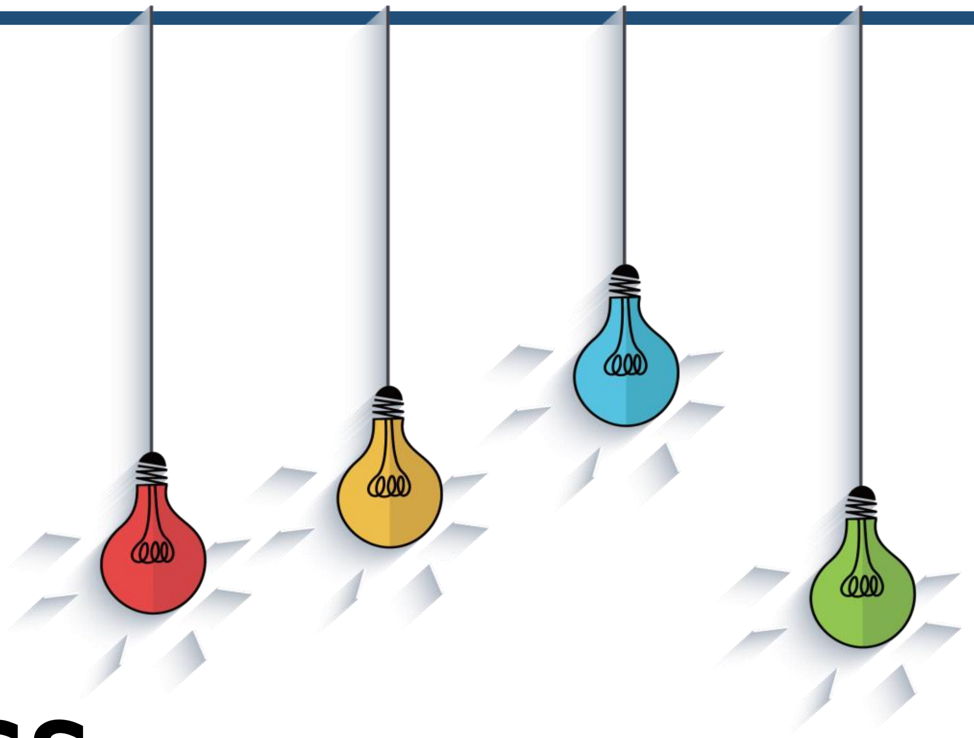

IF120
Discrete Mathematics

01 Sets

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DESCRIPTION

- Discrete Mathematics aims to teach the students to know and understand some basic concepts on Discrete Mathematics. Some topics being covered in this course are the concepts of sets, Mathematics logics and proofs, basic concept of functions, sequences, and series, relations and matrices of relations, number theory, counting methods, discrete probability, recurrence relations, graph theory, trees, combinatorial circuits and Boolean algebras.
- References:

1. Johnsonbaugh, R., 2005, *Discrete Mathematics*, New Jersey: Pearson Education, Inc.
2. Rosen, Kenneth H., 2005, *Discrete Mathematics and Its Applications*, 6th edition, McGraw-Hill.
3. Hansun, S., 2021, *Matematika Diskret Teknik*, Deepublish.

EVALUATION

- Attending lecture punctually is mandatory. If not, student could still attend the class but not considered present.
- Attending 14 class lectures is mandatory. Students have the right to attend Final Examination (Ujian Akhir Semester/ UAS) if they attend minimum 11 out of 14 lectures being held.
- Final grade is determined by following components:
  - Midterm Exam (UTS) : 30%
  - Final Exam (UAS) : 40%
  - Activities : 30%

OUTLINE

- Sets
- Basic Operation on Sets

SETS

- A **set** is a collection of objects; order is not taken into account.
- A set is simply a **collection** of objects.
- The objects are sometimes referred to as **elements** or **members**.

$$A = \{1, 2, 3, 4\}.$$

- A set is determined by its elements and not by any particular order in which the elements might be listed.
- The elements making up a set are assumed to be **distinct**, and although for some reason we may have duplicates in our list, only one occurrence of each element is in the set.

