

Undergraduate Final Project

Due May 6, 11:59pm

150 points

CS 4499/5531

Scientific Computing

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Write a C++ program that randomly samples from the Watt Fission Spectrum.

The Watt Distribution (Spectrum)

The distribution of the number of neutrons with energy in the fission spectrum is well represented by a mathematical function, the *Watt* function.

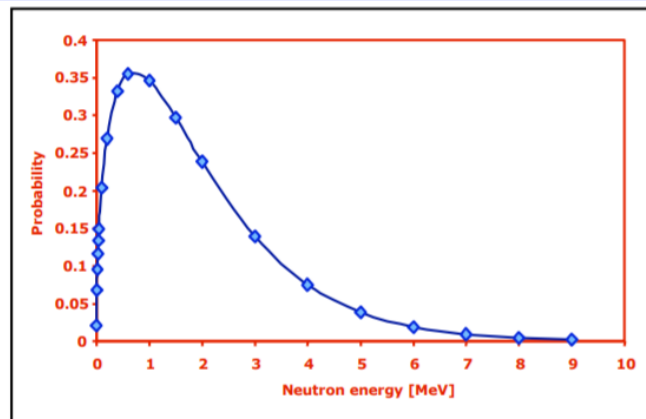
The probability of a neutron from fission having an energy between E and $E+dE$ is the function $P(E)dE$.

$$P(E) = 0.4865 \sinh(\sqrt{2E}) e^{-E} \text{ MeV}^{-1}$$

This is an *empirical* formula – it is just a convenient way of expressing the experimental distribution.

This distribution is for neutrons from the fission of ^{235}U with a slow neutron and is shown in the figure.

Note: It only changes slightly for other types of fission.



Say you have a Monte Carlo neutronics code. Part of this code will involve randomly sampling what energy a new neutron created from fission will have. This can be done using the Watt distribution. For this problem, we will assume the probability distribution of the energy is

$$P(E) = 0.4865 \sinh(\sqrt{2E}) e^{-E}$$

Our plan will be to utilize rejection sampling as this is a powerful technique which can be used to randomly sample from essentially any distribution. The rejection method requires knowledge of the min/max and range, so we will calculate these along the way.

1. Set up $f(x)$ (in this case $P(E)$).
 - a. First create a function **fx** (either global or a lambda in main) which is passed **E** and returns the value of $P(E)$ from the equation above.
 - b. Check that you have **fx** correct by integrating, using an integration method from your HW 7, from $[0,10]$ and showing your result is 1.00 (accurate to about the hundredth place). Print the integral to the screen.
2. Find the **maximum** of **fx** ($P(E)$).
 - a. You can do this either by finding the roots of the derivative of **fx** (ie using **finite_difference_derivative** and **bisect**) and evaluating **fx** at the root, or by using **minima**. Print the (E , $P(E)$) for the maximum to the screen.
3. Create a random point.
 - a. Randomly create a point in the range $x=[0,8]$ and $y=[0, \text{max from part 2}]$. You will need to randomly select two values.
4. Use the rejection method.
 - a. Check to see if the point selected in part 3 is under or equal to the $P(E)$ curve. If it is, save the energy to a vector of doubles called **sample**. If it is not, reject the point and randomly create a new one and check again. Repeat until a valid point is found and the selected energy is stored.
5. Create a sample.
 - a. Randomly sample 100,000 energies from the Watt Fission Spectrum. They should be stored in **sample**. Print the size of **sample** to the screen.
6. Binning.
 - a. Create a vector of 100 energy bins, from 0 to 8 MeV, called **bins**. Find the number of energies in **sample** that are in each bin. Print to the screen the number of selections found in each of the 100 bins.
7. Histogram plot.
 - a. Make sure your random sampler works by plotting a histogram of **sample**, by plotting the value in each bin of **bins**. You can use Excel, Python, C++, etc. If not in your C++ code, then attach the other file with the image of the plot.

Attach all relevant code and screenshots.