Probability and Random Process

Probability :

Probability is a numerical measure of the likelihood of occurence of an event. It is a measure of the degree of uncertainity associated with random events.

Probability finds application in Quality control, reliability theory, Queueing theory, system failure, radar detection, noise, birth and death rates, and games of chance....

Basic Concepts of Probability :

Random Experiment :

A random experiment is also called a trial and the outcomes are called events

eg: 1) Rolling a die is a trial, getting 2 is an event.

2) Tossing a coin is a trial, getting head is an event.

Sample space :

The collection of all possible outcomes or sample points of a random experiment is called a sample space, denoted as S.

eg: when a coin is tossed, there are two sample points Head (H) and Tail (T).

$\therefore S = \{H, T\}$, $n(S) = 2$

Mutually Exclusive Events :

(Disjoint or Incompatible events)

Two events A and B in a sample space are said to be mutually exclusive events if they cannot

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occur together, (ie) if one occurs, the other cannot

(ie) $A \cap B = \phi$, (ϕ - null set)

eg: when we toss a coin, either head or tail can be up, but both cannot be up at a time.

※ Mutually exclusive events are applicable for a single trial only.

Equally likely events :

Two events are said to be equally likely if either of them cannot be expected to occur in preference to the other.

eg: Cast a uniform die, then all the six sample points are equally likely events.

Favourable Events :

The number of outcomes favourable to an event in a experiment is the number of cases which entail the happening of the event.

eg: In tossing two coins the cases favourable to the event of getting a head are HT, TH, HH.

Exhaustive events :

Outcomes are said to be exhaustive when they include all possible outcomes.

eg: 1) In drawing three cards from a pack of 52 cards, the exhaustive number of cases is $52C_3$.

2) In the case of throwing two dice, the exhaustive number of cases is 36 ($= 6^2$).

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Independence Events :

(Suitable for two or more events)

Two or more events are considered to be independent if the occurence or non-occurence of an event does not affect the occurence or non-occurence of the other.

eg : If a coin is thrown twice, the result of the second throw is in no way affected by the result of first throw. Thus the events are independent events.

Mathematical Definition of Probability :

If P is the notation for probability of happening of the event, then,

$$P(A) = p = \frac{\text{No. of favourable cases}}{\text{Total Number of exhaustive cases}}$$

$$= \frac{m}{n}.$$

Axiomatic Treatment of Probability Theory :

Let 'S' be the sample space, and σ be the class of events, and let P be a real valued function defined on σ.

Then P(A) is called the probability of the event A if P satisfies the following axioms.

1) For every event, $A \in \sigma$, $0 \leq P(A) \leq 1$

2) $P(S) = 1$

3) For every finite sequence of disjoint events.

$A_1, A_2 \cdots A_n$.

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) \cdots + P(A_n).$$

(ie) $P(A_1 \cup A_n) = \leq P(A_n)$.

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NOTE :

1) If A is an event, then $0 \le P(A) \le 1$,
Probability of an event between 0 and 1.

2) $P(\overline{A}) = 1 - P(A)$ → (Probability of complement event $\overline{A}$ of A)

3) If $P(A) = 1$, then event A is called a certain or sure event.

4) If $P(A) = 0$, the event is called impossible event (or) $P(\phi) = 0$.

5) If A and B are any two events which are not disjoint, then
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

6) If A, B and C are three events, then
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

7) Two events A and B are independent if
$$P(A \cap B) = P(A) \cdot P(B).$$

Conditional Probability :

If A & B are dependent events in a sample space S, then,
$$P(A \cap B) = P(A) \cdot P(B/A), \quad P(A) \neq 0.$$
$$P(B \cap A) = P(B) \cdot P(A/B), \quad P(B) \neq 0.$$
where, $P(B/A)$ → conditional probability of B given A.
$P(A/B)$ → conditional probability of A given B.

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* Conditional Probability of A given B is defined by $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ if $P(B) \neq 0$

* conditional Probability of B given A is defined by $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$ if $P(A) \neq 0$.

* If the events $A_1, A_2, \ldots, A_n$ constitute a partition of sample space S and $P(A_i) \neq 0$, $i = 1, 2, \ldots n$, then for any event B in S, we have

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i) = \sum_{i=1}^{n} P(A_i) P(B/A_i)$$

Problem:

1) Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. S.T the chance of exactly two of them being children is $\frac{10}{21}$.

Solu: Total No. of persons $= 3 + 2 + 4 = 9$

Four persons can be selected in ${}^9C_4$ ways.

Probability of selecting exactly 2 children & remaining 2 from among 3 men & 2 women.

$$= \frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} \qquad \because {}^nC_k = \frac{n!}{k!(n-k)!}$$

$$= \frac{\dfrac{4!}{2!(4-2)!} \times \dfrac{5!}{2!(5-2)!}}{\dfrac{9!}{4!(9-4)!}} = \frac{10}{21} \;//$$

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2) If A and B are events with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4}$. find $P(A^c \cap B^c)$.

Soln: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3}{8} + \frac{1}{2} - \frac{1}{4}.$$

$$= \frac{5}{8}$$

$$P(A^c \cap B^c) = P[(A \cup B)^c] = 1 - P(A \cup B).$$

$$= 1 - \frac{5}{8}$$

$$= \frac{3}{8} \text{ //.}$$

3) If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have high fidelity and selectivity is 0.18, what is the probability that a system with high fidelity will also have selectivity?

Soln:

Let A be the event that system has selectivity

B be the event that the system has fidelity.

$P(B) = 0.81$, $P(A \cap B) = 0.18$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.18}{0.81}$$

$$= \frac{2}{9} \text{ //}$$

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Random Variable

A real variable 'x' whose value is determined by the outcome of a random experiment is called a random variable.

eg: 1) Tossing a coin

$S = \{H, T\}$, $n(S) = 2$

2) Tossing 2 coins simultaneously

$S = \{HH, HT, TH, TT\}$, $n(S) = 4$

3) Tossing 3 coins simultaneously

$S = \{HHH, THH, HTH, HHT, TTT, TTH, THT, HTT\}$

$n(S) = 8$

Random Variable

- Discrete Random Variable → Probability Mass function
- Continuous Random Variable → Probability density function

Discrete Random Variable :

A Random Variable can assume only a countable number of real values is called discrete random variable.

eg: 1) No. of telephone calls

2) No. of printing mistakes in each page of a book.

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Probability mass function :

Let X be a discrete random Variable which takes Values $x_1, x_2, x_3, \ldots$ then

$P(X = x_i) = p(x_i)$ be probability of x_i

Then fn: P is called probability mass function (p.m.f) of X.

The numbers $p(x_i)$, $i = 1, 2, \ldots$ must satisfy the conditions.

(i) $p(x_i) \geq 0$, $i = 1, 2, 3 \ldots$

(ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

Probability distribution function of X

If X is a random Variable, then the function $F(x)$, defined by $F(x) = P[X \leq x]$ is called the distribution function of X., $(-\infty < x < \infty)$ $F(x) = \sum_{t \leq x} f(t)$.

Note : 1

The set of ordered pairs of nos: $(x_i, p(x_i))$ is called the probability distribution of a random Variable X.

Note : 2

$F(-\infty) = 0$.

$F(\infty) = 1$.

$0 \leq F(x) \leq 1$.

$F(x_1) \leq F(x_2)$ if $x_1 < x_2$.

$P(x_1 < X \leq x_2)$ if $F(x_2) - F(x_1)$

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Result :

$P(X \leq \infty) = 1$

$P(X \leq -\infty) = 0$.

$P[X > x] = 1 - P[X \leq x]$

$P[X \leq x] = 1 - P[X > x]$

Mean (or) Expected Value of X of a discrete R.V X

Let X be a discrete random Variable taking Values $x_1, x_2, \ldots, x_n \ldots$ with probabilities $p(x_1), p(x_2) \ldots p(x_n)$, then expected value of X is defined as

$$E(X) = \sum_{i=1}^{\infty} x_i \, p(x_i)$$

Variance of a random variable X

$$Var(X) = E(X^2) - [E(X)]^2$$

$$SD = \sqrt{Var(X)}$$

NOTE :

1) $F(x) = P[X \leq x]$

(ie) $P[X \leq 4] = F[4]$

$P[X \leq 5] = F[5]$

and $F[1] = P[0] + P[1]$

$F[2] = P[0] + P[1] + P[2] = F[1] + P[2]$

- - - - -

2) $P[1] = F[1] - F[0]$

$P[2] = F[2] - F[1]$

$P[3] = F[3] - F[2]$

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3) $E(c) = c$, c - constant

4) $E[aX+b] = aE[X] + b$, a, b - constants.

5) $E[X^2] = \sum_{i=1}^{\infty} x^2 p(x_i)$

6) $Var(aX) = a^2 Var(X)$

7) $Var(aX+b) = a^2 Var(X)$.

8) $Var(c) = 0$, c - constant.

Continuous Random Variable :

A random variable X is said to be continuous if it take all possible values between certain limits, say from real number 'a' to 'b'.

eg: If X denotes the life time of a transistor then X is a continuous random variable.

Probability density function :

For a continuous random variable X, a probability density function is a function such that

1) $f(x) \geq 0$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Cumulative distribution function :

The cumlative distribution function of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt, \quad -\infty < x < \infty, \; t \leq x.$$

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Mean or Expected Value of a C.R.V - 'X' :

Let X be a continuous random variable with p.d.f 'f' is defined in $(-\infty, \infty)$, then expected value of X is defined as

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx .$$

Variance of C.R.V - 'X' :

$$Var(X) = E[X^2] - (E[X])^2$$

Variance $\rightarrow$ $Var(X)$ or σ^2.

$$SD\ (\sigma) = \sqrt{\sigma^2}.$$

NOTE :

1) $f(x) = \frac{d}{dx} F(x)$

2) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx .$

3) $0 \leq F(x) \leq 1.$

4) $P(X < a) = \int_{-\infty}^{a} f(x)\, dx .$

5) $P(X > a) = \int_{a}^{\infty} f(x)\, dx .$

6) $P(a \leq X \leq b) = \int_{a}^{b} f(x)\, dx$

7) If $b = a$, $P(X = a) = \int_{a}^{a} f(x)\, dx = 0 .$

8) Probability a continuous R.V at a point x is zero.

9) $P[a \leq X \leq b] = F(b) - F(a).$

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10) For a continuous Random Variable,

(i) $P(X \le a) = P(X < a)$

(ii) $P(X > a) = P(X \ge a)$

(iii) $P(a \le X \le b) = P(a \le X < b) = P(a < X \le b)$

$= P(a < X < b)$.

Problems in Probability mass function- Discrete R.V:

1) A Random Variable X has the following Probability fn:

x	0	1	2	3	4
$p(x)$	k	$3k$	$5k$	$7k$	$9k$

Find the value of k & $P(X \ge 3)$.

Soln:

(i) W.K.T, probability mass fn: $\sum_{i=1}^{\infty} p(x_i) = 1$.

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$.

$k + 3k + 5k + 7k + 9k = 1$.

$25k = 1$

$\boxed{k = 1/25}$

(ii) $P[X \ge 3] = 1 - P[X < 3]$

(ie) $\left(P[X \ge a] = 1 - P[X < a]\right)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - [k + 3k + 5k]$

$= 1 - 9k$

$= 1 - 9\left(\frac{1}{25}\right) = \frac{25-9}{25}$

$= \frac{16}{25}$ //.

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2) If x is a discrete R.V with Probability distribution.

