Unit - III (Sampling Distribution)

Sampling :-

A finite subset of the population is called sample and the process of selection of such samples is called sampling.

Parameter and Statistics :-

Statistical measures calculated on the basis of population are called parameters. Corresponding measures calculated on the basis of sample observation are called statistics.

Test of Hypothesis :-

A hypothesis which assume that there is no significant difference b/w the sample statistics and the corresponding population parameter or between two sample statistics is called null hypothesis and is denoted by H_0. A hypothesis which is different from the null hypothesis is called an alternate hypothesis and is denoted by H_1. A procedure for deciding whether to accept or to reject a null hypothesis is called test of hypothesis.

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Test of Significance :-

If θ_0 is a parameter of population and θ is the corresponding sample statistics, usually there will be some difference b/w θ_0 and θ. The procedure of testing whether the difference b/w θ_0 and θ is significant or not is called test of significance.

Critical Region and Level of Significance :-

If the difference b/w a sample statistics and the corresponding parameter is significant then the sample statistic lies in a certain region or interval, this region is called the critical region or region rejection.

The probability that a random value (x) of the statistic lies in the critical region is called level of significance (LOS).

One Tailed and Two - Tailed Test :-

If θ_0 is a population parameter and θ is the corresponding sample statistic and null hypothesis $H_0 : \theta_0 = \theta$, then the alternative hypothesis can be any one :

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(I) $H_1 : \theta_0 \neq \theta$ i.e. $\theta_0 > \theta$ or $\theta_0 < \theta$

(II) $H_1 : \theta_0 > \theta$ (Left tailed)

(III) $H_1 : \theta_0 < \theta$ (Right tailed)

H_1 given in (I) is called two-tailed hypothesis. Whereas H_1 in (II) and (III) is called one-tailed alternative hypothesis.

Critical Values or Significant Values :-

The critical values for some standard LOS, are as:

Nature of Test	Level of Significant			
	1% (0.01)	2% (0.02)	5% (0.05)	10% (0.1)
Two-tailed	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 2.33$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right Tailed	$Z_\alpha = 2.33$	$Z_\alpha = 2.055$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left Tailed	$Z_\alpha = -2.33$	$Z_\alpha = -2.055$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Note:- If the size of sample exceeds 30, it is called a large sample otherwise small sample.

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Test of Significance for Large Samples:-

Z-Test:-

Test 1:- Test of significance of the difference b/w sample proportion and population proportion.

$$\boxed{Z = \frac{p - P}{\sqrt{\left(\frac{PQ}{n}\right)}}}$$

$p \rightarrow$ sample proportion

$P \rightarrow$ Population proportion

$n \rightarrow$ size of sample

If P is unknown, then we assume that p is nearly equal to P.

$$Z = \frac{p - P}{\sqrt{\left(\frac{pq}{n}\right)}}$$

Test 2:- Test of significance of the difference between two samples.

$$\boxed{Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

If P is not known, then

$$\boxed{P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}}, \quad Q = 1 - P$$

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Ex-①:- Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20% was wrong. Based on the particular day's production, find also the 95% confidence limits for the percentage of top quality product.

Sol:- Let P and p denote the population and sample proportion respectively.

$H_0 : p = P$

$H_1 : p \neq P$, i.e. two tailed test

Here $P = 20\% = \frac{1}{5}$, $p = \frac{50}{400} = \frac{1}{8}$

$\therefore Q = 1 - P = 1 - \frac{1}{5} = \frac{4}{5}$, $n = 400$

$$Z = \frac{p - P}{\sqrt{\left(\frac{PQ}{n}\right)}} = \frac{\frac{1}{8} - \frac{1}{5}}{\sqrt{\frac{1}{400} \times \frac{1}{5} \times \frac{4}{5}}} = -3.75$$

$\therefore |Z| = 3.75 > 1.96 = |Z_\alpha|$

As $Z_{cal} > Z_{tab}$ $\therefore$ Null hypothesis i.e. H_0 is rejected.

Now, 95% confidence limits for P are given by

$$\frac{|p - P|}{\sqrt{\left(\frac{pq}{n}\right)}} \leq 1.96$$

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$$-1.96 \times \sqrt{\frac{pq}{n}} \leq |p - P| \leq 1.96 \times \sqrt{\frac{pq}{n}}$$

$$p - 1.96 \times \sqrt{\frac{pq}{n}} \leq P \leq p + 1.96 \times \sqrt{\frac{pq}{n}}$$

But $p = \frac{1}{8}$, $q = \frac{7}{8}$, $n = 400$

$$0.093 \leq P \leq 0.157$$

Therefore, 95% confidence limits for the percentage of top quality product are 9.3 and 15.7.

