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B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, NOVEMBER 2024

Fourth Semester

21MAB204T - PROBABILITY AND QUEUEING THEORY

(For the candidates admitted from the academic year 2021-2022 to 2023 - 2024)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 75

PART - A (20 x 1 = 20 Marks)
Answer ALL Questions

Marks BL CO PO

1. The cumulative distribution function of a random variable X is given as $F\left(x\right)=\begin{cases}\frac{x^3}{9}, -1<x<2\\ 0 \quad , otherwise\end{cases}$ Find the probability distribution function f(x). 1 1 1 1
   A) $\frac{x}{3},\ -1<x<2$
   B) $\frac{x}{2},\ -1<x<2$
   C) $\frac{x^2}{3},\ -1<x<2$
   D) $\frac{x^2}{2},\ -1<x<2$

2. Let the PDF of X be $f_X(x)$ and let $Y=g(x)$ be the given transformation. If $g(x)$ is strictly monotonic then the PDF of Y is $f_{Y(y)}=$ 1 1 1 1
   A) $\frac{dy}{dx}\ f_X(x)$
   B) $\frac{dx}{dy}\ f_X(x)$
   C) $\frac{dx}{dy}\ \frac{1}{f_X(x)}$
   D) $\frac{dy}{dx}\ \frac{1}{f_X(x)}$

3. The MGF of a random variable X is $M_X(t)=\frac{2}{2-t}$. Find the mean of X. 1 2 1 1
   A) 2
   B) 4
   C) $\frac{1}{2}$
   D) $\frac{1}{4}$

4. Variance of a random variable X is given by $\mu_2=$ 1 1 1 1
   A) $E(X^2)-E(X)$
   B) $E(X^2)^2-E(X)$
   C) $E(X^2)$
   D) $E(X^2)-(E(X))^2$

5. If the parameters n and p of a binomial distribution are 4, $\frac{1}{3}$ respectively, find the MGF. 1 2 2 1
   A) $\left(\frac{2}{3}+\frac{1}{3}e^t\right)^4$
   B) $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^4$
   C) $\left(\frac{2}{3}-\frac{1}{3}e^t\right)^4$
   D) $\left(\frac{1}{3}-\frac{2}{3}e^t\right)^4$

6. If X_1 and X_2 are 2 independent Poisson variates with parameters λ_1 and λ_2 respectively then X_1+X_2 is also a Poisson variate with parameter ______ 1 1 2 1
   A) $2(\lambda_1+\lambda_2)$
   B) $\lambda_1-\lambda_2$
   C) $\lambda_1+\lambda_2$
   D) $2(\lambda_1-\lambda_2)$

7. If the parameter of an exponential distribution is $\theta>0$, then its mean is _________ 1 1 2 1
   A) $\frac{2}{\theta}$
   B) $\frac{1}{\theta}$
   C) $\frac{1}{\theta^2}$
   D) $\frac{2}{\theta^2}$

8. If $Z=\frac{X-\mu}{\sigma}$, then $E(Z)$ and $Var(Z)$ are ______, ______ respectively. 1 1 2 1
   A) $1,0$
   B) $1,1$
   C) $0,1$
   D) $2,1$

9. If (X,Y) is a two dimensional discrete random variable, then $p(x/_{y=2}) =$ A) $\frac{p(x,2)}{p_Y(2)}$ B) $\frac{p(2,x)}{p_Y(2)}$ C) $\frac{p(x,2)}{p_X(2)}$ D) $\frac{p(2,x)}{p_X(2)}$	1 1 3 1	
10. $Cov(X+a, Y+b) =$ A) $a\, Cov(X,Y)$ B) $b\, Cov(X,Y)$ C) $Cov(X,Y)$ D) $a\, b\, Cov(X,Y)$	1 1 3 1	
11. If the regression coefficient $r =$ _______, then the regression lines are perpendicular. A) 1 B) -1 C) ± 1 D) 0	1 2 3 1	
12. The regression coefficient $r =$ _______ A) $\pm\, b_{yx}\, b_{xy}$ B) $\pm\sqrt{b_{yx}\, b_{xy}}$ C) $(b_{yx}\, b_{xy})^2$ D) $b_{yx}\, b_{xy}$	1 1 3 1	
13. Consider an $(M/M/1):(\infty/FIFO)$ queueing system. If $\lambda = 6$ and $\mu = 8$, find the expected waiting time of customer in the queue if he has to wait. A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{3}{4}$ D) $\frac{1}{2}$	1 2 4 1	
14. In an $(M/M/s):(\infty/FIFO)$ model, find the probability of the serving facilities being busy. A) $\frac{\lambda}{\mu s}$ B) $\frac{\lambda}{\mu}$ C) $\frac{s\lambda}{\mu}$ D) $\frac{2\lambda}{\mu s}$	1 2 4 1	
15. In an $(M/M/1):(\infty/FIFO)$ model, the probability that a customer is idle is given as _______ A) $\frac{\lambda}{\mu}$ B) $1-\frac{\lambda}{\mu}$ C) $\left(\frac{\lambda}{\mu}\right)^2$ D) $1-\left(\frac{\lambda}{\mu}\right)^2$	1 1 4 1	
16. In an $(M/M/1):(k/FIFO)$ model, if $\lambda = \mu$ then $E(N_s) =$ _______ A) $\frac{k}{2}$ B) $\frac{1}{2k}$ C) $2k$ D) $\frac{k}{4}$	1 1 4 1	
17. Consider a Markov chain with the state space $(0,1)$ and transition probability matrix $P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ Find $F_{ii} = \sum_{n=1}^{\infty} f_{ii}^{(n)}$ A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$	1 2 5 1	
18. Identify the one-step transition probability matrix. A) $\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ B) $\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 0 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ D) $\begin{pmatrix} 0 & \frac{1}{2} \\ 1 & 0 \end{pmatrix}$	1 1 5 1	

