CHAPTER

22

FUZZY LOGIC SYSTEMS

I never came upon any of my discoveries through the process of rational thinking.

—**Albert Einstein**
(1879–1955), German-born theoretical physicist

22.1 INTRODUCTION

A large amount of data can constitute a proportionately large amount of information. But this comes with a level of uncertainty. As we come to know more, we also know how much we do not know and our awareness of the concept of complexity seems to increase. We tend to forego some precise data and allow uncertainty to creep into our perception. This is when we start describing things in a slightly vague and fuzzy manner. For instance after seeing a long list of names, you tell your friend that his name was cited somewhere *near* the middle of the list. The word *near* seems to be comprehended effortlessly by the human brain but what of computing systems? What does *near* mean in this context? Two names below the middle or five above or...? Is there a way we can make these number crunching systems understand this concept?

Consider the case of the statement - "Drive slowly". Does it mean you should drive at 10, 20 or 20.5 km/hour? The answer could be any value or a very different one depending on the context. If it is ascertained in a machine that any speed less than or equal to 20 km/hour means *slow* speed and anything above is fast, then does it mean that 20.1 km/hour (or 20.01 km/hour for that matter) is fast? This is an exaggeration in the real world. Fuzzy logic deals with how we can capture this essence of comprehension and embed it on the system by allowing for a gradual transition from slow to high speeds. This comprehension, as per Lotfi Zadeh, the founder of the fuzzy logic concept, confers a higher *machine intelligence quotient* to computer systems.

22.2 CRISP SETS

The conventional machine uses crisp sets to take care of concepts like *fast* and *slow* speeds. It relates speed to crisp values thereby forming members that either belong to a group or do not belong to it. For example

$$\text{Slow} = \{0,5,10,15,20,25,30,35,40\}$$

We could mean a crisp set that says that when the value of speed is equal to either of those mentioned in the set then the speed is categorized as slow. This may be modified to a closed interval [0, 40] to include the complete range of values. However, when the speed crosses over to 40.1 it will be categorized as *not slow* (or maybe *fast*). Likewise 39.99 will be *slow*. It is thus easy to visualize that a physical system which has to apply the brakes when the speed is *fast* (current speed does not belong to [0, 40]) and release when otherwise, would continuously keep jerking if the speed oscillates in the interval [39, 41], a situation that could eventually cause harm and subsequent damage. In such situations, we have to think of some alternative to a crisp set definition of speed.

22.3 FUZZY SETS

Fuzzy sets introduce a certain amount of vagueness to reduce complexity of comprehension. This set consists of elements that signify the degree or grade of membership to a fuzzy aspect. Membership values usually use closed intervals and denote the sense of belonging of a member of a crisp set to a fuzzy set. To make the point clear consider a crisp set *A* comprising of elements that signify the ages of a set of people in years.

$$A = \{2,4,10,15,21,30,35,40,45,60,70\}$$

We could classify age in terms of what are known as fuzzy linguistic variables – *infant, child, adolescent, adult, young* and *old*. A person whose age is 15 is no doubt young but how would you categorize a person who is 30. If the latter is to be considered *young* what about the person who is 40? Is he old? How do we translate all these into numbers for efficiently making the computer understand what our feelings about age are?

Inspect the Table 22.1 giving ages and their membership to a particular set.

Table 22.1 *Ages and their memberships*

Age	*Infant*	*Child*	*Adolescent*	*Young*	*Adult*	*Old*
2	1	0	0	1	0	0
4	0.1	0.5	0	1	0	0
10	0	1	0.3	1	0	0
15	0	0.8	1	1	0	0
21	0	0	0.1	1	0.8	0.1
30	0	0	0	0.6	1	0.3
35	0	0	0	0.5	1	0.35
40	0	0	0	0.4	1	0.4
45	0	0	0	0.2	1	0.6
60	0	0	0	0	1	0.8
70	0	0	0	0	1	1

The values in the table indicate memberships to the fuzzy sets – *infant, child, adolescent, young, adult* and *old*. Thus a child of age 4 belongs only 50% to the fuzzy set *child* while when he is 10 years he is a 100% member. Note that membership is different from probabilities. Memberships do not necessarily add up to 1. The entries in the table have been made after a manual evaluation of the different ages.