$P(X=x) = kx, \quad x = 1,2,3,4.$

(i) Find k and (ii) $P(2 < x < 4)$

Soln:

Given, $P(X=x) = kx.$

$x = 1., \quad P(X=1) = k.$

$x = 2, \quad P(X=2) = 2k$

$x = 3, \quad P(X=3) = 3k$

$x = 4, \quad P(X=4) = 4k.$

x	1	2	3	4
$p(x)$	k	$2k$	$3k$	$4k$.

(i) W.K.T Probability mass fn: $\sum_{i=1}^{\infty} p(x_i) = 1.$

(ie) $k + 2k + 3k + 4k = 1.$

$10k = 1.$

$\boxed{k = 1/10}$

(ii) $P(2 < x < 4) \Rightarrow P(X=3)$

$= 3k$

$= 3\left(\frac{1}{10}\right)$

$= 3/10.$

3) If the range of X is the set $\{0,1,2,3,4\}$ and $P(X=x) = 0.2$. Determine the mean and Variance of X.

$P(X=0) = 0.2 \quad P(X=1) = 0.2 \quad P(X=2) = 0.2$

$P(X=3) = 0.2 \quad P(X=4) = 0.2$

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(ie)

x	0	1	2	3	4
$p(x)$	0.2	0.2	0.2	0.2	0.2

(i) Mean $= \sum x_i p(x_i)$

$= x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5)$

$= 0(0.2) + 1(0.2) + 2(0.2) + 3(0.2) + 4(0.2)$

$= 0.2 + 0.4 + 0.6 + 0.8$

$= 2$ //

$E(X^2) = \sum x_i^2 p(x_i)$

$= x_1^2 p(x_1) + x_2^2 p(x_2) + x_3^2 p(x_3) + x_4^2 p(x_4) + x_5^2 p(x_5)$

$= 0^2(0.2) + 1^2(0.2) + 2^2(0.2) + 3^2(0.2) + 4^2(0.2)$

$= 0.2 + 0.8 + 1.8 + 3.2$

$= 6$

$Var(X) = E[X^2] - [E(X)]^2$

4) If the probability mass fn/: of a random Variable X is given by $P(X = r) = Kr^3$, $r = 1, 2, 3, 4$.

1) Find K
2) $P[(1/2 < X < 5/2) / (X > 1)]$
3) Find mean and Variance of X.
4) Find Distribution fn/: of X.

Soln: Given $P(X = r) = Kr^3$

$r = 1$, $P(X = 1) = K$

$r = 2$, $P(X = 2) = 8K$

$r = 3$, $P(X = 3) = 27K$

$r = 4$, $P(X = 4) = 64K$

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(i) Find Value of k.

$$\Sigma p(x_i) = 1.$$

$$p(x_1) + p(x_2) + p(x_3) + p(x_4) = 1.$$

$$k + 8k + 27k + 64k = 1$$

$$100k = 1.$$

$$\boxed{k = 1/100}$$

(iii) To Find Mean / Variance / Distribution fn:

Mean: $E(X) = \Sigma x_i p(x_i)$

$E(X^2) = \Sigma x_i^2 p(x_i)$

x_i	$p(x_i)$	$x_i p(x_i)$	$x_i^2 p(x_i)$	F(x) - Dist. fn.
1	$k = 1/100$	k	k	$1/100.$
2	$8k = 8/100$	$16k$	$32k.$	$1/100 + 8/100 = 9/100$
3	$27k = \frac{27}{100}$	$81k$	$243k$	$9/100 + 27/100 = 36/100$
4	$64k = \frac{64}{100}$	$256k.$	$1024k$	$\frac{36}{100} + \frac{64}{100} = 1$
	Σ	$354k$	$1300k.$	

Mean $= \Sigma x_i p(x_i)$

$E(X) = 354k$

$= \frac{354}{100}.$

$= 3.54$ //.

$E(X^2) = \Sigma x_i^2 p(x_i)$

$= 1300k.$

$= 1300\left(\frac{1}{100}\right)$

$= 13$ //

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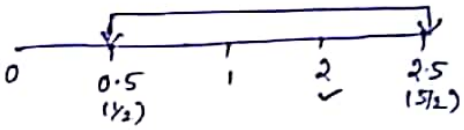
$$Var(X) = E(X^2) - (E(X))^2$$

$$= 13 - (3.54)^2$$

$$= 13 - 12.5316.$$

$$= 0.4684.$$

(ii) $P(1/2 < X < 5/2 \,/\, X > 1)$



$P(X>1) = P(X=2), P(X=3)$
$P(X=4).$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P[(1/2 < X < 5/2) \cap (X > 1)]}{P[X>1]}$$

$$= \frac{P[X=2]}{P[X>1]}$$

$$= \frac{P[X=2]}{P[X=2] + P[X=3] + P[X=4]}$$

$$= \frac{8/100}{8/100 + 27/100 + 64/100} = \frac{8/\cancel{100}}{99/\cancel{100}}$$

$$= 8/99 \;//$$

---

5) If X has distribution fuy: $F(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1/3 & \text{for } 1 \le x < 4 \\ 1/2 & \text{for } 4 \le x < 6 \\ 5/6 & \text{for } 6 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$

Find (i) Probability distribution of X

(ii) $P[2 < X < 6]$

(iii) Mean & Variance of X.

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6) A discrete Random Variable x has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Find 'a'.

(ii) Find $P(0<x<5)$, $P(X<3)$, $P(x\geq 3)$,

(iii) Find distribution function of X.

(i) If $p(x)$ is the pmf, then $\sum_{i=0}^{\infty} p(x_i) = 1$.

$$a+3a+5a+7a+9a+11a+13a+15a+17a = 1.$$

$$81a = 1$$

$$\boxed{a = 1/81}$$

Table values :

x	0	1	2	3	4	5	6	7	8
$p(x)$	$1/81$	$3/81$	$5/81$	$7/81$	$9/81$	$11/81$	$13/81$	$15/81$	$17/81$

(ii) $P(X<3) = p(0)+p(1)+p(2)$

$= 1/81 + 3/81 + 5/81$

$= 9/81$

$P(x\geq 3) = 1 - P(X<3)$

$= 1 - 9/81$

$= 72/81$

$P(0<x<5) = p(1)+p(2)+p(3)+p(4)$ (0 & 5 not included)

$= 3/81 + 5/81 + 7/81 + 9/81$

$= 24/81$

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(iii) To find distribution function of X.

$X = x$	$F(x) = P(X \le x)$
0	$F(0) = p(0) = 1/81$
1	$F(1) = p(0) + p(1)$ $= 1/81 + 3/81 = 4/81$
2	$F(2) = p(0) + p(1) + p(2)$ $= 1/81 + 3/81 + 5/81$ $= 9/81$
3	$F(3) = P(X \le 3)$ $= p(0) + p(1) + p(2) + p(3)$ $= 1/81 + 3/81 + 5/81 + 7/81 = 16/81$
4	$F(4) = p(0) + p(1) \cdots + p(4)$ $= \frac{25}{81}$
5	$F(5) = p(0) + p(1) + \cdots + p(5)$ $= \frac{36}{81}$
6	$F(6) = p(0) + p(1) + \cdots + p(6)$ $= \frac{49}{81}$
7	$F(7) = p(0) + p(1) + \cdots + p(7)$ $= 67/81$
8	$F(8) = p(0) + p(1) + \cdots + p(8)$ $= 1.$

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7) A Random Variable 'X' has the probability function

$X = x$	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

1) Find k.
2) Evaluate $P(X<6)$, $P(X \geq 6)$ & $P(0<X<5)$
3) If $P(X \leq k) > \frac{1}{2}$, Find minimum value of k.
4) Determine distribution function of X.

(1) Total probability is equal to 1.

$$\sum_{i=0}^{7} p(x_i) = 1.$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1.$$

$$10k^2 + 9k - 1 = 0.$$

$$k = \frac{-9 \pm \sqrt{81+40}}{20}$$

$$= \frac{-9 \pm \sqrt{121}}{20} = \frac{-9 \pm 11}{20}.$$

$$= \frac{-9+11}{20} \text{ (or) } \frac{-9-11}{20}.$$

$$= \frac{2}{20} \text{ (or) } \frac{-20}{20}.$$

$$k = \frac{1}{10} \text{ (or) } -1.$$

$k = -1$ is not possible $\because$ (probability cannot be a negative value)

$$\therefore \boxed{k = \frac{1}{10}}$$

Table value.

$X = x$	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

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2) $P(X<6) = P(X=0) + P(X=1) + P(X=2) + \cdots + P(X=5)$

$= 0 + 1/10 + 2/10 + 2/10 + 3/10 + 1/100$

$= 81/100$

$P(X \geq 6) = 1 - P(X<6)$

$= 1 - 81/100$

$= 19/100$

$P(0<X<5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 1/10 + 2/10 + 2/10 + 3/10$

$= 4/5$.

(4) Distribution function of X.

$X=x$	$F(x) = P(X \leq x)$
0	$F(0) = 0$
1	$F(1) = p(0) + p(1) = 1/10$
2	$F(2) = p(0) + p(1) + p(2) = 3/10$
3	$F(3) = p(0) + p(1) + p(2) + p(3)$ $= 5/10$
4	$F(4) = p(0) + p(1) + \cdots + p(4)$ $= 8/10$
5	$F(5) = p(0) + p(1) + \cdots + p(5)$ $= \frac{81}{100}$
6	$F(6) = p(0) + p(1) + \cdots + p(6)$ $= 83/100$
7	$F(7) = p(0) + p(1) + \cdots + p(7)$ $= 1$.

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3) If $P(X \le K) > 1/2$.

Minimum value of $K = 4$. (by distribution fn of X fn above).

8) The probability fn of a R.V X is gn by $P(X=x) = \frac{1}{2^x}$, $x = 1, 2, 3 \ldots$

Find 1) $P(X \text{ is even})$

2) $P(X \text{ is odd})$

3) $P(X \ge 4)$

5) $P(X \text{ is multiple of } 3)$.

Soln:

Gn $P(X=x) = \frac{1}{2^x}$

(i) $P(X \text{ is even}) = P(X=2) + P(X=4) + P(X=6) + \cdots$

$$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots$$

$$= \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{2^4} + \cdots\right]$$

$$= \frac{1}{2^2}\left[1 - \frac{1}{2^2}\right]^{-1} = \frac{1}{4}\left(\frac{3}{4}\right)^{-1} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3}$$

$$= \frac{1}{3}.$$

(ii) $P(X \text{ is odd})$

$= 1 - P(X \text{ is even})$.

$= 1 - 1/3$

$= 2/3$.

(iii) $P[X \ge 4] = P[X=4] + P[X=5] + P[X=6] + \cdots$

$$= \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \cdots$$

$$= \frac{1}{2^4}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots\right] = \frac{1}{16}\left[1 - \frac{1}{2}\right]^{-1}$$

$$= \frac{1}{16}\left[\frac{1}{2}\right]^{-1} = \frac{1}{\cancel{16}} \times \cancel{2} = \frac{1}{8}.$$

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(iv) $P[X \text{ is multiple of } 3] = P[X=3] + P[X=6] + P[X=9] + \cdots$

$$= \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \cdots$$

$$= \frac{1}{2^3}\left[1 + \frac{1}{2^3} + \frac{1}{(2^3)^2} + \cdots\right]$$

$$= \frac{1}{8}\left[1 - \frac{1}{8}\right]^{-1} = \frac{1}{8} \cdot \frac{8}{7} = \frac{1}{7} \,//.$$

Problems on a Continuous Random Variable

1) A continuous R.V x than can assume any value between $x=2$ and $x=5$ has a density function given by $f(x) = k(1+x)$. Find $P(X<4)$, $P(2<X<5)$

Gn $f(x) = k(1+x)$.

