CE232 DIGITAL SYSTEM

Topic 4. K-MAP

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Subtopic

4.1 Basic K-MAP

4.2 K-MAP Implementation

4.1 Basic K-MAP

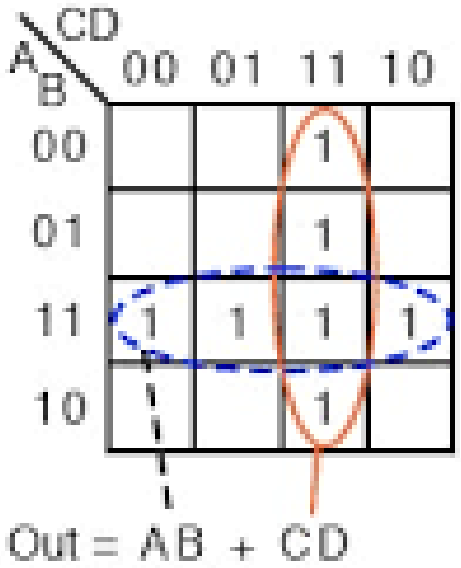
4.1 Basic K-MAP

- K-Map stands for Karnaugh Map, it is developed by Karnaugh in 1953
- Used to simplify Boolean algebraic expression without having to use Boolean algebra theorems and Laws
- It use pictorial method to solve the Boolean expression

4.1 Basic K-MAP

Example of K-MAP

$$Out = \overline{A}\overline{B}CD + \overline{A}BCD + ABCD + A\overline{B}CD + AB\overline{C}\overline{D} + AB\overline{C}D + ABC\overline{D}$$



4.2 K-MAP Implementation

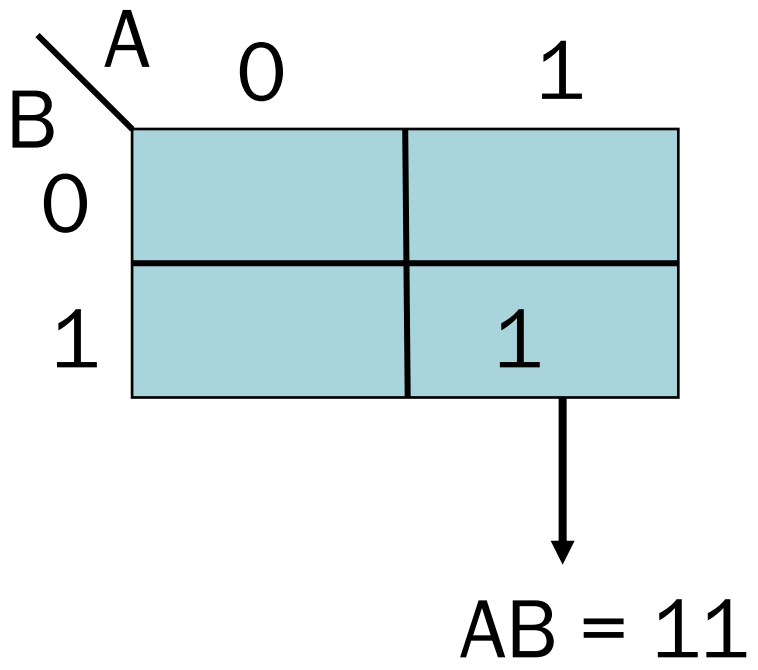
4.2 K-MAP Implementation

2 Variable K-MAP

- Number of cells = 2^n

where n= number of variable

- For 2 variable k-map, the number of cells is $2^2 = 4$
- Put “1” in the represented cell