Ans

Ex-Q:- The fatality rate of typhoid patients is belived to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient?

Sol:- Let P be the probability that hospital not efficient. consider

$H_0 : p = P$

$H_1 : p < P$

$\therefore$ One-tailed (left) test is to be used

Here $P = 0.1726$ $\therefore Q = 0.8274$, $p = \frac{63}{640} = 0.0984$

$n = 640$

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Now

$$Z = \frac{p - P}{\sqrt{\left(\frac{PQ}{n}\right)}}$$

$$= \frac{0.0984 - 0.1726}{\sqrt{\frac{0.1726 \times 0.8274}{640}}} = -4.96$$

Now

Z_α at 1% LOS $= -2.33$

$\therefore \quad |Z_{cal}| > |Z_{tab}|$

$\therefore$ The difference b/w p and P is significant i.e. H_0 is rejected.

That is, the hospital is efficient in bringing down the fatality rate of typhoid patients.

Answer

Ex-③ :- A salesman in a departmental store claims that at most 60 percent of the shoppers entering the store leave without making a purchase. A random sample of 50 shopers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman? Use an LOS of 0.05.

Sol:- Let P and p denote the population and sample proportion of shoppers not making a purchase.

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Let

$$H_0 : p = P$$

$$H_1 : p > P$$

One tailed (right) test is to be used

Here $P = 0.6 \quad \therefore Q = 0.4 \quad , \quad n = 50$

$$p = \frac{35}{50} = 0.7$$

Now

$$z = \frac{p - P}{\sqrt{\left(\frac{PQ}{n}\right)}}$$

$$= \frac{0.7 - 0.6}{\sqrt{\frac{0.6 \times 0.4}{50}}} = 1.443$$

At 5% LOS, $Z_\alpha = 1.645$

$\therefore \quad |Z| < Z_\alpha$

Therefore, the difference b/w p and P is not significant at 5% Level.

$\therefore$ H_0 is accepted

That is, the sample results are consistent with the claim of the salesman.

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Ex-(6):- In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5 percent of a random sample of 1600 boys (school) had the same defect. Is the difference b/w the proportion significant?

Sol:- We have, $H_0 : P_1 = P_2$ & $H_1 : P_1 \neq P_2$

Here $P_1 = 20\% = 0.2$ $\quad P_2 = 18.5\% = 0.185$

$n_1 = 900$ $\quad n_2 = 1600$

Let LOS be 5%, then $Z_\alpha = 1.96$

Now

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

As P, the population proportion is not given

$$\therefore P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{180 + 296}{900 + 1600} = 0.1904$$

$$\therefore Q = 1 - P = 0.8096$$

Now

$$Z = \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8096\left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.92$$

$\therefore |Z| < |Z_\alpha|$

$\therefore$ The difference b/w P_1 and P_2 is not significant at 5% level.

Answer

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Ex-(7):- Before an increase in excise duty on tea, 800 people out of a sample 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find wether there is a significant decrease in the consumption of tea after the increase in duty.

Sol:- Here

$$P_1 = \frac{800}{1000} = 0.8, \quad P_2 = \frac{800}{1200} = \frac{2}{3}, \quad n_1 = 1000, \quad n_2 = 1200$$

Consider the hypothesis

$$H_0 : P_1 = P_2 \quad \text{and} \quad H_1 : P_1 > P_2$$

So, one tailed (Right) test is to be used.

Let LOS be 1%. Therefore $Z_\alpha = 2.33$

Now

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{1000 \times 0.8 + 1200 \times \frac{2}{3}}{1000 + 1200} = 0.7273$$

$$\therefore \quad Q = 1 - P = 1 - 0.7273 = 0.2727$$

Now

$$Z = \frac{0.8 - \frac{2}{3}}{\sqrt{0.7273 \times 0.2727\left(\frac{1}{1000} + \frac{1}{1200}\right)}} = 6.82$$

$$\therefore \quad |Z| > Z_\alpha$$

$\therefore$ The difference b/w P_1 and P_2 is significant at 1% level.

i.e. H_0 is rejected and H_1 is accepted.