W.K.T $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$$\int_2^5 k(1+x)\,dx = 1.$$

$$k\left[\frac{(1+x)^2}{2}\right]_2^5 = 1.$$

$$\frac{k}{2}\left[(1+5)^2 - (1+2)^2\right] = 1$$

$$\frac{k}{2}\left[6^2 - 3^2\right] = 1.$$

$$\frac{k}{2}(36-9) = 1$$

$$\frac{k}{2}(27) = 1$$

$$\boxed{k = \frac{2}{27}}$$

W.K.T $P[X<a] = \int_{-\infty}^{a} f(x)\,dx$.

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$$P[X<4] = \int_{-\infty}^{4} f(x)\,dx$$

(or)

$$P[2<X<4] = \int_{-\infty}^{2} \cancel{f(x)}^{\,0}\,dx + \int_{2}^{4} f(x)\,dx$$

$$= K\int_{2}^{4} (1+x)\,dx$$

$$= \frac{2}{27}\int_{2}^{4} (1+x)\,dx$$

$$= \frac{2}{27}\left[\frac{(1+x)^2}{2}\right]_2^4$$

$$= \frac{\cancel{2}^{\,1}}{27}\left[(1+4)^2 - (1+2)^2\right]$$

$$= \frac{1}{27}(5^2 - 3^2)$$

$$= \frac{16}{27}$$

---

2) Given that the p.d.f of R.V 'X' is $f(x) = Kx,\ 0<x<1$, find K and $P(X>0.5)$

W.K.T $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$$K\int_0^1 x\,dx = 1$$

$$K\left(\frac{x^2}{2}\right)_0^1 = 1.$$

$$K\left(\frac{1}{2} - 0\right) = 1, \quad \boxed{K=2}$$

$$P[X>0.5] = \int_{0.5}^{\infty} f(x)\,dx.$$

$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} \cancel{f(x)}^{\,0}\,dx.$$

$$= \int_{0.5}^{1} 2x\,dx = 2\left(\frac{x^2}{2}\right)_{0.5}^{1}$$

$$= 1^2 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

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3) The diameter of an electric cable, say x is assumed to be a continuous random variable with p.d.f.

$f(x) = 6x(1-x)$, $0 \le x \le 1$.

1) check that above is a pdf

2) Determine a number 'b' such that $P(X < b) = P(X \ge b)$

Soln:

$$f(x) = 6x(1-x), \quad 0 \le x \le 1.$$

In the interval $0 \le x \le 1$, $f(x)$ is always +ve

(ie) in $0 \le x \le 1$, $f(x) \ge 0$.

i) $$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 f(x)\,dx$$

$$= \int_0^1 6x(1-x)\,dx$$

$$= 6\int_0^1 (x - x^2)\,dx = 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1$$

$$= 6\left(\frac{1}{2} - \frac{1}{3}\right) = 6\left(\frac{3-2}{6}\right) = 1.$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$\therefore$ $f(x)$ is a random variable X

ii) Given $P(X < b) = P(X \ge b)$

$$P(X < b) = 1 - P(X < b)$$

$$2P(X < b) = 1$$

$$P(X < b) = \frac{1}{2}.$$

Since, $\left(P[X < a] = \int_{-\infty}^{a} f(x)\,dx\right)$

$$\int_{-\infty}^{b} f(x)\,dx = \frac{1}{2}$$

$$6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b = \frac{1}{2}$$

$$6\left(\frac{b^2}{2} - \frac{b^3}{3}\right) = \frac{1}{2}.$$

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$$6\left(\frac{3b^2 - 2b^3}{6}\right) = \frac{1}{2}$$

$$3b^2 - 2b^3 = \frac{1}{2}$$

$$6b^2 - 4b^3 = 1$$

$$4b^3 - 6b^2 + 1 = 0$$

using synthetic division,

$$\begin{array}{c|cccc} \frac{1}{2} & 4 & -6 & 0 & 1 \\ & 0 & 2 & -2 & -1 \\ \hline & 4 & -4 & -2 & \boxed{0} \end{array}$$

$b = \frac{1}{2}$ is a root & also lies between $(0,1)$

$\therefore$ The value of $\boxed{b = \frac{1}{2}}$

Continuous distribution function

If $f(x)$ is a p.d.f of a continuous random variable 'X' then the function,

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx, \quad -\infty < x < \infty$$

is called the distribution function (or) cumulative distribution function of the random variable 'X'

1) The probability distribution function of a random variable X is $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$

Find the cumulative distribution function of X.

W.K.T c.d.f $F(x) = \int_{-\infty}^{x} f(x)\,dx$

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when x lies in $0 < x < 1$,

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

$$= \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{x} f(x)\, dx$$

$$= \left.\frac{x^2}{2}\right]_0^x = \frac{x^2}{2}$$

when x lies in $1 < x < 2$

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

$$= \int_{-\infty}^{0} \cancel{f(x)}^{0}\, dx + \int_{0}^{1} f(x)\, dx + \int_{1}^{x} f(x)\, dx$$

$$= \int_{0}^{1} x\, dx + \int_{1}^{x} (2-x)\, dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^x$$

$$= \left(\frac{1}{2} - 0\right) + \left(2x - \frac{x^2}{2} - \frac{3}{2}\right)$$

$$= 2x - \frac{x^2}{2} - 1$$

when $x > 2$,

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

$$= \int_{-\infty}^{0} \cancel{f(x)}^{0}\, dx + \int_{0}^{1} f(x)\, dx + \int_{1}^{2} f(x)\, dx + \int_{2}^{x} \cancel{f(x)}^{0}\, dx$$

$$= \int_{0}^{1} x\, dx + \int_{1}^{2} (2-x)\, dx$$

$$= \left.\frac{x^2}{2}\right]_0^1 + \left.2x - \frac{x^2}{2}\right]_1^2$$

$$= \left(\frac{1}{2}\right) + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right]$$

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$$= \frac{1+4-3}{2}$$

$$= 1$$

using ①, ②, ③.

$$F(x) = \begin{cases} x^2/2 & , \quad 0 < x < 1 \\ 2x - x^2/2 - 1 & , \quad 1 < x < 2 \\ 1 & , \quad x > 2 \end{cases}$$

2) The cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ x^2 & , \quad 0 \le x < 1/2 \\ 1 - \frac{3}{25}(3-x)^2 & , \quad 1/2 \le x < 3 \\ 1 & , \quad x \ge 3 \end{cases}$$

Find pdf of x and evaluate $P[|X| \le 1]$ using both pdf and cdf

Soln :

p.d.f is $f(x) = \frac{d}{dx}[F(x)]$

$$= \begin{cases} \frac{d}{dx}(0) & , \quad x < 0 \\ \frac{d}{dx}(x^2) & , \quad 0 \le x < 1/2 \\ \frac{d}{dx}\left[1 - 3/25 (3-x)^2\right] & , \quad 1/2 \le x < 3 \\ \frac{d}{dx}(1) & , \quad x \ge 3 \end{cases}$$

$$= \begin{cases} 0 & , \quad x < 0 \\ 2x & , \quad 0 \le x < 1/2 \\ 6/25 (3-x) & , \quad 1/2 \le x < 3 \\ 0 & , \quad x \ge 3 \end{cases}$$

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To find $P[|X| \le 1]$ using cdf

$$P[|X| \le 1] = P[-1 \le X \le 1]$$
$$= F(1) - F(-1)$$
$$= \left(1 - \tfrac{3}{25}(3-1)^2\right) - 0$$
$$= 1 - \tfrac{12}{25}$$
$$= \frac{13}{25}$$

To find $P[|X| \le 1]$ using pdf

$$P(|X| \le 1) = P(-1 \le X \le 1)$$
$$= \int_a^b f(x)\,dx = \int_{-1}^{1} f(x)\,dx.$$
$$= \int_{-1}^{0} f(x)\,dx + \int_0^{1/2} f(x)\,dx + \int_{1/2}^{1} f(x)\,dx.$$
$$= \int_0^{1/2} 2x\,dx + \int_{1/2}^{1} \tfrac{6}{25}(3-x)\,dx.$$
$$= \cancel{2}\left(\tfrac{x^2}{\cancel{2}}\right)_0^{1/2} + \tfrac{6}{25}\left[3x - \frac{x^2}{2}\right]_{1/2}^{1}$$
$$= \frac{1}{4} + \frac{6}{25}\left[3 - \frac{1}{2} - \frac{3}{2} + \frac{1}{8}\right]$$
$$= \frac{1}{4} + \frac{6}{25}\left[\frac{24 - 4 - 12 + 1}{8}\right]$$
$$= \frac{1}{4} + \frac{3}{100}(9)$$
$$= \frac{1}{4} + \frac{27}{100} = \frac{25 + 27}{100}.$$
$$= \frac{52}{100}$$
$$= \frac{13}{25}.$$

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Mathematical Expectations.

Averaging process, when applied to a R.V is called expectation. It is denoted by $E(X)$ and is read as expected value of X or the mean value of X.

$$E(X) = \begin{cases} \sum_i x_i p(x_i), & \text{discrete} \\ \int_{-\infty}^{\infty} x f(x)\,dx, & \text{continuous} \end{cases}$$

Expectations Table

Discrete R.V	Continuous R.V
$E(X) = \sum x p(x)$	$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$
$E(X^r) = \mu_r' = \sum_x x^r p(x)$ r^{th} moment (about origin)	$E(X^r) = \mu_r' = \int_{-\infty}^{\infty} x^r f(x)\,dx$
Mean $= \mu_1' = \sum x p(x)$	Mean $= \mu_1' = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$.
$E(X^2) = \mu_2' = \sum x^2 p(x)$	$E(X^2) = \mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\,dx$.
Variance $= \mu_2' - \mu_1'^2$ $= E(X^2) - (E(X))^2$	Variance $= \mu_2' - \mu_1'^2$ $= E(X^2) - (E(X))^2$
r^{th} moment (About mean) $\mu_r^* = \sum (x-\bar{x})^r p(x)$	$\mu_r^* = \int_{-\infty}^{\infty} (x-\bar{x})^r f(x)\,dx$.

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1) when a die is thrown 'X' denotes the number that turns up. Find $E(X)$, $E(X^2)$ and $Var(X)$.

Let X be the R.V denoting the number that turns up in a die.

'X' takes values 1, 2, 3, 4, 5, 6 with probability $1/6$ for each.

$X = x$	1	2	3	4	5	6
$p(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

$$E(X) = \sum_{i=1}^{6} x_i \, p(x_i)$$

$$= 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6}.$$

$$= \frac{7}{2}$$

$$E(X^2) = \sum_{i=1}^{6} x_i^2 \, p(x_i)$$

$$= 1(1/6) + 4(1/6) + 9(1/6) + 16(1/6) + 25(1/6) + 36(1/6)$$

$$= \frac{1+4+9+16+25+36}{6}$$

$$= \frac{91}{6}.$$

$$Var(X) = E(X^2) - (E(X))^2.$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2$$

$$= \frac{35}{12}$$

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2) The density function of a random variable 'X' is given by $f(x) = kx(2-x)$, $0 \le x \le 2$. Find k, mean, variance & r^{th} moment.

Given $f(x) = kx(2-x)$, $0 \le x \le 2$.