22.4 SOME FUZZY TERMINOLOGY

Now that you have a notion of fuzziness we could define some terms based on a Universal set *U*.

Universe of Discourse (U):

This is defined as the range of all possible values that comprise the input to the fuzzy system.

Fuzzy Set

Any set that empowers its members to have different grades of membership (based on a membership function) in an interval [0,1] is a fuzzy set.

Membership function

The membership function μ_A which forms the basis of a fuzzy set is given by

$$\mu_A : U \to [0,1]$$

where the closed interval is one that holds real numbers.

Support of a fuzzy set (S_f)

The support S of a fuzzy set f, in a universal crisp set U is that set which contains all elements of the set U that have a non-zero membership value in f. For instance, the support of the fuzzy set *adult* is

$$S_{adult} = \{21,30,35,40,45,60,70\}$$

Depiction of a fuzzy set

A fuzzy set f in a universal crisp set U, is written as

$$f = \mu_1/s_1 + \mu_2/s_2 + \mu_3/s_3 + \ldots + \mu_n/s_n$$

where μ_i is the membership and s_i is the corresponding term in the *support* set of f i.e. S_f.

This is however only a representation and has *no algebraic implication* (the slash and + signs do not have any meaning).

Accordingly,

$$\text{Old} = 0.1/21 + 0.3/30 + 0.35/35 + 0.4/40 + 0.6/45 + 0.8/60 + 1/70$$

Fuzzy Set Operations

- **Union:** The membership function of the union of two fuzzy sets A and B is defined as the maximum of the two individual membership functions. It is equivalent to the Boolean OR operation.

$$\mu_{A \cup B} = \max(\mu_A, \mu_B)$$

- **Intersection:** The membership function of the intersection of two fuzzy sets A and B is defined as the minimum of the two individual membership functions and is equivalent to the Boolean AND operation.

$$\mu_{A \cap B} = \min(\mu_A, \mu_B)$$

- **Complement:** The membership function of the complement of a fuzzy set A is defined as the negation of the specified membership function: $\mu_{\bar{A}}$. This is equivalent to the Boolean NOT operation

$$\mu_{\bar{A}} = \mu_{A \cup B} = (1 - \mu_A)$$

It may be further noted here that the laws of Associativity, Commutativity , Distributivity and De Morgan's laws hold in fuzzy set theory too.

22.5 FUZZY LOGIC CONTROL

The theory of a fuzzy logic based system could remain fuzzy till one discovers how to apply it to a problem. Fuzzy logic has been used in a broad spectrum of applications ranging from domestic appliances like washing machines and cameras, to more sophisticated ones that include turbine control, tracking, data classifiers, etc. Fuzzy logic by itself does not exhibit intelligence. Invariably systems that use fuzzy logic are augmented with techniques that facilitate learning and adaptation to the environment in question.

We discuss a traditional problem of controlling the speed of a motor based on two parameters temperature and humidity. Such a model fits snug into room coolers that use a tank of water and a fan to increase humidity to bring down temperature. Coolers like these are widely used in tropical high temperature and dry environments. The same could be extended for a wide range of applications. The following description explains how fuzzy logic works and how we model a system to use the concept. The logic explained herein is said to use the Mamdani style of fuzzy inference processing.

22.5.1 Fuzzy Room Cooler

We assume the conventional room cooler implemented using a fan encased in a box with wool or hay that is continuously moistened by a trickle of water. A motorized pump controls the rate of flow of water required for moistening. Two sensors mounted inside the cooler or in the room at strategic locations measure the fan motor speed and the temperature within the room. The fan speed could be varied either by a knob by the user or could be designed to change based on an appropriate parameter sensed (humidity, for instance). The basic aim here is to achieve a smooth control and also save on water, a precious resource. Figure 22.1 shows a typical setup.