Therefore, there is a significant decrease in consumption of tea after the increase in duty.

Ans

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Ex-(8):- 15.5% of a random sample of 1600 undergraduates were smokers, whereas 20% of a random sample of 900 postgraduate were smokers in a state. Can we conclude that less number of undergraduates are smokers than the postgraduates?

Sol:- Here

$P_1 = 15.5\% = 0.155$, $P_2 = 20\% = 0.2$, $n_1 = 1600$, $n_2 = 900$

Consider the hypothesis

$H_0 : P_1 = P_2$ and $H_1 : P_1 < P_2$

∴ One tailed (left) test is to be used.

Let LOS be 5%, Therefore $Z_\alpha = -1.645$

Now

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{1600 \times 0.155 + 900 \times 0.2}{1600 + 900} = 0.1712$$

$$\therefore Q = 1 - P = 1 - 0.1712 = 0.8288$$

Now

$$Z = \frac{0.155 - 0.2}{\sqrt{0.1712 \times 0.8288\left(\frac{1}{1600} + \frac{1}{900}\right)}} = -2.87$$

$\therefore |Z| > |Z_\alpha|$

∴ The difference b/w P_1 and P_2 is significant at 5% level i.e. H_0 is rejected and H_1 is accepted.

That is, the habit of smoking is less among the undergraduate that among the postgraduates.

Ans

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Test 3:- Test of significance of the difference b/w sample mean and population mean.

$$\boxed{Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

$\bar{x}$: Mean of sample

μ : Mean of Population

σ : Standard deviation of population

n : sample size

Test 4:- Test of significance of the differce b/w the mean of two samples.

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}}$$

$\bar{x}_1$: Mean of first sample

$\bar{x}_2$: Mean of second sample

n_1 : size of first sample

n_2 : size of second sample

σ_1 : SD of first population

σ_2 : SD of second population

Note: ①:- If two samples are draw from same population i.e. if $\sigma_1 = \sigma_2 = \sigma$

$$\boxed{Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$

② If σ_1 and σ_2 are not known and $\sigma_1 \neq \sigma_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

s_1 : SD of first sample

s_2 : SD of second sample

③ If σ_1 and σ_2 are not known and $\sigma_1 = \sigma_2 = \sigma$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}\right)}} \quad \text{and} \quad \sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

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Ex-(9):- A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the SD is 10 cm?

Sol:- Here $\bar{x} = 160$, $n = 100$, $\mu = 165$, $\sigma = 10$

Consider the hypothesis

$H_0 : \bar{x} = \mu$

$H_1 : \bar{x} \neq \mu$

$\therefore$ Two tailed test is to be used.

Let LOS be 1%. $\therefore$ $Z_\alpha = 2.58$

Now

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{160 - 165}{10/\sqrt{100}} = -5$$

$\therefore$ $|Z| > |Z_\alpha|$

$\therefore$ The difference between $\bar{x}$ and μ is significant at 1% level. i.e. H_0 is rejected.

That is, it is not statistically correct to assume that $\mu = 165$

Ans

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Ex-(10):- The mean breaking strength of the cables supplied by a manufacturer is 1800, with an SD of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS?

Sol:- Here $\mu = 1800$, $\sigma = 100$, $n = 50$, $\bar{x} = 1850$

Let $H_0 : \bar{x} = \mu$

$H_1 : \bar{x} > \mu$

$\therefore$ One right tailed test is to be used.

Let LOS be 1%. $\therefore Z_\alpha = 2.33$

Now

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1850 - 1800}{100/\sqrt{50}} = \frac{\sqrt{50}}{2} = 3.54$$

$\therefore |Z| > |Z_\alpha|$

$\therefore$ The difference b/w $\bar{x}$ and μ is significant at 1% level i.e. H_0 is rejected and H_1 is accepted.

So, we may support the claim of increase in breaking strength.

Ans

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Ex-(13):- A normal population has a mean 0.1 and an SD of 2.1. Find the probability that the mean of a sample of 900 drawn from this population will be negative.

