CE232 DIGITAL SYSTEM

Topic 1. Introduction and Number Systems

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Subtopic

1.1 Introduction to Digital System

1.2 Number Systems

1.3 Number Based Conversion

1.4 Arithmetic Number Systems

1.5 Binary Coding

1.1 Introduction to Digital Systems

1.1 Introduction to Digital Systems

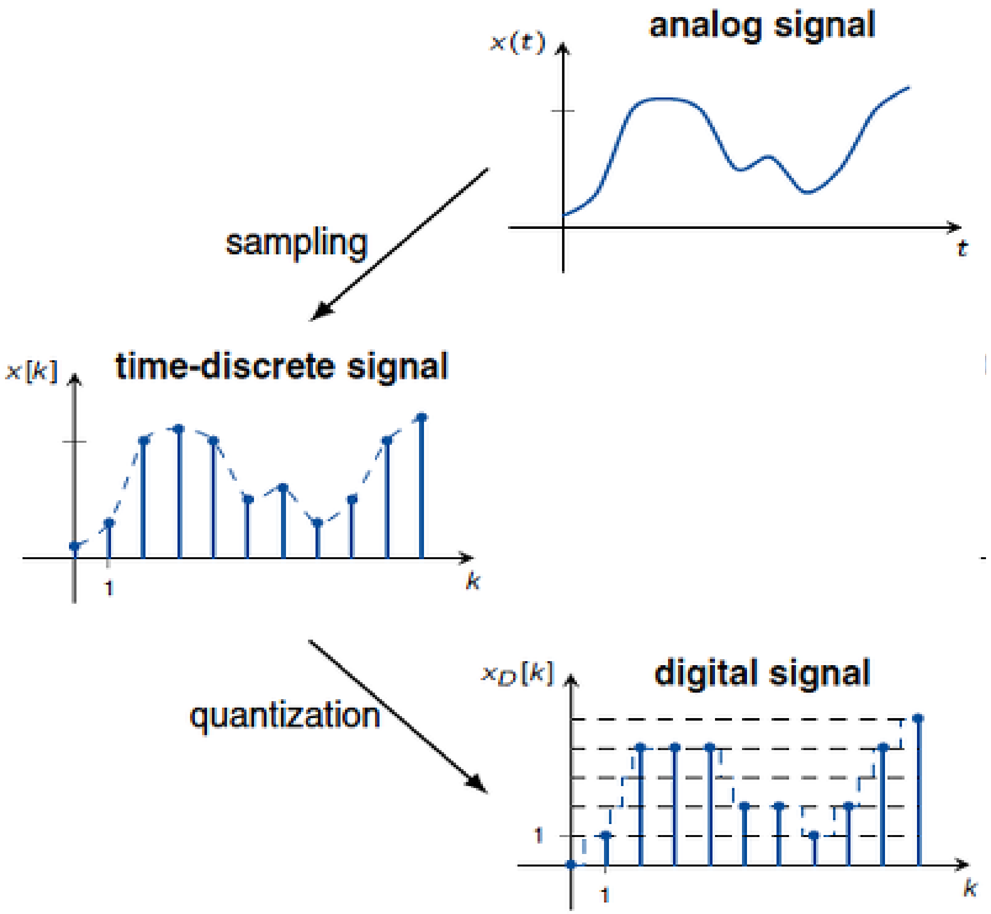
- Digital systems have such a prominent role in everyday life
- Digital systems can **represent and manipulate discrete elements of information**

  -- Examples of discrete sets are the 10 decimals digits, the 26 letters of the alphabet, etc

- Discrete elements of information are represented in a digital system by physical quantities called **signals**

1.1 Introduction to Digital Systems

Digital vs Analog signal



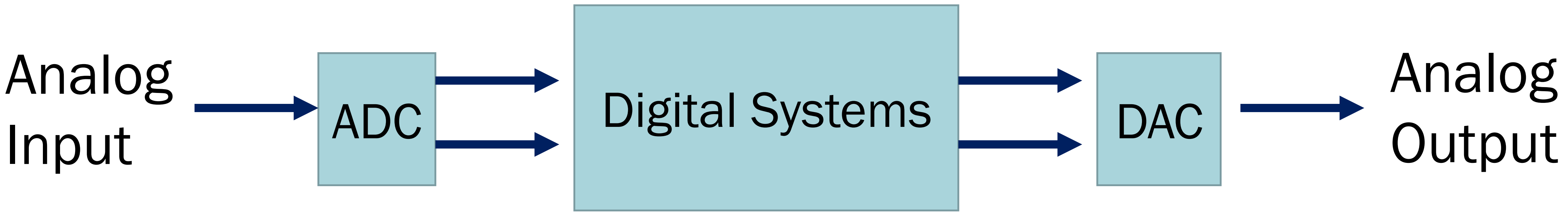
1.1 Introduction to Digital Systems

Why Digital signal?