$f(x)$ is p.d.f

To find k :

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$\int_0^2 kx(2-x)\,dx = 1$$

$$k\int_0^2 (2x - x^2)\,dx = 1.$$

$$k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1.$$

$$k\left[\left(4 - 8/3\right) - (0-0)\right] = 1.$$

$$k\left(4/3\right) = 1$$

$$\boxed{k = 3/4}$$

$$\therefore \quad f(x) = \frac{3}{4}\cdot x(2-x) \text{ in } 0 \le x \le 2.$$

Mean, $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$$= 3/4 \int_0^2 (2x^2 - x^3)\,dx$$

$$= 3/4\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= 3/4\left[\frac{16}{3} - \frac{16}{4}\right]$$

$$= \frac{3}{4}\left[16\left(1/3 - 1/4\right)\right]$$

$$\boxed{E(X) = 1}$$

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$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx.$$

$$= \frac{3}{4}\int_0^2 (2x^3 - x^4)\, dx.$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\left(\frac{2 \times 16}{4} - \frac{32}{5}\right) - 0\right]$$

$$= 24\left(\frac{5-4}{20}\right) = \frac{6}{5}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= \frac{6}{5} - 1$$

$$= \frac{1}{5}$$

The rth moment $= E(X^r)$

$$= \frac{3}{4}\int_0^2 (2x^{r+1} - x^{r+2})\, dx.$$

$$= \frac{3}{4}\left[\frac{2x^{r+2}}{r+2} - \frac{x^{r+3}}{r+3}\right]_0^2$$

$$= \frac{3}{4}\left[\left(2\left(\frac{2^{r+2}}{r+2}\right) - \frac{2^{r+3}}{r+3}\right) - 0\right]$$

$$= \frac{3}{4}\left[2^r \cdot 2^3\left(\frac{1}{r+2} - \frac{1}{r+3}\right)\right]$$

$$= 6 \cdot 2^r\left[\frac{\cancel{r}+3-\cancel{r}-2}{(r+2)(r+3)}\right]$$

$$\boxed{E(X^r) = \frac{6 \cdot 2^r}{(r+2)(r+3)}}$$

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Mean & Variance got from $E(x^r)$

$$\text{Mean} = E(x) = E(x^r)\Big]_{r=1}$$

$$= 6 \cdot \frac{2!}{(1+2)(1+3)}$$

$$\boxed{E(x) = 1.}$$

$$E(x^2) = 6 \cdot \frac{2^2}{(2+2)(2+3)}$$

$$= \frac{6 \times \cancel{4}}{\cancel{4} \times 5} = \frac{6}{5}$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$= \frac{6}{5} - 1$$

$$\boxed{Var(x) = \frac{1}{5}}$$

Moment Generating function

Moment generating function of a R.V x about the origin is defined as

$$M_x(t) = E(e^{tx}) = \begin{cases} \sum_x e^{tx} p(x), & \text{discrete} \\ \int_{-\infty}^{\infty} e^{tx} f(x)\, dx, & \text{continuous} \end{cases}$$

Note :

1) r^{th} moment = Coefficient of $\frac{t^r}{r!}$.

2) Mean using MGF :

$$\text{Mean} = M_x'(0) = \left[\frac{d}{dt} M_x(t)\right]_{t=0}.$$

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$$M_x''(0) = \mu_2' = \left[\frac{d^2}{dt^2}(M_x(t))\right]_{t=0}.$$

In general, $\mu_r' = \left[\frac{d^r}{dt^r} M_x(t)\right]_{t=0}$.

3) Variance about MGF:

$$Var(x) = \mu_2' - (\mu_1')^2.$$

4) MGF of x about the point $x = a$.

$$\left[M_x(t)\right]_{t=a} = 1 + t\mu_1' + \frac{t^2}{2!}\mu_2' + \cdots + \frac{t^r}{r!}\mu_r' + \cdots$$

---

1) The probability function of an infinite discrete distribution is given by $P(X=x) = \frac{1}{2^x}$, $x = 1, 2 \ldots \infty$.
Find the mean and variance of the distribution.
Also find $P(X \text{ is even})$. Find MGF.

W.K.T $M_x(t) = \sum_{x=1}^{\infty} e^{tx} p(x)$

$$= \sum_{x=1}^{\infty} e^{tx} \cdot \frac{1}{2^x}$$

$$= \sum_{x=1}^{\infty} \left(\frac{e^t}{2}\right)^x.$$

$$= \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \cdots$$

$$= \frac{e^t}{2}\left[1 + \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \cdots\right]$$

$$= \frac{e^t}{2}\left[1 - \frac{e^t}{2}\right]^{-1} = \frac{e^t}{\cancel{2}} \cdot \frac{(2-e^t)^{-1}}{\cancel{2^{-1}}}$$

using $(1-x)^{-1} = 1 + x + x^2 + \cdots$

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$$M_x(t) = \frac{e^t}{2-e^t}$$

$$= (2-e^t)^{-1} e^t$$

$$M_x'(t) = -e^t(2-e^t)^{-2}(-e^t) + (2-e^t)^{-1} e^t$$

$$= e^{2t}(2-e^t)^{-2} + (2-e^t)^{-1} e^t$$

$$M_x''(t) = 2(2-e^t)^{-2} e^{2t} + e^{2t}(-2)(2-e^t)^{-3}(-e^t)$$

$$+ (2-e^t)^{-1} e^t + e^t(-1)(2-e^t)^{-2}(-e^t)$$

Now, $E(X) = \text{Mean} = M_x'(0)$

$= 1+1 = 2.$

$E(X^2) = M_x''(0) = 6.$

Put $t=0$ in $M_x'(t)$ & $M_x''(t)$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 6 - 4$$

$$= 2.$$

Now $P(X = \text{even}) = P(X=2) + P(X=4) + \cdots$

$$= \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^4 + \cdots$$

$$= \left(\tfrac{1}{2}\right)^2 \left[1 + \left(\tfrac{1}{2}\right)^2 + \cdots\right]$$

$$= \tfrac{1}{4}\left[1 - \tfrac{1}{2^2}\right]^{-1}$$

$$= \tfrac{1}{4}\left(\tfrac{3}{4}\right)^{-1} = \tfrac{1}{4} \times \tfrac{4}{3}$$

$$= \tfrac{1}{3}.$$

---

2) A random variable has the pdf given by

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

a) Find MGF b) First four moments about origin.

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Given $f(x) = 2e^{-2x}, \quad x \geq 0.$

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx.$$

$$= 2\int_{-0}^{\infty} e^{tx} e^{-2x}\, dx. \qquad , x \geq 0.$$

$$= 2\int_{0}^{\infty} e^{-(2-t)x}\, dx.$$

$$= 2\left[\frac{e^{-(2-t)x}}{-(2-x)}\right]_0^{\infty} \qquad \because \int e^{-ax}dx = \frac{e^{-ax}}{-a}.$$

$$= -\frac{2}{2-t}\left[-e^{-\infty} - e^{-0}\right]$$

$$= -\frac{2}{2-t}(0-1) = \frac{2}{2-t}.$$

MGF is $M_x(t) = 2(2-t)^{-1}$

$$M_x'(t) = -2(2-t)^{-2}(-1)$$

$$= 2(2-t)^{-2}$$

1st Moment, $M_x'(0) = \frac{2}{4} = \frac{1}{2}.$

$$M_x''(t) = 2(-2)(2-t)^{-3}(-1)$$

$$= 4(2-t)^{-3}.$$

2nd Moment, $M_x''(0) = \frac{4}{8} = \frac{1}{2}.$

$$M_x'''(t) = -12(2-t)^{-4}(-1)$$

$$= 12(2-t)^{-4}$$

3rd Moment, $M_x'''(0) = \frac{12}{16} = \frac{3}{4}$

$$M_x^{iv}(t) = 48(2-t)^{-5}$$

4th Moment, $M_x^{iv}(0) = 3/2.$

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3) For a triangular distribution

$$f(x) = \begin{cases} x & , 0 \le x \le 1 \\ 2-x & , 1 \le x \le 2 \\ 0 & , \text{other} \end{cases}$$

Find $E(x)$, $V(x)$ & MGF.

4) Let R.V x have pdf $f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & , x > 0 \\ 0 & , x < 0 \end{cases}$

Find MGF, Mean, Var(x).

$$M_x(t) = \frac{1}{2} \int_{-\infty}^{\infty} e^{tx} \cdot e^{-x/2} \, dx.$$

$$= \frac{1}{2} \int_{0}^{\infty} e^{-(\frac{1}{2}-t)x} \, dx.$$

$$= \frac{1}{2} \left[\frac{e^{-(\frac{1}{2}-t)}}{-(\frac{1}{2}-t)} \right]_0^{\infty}$$

$$= \frac{1}{\cancel{2}} \left[\cancel{e^{0-\infty}} + \frac{\cancel{2}}{1-2t} \right]$$

$$= \frac{1}{1-2t}$$

Mean : $E(x) = \mu_1' = M_x'(0) = \frac{d}{dt}(M_x(t))_{t=0}.$

$$= \left[\frac{(1-2t)0 - 1(-2)}{(1-2t)^2} \right]_{t=0}$$

$$= \left[\frac{2}{(1-2t)^2} \right]_{t=0}$$

$$\boxed{\mu_1' = 2}$$

$$\mu_2' = M_x'(0) = \frac{d^2}{dt^2} \left(\frac{1}{1-2t} \right)\Big]_{t=0}$$

$$= \left[\frac{(1-2t)^2(0) - 2(2)(1-2t)(-2)}{(1-2t)^4} \right]_{t=0}.$$

$$\boxed{\mu_2' = 8}$$

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$$Var(x) = \mu_2' - (\mu_1')^2$$
$$= 8 - 4$$
$$= 4$$

5) First four moments at $X = 4$ are 1, 4, 10, 45.
Prove that Mean = 5, $Var(X) = 3$, $\mu_3 = 0$, $\mu_4 = 26$.

Given $\mu_1' = 1$, $\mu_2' = 4$, $\mu_3' = 10$, $\mu_4' = 45$.

$$\mu_2 = \mu_2' - (\mu_1')^2.$$
$$= 4 - 1 = 3.$$
$$\boxed{\mu_2 = Var(X) = 3}$$

$$\mu_1' = \Sigma x \, p(x).$$
$$= \Sigma (x-4) \, p(x) \quad \because \text{(Gn Moments about a point } X = 4)$$
$$= \Sigma x \, p(x) - \Sigma 4 p(x)$$
$$1 = \text{Mean} - 4 \Sigma p(x). \quad \because (\Sigma p(x) = 1)$$
$$\text{Mean} = 1 + 4 = 5$$
$$\boxed{\text{Mean} = 5}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$= 10 - 12 + 2$$
$$\boxed{\mu_3 = 0}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$
$$= 45 - 4(10) + 6(4)(1) - 3.$$
$$= 45 - 40 + 24 - 3.$$
$$\boxed{\mu_4 = 26}$$

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Relationship between Raw moments (Moments about origin) and central moments (Moments about mean)

$$\mu_1 = 0 \quad \text{(always)}$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

Discrete Distributions.

Important discrete distributions of a random Variable 'x' are

* Binomial distribution
* Poisson distribution
* Geometric distribution.

Binomial distributions :

A random Variables x is said to follow Binomial distribution with parameter 'n' and 'p' if its pmf is given by

$$P(X=x) = \begin{cases} nC_x \, p^x q^{n-x}, & x = 0, 1, 2 \ldots n \\ 0, & \text{other} \end{cases}$$

X is called Binomial R.V ; n, p are parameters.

Note :

$q = 1-p$ / $p+q = 1$.

* $n \rightarrow$ no. of trials
  $p \rightarrow$ no. of success.
  $q \rightarrow$ no. of failures.

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Assumptions in Binomial Distribution.

a) There are only two possible outcomes for each trial (success or failure).

b) The probability of a success is the same for each trial.

c) There are 'n' trials, where 'n' is a constant.

d) The 'n' trials are independent.

uses :

* used in games of chance
* military defence
* failure analysis.

