ATENEO DE MANILA UNIVERSITY
Loyola Schools

Department of Physics

PHYS 33.02 Section LABEF

Experiment No. 1

"Mapping of Equipotential Lines"

Group Name

Group 5 - Jay

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II. Objectives

The experiment documented in this paper aimed to:

a. characterize the relative shapes of the electric field lines and equipotential lines observed between two conductors of specified geometries that are across a 12-V constant-voltage source and are connected to a rectangular resistive board measuring 14 cm × 20 cm, and
b. compare the shapes of the observed electric field lines and equipotential lines to those predicted by applying Coulomb's law and related equations via a numerical computer-based method.

III. Theoretical Discussion

In the study of electromagnetism, any two charged objects in the universe can interact with each other via the electromagnetic force. This force acts even without contact between charges, and it is divided into two components, namely the electric force $\mathbf{F_e}$ acting between any two charges, and the magnetic force $\mathbf{F_b}$ acting between two moving charges in space (Ulaby & Ravaioli, 2019; Young & Freedman, 2020). The forces that act on all charged particles are mediated by electric fields and magnetic fields, respectively, which are generated by all charges and moving charges in proximity to the test charge, that is, the charge being studied. These fields can interconvert and be intrinsically linked together in the presence of time-varying sources of charge. Under static conditions, that is, when all charges in a system experience zero acceleration, electric and magnetic fields occur independently of one another. Consequently, they can be studied as separate phenomena.

In systems containing charges under static conditions, each charge generates an electric field $\mathbf{E}_i$ that emanates throughout the surrounding space, with a direction dependent on the sign of the said charge. The electric field obeys the principle of linear superposition, that is, the net electric field $\mathbf{E_{net}}$ at any point is the vector sum of all individual electric fields generated by each of the n charges (Equation 1) (Young & Freedman, 2020):

$$\mathbf{E}_{\text{net}} = \sum \mathbf{E}_i = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots + \mathbf{E}_n \tag{1}$$

The dispersion and behavior of electric fields within a given material in static conditions are governed by three equations, namely Gauss's law (Equation 2a), the condition for irrotational vector fields (Equation 2b), and the Lorentz force law (Equation 2b) (Ulaby &

Ravaioli, 2019). As shown below, ρ_v represents the charge density per unit volume, q is the test charge affected by the electric field **E**, $\mathbf{F}_e$ represents the electrostatic force experienced by the test charge, $\nabla \cdot$ and $\nabla \times$ represent the divergence and curl operators on vector field **E**, respectively, and ε is the permittivity of the material or medium where **E** is set within.

(2a) $$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon}$$

(2b) $$\nabla \times \mathbf{E} = 0$$

(2c) $$\mathbf{F}_e = q\mathbf{E}$$

According to Equation 2a, in static conditions, the electric field diverges from or converges toward locations occupied by charges; whereas positive charges cause the electric field to point away from the source, negative charges cause the electric field to point towards said source. Moreover, the magnitude of an electric field around a charge is related to the charge density ρ_v within the volume of any closed surface enclosing the charge. Thus, for a single point charge Q and any point P in space, the magnitude $|\mathbf{E}|$ of the electric field can be shown to be directly proportional to the magnitude of the source charge $|Q|$ and inversely proportional to the square of the distance r between the point charge and P; this can be expressed as Coulomb's law as in Equation 3, which may be derived from Equation 2a by converting it to a surface integral and applying this surface integral to the case of a spherical imaginary surface around a point charge (Young & Freedman, 2020).

(3) $$|\mathbf{E}| = \frac{k\,|Q|}{r^2}, \qquad k = \frac{1}{4\pi\varepsilon}$$

Finally, the greater the material’s permittivity ε, the weaker the electric field that can be dispersed within it. In vacuum, air, and most electrical conductors, ε can be approximated as the permittivity of free space $\varepsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$ (Ulaby & Ravaioli, 2019). However, the same is not true for most insulating materials, as they often respond to and opposite external electric fields via polarization (Askeland, *et al.* 2010)

Both the relative magnitude and direction of the electric field, particularly in two-dimensional space, may be visualized as electric field lines (Young & Freedman, 2019), which are drawn tangent to the direction of the electric field at selected areas. For an electric dipole, that is, a pair of (+) and (–) charges, field lines are drawn from the positive charge to the negative charge (Figure 1a). Secondly, stronger electric fields are highlighted in areas where field lines are closer together, whereas weaker electric fields have more dispersed field lines. Around a point charge, for example, the field lines are spaced farther apart in areas that are at greater distances r from the charge, which reflects the inverse dependence of the magnitude of the electric field on the distance from the charge (Figure 1b). Finally, while different materials can affect the density of field lines depending on their permittivities, electric field lines are always perpendicular to boundaries between materials (Ulaby & Ravaioli, 2019).