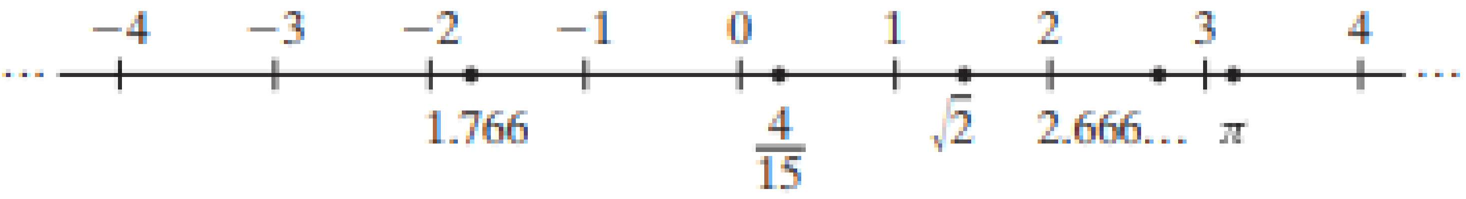
$$A = \{1, 2, 2, 3, 4\}.$$

- If a set is a large finite set or an infinite set, we can describe it by listing a **property necessary** for membership.

$$B = \{x \mid x \text{ is a positive, even integer}\}$$

SETS OF NUMBERS

Symbol	*Set*	*Example of Members*
Z	Integers	$-3, 0, 2, 145$
Q	Rational numbers	$-1/3, 0, 24/15$
R	Real numbers	$-3, -1.766, 0, 4/15, \sqrt{2}, 2.666\ldots, \pi$



- Z^{-}
- Q^{+}
- Z^{nonneg}

- If X is a finite set, we let

|X| = number of elements in X

- We call |X| the cardinality of X.

ELEMENTS, EMPTY, EQUAL

- If x is in the set X, we write $x \in X$, and if x is not in X, we write $x \notin X$

$$3 \notin \{ x \mid x \textit{ is a positive, even integer} \}$$

- The set with no elements is called the **empty** (or null or void) set and is denoted $\emptyset$.
- Two sets X and Y are **equal** and we write $X = Y$ if X and Y have the same elements.
    - For every x, if x ∈ X, then x ∈ Y , and
    - For every x, if x ∈ Y , then x ∈ X.

❑ Let us verify that if $A = \{x \mid x^2 + x - 6 = 0\}$ and $B = \{2, -3\}$, then A = B.

SUBSET

- Suppose that X and Y are sets.
- If every element of X is an element of Y , we say that X is a **subset** of Y and write X ⊆ Y.
- In other words, X is a subset of Y if for every x, if x ∈ X, then x ∈ Y.
- If C = {1, 3} and A = {1, 2, 3, 4}, by inspection, every element of C is an element of A. Therefore, C is a subset of A and we write C ⊆ A.

❑ Let $X = \{x \mid x^2 + x - 2 = 0\}$. We show that X ⊆ Z.

❑ Let $X = \{x \mid 3x^2 - x - 2 = 0\}$. We show that X is not a subset of Z.

PROPER SUBSET, POWER SET

- If X is a subset of Y and X does not equal Y, we say that X is a **proper subset** of Y and write X ⊂ Y .
- Let C = {1, 3} and A = {1, 2, 3, 4}. Then C is a proper subset of A since C is a subset of A but C does not equal A. We write C ⊂ A.
- The set of all subsets (proper or not) of a set X, denoted $P(X)$, is called the **power set** of X.
- If A = {a, b, c}, the members of P(A) are $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$. All but {a, b, c} are proper subsets of A.
- The power set of a set with n elements has 2^n elements.