Derive MGF of a Binomial distribution.

$$MGF = M_x(t) = E(e^{tx}) = \sum_{x=0}^{n} e^{tx} p(x)$$

$$= \sum_{x=0}^{n} e^{tx} \cdot nC_x p^x q^{n-x}.$$

$$= \sum_{x=0}^{n} nC_x (pe^t)^x q^{n-x}.$$

$$= nC_0 (pe^t)^0 q^n + nC_1 (pe^t)^1 q^{n-1} + nC_2 (pe^t)^2 q^{n-2} + \cdots + nC_n (pe^t)^n q^0.$$

$$nC_0 = 1, \quad nC_n = 1, \quad nC_1 = n, \quad nC_2 = \frac{n(n-1)}{2!} \cdots$$

$$= q^n + n(pe^t)q^{n-1} + \frac{n(n-1)}{2!}(pe^t)^2 q^{n-2} + \cdots (pe^t)^n$$

$$= (q + pe^t)^n.$$

By expansion of $(a+b)^n$

$$(a+b)^n = b^n + nC_1 a^n b^{n-1} + \frac{n(n-1)}{2!} a^2 b^{n-2} + \cdots a^n.$$

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Mean and Variance of B.D using MGF.

$$\text{Mean} = E(X) = \left[\frac{d}{dt}(M_x(t))\right]_{t=0}$$

$$= \left[\frac{d}{dt}(q+pe^t)^n\right]_{t=0}$$

$$= n(pe^t+q)^{n-1}pe^t\Big]_{t=0}$$

$$= np(p+q)^{n-1}.$$

$$\boxed{E(X) = np} \qquad \because (p+q=1)$$

$$E(X^2) = \left[\frac{d^2}{dt^2}(M_x(t))\right]_{t=0}$$

$$= \left[\frac{d^2}{dt^2}(pe^t+q)^n\right]_{t=0}$$

$$= \left(\frac{d}{dt}\left[npe^t(pe^t+q)^{n-1}\right]\right)_{t=0}$$

$$= np\left[(n-1)(pe^t+q)^{n-2}pe^t\, e^t + e^t(pe^t+q)^{n-1}\right]_{t=0}$$

$$= np\left[(n-1)(\cancel{p+q})^{1}p + (\cancel{p+q})^{1}\right]$$

$$= np\left[(n-1)p+1\right]$$

$$= n(n-1)p^2 + np.$$

$$E(X^2) = n^2p^2 - np^2 + np$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np(1-p)$$

$$\boxed{Var(X) = npq}$$

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Problems :

1) A machine manufacturing screws is known to produce 5% defective, In a Random sample of 15 screws. What is the probability that there are

(i) exactly 3 defectives

(ii) Not more than 3 defectives.

Soln :

Defective is success.

Given $p = 5\% = \frac{5}{100} = 0.05$

$q = 1 - p \Rightarrow q = 0.95$.

& $n = 15$

B.D, $P[X = x] = nC_x \, p^x \, q^{n-x}$, $x = 0, 1, 2 \ldots 15$.

$P[X = x] = 15C_x \, (0.05)^x \, (0.95)^{15-x}$.

(i) Exactly 3 defectives. Exactly $=$

$P[X = 3] = 15C_3 \, (0.05)^3 \, (0.95)^{12}$.

$= 455 \times 0.000125 \times 0.54036$

$= 0.0307$.

(ii) Not more than 3 defectives.

Not more than $\leq$

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.9945$.

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2) 4 coins were tossed simultaneously, what is the probability of getting
1) 2 heads
2) atleast 2 heads
3) atmost 2 heads.

Atleast $\geq$
Atmost $\leq$

Soln:

Getting head is success.

Probability of getting head, $p = 1/2$

$q = 1/2$

$n = 4$

B.D, $P(X=x) = nC_x \, p^x q^{n-x}$

$P(X=x) = 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$.

(i) 2 Heads:

$P(X=2) = 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = 3/8$

(ii) Atleast 2 Heads:

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$.

(or)

$P(X \geq 2) = 1 - P(X < 2)$.

$= 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \left[\frac{1}{2^4} + 4 \cdot \frac{1}{2^4}\right]$

$= 1 - \left[1/16 + 4 \cdot \frac{1}{16}\right]$

$= 1 - 5/16$.

$= 11/16$

(iii)

$P[X \leq 2] = P[X=0] + P[X=1] + P[X=2]$

$= 11/16$

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3) Find B.D for which mean is 4 & Var is 3.

Mean $= np = 4$

Variance $= npq = 3$

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{3}{4}, \quad \boxed{q = 3/4}$$

$$q = 1 - p, \quad \boxed{p = 1 - 3/4} \Rightarrow \boxed{p = 1/4}$$

Gn $np = 4$, $n(1/4) = 4$

$$\boxed{n = 16}$$

B.D $\quad P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$$= {}^{16}C_{x} \left(\tfrac{1}{4}\right)^{x} \left(\tfrac{3}{4}\right)^{n-x}.$$

4) Find P for a Binomial R.V x if $n = 6$ if.

$9\,P(X = 4) = P(X = 2)$.

$$P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}.$$

$$9P(X = 4) = P(X = 2)$$

$\because \left({}^{6}C_{4} = 15 \right.$
$\left. {}^{6}C_{2} = 15 \right)$

$$9 \cdot \cancel{{}^{6}C_{4}}\, p^{4} q^{2} = \cancel{{}^{6}C_{2}}\, p^{2} q^{4}$$

$$9p^{2} = q^{2}$$

$$9p^{2} = (1-p)^{2} = 1 + p^{2} - 2p$$

$$8p^{2} + 2p - 1 = 0.$$

$$p = \frac{-2 \pm \sqrt{4 + 32}}{16}.$$

$$= \frac{-2 \pm 6}{16} = \frac{1}{4} \text{ (or) } -\frac{1}{2}.$$

$\therefore \boxed{p = 1/4}$

Hence $q = 1 - p$, $\boxed{q = 3/4}$

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5) In a large consignment of electric bulbs 10% are defective. A random sample of 20 is taken for inspection. Find the probability that :

(1) All are good bulbs

(2) Atmost there are 3 defective bulbs

(3) Exactly there are 3 defective bulbs.

Let X be R.V denoting no. of defective bulbs.

$p = \frac{10}{100} = 0.1, \quad q = 1-p = 0.9, \quad n = 20.$

$P[X = x] = {}^nC_x \, p^x \, q^{n-x}$

$= {}^{20}C_x \, (0.1)^x \, (0.9)^{20-x}.$

1) $P[\text{All are good bulbs}] = P[\text{None are defective}]$

$= P[X = 0]$

$= {}^{20}C_0 \, (0.1)^0 \, (0.9)^{20-0}.$

$= (0.9)^{20} = 0.121.$

2) $P[\text{Atmost there are 3 defective bulbs}] = P[X \leq 3]$

$= P[X=0] + P[X=1] + P[X=2] + P[X=3]$

$= 0.121 + 0.27 + 0.285 + 0.19$

$= 0.8666$

3) $P[\text{Exactly 3 defective bulbs}] = P[X=3]$

$= {}^{20}C_3 \, (0.1)^3 \, (0.9)^{17}$

$= 0.19.$

6) The probability of a bomb hitting a target $1/5$. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the prob/: that the bridge is destroyed.

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Soln :

Let X be a R.V of hitting the target.

P[hitting the target] = $1/5$.

$p = 1/5$

$q = 1 - 1/5 = 4/5$

$n = 6$.

$P[X = x] = {}^nC_x \, p^x \, q^{n-x}$.

$P[X = 2] = P[\text{bridge is destroyed}]$

$= {}^6C_2 \, (1/5)^2 \, (4/5)^{6-2}$

$= 0.25$.

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Poisson Distribution :

P.D is a limiting case of B.D under the following conditions

→ No. of trials 'n' is indefinitely large (ie) $n \to \infty$

→ the constant prob. of success p for each trial is indefinitely small (ie) $p \to 0$.

→ $np = \lambda$ is finite, $\boxed{p = \lambda/n}$

Definition :

A R.V X taking non-negative integral values is said to follow P.D if its pmf is given by

$$P[X = x] = \frac{e^{-\lambda} \lambda^{x}}{x!}, \quad x = 0, 1, 2 \ldots \infty.$$

This dis/ has only one parameter $\lambda > 0$.

Note :

P.D occurs when there are events which do not occur as outcomes of a definite no. of trials of an experiment but which occur at random points of time & space.

Examples :

→ No of suicides reported on a particular day.

→ No. of air accidents in some unit (interval) of time

→ No. of printing mistakes at each page of the book.

→ No. of faculty blades in a packet of 100.

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MGF of Poisson Distribution.

$$M_x(t) = E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} p(x).$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda e^t}{1!} + \frac{(\lambda e^t)^2}{2!} + \cdots\right]$$

$$= e^{-\lambda} e^{\lambda e^t} = \cancel{e^{\lambda(e^t-1)}} \; e^{\lambda(e^t-1)}$$

Mean and Variance of poisson distribution

$$E(x) = M_x'(0) = \frac{d}{dt}\left(e^{\lambda(e^t-1)}\right)\Big]_{t=0}$$

$$= e^{\lambda(e^t-1)} \lambda e^t \Big]_{t=0}$$

$$= e^{\lambda(e^0-1)} \lambda e^0.$$

Mean, $\boxed{E(x) = \lambda}$

$$E(x^2) = M_x''(0) = \frac{d^2}{dt^2} M_x(t)\Big]_{t=0}.$$

$$= \frac{d}{dt}\left[e^{\lambda(e^t-1)} \lambda e^t\right]_{t=0}.$$

$$= e^{\lambda(e^t-1)}(\lambda e^t)^2 + e^{\lambda(e^t-1)} \lambda e^t \Big]_{t=0}$$

$$= e^0 \lambda^2 e^0 + e^0 \lambda e^0$$

$$\boxed{E(x^2) = \lambda^2 + \lambda}$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$\boxed{Var(x) = \lambda}$$

Mean & Variance of P.D are same.

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1) If x is a poisson variate such that $P[x=2] = 9P[x=4] + 90\,P[x=6]$. Find Variance, $P[x \geq 2]$

$$P[X=x] = p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$\frac{e^{-\lambda}\lambda^{2}}{2!} = 9\left[\frac{e^{-\lambda}\lambda^{4}}{4!}\right] + 90\left[\frac{e^{-\lambda}\lambda^{6}}{6!}\right]$$

$$\frac{\cancel{e^{-\lambda}}\cancel{\lambda^{2}}}{2} = \cancel{e^{-\lambda}}\cancel{\lambda^{2}}\left[\frac{\overset{3}{\cancel{9}}\lambda^{2}}{\underset{8}{\cancel{24}}} + \frac{\overset{1}{\cancel{90}}\lambda^{4}}{\underset{8}{\cancel{720}}}\right]$$

$$\frac{1}{2} = \frac{3}{8}\lambda^{2} + \frac{1}{8}\lambda^{4}$$

$$1 = \frac{3}{4}\lambda^{2} + \frac{1}{4}\lambda^{4}$$

$$\lambda^{4} + 3\lambda^{2} - 4 = 0$$

$$(\lambda^{2})^{2} + 3\lambda^{2} - 4 = 0$$

$$\begin{array}{c} -4 \\ \diagup \quad \diagdown \\ -1 \quad\quad 4 \end{array}$$

$$(\lambda^{2}-1)(\lambda^{2}+4) = 0$$

$$\lambda^{2}-1 = 0, \quad \lambda^{2}+4 = 0$$

$$\lambda^{2} = 1, \quad \lambda^{2} = -4$$

$$\lambda = \pm 1, \quad \lambda = \pm 2i$$

$\therefore$ $\boxed{\lambda = 1}$ is only possible.