Sol:- As $\bar{x}$ follows normal distribution

we have, $\mu = 0.1$, $\sigma = 2.1$, $n = 900$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 0.1}{2.1/\sqrt{900}}$$

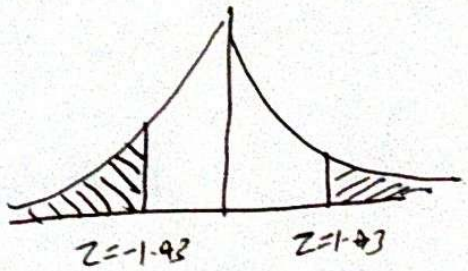
$$= \frac{\bar{x} - 0.1}{0.07}$$

Now

$$P(\bar{x} < 0) = P\{\bar{x} - 0.1 < -0.1\}$$

$$= P\left\{\frac{\bar{x} - 0.1}{0.07} < -\frac{0.1}{0.07}\right\}$$

$$= P\{Z < -1.43\}$$



$$= P\{Z > 1.43\}$$

$$= 0.5 - P\{0 < Z < 1.43\}$$

$$= 0.5 - 0.4236$$

$$= 0.0764$$

Ans

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Ex-(13):- In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the samples have been drawn from the same population with SD. 4?

Sol:- Here $\bar{x}_1 = 20$, $n_1 = 500$, $\bar{x}_2 = 15$, $n_2 = 400$, $\sigma = 4$

Consider the hypothesis

$H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : x_1 \neq \bar{x}_2$

$\therefore$ Two tailed test is to be used.

Let LOS be 1%. $\therefore$ $Z_\alpha = 2.58$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{20-15}{4\sqrt{\frac{1}{500} + \frac{1}{400}}} = 18.6$$

$\therefore$ $|Z| > |Z_\alpha|$

i.e. The difference b/w $\bar{x}_1$ and $\bar{x}_2$ is significant at 1% level i.e. H_0 is rejected.

Therefore, the samples could not have been drawn from the same population.

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Ex-(14):- A simple sample of heights of 6400 English men has a mean of 170 cm and an SD of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and an SD of 6.3 cm. Do the data indicate the Americans are on the average, taller than the Englishmen?

Sol:- Here

$n_1 = 6400$, $\bar{x}_1 = 170$, $s_1 = 6.4$

$n_2 = 1600$, $\bar{x}_2 = 172$, $s_2 = 6.3$

Let $H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 < \bar{x}_2$

i.e. Left tailed test is to be used.

Let LOS be 1%. Therefore, $Z_\alpha = -2.33$

Now

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{170 - 172}{\sqrt{\frac{(6.4)^2}{6400} + \frac{(6.3)^2}{1600}}} = -11.32$$

$\therefore |Z| > |Z_\alpha|$

So, the difference b/w $\bar{x}_1$ and $\bar{x}_2$ is significant at 1% level i.e. H_0 is rejected and H_1 is accepted

That is, the Americans are, on average, taller than Englishmen.

Answer

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Ex-(15):- Test the significance of the difference between the means of the samples, drawn from two normal populations with the same SD using the following data:

	Size	Mean	SD
Sample 1	100	61	4
Sample 2	200	63	6

Sol:-

Consider

$$H_0 : \bar{x}_1 = \bar{x}_2$$

$$H_1 : \bar{x}_1 \neq \bar{x}_2$$

$\therefore$ Two tailed test is to be used.

Let LOS be 5%. $\therefore Z_\alpha = 1.96$

Now

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}}} \qquad (\because \sigma_1 = \sigma_2)$$

$$= \frac{61 - 63}{\sqrt{\frac{4^2}{200} + \frac{6^2}{100}}} = -3.02$$

$\therefore |Z| > |Z_\alpha|$

$\therefore$ The difference b/w $\bar{x}_1$ and $\bar{x}_2$ is significant at 5% level. i.e. H_0 is rejected and H_1 is accepted.

$\therefore$ Two sample have not same mean.

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Test - (5):-

Test of significance of the difference between sample SD and population SD.

$$\boxed{Z = \frac{s - \sigma}{\sigma/\sqrt{2n}}}$$

$s \rightarrow$ SD of sample

$\sigma \rightarrow$ SD of population

$n \rightarrow$ size of sample

Test - (6):-

Test of significance of the difference between SDs of two samples:

$$\boxed{Z = \frac{s_1 - s_2}{\sigma\sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}}}$$

$s_1 \rightarrow$ SD of Ist sample

$s_2 \rightarrow$ SD of IInd sample

$\sigma \rightarrow$ SD of population

$n_1 \rightarrow$ size of Ist sample

$n_2 \rightarrow$ size of IInd sample

Note:- If σ is not known.

$$\boxed{Z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_2} + \frac{s_2^2}{2n_1}}}}$$

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Ex-(19):- A manufacturer of electric bulbs, according to a certain process, find the SD of the life of lamps to be 100 hrs. He want to change the process, if the new process results in a smaller variation in the life of lamps. In adopting a new process, a sample of 150 bulbs gave an SD of 95 hrs. Is the manufacturer justified in changing the process?

