Reg. No.															

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, DECEMBER 2023

Fourth Semester

21MAB204T – PROBABILITY AND QUEUEING THEORY

(For the candidates admitted from the academic year 2021 – 2022 & 2022-2023)

(Data books, tables, graph sheet to be provided)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours

Max. Marks: 75

PART – A (20 × 1 = 20Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
1.	A random variable X has the following probability function. The value of k is	1	1	1	2

X	0	1	2	3	4
P(X)	k	2k	5k	7k	9k

(A) 2/24
(B) 21/24
(C) 7/12
(D) 1/24

2. If k is a constant (non random variable) then E(k) is — Marks 1, BL 1, CO 1, PO 2

(A) k
(B) 0
(C) 1
(D) k^2

3. If $E\left(X^2\right)=6, E(X)=2$ then $Var(X)$ is — Marks 1, BL 1, CO 1, PO 1

(A) 2
(B) 4
(C) 6
(D) 8

4. If F is the distribution function of a continuous random variable X and if a<b then P(a<X<b) — Marks 1, BL 1, CO 1, PO 2

(A) F(a)+F(a)
(B) aF(b)–bF(a)
(C) aF(b)+bF(a)
(D) F(b)–F(a)

5. A coin is tossed 300 times. Find the mean of the binomial distribution. — Marks 1, BL 2, CO 2, PO 1

(A) 100
(B) 75
(C) 300
(D) 150

6. The moment generating function of Poisson distribution is — Marks 1, BL 1, CO 2, PO 2

(A) $e^{\lambda\left(1-e^t\right)}$
(B) $e^{\lambda\left(e^t-1\right)}$
(C) $e^{\left(e^t-1\right)}$
(D) $e^{-\lambda\left(e^t-1\right)}$

7. The mean of exponential distribution is — Marks 1, BL 2, CO 2, PO 1

(A) λ
(B) λ/λ–t
(C) 1/λ
(D) $1/\lambda^2$

8. The normal curve is 1 2 2 2

(A) Very flat
(B) Bell shaped symmetrical about mean
(C) Very peaked
(D) Smooth

9. The conditional probability density function of X given Y is 1 2 3 1

(A) $f(x,y)f(x)$
(B) $f(x,y)f(y)$
(C) $\dfrac{f(x,y)}{f(x)}$
(D) $\dfrac{f(x,y)}{f(y)}$

10. The coefficient of correlation lies between 1 2 3 2

(A) -1 and 1
(B) 0 and 1
(C) 0
(D) -1

11. If X and Y have joint probability density function 1 1 3 2

$$f(x,y)=\begin{cases} k\,xy, 0<x<1, 0<y<1 \\ 0, \quad \text{otherwise} \end{cases}$$

Find the value of k.

(A) 2
(B) 1
(C) 4
(D) 3

12. The Karl Pearson's coefficient of correlation between X and Y is 1 2 3 1

(A) $r(x,y)=\dfrac{\operatorname{cov}(x,y)}{\sigma_x^2\sigma_y^2}$
(B) $r(x,y)=\dfrac{\operatorname{cov}(x,y)}{\sigma_x\sigma_y}$
(C) $r(x,y)=\dfrac{\operatorname{cov}^2(x,y)}{\sigma_x\sigma_y}$
(D) $r(x,y)=\dfrac{\operatorname{cov}^2(x,y)}{\sigma_x^2\sigma_y^2}$

13. What stands for 'c' in the queue model (a/b/c:d/e) 1 2 4 1

(A) Queue discipline
(B) System capacity
(C) Number of server
(D) Service time

14. The interval between two consecutive arrival of a Poisson process follows __________ distribution. 1 4 4 1

(A) Binomial
(B) Uniform
(C) Normal
(D) Exponential

15. The average waiting time of a customer in the system (M|M|1:∞|FIFO) model __________ 1 1 4 2

(A) $\dfrac{\mu}{\lambda+\mu}$
(B) $\dfrac{1}{\lambda-\mu}$
(C) $\dfrac{1}{\lambda+\mu}$
(D) $\dfrac{1}{\mu-\lambda}$