1) Mean of P.D $E(x) = \lambda = 1$

2) Variance of P.D $Var(x) = \lambda = 1$

3) $P[x \geq 2] = 1 - P[x < 2]$

$$= 1 - \left(P[x=0] + P[x=1]\right)$$

$$= 1 - \left(\frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!}\right)$$

$$= 1 - e^{-1}(1+1) \qquad \because (\lambda = 1)$$

$$= 1 - 2/e$$

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2) In a certain factory turning razor blades there is a small chance of $1/500$ for any blade to be defective. The blades are in packets of 10. use Poisson dis/: to calculate the approximate no. of packets containing

1) No defective blade
2) 2 Defectives blades res/:
3) 1 defective

in a consignment of 10,000 packets.

Gn $p = 1/500$, $n = 10$.

$\lambda = np = 1/500 \times 10 = 0.02$.

$P[X = x] = \dfrac{e^{-\lambda} \lambda^x}{x!}$ (using Poisson dis/:)

1) $P[\text{No defective}] = P[X = 0] = \dfrac{e^{-\lambda} \lambda^0}{0!}$

$= \dfrac{e^{-0.02} (0.02)^0}{0!} = 0.9802$

2) $\therefore$ No of packets contains no defectives

$= N \times P[X = 0] = 10000 \times 0.9802$

$= 9802$ packets.

2) $P[1 \text{ defective}] = P[X = 1]$

$= \dfrac{e^{-0.02} (0.02)^1}{1!} = 0.0196$.

No. of packets containing 1 def. $= N \times P[X = 1]$

$= 196$ packets

3) $P[X = 2] = \dfrac{e^{-0.02} (0.02)^2}{2!} = 0.000196$.

No. of packets containing 2 defectives $= N \times P[X = 2]$

$= 10000 \times 0.000196$

$\simeq 2$ packets.

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3) The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on an average, emits 3.9 alpha particles per sec, what is the probability that during the next second, the no. of alpha particles emitted from 1 gm is

1) atmost 6 2) atleast 2 3) atleast 3 & atmost 6.

Gn $\lambda = 3.9$ (Average / Mean).
(Mean of poisson dis/: is λ)

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

1) $P(\text{Almost } 6) = P(X \leq 6)$

$= P(X=0) + P(X=1) + P(X=2) + \cdots + P(X=6)$

$= 0.8995$

2) $P(\text{Atleast } 2) = P[X \geq 2]$

$= 1 - (P[X < 2])$

$= 1 - [P(X=0) + P(X=1)]$

$= 0.9008$

3) $P[\underbrace{\text{Atleast } 3}_{X \geq 3} \text{ and } \underbrace{\text{Almost } 6}_{X \geq 6}] = P[3 \leq X \leq 6]$

$= P[X=3] + P[X=4] + P[X=5] + P[X=6]$

$= 0.6464$

4) In a company, the monthly breakdown of a machine is a random variable with Poisson dis/: with an average of 1.8. Find the probability that the machine will function for a month

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1) without break down
2) with exactly one break down.
3) with atleast one break down.

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Gn $\lambda = 1.8$.

1) $P[\text{without break down}] = P[X=0]$

$$= \frac{e^{-1.8}(1.8)^{0}}{0!} = 0.1653$$

2) $P[\text{with exactly one break down}] = P[X=1]$

$$= 0.2975$$

3) $P[\text{with atleast one break down}] = P[X \geq 1]$

$$= 1 - P[X<1]$$

$$= 1 - P[X=0]$$

$$= 0.8347.$$

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Geometric Distribution :

The Geometric Random Variable are characterised by

- The trials are Bernoullis trials with 2 outcomes, success (s) and failure 'f'
- The trials are identical and independent. The probability of success remains the same from trial to trial.
- Random Variable X denotes no. of trials needed to obtain first success.

Definition :

A Random Variable X is said to have geometric distribution if it assumes only non-negative values & its p.m.f is given by

$$P(X=x) = \begin{cases} pq^{x-1}, & x = 1,2,3 \ldots \\ 0, & \text{otherwise} \end{cases} \quad (0<p<1 \quad q=1-p)$$

MGF of Geometric Distribution :

$$M_X(t) = E(e^{tX}) = \sum_{x=1}^{\infty} e^{tx} pq^{x-1}$$

$$= \sum_{x=1}^{\infty} p(qe^t)^x q^{-1}$$

$$= \frac{p}{q}\left[qe^t + (qe^t)^2 + (qe^t)^3 + \cdots\right]$$

$$= \frac{p}{q}(qe^t)\left[1 + (qe^t) + (qe^t)^2 + \cdots\right]$$

$$= pe^t(1-qe^t)^{-1}$$

$$M_X(t) = \frac{pe^t}{1-qe^t}$$

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Mean and Variance :

$$E(X) = \frac{d}{dt}\left(M_X(t)\right)_{t=0} = \frac{d}{dt}\left(\frac{pe^t}{1-qe^t}\right)_{t=0}$$

$$= \left.\frac{(1-qe^t)pe^t - pe^t(-qe^t)}{(1-qe^t)^2}\right]_{t=0}$$

$$= \left.\frac{pe^t}{(1-qe^t)^2}\right]_{t=0} = \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$\because (q = 1-p)$

$$\boxed{\text{Mean} = 1/p}$$

$$E(x^2) = \left.\frac{(1-qe^t)^2(pe^t) - 2(1-qe^t)pe^t(-qe^t)}{(1-qe^t)^4}\right]_{t=0}$$

$$= \frac{(1-q)^2 p - 2(1-q)(-pq)}{(1-q)^4} = \frac{p^3 + 2p^2q}{p^4}$$

$$= \frac{\cancel{p^2}(p+2q)}{\cancel{p^4}} = \frac{p+2q}{p^2} = \frac{\cancel{p+2(1-p)}}{\cancel{p^2}}$$

$$= \frac{\cancel{p+2-2p}}{\cancel{p^2}} = \frac{p+q+q}{p^2} \qquad \because \quad p+q=1$$

$$= \frac{1+q}{p^2}$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= \frac{1+q}{p^2} - \left(\frac{1}{p}\right)^2$$

$$= \frac{\cancel{1}+q-\cancel{1}}{p^2}$$

$$\boxed{Var(x) = \frac{q}{p^2}}$$

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Memoryless property (or) Forgetfulness property of Geometric Distribution.

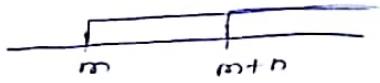
If a R.V X has geometric dis/: , then for any two +ve integers m & n,

$$P[X>m+n / X>m] = P[X>n]$$

LHS:

$$P[X>m+n / X>m] = \frac{P[(X>m+n] \cap P[X>m)]}{P[X>m]}$$

$$= \frac{P[X>m+n]}{P[X>m]}$$



$$P[X=x] = pq^{x-1}, \quad x = 1,2,3\ldots$$

$$P[X>x] = \sum_{x+1}^{\infty} pq^{x-1}$$

$$= pq^{x} + pq^{x+1} + pq^{x+2} + \cdots$$

$$= pq^{x}[1+q+q^2+\cdots]$$

$$= pq^{x}(1-q)^{-1}$$

$$= pq^{x}p^{-1}$$

$$P[X>x] = q^{x}$$

(ie) $P[X>m+n] = q^{m+n}$, $P[X>m] = q^{m}$

LHS: $P[X>m+n / X>m] = \dfrac{q^{m+n}}{q^{m}}$

$$= q^{n}.$$

RHS: $P[X>n] = q^{n}$.

LHS = RHS.

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1) A & B shoot independently until each has his own target. The probability of their hitting the target at each shot are $3/5$ & $5/7$ respectively. Find the probability that B will require more shots than A.

Let X denote the no. of trials required by A to get the no. of hits to get success.

Then X follows G D given by

$$P(X=x) = pq^{x-1}, \quad x = 1,2,3\ldots \qquad \text{Gn } p = 3/5, \quad q = 1-p = 2/5$$

$$= (3/5)(2/5)^{x-1}$$

$$P(Y=y) = pq^{y-1}$$

$$= (5/7)(2/7)^{y-1}, \quad y = 1,2,3\ldots$$

P[B required more shots than A to get his 1st success]

(Gn A & B are independent)

$$P[X=x, Y>X] = \sum_{x=1}^{\infty} (3/5)(2/5)^{x-1} \cdot \sum_{y=x+1}^{\infty} (5/7)(2/7)^{y-1}$$

$$= 3/7 \sum_{x=1}^{\infty} \sum_{y=x+1}^{\infty} (2/5)^{x-1} (2/7)^{y-1}$$

$$= \frac{3}{7}\left[\sum_{x=1}^{\infty} (2/5)^{x-1}\right]\left[\sum_{y=x+1}^{\infty} (2/7)^{y-1}\right]$$

$$= \frac{3}{7}\left[\left(\sum_{x=1}^{\infty} (2/5)^{x-1}\right)\left((2/7)^{x} + (2/7)^{x+1} + (2/7)^{x+2} + \cdots\right)\right]$$

$$= \frac{3}{7}\left[\sum_{x=1}^{\infty} (2/5)^{x-1}\right]\left(\frac{2}{7}\right)^{x}\left[1 + (2/7) + (2/7)^2 + \cdots\right]$$

$$= 3/7 \sum_{x=1}^{\infty} (2/5)^{x-1} (2/7)^{x} \underbrace{(1 - 2/7)^{-1}}_{\Rightarrow 5/7}$$

$$= \frac{3}{\cancel{7}} \times \frac{\cancel{7}}{5} \sum_{x=1}^{\infty} (2/5)^{x-1} (2/7)^{x}$$

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$x \Rightarrow x-1+1.$

$$= \frac{3}{5} \sum_{x=1}^{\infty} (2/5)^{x-1} \underbrace{(2/7)^{x-1} \cdot 2/7}.$$

$$= \left(\frac{3}{5}\right)\left(\frac{2}{7}\right) \cdot \sum_{x=1}^{\infty} (4/35)^{x-1}$$

$$= \frac{6}{35}\left[\sum_{x=1}^{\infty} (4/35)^{x-1}\right]$$

$$= \frac{6}{35}\left[\left(\frac{4}{35}\right)^0 + \left(\frac{4}{35}\right)^1 + \left(\frac{4}{35}\right)^2 + \cdots\right]$$

$$= \frac{6}{35}\left[1 + \frac{4}{35} + \left(\frac{4}{35}\right)^2 + \cdots\right]$$

$$= \frac{6}{35}\left[1 - \frac{4}{35}\right]^{-1} = \frac{6}{35}\left(\frac{31}{35}\right)^{-1}$$

$$= \frac{6}{\cancel{35}} \times \frac{\cancel{35}}{31} = \frac{6}{31}$$

---

2) Suppose that a trainee soldier shots a target in an independent fashion. If the prob. that the target is shot on any one shot is 0.8.

1) what is the prob. that target would be hit on 6th attempt.

2) what is the prob that it takes him less than 5 shots.

3) what is the prob that it takes him an even. no. of shots

$$p = 0.8, \quad q = 1 - 0.8 = 0.2$$

$$P(X=x) = pq^{x-1} = (0.8)(0.2)^{x-1}, \quad x = 1, 2, 3 \cdots$$

1) $P(X=6) = (0.8)(0.2)^{6-1} = 0.000256.$

2) $P(X<5) = P[(X=1)] + P(X=2) + P(X=3) + P(X=4)$

$$= 0.9984.$$

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2) $P(X=2) + P(X=4) + P(X=6) + \cdots$

$= (0.8)(0.2)^1 + (0.8)(0.2)^3 + (0.8)(0.2)^5 + \cdots$

$= (0.8)(0.2)\left[1 + (0.2)^2 + (0.4)^4 + \cdots\right]$

$= (0.8)(0.2)\left[1 + 0.04 + (0.04)^2 + \cdots\right]$

$= (0.8)(0.2)\left[1 - 0.04\right]^{-1}$

$= (0.8)(0.2)(0.96)$

$= 0.1667$ (approximately)

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Continuous Distribution :

- Exponential Distribution .
- Normal Distribution .

