



UNIVERSIDAD DEL BÍO-BÍO

FACULTAD DE CIENCIAS

DEPARTAMENTO DE MATEMÁTICA

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**Formativo N°2 - Álgebra I (220155) - MÓDULO 2**

Nombre: \_\_\_\_\_ Rut: ..... Fecha: .....

**Pregunta 1** Efectuar las siguientes operaciones y expresar el resultado en forma binómica.

a)  $\left(1 + \frac{1}{2}i\right) \cdot (-2 + i)$

$= -\frac{5}{2}$

b)  $\frac{1 - 4i}{\sqrt{2} - i}$

$\frac{\sqrt{2} + 4}{3} + \frac{4 - 4\sqrt{2}}{3}i$

c)  $\frac{1}{i} + \frac{3}{1+i} - \frac{(1-i)(2+i)}{3-i} + \left(\frac{1}{3} - 2i\right)$

d)  $i^{14} - i^9 + 3i^5 - i^3 + 1$

e)  $(3i - 2)^2 - \frac{1}{i^3} + \frac{1}{4}i(-\sqrt{2} + 5i) - 1$

**Pregunta 2** Hallar las soluciones en  $\mathbb{C}$  del siguiente sistema de ecuaciones:

$$\begin{cases} (1+i)x + (1+2i)y = 1+5i \\ (3-i)x + (4-2i)y = 2-i \end{cases}$$

$\begin{bmatrix} x & y \\ x & y \\ x & y \end{bmatrix} \begin{bmatrix} 1+5i \\ 2-i \end{bmatrix}$   $\Delta R =$   
 $\Delta x =$   
 $\Delta y =$

**Pregunta 3** Usando las representaciones en forma polar y sus propiedades, efectuar las siguientes operaciones, expresando el resultado final en forma binómica.

1.  $(-1 + \sqrt{3}i)(-3i)$

2.  $(-\sqrt{3} - i) : \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

3.  $(1 - 2i)^3$

4.  $(1 - i)^{47}$

5.  $(-\sqrt{3} - i)^{100}$

**Pregunta 4** Aplicar la fórmula de De Moivre para hallar:

a) las 3 raíces cúbicas de  $-i$ .

b) las 2 raíces cuadradas de  $-1 - \sqrt{3}i$

c) Las 4 raíces cuartas de  $-1$ .

$$a) \left(1 + \frac{1}{2}\right) \cdot (-2+i) = -2+i-i+\frac{1}{2}i^2$$

$$-\frac{2}{1} - \frac{1}{2} = \boxed{-\frac{5}{2}}$$

$$b) \frac{2-4i}{\sqrt{2}-i} \cdot \frac{\sqrt{2}+i}{\sqrt{2}+i} = \frac{\sqrt{2}+i-4i\sqrt{2}+4}{3} = \frac{4+\sqrt{2}-(4\sqrt{2}i+i)}{3} = \frac{4+\sqrt{2}+1-4\sqrt{2}i}{3} = \frac{5+\sqrt{2}}{3} + \frac{1-4\sqrt{2}i}{3}$$

$$c) \frac{1}{i} + \frac{3}{2+i} - \frac{(1-i)(2+i)}{3-i} + \left(\frac{1}{3} - 2i\right) = \frac{i}{-1} + \frac{3-3i}{2} - \frac{1}{1} + \frac{1}{3} - \left(\frac{6i}{3}\right) \rightarrow 2i$$

$$-i + \frac{3}{2} - \frac{3i}{2} - \frac{2}{3} - 2i = z = \frac{5}{6} - \frac{9i}{2} \quad \bar{z} = \frac{5}{6} + \frac{9i}{2}$$

$$d) i^{24} - i^9 + 3i^6 - i^3 + 1 = \cancel{-1} - \cancel{i} + 3i^5 \cancel{+ i} \cancel{+ 1}$$

$i = i$	$i^4 = i$	$i$	
$i^2 = -1$	$i^8 = -1$	$-1$	$3i$
$i^3 = -i$	$i^{12} = -i$	$-i$	
$i^4 = 1$	$i^{16} = 1$	$1$	
$i^5 = i$	$i^{20} = i$		
$i^6 = -1$	$i^{24} = -1$		
$i^7 = -i$			
$i^8 = 1$			

$$e) (3i-2)^2 - \frac{1}{i^3} + \frac{1}{4}i(-\sqrt{2}+5i) - 1 = -\frac{29}{4} + \left(-13 - \frac{\sqrt{2}}{4}i\right)$$

$$(3i-2)(3i-2) = \frac{9i^2-6i-6i+4}{4-12i-9} \left| \frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = i \right|$$

$$-5-12i - i + \frac{1}{4}i(-\sqrt{2}+5i) - 1$$

$$-5-12i-i-\frac{5}{4}-\frac{\sqrt{2}}{4}i-1$$

$$-6-13i-\frac{5}{4}-\frac{\sqrt{2}}{4}i = \frac{-6}{1} - \frac{5}{4} = \frac{-24-5}{4} = -\frac{29}{4} \quad \left| \frac{-13i}{1} - \frac{\sqrt{2}}{4}i = \left(-13 - \frac{\sqrt{2}}{4}i\right) \right|$$