Sol:- Here $\sigma = 100$, $n = 150$, $s = 95$

let

$H_0 : s = \sigma$

$H_1 : s_1 < \sigma$

$\therefore$ Left tailed test is to be used.

let LOS be 5%. $\therefore Z_\alpha = -1.645$

Now

$$Z = \frac{s-\sigma}{\sigma/\sqrt{2n}} = \frac{95-100}{100/\sqrt{2\times150}} = -0.866$$

$\therefore |Z| < |Z_\alpha|$

$\therefore$ The difference b/w s and σ is not significant at 5% level. i.e. H_0 is accepted.

i.e., the manufacturer is not justified in changing the process.

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Ex-(20): The SD of a random sample of 1000 is found to be 2.6 and the S.D of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same SD.

Soln: Here $n_1 = 1000$, $s_1 = 2.6$, $n_2 = 500$, $s_2 = 2.7$

Consider

$H_0 : s_1 = s_2$

$H_1 : s_1 \neq s_2$

$\therefore$ Two tailed test is to be used.

Let LOS be 5%. $\therefore$ $Z_\alpha = 1.96$

Now

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_2} + \frac{s_2^2}{2n_1}}} \quad (\because \sigma \text{ is not known})$$

$$= \frac{2.6 - 2.7}{\sqrt{\frac{(2.6)^2}{2\times500} + \frac{(2.7)^2}{2\times1000}}} = -0.98$$

$\therefore$ $|Z| < |Z_\alpha|$ i.e. the difference b/w s_1 and s_2 is not significant at 5% level. $\therefore$ H_0 is accepted.

$\therefore$ Two samples could have come from same population.

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Test of Significance for Small samples ($n < 30$)

Student's t - distribution :-

A RV T is said to follow Student's t-distribution if its probability density function is

$$f(t) = \frac{1}{\sqrt{\nu}\, \beta\left(\frac{\nu}{2}, \frac{1}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\left(\frac{\nu+1}{2}\right)}, \quad -\infty < t < \infty$$

where

ν denote the number of degree of freedom of t - distribution.

Properties of t - Distribution :-

① The probability curve of t-distribution is similar to the standard normal curve and is symmetric about $t = 0$.

② For sufficient large value of ν, t-distribution tends to the standard normal distribution

③ The mean of t-distribution is zero.

④ The variance of t distribution is $\frac{\nu}{\nu - 2}$, if $\nu > 2$ and is greater than 1, but it tends to 1 as $\nu \to \infty$.

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Use of t - Distribution :-

The t-distribution is used to test the significance of the difference between

① The mean of small sample and mean of population.

② The mean of two small samples

Degree of Freedom:- The number of degree of freedom is given by $\nu = n - k$, where n is the number of observation and k is the number of values that have been found out and specified by prior calculations.

Test ①:- Test of significance of the difference b/w sample mean and population mean

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}}$$

$\bar{x} \rightarrow$ mean of sample

$\mu \rightarrow$ mean of population

$n \rightarrow$ sample size

where $\boxed{\nu = n-1}$ $\quad s \rightarrow$ SD of sample.

Test ②:- Test of significance of the difference b/w means of two small samples.

$$\boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}, \quad \boxed{\nu = n_1 + n_2 - 2}$$

Note:- If $n_1 = n_2 = n$

$$\boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2 + s_2^2}{n-1}}}} \quad \boxed{\nu = 2n - 2}$$

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Ex-(1):- Test made on the breaking strength of 10 pieces of a metal gave the following results: 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 Kg. Test if the mean breaking strength of the wire can be assume as 577 Kg.

