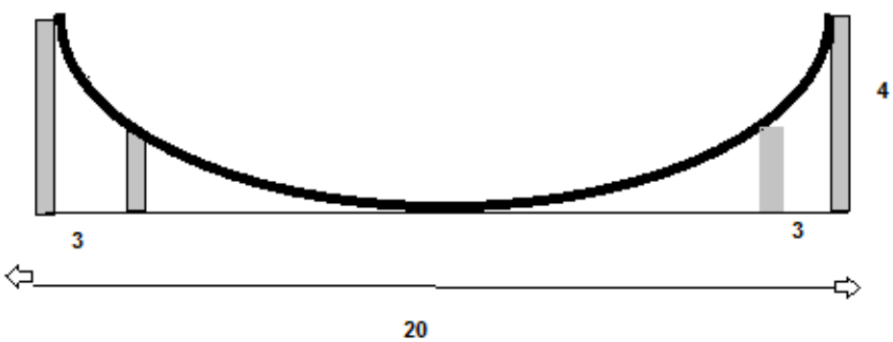
Formativa 1: Cálculo I MOD2 220157

1. Determine la ecuación principal de la elipse usando completación de cuadrados y obtenga centro, vértices, focos excentricidad y longitud lado recto. Grafique.

$$9x^2 + 13y^2 + 18x + 208y + 724 = 0.$$

2. Se construye una plataforma de skate con forma de arco semielíptico de 20 metros lineales de largo y una profundidad de 4 metros. Para su construcción se consideran 4 pilares, 2 en los extremos y dos postes interiores ubicados a tres metros de éstos, como se muestra en la figura. Determinar la altura de los postes interiores



3. Resuelva

   a) $\lim\limits_{x\to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3} \right)$

   b) $\lim\limits_{x\to 0} \dfrac{1-\sqrt[3]{1+x}}{1-\sqrt{1+x}}$

   c) $\lim\limits_{x\to +\infty} \dfrac{6x+1}{\sqrt{3x^2-7}}$

   d) $\lim\limits_{x\to -\infty} (\sqrt{x^2+x}+x)$

4. Encontrar las asíntotas verticales, horizontal u oblicuas, si existen, de la función

$$f(x) = \frac{2x^4 - 8x^3 - 10x^2}{6x^3 - 6x^2 - 12x}.$$

1. Determine la ecuación principal de la elipse usando completación de cuadrados y obtenga centro, vértices, focos excentricidad y longitud lado recto. Grafique.

$$9x^2 + 13y^2 + 18x + 208y + 724 = 0.$$

$9x^2 + 18x + 13y^2 + 208y + 724 = 0$

$9(x^2 + 2x) + 13(y^2 + 16y) + 724 = 0$

$9(x^2 + 2x + 1) - 9 + 13(y^2 + 16y + 64) - 832 + 724 = 0$

$9(x+1)^2 + 13(y+8)^2 - 9 - 832 + 724 = 0$

$9(x+1)^2 + 13(y+8)^2 = 117 \quad / 117$

$\dfrac{(x+1)^2}{13} + \dfrac{(y+8)^2}{9} = 1$

$C(h,k) = (-1,-8)$

$A_1(h+a,k),\ A_2(h-a,k) = A_1(-1+\sqrt{13}, -8),\ A_2(-1-\sqrt{13}, -8)$

$B_1(h, k+b),\ B_2(h, k-b) = B_1(-1,-5),\ B_2(-1,-11)$

$F_1(h+c,k),\ F_2(h-c,k) : F_1(1,-8),\ F_2(-3,-8)$

$e = \dfrac{c}{a} = \dfrac{2}{\sqrt{13}}$

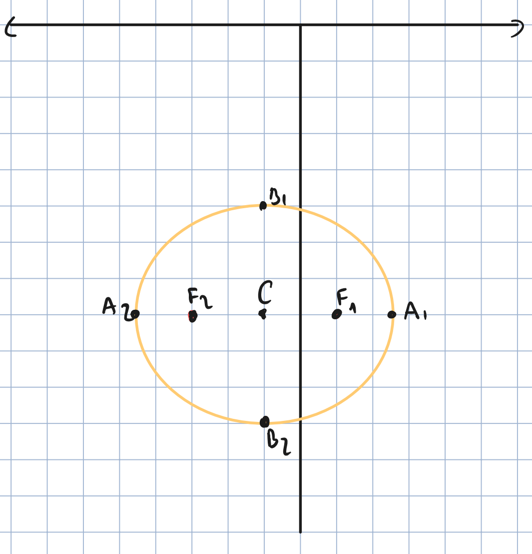
$LR = \dfrac{2b^2}{a} = \dfrac{18}{\sqrt{13}}$