UNION, INTERSECTION, DIFFERENCE

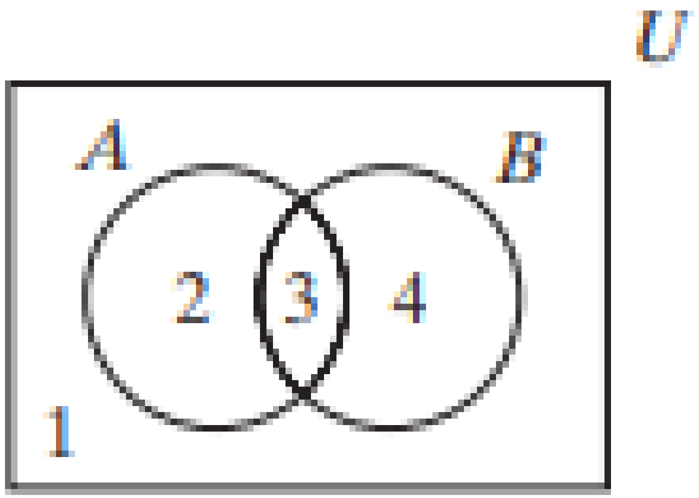
- Given two sets X and Y, there are various set operations involving X and Y that can produce a new set.
- The set $X \cup Y = \{x \mid x \in X \; or \; x \in Y\}$ is called the **union** of X and Y. The union consists of all elements belonging to either X or Y (or both).
- The set $X \cap Y = \{x \mid x \in X \; and \; x \in Y\}$ is called the **intersection** of X and Y. The intersection consists of all elements belonging to both X and Y.
- The set $X - Y = \{x \mid x \in X \; and \; x \notin Y\}$ is called the **difference** (or relative complement). The difference X – Y consists of all elements in X that are not in Y.
- If A = {1, 3, 5} and B = {4, 5, 6}, then
  - A ∪ B = {1, 3, 4, 5, 6}
  - A ∩ B = {5}
  - A – B = {1, 3}
  - B – A = {4, 6}.