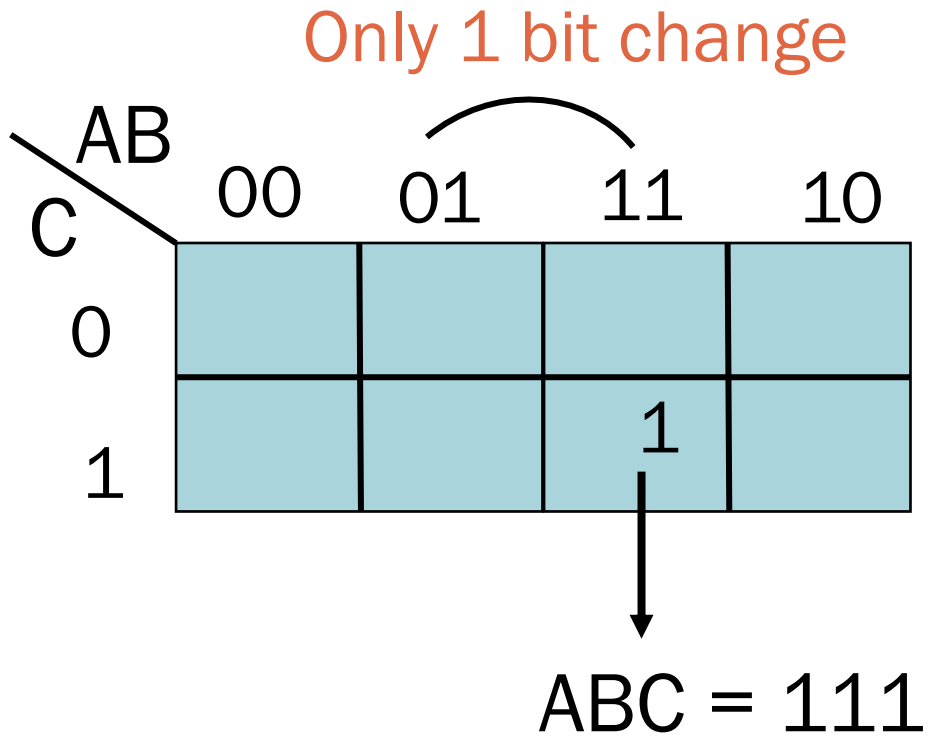


4.2 K-MAP Implementation

3 Variable K-MAP

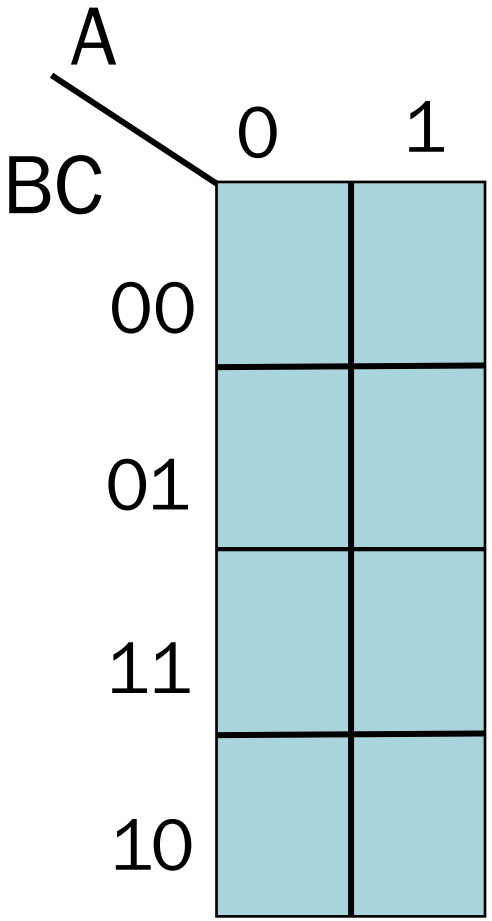
Can be represented horizontally or vertically

For 3 variable k-map, the number of cells is $2^3 = 8$



4.2 K-MAP Implementation

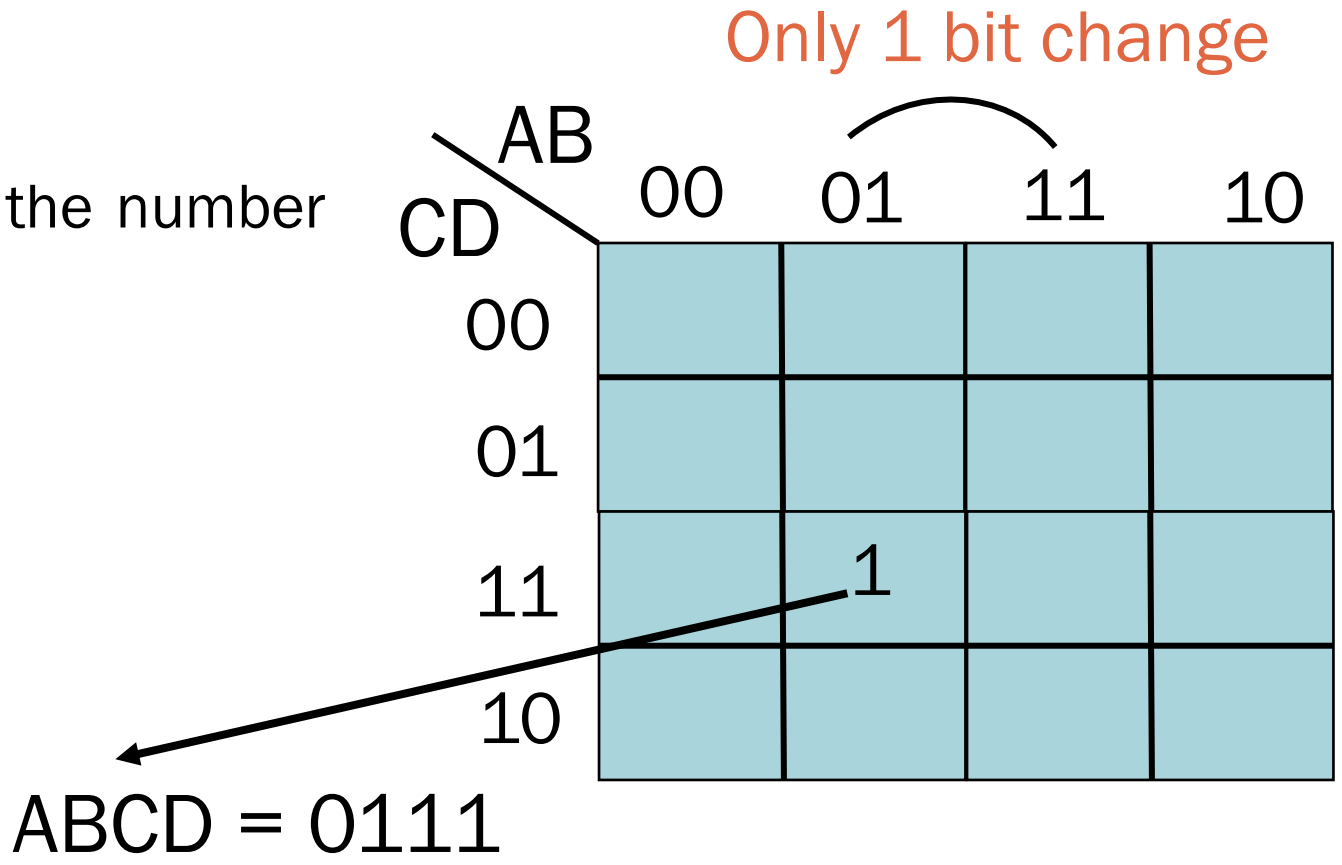
Another representation on K-Map with 3 Variable



4.2 K-MAP Implementation

4 Variable K-MAP

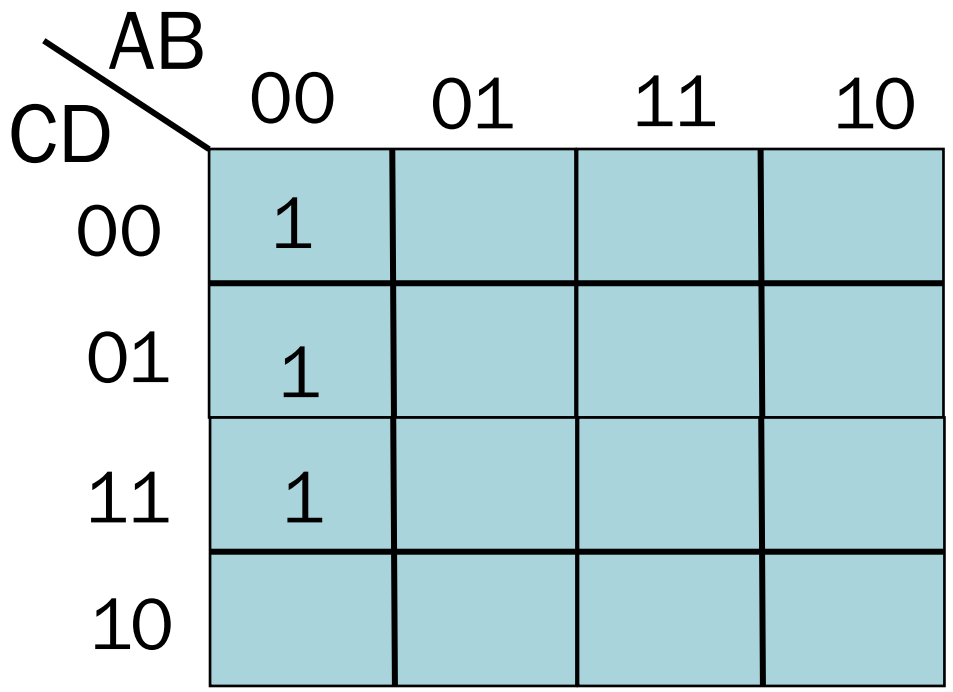
For 4 variable k-map, the number of cells is $2^4 = 16$