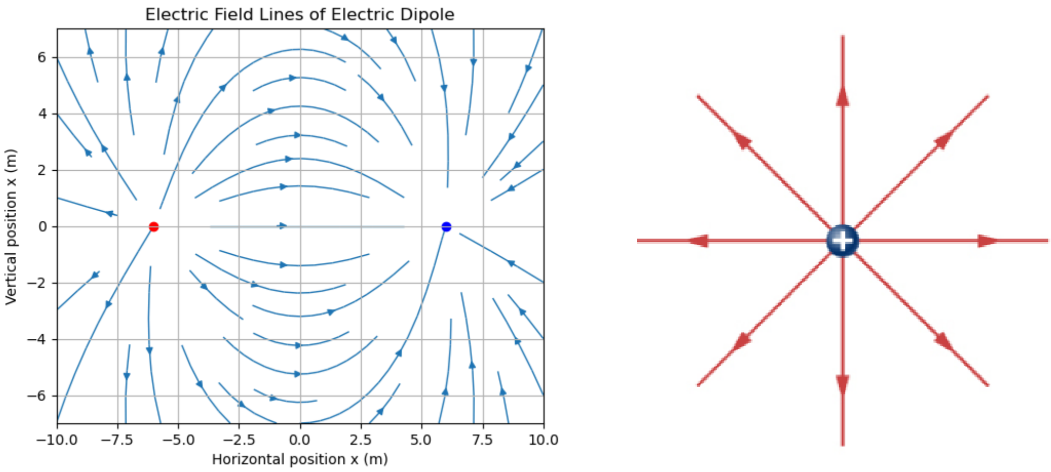

***Figure 1**. (a) Electric Field Lines of an Electric Dipole, with Red and Blue as Positive and Negative Charges, respectively* (*Sanguyo, 2024*)*; (b) Electric Field Lines of a Positive Test Charge* (*Ling, et al., 2016)*

Other aspects of the electric field are revealed through Equations 2b and 2c. For instance, Equation 2c states that an electric field **E** exerts an electrostatic force $\mathbf{F}_\mathbf{e}$ on any charged particle, with a direction parallel or antiparallel to **E** and a magnitude that is directly proportional to both the magnitude of **E** and the magnitude $|q|$ of the affected charge. More importantly, it can be proved from Equation 2b and Stokes' theorem that the electric field **E** is a conservative or irrotational vector field, that is, any line integral of **E** with differential length **dL** around a closed path is always zero, as is shown in Equation 4a. Another implication is that the line integral evaluated on any acyclic path only depends on the path's initial and final endpoints. The negative of this line integral can be defined as the change in electric potential or potential difference V and is measured in volts. The conservative property of static electric fields implies that they can be associated with doing work W on a charged particle and with a form of potential energy U_E. By combining the conservation of energy and Equations 2b and 2c, the potential difference V between point A to point B within the electric field **E** can be interpreted as (i) the difference in electric potential energy U_E per unit charge incurred from moving charges from A to B, and as (ii) the amount of work per unit charge that needs to be applied by a force **F** in opposition to **E** to move charges from A to B (Equation 4b).

(4a)
$$\Delta V = \oint \mathbf{E} \cdot \mathbf{dL} = 0$$

(4b)
$$\Delta V = \int_A^B \mathbf{E} \cdot \mathbf{dL} = \frac{1}{q}\int_A^B \mathbf{F} \cdot \mathbf{dL} = \frac{-W}{q} = \frac{\Delta U_E}{q}$$

The spatial distribution of the potential energy stored within an electric field may be visualized by drawing equipotential lines. By plotting the electric potential V on each position in two-dimensional space, equipotential lines may be drawn as curves where V remains constant. Unlike electric field lines, equipotential lines can form loops and generally encircle regions of charge, with each succeeding equipotential line being spaced farther away; one example can be drawn around an electric dipole, as shown in Figure 2. The shapes of these successive lines indicate that as one goes farther away from a region of charge, V increases away from negative charges, but decreases away from positive source charges. Note that the values of V are defined from a reference point called the ground, which is arbitrarily assigned a potential value of 0 volts. In Figure 2, for instance, V = 0 at infinite distances from both charges.

