

**PROGRAM STUDI TEKNIK KOMPUTER
FAKULTAS TEKNIK DAN INFORMATIKA
UNIVERSITAS MULTIMEDIA NUSANTARA
SEMESTER GANJIL TAHUN AJARAN 2024/2025**



CE 121 – LINEAR ALGEBRA

Pertemuan 4 Sistem Persamaan Linier 1

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Capaian Pembelajaran Mingguan Mata Kuliah (Sub-CPMK)

1. Mahasiswa mampu menentukan solusi sistem persamaan linier dengan menggunakan metode Cramer dan Invers Matrik (C3)

Sub-Pokok Bahasan

1. SPL homogen vs non homogen
2. SPL konsisten vs inkonsisten
3. Penyelesaian SPL dengan metode Cramer
4. Penyelesaian SPL dengan metode Invers Matrik

Sistem Persamaan Linier

Sistem persamaan linier dapat diselesaikan menggunakan metode:

- Eliminasi
- Cramer
- Eliminasi Gauss
- Eliminasi Gauss-Jordan

Metode Cramer

SPL bisa diselesaikan dengan menggunakan matriks

$$a_1x + a_2y + a_3z = K$$

$$b_1x + b_2y + b_3z = L$$

$$c_1x + c_2y + c_3z = M$$

Persamaan di atas dapat dibuat menjadi:

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K \\ L \\ M \end{bmatrix}$$

Metode Cramer

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K \\ L \\ M \end{bmatrix}$$

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad C = \begin{bmatrix} K \\ L \\ M \end{bmatrix} \quad \longrightarrow \quad AB = C$$

$$A_x = \begin{bmatrix} K & a_2 & a_3 \\ L & b_2 & b_3 \\ M & c_2 & c_3 \end{bmatrix} \quad A_y = \begin{bmatrix} a_1 & K & a_3 \\ b_1 & L & b_3 \\ c_1 & M & c_3 \end{bmatrix} \quad A_z = \begin{bmatrix} a_1 & a_2 & K \\ b_1 & b_2 & L \\ c_1 & c_2 & M \end{bmatrix}$$

$$x = \frac{|A_x|}{|A|}$$

$$y = \frac{|A_y|}{|A|}$$

$$z = \frac{|A_z|}{|A|}$$

Metode Cramer

Selesaikan persamaan linier berikut:

$$-2x + 3y - z = 4$$

$$3x + 2y + 3z = 1$$

$$2x - y + 3z = 2$$

Metode Cramer

$$\begin{pmatrix} -2 & 3 & -1 \\ 3 & 2 & 3 \\ 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \rightarrow AB = C$$

$$|A| = \begin{vmatrix} -2 & 3 & -1 \\ 3 & 2 & 3 \\ 2 & -1 & 3 \end{vmatrix} = -12 + 18 + 3 + 4 - 6 - 27 = -20$$

$$|A_x| = \begin{vmatrix} 4 & 3 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{vmatrix} = 24 + 18 + 1 + 4 + 12 - 9 = 50$$

$$|A_y| = \begin{vmatrix} -2 & 4 & -1 \\ 3 & 1 & 3 \\ 2 & 2 & 3 \end{vmatrix} = -6 + 24 - 6 + 2 + 12 - 36 = -10$$

$$|A_z| = \begin{vmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = -8 + 6 - 12 - 16 - 2 - 18 = -50$$

$$x = \frac{|A_x|}{|A|} = \frac{50}{-20} = -\frac{5}{2}$$

$$y = \frac{|A_y|}{|A|} = \frac{-10}{-20} = \frac{1}{2}$$

$$z = \frac{|A_z|}{|A|} = \frac{-50}{-20} = \frac{5}{2}$$

Latihan

Tentukan nilai x , y , dan z dari sistem persamaan linier berikut:

a.
$$\begin{aligned} 2x + 5y + 3z &= 1 \\ -x + 2y + z &= 2 \\ x + y + z &= 0 \end{aligned}$$

b.
$$\begin{bmatrix} 2 & 3 & -1 \\ 2 & 3 & 2 \\ 4 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}$$