4.2 K-MAP Implementation

Representation using K-MAP

Output : A’B’C’D+A’B’C’D+A’B’CD



4.2 K-MAP Implementation

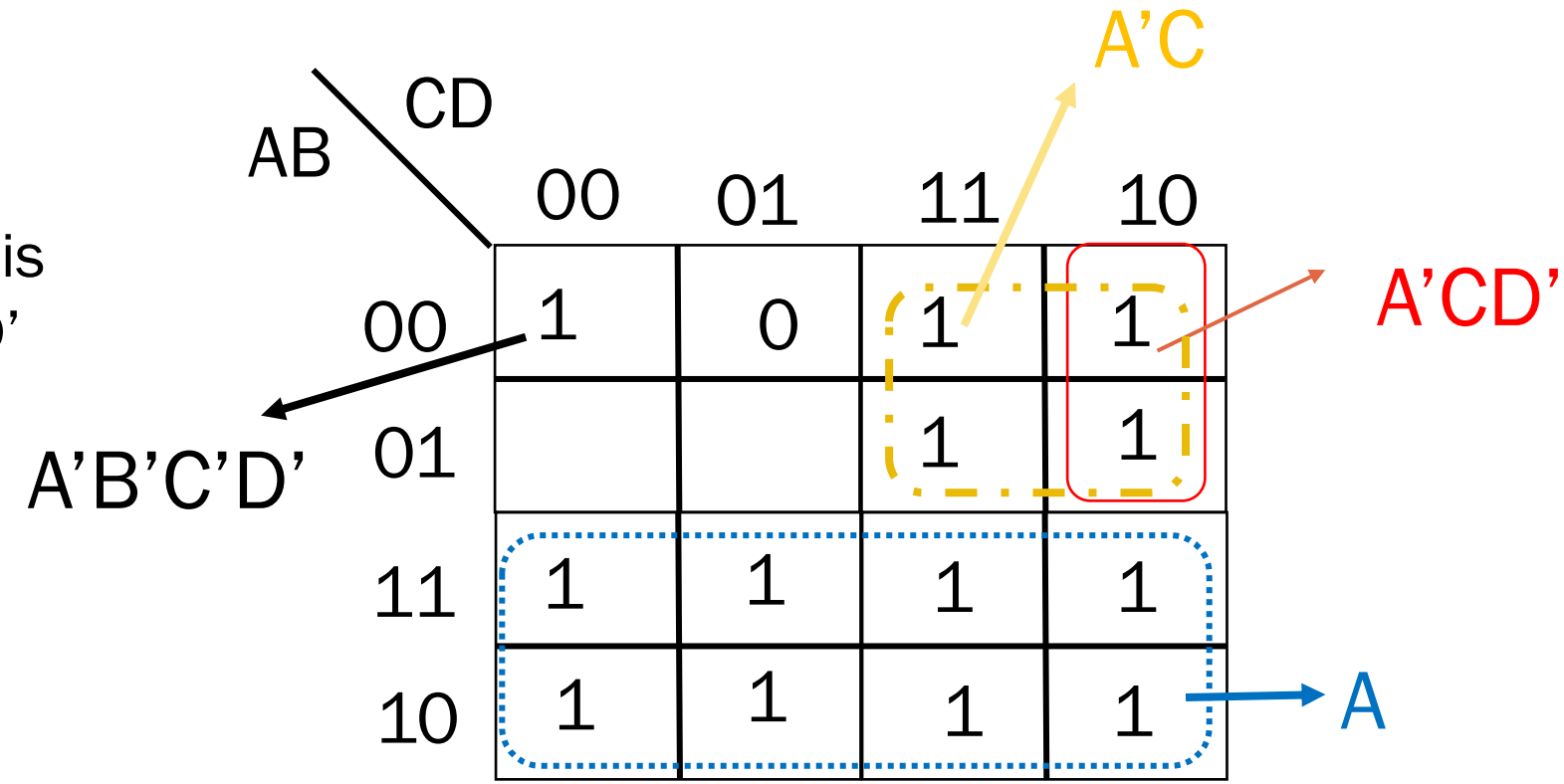
Rules for K-MAP Simplification (For SOP Expression)

- Groups may not contain 0 (in POS groups may not contain 1)
- We can group 1,2,4,8, or 2^n cells
- Each group should be as large as possible
- Cells containing 1 must be grouped
- Groups may overlap
- Opposite grouping and corner grouping is allowed
- There should be as few groups as possible

