UNIT III - IMAGE RESTORATION

Image Restoration - degradation model, Properties, Noise models – Mean Filters – Order Statistics – Adaptive filters – Band reject Filters – Band pass Filters – Notch Filters – Optimum Notch Filtering – Inverse Filtering – Wiener filtering.

PART A-TWO MARKS

1. What is meant by image restoration?

Restoration attempts to reconstruct or recover an image that has been degraded, by using a clear knowledge of the degrading phenomenon.

The restoration process the degradation images and apply inverse process to that image to recover the original image.

2. What are the types of noise models?

(a) Gaussian noise
(b) Exponential noise
(c) Rayleigh noise
(d) Uniform noise
(e) Erlang noise
(f) Impulse noise

3. What is salt and pepper noise? Suggest a filter to remove salt and pepper noisein images. [OR] Which filter will be effective in minimizing the impact of salt and pepper noise in an image? (NOV/DEC-17)

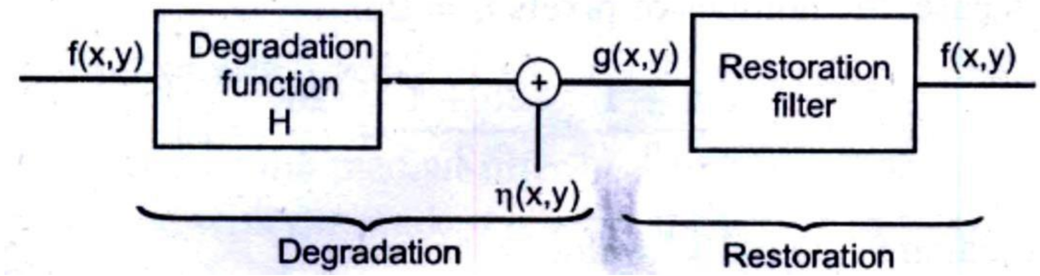
Bipolar impulse noise is called as Salt and Pepper Noise. Median filter is the most suitable filter to remove salt and pepper noise in images.

4. What is periodic noise?

Periodic noise arises typically from electrical or electromechanical interference during image acquisition.

5. How a degradation process is modeled?

The Image degradation/ restoration model is given as,



Degradation: A system operator H (degradation function), together with an additive white noise term η(x, y) operates on an input image f(x, y) to produce a degraded image g(x, y).

Restoration: The restoration filter which is inverse of the degradation function is used to obtain an estimate of the original image from the degraded image.

6. What is fredholm integral of first kind?

The equation,

$$g(x,y) = \iint_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)d\alpha d\beta$$

which is called the superposition or convolution or fredholm integral of first kind. It states that if there sponse of H to an impulse is known, the response to any input f (α, β) can be calculated by means of fredholm integral.

7. What is a median filter? What are its properties?

The median filter replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel.

Median filter has the following properties:

(a) A median filter smoothens additive white noise

(b) A median filter does not degrade edges

(c) A median filter is effective in removing impulses. (Salt and pepper noise).

8. What is maximum filter and minimum filter?

- The 100^{th} percentile is maximum filter that is a Max filter replaces the value of a pixel by the maximum value of the gray levels in the neighborhood of that pixel. It is used in finding brightest points in an image and is effective in removing pepper noise.
- The 0^{th} percentile filter is minimum filter that is a Min filter replaces the value of a pixel by the minimum value of the gray levels in the neighborhood of that pixel. It is used in finding darkest points in an image and is effective in removing salt noise.

9. What is geometric mean filtering?

Geometric mean filter achieves smoothing better than the arithmetic mean filter. The amount of details lost in the geometric mean filtering is lesser.

The filter is given by,

$$f(x,y) = \left[\prod_{(s,t)\in s_{xy}} g(s,t) \right]^{1/mn}$$

The product of the pixel values in the defined neighborhood raised to the power of l/mn, gives the restore pixel value.

10. What is the need for adaptive median filter?

Though median filter is effective in removing impulse noise, the fine details in the image are lost since the impulses are replaced with median values. Thus adaptive median filters are used to overcome this disadvantage of median filter.

11. What are the filters that can be used to remove periodic noise?

Frequency domain filters are to be used to remove periodic noise in images

The commonly used frequency domain filters are,

(a) Notch Filters

(b) Notch Reject filters

(c) Notch Pass filters

(d) Optimum notch filters

12. Is 100% restoration possible. Justify.

100% restoration is possible only if the true degradation function is known and the image is degraded only due to this degradation function. If the image is degraded by additive noise, 100% restoration is not possible.

13. Differentiate image enhancement and image restoration.(May/June 2017)

Image Enhancement	Image Restoration
Image enhancement is a subjective process i.e., it is a heuristic procedure designed to manipulate an image in order to please the viewer.	Restoration techniques are oriented towards modeling the degradation and applying the inverse process in order to recover the original image.
Modeling of degradation process is not required.	Modeling of degradation is a must.
Apriori knowledge of the degradation is not required.	Apriori knowledge of the degradation function is required to model the degradation function.
Ex: Contrast stretching	Ex: Removal of motion blur

14. Write the equation for discrete degradation model?

$$g_e(x,y) = \sum_{m=0}^{m-1}\sum_{n=0}^{n-1} f_e\,(m-n)h_e(x-m,y-n) + n_e(x,y)$$

For x=0.................M – 1
Y=0………… N-1

15. What are the forms of degradation? (or) State the causes of degradation in an image. (Nov/Dec 16)

(a) Sensor noise
(b) Blur due to camera misfocus
(c) Relative object camera motion
(d) Random atmosphere turbulence
(e) Thermal noise etc.

16. List the properties involved in degradation model?

(a) Linearity
(b) Additivity
(c) Homogeneity
(d) Position / space invariant

17. Give the noise probability density function?

a) Gaussian noise.
b) Rayleigh noise.
c) Erlang or Gamma noise
d) Exponential noise.
e) Uniform noise.
f) Impulse noise.

18. Define pseudo inverse filter? (Nov 2008 & Nov 2011)

- Stabilized or generalized version of inverse filter.
- For linear shift invariant system with frequency response H (u,v) is

$$H^{-1}\,(u,v) = \begin{Bmatrix} \dfrac{1}{H\,(u,v)}, & H \neq 0 \\ 0, & H = 0 \end{Bmatrix}$$

19. What are the assumptions in Wiener filter?

- Noise and image are uncorrelated and has zero mean.

- Gray levels in the estimate are linear functions of leads in degraded image

20. State the concept of inverse filtering? (or) What is the principle of inverse filtering? (May-2014)

The inverse filtering divides the transform of the degraded image G(u, v) by the degradation function H(u, v) and determines an approximation of the transform of the original image.

$$F(u,v) = \frac{G(u,v)}{H(u,v)}$$

Limitation: **-** It has no provision for handling noise

21. Define spatial transformation?

Spatial transformation is defined as the rearrangement of the pixels on the image plane.

22. Define gray level interpolation? (June 2010)

It deals with the assignment of gray levels to pixels in the spatially transformed image.

23. What is constrained restoration with Lagrange multiplier?

$$\hat{f} = H^T g\,(\gamma\, Q\, Q^t + H^T H)^{-1}$$

This quantity must be adjusted, so that the constrained is satisfied.

24. What are the limitations of inverse filtering? (or) Mention the drawbacks of inverse filtering. (June 2011, Nov-2013)[NOV/DEC-17]

1. Inverse filtering is highly sensitive to noise.
2. It has Zero or small value problem.

If degradation function H (u, v) has zero or small value then the ratio N(u,v) / H(u,v) dominates the value of restored image.