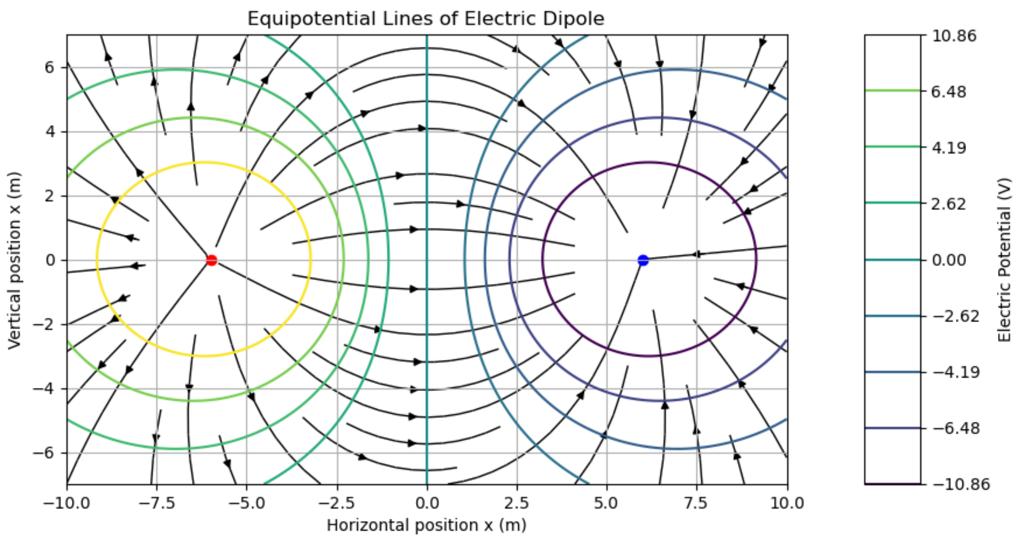

Figure 2. *Equipotential Lines and Electric Field Lines of an Electric Dipole, with Red and Blue as Positive and Negative Charges, respectively (Sanguyo, 2024)*

As shown in Figure 2, equipotential lines are always drawn perpendicular to electric field lines, and consequently become parallel when near boundaries between materials (Figure 2b). Conversely, the direction of the field lines indicates where the electric potential decreases most quickly. These observations are a consequence of the relationship that the electric field **E** is the gradient of electric potential V, meaning that **E** always indicates the direction and magnitude at any point which leads to the greatest infinitesimal decrease in V (Equation 5). This statement can be shown to be mathematically equivalent to Eqs. 2c and 4a, and is thus associated with the conservative property of **E**.

$$\mathbf{E} = \nabla V \tag{5}$$

While they serve as conceptual explanations for the effects of the electromagnetic force on charged particles, both electric and magnetic fields cannot be directly observed and measured. There are, however, indirect methods involving the measurement of electrostatic force $\mathbf{F}_e$, potential difference ΔV, and other related quantities which allows the electric field to be mapped or estimated from the above equations. One such method, which will be utilized throughout this experiment, involves a simple direct-current (DC) electrical circuit involving a resistor that covers a wide area. The circuit schematics are given in Figures 3a and 3b.

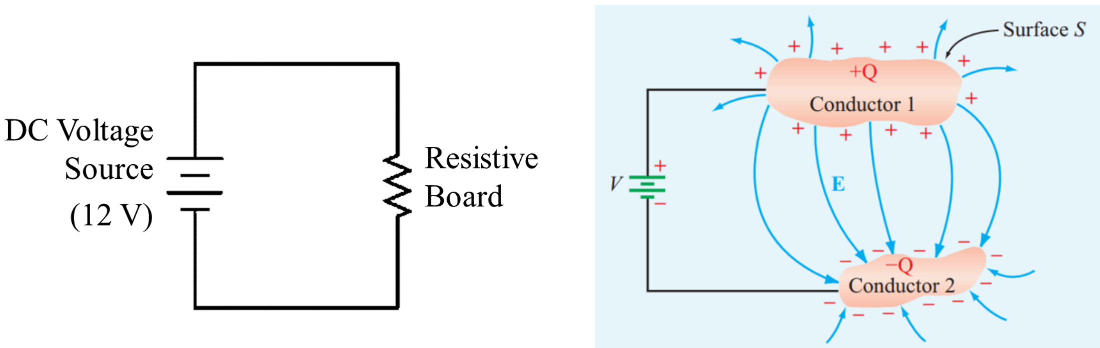

Figure 3. *(a) Circuit Schematic of a 12 V DC Circuit and a Resistive Board; (b) An electric field is generated between the two conductors of the resistive board (Ulaby & Ravaioli, 2019)*