4.2 K-MAP Implementation

Example

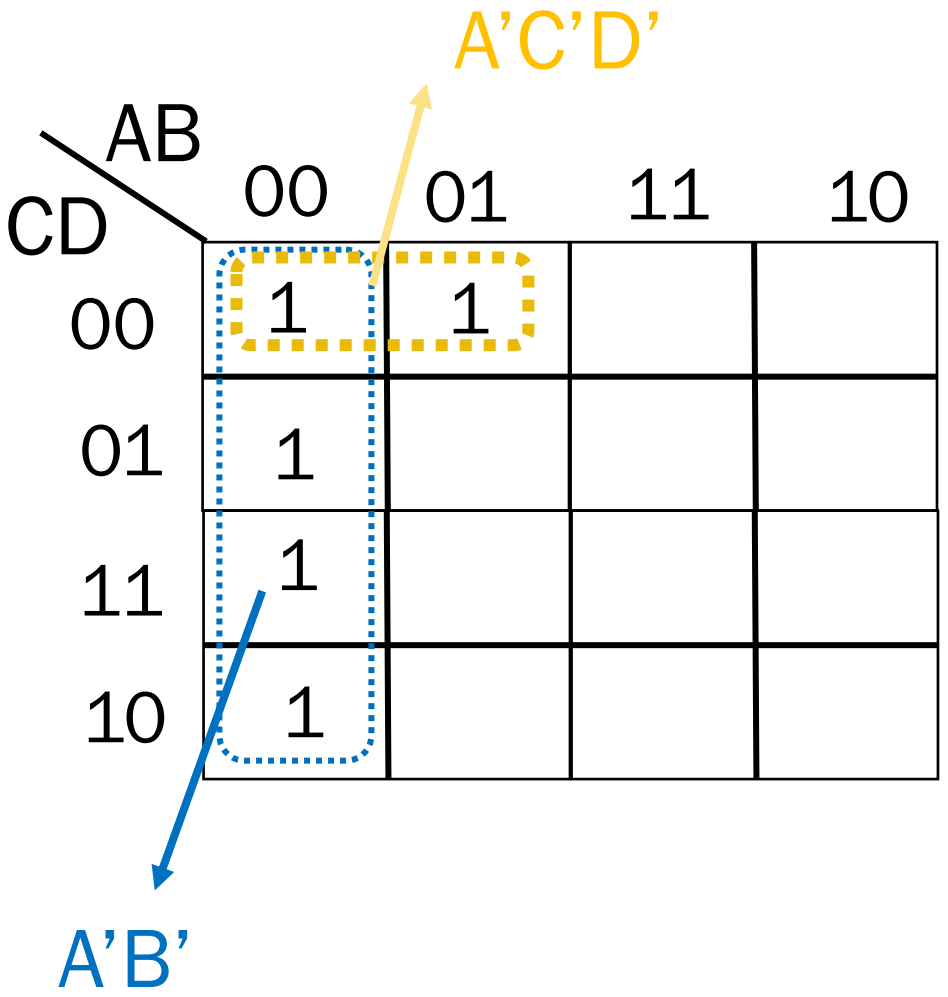
The expression is
=A+A’C+A’B’C’D’



4.2 K-MAP Implementation

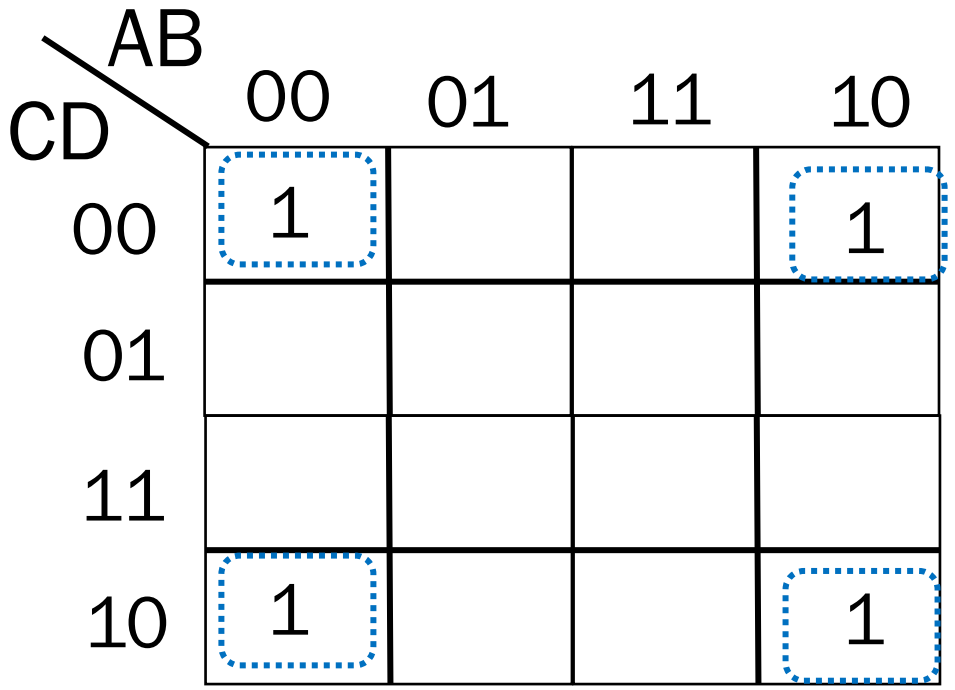
Example.

A’B’+A’C’D’



4.2 K-MAP Implementation

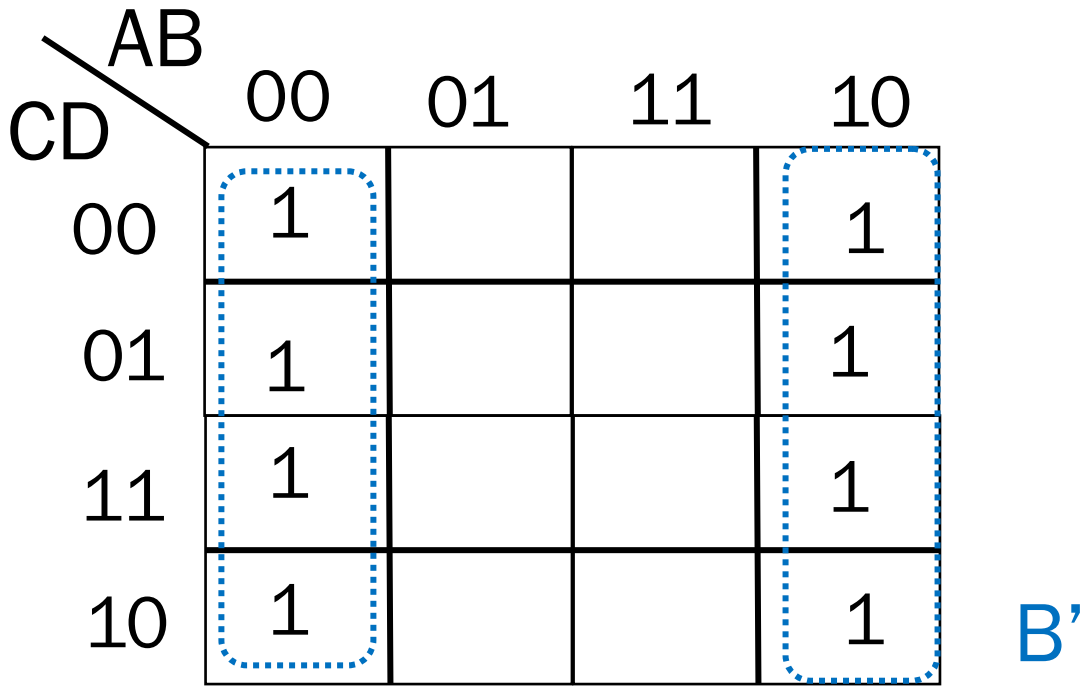
Example corner grouping



B’D’