This implies a poor performance of the system and results in bad approximation of the original function. This is known as zero or small value problem.

25. Difference between Wiener and inverse filtering? (June 2011)

Wiener filtering	**Inverse filtering**
When noise level is increased Wiener filter works.	With small amount of noise inverse filtering works.
It has no zero or small value problem.	It has zero or small value problem.
The result obtained is closer to the original image.	The result obtained is not closer to the original image.

26. Give the transfer function of Wiener filtering? (June 2009)

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] G(u,v)$$

Where, $|H(u, v)|^2 = H^*(u, v) H(u,v)$

$H^*(u, v)$ – conjugate of $H(u, v)$

$S_f(u,v)$ - Power spectrum of original image.

$S_\eta(u,v)$ - Power spectrum of noise.

27. Define rubber sheet transformation. (or) Geometric transformation :(May 2013 & 2015)

Geometric transformation:

It is generally modify spatial relationship between pixels in an image. It is also called as "Rubber sheet transformation" because they may be viewed as the process of printing the image on a sheet of rubber.

28. What is meant by bilinear interpolation? (April 2011)

- Bilinear interpolation is used when we need to know values at random position on a regular 2D grid. Note that this grid can as well be an image or a texture map.
    - Interpolation Techniques:
        - 1D linear interpolation (elementary algebra)
        - 2D -2 sequential 1D (divide-and-conquer)
        - Directional (adaptive) interpolation.
    - Interpolation Applications:
        - Digital zooming (resolution enhancement)
        - Image imprinting (error concealment)
        - Geometric transformations

29. What are the basic transformations that can be applied on the images?

Applying some basic transformation to a uniformly distorted image can correct for a range of perspective distortions by transforming the measurements from the ideal coordinates to those actually used. (For example, this is useful in satellite imaging where geometrically correct ground maps are desired.)

Translation

Scaling

Rotation

Concatenation

30. Write the equation for continuous degradation model?

$$g_e(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_e(\alpha,\beta)\, h(x,\alpha,y,\beta)\, d\alpha\, d\beta + n_e(x,y)$$

31. What is Lagrange multiplier? Where it is used? (nov 2014)

The Lagrange multiplier is a strategy for finding the local maxima and minima of a function subject to equality constraints. This is mainly used in the image restoration process like image acquisition, image storage and transmission.

32. Why blur is to be removed from images? (nov 2014)

The blur is caused by lens that is in improper manner, relative motion between camera and scene and atmospheric turbulence.It will introduce bandwidth reduction and make the image analysis as complex.To prevent these problems only the blur is removed from images.

33. Define Wiener filter? (OR)What are the functions of wiener filter? (June 2011)

Wiener filtering is a method of restoring images in the presence of blur and noise.

34. Why the restoration is called as unconstrained restoration? (May/June 17)

In the absence of any knowledge about the noise 'n', a meaningful criterion function is to seek an f^ such that H f^ approximates of in a least square sense by assuming the noise term is as small as possible.

Where H = system operator.

f^ = estimated input image.

g = degraded image.

35. How the derivatives are obtained in edge detection during formulation? (APR/MAY-18)

The first derivative at any pointin an image is obtainedby using the magnitude of the gradient at that point.similarly, the second derivatives are obtained by using the laplacian**.**

36. How the discontinuity is detected in an image using segmentation? (APR/MAY-18)

The steps used to detect the discontinuity in an image using segmentation are

a) Compute the sum of products of the coefficient with the gray levels contained in the region encompassed by the mask.
b) The response of the mask at any point in the image is R – W1Z1+W2Z2+W3Z3…..
   Z= gray levels of the pixels associated with mass coefficient W
c) The response of the mask is defined with respect to its centre location.

37. Give the relation for Gamma noise and exponential noise?(NOV/DEC-18)

GAMMA NOISE	EXPONENTIAL NOISE
The PDF is $P(z) = \left\{ \begin{matrix} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & ; z \geq a \\ & ; z < 0 \end{matrix} \right\}$	**The PDF is** $P(z) = \left\{ \begin{matrix} ae^{-az} & ; z \geq a \\ 0 & ; z < 0 \end{matrix} \right\}$
Mean, μ = b/a	**Mean, μ = 1/a**
Standard deviation, σ^2 = b/a2	**Standard deviation, $\sigma^2 = 1/a^2$**

38. Give the 3 x 3 mask to detect horizontal line in an image(APR/MAY-19)

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

39. Compute the restored image pixel value (center pixel only) for the following 3 × 3 grayscale 8 bit image by using arithmetic mean filter.(A/M 2021)

50	100	50
100	**150**	100
100	100	150

Mean value = $\frac{50+100+50+100+150+100+100+100+150}{9} = \frac{900}{9} = 100$

Total number of pixel

50	100	50
100	**100**	100
100	100	150

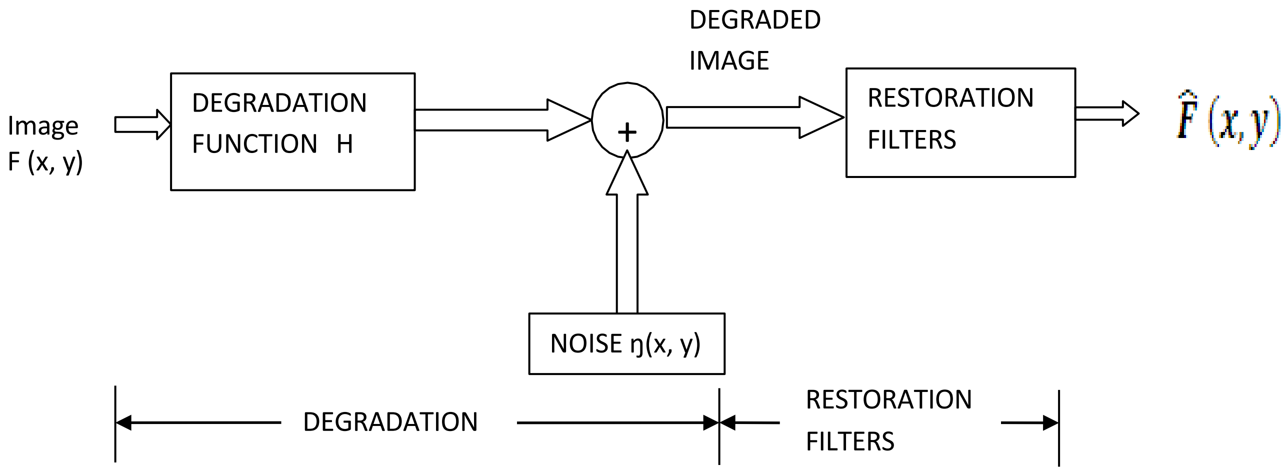
40. How to achieve restoration by using direct inverse filtering ?(A/M 2021)

However, even if H is known completely, the undegraded image cannot be recovered exactly due to noise N

$$F(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Evenworse when H has zero or very small values N/H would dominates the estimated image. One way to get around this problem is to limit the filter frequencies to values near the origin where H is large in general.

41. Draw the model of image degradation / restoration process? (Nov 2008 & Nov 2011 & May 2013)



PART-B

1. Explain in detail about noise distributions or noise models? (Apr2010) (A/M 2021)

Noise distribution:

The noise in the digital image arises due to image acquisition or transmission while image acquisition, the image sensor is affected by environmental factors.
While transmission of images are corrupted due to interference in the channel.