$a^2 = 13,\ a = \sqrt{13}$

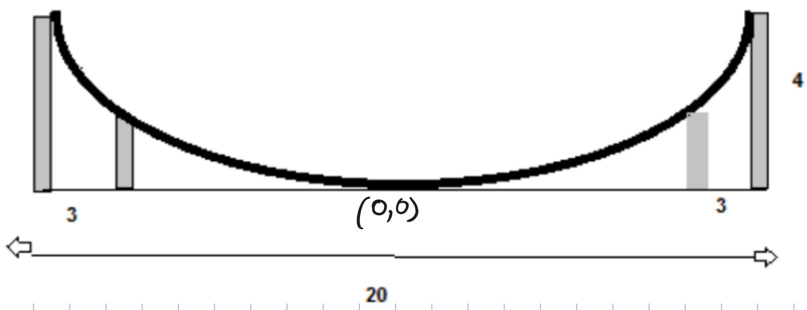
$b^2 = 9 \quad b = 3$

$c = \sqrt{a^2 - b^2} = 2$

Grafica:



2. Se construye una plataforma de skate con forma de arco semielíptico de 20 metros lineales de largo y una profundidad de 4 metros. Para su construcción se consideran 4 pilares, 2 en los extremos y dos postes interiores ubicados a tres metros de éstos, como se muestra en la figura. Determinar la altura de los postes interiores



largo $= 20$, $2a = 20$, $a = 10$
profundidad $= 4$, por tanto $b = 4$
$C(0,4)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{100} + \frac{(y-4)^2}{16} = 1$$

$$\frac{(10-3)^2}{100} + \frac{(y-4)^2}{16} = 1$$

$$\frac{(y-4)^2}{16} = 1 - \frac{7^2}{100}$$

$$\frac{(y-4)^2}{16} = \frac{51}{100}$$

$$(y-4)^2 = \frac{51}{100} \cdot 16$$

$$(y-4)^2 = \frac{204}{25}$$

$$y - 4 = \pm\sqrt{8{,}16}$$

$$y = \pm\sqrt{8{,}16} + 4$$

$y_1 = -1{,}14$
$y_2 = 6{,}9$

3. Resuelva

a) $\lim\limits_{x\to 1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right)$

b) $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1+x}}{1-\sqrt{1+x}}$

c) $\lim\limits_{x\to +\infty}\dfrac{6x+1}{\sqrt{3x^2-7}}$

d) $\lim\limits_{x\to -\infty}(\sqrt{x^2+x}+x)$

a) $$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$$

$$(-x^3+1) = -(x^3-1) = -(x-1)(x^2+x+1)$$

$$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{(1-x)(1^2+x+x^2)}\right)$$

$$\lim_{x\to 1}\frac{-(x-1)(x^2+x+1)-3(x-1)}{-(x-1)\,(x-1)(x^2+x+1)}$$

$(x-1)^2$

$$\lim_{x\to 1}\frac{(x^3-1)-3x+3}{-(x-1)^2(x^2+x+1)}$$

$$\lim_{x\to 1}\frac{x^3-3x+2}{-(x-1)^2(x^2+x+1)} \longrightarrow (x-1)^2(x+2)$$

$$\lim_{x\to 1}\frac{(x-1)^2(x+2)}{-(x-1)^2(x^2+x+1)}$$

$$\lim_{x\to 1}-\frac{x+2}{x^2+x+1} = -\frac{3}{3} = -1$$

b) $$\lim_{x\to 0}\frac{1-\sqrt[3]{1+x}}{1-\sqrt{1+x}} \quad \cdot\ \frac{1+\sqrt[3]{1+x}+\sqrt[3]{1+x}^{\,2}}{1+\sqrt[3]{1+x}+\sqrt[3]{1+x}^{\,2}}\cdot\frac{1+\sqrt{1+x}}{1+\sqrt{1+x}}$$