Sol:- We have

$\mu = 577$, $n = 10$ let $d_i = x_i - 582$

$$\bar{x} = \frac{1}{n}\Sigma x_i = A + \frac{1}{n}\Sigma d_i = \frac{1}{10}(-68) + 582 = 575.2$$

$$s = \sqrt{\frac{1}{n}\Sigma d_i^2 - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{1}{10}\times 1144 - \left(\frac{-68}{10}\right)^2}$$

$$= 8.26$$

x_i	$d_i = x_i - 582$	d_i^2
578	−4	16
572	−10	100
570	−12	144
568	−14	196
572	−10	100
570	−12	144
570	−12	144
572	−10	100
596	14	196
584	2	4
	−68	1144

Now

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}, \quad \nu = n-1$$

$$= \frac{575.2 - 577}{8.26/\sqrt{9}} = -0.65, \quad \nu = 10 - 1 = 9$$

let

$H_0: \bar{x} = \mu$ and $H_1: \bar{x} \neq \mu$

∴ Two tailed test is to be used. Let LOS be 5%.

From table $t_{0.05}(\nu = 9) = 2.26$

∴ $|t| < t_{0.05}$. There is no significance difference b/w $\bar{x}$ and μ. i.e. H_0 is accepted.

i.e. The mean strength of wire can be assume as 577 kg at 5% level.

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Ex-②:- A machinist is to expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter of 1.85 cm, with an SD of 0.1 cm. On the basis of this sample, would you say that the work of machinist is inferior?

Sol:- Here

$\bar{x} = 1.85$, $s = 0.1$, $n = 10$ and $\mu = 1.75$

Let

$H_0 : \bar{x} = \mu$

$H_1 : \bar{x}_1 \neq \mu$

∴ Two tailed test is to be used. Let LOS be 5%.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}, \quad \nu = n-1$$

$$= \frac{1.85 - 1.75}{0.1/\sqrt{10-1}} = 3, \quad \nu = 10-1 = 9$$

From table

$t_{0.05}$ (at $\nu = 9$) $= 2.26$ & $t_{0.01}$ (at $\nu = 9$) $= 3.25$

∴ $|t| > t_{0.05}$ and $|t| < t_{0.01}$

H_0 is rejected at 5%. So, work is inferior at 5% level.

H_0 is accepted at 1%. So, work is not inferior at 1% level.

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Ex-③:- A certain injection administered to each of the 12 patients resulted in the following increase of blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4

Can it can be conclude that the injection will be in general, accompainted by an increase in BP?

Sol:- we have

$$\bar{x} = \frac{1}{n} \Sigma x = \frac{31}{12} = 2.58$$

$$s^2 = \frac{1}{n} \Sigma x_i^2 - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{1}{12} \times 189 - (2.58)^2 = 8.76$$

$$\therefore s = \sqrt{8.76} = 2.96 \quad , \quad n = 12 \quad , \quad \mu = 0$$

Let $H_0 : \bar{x} = \mu$ and $H_1 : \bar{x} > \mu$

Right tailed test is to be used. Let LOS be 5%.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{2.58 - 0}{2.96/\sqrt{11}} = 2.89 \quad , \quad \nu = n-1 = 11$$

$\therefore t_{0.05} (\nu = 11)$ for one tailed $= t_{0.1} (\nu = 11)$ for two tailed

$= 1.80$

$\therefore |t| > t_{0.05} (\nu = 11)$ for one tailed.

$\therefore H_0$ is rejected and H_1 is accepted.

Thus, we conclude that the injection is accompanied by an increase in BP.

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Ex-④:- The mean lifetime of a sample 25 bulbs is found to as 1550 hrs and with an SD of 120 hrs. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hrs. Is the claim acceptable at 5%. LOS?

Sol:- Here $\bar{x} = 1550$, $s = 120$, $n = 25$ and $\mu = 1600$

Let

$H_0 : \bar{x} = \mu$ and $H_1 : \bar{x} < \mu$

Left-tailed test is to be used.

Let LOS be 5%.

Now

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \quad \& \quad \nu = n-1$$

$$= \frac{1550 - 1600}{120/\sqrt{24}} = -2.04 \quad \& \quad \nu = 24$$

From table

$t_{0.05}$ for one tailed $= t_{0.1}$ for two tailed (for $\nu = 24$) $= 1.71$

$\therefore \quad |t| > |t_{0.05}|$

$\therefore$ H_0 is rejected and H_1 is accepted at 5% level.

$\therefore$ The company claim is accepted at 5% level

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Ex-⑥:- Two independent samples of size 8 and 7 contained the following values:

Sample 1: 19 17 15 21 16 18 16 14

Sample 2: 15 14 15 19 15 18 16

Is the difference b/w the samples mean significant?

