Unit – V

Markov Process

The random process which depends only upon the most recent previous value and is independent of all values in the more distant past. Models in which the future depends only upon the present are common among electrical engineering models.

Markov Chain :-

If for all n,

$$P\{X_n = a_n / X_{n-1} = a_{n-1}, X_{n-2} = a_{n-2}, \ldots, X_0 = a_n\} = P\{X_n = a_n / X_{n-1} = a_{n-1}\}$$

then process $\{X_n\}$, $n = 0, 1, 2, \ldots$ is called a Markov Chain.

Note:-

① $(a_0, a_1, \ldots, a_n, \ldots)$ are called state of the markov chain.

② The conditional probability $P\{X_n = a_j / X_{n-1} = a_i\}$ is called one-step transition probability from state a_i to state a_j at the nth step and denoted by $P_{ij}(n-1, n)$.

③ If one-step transition probability does not depend on the step i.e. $P_{ij}(n-1, n) = P_{ij}(m-1, m)$, then, the

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Markov chain is called a homogeneous Markov chain or stationary transition probability.

④ When the Markov chain is homogeneous, the one-step transition probability is denoted by P_{ij}, the matrix $P = \{P_{ij}\}$ is called (one-step) transition probability matrix, shortly, tpm.

⑤ The tpm of a Markov chain is stochastic matric, since $P_{ij} \geq 0$ and $\sum P_{ij} = 1$ for all i i.e. the sum of all elements of any row of the tpm is 1.

⑥ The conditional probability that the process is in state a_j at step n, given that it was in state a_i at step 0. i.e.

$P\{X_n = a_j / X_0 = a_i\}$ is called n-step transition probability and denoted by $P_{ij}(n)$.

⑦ If the probability that the process is in state a_i is P_i $(i = 1, 2, \ldots, k)$ at any arbitrary step, then the row vector $p = \{P_1, P_2, \ldots, P_k\}$ is called the probability distribution of the process at that time.

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Chapman - Kolmogorow Theorem :-

If P is the transition probability matrix (tpm) of a homogeneous Markov chain, then the n step tpm $P^{(n)}$ is equal to P^n.

i.e. $$\left[P_{ij}^{(n)}\right] = \left[P_{ij}\right]^n.$$

Regular Matrix :-

A stochastic matrix P is said to be regular matrix, if all the entries of P^m (for some positive integer) are positive. A homogeneous Markov chain is said to be regular if its tpm is regular.

Classification of States of Markov chain :-

If $P_{ij}^{(n)} > 0$ for some n and for all i and j, then every state can reached from every other state. When this condition is satisfied the Markov chain is said to be irreducible. The tpm of an irreducible chain is an irreducible matrix otherwise, the chain is said to be non-irreducible or reducible.

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Note:-

① State i of a Markov chain is called return state if $P_{ij}^{(n)} > 0$ for some $n > 1$.

② The period d_i of a return i is defined as the greatest common divisor of all m such that $P_{ij}^{(m)} > 0$ i.e. $d_i = GCD\{m : P_{ij}^{(m)} > 0\}$

③ State i is said to be periodic with period d_i, if $d_i > 1$ and aperiodic if $d_i = 1$.

④ Obviously state i is aperiodic if $P_{ii} \neq 0$. The prob- that the chain returns to state i, having started from state i, for the first time at the nth step is denoted by $f_{ii}^{(n)}$ and called the first return time probability.

⑤ A state i is said to be persistent or recurrent if the return to state i is certain i.e. if $F_{ii} = 1$

⑥ The state i is said to be transient if the return to state i is uncertain i.e. if $F_{ii} < 1$.

⑦ The state i is said to be nonnull persistent if its mean recurrence time μ_{ii} is finite and null persistent if $\mu_{ii} = \infty$.

⑧ A nonnull persistent and aperiodic state is called ergodic.

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Ex-①:- The transition probability matrix of a Markov chain $\{X_n\}$, $n=1,2,3,\ldots$ having 3 states 1, 2 and 3 is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (I) $P\{X_2=3\}$ and (II) $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$.

Sol:- We have

$$P^{(2)} = P^2 = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$$

(I) $$P\{X_2=3\} = P_{13}^{(2)} P(X_0=1) + P_{23}^{(2)} P(X_0=2) + P_{33}^{(2)} P(X_0=3)$$

$$= 0.26 \times 0.7 + 0.34 \times 0.2 + 0.29 \times 0.1$$

$$= 0.182 + 0.068 + 0.029 = 0.279$$

(II) $$P(X_3=2, X_2=3, X_1=3, X_0=2)$$

$$= P\{X_3=2/X_2=3\}\, P\{X_2=3/X_1=3\}\, P\{X_1=3/X_0=2\}\, P\{X_0=2\}$$

$$= 0.4 \times 0.3 \times 0.2 \times 0.2 \quad \text{(By Markov property)}$$

$$= 0.0048$$

Answer

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Ex-③: A fair dice is tossed repeatedly. If X_n denote the maximum of the numbers occuring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Find also $p^{(2)}$ and $P(X_2=6)$.