Spatial property of noise:

The noise is independent of spatial co-ordinates. However in some application,it is invalid and so we deal with spatial dependent.

PDF of noise:

(i). Gaussian noise:

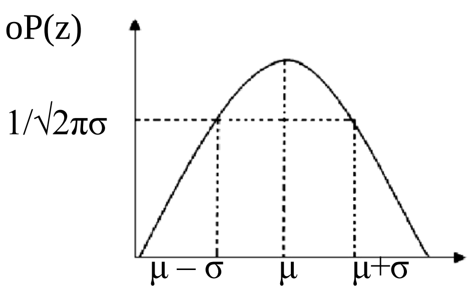
The mathematical expression for Gaussian noise is in both spatial of frequency domains. This models are convenient because they are marginally applicable at best.

The PDf of Gaussian noise is,

$$P(z) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-2-\mu)^2/2\sigma^2$$

Where,

M – means of z   σ – SD
σ^2 - variance   z – gray level



When Z is described 70% of its value will be in the range of μ – σ, μ+σ.

(ii) Rayleigh noise:

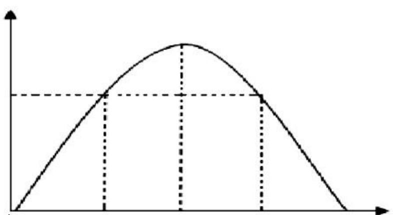
The PDF of Ray – leigh noise is

$$P(x) = \begin{cases} \frac{2}{b}(z-a)e^{\frac{-(z-a)^2}{b}} & ; z \geq a \\ & ; z < a \end{cases}$$

$$mean\ \mu = a + \sqrt{\frac{\pi b}{4}}$$

$$variance\ \sigma^2 = \frac{b\ (4-\pi)}{4}$$

P(2)
0.607
√2/b



aa+ √b/2 b

(iii) Gamma noise:

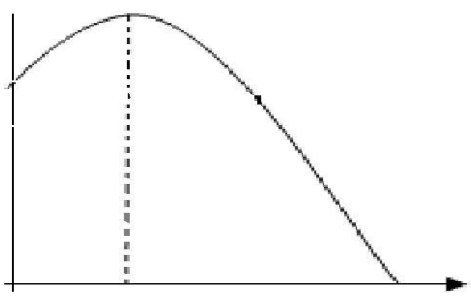
The PDF of gamma noise is given by

$$P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & ; z \geq a \\ & ; z < 0 \end{cases}$$

$$\mu = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$

P(2)



z

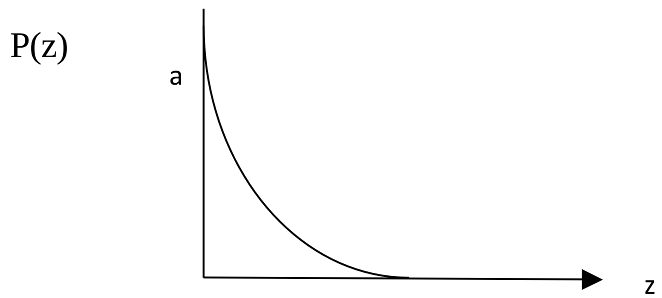
(b-1)/a

(iv) Exponential Noise:

The PDF of exponential noise is given by

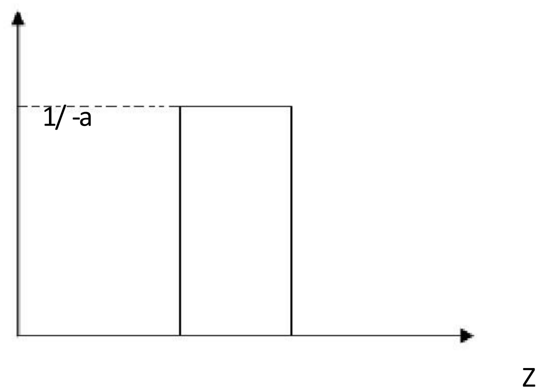
$$P(z) = \begin{cases} ae^{-az} & ; z \geq a \\ 0 & ; z < 0 \end{cases}$$

$$\mu = \frac{1}{a} \quad ; \sigma^2 = \frac{1}{a^2}$$

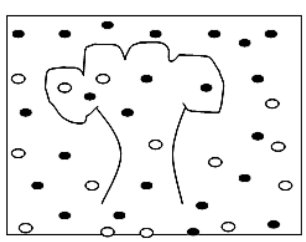


(v) Uniform Noise: The PDF of uniform noise is given by,

$$\mu = \frac{a+b}{2} \quad ; \sigma^2 = -\frac{(b-a)^2}{12}$$



(vi). Impulse noise distribution or Salt & Pepper noise:



The PDF impulse noise is given by,

$$P(z) = \begin{cases} P_a, & for \quad z = a \\ P_b, & for \quad z = b \\ 0, & other\ wise \end{cases}$$

➢ If b>a, there will be dark dot in the image and will appear like a black dots.

➢ If a>b, these will be white dots appearing in the image. If either P_a or P_b is zero then the impulse noise is called as “Unipolar noise”.

- If neither P_a nor P_b is zero, then the noise is called as bipolar noise. It means salt & pepper granules distributed over the images so the bipolar noise is known as "salt &pepper noise".
- It is also called as short spike noise. Negative impulse appears in black and positive impulse appears in white dots.

2. <u>Explain the various types of mean filters in detail.</u> (NOV/DEC-18)

(i) Arithmetic Mean filters

The arithmetic mean filter computes the average value of the corrupted **image *g(x,*** y) in the area defined by ***S***S_{xr}The value of the restored image f at **point *(x,y)*** is simply the arithmetic mean computed using the pixels in the region **defined by** S_{xy}**.**

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s, t)$$

(ii) Geometric mean filtering:

$$\hat{f}(x, y) = \left[\underset{(s,t) \in Sxy}{\pi\, g(s,t)} \right]^{\frac{1}{mn}}$$

- The geometric mean filter is performed by restored pixel is by the product of pixel is the sub image window. It rises to the power of 1/mn.
- The image analysis includes measurement of shape, size, texture and color of the objects present in the image. Here the input is image and produces numerical and graphical information based on the characteristics of the image data.
- Based upon the filter results, the analysis of an image is performed and identify which pixel need to be improved. These are done based upon the statistical measurements.

(iii) Harmonic mean filter: (Nov 2014)

It can be used to reduce noise such as salt noise and Gaussian noise, The restored image using harmonic mean filter is given by,

$$\hat{f}(x,y) = \frac{mn}{\sum_{S,t=Sxy} \frac{1}{g(S,t)}}$$

(iv) Contra harmonic mean filter:

It is used to reduce salt and pepper noise. The restored image is given by,

$$\hat{f}(x,y) = \frac{\sum_{S,t \in Sxy} g(S,t)^{Q+1}}{\sum_{S,t \in Sxy} g(S,t)^{Q}}$$

Where q – order of filter.

For positive values of Q, it eliminates pepper noise and for negative values of q it eliminates salt noise.
It can't do both simultaneously.
If Q=0, then the contra harmonic mean filter reduces to arithmetic mean filter.
If Q=-1, it reduces to harmonic mean filter. Thus it is helpful restore the images.