Exponential Distribution :

A continuous R.V X is said to follow an exponential distribution . with parameter $\lambda > 0$. if its prob. density function is given by .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} , & x > 0 , \quad \lambda > 0 , \quad 0 < x < \infty \\ 0 , & \text{otherwise} \end{cases}$$

MGF of exponential distribution :

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \int_{0}^{\infty} e^{tx} \lambda e^{-\lambda x}\, dx , \quad x > 0 .$$

$$= \lambda \int_{0}^{\infty} e^{-(\lambda - t)x}\, dx$$

$$= \lambda \left[\frac{e^{-(\lambda - t)x}}{-(\lambda - t)} \right]_0^{\infty}$$

$$= \frac{\lambda}{-(\lambda - t)} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{\lambda}{\lambda - t}$$

Mean and Variance of Exponential Distribution :

$$M_X(t) = \frac{\lambda}{\lambda - t}$$

$$E(X) = \frac{d}{dt} \left(\frac{\lambda}{\lambda - t} \right)_{t=0} .$$

$$= \left. \frac{(\lambda - t)0 - \lambda(-1)}{(\lambda - t)^2} \right]_{t=0} = \left. \frac{\lambda}{(\lambda - t)^2} \right]_{t=0} = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$$

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$$\boxed{\text{Mean} = 1/\lambda}$$

$$E(x^2) = \frac{d^2}{dt}\left[\frac{\lambda}{(\lambda-t)^2}\right]_{t=0}$$

$$= \frac{(\lambda-t)^2 0 - \lambda\left(2(\lambda-t)(-1)\right)}{(\lambda-t)^4}\Bigg]_{t=0}$$

$$= \frac{2\lambda(\lambda-t)}{(\lambda-t)^4}\Bigg]_{t=0}$$

$$= \frac{2\lambda}{(\lambda-t)^3}\Bigg]_{t=0} = \frac{2\lambda}{\lambda^3} = \frac{2}{\lambda^2}$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= 2/\lambda^2 - 1/\lambda^2$$

$$\boxed{Var(x) = 1/\lambda^2}$$

Memoryless property of exponential distribution.

If x is exponentially distributed with parameter λ, then for any 2 positive integer 'm' & 'n'

$$P[x > m+n / x > m] = P[x > n].$$

Proof:

p.d.f of exponential dis/:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0. \\ 0, & \text{otherwise}. \end{cases}$$

$$P[X > k] = \int_k^\infty \lambda e^{-\lambda x}\, dx = \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_k^\infty$$

$$= -\left[e^{-\infty} - e^{-\lambda k}\right]$$

$$= e^{-\lambda k}$$

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LHS : $P[X > m+n / X > m] = \dfrac{P[X > m+n]}{P[X > m]}$

$$= \frac{e^{-\lambda(m+n)}}{e^{-\lambda m}} = e^{-\lambda n}.$$

RHS : $P[X > n] = e^{-\lambda n}$

$$\boxed{LHS = RHS}$$

1) Suppose that during a rainy season in a tropical island, the length of the shower has an exponential distribution with average 2 min.

1) Find the prob. that the shower will be there for more than 3 minutes.

2) If the shower has already lasted for 2 min, what is the prob. that it will last for atleast one more minute.

Let X be R.V representing the length of shower in minutes.

Gn X follows exp. dis/:

Mean (Avg length) = 2.

Mean = $1/\lambda$

$\lambda = 1/\text{Mean} = 1/2$

$$\boxed{\lambda = 1/2}$$

p.d.f $\rightarrow$ $f(x) = \lambda e^{-\lambda x} = \frac{1}{2} e^{-\frac{1}{2}x}, \quad x \geq 0$

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1) P[shower will last more than 3 min)

$$P[X>3] = \int_3^\infty \tfrac{1}{2} e^{-\frac{1}{2}x}\,dx = \tfrac{1}{2}\left[\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\right]_3^\infty$$

$$= -\left[e^{-\infty} - e^{-3/2}\right]$$

$$= e^{-3/2} = 0.2231.$$

2) P[shower will last at least one more min, given that it has lasted 2 min]

$$P[X>3 / X>2] = P[X>1]$$

By Memoryless property,

$$P[X>m+n / X>m] = P[X>n]$$

$(m=2, n=1)$

$$= P[X>1]$$

$$= \tfrac{1}{2}\int_1^\infty e^{-\frac{1}{2}x}\,dx$$

$$= \tfrac{1}{2}\left[\frac{e^{-\frac{1}{2}}}{-\frac{1}{2}}\right]_1^\infty = e^{-\frac{1}{2}} = 0.6065$$

---

2) The time (in hrs) required to repair a machine is exp. dis/: with parameter $\lambda = \frac{1}{2}$.

1) what is the prob. that repair time exceeds 2 hrs?

2) what is the conditional prob. that the repair time takes atleast 10 hrs given that its duration exceeds 9 hrs?

Gn $\lambda = \frac{1}{2}$

1) $P(X>2) = \int_2^\infty \lambda e^{-\lambda x}\,dx = e^{-1} = 0.3679.$

2) $P[X>10 \mid X>9] = P[X>1]$ (by Memoryless property)

$$= \int_1^\infty \lambda e^{-\lambda x}\,dx = e^{-\frac{1}{2}} = 0.6065.$$

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3) If X is R.V which follows exponential distribution with parameter λ. with $P[X \le 1] = P[X > 1]$. Find $Var(X)$.

$$P[X \le 1] = P[X > 1]$$

$$1 - P[X > 1] = P[X > 1].$$

$$2P[X > 1] = 1$$

$$\boxed{P[X > 1] = 1/2}.$$

W.K.T, $P[X > K] = e^{-\lambda K}$, $(K = 1)$

$$e^{-\lambda} = 1/2$$

$$\frac{1}{e^{\lambda}} = \frac{1}{2}.$$

If $e^{\lambda} = 2$

$$\log e^{\lambda} = \log 2.$$

$$\boxed{\lambda = \log_e 2}$$

$$Var(X) = \frac{1}{\lambda^2} = \frac{1}{(\log_e 2)^2}$$

---

4) Average length of time that a certain lady speaks on the telephone is a R.V specified by the pdf, $f(x) = \begin{cases} Ae^{-x/5}, & x > 0. \\ 0, & \text{other}. \end{cases}$

Find 1) A 2) More than 10 min 3) Less than 5 min.

4) Between 5 & 10 min.

1) $f(x) = \lambda e^{-\lambda x} \Rightarrow \lambda = A \Rightarrow A = 1/5$

2) $P[X > 10] = \int_{10}^{\infty} \frac{1}{5} e^{-\frac{1}{5}x} dx = 0.1353$

3) $P[X < 5] = 1 - \left(\int_{0}^{\infty} \frac{1}{5} e^{-\frac{1}{5}x} dx\right) = 0.6321$

4) $P[5 \le X \le 10]$

$= \int_{5}^{10} \lambda e^{-\lambda x} dx$

$= 0.2326.$

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Normal distribution :

A continuous R.V x with p.d.f is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty. \quad -\infty < \mu < \infty. \quad \sigma > 0$$

where μ and σ are parameters.

with (ie) 'μ' → mean & standard deviation (S.D) → 'σ'

Note :

Normal dis/: is denoted by $X \sim N(\mu, \sigma)$ (or) $X \sim N(\mu, \sigma^2)$.

Characteristics of Normal distribution :

→ curve is bell shaped and symmetrical about the line $x = \mu$

→ Mean = Median = Mode

→ As x increases numerically, $f(x)$ decreases rapidly the maximum prob. occuring at the point $x = \mu$, given by

$$p(x)\Big]_{max} = \frac{1}{\sigma\sqrt{2\pi}}$$

→ Since $f(x)$ is non-negative, the curve will not go below x-axis.

→ x-axis is an asymptote of the curve.

→ $\mu_{2r+1} = 0$, $r = 0, 1, 2, 3 \cdots$ $\quad \mu_{2r} = 1 \cdot 3 \cdot 5 \cdots (2r-1)\sigma^{2r}$ $(r = 0, 1, 2 \cdots)$

→ $f(x)$ is +ve, curve is above x-axis.

Standard Normal dis/:

If X is a R.V following normal dis/: with parameters μ & σ, then $Z = \frac{X-\mu}{\sigma}$ is called standard normal variate & pdf is $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$, $-\infty < z < \infty$

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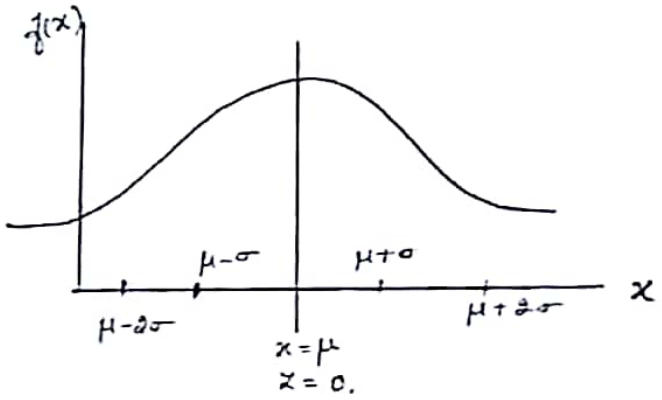
Basic properties of standard Normal curve:

→ Total area under the standard normal curve is equal to 1.

→ The std. normal curve is asymptotic to x-axis.

→ The std. normal curve is symmetric about zero.

Normal density function.



Note:

The normal dis/ whose mean is zero & Variance is 1.

(i.e) $N(0,1)$ is called standard normal dis/.

If $z = \frac{x-\mu}{\sigma}$.

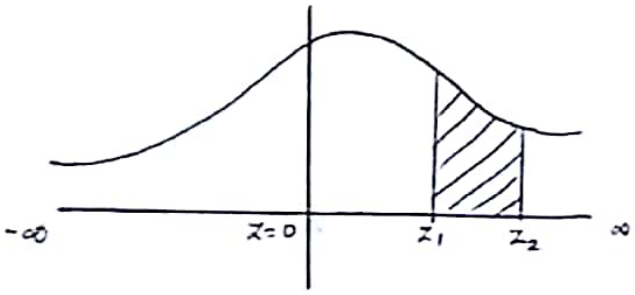
Mean: $E(z) = E\left(\frac{x-\mu}{\sigma}\right) = \frac{E(x)-\mu}{\sigma} = \frac{\mu-\mu}{\sigma} = 0.$

Variance: $Var(z) = Var\left(\frac{x-\mu}{\sigma}\right) = \frac{Var(x)}{\sigma^2} = \frac{\sigma^2}{\sigma^2} = 1.$

By property $Var(A \times B) = A^2 Var(x)$

Note:

1) $\boxed{P[z_1 < z < z_2] = P[0 < z < z_2] - P[0 < z < z_1]}$



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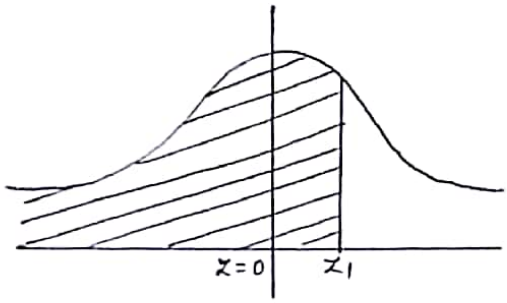
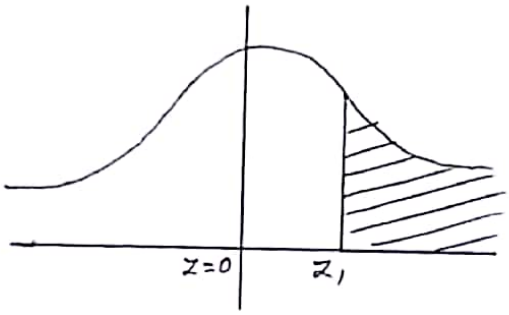
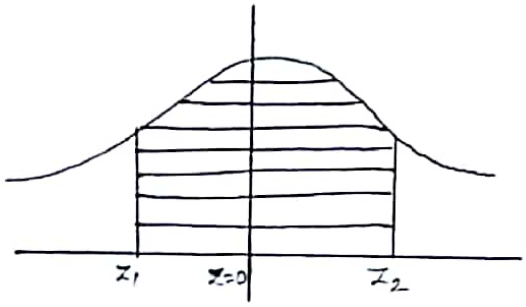
2) $\boxed{P(Z < z_1) = 0.5 + P(0 < Z < z_1)}$



3) $\boxed{P(Z > z_1) = 0.5 - P(0 < Z < z_1)}$



4) $\boxed{P(z_1 < Z < z_2) = P(0 < Z < z_1) + P(0 < Z < z_2)}$



MGF of Normal distribution :

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx = \int_{-\infty}^{\infty} e^{tx} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$$Z = \frac{x-\mu}{\sigma}, \quad x = \sigma z + \mu.$$

$$dz = \frac{dx}{\sigma}$$

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$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} e^{-z^2/2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\mu t} \cdot e^{\sigma z t} e^{-z^2/2} dz$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[z^2 - 2\sigma t z]} dz$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(z-\sigma t)^2 - \sigma^2 t^2]} dz$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2} e^{-\sigma^2 t^2/2} dz.$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} e^{+\sigma^2 t^2/2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2} dz$$

Let $(z - t\sigma) = u$

$du = dz$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu + \sigma^2 t^2/2} \int_{-\infty}^{\infty} \underbrace{e^{-u^2/2}}_{\text{even}} du.$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu + \sigma^2 t^2/2} \, 2 \int_{0}^{\infty} e^{-u^2/2} du$$

Let $u^2/2 = t^2$

$u = \sqrt{2}\, t \longrightarrow u^2 = 2t^2, \quad du = \sqrt{2}\, dt$

$$= \frac{1}{\sqrt{\cancel{2}\pi}} e^{t\mu + \sigma^2 t^2/2} \, 2 \int_{0}^{\infty} e^{-t^2} \cancel{\sqrt{2}}\, dt$$

$$= \frac{1}{\sqrt{\cancel{\pi}}} e^{\mu t + \sigma^2 t^2/2} \cancel{2} \cdot (\sqrt{\cancel{\pi}})/\cancel{2}$$

$$= e^{\mu t + \sigma^2 t^2/2}$$

Take $\int_0^{\infty} e^{-t^2} dt$

W.K.T $\int_0^{\infty} e^{-x} x^{n-1} dx = \Gamma(n)$

$x = t^2, \quad t = \sqrt{x}$

$dx = 2t\,dt = \quad dt = \frac{dx}{2\sqrt{x}}.$

$$\int_0^{\infty} e^{-x} \frac{dx}{2\sqrt{x}} = \frac{1}{2}\int e^{-x} x^{-1/2} dx$$

$(n = 1/2)$

$= \frac{1}{2}\Gamma(1/2)$

$= \frac{1}{2}(\sqrt{\pi})$

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Mean and Variance of Normal distribution.

$$E(X) = \frac{d}{dt}\left(M_X(t)\right)_{t=0} =$$

$$= e^{(\mu t + \sigma^2 t^2/2)}\left(\mu + \frac{\sigma^2 2t}{2}\right)\Big]_{t=0}$$

$$= \cancel{e^{\mu \times 0}} \quad e^0(\mu + 0) = \mu.$$

$$\boxed{E(X) = \mu}$$

$$E(X^2) = \frac{d^2}{dt^2} M_X(t)\Big]_{t=0}$$

$$= \left(e^{\mu t + \sigma^2 t^2/2}\right)(0 + \sigma^2) + (\mu + \sigma^2 t)\, e^{\mu t + \sigma^2 t^2/2}\left(\mu + \sigma^2 t\right)\Big]_{t=0}$$

$$= e^0(\sigma^2) + \mu^2 e^0 = \sigma^2 + \mu^2$$

$$\text{Variance} = \sigma^2 + \cancel{\mu^2} - \cancel{\mu^2} = \sigma^2$$

$$\boxed{Var(X) = \sigma^2}$$

---

1) An electrical firm manufacturers light bulbs that have a length of life which is normally distributed with $\mu = 800$ hrs and $\sigma = 40$. Find proba. that a bulb burns between 778 & 834 hrs.

$$Z = \frac{X - \mu}{\sigma}, \qquad Z = \frac{X - 800}{40}.$$

when $X = 778$, $Z = \frac{778 - 800}{40} = -0.55$

$X = 835$, $Z = \frac{835 - 800}{40} = 0.85$

$$P[778 < X < 834] = P[-0.55 < Z < 0.85]$$

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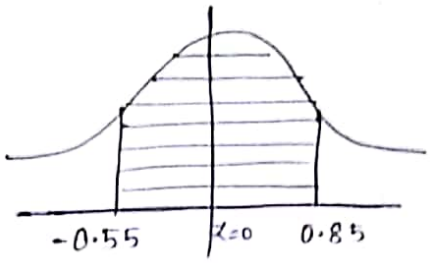


$= P[0 < z < 0.55] + P[0 < z < 0.85]$

$= 0.2088 + 0.3023$

$= 0.5111$

2) Mean yield for one acre plots is 662 kgs (μ) with S.D (σ) is 32. Assuming Normal dis/: how many one acre plots in a batch of 1000 plots would you expect to yield 1) over 700 kgs
2) below 650 kgs.

$$z = \frac{x-\mu}{\sigma} = \frac{x-662}{32}.$$

1) $P[x > 700] = P\left[\frac{x-\mu}{\sigma} > \frac{700-\mu}{\sigma}\right]$

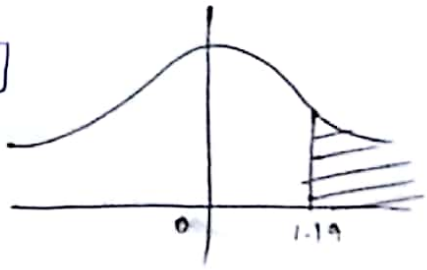
$= P\left[z > \frac{700-662}{32}\right]$

$= P[z > 1.19]$

$= 0.5 - P[0 < z < 1.19]$

$= 0.5 - 0.883$

$= 0.117$



Number of plots have yield over 700 kgs - 117

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2) below 650 kgs

$$P[x < 650] = P\left[\frac{x-\mu}{\sigma} < \frac{650-\mu}{\sigma}\right]$$

$$= P\left[z < \frac{650-662}{32}\right]$$

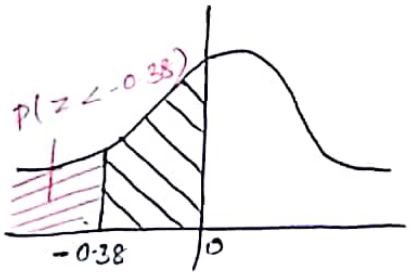
$$= P[z < -0.375]$$

$$= P[z < -0.38]$$

$$= 0.5 - P(0 < z < 0.38)$$

$$= 0.5 - 0.148$$

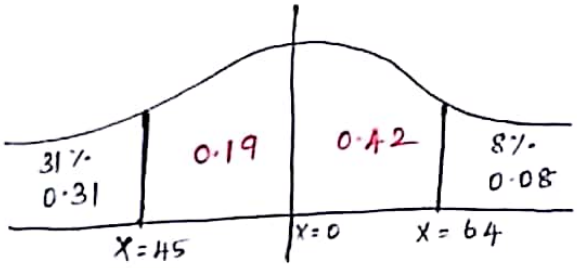
$$= 0.352$$



No. of plots have yield below 650 kgs = 352.

3) In a Normal dis/: 31% of items are under 45 & 8% are over 64. Find mean & S.D of dis/:

Let the mean & S.D of N.D be μ & σ



The area lying to left of the ordinate $x=45$ is 0.31

Area lying to right of ordinate upto mean,

$$= 0.5 - 0.31$$

$$= 0.19$$

Value of z to area 0.19 $\Big\}$, $z = \frac{45-\mu}{\sigma} = -0.5$

(ie) $\boxed{-0.5\sigma + \mu = 45} \rightarrow (1)$.

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Area to the left of ordinate $x = 64$ is $0.5 - 0.08$.

$= 0.42$.

Value of z corresponding to area is 1.4 $\Big\} \Rightarrow z = \dfrac{64 - \mu}{\sigma} = 1.4$

$$\boxed{1.4\sigma + \mu = 64} \longrightarrow (2)$$

Solving (1) & (2), $\boxed{\mu = 50, \ \sigma = 10}$

4) In an examination the marks obtained by the students in Maths, Physics & chemistry are normally dis/: about mean – 50, 52, 48 and S.D → 15, 12, 16 res/: Find the prob. of securing a total mark of

1) 180 or above
2) 90 or below.

Let x, y, z be marks of respective subjects

Total marks $(T) = x + y + z$.

$$\mu = E(T) = E(x + y + z)$$
$$= E(x) + E(y) + E(z)$$
$$= 50 + 48 + 52$$
$$\boxed{\mu = 150}$$

$$\sigma^2 = Var(x) = Var(x + y + z)$$
$$= Var(x) + Var(y) + Var(z)$$
$$= (15)^2 + (12)^2 + (16)^2$$
$$= 625 \quad \therefore \boxed{S.D\ (\sigma) = 25}$$

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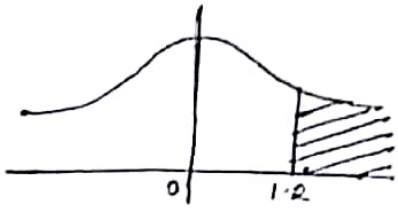
1) $P[T > 180] = P\left[\frac{T-150}{25} > \frac{180-150}{25}\right]$

$= P[Z > 1.2]$



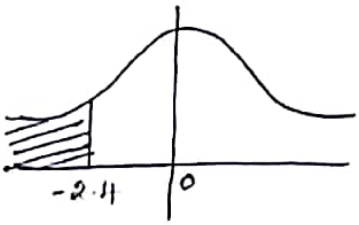
$= 0.5 - P[0 < Z < 1.2]$

$= 0.5 - 0.3849$

$= 0.1151$

2) $P[T < 90] = P\left[\frac{T-150}{25} < \frac{90-150}{25}\right]$

$= P[Z < -2.4]$



$= 0.5 - P(0 < Z < 2.4)$

$= 0.5 - 0.4918$

$= 0.0082$

---

5) The marks obtained by a large group of students in a final exam in statistics have a mean 58 & SD 8.5 Assuming that these marks are approximately normally distributed. what percentage of the students can be expected to have obtained marks 60 to 69, both inclusive?

Gn $\mu = 58$, $\sigma = 8.5$

$Z = \frac{X-\mu}{\sigma} = \frac{X-58}{8.5}$

$P[60 < X < 69] = P\left[\frac{60-58}{8.5} < \frac{X-58}{8.5} < \frac{69-58}{8.5}\right]$

$= P[0.24 < Z < 1.29]$

$= P[0 < Z < 1.29] - P[0 < Z < 0.24]$

$= 0.4015 - 0.0948 = 0.3069$.

30.67% of students expected to have obtained marks btn 60 & 69.

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