Sol:- Here, the state space: $\{1, 2, 3, 4, 5, 6\}$

Consider the analysis

def X_n = the max. no occuring in first n trials $= 3$ say

Then $X_{n+1} = 3$ if $(n+1)$th trial results in 1, 2 or 3

$X_{n+1} = 4$ if $(n+1)$th trial results in 4

$X_{n+1} = 5$ if $(n+1)$th trial results in 5 etc.

$$\therefore P\{X_{n+1}=3/X_n=3\} = \frac{1}{6}+\frac{1}{6}+\frac{1}{6} = \frac{3}{6}$$

$$P\{X_{n+1}=i/X_n=3\} = \frac{1}{6} \quad \text{when } i = 4, 5, 6$$

$\therefore$ The transition probability matrix is

$$P = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 2/6 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{3}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 4/6 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 5/6 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore p^2 = \frac{1}{36}\begin{bmatrix} 1 & 3 & 5 & 7 & 9 & 11 \\ 0 & 4 & 5 & 7 & 9 & 11 \\ 0 & 0 & 9 & 7 & 9 & 11 \\ 0 & 0 & 0 & 16 & 9 & 11 \\ 0 & 0 & 0 & 0 & 25 & 11 \\ 0 & 0 & 0 & 0 & 0 & 36 \end{bmatrix}$$

Also, the initial probability distribution is

$$p^{(0)} = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$$

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Now

$$P(X_2=6) = P_{16}^{(2)} P(X_0=1) + P_{26}^{(2)} P(X_0=2) + P_{36}^{(2)} P(X_0=3) + P_{46}^{(2)} P(X_0=4)$$
$$+ P_{56}^{(2)} P(X_0=5) + P_{66}^{(2)} P(X_0=6)$$

$$= \frac{11}{36}\times\frac{1}{6} + \frac{11}{36}\times\frac{1}{6} + \frac{11}{36}\times\frac{1}{6} + \frac{11}{36}\times\frac{1}{6} + \frac{11}{36}\times\frac{1}{6} + \frac{36}{36}\times\frac{1}{6}$$

$$= \frac{1}{36\times 6}(11+11+11+11+11+36)$$

$$= \frac{91}{216}$$ Answer

Ex-(3):- A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a six appeared. Find (I) the probability that he takes a train on the third day and (II) the probability that he drive to work in the long run.

Sol:- Clearly, the travel pattern is a Markov chain, with state space = (train, car)

The tpm of the chain is

$$P = \begin{array}{c c} & \begin{array}{cc} T & C \end{array} \\ \begin{array}{c} T \\ C \end{array} & \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

and

The initial state probability distribution is

$$p^{(1)} = \left(\frac{5}{6}, \frac{1}{6}\right)$$

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Now

$$P^{(2)} = P^{(1)} P$$

$$= \left(\frac{5}{8}, \frac{1}{8}\right)\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \left(\frac{1}{12}, \frac{11}{12}\right)$$

$$P^{(3)} = P^{(2)} P$$

$$= \left(\frac{1}{12}, \frac{11}{12}\right)\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \left(\frac{11}{24}, \frac{13}{24}\right)$$

$\therefore$ P(the man travels by train on the third day)

$$= \frac{11}{24}$$

Let $\pi = (\pi_1, \pi_2)$ be the limiting form of the state probability distribution. Then by property

$$\pi P = \pi$$

$$(\pi_1, \pi_2)\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = (\pi_1, \pi_2)$$

$\therefore$ $\frac{1}{2}\pi_2 = \pi_1$ and $\pi_1 + \frac{1}{2}\pi_2 = \pi_2$ ——— (1)

But $\pi_1 + \pi_2 = 1$ ——— (1), since π is a probability distribution.

By solving (1) & (2), we get

$$\pi_1 = \frac{1}{3} \quad \text{and} \quad \pi_2 = \frac{2}{3}$$

$\therefore$ P(the man travels by car in the long run)

$$= \frac{2}{3}$$

Ans

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Ex-(5):- A gambler has Rs 2. He bets Rs 1 at a time and wins Rs 1 with probability $\frac{1}{2}$. He stop playing if he loses Rs 2 or wins Rs. 4. (a) What is the tpm of the related Markov chain (b) What is the probability that he has lost his money at the end of 5 plays (c) What is the probability that the game lasts more than 7 plays.