3. <u>Explain various order-statistic filters:</u>

Order-statistic filters are spatial filters whose response is based on ordering (ranking) the values of the pixels contained in the image area encompassed by the filter,

(i) Median filtering:

The best-known order-statistic filter is the ***median filter,*** which, as its name implies, replaces the value of a pixel by the median of the intensity levels in the neighborhood of that pixel:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$$

- The value of the pixel at ***(x,y)*** is included in the computation of the median.
- Median filters are quite popular because, for certain types of random noise, they provide excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters of similar size.
- Median filters are particularly effective in the presence of both bipolar and unipolar impulse noise.
- The median filter yields excellent results for images corrupted by this type of noise.

(ii) Max and min filter:
100th percentile results in the so-called *max filter,* given by

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

This filter is useful for finding the brightest points in an image. Also, because pepper noise has very low values, it is reduced by this filter as a result of the

The 0th percentile filter is the ***min filter.***

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s, t)\}$$

(iii) Midpoint filter

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter:

$$\hat{f}(x, y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s, t)\} + \min_{(s,t)\in S_{xy}} \{g(s, t)\}\right]$$

This filter combines order statistics and averaging. It works best for randomly distributed noise, like Gaussian or uniform noise.

(iv) Alpha-trimmed mean filter

➢ Suppose that we delete the $d/2$ lowest and the $d/2$ highest intensity values of $g(s,t)$ in the neighborhood S_{xy}. Let $g_r(s, t)$ represent the remaining ***mn-d*** pixels.

➢ A filter formed by averaging these remaining pixels is called an ***alpha-trimmed mean*** filter:

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t)\in S_{xy}} g_r(s, t)$$

➢ where the value of ***d*** can range from 0 to ***mn* - 1.** When ***d*** = 0, the alpha-trimmed filter reduces to the arithmetic mean filter discussed in the previous section. If we choose ***d*** = ***mn* -** 1, the filter becomes a median filter.

➢ For other values of ***d*,** the alpha-trimmed filter is useful in situations involving multiple types of noise, such as a combination of salt-and-pepper and Gaussian noise.

4. <u>Explain Adaptive filters in detail?OrAdaptive, local noise reduction filter[A/M-18][A/M 2021]</u>

(i) Adaptive, local noise reduction filter

➢ **The** simplest statistical measures of a random variable are its mean and vari**ance.**

➢ These are reasonable parameters on which to base an adaptive filter be**cause** they are quantities closely related to the appearance of an image.

- The **mean** gives a measure of average intensity in the region over which the mean **is** computed, and the variance gives a measure of contrast in that region.
- **Our** filter is to operate on a local region, S_{xv}.

The response of the filter at **any** point (x, y) on which the region is centered is to be based on four quanti**ties:**

1. $g(x, y)$, the value of the noisy image at (x, y);
2. (b)σ_η^2, the variance of **the** noise corrupting f(x, y) to form $g(x, y)$;
3. m_L, the local mean of the pix**els** in S_{xv}; and
4. σ_L^2, the local variance of the pixels in S_{xv}.

We want the behavior of the filter to be as follows:

1. If σ_η^2 is zero, the filter should return simply the value of $g(x, y)$. This is the trivial, zero-noise case in which $g(x,y)$ is equal to $f(x,y)$.

2. If the local variance is high relative to σ_η^2, thefilter should return a value close *to* $g(x, y)$. ***A*** high local variance typically is associated with edges, and these should be preserved.

3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in S_{xv}.

This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced simply by averaging.

An adaptive expression for obtaining $\hat{f}(x, y)$ based on these assumptions may be written as

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]$$

- The only quantity that needs to be known or estimated is the variance of the overall noise, σ_η^2.
- The other parameters are computed from the pixels in S_{xy} at each location (x, y) on which the filter window is centered.
- A tacit assumption in Eq. (5.3-12) is that $\sigma_\eta^2 \leq \sigma_L^2$.
- The noise in our model is additive and position independent, so this is a reasonable assumption to make because S_{xy} is a subset of $g(x,y)$.
- However, we seldom have exact knowledge of σ_η^2. Therefore, it is possible for this condition to be violated in practice.
- For that reason, a test should be built into an implementation of Eq. (5.3-12) so that the ratio is set to 1 if the condition $\sigma_\eta^2 > \sigma_L^2$ occurs. This makes this filter nonlinear.

➢ However, it prevents nonsensical results (i.e., negative intensity levels, depending on the value of m_L) due to a potential lack of knowledge about the variance of the image noise. Another approach is to allow the negative values to occur, and then rescale the intensity values at the end. The result then would be a loss of dynamic range in the image.

(ii) Adaptive median filter<u>(OR) Apply suitable filter for the marked pixels in the image which is corrupted by salt and pepper noise [APR/MAY-19]</u>

$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 255 & \boxed{0} \\ 3 & \boxed{255} & 6 \end{pmatrix}$$

➢ The median filter performs well if the spatial density of the impulse noise is not large (as a rule of thumb, P_a and P_b less than 0.2).

➢ An additional benefit of the adaptive median filter is that it seeks to preserve detail while smoothing nonimpulse noise, something that the "traditional" median filter does not do.

➢ As in all the filters discussed in the preceding sections, the adaptive median filter also works in a rectangular window area S_{xy}.

➢ Unlike those filters, however, the adaptive median filter changes (increases) the size of S_{xv} during filter operation, depending on certain conditions listed in this section.

➢ Keep in mind that the output of the filter is a single value used to replace the value of the pixel at (x,y), the point on which the window S_{xv} is centered at a given tune.

Consider the following notation:

z_{min} = minimum intensity value in S_{xy}
Z_{max} = maximum intensity value in S_{xv}
z_{med} = median of intensity values in S_{xy}
z_{xy} =intensity value at coordinates (x, y)
S_{max} = maximum allowed size of S_{xy}

The adaptive median-filtering algorithm works in two stages, denoted stage A and stage B, as follows:

Staged: $A1 = Z_{med} - Z_{min}$
$A2 = Z_{med} - Z_{max}$
If $A1 > 0$ AND $A2 < 0$, go to stage B

Else increase the window size

If window size $\leq S_{max}$ repeat stage ***A***
Else output Z_{med}

Stage ***B:*** $B1=Z_{xy}-Z_{min}$
$B2=Z_{xy}-Z_{max}$
If B1 > 0 AND ***Bl***< 0, output z_{xy}
Else output z_{med}

- The key to understanding the mechanics of this algorithm is to keep in mind that it has three main purposes: to remove salt-and-pepper (impulse) noise, to provide smoothing of other noise that may not be impulsive, and to reduce distortion, such as excessive thinning or thickening of object boundaries.
- The values z_{min} and $z_{max}$are considered statistically by the algorithm to be "impulse-like" noise components, even if these are not the lowest and highest possible pixel values in the image.
- With these observations in mind, we see that the purpose of stage ***A*** is to determine if the median filter output, z_{med}, is an impulse (black ***or*** white) or not.
- If the condition $z_{min}<z_{med}<z_{max}$ holds, then z_{med} cannot be an impulse for the reason mentioned in the previous paragraph.
- In this case, we go to stage ***B*** and test to see if the point in the center of the window,Z_{xy}, is itself an impulse (recall that $z_{xy}$is the point being processed).
- If the condition ***B1***>0 AND ***B2*** <0 is true, then $z_{min}<z_{xy}<z_{max}$,and z_{xv}cannot be an impulse for the same reason that $z_{med}$was not.
- In this case, the algorithm outputs the unchanged pixel value, z_{xy}.By not changing these "intermediate-level" points, distortion is reduced in the image.
- If the condition B1> 0 AND ***B2*** <0 is false, then either Zxy-Zminor Zxy= Zmax.In either case, the value of the pixel is an extreme value and the algorithm outputs the median value z_{med}, which we know from stage ***A*** is not a noise impulse.
- The last step is what the standard median filter does. The problem is that the standard median filter replaces every point in the image by the median of the corresponding neighborhood. This causes unnecessary loss of detail.
- Continuing with the explanation, suppose that stage ***A does*** find an impulse (i.e., it fails the test that would cause it to branch to stage B).
- The algorithm then increases the size of the window and repeats stage A. This looping continues untilthe algorithm either finds a median value that is not an impulse (and branches to stage ***B),***or the maximum window size is reached.