16. What is the mean of the Poisson process? 1 1 4 2

(A) λ
(B) λt
(C) λ/t
(D) t/λ

17. A stochastic matrix P is said to be regular matrix if all the entries of p^m for some m>0 are (A) Positive (B) Negative (C) Integer (D) Real	1	2	5	1
18. A state i is said to be non null persistent if its mean recurrence time list is (A) Finite (B) Infinite (C) Empty (D) 1	1	2	5	2
19. The state i is said to be transient if the return to state i is uncertain if (A) $F_{ii}=0$ (B) $F_{ii}=1$ (C) $F_{ii}>1$ (D) $F_{ii}<1$	1	2	5	2
20. In Markov analysis, state probabilities must (A) Sum to zero (B) Sum to one (C) Less than one (D) Greater than one	1	2	5	3

PART – B (5 × 8 = 40 Marks)

Answer **ALL** Questions

	Marks	BL	CO	PO
21. a. A fair die is tossed 720 times. Use Tchebycheff inequality to find a lower bound for the probability of getting 100 to 140 sixes.	8	4	1	2

(OR)

	Marks	BL	CO	PO
b. A discrete random variable X has the following probability distribution.	8	4	1	2

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Find the value of a
(ii) Find $P(X<3)$
(iii) Find $P(0<X<3)$
(iv) Find the distribution function of X

	Marks	BL	CO	PO
22. a. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and the variance of the distribution.	8	3	2	1

(OR)

	Marks	BL	CO	PO
b. Fit a binomial distribution for the following data. Find the parameters of the distribution.	8	4	2	2

x	0	1	2	3	4	5	6	Total
f	5	18	28	12	7	6	4	80

	Marks	BL	CO	PO
23. a. The joint probability distribution of the random variables X and Y is given below.	8	3	3	1

X \ Y	1	2	3	4	5	6
0	0	0	2k	4k	4k	6k
1	4k	4k	8k	8k	8k	8k
2	2k	2k	k	k	0	2k

Find, (i) k (ii) the marginal probability distribution of X and Y (iii) $P(X\leq 1 \mid Y=2)$.

(OR)

								Marks	BL	CO	PO

b. Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y). — 8 4 3 2

X	22	26	29	30	31	31	34	35
Y	20	20	21	29	27	24	27	31

24. a. A departmental store has single cashier. During the rush hours, customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate the following, — 8 3 4 2
   (i) What is the probability that the cashier is idle?
   (ii) What is the average number of customers in the queuing system
   (iii) What is the average time a customer spends in the system
   (iv) What is the average number of customers in the queue?

(OR)

b. There are 3 typists in an office. Each typist can type an average of 6 letter per hour. If letters arrive for being typed at the rate of 15 letters per hour — 8 4 4 1
   (i) What is the average number of letters waiting to be typed?
   (ii) What is the average time a letter has to speed waiting and for being typed?

25. a. A college study X has the following study habits. If he studies one night, he is 70% sure not to study the next night. If he does not study one night, he is only 60% sure not to study the next night also. Find the transition probability matrix and how often he studies in the long run. — 8 3 5 2

(OR)

b. The transition probability matrix of a Markov chain $\{X_n\}$, n=1,2,...having 3 states 1,2,3 is $P=\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $p^{(0)}=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ — 8 4 5 2

Find, (i) $P(X_2=2, X_1=1, X_0=2)$ (ii) $P(X_3=1, X_2=2, X_1=1, X_0=2)$.

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Questions

	Marks	BL	CO	PO

26. Three boys A, B, C are throwing a ball to each other. A always throw the ball to B and B always throws to C but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. — 15 3 5 1

27. Marks obtained by 10 students in mathematics (X) and statistics (Y) are given below. — 15 4 3 1

X	60	34	40	50	45	40	22	43	42	64
Y	75	32	33	40	45	33	12	30	34	51

Find the regression lines.

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