4.2 K-MAP Implementation

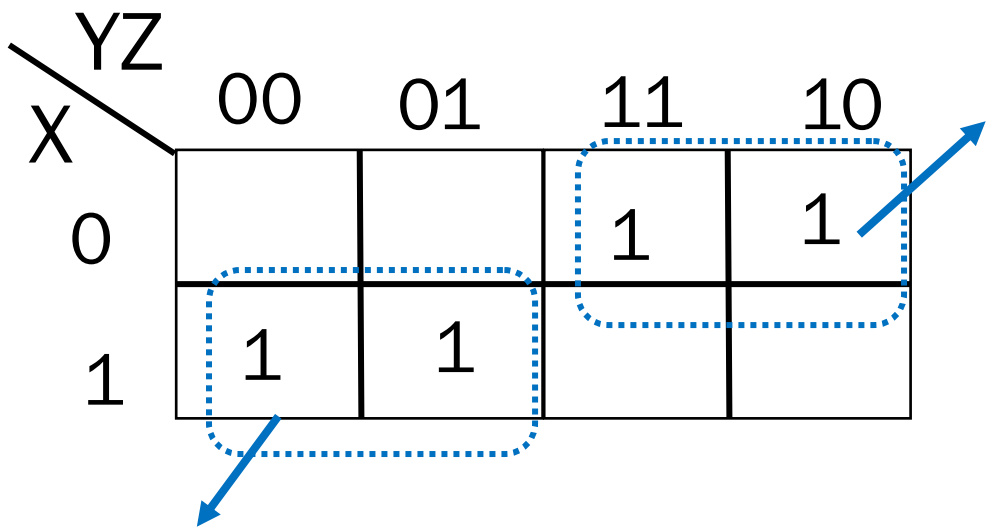
Example opposite grouping



4.2 K-MAP Implementation

Example.

In the sum of products function $f(x, y, z) = \sum m(2,3,4,5)$, find the prime implicants



The prime implicant = X'Y+XY'

4.2 K-MAP Implementation

Example.

K-MAP for Boolean example is attached. Find the number of essential prime implicants

Number of prime implicants = 4

CD \ AB	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	1	0	0	0
10	1	0	0	1

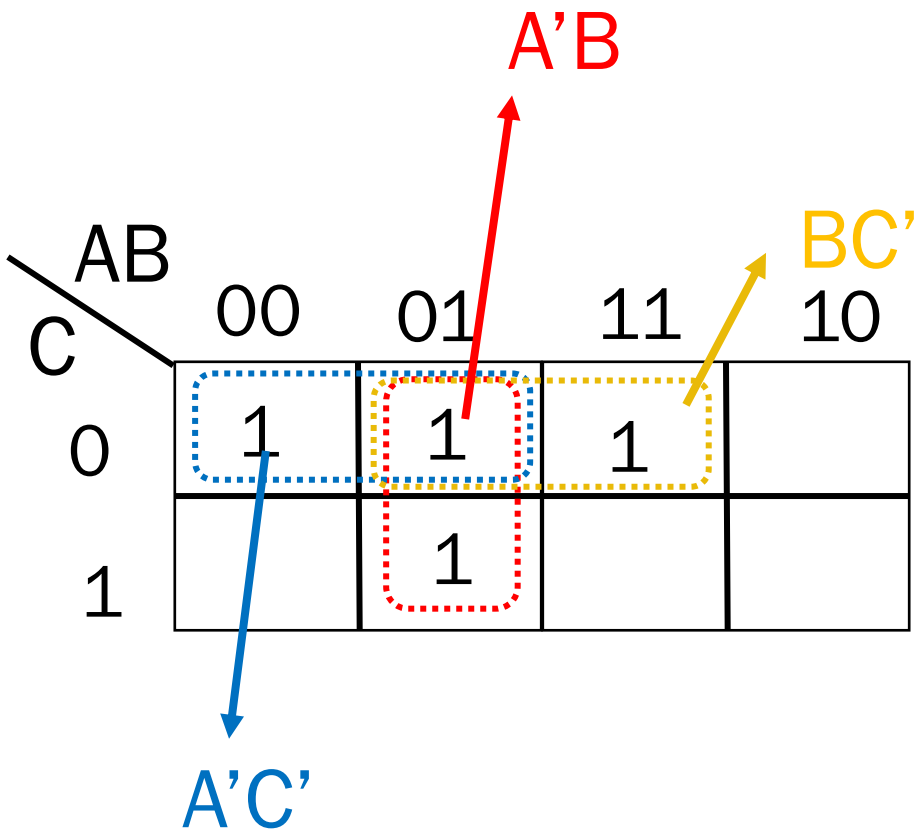
4.2 K-MAP Implementation

Example.

Find the minimized form of
A’B’C’+A’BC’+A’BC+ABC’

000 010 011 110

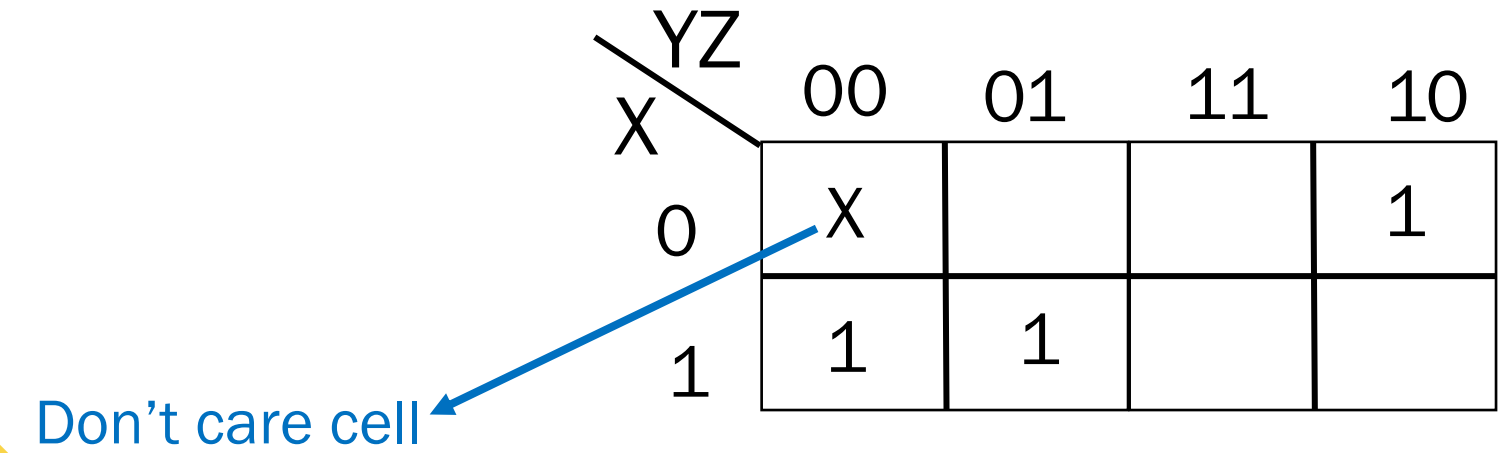
= A’C’+A’B+BC’



