CE232 DIGITAL SYSTEM

Topic 5. Simplification of Logical Equations using Tabular Method

Prepared by Nabila Husna Shabrina

Contact : nabila.husna@umn.ac.id

Subtopic

5.1 Tabular Method

5.2 Quine-McCluskey Method

5.1 Tabular Method

5.1 Tabular Method

- The tabular method (also known as the Quine-McCluskey method) is particularly useful when minimizing functions having a large number of variables, for example for 6 variables functions
- The method reduces a function in standard sum of products (SSOP) form to a set of prime implicants from which as many variables are eliminated as possible
- The tabular method makes repeated use of the law $A + \bar{A} = 1$

5.2 Quine-McCluskey Method

5.2 Quine-McCluskey Method

- **The Quine-McCluskey method** is an exact algorithm which **finds a minimum-cost sum-of-products** implementation of a Boolean function
- **2 basic concept in Quine-McCluskey Method :**
  - **Prime Implicants**

    Implicants : groups of ‘1’

    Prime implicants : largest possible groups of ‘1’
  - **Essential Prime Implicants**

    Prime implicants having at least 1 minterms cannot be combine in any other way

5.2 Quine-McCluskey Method

Step

1. Arrange the given minterms/maxterms in an ascending order and make groups based on the number of ‘1’
2. Compare in successive groups to find **“matched pair”**
3. Repeat step 2 with newly formed tems till we get all prime implicants
4. Formulate prime implicants table and find essential prime implicants

5.2 Quine-McCluskey Method

Example 1.

$F(A, B, C, D) = \sum m(0,1,3,7,8,9,11,15)$

Binary representation

0 – 0000

1 – 0001

3 – 0011

7 – 0111

8 – 1000

9 – 1001

11 – 1011

15 – 1111

5.2 Quine-McCluskey Method

Step 1 : Arrange the given minterms in an ascending order and make groups based on the number of ‘1’

Table Result

Group	Minterm	A B C D
0	m0	0 0 0 0
1	m1	0 0 0 1
	m8	1 0 0 0
2	m3	0 0 1 1
	m9	1 0 0 1
3	m7	0 1 1 1
	m11	1 0 1 1
4	m15	1 1 1 1

5.2 Quine-McCluskey Method

Step 2 : Compare in successive groups to find **“matched pair”**

- **Matched pair : pair of minterms with only differ with 1 bit position**
- If there is a change in only one-bit position, take the pair of those two min terms. Place symbol ‘-’ in the differed bit position and keep the remaining bits as it is

Group	Minterm	A B C D
0	m0	0 0 0 0
1	m1	0 0 0 1
	m8	1 0 0 0
2	m3	0 0 1 1
	m9	1 0 0 1
3	m7	0 1 1 1
	m11	1 0 1 1
4	m15	1 1 1 1

Differ in 1 bit

compare n and n+1 groups, For example, compare group of 0 with group of 1

5.2 Quine-McCluskey Method

Step 2 : Compare in successive groups to find **“matched pair”**

Table Result

Group	Matched pair	A B C D
0	m0 – m1	0 0 0 -
	m0 – m8	- 0 0 0
1	m1 – m3	0 0 – 1
	m1 – m9	- 0 0 1
	m8 – m9	1 0 0 -
2	m3 – m7	0 - 1 1
	m3 – m11	- 0 1 1
	m9 – m11	1 0 – 1
3	m7 – m15	- 1 1 1
	m11 – m15	1- 1 1

5.2 Quine-McCluskey Method

Step 3 : Repeat step 2 with newly formed terms till we get all prime implicants

- m0-m1-m8-m3 not matched pair because there is 2 bit changing
- m0-m1-m8-m9 is matched pair

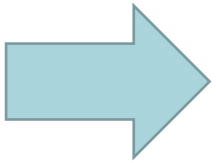
Group	Matched pair	A B C D
0	m0 – m1	0 0 0 -
	m0 – m8	- 0 0 0
1	m1 – m3	0 0 – 1
	m1 – m9	- 0 0 1
	m8 – m9	1 0 0 -
2	m3 – m7	0 - 1 1
	m3 – m11	- 0 1 1
	m9 – m11	1 0 – 1
3	m7 – m15	- 1 1 1
	m11 – m15	1- 1 1

Differ in 1 bit

5.2 Quine-McCluskey Method

Step 3 : Repeat step 2 with newly formed terms till we get all prime implicants

- 0 0 – $\rightarrow \bar{B}\bar{C}$
- 0 – 1 $\rightarrow \bar{B}D$
- - 1 1 $\rightarrow CD$



PRIME IMPLICANTS

Table Result

Group	Matched pair	A B C D
0	m0 – m1 – m8 – m9	- 0 0 -
	m0 – m8 – m1 – m9	- 0 0 -
1	m1– m3 – m9 – m11	- 0 – 1
	m1 – m9 – m3 – m11	- 0 – 1
2	m3 – m7 – m 11 – m15	0 - 1 1
	m3 – m11 – m7 – m15	- 0 1 1

5.2 Quine-McCluskey Method

Step 4 : Formulate prime implicants table and find essential prime implicants

→ $\bar{B}\bar{C} + CD$

P.I.	Minterms Involved	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0, 1, 8, 9	x	x			x	x		
$\bar{B}D$	1,3,9, 11		x	x			x	x	
CD	3, 7, 11, 15			x	x			x	x

5.2 Quine-McCluskey Method

Example 2. (POS form)