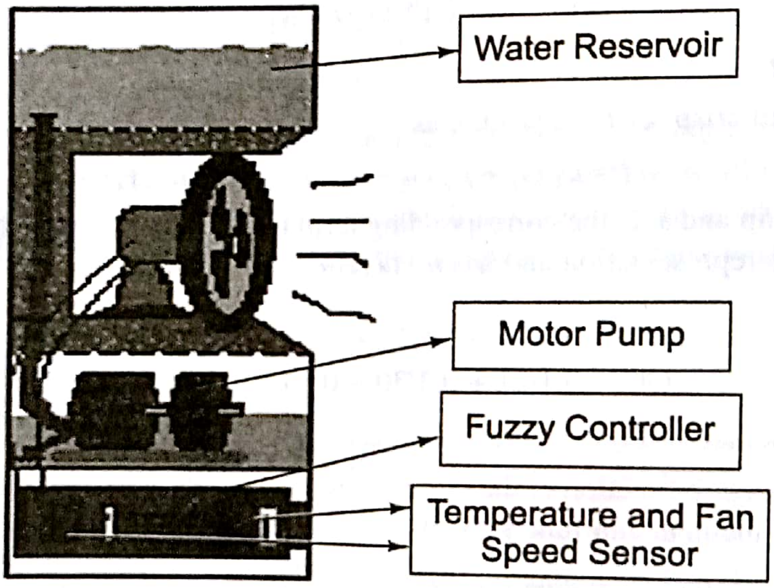


Fig. 22.1 *A Sectional View of a Fuzzy Room Cooler*

For simplicity we assume that to maintain the temperature of the room, only the rate of flow of water needs to be controlled based on the speed of the fan and the temperature. With this infrastructure we move to design the fuzzy engine to control this system.

Fuzzy regions

Two parameters (viz. temperature and pressure) decide the water flow rate. We define fuzzy terms for temperature as—*Cold*, *Cool*, *Moderate*, *Warm* and *Hot* while those for fan speed (measured in rotations per minute) as—*Slack*, *Low*, *Medium*, *Brisk*, *Fast*.

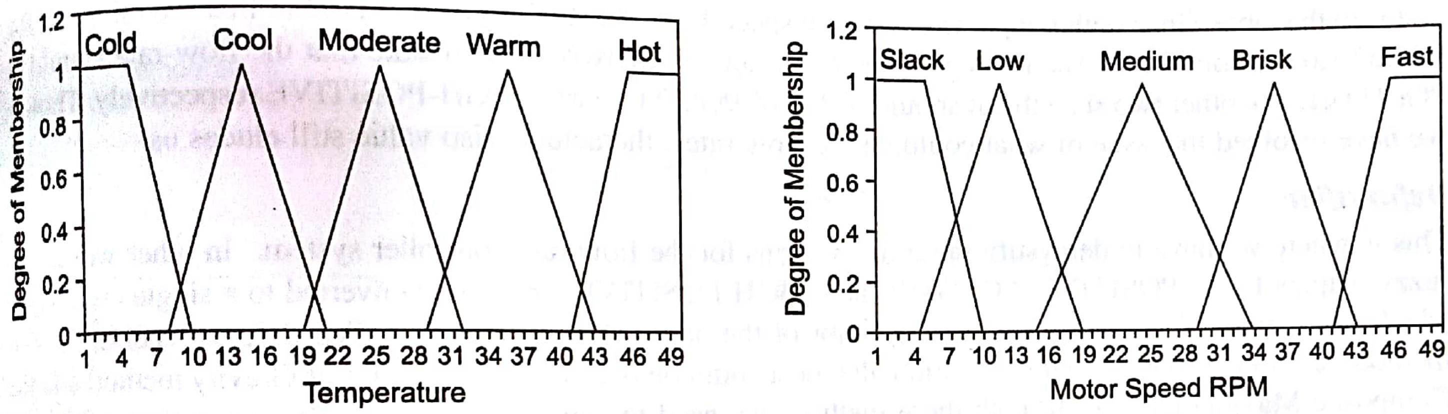
Thus temperature in the room could be defined as *Cold* or *Cool* or by any of the corresponding fuzzy linguistic variables. Likewise, the fan speed too could be defined by any of the latter variables.

The output of the system, which is the flow-rate of the water controlled by the motorized pump, could also be defined accordingly by yet another set of fuzzy terms—*Strong-Negative (SN)*, *Negative (N)*, *Low-Negative (LN)*, *Medium (M)*, *Low-Positive (LP)*, *Positive (P)* and *High-Positive (HP)*.