- If the maximum window size is reached, the algorithm returns the value of z_{med}. Note that there is no guarantee that this value is not an impulse.
- The smaller the noise probabilities P_a and/or P_b are, or the larger S_{max} is allowed to be, the less likely it is that a premature exit condition will occur.
- This is plausible. As the density of the impulses increases, it stands to reason that we would need a larger window to "clean up" the noise spikes.
- Every time the algorithm outputs a value, the window S_{xy} is moved to the next location in the image. The algorithm then is reinitialized and applied to the pixels in the new location.

5. Explain Periodic Noise Reduction by Frequency Domain Filtering(or) various types of filter for periodic noise reduction.[A/M 2021]

- The basic idea is that periodic noise appears as concentrated bursts of energy in the Fourier transform, at locations corresponding to the frequencies of the periodic interference.
- The approach is to use a selective filter to isolate the noise.
- The three types of selective filters bandreject, bandpass, and notchare used for basic periodic noise reduction.

5.1 Bandreject Filters

- One of the principal applications of bandreject filtering is for noise removal in applications where the general location of the noise component(s) in the frequency domain is approximately known.
- A good example is an image corrupted by additive periodic noise that can be approximated as two-dimensional sinusoidal functions.
- It is not difficult to show that the Fourier transform of a sine consists of two impulses that are mirror images of each other about the origin of the transform.
- The impulses are both imaginary (the real part of the Fourier transform of a sine is zero) and are complex conjugates of each other.

- Figure 5.16(a), which is the same as Fig. 5.5(a), shows an image heavily corrupted by sinusoidal noise of various frequencies.
- The noise components are easily seen as symmetric pairs of bright dots in the Fourier spectrum shown in Fig. 5.16(b).
- In this example, the components lie on an approximate circle about the origin of the transform, so a circularly symmetric bandreject filter is a good choice.
- Figure 5.16(c) shows a Butterworth bandreject filter of order 4, with the appropriate radius and width to enclose completely the noise impulses.

- Since it is desirable in general to remove as little as possible from the transform, sharp, narrow filters are common in bandreject filtering.
- The result of filtering Fig. 5.16(a) with this filter is shown in Fig. 5.16(d). The improvement is quite evident.
- Even small details and textures were restored effectively by this simple filtering approach.
- It is worth noting also that it would not be possible to get equivalent results by a direct spatial domain filtering approach using small convolution masks.

5.2 Bandpass Filters

A ***bandpass***filter performs the opposite operation of a bandreject filter.
We showed in Section 4.10.1 how the transfer function $H_{BP}(U,V)$ of a bandpassfilter is obtained from a corresponding bandreject filter with transfer function $H_{BR}(u, v)$ by using the equation

$$H_{BP}(u, v) = 1 - H_{BR}(u, v)$$

5.3 Notch filters

- A ***notch*** filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency.
- Equations for notch filtering are detailed in Section 4.10.2. Figure 5.18 shows 3-D plots of ideal, Butterworth, and Gaussian notch (reject) filters.
- Due to the symmetry of the Fourier transform, notch filters must appear in symmetric pairs about the origin in order to obtain meaningful results.
- The one exception to this rule is if the notch filter is located at the origin, in which case it appears by itself.
- Although we show only one pair for illustrative purposes, the number of pairs of notch filters that can be implemented is arbitrary.
- The shape of the notch areas also can be arbitrary (e.g., rectangular). Wecan obtain notch filters that *pass*,rather than suppress, the frequencies contained in the notch areas.
- Since these filters perform exactly the opposite function as the notch reject filters, their transfer functions are given by

$$H_{NP}(u, v) = 1 - H_{NR}(u, v)$$

- where$H_{NP}(u,v)$ is the transfer function of the notch pass filter corresponding to the notch reject filter with transfer function $H_{NR}(u, v)$.

5.4 Optimum Notch Filtering

- Alternative filtering methods that reduce the effect ofthese degradations are quite useful in many applications.
- The method discussed here is optimum, in the sense that it minimizes local variances of the restored estimate ***f(x, y).***
- The procedure consists of first isolating the principal contributions of the interference pattern and then subtracting a variable, weighted portion of the pattern from the corrupted image.
- Although we develop the procedure in the context of a specific application, the basic approach is quite general and can be applied to other restoration tasks in which multiple periodic interference is a problem.
- The first step is to extract the principal frequency components of the interference pattern.
- As before, this can be done by placing a notch pass filter. H_{NP} ***(u, v),*** at the location of each spike.
- If the filter is constructed to pass only-components associated with the interference pattern, then the Fourier transform of the interference noise pattern is given by the expression

$$N(u, v) = H_{\mathrm{NP}}(u, v)G(u, v)$$

- where, as usual, *G(u,v)*,denotes the Fourier transform of the corrupted image. Formation of H_{NP} *(u,v)* requires considerable judgment about what is or is not an interference spike.
- For this reason, the notch pass filter generally is constructed interactively by observing the spectrum of *G (u,v)* on a display.
- After a particular filter has been selected, the corresponding pattern in thespatial domain is obtained from the expression

$$\eta(x, y) = \Im^{-1}\{H_{\mathrm{NP}}(u, v)G(u, v)\}$$

- Because the corrupted imageisassumed to beformed by the addition ofthe uncorrupted image *f (x ,* y) andtheinterference, if ŋ(x,y)wereknowncompletely, subtracting (hepattern from*g(x,* y) toobtainf(x, y) wouldbeasimple matter.
- The problem,of course,is that (hisfiltering procedureusually yields only anapproximation of the truepattern.The effect of components not present in the estimate of η(x,y) can be minimized instead of subtracting from g(x,y) a weighted portion of η(x,y) to obtain tan estimate of f(x,y)

$$\hat{f}(x, y) = g(x, y) - w(x, y)\eta(x, y)$$

The function w(x,y)is called a weighing function. Consider a neighborhood of size (2a+1) by (2b+1) about a point (x.y).The local variance of f(x,y) at coordinates (x,y) can be estimated from the sample as follows

$$\sigma^2(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x, y) \right]^2 \quad (5.4\text{-}6)$$

where $\bar{\hat{f}}(x, y)$ is the average value of $\hat{f}$ in the neighborhood; that is,

$$\bar{\hat{f}}(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \hat{f}(x+s, y+t) \quad (5.4\text{-}7)$$

Points on or near the edge of the image can be treated by considering partial neighborhoods or by padding the border with 0s.