4.2 K-MAP Implementation

Don’t Care

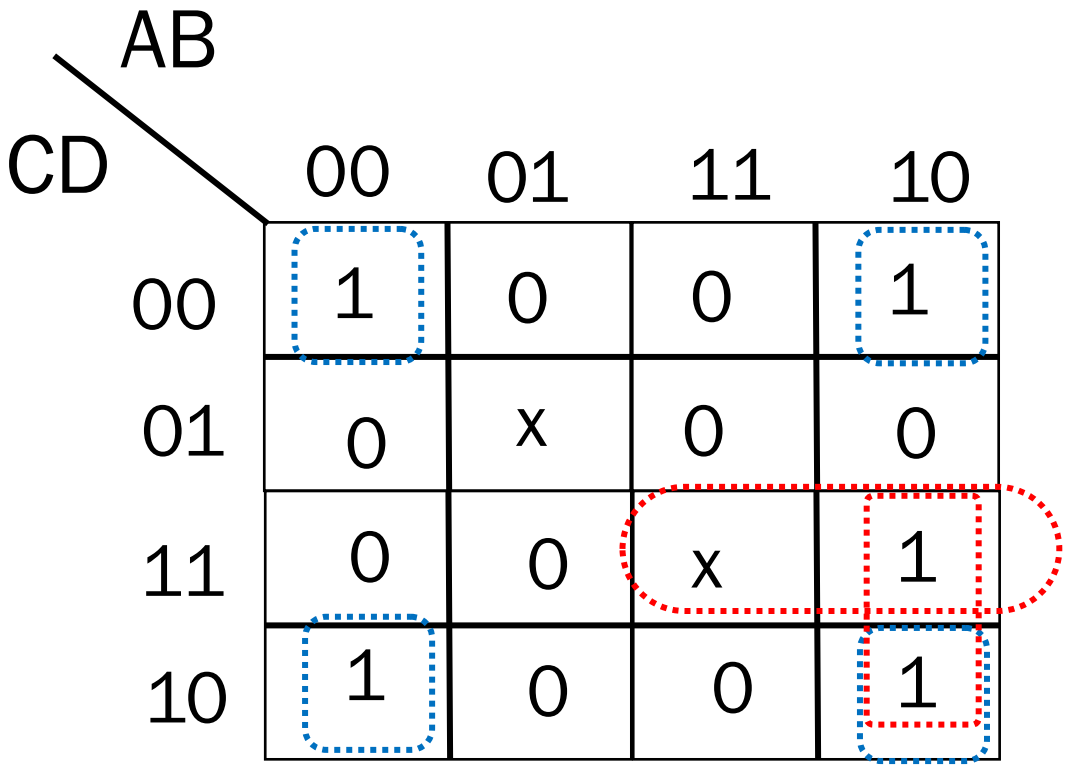
- **A don’t care cell** can be represented by a cross (X) or minus (-)
- We can consider a don’t care cell as 1 or 0



4.2 K-MAP Implementation

Example.

Find the number of product terms in the minimized sum of product (SOP) expression obtained through the following K-MAP ➔ 2



4.2 K-MAP Implementation

Example.

Minimize the following Boolean expression using K-MAP

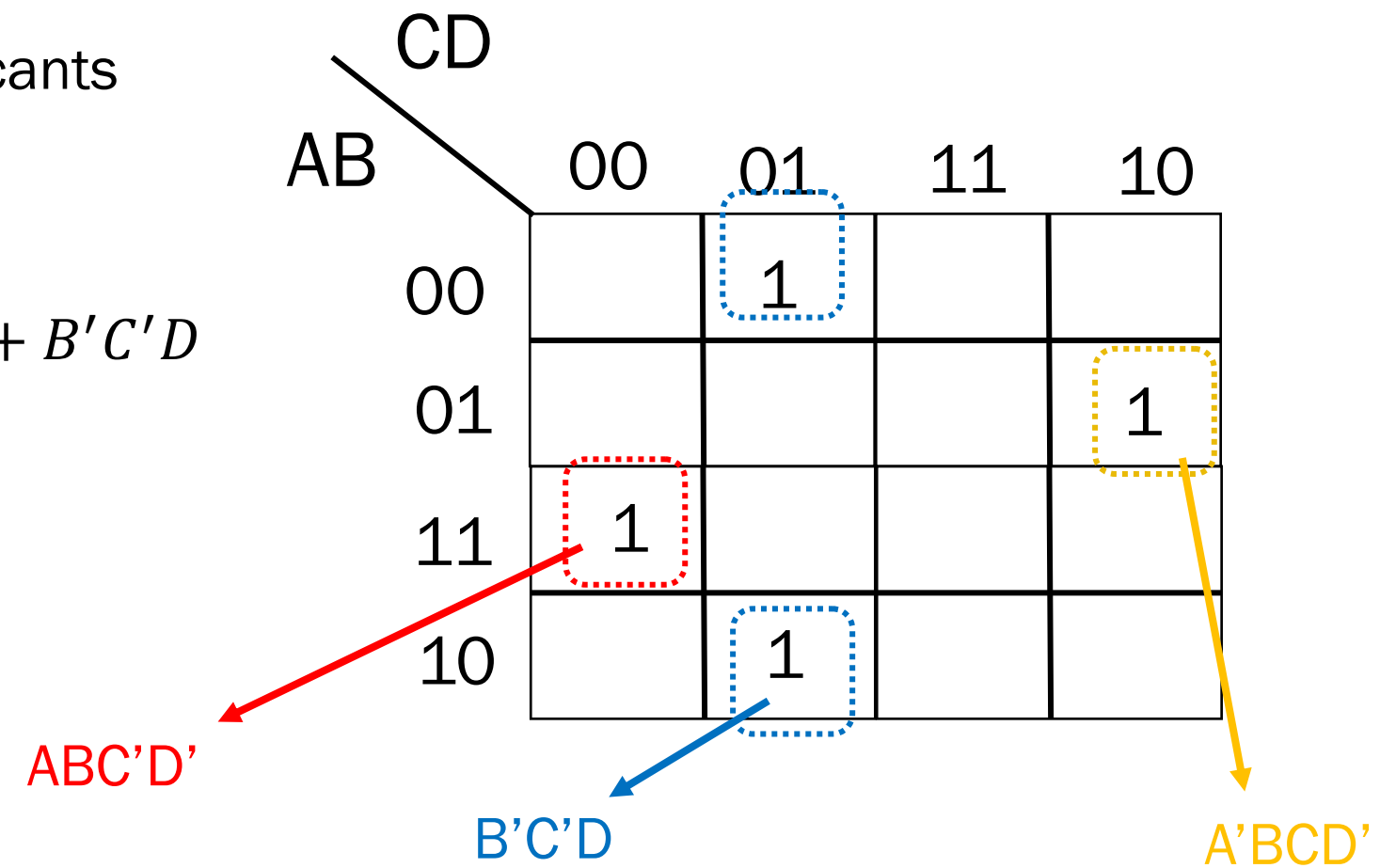
$$Y = A'B'C'D + A'BCD' + AB'C'D + ABC'D'$$