Fuzzy profiles

With real data available we now define profiles for each of these parameters (viz. temperature, fan motor speed and flow rate) by assigning memberships to their respective values. The graphs shown in Fig. 22.2 depict this relationship for the inputs temperature and fan speed while Fig. 22.3 reveals this for the output flow rate.

Observe that the regions for each of the sets for both the input parameters, temperature and fan motor speed, as also the output have a common intersection area. For example, we may say that when the temperature is 25 degrees, its membership to the fuzzy set *moderate* is 1 (100%). But as we drift away to 30 degrees, its membership to this set decreases while the same to the set *warm* starts to increase. Thus when the temperature is 30 degrees it is neither fully *moderate* nor *warm*. These profiles have to be carefully designed after studying the nature and desired behaviour of the system.



(a) Temperature *(b) Fan Motor Speed*

Fig. 22.2

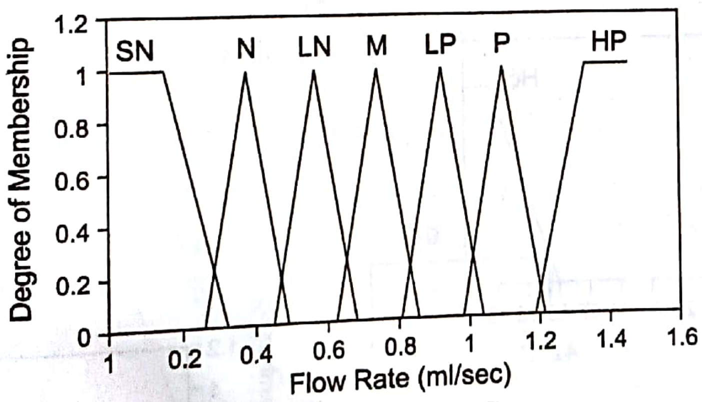


Fig. 22.3 *Water Flow Rate*

Fuzzy Rules

The fuzzy rules form the triggers of the fuzzy engine. After a study of the system we could write linguistic rules (so akin to natural language) such as –

R1: If temperature is HOT ***and*** fan motor speed is SLACK then flow-rate is HIGH-POSITIVE.
R2: If temperature is HOT ***and*** fan motor speed is LOW then flow-rate is HIGH-POSITIVE
R3: If temperature is HOT ***and*** fan motor speed is MEDIUM then the flow-rate is POSITIVE.
R4: If temperature is HOT ***and*** fan motor speed is BRISK then the flow-rate is HIGH-POSITIVE.
R5: If temperature is WARM ***and*** fan motor speed is MEDIUM then the flow-rate is LOW-POSITIVE,
R6: If temperature is WARM ***and*** fan motor speed is BRISK then the flow-rate is POSITIVE.
R7: If temperature is COOL ***and*** fan motor speed is LOW then flow-rate is NEGATIVE.
R8: If temperature is MODERATE ***and*** fan motor speed is LOW then flow-rate is MEDIUM.

The reader is urged to write the remaining set of rules based on the requirement of the system.

Fuzzification

The fuzzifier forms the heart of the fuzzy engine. Whenever the sensors report the values of temperature and fan speed, they are mapped based on their memberships to the respective fuzzy regions they belong to. For instance if at some instance of time t the temperature is 42 degrees and fan speed is 31 rpm, the corresponding membership values and the associated fuzzy regions are mentioned below

Parameter	*Fuzzy Regions*	*Memberships*
Temperature Fan speed	warm, hot medium, brisk	0.142, 0.2 0.25,0.286

From the table, since both temperature and fan speed belong to two regions, it is clear that the rules R3, R4, R5 and R6 are applicable. The rules indicate a conflict. While two of them state that the flow-rate should be POSITIVE, the other two state that it should be LOW-POSITIVE and HIGH-POSITIVE respectively. Though we have resolved the issue of what could be the flow rates, the actual crisp value still eludes us.