Metode Cramer

$$\begin{pmatrix} 2 & 5 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \rightarrow AB = C$$

$$|A| = \begin{vmatrix} 2 & 5 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 4 + 5 - 3 - 6 - 2 + 5 = 3$$

$$|A_x| = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 2 + 0 + 6 - 0 - 1 - 10 = -3$$

$$|A_y| = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 4 + 1 + 0 - 6 - 0 + 1 = 0$$

$$|A_z| = \begin{vmatrix} 2 & 5 & 1 \\ -1 & 2 & 2 \\ 1 & 1 & 0 \end{vmatrix} = 0 + 10 - 1 - 2 - 4 - 0 = 3$$

$$x = \frac{|A_x|}{|A|} = \frac{-3}{3} = -1$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{3} = 0$$

$$z = \frac{|A_z|}{|A|} = \frac{3}{3} = 1$$

Metode Cramer

$$\begin{pmatrix} 2 & 3 & -1 \\ 2 & 3 & 2 \\ 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix} \rightarrow AB = C$$

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 2 & 3 & 2 \\ 4 & 1 & -1 \end{vmatrix} = -6 + 24 - 2 + 12 - 4 + 6 = 30$$

$$|A_x| = \begin{vmatrix} 3 & 3 & -1 \\ 0 & 3 & 2 \\ 5 & 1 & -1 \end{vmatrix} = -9 + 30 + 0 + 15 - 6 - 0 = 30$$

$$|A_y| = \begin{vmatrix} 2 & 3 & -1 \\ 2 & 0 & 2 \\ 4 & 5 & -1 \end{vmatrix} = 0 + 24 - 10 - 0 - 20 + 6 = 0$$

$$|A_z| = \begin{vmatrix} 2 & 3 & 3 \\ 2 & 3 & 0 \\ 4 & 1 & 5 \end{vmatrix} = 30 + 0 + 6 - 36 - 0 - 30 = -30$$

$$x = \frac{|A_x|}{|A|} = \frac{30}{30} = 1$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{30} = 0$$

$$z = \frac{|A_z|}{|A|} = \frac{-30}{30} = -1$$

Metode Eliminasi Gauss-Jordan

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K \\ L \\ M \end{bmatrix}$$

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad C = \begin{bmatrix} K \\ L \\ M \end{bmatrix} \quad \longrightarrow \quad AB = C$$

$$A^{-1}AB = A^{-1}C$$

Karena $A^{-1}A = I$

Maka $B = A^{-1}C$

$$\begin{aligned} BA &= C \\ BAA^{-1} &= CA^{-1} \\ B &= CA^{-1} \end{aligned}$$

Mencari A^{-1} bisa menggunakan metode Eliminasi Gauss-Jordan

Metode Eliminasi Gauss-Jordan

$$[A|I] \xrightarrow{\text{red arrow}} [I|A^{-1}]$$

$$B = A^{-1}C$$

Tentukan nilai x , y , dan z menggunakan metode Eliminasi Gauss-Jordan

$$x - 3y + 5z = 10$$

$$3x + 4y + 2z = 17$$

$$2x + 5y - 2z = 6$$

$$\begin{aligned}
& \begin{pmatrix} 1 & -3 & 5 & | & 1 & 0 & 0 \\ 3 & 4 & 2 & | & 0 & 1 & 0 \\ 2 & 5 & -2 & | & 0 & 0 & 1 \end{pmatrix} b_2 - 3b_1 \\
& \begin{pmatrix} 1 & -3 & 5 & | & 1 & 0 & 0 \\ 0 & 13 & -13 & | & -3 & 1 & 0 \\ 0 & 11 & -12 & | & -2 & 0 & 1 \end{pmatrix} b_2 \times \left(\frac{1}{13}\right) \\
& \begin{pmatrix} 1 & -3 & 5 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -3/13 & 1/13 & 0 \\ 0 & 11 & -12 & | & -2 & 0 & 1 \end{pmatrix} b_3 - 11b_2 \\
& \begin{pmatrix} 1 & -3 & 5 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -3/13 & 1/13 & 0 \\ 0 & 0 & -1 & | & 7/13 & -11/13 & 1 \end{pmatrix} b_3 \times (-1)
\end{aligned}$$