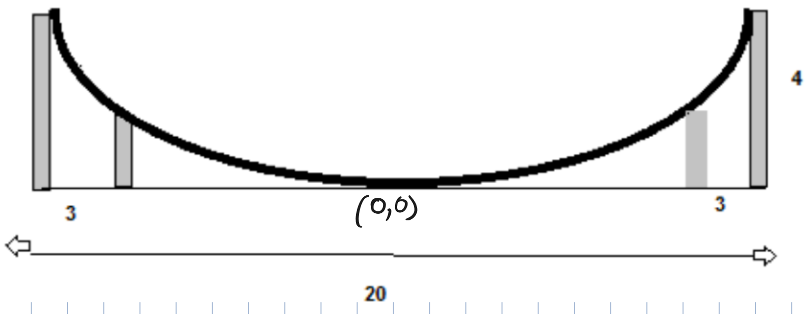
$$\lim_{x\to 0}\frac{(1^3-\cancel{\sqrt[3]{1+x}}^{\,\cancel{3}})\cdot(1+\sqrt{1+x})}{(1^2-\sqrt{1+x}^{\,2})(1+\sqrt[3]{1+x}+\sqrt[3]{1+x}^{\,2})}$$

$$\lim_{x\to 0}\frac{(1-(1+x))(1+\sqrt{1+x})}{(1-(1+x))(1+\sqrt[3]{1+x}+\sqrt{1+x}^{\,2})}$$

$$\lim_{x\to 0}\frac{\cancel{-x}\,(1+\sqrt{1+x})}{\cancel{-x}\,(1+\sqrt[3]{1+x}+\sqrt{1+x}^{\,2})}$$

$$=\frac{2}{3}$$

2. Se construye una plataforma de skate con forma de arco semielíptico de 20 metros lineales de largo y una profundidad de 4 metros. Para su construcción se consideran 4 pilares, 2 en los extremos y dos postes interiores ubicados a tres metros de éstos, como se muestra en la figura. Determinar la altura de los postes interiores



Largo = 20 , 2a = $\quad 2a = 20$, $b = 4$

$a = 10$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{100} + \frac{(y-4)^2}{16} = 1$$

$$\frac{(10-3)^2}{100} + \frac{(y-4)^2}{16} = 1$$

$$\frac{49}{100} + \frac{(y-4)^2}{16} = 1$$

$$\frac{y-4^2}{16} = 1 - \frac{49}{100}$$

$$\frac{(y-4)^2}{16} = \frac{51}{100}$$

$$(y-4)^2 = \frac{204}{25}$$

$$y-4 = \pm\sqrt{\frac{204}{25}}$$

$$y = \pm\sqrt{\frac{204}{25}} + 4$$

$y_1 = 6{,}8$

$y_2 = 1{,}12$

$$\lim_{x \to 7} \frac{x-7}{3x^2-21x} = \frac{0}{0}$$

$$\lim_{x \to 7} \frac{\cancel{x-7}}{3x\cancel{(x-7)}} = \frac{1}{3x} = \frac{1}{27}$$

$$\lim_{x \to -3} \frac{5x^2+15x}{x+3} = \frac{0}{0}$$

$$\lim_{x \to -3} \frac{5x\cancel{(x+3)}}{\cancel{x+3}} = 5x = -15$$

$$\lim_{x \to -3} \frac{x^2+x-6}{x+3} = \frac{0}{0}$$

$$\lim_{x \to -3} \frac{(x-2)\cancel{(x+3)}}{\cancel{x+3}} = x-2 = -3-2 = -5$$

$$\lim_{x \to -1} \frac{5x^2+7x+2}{x+1} = \frac{5(-1)^2+7(-1)+2}{-1+1} = \frac{0}{0}$$

$$\lim_{x \to -1} \frac{(x+2/5)\cancel{(x+1)}}{\cancel{x+1}} = x+\frac{2}{5} = -1+\frac{2}{5} = \frac{-3}{5}$$

$$\lim_{x \to 3} \frac{x-3}{3x^2-5x-12} = \frac{0}{0}$$

$$\lim_{x \to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(4x-3)} = \frac{1}{3x-3}$$

$$\lim_{x \to 3} \frac{1}{3x-3} = \frac{1}{9}$$

$$b)\ \lim_{x\to 0} \frac{1-\sqrt[3]{1+x}}{1-\sqrt{1+x}}$$

1) $= \frac{0}{0}$

2) $\frac{1-\sqrt[3]{1+x}}{1-\sqrt{1+x}} \cdot \frac{1+\sqrt{1+x}}{1+\sqrt{1+x}}$