Substituting Eq. (5.4-5) into Eq. (5.4-6) yields

$$\begin{aligned}\sigma^2(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{ [g(x+s, y+t) \\ - w(x+s, y+t)\eta(x+s, y+t)] \\ - [\bar{g}(x, y) - \overline{w(x, y)\eta(x, y)}] \}^2\end{aligned} \quad (5.4\text{-}8)$$

Assuming that $w(x, y)$ remains essentially constant over the neighborhood gives the approximation

$$w(x+s, y+t) = w(x, y) \quad (5.4\text{-}9)$$

for $-a \leq s \leq a$ and $-b \leq t \leq b$. This assumption also results in the expression

$$\overline{w(x, y)\eta(x, y)} = w(x, y)\bar{\eta}(x, y) \quad (5.4\text{-}10)$$

in the neighborhood. With these approximations, Eq. (5.4-8) becomes

$$\begin{aligned}\sigma^2(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{ [g(x+s, y+t) \\ - w(x, y)\eta(x+s, y+t)] \\ - [\bar{g}(x, y) - w(x, y)\bar{\eta}(x, y)] \}^2\end{aligned} \quad (5.4\text{-}11)$$

To minimize $\sigma^2(x, y)$, we solve

$$\frac{\partial \sigma^2(x, y)}{\partial w(x, y)} = 0 \qquad (5.4\text{-}12)$$

for $w(x, y)$. The result is

$$w(x, y) = \frac{\overline{g(x, y)\eta(x, y)} - \overline{g}(x, y)\overline{\eta}(x, y)}{\overline{\eta^2}(x, y) - \overline{\eta}^2(x, y)} \qquad (5.4\text{-}13)$$

To obtain the restored image $\hat{f}(x, y)$, we compute $w(x, y)$ from Eq. (5.4-13) and then use Eq. (5.4-5). As $w(x, y)$ is assumed to be constant in a neighborhood, computing this function for every value of x and y in the image is unnecessary. Instead, $w(x, y)$ is computed for *one* point in each nonoverlapping neighborhood (preferably the center point) and then used to process all the image points contained in that neighborhood.

6. Explain the image Restoration filters in details with its types.[APR/MAY-18]

1. Inverse filter
2. Wiener filter

6.1 Explain the image restoration using inverse filtering. What are its limitations? Why inverse filtering approach fails in the presence of noise? (NOV/DEC-17)
(or)Describe inverse filtering for removal of blur caused by any motion and describe how it restore the image. (Nov 2010, April 2010, May-2014&May 2015)

Inverse filtering:-

It is the process of recovering the input of a system from its output. They are useful for recorrecting an input signal in anticipation of the degradation caused by the system such as correcting a non – linearity of the display.

The inverse filtering divides the transform of the degraded image by the degradation function.

w.k.t unconstrained restoration,

$$\boxed{\hat{f} = H^{-1} g}$$

$$\hat{f}(u,v) = \frac{G(u,v)}{H(u,v)}$$

We know that, g = Hf + ŋ

$$G(u, v) = H(u, v) F(u, v) + N(u, v)$$

Restored image is given by,

$$\hat{f}(u,v) = \frac{H(u,v)\ F(u,v)\ +\ N(u,v)}{H\ (u,v)}$$

$$\hat{f}(u,v) = \frac{H(u,v)\ F(u,v)}{H\ (u,v)} + \frac{N(u,v)}{H\ (u,v)}$$

$$\hat{f}(u,v) = F(u,v) + \frac{N(u,v)}{H\ (u,v)}$$

To find the original image,

$$\hat{f}(x,y) = F^{-1}[\hat{f}\ (u,v)]$$

Drawbacks:

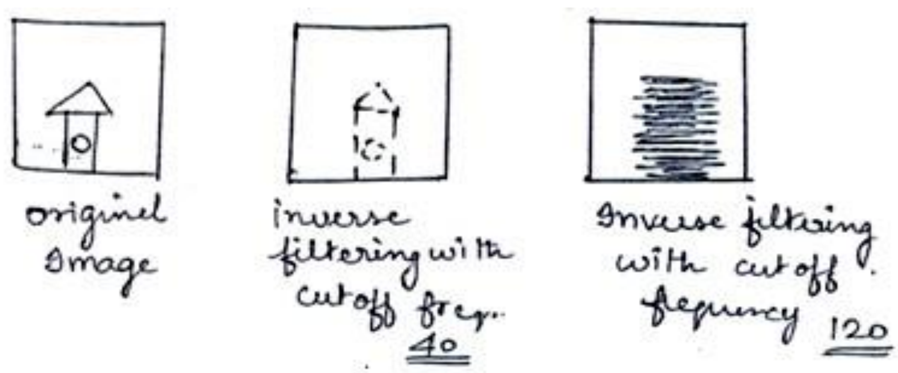
Inverse filtering is highly sensitive to noise.

Zero or Small value problem:-

If t he degradation function H(u, v) has zero or small value, then the ratio of N(u, v)/H(u, v) dominates the value of restored image.

This implies a poor performance of the system and results in best.

Approximation of the original image function. This is known as zero or small value problem.



The above drawbacks can be overcome by limiting the filter frequencies to only the values around the origin. This will decrease the probability of zero occurrence value degradation function.

6.2. Wiener filter: Explain the function of Wiener filter for image restoration inpresence of additive noise? (or) Explain the principle of least square filter and state its limitation. (Nov 2012, June 2011, May 2013, May-2014) (or) minimum mean square error filtering?(Nov 2014 & May 2015)(May/June 17)[APR/MAY-19][A/M 2021]

Wiener filtering or LMS filter – Least Mean square filtering:

For the restoration of an image, this method considers the degradation function as well as statistical properties of noise.

Objective:

It is to approximate the original image in such a way the mean square error between original and approximated image will be minimized.

LMS Value:

$$e^2 = E\left\{\left[f - \hat{f}\right]^2\right\}$$

Where,

f – Original image.

$\hat{f}$ - Restored image.

Assumptions:

(i). The image of noise are uncorrelated (no relation).

(ii). Either image or the noise has zero mean.

(iii). Approximated gray level for a linear function of degraded gray level.

Approximated image,

$$\hat{f}(u,v) = \left[\frac{H^*(u,v)s_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right] X\, G(u,v)$$

Where,

$H^*(u,v)$ – conjugate of H (u,v)

$S_f(u, v)$ – Power spectrum of original image.

$S_\eta(u,v)$ – Power density spectrum of noise.

H (u, v) – Linear operator.

$$\hat{f}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] X\, G(u,v)$$

Multiply & divide by H (u, v)

Wiener filtering equation is,

$$\hat{f}(u,v) = \left[\frac{H^*(u,v)\ H(u,v)}{H(u,v)|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] X\ G(u,v)$$

Case 1:

If noise = 0

$$\hat{f}(u,v) = \left[\frac{|H(u,v)|^2}{H(u,v)|H(u,v)|^2 + \frac{0}{S_f(u,v)}}\right] X\ G(u,v)$$

$$\boxed{\hat{f}(u,v) = \frac{G(u,v)}{H(u,v)}}$$

the***signal-to-noise ratio,***approximated using frequency domain quantities such as

$$\text{SNR} = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2}$$

The ***meansquareerror***can be approximated also in terms a-summation involving the original and restored images:

$$\text{MSE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

considers the restored image to be "signal" and the difference between this image and the original to be noise, signal-to-noise ratio in the spatial domain as

$$\text{SNR} = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2}$$

Case 2:

If noise = unknown quantity

$$\boxed{\hat{f}(u,v) = \left[\frac{|H(u,v)|^2}{H(u,v)|H(u,v)|^2 + k}\right] X\ G(u,v)}$$

Advantages of Wiener filtering over inverse filtering:

(i). Wiener filtering has no zero or small value problem.

(ii). The results obtained in Wiener filtering are more closer to the original image than inverse filtering.