4.2 K-MAP Implementation

There are 3 prime implicants

Therefore, the answer is

$$Y = A'BCD' + ABC'D' + B'C'D$$



4.2 K-MAP Implementation

Example for Product of Sum

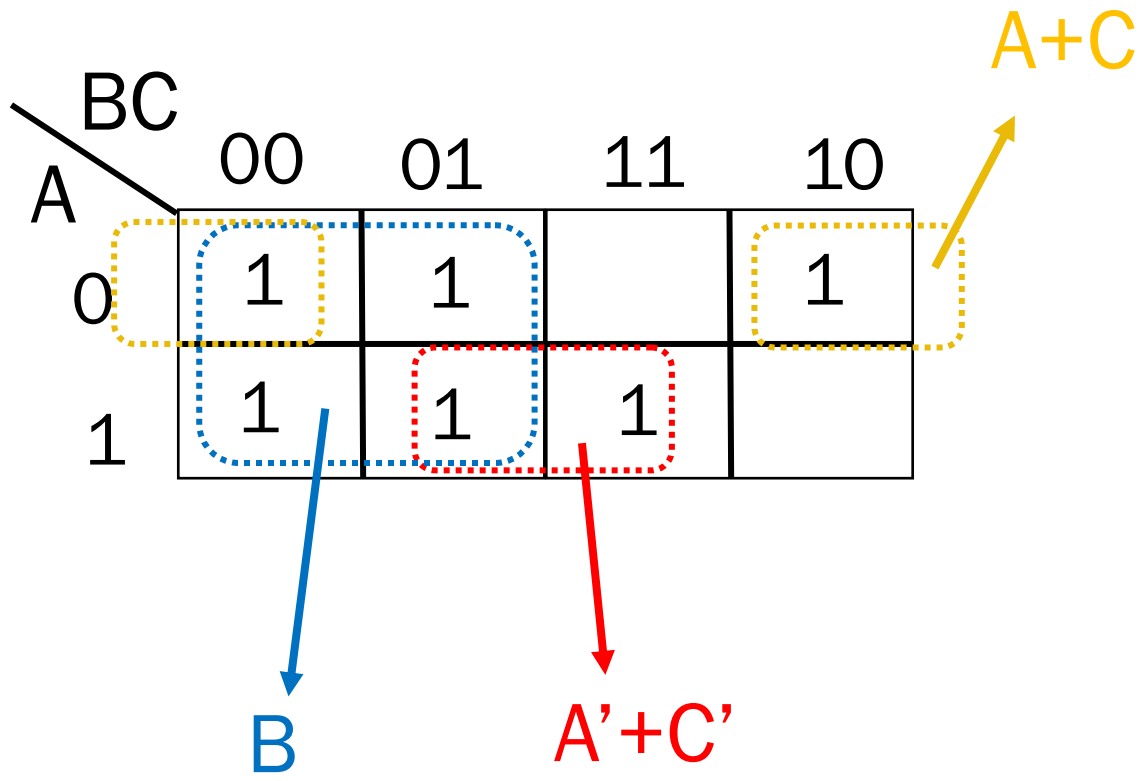
Find the Boolean expression for the given truth table

A	B	C	OUTPUT
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

4.2 K-MAP Implementation

In POS consider 0
(instead of 1)