For context, electricity can be viewed as an emergent phenomenon arising from the interaction of an applied electric field **E** and the free, delocalized, conducting electrons between immobile, positively-charged cations within a conducting metal (Sengupta & Wilensky, 2009). Because this electric field exerts a force and does work on each free electron (Equations 2c and 4b), (i) a net flow of electrons or electric current I can occur as a result of the collective motion of the particles, although (ii) these electrons can still lose their energy by colliding with immobile cations, giving rise to electrical resistance R. Note that quantities I and R are measured in amperes and ohms, respectively. This occurrence, where an electric field gives rise to electric current, is what runs batteries and electric generators. In either case, a potential difference ΔV is generated between the two terminals of either device. Note however, that under static conditions, no electron can accelerate and thus electrostatic equilibrium must be maintained by having $\mathbf{E} = \mathbf{0}$ within the volume of the conductor (Young & Freedman, 2020). Thus, all charges that will contribute to electric current will reside at or near the surface of the conductor, and the electric field **E** associated with said current is detectable only at the conductor surface.

When this electric current is passed through a material with a high resistivity ρ, say the material making up the resistive board in Figure 3a, more electron-cation collisions occur and the conducting electrons gradually lose their potential energy U_e via Joule heating. Regardless of what path is taken across the resistive board, however, Equation 4b requires that this change in U_e be the same between the two conductors attached to the board. Because there is a change in U_e between the two conductors, this occurrence can conversely be associated with the generation of a potential difference ΔV and an electric field **E** between the conductors on the resistive board (Figure 3b). By setting the reference for ground, measuring the potential difference ΔV at given points of the surface of the resistive board, and tracing the points on the board surface where ΔV remains constant, one can infer the shapes of the equipotential lines, and thus the directions of the electric field lines for the electric field **E**.

IV. Data and Results

Figures 4, 5, and 6 show the equipotential lines around two charged circular conductors, two bar-shaped conductors, and one charged circular conductor and one bar-shaped conductor respectively as empirically determined in the experiment, as well as the field lines surrounding each conductor. In each figure, the equipotential lines are shown as solid curved lines, and the electric field lines are shown as broken lines with arrowheads near the middle of the curve.

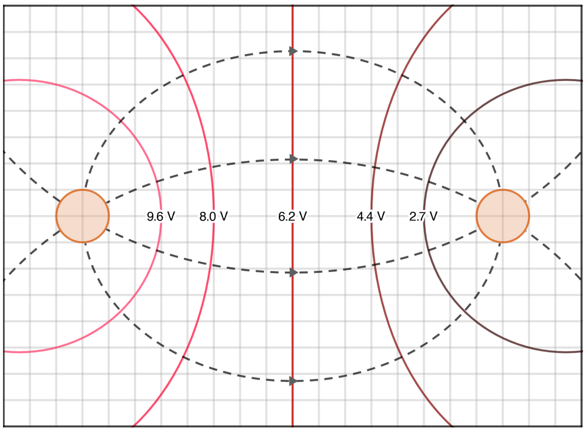

Figure 4. *Equipotential and Field Lines of Two Charged Circular Conductors*

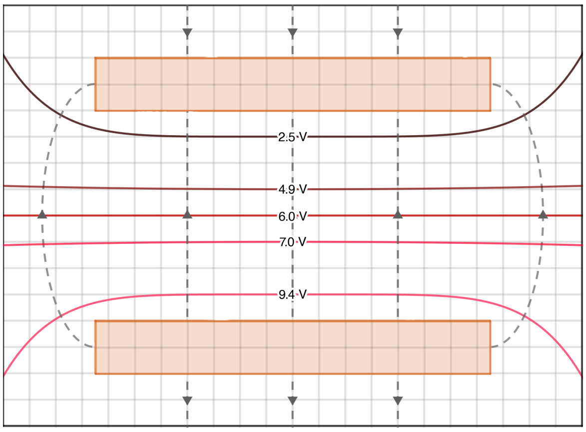

Figure 5. *Equipotential and Field Lines of Two Bar-shaped Conductors*

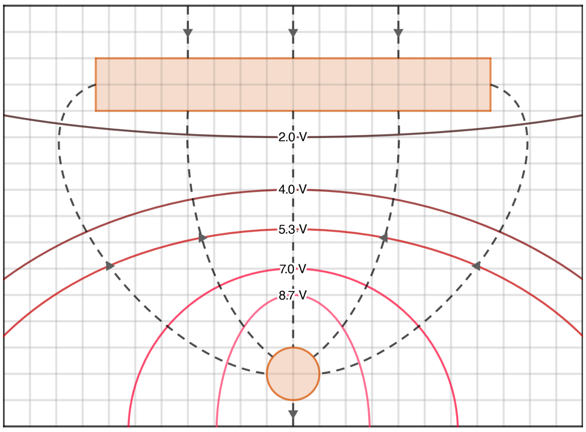

Figure 6. *Equipotential and Field Lines of a Charged Circular and a Bar-shaped Conductor*