19. According to Chapman Kolmogorov's theorem 1 1 5 1

A) $[p_{ij}^{(n)}] = [p_{ij}]$ B) $[p_{ij}^{(n)}] = [p_{ij}]^n$

C) $[p_{ij}^{(n)}] = [p_{ij}]^2$ D) $[p_{ij}] = [p_{ij}]^2$

20. In a Markov chain with state space (1, 2, 3), $P(X_2 = 1, X_1 = 2, X_0 = 3) =$ _______ 1 2 5 1

A) $p_{32}^{(1)} p_{21}^{(1)} P(X_0 = 2)$ B) $p_{32}^{(1)} p_{21}^{(1)} P(X_0 = 3)$

C) $p_{32}^{(1)} p_{12}^{(1)} P(X_0 = 3)$ D) $p_{23}^{(1)} p_{21}^{(1)} P(X_0 = 2)$

PART - B (5 x 8 = 40 Marks)
Answer ALL Questions

Marks BL CO PO

21 a. A random variable X has the following probability distribution. 8 3 1 2

x	15	20	25	30
$p(x)$	$7k$	$21k$	$14k$	$2k$

Find (i) the value of k (ii) the cumulative distribution function of X (iii) the mean of X (iv) $P(X \leq 30 | X > 20)$.

(OR)

b. An unbiased die is rolled 720 times. Find a lower bound for the probability of getting 100 to 140 sixes using Chebycheff's inequality. 8 3 1 2

22 a. The length of time a person speaks over phone follows an exponential distribution with mean 6 minutes. What is the probability that the person will speak for (i) more than 8 minutes? (ii) between 4 and 8 minutes? (iii) If the person speaks more than 8 minutes what is the probability that he speaks atleast one more minute? 8 3 2 2

(OR)

b. If X is a normal random variable with $\mu = 3$ and standard deviation $\sigma = 4$, find (i) $P(X < 1)$ (ii) $P(X > -1)$ and (iii) $P(2 < X < 7)$. 8 3 2 2

23 a. From the following joint distribution of X and Y find (i) the marginal distributions of X and Y (ii) the covariance of X and Y 8 3 3 2

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$
$Y = 1$	$\frac{6}{28}$	$\frac{6}{28}$	0
$Y = 2$	$\frac{1}{28}$	0	0

(OR)

b. If the joint probability density function of (X, Y) is $f(x, y) = \begin{cases} \frac{1}{4}, & 0 < x < 2;\ 0 < y < 2 \\ 0, & otherwise \end{cases}$ find $P\left(X < \frac{1}{2}\ Y < 1\right)$ 8 3 3 2

24 a. A super market has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Find (i) the average number of customers in the queue (ii) average time a customer spends in the system (iii) the probability that the number of customers in the system is more than 5 (iv) Average length of the queue that forms from time to time. 8 3 4 2

(OR)

b. A petrol pump has 2 pumps. The service times follow the exponential distribution distribution and cars are serviced at the rate of 15/ hr. The cars arrive for service on a Poisson process at the rate of 10 cars/hr. Find the probability that a customer has to wait for service. 8 3 4 2

25 a. Three girls G_1, G_2, G_3 are throwing a ball to each other. G_1 always throws the ball to G_2 and G_2 always throws to G_3 but G_3 is just as likely to throw the ball to G_2 as to G_1. (i) Find the transition probability matrix. (ii) In the long run how often does each throw the ball to the others. 8 3 5 2

(OR)

b. Let $\{X_n : n = 1, 2, \ldots\}$ be a Markov chain with state space $S = \{1, 2, 3\}$ and one -step transition probability $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}$. Classify the states of the Markov chain. 8 3 5 2

PART - C (1 x 15 = 15 Marks)
Answer ANY ONE Question

Marks BL CO PO

26. i) A machine manufacturing screws is known to produce 5% defective . In a random sample of 15 screws what is the probability that there are (i) exactly 3 defectives (ii) not more than 3 defectives. 15 3 2 2

(5 marks)

ii) Fit a Poisson distribution to the given data and find the expected frequencies.

(10 marks)

x	0	1	2	3	4	5
f	142	156	69	27	5	1

27. A one-man barber shop can accommodate a maximum of 5 people at a time, 4 waiting and 1 getting hair cut. Customers arrive following Poisson distribution with an average of 5 per hour and service is rendered according to exponential distribution at an average rate of 4 per hour. 15 4 5 2

Find (i) the percentage of idle time? (ii) the probability of a potential customer turned away. (iii) the expected number of customers in the queue and (iv) the expected time spent in the shop.

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