$(1-\sqrt{1+x})\cdot(1+\sqrt{1+x})$

$1-(1+x)$

$1-(1+x)$

$-x$

$$\frac{(1-\sqrt[3]{1+x} \cdot 1+\sqrt[3]{1+x}) \cdot 1 + \sqrt{1+x}}{-x\cdot(1+\sqrt[3]{1+x})}$$

$$\frac{(1-\sqrt[3]{1+x})\cdot(1-\sqrt[3]{1+x})}{-x\cdot(1+\sqrt[3]{1+x})} = 1-2\sqrt[3]{1+x} = -1$$

$$\frac{-x+\sqrt{1+x}}{} = \frac{1}{2} \neq \frac{2}{3}$$

$$\lim_{x \to \infty} \frac{5}{\infty} = 0$$

$$\lim_{x \to \infty} \frac{10}{\infty^2} = 0$$

$$\lim_{x \to \infty} \frac{x}{12} = \infty$$

$$\lim_{x \to \infty} \frac{3\cancel{x^2}}{5\cancel{x^2}} = \frac{3}{5}$$

$$\lim_{x \to \infty} \frac{-3x^4}{x^2} = -3x^2 = -\infty$$

$$\lim_{x \to \infty} \frac{2x^5}{x^5} = 2$$

$$\lim_{x \to \infty} \frac{3x}{x^2} = \frac{3}{x} = 0$$

$$\lim_{x \to \infty} \frac{5x^3}{x} = 5x^2 = \infty$$

$$\lim_{x \to \infty} \frac{250}{\infty^2} = 0$$

$$\lim_{x \to \infty} \frac{x^3}{12} \quad \infty$$

$$\lim_{x \to \infty} \frac{2x^6}{3x^6} = \frac{2}{3}$$

$$\lim_{x \to \infty} \frac{x}{x} = 0$$

1) $$\lim_{x \to \infty} \frac{5x^2 - 3x}{2 + x + 10x^2} =$$

$$\lim_{x \to \infty} \frac{\frac{5x^2}{x^2} - \frac{3x}{x^2}}{\frac{2}{x^2} + \frac{x}{x^2} + \frac{10x^2}{x^2}}$$

$$\lim_{x \to \infty} \frac{5 - \cancel{\frac{3}{x}}^{0}}{\cancel{\frac{2}{x^2}}^{0} + \cancel{\frac{1}{x}}^{0} + 10} \quad \frac{5}{10} = \frac{1}{2}$$

$$\frac{\infty}{x} = \infty$$

$$\frac{x}{\infty} = 0$$

$$\frac{x}{0} = \infty$$

$$\lim_{x \to \infty} \frac{3x^3 + 5x^2 + 2x^3}{5x^3 - 3x^2 + 2x - 1}$$

$$\lim_{x \to \infty} \frac{5x^3 + 5x^2}{5x^3 - 3x^2 + 2x - 1}$$

$$\lim_{x \to \infty} \frac{\frac{5x^3}{x^3} + \frac{5x^2}{x^3}}{\frac{5x^3}{x^3} - \frac{3x^2}{x^3} + \frac{2x}{x^3} - \frac{1}{x^3}}$$

$$\lim_{x \to \infty} \frac{5 + \cancel{\frac{5}{x}}^{0}}{5 - \cancel{\frac{3}{x}}^{0} + \cancel{\frac{2}{x^2}}^{0} - \cancel{\frac{1}{x^3}}^{0}} \quad \frac{5}{5} = 1$$