$$\begin{cases} (1+i)x + (1+2i)y = 1+5i \\ (3-i)x + (4-2i)y = 2-i \end{cases}$$

$$x = \frac{1+5i - (1+2i)y}{2+i}$$

$$x = 1+5i - (y+2iy)$$

$$x = \frac{1+5i-y-2iy}{1+i} \cdot \frac{1-i}{1-i} =$$

$$x = \frac{1+5i-y-2iy-i-5i^2+iy+2i^2y}{2}$$

$$x = \frac{6+4i-y-iy-2y}{2}$$

$$x = \frac{6+4i-y-iy-2y}{2}$$

$$x = \frac{6-3y}{2} + \frac{4-y}{2}i$$

$$x = \frac{2-i - (4-2i)y}{3-i}$$

$$x = 2-i - (4y-2iy)$$

$$x = \frac{2-i-4y+2iy}{3-i} \cdot \frac{3+i}{3+i} =$$

$$x = \frac{6-3i-12y+6iy+i^2-4yi+2i^2y}{10}$$

$$x = \frac{7-14y}{10} + \frac{-1+2y}{10}i$$

igualar x's

$$\frac{6-3y}{2} + \frac{4-y}{2}i = \frac{7-14y}{10} + \frac{-1+2y}{10}i$$

$$\frac{6-3y}{2} - \frac{7-14y}{10} = \frac{-1+2y}{10} - \frac{4-y}{2}i$$

$$\frac{30-15y-7+14y}{10} = \frac{-1+2y-20+5y}{10} = \frac{-1+2y-20+5y}{10} = \frac{-21+7y}{10}$$

$$\frac{23+y}{10} = \frac{-21+7y}{10}$$

$$\frac{23-y}{10} = \frac{-41+12y}{20}$$

$$\frac{6-3y}{2} - \frac{7+14y}{10} = \frac{-1+2yi}{10} - \frac{4-y}{2} i$$

$$\frac{30-15y-7+14y}{10} = \frac{-1+2yi+20-5y}{10} i$$

$$\frac{6-3y}{2} - \frac{7+14y}{10} = \frac{-1+2yi}{10} - \frac{4-y}{2} i$$

$$60-30y-14+28y = -2i+4yi-40i+10yi$$

$$46-2y = \frac{-42i+14yi}{-2-14yi} \quad / -46-14yi$$

$$-2y-14yi = -46-42i$$

$$y(-2-14i) = -46-42i \quad / : -2-14i$$

$$y = \frac{-46-42i}{-2-14i} = \frac{-2+14i}{-2+14i}$$

$$y = \frac{47}{5} - \frac{14}{5}i$$

La solución por la regla de Cramer

$$\begin{cases} (1+i) \cdot x + (1+2i) \cdot y = 1+5i \\ (3-i) \cdot x + (4-2i) \cdot y = 1+5i \end{cases}$$

$$\Delta = \begin{vmatrix} 1+i & 1+2i \\ 3-i & 4-2i \end{vmatrix} = 1-3i$$

▼ Los detalles

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} (?)$$

$$\begin{vmatrix} 1+i & 1+2i \\ 3-i & 4-2i \end{vmatrix} = (1+i) \cdot (4-2i) - (1+2i) \cdot (3-i) = 1-3i$$

$$\Delta_1 = \begin{vmatrix} 1+5i & 1+2i \\ 2-i & 4-2i \end{vmatrix} = 10+15i$$

▼ Los detalles

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} (?)$$

$$\begin{vmatrix} 1+5i & 1+2i \\ 2-i & 4-2i \end{vmatrix} = (1+5i) \cdot (4-2i) - (1+2i) \cdot (2-i) = 10+15i$$

$$\Delta_2 = \begin{vmatrix} 1+i & 1+5i \\ 3-i & 2-i \end{vmatrix} = -5-13i$$

▼ Los detalles

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} (?)$$

$$\begin{vmatrix} 1+i & 1+5i \\ 3-i & 2-i \end{vmatrix} = (1+i) \cdot (2-i) - (1+5i) \cdot (3-i) = -5-13i$$

$$x = \Delta_1 / \Delta = \frac{10+15i}{1-3i} = \frac{-7+9i}{2}$$

$$y = \Delta_2 / \Delta = \frac{-5-13i}{1-3i} = \frac{17-14i}{5}$$

La respuesta:

$$x = \frac{-7+9i}{2}$$

$$y = \frac{17-14i}{5}$$

$$1. (-1 + \sqrt{3}i)(-3i)$$

$r_1: 2$   
 $r_2: 3$   
 $\rightarrow -\sqrt{3} = -\frac{\pi}{3}$   
 $= -\frac{\pi}{2}$

$$r_1 \cdot r_2 \cdot e^{i(\theta_1 + \theta_2)} = 6 \cdot e^{i(-\frac{5}{6}\pi)}$$

$$\frac{-\pi}{3} - \frac{\pi}{2} = \frac{-2\pi - 3\pi}{6} = \frac{-5\pi}{6}$$

$$6 \cdot e^{i(-\frac{5}{6}\pi)} = 6 \left( \cos(-\frac{5}{6}\pi) + i \cdot \sin(-\frac{5}{6}\pi) \right)$$

$$2. (-\sqrt{3} - i) : \left( -\frac{\sqrt{3}}{2} + \frac{1}{2}i \right)$$

$$3. (1 - 2i)^3$$

$$4. (1 - i)^{47}$$

$$5. (-\sqrt{3} - i)^{100}$$