Sol:- We have

Sample 1

x_i	$d_i = x_i - 18$	d_i^2
19	1	1
17	-1	1
15	-3	9
21	3	9
16	-2	4
18	0	0
16	-2	4
14	-4	16
	-8	44

Sample 2

x_i	$d_i = x_i - 16$	d_i^2
15	-1	1
14	-2	4
15	-1	1
19	3	9
15	-1	1
18	2	4
16	0	0
	0	20

$\bar{x}_1 = 18 + \frac{\Sigma d_i}{8} = 18 + \left(\frac{-8}{8}\right) = 17$ $\qquad$ $\bar{x}_2 = 16 + \frac{\Sigma d_i}{7} = 16 + \frac{0}{7} = 16$

$s_1 = \sqrt{\frac{1}{n}\Sigma d_i^2 - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{4.5} = 2.12$, $\quad s_2 = \sqrt{\frac{1}{n}\Sigma d_i^2 - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{2.857} = 1.69$

Now $H_0: \bar{x}_1 = \bar{x}_2$ and $H_1: \bar{x}_1 \neq \bar{x}_2$

$\therefore$ Two tailed test is to be used. Let LOS be 5%.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n}\right)}}, \quad \nu = n_1 + n_2 - 2 = 8 + 7 - 2 = 13$$

$= 0.95$ $\qquad$ But $t_{0.05}(\nu = 13) = 2.16$

$\therefore |t| < (t_\alpha)$ $\quad \therefore$ H_0 is accepted. There is no significant difference b/w the samples mean.

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Ex-(7):- The following data represent the biological values of protein from cow's milk and buffalo's milk at a certain level.

Cow's milk :	1.82	2.02	1.88	1.61	1.81	1.54
Buffalo's milk :	2.00	1.83	1.86	2.03	2.19	1.88

Examine if the average values of protein in the two samples significantly differ.

Soln:- Here $n = 6$

$$\bar{x}_1 = \frac{\Sigma x_i}{n} = \frac{1}{6} \times 10.68 = 1.78$$

$$s_1^2 = \frac{1}{6}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{6}\right)^2 = \frac{1}{6} \times 19.167 - (1.78)^2 = 0.0261$$

$$\bar{x}_2 = \frac{\Sigma x_i}{n} = \frac{1}{6} \times 11.79 = 1.965$$

$$s_2^2 = \frac{1}{6}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{6}\right)^2 = \frac{1}{6} \times 23.2599 - (1.965)^2 = 0.0157$$

As here $n_1 = n_2 = n = 6$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2 + s_2^2}{n-1}\right)}}, \quad \nu = 2n-2 = 12-2 = 10$$

$$t = \frac{1.78 - 1.965}{\sqrt{\frac{0.0261 + 0.0157}{6-1}}} = -2.03, \quad \nu = 10$$

det

$H_0 : \bar{x}_1 = \bar{x}_2$, $H_1 : \bar{x}_1 \neq \bar{x}_2$

$\therefore$ Two tailed test is to be used. Let LOS 5%.

$t_{0.05}\ (\nu = 10) = 2.23$

Now $|t| < |t_\alpha|$. H_0 is accepted.

$\therefore$ There is no significant difference [illegible] value of protein in two milk

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Ex-(7):- Samples of two types of electric bulbs, were tested for length of life and the following data were obtained

	size	mean	SD
Sample 1	8	1234 hrs	36 hrs
Sample 2	7	1036 hrs	40 hrs

Is the difference in the mean sufficient to warrant that types I bulbs are superior to types 2 bulbs?

Sol:- Here

$n_1 = 8, \quad \bar{x}_1 = 1234, \quad s_1 = 36, \quad n_2 = 7, \quad \bar{x}_2 = 1036, \quad s_2 = 40$

let $H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 > \bar{x}_2$

$\therefore$ The right tailed test is to be used.

Let LOS be 5%.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad V = n_1 + n_2 - 2$$

$t = 9.39 \quad , \quad \nu = 8 + 7 - 2 = 13$

$t_{0.05}(\nu = 13)$ for one tailed $= t_{0.1}(\nu = 13)$ for two tailed $= 1.77$

$\therefore |t| > |t_\alpha| \quad \therefore H_1$ is accepted.

$\therefore$ Type 1 bulbs may be regards superior to type 2 bulbs at 5% level.

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