Sol:- Let X_n be the amount with the player at the end of nth round of the play.

Here, the state space is $\{0, 1, 2, 3, 4, 5, 6\}$

Acc^n to question the tpm of the Markov chain is

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The initial state probability distribution is $p^{(0)} = (0, 0, 1, 0, 0, 0, 0)$

Now $p^{(1)} = p^{(0)} P = (0, \frac{1}{2}, 0, \frac{1}{2}, 0, 0, 0)$

$p^{(2)} = p^{(1)} P = (\frac{1}{4}, 0, \frac{1}{2}, 0, \frac{1}{4}, 0, 0)$

$p^{(3)} = p^{(2)} P = (\frac{1}{4}, \frac{1}{4}, 0, \frac{3}{8}, 0, \frac{1}{8}, 0)$

$p^{(4)} = p^{(3)} P = (\frac{3}{8}, 0, \frac{5}{16}, 0, \frac{1}{4}, 0, \frac{1}{16})$

$p^{(5)} = p^{(4)} P = (\frac{3}{8}, \frac{5}{32}, 0, \frac{9}{32}, 0, \frac{1}{8}, \frac{1}{16})$

$p^{(6)} = p^{(5)} P = (\frac{29}{64}, 0, \frac{7}{32}, 0, \frac{13}{64}, 0, \frac{1}{8})$

$p^{(7)} = p^{(6)} P = (\frac{29}{64}, \frac{7}{64}, 0, \frac{27}{128}, 0, \frac{13}{128}, \frac{1}{8})$

$\therefore P(\text{The man lost his money at the end of 5 plays}) = P(X_5 = 0) = \frac{3}{8}$

and $P(\text{The game more than 7 round}) = P(X_7 = 1, 2, 3, 4 \text{ or } 5)$

$= \frac{7}{64} + 0 + \frac{27}{128} + 0 + \frac{13}{128} = \frac{27}{64}$

Ans

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Ex-(6):- There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are two red marbles in A after 3 steps. In the long run, what is the probability that there are 2 red marbles in urn A?

Sol:- We have

The state space of the chain $\{X_n\} = (0, 1, 2)$, since the number of balls in the urn A is always 2.

The tpm of the chain $\{X_n\}$ be

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

$P_{00} = 0$, $P_{01} = 1$, $P_{02} = 1 - P_{00} - P_{01} = 0$

$P_{10} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, $P_{12} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}$

$\therefore P_{11} = 1 - P_{10} - P_{12} = \frac{1}{2}$

$P_{20} = 0$, $P_{21} = \frac{2}{3}$

$P_{22} = 1 - P_{20} - P_{21} = \frac{1}{3}$

Now $p^{(0)} = (1, 0, 0)$, as there is no red marbles in A in the begianing.

Now

$$p^{(1)} = p^{(0)} P = (0, 1, 0)$$

$$p^{(2)} = p^{(1)} P = \left(\frac{1}{6}, \frac{1}{2}, \frac{1}{3}\right)$$

$$p^{(3)} = p^{(2)} P = \left(\frac{1}{12}, \frac{23}{36}, \frac{5}{18}\right)$$

$\therefore$ P(there are 2 red marbles in A after 3 steps)

i.e. $P(X_3 = 2) = \frac{5}{18}$

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Let $\pi = (\pi_0, \pi_1, \pi_2)$ be the stationary probability distribution of the chain. By property

$$\pi P = \pi \quad \text{and} \quad \pi_0 + \pi_1 + \pi_2 = 1$$

$$(\pi_0, \pi_1, \pi_2) \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix} = (\pi_0, \pi_1, \pi_2)$$

i.e. $\frac{1}{6}\pi_1 = \pi_0$, $\pi_0 + \frac{1}{2}\pi_1 + \frac{2}{3}\pi_2 = \pi_1$, $\frac{1}{3}\pi_1 + \frac{1}{3}\pi_2 = \pi_2$

Solving $\pi_0 = \frac{1}{10}$, $\pi_1 = \frac{6}{10}$, $\pi_2 = \frac{3}{10}$

$\therefore$ P (there are 2 red marbles in A in the long run)

$$= \frac{3}{10} = 0.3$$

Answer

Ex-(7):- Find the nature of the states of the Markov chain with the tpm

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 2 & 0 & 1 & 0 \end{array}$$

Sol:- We have

$$P^2 = PP = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}, \quad P^3 = P$$

$\therefore P^4 = P^2$ i.e. $P^{2n} = P^2$ and $P^{2n+1} = P$

Noted

$$\left.\begin{array}{lll} P_{00}^{(2)} > 0, & P_{01}^{(2)} > 0, & P_{02}^{(2)} > 0 \\ P_{10}^{(1)} > 0, & P_{11}^{(2)} > 0, & P_{12}^{(1)} > 0 \\ P_{20}^{(2)} > 0, & P_{21}^{(1)} > 0, & P_{22}^{(2)} > 0 \end{array}\right\} \therefore \text{The Markov chain is irreducible.}$$

Also $P_{ii}^{(2)} = P_{ii}^{(4)} = P_{ii}^{(6)} = \cdots > 0$ for all i and $GCD(2, 4, 6, \ldots) = 2$

So, all states of the chain are periodic, with period 2.