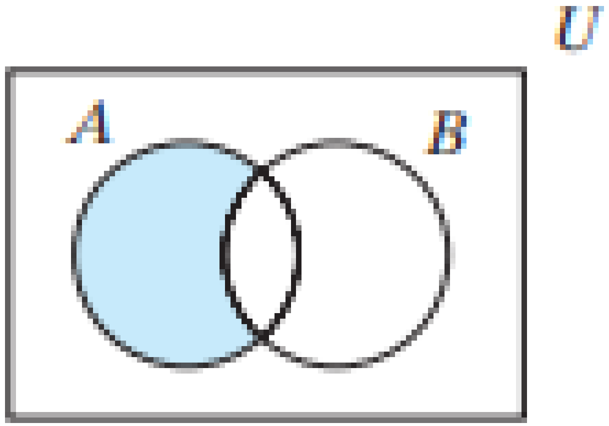
DISJOINT, UNIVERSE, COMPLEMENT

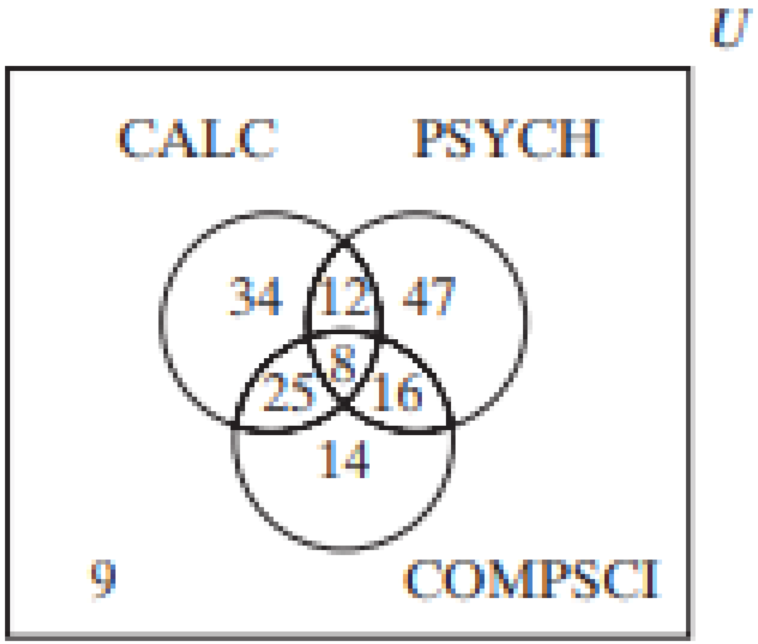
- Sets X and Y are **disjoint** if X ∩ Y = ∅. A collection of sets S is said to be **pairwise disjoint** if, whenever X and Y are distinct sets in S, X and Y are disjoint.
- The sets {1, 4, 5} and {2, 6} are disjoint. The collection of sets S = {{1, 4, 5}, {2, 6}, {3}, {7, 8}} is pairwise disjoint.
- Sometimes we are dealing with sets, all of which are subsets of a set U. This set U is called a **universal set** or a **universe**. The set U must be explicitly given or inferred from the context.
- Given a universal set U and a subset X of U, the set U − X is called the **complement** of X and is written $\bar{X}$.
- Let A = {1, 3, 5}. If U, a universal set, is specified as U = {1, 2, 3, 4, 5}, then $\bar{A}$ = {2, 4}. If, on the other hand, a universal set is specified as U = {1, 3, 5, 7, 9}, then $\bar{A}$ = {7, 9}.

VENN DIAGRAM

- Venn diagrams provide pictorial views of sets.
- In a Venn diagram, a rectangle depicts a universal set. Subsets of the universal set are drawn as circles. The inside of a circle represents the members of that set. In the figure, we see two sets A and B within the universal set U.







SETS' PROPERTIES

- Let U be a universal set and let A, B, and C be subsets of U. The following properties hold.

Identity	Name
$A \cup \phi = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \phi = \phi$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{\left(\overline{A}\right)} = A$	Complementation (involution) laws
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \phi$	Complement laws
$\overline{\phi} = U$ $\overline{U} = \phi$	0/1 laws

PARTITION, ORDERED PAIR, CARTESIAN PRODUCT

- A collection S of nonempty subsets of X is said to be a **partition** of the set X if every element in X belongs to exactly one member of S. Notice that if S is a partition of X, S is pairwise disjoint and $\cup S = X$.
- Since each element of X = {1, 2, 3, 4, 5, 6, 7, 8} is in exactly one member of S = {{1, 4, 5}, {2, 6}, {3}, {7, 8}}, S is a partition of X.
- An **ordered pair** of elements, written (a, b), is considered distinct from the ordered pair (b, a), unless, of course, a = b.
- If X and Y are sets, we let X × Y denote the set of all ordered pairs (x, y) where x ∈ X and y ∈ Y . We call X × Y the **Cartesian product** of X and Y.
- If X = {1, 2, 3} and Y = {a, b}, then what are X × Y, Y × X, X × X, Y × Y ?

N-TUPLE

- Ordered lists need not be restricted to two elements.
- An **n-tuple**, written $(a_1, a_2, \dots, a_n)$, takes order into account; that is, $(a_1, a_2, \dots, a_n) = (b_1, b_2, \dots, b_n)$ precisely when $a_1 = b_1, a_2 = b_2, \dots, a_n = b_n$.
- The Cartesian product of sets $X_1, X_2, \dots, X_n$ is defined to be the set of all n-tuples $(x_1, x_2, \dots, x_n)$ where $x_1 \in X_1$ for $i = 1, \dots, n$; it is denoted $X_1 \times X_2 \times \cdots \times X_n$.
- If X = {1, 2}, Y = {a, b}, Z = {α, β}, then

  X × Y × Z = {(1, a, α), (1, a, β), (1, b, α), (1, b, β), (2, a, α), (2, a, β), (2, b, α), (2, b, β)}
- Notice that |X × Y × Z|=|X| ·|Y | ·|Z|. In general, $|X_1 \times X_2 \times \cdots \times X_n| = |X_1| \cdot |X_2| \cdots |X_n|$.

