and inference, it might help to think of the set of all consequences of KB as a haystack and of α as a needle. Entailment is like the needle being in the haystack; inference is like finding it. This distinction is embodied in some formal notation: if an inference algorithm i can derive α from KB , we write

$$KB \vdash_i \alpha ,$$

which is pronounced “ α is derived from KB by i ” or “ i derives α from KB .”

SOUND

TRUTH-PRESERVING

An inference algorithm that derives only entailed sentences is called **sound** or **truth-preserving**. Soundness is a highly desirable property. An unsound inference procedure essentially makes things up as it goes along—it announces the discovery of nonexistent needles. It is easy to see that model checking, when it is applicable,⁴ is a sound procedure.

COMPLETENESS

The property of **completeness** is also desirable: an inference algorithm is complete if it can derive any sentence that is entailed. For real haystacks, which are finite in extent, it seems obvious that a systematic examination can always decide whether the needle is in the haystack. For many knowledge bases, however, the haystack of consequences is infinite, and completeness becomes an important issue.⁵ Fortunately, there are complete inference procedures for logics that are sufficiently expressive to handle many knowledge bases.

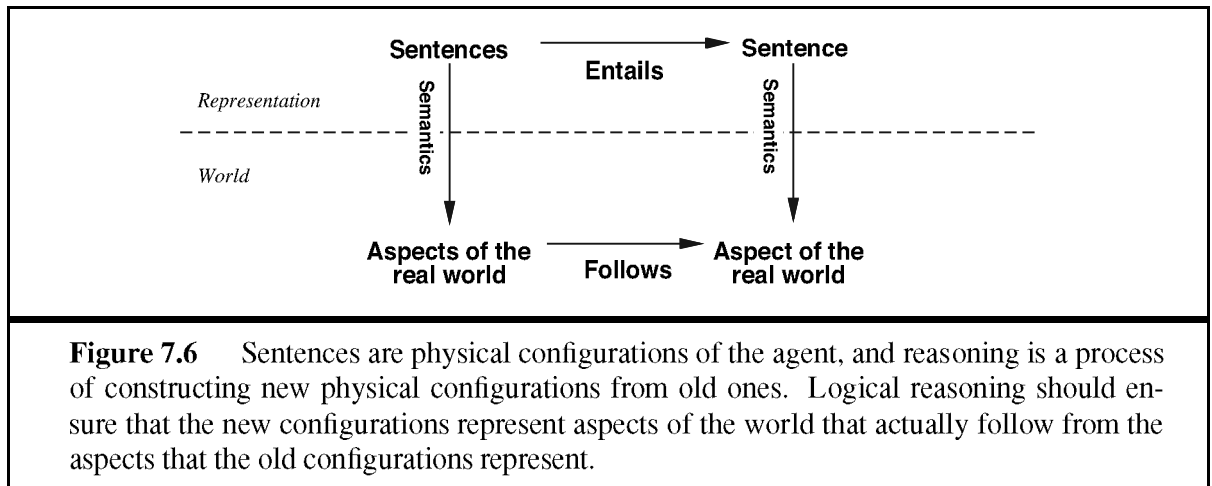


We have described a reasoning process whose conclusions are guaranteed to be true in any world in which the premises are true; in particular, *if KB is true in the real world, then any sentence α derived from KB by a sound inference procedure is also true in the real world*. So, while an inference process operates on “syntax”—internal physical configurations such as bits in registers or patterns of electrical blips in brains—the process *corresponds*

³ The agent can calculate the *probability* that there is a pit in [2,2]; Chapter 13 shows how.

⁴ Model checking works if the space of models is finite—for example, in wumpus worlds of fixed size. For arithmetic, on the other hand, the space of models is infinite: even if we restrict ourselves to the integers, there are infinitely many pairs of values for x and y in the sentence $x + y = 4$.

⁵ Compare with the case of infinite search spaces in Chapter 3, where depth-first search is not complete.



to the real-world relationship whereby some aspect of the real world is the case⁶ by virtue of other aspects of the real world being the case. This correspondence between world and representation is illustrated in Figure 7.6.

GROUNDING



The final issue to consider is **grounding**—the connection between logical reasoning processes and the real environment in which the agent exists. In particular, *how do we know that KB is true in the real world?* (After all, *KB* is just “syntax” inside the agent’s head.) This is a philosophical question about which many, many books have been written. (See Chapter 26.) A simple answer is that the agent’s sensors create the connection. For example, our wumpus-world agent has a smell sensor. The agent program creates a suitable sentence whenever there is a smell. Then, whenever that sentence is in the knowledge base, it is true in the real world. Thus, the meaning and truth of percept sentences are defined by the processes of sensing and sentence construction that produce them. What about the rest of the agent’s knowledge, such as its belief that wumpuses cause smells in adjacent squares? This is not a direct representation of a single percept, but a general rule—derived, perhaps, from perceptual experience but not identical to a statement of that experience. General rules like this are produced by a sentence construction process called **learning**, which is the subject of Part V. Learning is fallible. It could be the case that wumpuses cause smells *except on February 29 in leap years*, which is when they take their baths. Thus, *KB* may not be true in the real world, but with good learning procedures, there is reason for optimism.

7.4 PROPOSITIONAL LOGIC: A VERY SIMPLE LOGIC

PROPOSITIONAL LOGIC

We now present a simple but powerful logic called **propositional logic**. We cover the syntax of propositional logic and its semantics—the way in which the truth of sentences is determined. Then we look at **entailment**—the relation between a sentence and another sentence that follows from it—and see how this leads to a simple algorithm for logical inference. Everything takes place, of course, in the wumpus world.

⁶ As Wittgenstein (1922) put it in his famous *Tractatus*: “The world is everything that is the case.”