Since the chain is finite and irreducible, all the states are recurrent persistent. Also all states are ergodic

Ans

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Ex-(8):- Three boys A, B and C throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the tpm and classify the states.

Sol:- Acc^n to question tpm is given by

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$

As state X_n depends only on state X_{n-1}, but not on $X_{n-2}, X_{n-3}, \ldots$ etc. Therefore, $\{X_n\}$ is a Markov chain.

Now

$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad P^3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

As all $P_{ij}^{(m)} > 0$ for at least one $m = 1, 2, 3$

$\therefore$ The chain is irreducible.

Again

$$P^4 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}, \quad P^5 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{2} \end{bmatrix}, \quad P^6 = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \end{bmatrix}$$

and so on.

Now

$P_{ii}^{(2)}, P_{ii}^{(3)}, P_{ii}^{(5)}, \ldots$ etc > 0 for $i = 2, 3$ and $GCD(2, 3, 5 \ldots) = 1$

$\therefore$ States 2 and 3 (i.e. B and C) are periodic with period 1 i.e. aperiodic

Also $P_{11}^{(3)}, P_{11}^{(5)}, P_{11}^{(6)}$ etc > 0 and $GCD(3, 5, 6, \ldots) = 1$

$\therefore$ State 1 (i.e A) is periodic with period 1 i.e. aperiodic.

Since the chain is finite and irreducible, all its states are nonnull persistent. Moreover all states are ergodic.

Ans

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Q-(15):- If the tpm of a Markov chain is $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find the steady-state distribution of the chain.

Sol:- Let $\pi = (\pi_1, \pi_2)$ is the stationary state distribution of the chain.

Then, by property

$$\pi P = \pi \quad \text{and} \quad \pi_1 + \pi_2 = 1$$

$$(\pi_1, \pi_2) \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = (\pi_1, \pi_2)$$

$$\Rightarrow \quad \frac{1}{2}\pi_2 = \pi_1$$

and $\pi_1 + \frac{1}{2}\pi_2 = \pi_2$

So, $\pi_2 = 2\pi_1$ but $\pi_1 + \pi_2 = 1$

$$\therefore \quad \pi_1 + 2\pi_1 = 1$$

$$\Rightarrow \quad \pi_1 = \frac{1}{3}$$

and $\pi_2 = \frac{2}{3}$

The steady-state distribution of the Markov chain is

$$\left(\frac{1}{3}, \frac{2}{3}\right)$$

Ans

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(17) Prove that the matrix

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

is the tpm of an irreducible Markov chain.

Sol:- we have

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$\therefore P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

and

$$P^3 = P^2 P = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

As $p_{11}^{(3)} > 0$, $p_{12}^{(1)} > 0$, $p_{13}^{(2)} > 0$

$p_{21}^{(2)} > 0$, $p_{22}^{(2)} > 0$, $p_{23}^{(1)} > 0$

$p_{31}^{(1)} > 0$, $p_{32}^{(1)} > 0$, $p_{33}^{(2)} > 0$

$\therefore$ The chain is an irreducible Markov chain

Proved

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(19) If the initial state probability distribution of a Markov chain is $p^{(0)} = \left(\frac{5}{6}, \frac{1}{6}\right)$ and the tpm of the chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Find the probability distribution of the chain after two steps.

<u>Sol:</u> We have

$$P = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Now

$$p^{(1)} = p^{(0)} P$$

$$= \left(\frac{5}{6} \quad \frac{1}{6}\right) \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \left(\frac{1}{12}, \frac{11}{12}\right)$$

and

$$p^{(2)} = p^{(1)} P$$

$$= \left(\frac{1}{12} \quad \frac{11}{12}\right) \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \left(\frac{11}{24}, \frac{13}{24}\right)$$

Ans

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Note:-

① A square matrix, in which the sum of all the elements of each row is 1, is called stochastic matrix. A stochastic matrix P is said to be regular if all the entries of P^m (for some positive integer m) are positive.

② A Markov chain is completely specified when the initial probability distribution and the tpm are given.

③ A state i of a markov chain is said to be an absorbing state if $P_{ii} = 1$ i.e. if it is impossible to leave it.

④ A Markov chain is said to be absorbing if it has at least one absorbing state.

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