- Can convey information with less noise, distortion, and interference
- More flexible
- More secure
- More accurate

1.1 Introduction to Digital Systems

Digital systems



Example of digital systems : computer, calculator, digital watch etc

1.1 Introduction to Digital Systems

- **A digital system** is a system that manipulates discrete elements of information
- Commercial product are made with digital circuits, because, like digital computer, most digital devices are programmable
- By changing the program in a programmable device, the same underlying hardware can be used for many different applications, therefore **dramatic cost reduction can be achieved**
- Equipment built with digital integrated circuits can perform at a speed of hundreds of millions of operations per second

1.1 Introduction to Digital Systems

- **A digital system** is an interconnection of digital modules
- To understand the operation of each digital module, **it is necessary to have a basic knowledge of digital circuits and their logical function**

1.2 Number Systems

1.2 Number Systems

Commonly occurring number

Name	Radix	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

1.2 Number Systems

Number System and Codes

- **Weighted :** there is weight in the position

  Example : Decimal, binary, octal, hexadecimal

- **Unweighted**

  Example : Gray code

1.2 Number Systems

Decimal Number

- Base (also called radix) = 10 → 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- Digit Position → integer & fraction
- Digit Weight → Weight = $(Base)^{Position}$
- Magnitude → Sum of "Digit x Weight"

1.2 Number Systems

Example

$$(7392)_{10} = 7000 + 300 + 90 + 2$$
$$= 7\ x\ 10^3 + 3\ x\ 10^2 + 9\ x\ 10^1 + 2\ x\ 10^0$$

Generally, the notation can be written as

$$10^5 a_5 + 10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10^1 a_1 + 10^0 a_0 + 10^{-1} a_{-1} + 10^{-2} a_{-2} + 10^{-3} a_{-3}$$

With the coefficient a_j are any of the 10 digit (0,1,2,…9) and the subscript j gives the place value

1.2 Number Systems

Example

$$(523.74)_{10} = 500 + 20 + 3 + 0.7 + 0.04$$
$$= 5\ x\ 10^{2} + 2\ x\ 10^{1} + 3\ x\ 10^{0} + 7\ x\ 10^{-1} + 4\ x\ 10^{-2}$$

1.2 Number Systems

Binary

- Base = 2 → 2 digits { 0, 1 }, called binary digits or "bits"
- Weights → Weight = $(Base)^{Position}$
- Magnitude → Sum of "Bit x Weight"
- Groups of bits
  - 4 bits = Nibble
  - 8 bits = Byte

1.2 Number Systems

Example

11010.11, can be written as

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$

In general, a number expressed in a base r-system has coefficient multiplied by powers of r

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

1.2 Number Systems

Special powers of 2

- 2^{10} (1024) is Kilo, denoted "K"
- 2^{20} (1,048,576) is Mega, denoted "M"
- 2^{30} (1,073, 741,824)is Giga, denoted "G"
- 2^{40} (1,099,511,627,776) is Tera, denoted “T"

1.2 Number Systems

Octal

- Base = 8 → 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }
- Weight = (Base) Position
- Magnitude → Sum of "Digit x Weight"
- For example $(127.4)_8$ can be written as

$$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

1.2 Number Systems

Hexadecimal

- Base = 16 → 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }
- Weight = (Base) Position
- Magnitude → Sum of "Digit x Weight"
- For example, $(B65F)_{16}$ can be written as

$$(\mathrm{B65F})_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46{,}687)_{10}$$

1.3 Number Based Conversion

1.3 Number Based Conversion

Convert decimal to binary

	Integer Quotient		Remainder	Coefficient
$41/2 =$	20	+	1	$a_0 = 1$
$20/2 =$	10	+	0	$a_1 = 0$
$10/2 =$	5	+	0	$a_2 = 0$
$5/2 =$	2	+	1	$a_3 = 1$
$2/2 =$	1	+	0	$a_4 = 0$
$1/2 =$	0	+	1	$a_5 = 1$

Therefore, the answer is $(41)_{10} = (101001)_2$

1.3 Number Based Conversion

The process can be written more conveniently as follows

Integer	Remainder	
41		
20	1	
10	0	
5	0	
2	1	
1	0	
0	1	101001 = answer

1.3 Number Based Conversion

Conversion of decimal fraction to binary

- Multiplication is used instead of division
- Integers are used instead of remainders