Defuzzifier

This is where we have to demystify these fuzzy terms for the flow rate controller system. In other words, the fuzzy outputs LOW-POSITIVE, POSITIVE and HIGH-POSITIVE are to be converted to a single crisp value which can then be delivered to the final actuator of the pump. This process is called defuzzification. Several methods are used to achieve defuzzification, the most common ones being the Centre of Gravity method and the Composite Maxima method. In both these methods we need to compute the composite region formed by the portions A, B, C and D (See Fig. 22.4) on the output profile. Figure 22.4 shows how this is calculated. In case of parameters whose premises are connected by an *AND*, the minimum of their memberships is first found. This

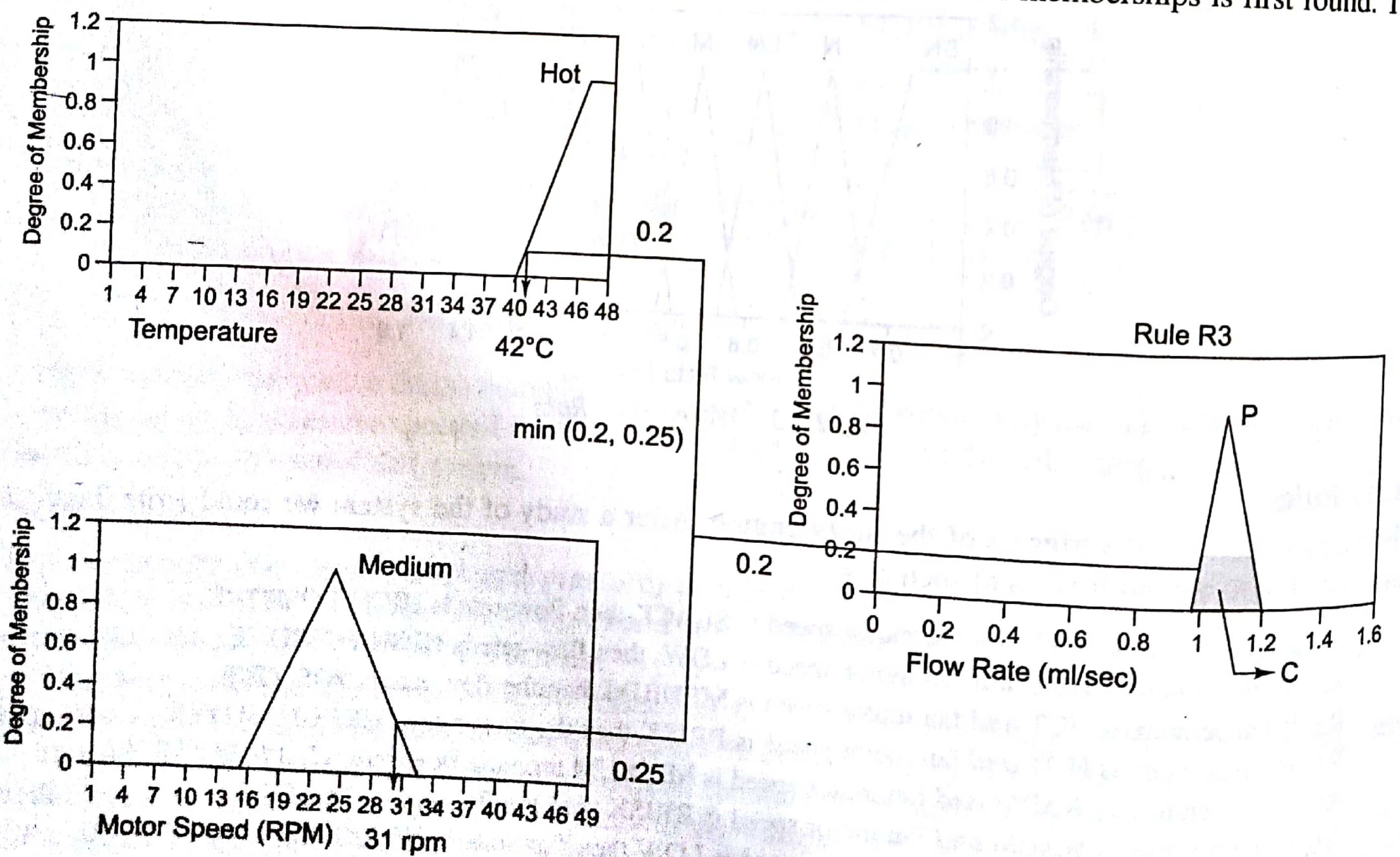


Fig. 22.4 *Defuzzification (contd.)*