2) $$\lim_{x \to \infty} \frac{2x + 3}{3x + 1}$$

$$\lim_{x \to \infty} \frac{\frac{2x}{x} + \frac{3}{x}}{\frac{3x}{x} + \frac{1}{x}}$$

$$\lim_{x \to \infty} \frac{2 + \cancel{\frac{3}{x}}^{0}}{3 + \cancel{\frac{1}{x}}^{0}} = \frac{2}{3}$$

1) $$\lim_{x \to \infty} \frac{3x^2 + 5x^3}{2x^2 - 3x}$$

$$\lim_{x \to \infty} \frac{\frac{3x^2}{x^3} + \frac{5x^3}{x^3}}{\frac{2x^2}{x^3} - \frac{3x}{x^3}}$$

$$\lim_{x \to \infty} \frac{\cancel{\frac{3}{x}}^{0} + 5}{\cancel{\frac{2}{x}}^{0} - \cancel{\frac{3}{x^2}}^{0}} = \frac{5}{0} = \infty$$

2) $$\lim_{x \to \infty} \frac{4x^5 - 2x^3 + 8x^2 - 6}{2x^4 - 3x^5 + 7x^6}$$

$$\lim_{x \to \infty} \frac{\frac{4x^5}{x^6} - \frac{2x^3}{x^6} + \frac{8x^2}{x^6} - \frac{6}{x^6}}{\frac{2x^4}{x^6} - \frac{3x^5}{x^6} + \frac{7x^6}{x^6}}$$

$$\lim_{x \to \infty} \frac{\cancel{\frac{4}{x}}^{0} - \cancel{\frac{2}{x^3}}^{0} + \cancel{\frac{8}{x^4}}^{0} - \cancel{\frac{6}{x^6}}^{0}}{\cancel{\frac{2}{x^2}}^{0} - \cancel{\frac{3}{x}}^{0} + 7} = \frac{0}{7} = 0$$

3) $$\lim_{x \to \infty} \frac{2x}{3x - 5x^3}$$

$$\lim_{x \to \infty} \frac{\frac{2x}{x^3}}{\frac{3x}{x^3} - \frac{5x^3}{x^3}}$$

$$\lim_{x \to \infty} \frac{\cancel{\frac{2}{x^2}}^{0}}{\cancel{\frac{3}{x^2}}^{0} - 5} = \frac{0}{-5} = 0$$

4) $$\lim_{x \to \infty} \frac{x^6 - 2x^7 - 5x^8}{3x^6 + 2x^2 - 5x + 1}$$

$$\lim_{x \to \infty} \frac{\frac{x^6}{x^8} - \frac{2x^7}{x^8} - \frac{5x^8}{x^8}}{\frac{3x^6}{x^8} + \frac{2x^2}{x^8} - \frac{5x}{x^8} + \frac{1}{x^8}}$$

$$\lim_{x \to \infty} \frac{\cancel{\frac{1}{x^2}}^{0} - \cancel{\frac{2}{x}}^{0} - 5}{\cancel{\frac{3}{x^2}}^{0} + \cancel{\frac{2}{x^6}} - \cancel{\frac{5}{x^7}} + \cancel{\frac{1}{x^8}}} = \frac{-5}{0} = -\infty$$

1) $$\lim_{x \to \infty} \sqrt{\frac{9x^2 - 3x + 2}{4x^2 - x}}$$

$$\lim_{x \to \infty} \sqrt{\frac{\frac{9x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2}}{\frac{4x^2}{x^2} - \frac{x}{x^2}}}$$

$$\lim_{x \to \infty} \sqrt{\frac{9 - \cancelto{0}{\frac{3}{x}} + \cancelto{0}{\frac{2}{x^2}}}{4 - \cancelto{0}{\frac{1}{x}}}}$$

$$\lim_{x \to \infty} \sqrt{\frac{9}{4}} = \frac{3}{2}$$

2) $$\lim_{x \to \infty} \sqrt{\frac{16x^4 - 2x^3 + x^2 - 3}{x^2 - 3x^3 + x^4}}$$

$$\lim_{x \to \infty} \sqrt{\frac{\frac{16x^4}{x^4} - \frac{2x^3}{x^4} + \frac{x^2}{x^4} - \frac{3}{x^4}}{\frac{x^2}{x^4} - \frac{3x^3}{x^4} + \frac{x^4}{x^4}}}$$