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

Therefore, the answer is $(0.6875)_{10} = (0.1011)_2$

1.3 Number Based Conversion

Convert Decimal to Octal

The division is done by 8

Integer	Remainder
153	
19	1
2	3
0	2 = $(231)_8$

1.3 Number Based Conversion

Conversion of decimal fraction to octal

- Example : convert $(0.513)_{10}$ to octal

$$0.513 \times 8 = 4.104$$
$$0.104 \times 8 = 0.832$$
$$0.832 \times 8 = 6.656$$
$$0.656 \times 8 = 5.248$$
$$0.248 \times 8 = 1.984$$
$$0.984 \times 8 = 7.872$$

The answer, to seven significant figures, is **obtained from the integer** part of the products. Therefore, the answer is $(0.513)_{10} = (0.406517\ldots)_8$

1.3 Number Based Conversion

- The conversion of decimal numbers with both integer and fraction parts is done by combining the integer and the fraction separately and then combining the answer

For example

$$(41.6875)_{10} = (101001.1011)_2$$

$$(153.513)_{10} = (231.406517)_8$$

1.3 Number Based Conversion

Convert Decimal to Hexadecimal

Key point : divide integer part by 16 and multiply fractional part by 16

Example. Convert $(254)_{10}$ to hexadecimal

Integer	Remainder
254	14
15	15

➔ $(FE)_{16}$

1.3 Number Based Conversion

Convert Decimal fraction to Hexadecimal

Key point : divide integer part by 16 and multiply fractional part by 16

Example. Convert $(25.625)_{10}$ to hexadecimal

Integer	Remainder
25	9
1	1

Fractional	
0.625 x 16	10.00

➔ $(19.A)_{16}$

1.3 Number Based Conversion

Convert Binary to Octal / Octal to Binary

- $8 = 2^3$, group of 3 bits represent an octal digit

$(10110.11)_2 \rightarrow (26.6)_8$

$(37.45)_8 \rightarrow (011111100101)_2$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

1.3 Number Based Conversion

Convert Binary to Hexadecimal/Hexadecimal to Binary

- $16 = 2^4$, group of 4 bits represent an octal digit

$$(\,000110001001.11\,)_2 \rightarrow\ (189.C)_{16}$$

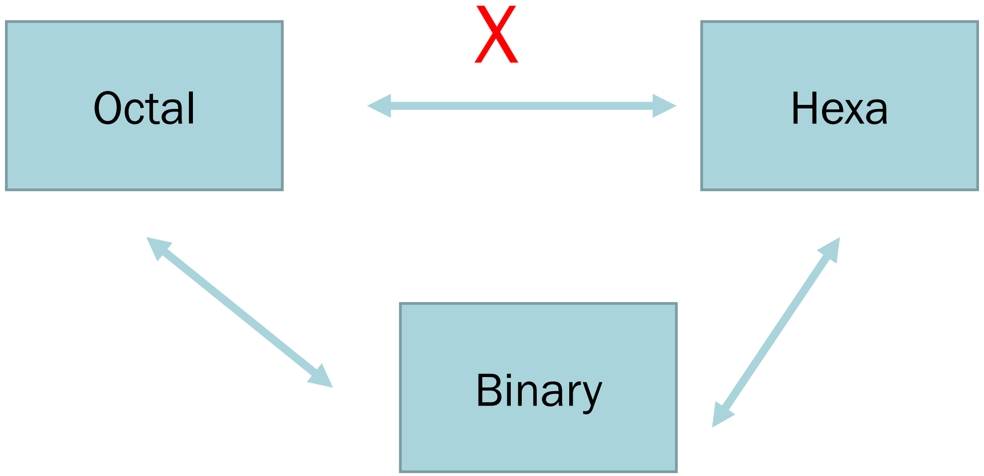
$$(CAFE.31)_{16} \rightarrow\ (1100101011111110.00110001)_2$$

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

1.3 Number Based Conversion

Convert Octal to Hexadecimal/Hexadecimal to Octal

Key point : use binary as intermediate step



1.3 Number Based Conversion

Example.

$(CAD)_{16} \rightarrow (\quad)_8$?