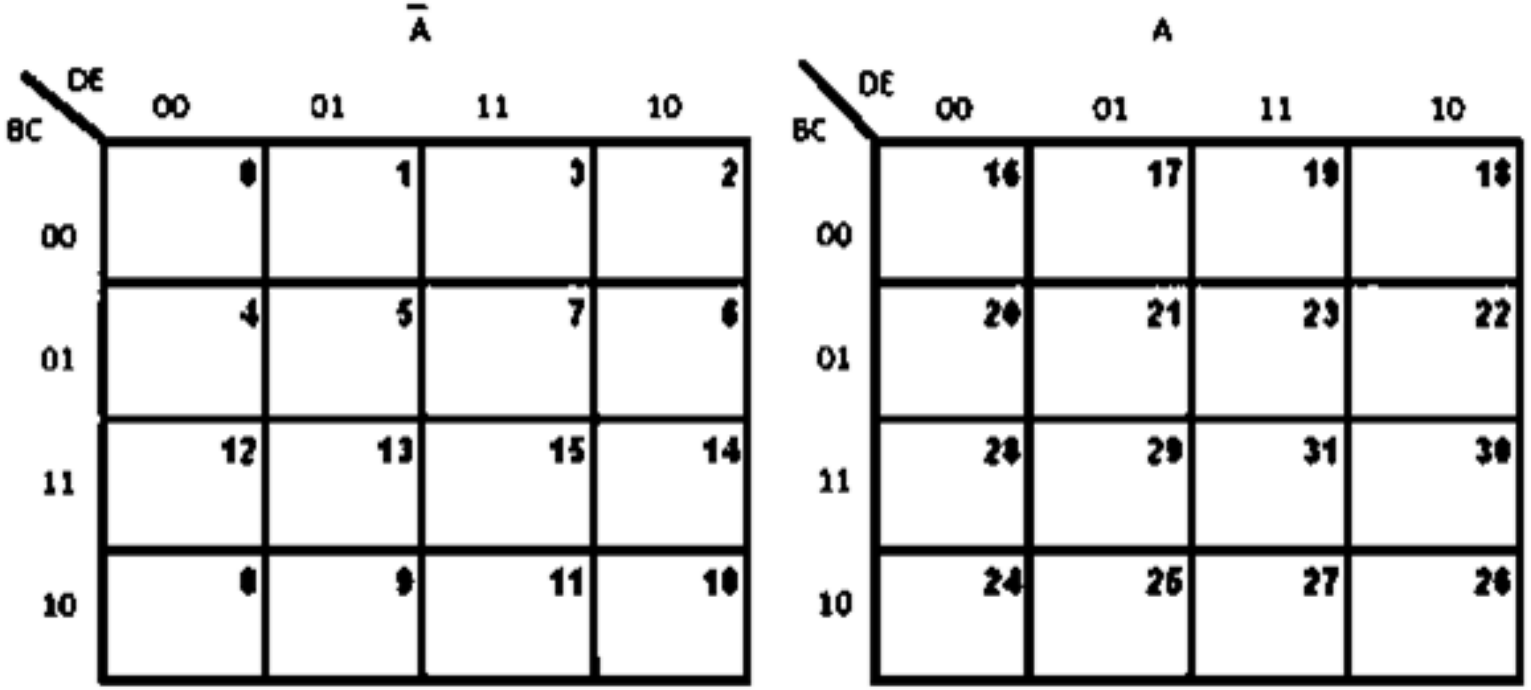
→ $B(A' + C')(A + C)$



4.2 K-MAP Implementation

5 Variable K-MAP

- For 5 variable, we use 2 adjacent K-maps
- Example for ABCDE variable



4.2 K-MAP Implementation

- It can also be represented as the following K-MAP

A = 0

BC \ DE	00	01	11	10
00	B'C'D'E' 0	B'C'D'E 1	B'C'DE 3	B'C'DE' 2
01	B'CD'E' 4	B'CD'E 5	B'CDE 7	B'CDE' 6
11	BCD'E' 12	BCD'E 13	BCDE 15	BCDE' 14
10	BC'D'E' 8	BC'D'E 9	BC'DE 11	BC'DE' 10

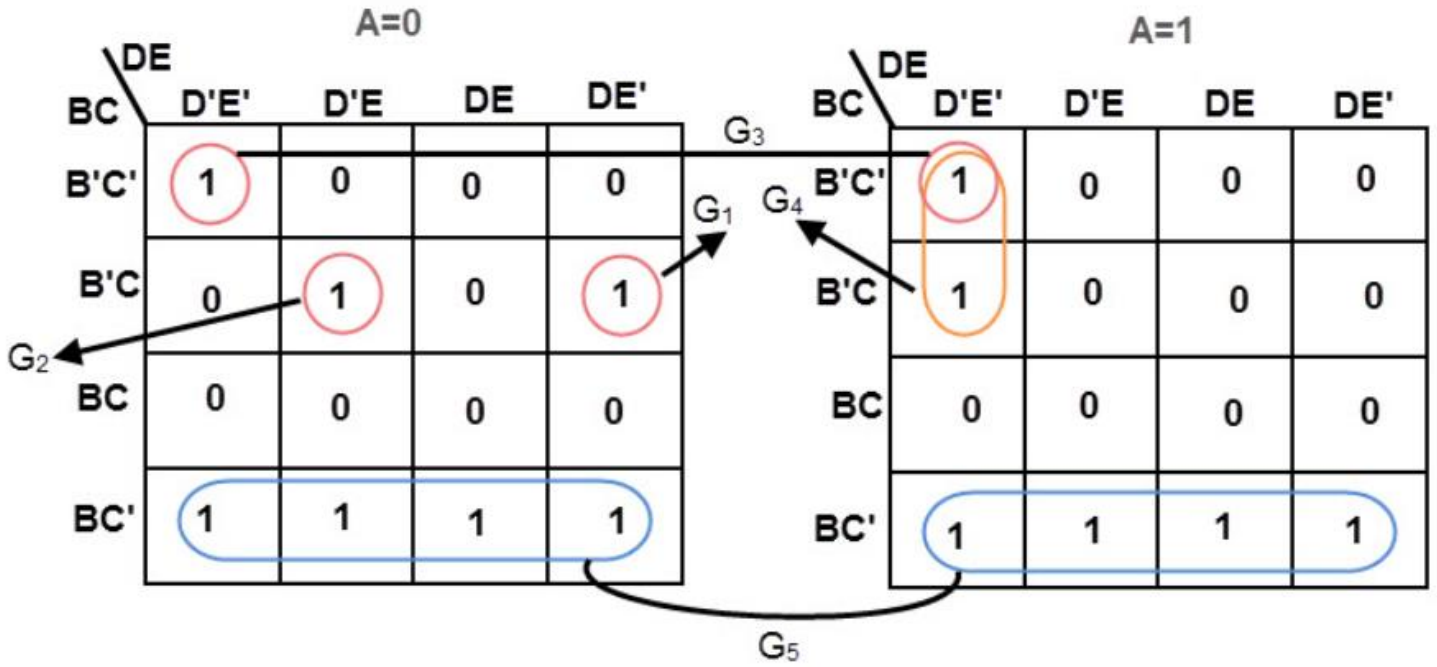
A = 1

BC \ DE	00	01	11	10
00	B'C'D'E' 16	B'C'D'E 17	B'C'DE 19	B'C'DE' 18
01	B'CD'E' 20	B'CD'E 21	B'CDE 23	B'CDE' 22
11	BCD'E' 28	BCD'E 29	BCDE 31	BCDE' 30
10	BC'D'E' 24	BC'D'E 25	BC'DE 27	BC'DE' 26

4.2 K-MAP Implementation

Example.

Simplify F(A, B, C, D, E) = $\sum$ m (0, 5, 6, 8, 9, 10, 11, 16, 20, 42, 25, 26, 27)



References

M. Morris Mano, Digital Design, 5th ed, Prentice Hall, 2012, **Chapter 3**

Next Topic : Tabular Method