$$\begin{aligned}
& \begin{pmatrix} 1 & -3 & 5 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -3/13 & 1/13 & 0 \\ 0 & 0 & 1 & | & -7/13 & 11/13 & -1 \end{pmatrix} b_1 - 5b_3 \\
& \begin{pmatrix} 1 & -3 & 0 & | & 48/13 & -55/13 & 5 \\ 0 & 1 & 0 & | & -10/13 & 12/13 & -1 \\ 0 & 0 & 1 & | & -7/13 & 11/13 & -1 \end{pmatrix} b_1 + 3b_2 \\
& \begin{pmatrix} 1 & 0 & 0 & | & 18/13 & -19/13 & 2 \\ 0 & 1 & 0 & | & -10/13 & 12/13 & -1 \\ 0 & 0 & 1 & | & -7/13 & 11/13 & -1 \end{pmatrix} \\
& A^{-1} = \begin{bmatrix} 18/13 & -19/13 & 2 \\ -10/13 & 12/13 & -1 \\ -7/13 & 11/13 & -1 \end{bmatrix}
\end{aligned}$$

$$A^{-1} = \begin{bmatrix} 18/13 & -19/13 & 2 \\ -10/13 & 12/13 & -1 \\ -7/13 & 11/13 & -1 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 18 & -19 & 26 \\ -10 & 12 & -13 \\ -7 & 11 & -13 \end{bmatrix}$$

$$B = A^{-1}C = \frac{1}{13} \begin{bmatrix} 18 & -19 & 26 \\ -10 & 12 & -13 \\ -7 & 11 & -13 \end{bmatrix} \begin{bmatrix} 10 \\ 17 \\ 6 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 180 - 323 + 156 \\ -100 + 204 - 78 \\ -70 + 187 - 78 \end{bmatrix}$$

$$B = \frac{1}{13} \begin{bmatrix} 13 \\ 26 \\ 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \longrightarrow \boxed{x = 1} \quad \boxed{y = 2} \quad \boxed{z = 3}$$

Tentukan x, y , dan z dari sistem persamaan linier berikut:

$$2x + 5y + 3z = 1$$

$$-x + 2y + z = 2$$

$$x + y + z = 0$$

$$\left(\begin{array}{ccc|ccc} 2 & 5 & 3 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right) b_3 = b_1$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 5 & 3 & 1 & 0 & 0 \end{array} \right) b_2 + b_1$$

$$b_3 - 2b_1$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 1 \\ 0 & 3 & 1 & 1 & 0 & -2 \end{array} \right) b_3 - b_2$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -1 & -3 \end{array} \right) b_3 \times (-1)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 3 \end{array} \right) b_1 - b_3$$

$$b_2 - 2b_3$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & -1 & -2 \\ 0 & 3 & 0 & 2 & -1 & -5 \\ 0 & 0 & 1 & -1 & 1 & 3 \end{array} \right) b_2 \times \left(\frac{1}{3} \right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & -1 & -2 \\ 0 & 1 & 0 & 2/3 & -1/3 & -5/3 \\ 0 & 0 & 1 & -1 & 1 & 3 \end{array} \right) b_1 - b_2$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & -1/3 \\ 0 & 1 & 0 & 2/3 & -1/3 & -5/3 \\ 0 & 0 & 1 & -1 & 1 & 3 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} 1/3 & -2/3 & -1/3 \\ 2/3 & -1/3 & -5/3 \\ -1 & 1 & 3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & -2 & -1 \\ 2 & -1 & -5 \\ -3 & 3 & 9 \end{pmatrix}$$

$$B = A^{-1}C = \frac{1}{3} \begin{pmatrix} 1 & -2 & -1 \\ 2 & -1 & -5 \\ -3 & 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 - 4 - 0 \\ 2 - 2 - 0 \\ -3 + 6 + 0 \end{pmatrix}$$

$$B = \frac{1}{3} \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \boxed{x = -1} \quad \boxed{y = 0} \quad \boxed{z = 1}$$

Latihan

Gunakan metode Eliminasi Gauss-Jordan untuk sistem persamaan linier berikut:

$$3u + 4v + 2w = 1$$

$$u - 3v + 5w = 22$$

$$2u + 5v - 2w = -14$$



$$\left[\begin{array}{ccc|c} 3 & 4 & 2 & 1 \\ 1 & -3 & 5 & 22 \\ 2 & 5 & -2 & -14 \end{array} \right]$$

$$3x - y + 2z = 11$$

$$-4x + 2y + 7z = 13$$

$$11x + 5y + 9z = 28$$



$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 11 \\ -4 & 2 & 7 & 13 \\ 11 & 5 & 9 & 28 \end{array} \right]$$

Terima Kasih

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di Pertemuan Selanjutnya**