The field lines were constructed by drawing curves from the conductor connected to the 12-V supply to the conductor connected to the ground such that these curves are perpendicular to the determined equipotential lines at intersection points. It can be observed that the equipotential lines determined in the experiment generally encircle the charged conductors, with a tendency to appear linear at some point between the two conductors.

V. Computations

All computations performed and plots for the electric field lines and equipotential lines generated in this experiment were done on an in-house-developed simulation program coded in Python 3.7. A numerical method using various equations discussed in the theoretical discussion was employed The program used may be accessed through this link: https://github.com/NotAMadTheorist/Electric-Field-Simulator (Sanguyo, 2024), The outline below shows a rough pseudo-code on the overall execution of the program:

- Define the region $R = [-10, 10] \times [-7, 7]$ on the *x-y* Cartesian coordinate plane, then divide the plane into a grid of discrete, evenly-spaced points P*(x, y)*.
- Define a group or groups of discrete point charges Q which approximate the shapes of the two conductors used for each setup. Specify the charge and positions of each point charge such that the electric potential is from 0 to 12 V.
- For each point P(*x, y*) in region *R*:
  - Compute the *x* and *y* components for the electric field vector **E** at each point P(*x, y*) in region *R*:
    - For each existing point charge Q with coordinates R(*x, y*):
      - Compute the magnitude |**E**| of the electric field via eq. (3), then determine the *x* and *y* components of **E** using the sign of Q and the coordinates P(*x, y*) and R(*x, y*):

$$|\mathbf{E}| = \frac{k\,|Q|}{r^2}, \qquad k = \frac{1}{4\pi\varepsilon} \tag{3}$$

      - Per eq. (1), add the *x* and *y* components of **E** due to point charge Q to the running total of *x* and *y* components of **E** for the point P(*x, y*).
  - Compute the electric potential V at point P(*x, y*) in region *R*:
    - For each existing point charge Q with coordinates R(*x, y*):
      - Compute the electric potential V due to point charge Q via Equation 6. Note that eq. 6 can be derived by substituting eq. (3) into eq. (4b) and integrating from infinity to point P:

$$V = \frac{kQ}{r}, \qquad k = \frac{1}{4\pi\varepsilon} \tag{6}$$

      - Add the electric potential due to point charge Q to the running total of the electric potential for P(*x, y*)

  - Eliminate the most extreme values for V and set the lowest value for V as the ground ($V = 0$) for the simulation specifically.
- Plot the electric field lines as a stream plot based on the x and y components of the electric field $\mathbf{E}$ at each point P(x, y). Plot positive point charges as red scatter points and plot negative point charges as blue scatter points.
- Plot the equipotential lines as a contour plot of the electric potential V as a multivariable function of $V(x, y)$. Plot positive point charges as red scatter points and plot negative point charges as blue scatter points.
- Plot the point charges, electric field lines, and equipotential lines for region R in a single combined diagram.

VI. Analysis and Conclusion

As shown in Figures 4, 5, and 6, the equipotential lines mapped in the experiment tended to form loops around the charged conductors, being spaced more closely around charged regions. This behavior can be attributed to the dependence of electric potential at point P around a charge Q on $1/r$, where r is the distance from the point P to the charge Q; this implies that the absolute gradient of electric potential is proportional to $1/r^2$, which manifests as an increase in rate at which the distance from Q required to attain a given potential V changes. Moreover, in a setup with n like charges Q_1, Q_2, …, Q_n, assuming without loss of generality that these charges are all positive, the potential at point P increases arbitrarily and continuously as P is moved closer to any given charge Q_i regardless of the direction from which P approaches Q_i; as such, an equipotential line with arbitrary potential can always be drawn around Q_i. Since V is a continuous function of P over its domain, it must only attain a single value at any point P. As such, any equipotential line with potential $V - \varepsilon$ for arbitrarily small $\varepsilon > 0$ must completely enclose an equipotential line with potential V, as otherwise the intersection points of the two equipotential lines must have two different values of V; by the same logic, any equipotential line with potential $V + \varepsilon$ for arbitrarily small $\varepsilon > 0$ must be completely enclosed by an equipotential line with potential V. Hence, any equipotential line around the setup of like charges must enclose the charges. In the experiment, however, linear equipotential lines that did not enclose the conductors were also observed between the two charged conductors, such as in the setup with two circular conductors. In this case, this can be attributed to the opposite charges of the conductors; in the specific case of the two circular conductors of opposite charges, there exists a line between the two conductors along which any point is equidistant to the two conductors, which implies that the two conductors contribute electric potentials that are equal and opposite for a total $V = 0$ uniform throughout the line.