Disadvantages: It requires power spectrum of ungraded image of noise to be known which makes the implementation more difficult.

13. A blur filter h(m,n) is given by

$$\begin{vmatrix} 0.1 & 0.1 & 0.1 & 0 \\ 0.1 & 0.1 & 0.1 & 0.1 \\ 0.05 & 0.1 & 0.1 & 0.05 \\ 0 & 0.05 & 0.05 & 0 \end{vmatrix}.$$

Find the deblur filter using inverse filtering? (Nov-2013)

Solution:

Find the Fourier transform =(4x4)DFT kernel *i/p image*DFT (kernel)^T

H(k,l)= $$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{vmatrix} \begin{vmatrix} 0.1 & 0.1 & 0.1 & 0 \\ 0.1 & 0.1 & 0.1 & 0.1 \\ 0.05 & 0.1 & 0.1 & 0.05 \\ 0 & 0.05 & 0.05 & 0 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{vmatrix}$$

H(k,l) = $$\begin{vmatrix} 1 & -0.2-0.2j & 0 & -0.2+2j \\ -0.1-0.3j & -0.1j & 0 & -0.1 \\ 0 & -0.1-0.1j & 0 & -0.1+0.1j \\ -0.1+3j & -0.1 & 0 & 0.1j \end{vmatrix}$$

G(k,l) = $\frac{1}{H(k,l)}$ is given by inverse filtering

$$= \begin{vmatrix} 1 & -2.5+2.5j & \infty & -2.5-2.5j \\ -1+3j & 10j & \infty & -10 \\ \infty & -5+5j & \infty & -5-5j \\ -1-3j & -10 & \infty & -10j \end{vmatrix}$$

14. Compare restoration with image enhancement. (nov 2014) (8 marks)

S.NO	**IMAGE ENHANCEMENT**	**IMAGE RESTORATION**
1.	It’s a subjective process.	It’s objective is based on sound mathematical principles.
2.	It involves only cosmetic changes in the brightness and contrast.	It requires modeling of the degradations.
3.	Often this is trial and error	The restoration algorithm is well defined.

	process.Theenchancement procedure is heuristic.	
4.	The procedure is very simple.	The procedure is complex.
5.	It increases the quality of an image.	It's related to image enchancement.
6.	Have prior knowledge about the information.	Does not need the prior information.
7.	Computation speed is low.	Computation speed is high.
8.	Identifying and analysing of degraded pixel is easy.	Difficult.
9.	Losses are minimum.	Losses are high compare to enchancement.

15. For the given image matrix, compute the new pixel value for the marked pixel using the following filters (i) Mean of filter, (ii) Max filter, (iii) Min Filter, (iv) Median filter of size 3x3.

(or)

Apply order statistics filter on the selected pixel in the image(NOV/DEC 16)

Problem 3.1

For the given image matrix, compute the new pixel value for the marked pixel using the following filters (i) Mean of filter, (ii) Max filter, (iii) Min filter, (iv) Median filter of size 3 × 3.

$$\begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 3 & 8 \\ 5 & 3 & 2 & 1 & 3 \\ 4 & 1 & 2 & 3 & 2 \\ 3 & 2 & 1 & 4 & 2 \end{bmatrix}$$

Solution

Consider 3 × 3 neighbourhood of the marked pixel,

$$\begin{bmatrix} 4 & 5 & 6 \\ 2 & 3 & 8 \\ 2 & 1 & 3 \end{bmatrix}$$

Mean filter:

Mean filter mask of size 3×3 is given by $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Applying the mask on the neighbourhood results in

$$\frac{1}{9}[4 \times 1 + 5 \times 1 + 6 \times 1 + 2 \times 1 + 3 \times 1 + 8 \times 1 + 2 \times 1 + 1 \times 1 + 3 \times 1]$$

$$= \frac{34}{9} = 3.77 \simeq 4$$

Obtained mean value 3.77 is rounded as '4'.

$\boxed{\text{Mean value} = 4}$

Max filter:

Arrange the pixels in the ascending order.

1 2 2 3 3 4 5 6 8

From the above order, max filter value is '8'.

$\boxed{\text{Max value} = 8}$

Min filter:

Arrange the pixel in the ascending order

1 2 2 3 3 4 5 6 8

From the above order, min filter value is 1

Min value = 1

Median filter:

Arrange the pixel in the ascending order

1 2 2 3 3 4 5 6 8

Number of pixels n = 9

Median value = n+1 /2 = 10 /2 = 5^{th} value in the order

Median value = 3

16. Problem 3.2

Compute the median value of the marked pixel shown in the image using (i) median filter of size 3×3, (ii) median filter of size 5×5.

$$\begin{bmatrix} 1 & 5 & 7 & 6 & 2 \\ 4 & 2 & 2 & 3 & 8 \\ 3 & 4 & 5 & 7 & 6 \\ 4 & 3 & 3 & 6 & 6 \\ 4 & 7 & 6 & 6 & 4 \end{bmatrix}$$

Solution

(i) Medina filter of size 3×3.

Step 1: For applying the median filter of size 3×3, consider the 3×3 neighbourhood with the marked pixel in the centre as shown,

$$\begin{bmatrix} 2 & 2 & 3 \\ 4 & 5 & 7 \\ 3 & 3 & 6 \end{bmatrix}$$

Step 2: Arrange the pixels in ascending order

2 2 3 3 3 4 5 6 7

Number of pixels $n = 9$

$$\therefore \quad \frac{n+1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5$$

5^{th} value in the order is the median value.

$\therefore$ Median value = 3.

Now the marked pixel value '5' in the original image is replaced with the median value '3'.

$$\underset{\text{Original image data}}{\begin{bmatrix} 2 & 2 & 3 \\ 4 & 5 & 7 \\ 3 & 3 & 6 \end{bmatrix}} \longrightarrow \underset{\text{After median filtering}}{\begin{bmatrix} 2 & 2 & 3 \\ 4 & 3 & 7 \\ 3 & 3 & 6 \end{bmatrix}}.$$

Step 1: For the median filter of size 5 × 5, consider the neighbourhood of size 5 × 5 with the marked pixel as the center pixel

$$\begin{bmatrix} 1 & 5 & 7 & 6 & 2 \\ 4 & 2 & 2 & 3 & 8 \\ 3 & 4 & 5 & 7 & 6 \\ 4 & 3 & 3 & 6 & 6 \\ 4 & 7 & 6 & 6 & 4 \end{bmatrix}$$

Step 2: In this case, the number of pixels $n = 25$.

$$\frac{n+1}{2} = \frac{25+1}{2} = \frac{26}{2} = 13$$

The median value is 13^{th} value in the order

1 2 2 2 3 3 3 4 4 4 4 5 5 5 6 6 6 6 6 7 7 7 8

13^{th} value in the order is 4. Now the marked pixel value '5' in the original image is replaced with the median value '4'.

$$\underset{\text{Original image data}}{\begin{bmatrix} 1 & 5 & 7 & 6 & 2 \\ 4 & 2 & 2 & 3 & 8 \\ 3 & 4 & 5 & 7 & 6 \\ 4 & 3 & 3 & 6 & 6 \\ 4 & 7 & 6 & 6 & 4 \end{bmatrix}} \longrightarrow \underset{\text{After median filtering}}{\begin{bmatrix} 1 & 5 & 7 & 6 & 2 \\ 4 & 2 & 2 & 3 & 8 \\ 3 & 4 & 4 & 7 & 6 \\ 4 & 3 & 3 & 6 & 6 \\ 4 & 7 & 6 & 6 & 4 \end{bmatrix}}$$