PRACTICE

PRACTICE 1

- Let the universe be the set U = {1, 2, 3, ... , 10}. Let A = {1, 4, 7, 10}, B = {1, 2, 3, 4, 5}, and C = {2, 4, 6, 8}. List the elements of each set.

a. $A \cup B$

b. $B - A$

c. $A \cup \emptyset$

d. $\overline{A \cap B} \cup C$

e. $(A \cup B) - (C - B)$

PRACTICE 2

- Show that $A = B$.

a. $C = \{1, 2, 3\}, D = \{2, 3, 4\}, A = \{2, 3\}, B = C \cap D$

b. $A = \{x \mid x^2 - 4x + 4 = 1\}, B = \{1, 3\}$

- Show that $A \neq B$.

a. $B = \{1, 2, 3, 4\}, C = \{2, 4, 6, 8\}, A = B \cap C$

b. $A = \{1, 2\}, B = \{x \mid x^3 - 2x^2 - x + 2 = 0\}$

PRACTICE 3

- Show that $A \subseteq B$ or $A \nsubseteq B$.

a. $A = \{1, 2\}, B = \{x \mid x^3 - 6x^2 + 11x = 6\}$

b. $A = \{1\} \times \{1, 2\}, B = \{1\} \times \{1, 2, 3\}$

c. $A = \{x \mid x^3 - 2x^2 - x + 2 = 0\}, B = \{1, 2\}$

d. $A = \{1, 2, 3, 4\}, C = \{5, 6, 7, 8\}, B = \{n \mid n \in A \; and \; n + m = 8 \; for \; some \; m \in C\}$

PRACTICE 4

- There is a group of 191 students, of which 10 are taking French, business, and music; 36 are taking French and business; 20 are taking French and music; 18 are taking business and music; 65 are taking French; 76 are taking business; and 63 are taking music.

a. How many are taking French and music but not business?
b. How many are taking business and neither French nor music?
c. How many are taking French or business (or both)?
d. How many are taking music or French (or both) but not business?
e. How many are taking none of the three subjects?

NEXT WEEK’S OUTLINE

- Propositions
- Logics Operators and Truth Table
- Conditional Propositions and Logical Equivalence
- Arguments and Rules of Inference
- Quantifiers

REFERENCES

- Johnsonbaugh, R., 2005, *Discrete Mathematics*, New Jersey: Pearson Education, Inc.
- Rosen, Kenneth H., 2005, *Discrete Mathematics and Its Applications*, 6th edition, McGraw-Hill.
- Hansun, S., 2021, *Matematika Diskret Teknik*, Deepublish.
- Lipschutz, Seymour, Lipson, Marc Lars, *Schaum's Outline of Theory and Problems of Discrete Mathematics*, McGraw-Hill.
- Liu, C.L., 1995, *Dasar-Dasar Matematika Diskret*, Jakarta: Gramedia Pustaka Utama.
- Other offline and online resources.

Visi

Menjadi Program Studi Strata Satu Informatika **unggulan** yang menghasilkan lulusan **berwawasan internasional** yang **kompeten** di bidang Ilmu Komputer (*Computer Science*), **berjiwa wirausaha** dan **berbudi pekerti luhur**.

INFORMATIKA
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Misi

1. Menyelenggarakan pembelajaran dengan teknologi dan kurikulum terbaik serta didukung tenaga pengajar profesional.
2. Melaksanakan kegiatan penelitian di bidang Informatika untuk memajukan ilmu dan teknologi Informatika.
3. Melaksanakan kegiatan pengabdian kepada masyarakat berbasis ilmu dan teknologi Informatika dalam rangka mengamalkan ilmu dan teknologi Informatika.