$$\lim_{x \to \infty} \sqrt{\frac{16 - \cancelto{0}{\frac{2}{x}} + \cancelto{0}{\frac{1}{x^2}} - \cancelto{0}{\frac{3}{x^4}}}{\cancelto{0}{\frac{1}{x^2}} - \cancelto{0}{\frac{3}{x}} + 1}}$$

$$\lim_{x \to \infty} \sqrt{\frac{16}{1}} = \sqrt{16} = 4$$

3) $$\lim_{x \to \infty} \sqrt{\frac{27x^2 + 3x + 5}{-x^2 + x + 1}}$$

$$\lim_{x \to \infty} \sqrt{\frac{\frac{27x^2}{x^2} + \frac{3x}{x^2} + \frac{5}{x^2}}{\frac{-x^2}{x^2} + \frac{x}{x^2} + \frac{1}{x^2}}}$$

$$\lim_{x \to \infty} \sqrt{\frac{27 + \cancelto{0}{\frac{3}{x}} + \cancelto{0}{\frac{5}{x^2}}}{-1 + \cancelto{0}{\frac{1}{x}} + \cancelto{0}{\frac{1}{x^2}}}}$$

$$\lim_{x \to \infty} \sqrt{\frac{27}{-1}} = -\sqrt{27} = -3$$

4. Encontrar las asíntotas verticales, horizontal u oblicuas, si existen, de la función

$$f(x)=\frac{2x^4-8x^3-10x^2}{6x^3-6x^2-12x}.$$

$$f(x)=\frac{2x^4-8x^3-10x^2}{6x^3-6x^2-12x}$$

$$=\frac{\frac{2\cancel{x^4}}{\cancel{x^4}}-\frac{8x^3}{x^4}-\frac{10x^2}{x^4}}{\frac{6x^3}{x^4}-\frac{6x^2}{x^4}-\frac{-12x}{x^4}}$$

$$=\frac{2-\cancel{\frac{8}{x}}^{0}-\cancel{\frac{10}{x^2}}^{0}}{\cancel{\frac{6}{x}}^{0}-\cancel{\frac{6}{x^2}}^{0}-\cancel{\frac{-12}{x^3}}^{0}}=\frac{2}{0}=+\infty$$

$$f(x)=\frac{2x^4-8x^3-10x^2}{6x^3-6x^2-12x}=\frac{2x^2(x^2-4x-5)}{6x(x^2-x-2)}=\frac{2x^2(x^2-4x-5)}{6x(x^2-x-6}=\frac{x(x^2-4x-5)}{3(x^2-x-6)}$$

$$=\frac{x(x^2-4x-5)}{3(x^2-x-2)}=\frac{x(x-5)\cancel{(x+1)}}{3(x-2)\cancel{(x+1)}}=\frac{x(x-5)}{3(x-2)}$$

= asíntota vertical son los puntos no definidos, o del denominador

$3(x-2) > x=2$ (Restricción).

5) $$\frac{x(x-5)}{3(x-2)}=\frac{x^2-5x}{3x-6}\cdot\frac{1}{x}=\frac{(x^2-5x)\cdot(1)}{3x-6\cdot(x)}=\frac{x^2-5x}{3x^2-6x}$$

$$\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\cancel{\frac{5x}{x^2}}^{0}}{\frac{3x^2}{x^2}-\cancel{\frac{6x}{x^2}}^{0}}\qquad \cancel{\frac{5}{x}}^{0}$$

$f(x)-mx$

$$M=\lim_{x\to\infty}\frac{1}{3}\qquad \lim_{x\to\infty} n=\frac{x^2-5x}{3x-6}-\frac{1x}{3}=\frac{3(x^2-5x)-x(3x-6)}{3(3x-6)}=\cancel{3x^2}-15x-\cancel{3x^2}+6x$$

$$=\frac{-9x}{9x-18}=\frac{\frac{-9\cancel{x}}{\cancel{x}}}{\frac{9\cancel{x}}{\cancel{x}}-\cancel{\frac{18}{x}}^{0}}=\frac{-9}{9}=-1$$