When this experiment is simulated, the same general shapes can be observed for the equipotential and field lines, particularly with the field lines encircling the charged conductor and the field lines occurring perpendicular to the equipotential lines. However, some visual differences, such as in the spacing of the equipotential and electric field lines, can be observed. Figures 7, 8, and 9 below show the expected equipotential and field lines around the three setups by simulation.

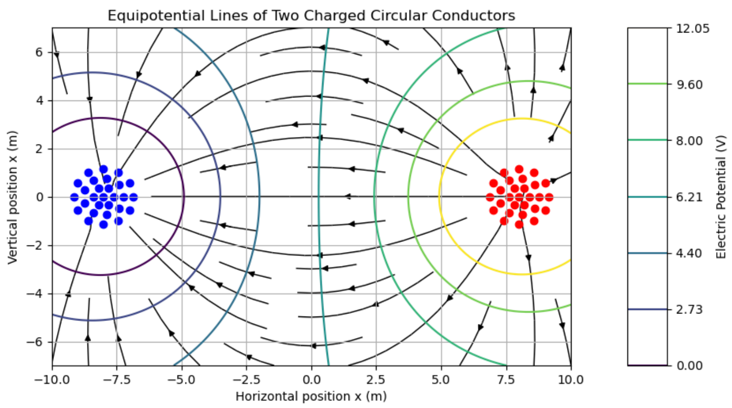

Figure 7. *Simulated Electric Field and Equipotential Lines of Two Charged Circular Conductors, with Positive Charges in Red and Negative Charges in Blue (Sanguyo, 2024)*

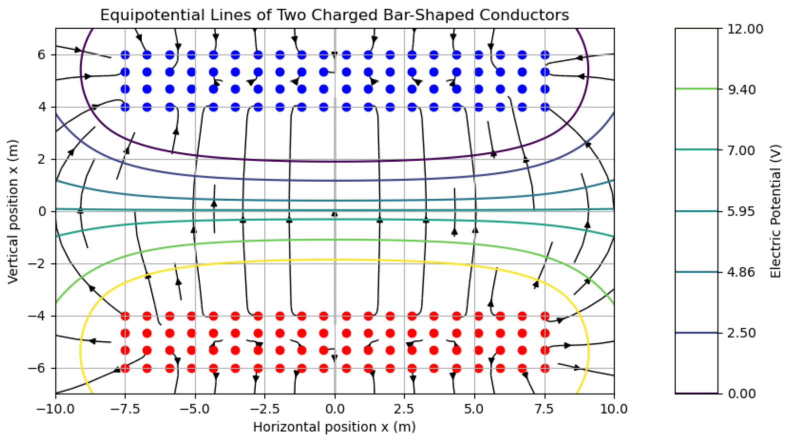

Figure 8. *Simulated Electric Field and Equipotential Lines of Two Bar-shaped Conductors (Sanguyo, 2024)*

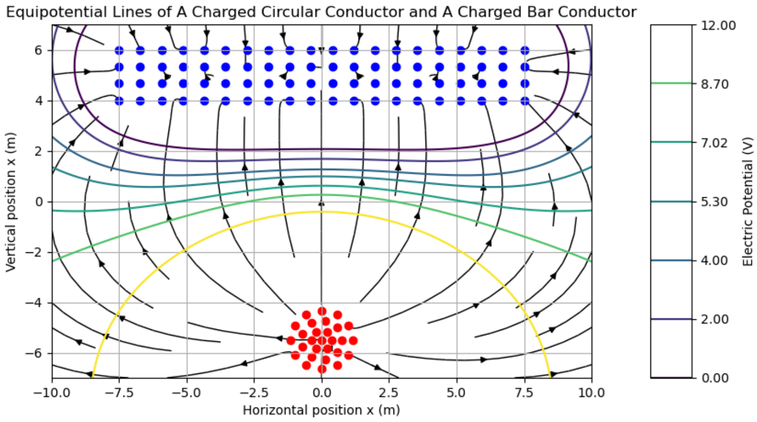

Figure 9. *Simulated Electric Field and Equipotential Lines of a Charged Circular and a Bar-shaped Conductor (Sanguyo, 2024)*