$F(A,B,C,D) = \prod M(0,1,3,7,8,9,11,15)$

Binary representation

0 – 0000

1 – 0001

3 – 0011

7 – 0111

8 – 1000

9 – 1001

11 – 1011

15 – 1111

5.2 Quine-McCluskey Method

Step 1 : Arrange the given maxterms in an ascending order and make groups based on the number of ‘1’

Group	Maxterms	A B C D
0	M0	0 0 0 0
1	M1	0 0 0 1
	M8	1 0 0 0
2	M3	0 0 1 1
	M9	1 0 0 1
3	M7	0 1 1 1
	M11	1 0 1 1
4	M15	1 1 1 1

5.2 Quine-McCluskey Method

Step 2 : Compare in successive groups to find **“matched pair”**

Group	Matched pair	A B C D
0	M0 – M1	0 0 0 -
	M0 – M8	- 0 0 0
1	M1 – M3	0 0 – 1
	M1 – M9	- 0 0 1
	M8 – M9	1 0 0 -
2	M3 – M7	0 - 1 1
	M3 – M11	- 0 1 1
	M9 – M11	1 0 – 1
3	M7 – M15	- 1 1 1
	M11 – M15	1- 1 1

5.2 Quine-McCluskey Method

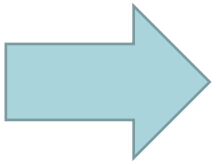
Step 3 : Repeat step 2 with newly formed terms till we get all prime implicants

Group	Matched pair	A B C D
0	M0 – M1 – M8 – M9	- 0 0 -
	M0 – M8 – M1 – M9	- 0 0 -
1	M1– M3 – M9 – M11	- 0 – 1
	M1 – M9 – M3 – M11	- 0 – 1
2	M3 – M7 – M 11 – M15	0 - 1 1
	M3 – M11 – M7 – M15	- 0 1 1

- 0 0 – → $B + C$

- 0 – 1 → $B + \overline{D}$

- - 1 1 → $\bar{C} + \overline{D}$



PRIME IMPLICANTS

5.2 Quine-McCluskey Method

Step 4 : Formulate prime implicants table and find essential prime implicants

→ $(B + C)\ (\bar{C} + \bar{D})$

P.I.	Maxterms Involved	0	1	3	7	8	9	11	15
$B + C$	0, 1, 8, 9	x	x			x	x		
$B + \bar{D}$	1,3,9, 11		x	x			x	x	
$\bar{C} + \bar{D}$	3, 7, 11, 15			x	x			x	x

5.2 Quine-McCluskey Method

Example 3.

Simplify F(A,B,C,D) = $\sum m(0,1,2,3,4,6,7,11,12,15)$

Binary representation

0 – 0000	6 – 0110
1 – 0001	7 – 0111
2 – 0010	11 – 1011
3 – 0011	12 – 1100
4 – 0100	15 – 1111

5.2 Quine-McCluskey Method

Step 1 : Arrange the given minterms in an ascending order and make groups based on the number of ‘1’

Group	Minterm	A B C D
0	m0	0 0 0 0
1	m1	0 0 0 1
	m2	1 0 0 0
	m4	0 1 0 0
2	m3	0 0 1 1
	m6	0 1 1 0
	m12	1 1 0 0
3	m7	0 1 1 1
	m11	1 0 1 1
4	m15	1 1 1 1

5.2 Quine-McCluskey Method

Step 2 : Compare in successive groups to find **“matched pair**

Group	Matched pair	A B C D
0	m0 – m1	0 0 0 –
	m1 – m2	0 0 – 0
	m0 – m4	0 – 0 0
1	m1 – m3	0 0 – 1
	m2 – m3	0 0 1 -
	m2 – m6	0 – 1 0
	m4 – m6	0 1 – 0
	m4 – m12	**- 1 0 0**
2	m3 – m7	0 – 1 1
	m3 – m11	- 0 1 1
	m6 – m7	0 1 1 -
3	m7 – m15	- 1 1 1
	m11 – m15	1 – 1 1

5.2 Quine-McCluskey Method

Step 3 : Repeat step 2 with newly formed terms till we get all prime implicants

0 0 - - → $\bar{A}\bar{B}$

0 - - 0 → $\bar{A}\bar{D}$

0 - 1 - → $\bar{A}C$

- - 1 1 → CD

And – 1 0 0 → B$\bar{C}\bar{D}$

PRIME IMPLICANTS

Group	Matched pair	A B C D
0	m0 – m1 – m2 – m3	0 0 - -
	m0 – m2 – m4 – m6	0 - - 0
	m0– m2 – m1 – m3	0 0 - - (redundant)
	m0 – m4 – m2 – m6	0 - - 0 (redundant)
1	m2 – m3 – m 6 – m7	0 – 1 –
	m2 – m6 – m3 – m7	0 – 1 – (redundant)
2	m3 – m7 – m 11 – m15	- - 1 1
	m3 – m11 – m7 – m15	- - 1 1 (redundant)

5.2 Quine-McCluskey Method

Step 4 : Find essential prime implicants

$B\bar{C}\bar{D} + \bar{A}\bar{B} + \bar{A}\bar{D}$ + CD

P.I.	Minterms Involved	0	1	2	3	4	6	7	11	12	15
$B\bar{C}\bar{D}$	4, 12					X				12	
$\bar{A}\bar{B}$	0, 1, 2, 3	X	X	X	X						
$\bar{A}\bar{D}$	0, 2, 4, 6	X		X		X	X				
$\bar{A}C$	2, 3, 6, 7			X	X		X	X			
CD	3, 7, 11, 15				X			X	X		X

References

M. Morris Mano, Digital Design, 5th ed, Prentice Hall, 2012, **Chapter 4**

Next Topic : Signed Number Format