$(CAD)_{16} \rightarrow (110010101101)_2$

$(110010101101)_2 \rightarrow (6255)_8$

1.3 Number Based Conversion

Example of numbers with different bases

Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

1.4 Arithmetic Number Systems

1.4 Arithmetic Number Systems

Decimal Addition and Subtraction

$$\begin{array}{r} {}^{1} \\ 25 \\ 47 \\ \hline 72 \end{array} +$$

1 ← carry

$2 \rightarrow 12 \geq$ base (base 10)

↳ substract with base

$$\begin{array}{r} {}^{10} \\ \cancel{1}25 \\ 72 \\ \hline 53 \end{array} -$$

10 ← borrow

1.4 Arithmetic Number Systems

Binary Addition and Subtraction

$$
\begin{array}{rl}
1 & \longrightarrow \text{carry} \\
110 & \\
101 & \\
\hline
1011 & +
\end{array}
\qquad\qquad
\begin{array}{rl}
2 & \longleftarrow \text{borrow} \\
1\cancel{1}011 & \\
10110 & \\
\hline
00101 & -
\end{array}
$$

1.4 Arithmetic Number Systems

Octal Addition and Subtraction

$$
\begin{array}{cccccl}
 & 1 & 1 & & & \longrightarrow \text{carry} \\
 & 5 & 6 & 7 & & \\
 & 2 & 4 & 3 & + & \\
\hline
1 & 0 & 3 & 2 & &
\end{array}
\qquad\qquad
\begin{array}{cccl}
 & {}^{+8} & & \\
6 & 3 & 8 & \longrightarrow \text{borrow} \\
\cancel{7} & \cancel{4} & 3 & \\
5 & 6 & 4 & - \\
\hline
1 & 5 & 7 &
\end{array}
$$

1.4 Arithmetic Number Systems

Hexadecimal Addition and Subtraction

$$\begin{array}{cccc} 5 & 6 & 8 & 9 \\ 4 & 5 & 7 & 4 \\ \hline 9 & B & F & D \end{array} \; +$$

$$\begin{array}{cccc} 1 & 1 & 1 & \longrightarrow \text{carry} \\ & A & D & D \\ & D & A & D \\ \hline 1 & 8 & 8 & A \end{array} \; +$$

1.4 Arithmetic Number Systems

Hexadecimal Addition and Subtraction

$$
\begin{array}{cccccc}
8 & 6+16 & 3+16 & 16 & \rightarrow & \text{borrow} \\
\cancel{9} & \cancel{7} & \cancel{4} & B & & \\
5 & 8 & 7 & C & & - \\
\hline
3 & E & C & F & &
\end{array}
$$

1.5 Binary Coding

1.5 Binary Coding

- The digital data is represented, stored and transmitted as group of binary bits. This group is also called as **binary code.**
- It is a representation for numbers, letters or words
- Advantages of binary codes

  - suitable for the computer applications.

  - suitable for the digital communications

1.5 Binary Coding

BCD

- Each **decimal digit** is represented by a **4-bit binary number**
- In BCD code only first ten of these are used (0000 to 1001)
- The remaining 1010 to 1111 are invalid in BCD
- BCD is **a fast system** but less the code is less efficient compared to binary

1.5 Binary Coding

BCD Addition

- Sum ≤ 9, Final carry 0 → answer is correct
- Sum ≤ 9, Final carry 1 → answer is incorrect → add 6 (0110)
- Sum > 9, Final carry 0 → answer is incorrect → add 6 (0110)

Question, Why add by 6?

1.5 Binary Coding

Example of decimal to BCD conversion

$$(123)_{10} = (0001\ 0010\ 0101)_{BCD}$$

BCD addition

$$\begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array} \quad \begin{array}{r} 0100 \\ +0101 \\ \hline 1001 \end{array} \qquad \begin{array}{r} 4 \\ +8 \\ \hline 12 \end{array} \quad \begin{array}{r} 0100 \\ +1000 \\ \hline 1100 \\ +0110 \\ \hline 10010 \end{array} \qquad \begin{array}{r} 8 \\ +9 \\ \hline 17 \end{array} \quad \begin{array}{r} 1000 \\ 1001 \\ \hline 10001 \\ +0110 \\ \hline 10111 \end{array}$$

Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

1.5 Binary Coding

Gray code

- In Gray Code only one bit will change each time the decimal number is incremented
- The gray code is called as a unit distance code
- Also known as reflected binary code (RBC)
- Two successive values differ in only 1 bit
- Binary is converted to Gray code to reduce switching operation

1.5 Binary Coding

Gray Code

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

1.5 Binary Coding

Decimal	BCD	Gray
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 0 1 0
4	0 1 0 0	0 1 1 0
5	0 1 0 1	0 1 1 1
6	0 1 1 0	0 1 0 1
7	0 1 1 1	0 1 0 0
8	1 0 0 0	1 1 0 0
9	1 0 0 1	1 1 0 1

References

M. Morris Mano, Digital Design, 5th ed, Prentice Hall, 2012, **Chapter 1**

Assignment 1

Access: https://quizizz.com/join?gc=07358489
Deadline Wednesday, 26th 2021 23.45 PM
1 attempt only
Participant's name : NIM(without zero)_Name

Next Topic : Boolean Algebra and Canonical Form