In contrast, the experiment proper introduced several complexities and sources of error. First, the presence of a sharp boundary between materials and the surrounding insulating material in the resistive board distorts the shape of the equipotential lines and the electric field lines, particularly near its edges. Since the wood comprising the frame of the board is an electrical insulator, electric current cannot flow beyond the resistive area of the board. Thus, the observed electric field **E** was distorted near, on, and beyond the insulating boundary in such a way that **E** becomes perpendicular near the boundary (Ulaby & Ravaioli, 2019).

Secondly, it is possible that the resistive board is not completely homogenous in terms of its resistance and therefore might have contributed to deviations in the measurement of potential V relative to ground. If the resistance of the board is uniform, then at any point P on the board, the voltage drop is expected to be influenced solely by the charges of the conductors and the boundary condition. However, if the resistance is not uniform throughout the board, regions of greater resistance between P and the conductors can cause a greater voltage drop than expected, leading to deviations in the voltage reading; this can consequently distort the empirical equipotential lines within the resistive board.

In the context of obtaining the experimental equipotential lines, several points on the board were selected and interpolated as they had relatively similar measured potentials. The mean average potential of these points are shown in Figures 4 through 6. However, significant but random variations in the measured potential as much as ± 0.10 V were observed along these points. Although some variation may be attributed to the distortion of **E** caused by the insulating boundaries, the random variation in V may be somewhat attributed to the uneven resistances of the resistive board along the interpolated equipotential lines. By contrast, if the resistivity of the board was completely uniform, the measured potentials should not have said errors, and the equipotential lines should look similar to the results obtained from the simulation.

Other sources of error may stem from measurement inaccuracies, which can result from the precision limitations or fluctuations in the readings of the voltmeter used or simple human error in reading and recording measurements. Additionally, positioning errors can occur if probes are not placed correctly (i.e., slight deviations from perpendicularity of the contact probe with the mapping board) or if the exact coordinates of the measurement points are inaccurately recorded, leading to distortions in the mapped lines. This necessitated the use of digital software to accurately map the equipotential lines. Errors in the experimental setup, such as improper connections, electrical interference by other devices, and fluctuations in power supply, could have also compromised the results.

From the data gathered in the experiment, it can be concluded that the general shapes of the electric field and equipotential lines between two conductors of specified geometries were characterized effectively, with main observations including the formation of equipotential loops around the conductors, the divergence and convergence of the electric field around the charged conductors, and the dependence of the direction and spacing of the derived electric field on that of the measured equipotential lines. Moreover, it can be concluded that the electric field and equipotential lines determined empirically by mapping were found to be substantially similar in general visual appearance to those determined by simulation, with deviations such as in electric field spacing primarily arising due to errors such as in the production of the hand-sketched empirical mapping and the high sensitivity of the voltmeter to slight changes in the position of the probe.

VII. Answers to Questions and Problems

1. Why didn't have to draw the equipotential lines just outside the conductors?

Ans: Equipotential lines are not typically drawn just outside the conductors mainly due to two reasons: charge distribution on the surface of the conductors and field distortion close to conductors. In particular, conductors in electrostatic equilibrium, which is an approximation of the conditions in the experiment, have a uniform potential on their surface. Drawing equipotential lines immediately outside the conductor would not provide any additional useful information as the lines would overlap with the surface of the conductor, which is expected to yield approximately the same potential value; moreover, the equipotential lines are expected to be closely parallel to the surface of the conductors. Moreover, the electric field, when close to conductors, can be highly non-uniform and complex due to edge effects and other local irregularities, making it difficult to accurately map the equipotential lines (Urone & Hinrichs, 2022).

2. Does your prediction of the field lines exactly agree with your experimental observation? Could you explain any difference?

Ans: It was predicted that the electric field lines would lie perpendicular to the mapped equipotential lines, which was indeed constructed in the empirical electric potential maps as well as observed in the simulations (Ulaby & Ravaioli, 2019). The general spacing of the electric field lines was also predicted to increase in regions of low electric field magnitude, which was deduced to occur in regions of greater separation between equipotential lines; this was especially prominent in Figure 4, but the proper spacing of the electric field lines in the figure as well as in Figures 5 and 6 may have been influenced by variations in the gradient between consecutive equipotential lines and by space constraints on the map sketches. The electric field was also predicted to point from higher to lower potential, which was also observed in the simulations. However, the predictions made were imprecise, which can potentially be attributed to variations in the conductors used as well as fluctuations in the voltmeter readings.