16. What is image restoration? Explain the degradation model for continuous function in detail? (Or)Explain the model of image degradation process and discuss its role in image restoration. (Nov 2010 & Nov 2012)

Introduction:-

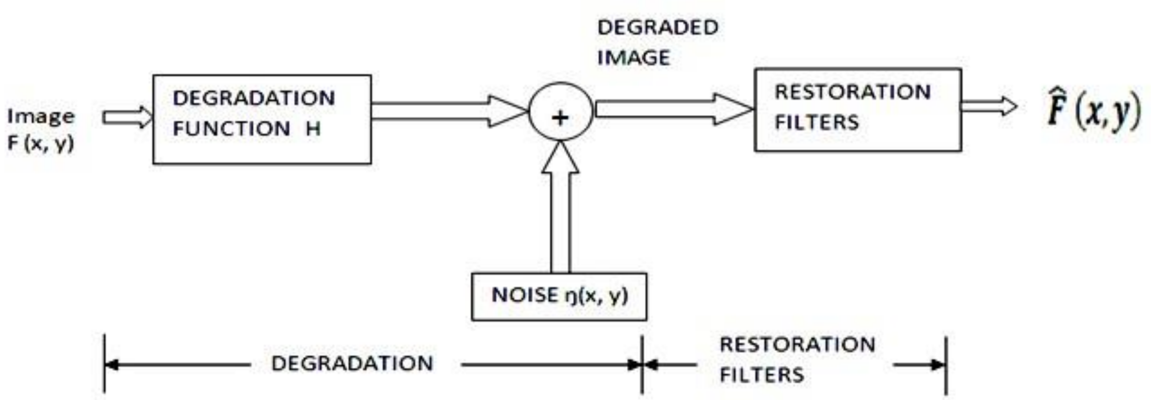
The restoration technique is used to reconstruct or recover the image (ie) already degraded by degradation phenomena.

In restoration process the degradation images and apply inverse process to that image to recover the original image.

Image restoration or degradation model:

The degradation function together with additive noise operates on an input image f(x, y) to give a degraded image g(x, y). Then image g(x, y) is given as input to the restoration filter to produce $(\hat{f}(x, y)$.

Image degradation / Restoration model



- The degradation function together with additive noise operates on an input image f(x, y) to give a degraded image g(x, y). Then image g(x, y) is given as a input to the restoration filter to produce $(\hat{F}(x, y)$.
- F(x, y) is similar to $(\hat{F}(x, y)$. The equation for restoration is given as a input to the restoration filter to produce $(\hat{F}(x, y)$.
- The equation for restoration is given by,

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

- Write the above equation in Fourier transform we get,

$$G(u, v) = H(u, v)\,F(u, v) + N(u, v)$$

- Therefore the restoration process is to reconstruct the image that has been degraded by a degradation function.

Properties of degradation model:

(a) Linearity property:

If $\eta(x, y) = o$,

$$G(x, y) = h(x, y)\,f(x, y)$$
$$= H[f(x, y)]$$

If H is linear,

$$H[k_1 f_1(x, y) + k_2 f_2(x, y)] = k_1 H[f_1(x, y) + k_2 H(f_2(x, y)] \quad \text{———} \; 1$$

Where,

k_1 & k_2 – constant

$f_1(x, y)$ & $f_2(x, y)$ – two input images.

(b) Additive property:

If $k_1 = K_2 = 1$, in equ 1

$$H[f_1(x, y) + f_2(x, y)] = H[f_1(x, y) + H(f_2(x, y)] \quad \text{———} \; 2$$

(c) Homogeneity property:

Consider $f_2(x, y) = 0$ in equation 1

$$H[k_1 f_1(x, y)] = k_1 H[f_1(x, y)] \quad \text{———} \; 3$$

- It states that a response to a constant multiple of any input image is equal to the response to that input image multiplied by the same constant.

Position invariant or space invariant:-

w.k.t H[f(x, y)] = g(x, y)

H[f(x-α, y-β)] = g(x-α, y-β)

Position invariant indicates that the response at any point in the image depend only on the value of the input at that point and not on the position of the point.

Degradation in continuous function:

$$f(x,y) = \int_{-\infty}^{\infty}\int f(\alpha,\beta)S(x-\alpha,y-\beta)\,d\alpha\,d\beta \quad (4)$$

Where,

α, β – constant

S – Impulse function

Wkt, g(x, y) = H[f(x, y)]+ ŋ(x, y)

Consider noise function ŋ(x, y) = 0

So g(x, y) = H[f(x, y)]

$$= H\left[\int_{-\infty}^{\infty}\int f(\alpha,\beta)S(x-\alpha,y-\beta)\,d\alpha\,d\beta\right]$$

According to homogeneity principle f(α, β) is independent of x, y

$$g(x,y) = \int_{-\infty}^{\infty}\int f(\alpha,\beta)H[S(x-\alpha,y-\beta)]\,d\alpha\,d\beta$$

H – Linear operator

H[s(x-α, y-β)] = h(x, α, y, β)

= h(x-α, y-β)

It is called as impulse response of H & also it is referred as PSF (Point spread function).

So the degraded image f(x, y) is given by,

$$g(x,y) = \int_{-\infty}^{\infty}\int f(\alpha,\beta)H[S(x,\alpha,y,\beta)]\,d\alpha\,d\beta$$

(or)

$$g(x,y) = \int_{-\infty}^{\infty}\int f(\alpha,\beta)H[S(x-\alpha,y-\beta)]\,d\alpha\,d\beta$$

Advantages:-

(i). Extensive tool for linear system theory.

(ii). Non linear & space variant introduced, difficulties, but restoration focus on linear & space invariant technique. [So simplification is obtained in result].

Degradation in discrete formulation:

$$\mathbf{1\ Dimensional:}\ g_e(m) = \sum_{m=0}^{m-1} f_e(m)\, h_e(x-m)$$

$$2\ Dimensional: g_e(m,n) = \sum_{m=0}^{m-1} f_e(m)\, h_e(x-m, y-n)$$

Vector matrix form,

G=Hf+ŋ [noise ŋ = 0]

So g= Hf

$$g = \begin{bmatrix} g_e(0) \\ g_e(1) \\ g_e(m-1) \end{bmatrix} ; \quad f = \begin{bmatrix} f_e(0) \\ f_e(1) \\ f_e(m-1) \end{bmatrix}$$

Block circular matrix:

Mxn matrix,

$$H = \begin{bmatrix} H_0 & H_{m-1} & H_{m-2} \cdots \cdots & H_1 \\ H_1 & H_0 & H_{m-1} \cdots \cdots & H_2 \\ H_2 & H_1 & H_0 \cdots \cdots & H_3 \\ & & \vdots & \\ H_{m-1} & H_{m-2} & \cdots \cdots & H_0 \end{bmatrix} \tag{*}$$

M≥A+B-1

$$Hj = \begin{bmatrix} h_e(j,0) & h_e(j,N-1) & h_e(j,1) \\ h_e(j,1) & h_e(j,0) & h_e(j,2) \\ h_e(j,2) & \cdots & \cdots \\ & \vdots & \\ h_e(j,N-1) & h_e(j,N-2) \ldots & h_e(j,0) \end{bmatrix}$$

Definition:

A square matrix in which each row is a circular shift of a proceeding row & the first row is circular shift of the last row is called as "circular matrix".

The equ is called as block circular matrix because of block of H subscripted in circular manner.