3. Can equipotential lines intersect? Give an example of a configuration where this happens.

Ans: In a typical mapping setup, equipotential lines do not intersect. This is because each equipotential line represents a location of constant electric potential. If two equipotential lines were to intersect, it would imply that a single point in space is at two different potentials simultaneously (Young & Freedman, 2020). However, there are certain special cases where equipotential lines can intersect, which are only possible when the net electric field at a specific point is zero.

Equation 5 shows the relationship between the electric field and the potential gradient:

$$\mathbf{E} = \nabla V \tag{5}$$

This implies that the electric field follows the direction of decreasing electric potential.

If the net electric field at a point is zero, this implies that the gradient of the potential at that point is also zero. In this scenario, equipotential lines can intersect because the potential does not change in any direction from that point, analogous to saddle points in a potential energy diagram.

An example would be two like charges (both positive or both negative) placed at a certain distance from each other. The midpoint between these two charges is a point of zero electric field. At this midpoint, the equipotential lines originating from each charge can intersect because the net electric field is zero. The given figure below demonstrates this intersection between two equipotential lines.

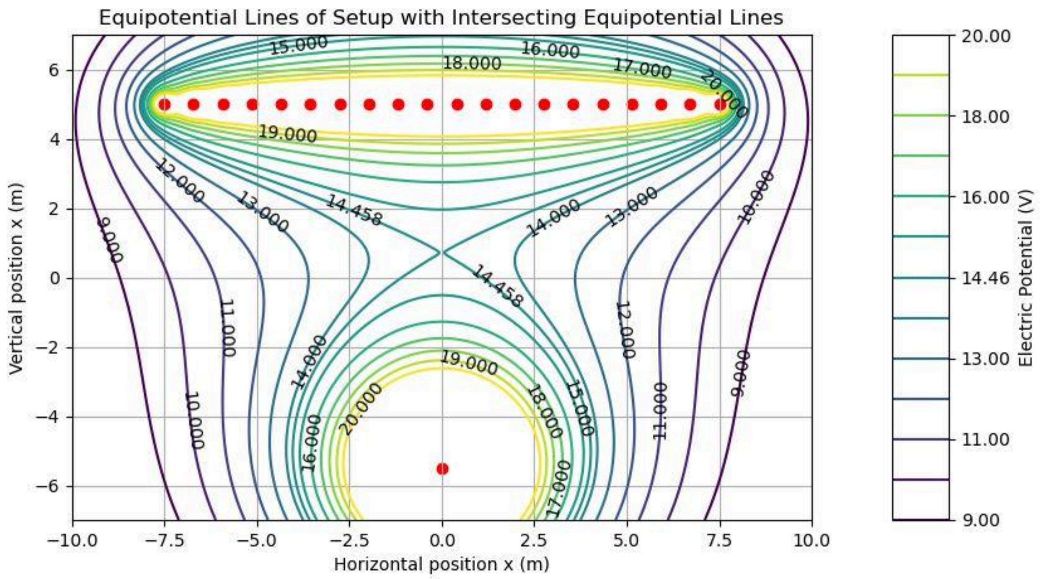

Figure 10. *Example of Crossing Equipotential Lines at 17.458 V at the point (0 m, 0.70 m) (Sanguyo, 2024)*

Another example is inside a uniformly charged spherical shell or inside a conductor in static condition where the whole region is an equipotential volume. In such a volume, an assumption can be drawn where any number of planes will be intersecting each other inside the volume. As a result, the potential on each such plane will be the same (Ling et al., 2016).

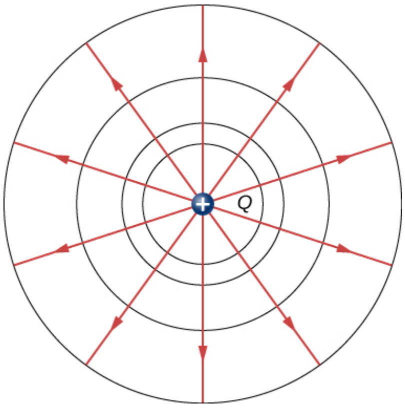

Figure 11. *Crossing Equipotential Lines in a Uniformly Charged Spherical Shell (Ling et al., 2016)*

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