Reconstruction of TLAPS proofs solved by SMT in Lambdapi

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Outline

TLA$^+$ at a glance

- Specification language to design and verify reactive systems
- Systems are described as state machines

$$
\begin{array}{l}
\textit{VARIABLE } x \\
\textit{CONSTANT } N \\
\textit{ASSUME } N \in \textit{Nat}
\end{array}
$$

$$\textit{Init} \triangleq \quad \land\, x = 0$$

$$\textit{Next} \triangleq \quad \begin{array}{l} \land\, x < N \\ \land\, x' = x + 1 \end{array}$$

$$\textit{Spec} \triangleq \textit{Init} \land \Box[\textit{Next}]_{\langle x \rangle}$$

TLAPS proof example

---------------- MODULE Cantor1 ----------------

THEOREM cantor ==

$\forall S :$

$\forall f \in [S \rightarrow \text{SUBSET } S] :$

$\exists A \in \text{SUBSET } S :$

$\forall x \in S :$

$f[x] \# A$

PROOF

<1>1. TAKE S

<1>2. TAKE $f \in [S \rightarrow \text{SUBSET } S]$

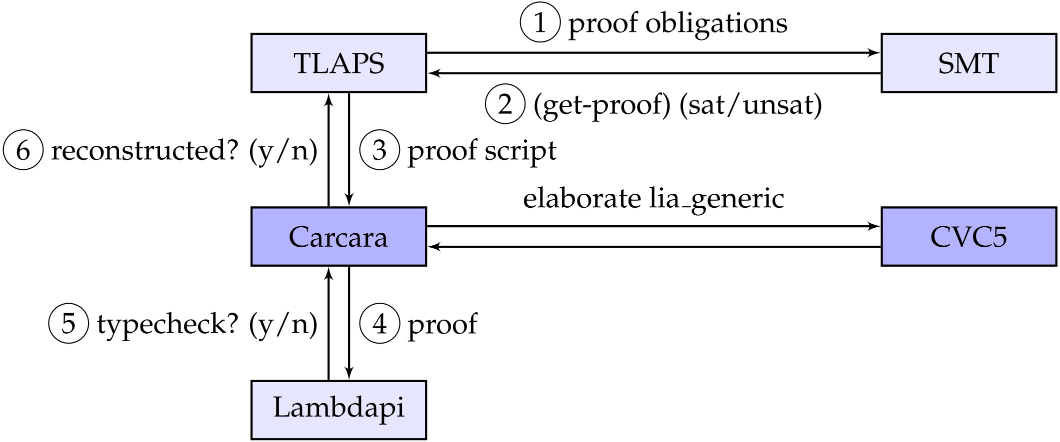
<1>3. DEFINE $T == \{ z \in S : z \notin f[z] \}$

<1>4. WITNESS $T \in \text{SUBSET } S$

<1>5. TAKE $x \in S$

<1>6. QED BY $x \in T \lor x \notin T$

Proposed solution



Simple example

```
(set-logic QF_UF)
(declare-sort U 0)
(declare-fun a () U)
(declare-fun b () U)
(declare-fun p (U) Bool)
(assert (p a))
(assert (= a b))
(assert (not (p b)))
(get-proof)
```

Alethe SMT proof

```
(assume a0 (p a))
(assume a1 (= a b))
(assume a2 (not (p b)))
(step t1 (cl (not (= (p a) (p b))) (not (p a)) (p b)) :rule equiv_pos2)
(step t2 (cl (= (p a) (p b))) :rule cong :premises (a1))
(step t3 (cl (p b)) :rule resolution :premises (t1 t2 a0))
(step t4 (cl) :rule resolution :premises (a2 t3))
```

Alethe format

Definition (Alethe step notation)

A proof in the Alethe language is an indexed list of step following the format:

$$j. \quad \Delta \quad \vdash \ \varphi \quad (R; p_1 \dots p_n)[a_1, \dots, a_n]$$

With $i \in \mathbb{I}$ where $\mathbb{I}$ is a countable infinite set of valid indices,

a formula φ,

a rule name $\mathcal{R}$ from a set of possible rules,

a possible empty sets $\{p_1 \dots p_n\} \subseteq \mathbb{I}$ of premises (previous steps),

a possible empty list of arguments $[a_1 \dots a_n]$ where $a_i = (x_i, t_i)$
with x_i a variable and t_i a term,

and a context Δ.

Overview of rules

1. Special rules
   * $\vdash \varphi$ `asssume`
   * $\vdash \varphi$ (hole; $p_1 \dots p_n$)[$a_1 \dots a_n$]
   * $\varphi_1 \dots \varphi_n,\ \psi \vdash \neg\varphi_1 \dots \neg\varphi_n \psi$ (subproof; $p_1 \dots p_n$)
2. Resolution rules
   * th_resolution, resolution
   * contraction
3. Introducing tautologies
   * $\vdash \neg(\neg\neg\varphi) \vee \varphi$ (not_not)
   * $\vdash \neg(\varphi_1 \approx \varphi_2) \vee \neg\varphi_1 \vee \varphi_2$ (equiv_pos2)
   * $\vdash \neg(\varphi_1 \wedge \cdots \wedge \varphi_n) \vee \varphi_k$ (and_pos)
4. Linear arithmetic
   * lia_generic, la_generic
   * $\vdash t_1 \leq t_2 \vee t_2 \leq t_1$ (la_totality)
5. Quantifier handling
   * $j.\ \Delta,\ x_i \mapsto y_i \vdash \varphi \approx \varphi'$
     $i. \vdash \forall x_1 \dots x_n, \varphi \approx \forall y_1 \dots y_n, \varphi'$ (bind)
   * forall_inst
6. Skolemization
   * sko_ex
   * sko_forall
7. Clausification rules
   * **let**
   * distinct_elim
8. Simplification rules
   * and_simplify
   * bool_simplify
   * eq_simplify
   * sum_simplify

Alethe proof as derivation tree

$$
t_i \;\dfrac{t_j \;\dfrac{\vdots}{\Delta'' \vdash p_1}\;\mathrm{Rule}(\dots) \qquad \dots \qquad t_k \;\dfrac{\vdots}{\Delta' \vdash p_n}\;\mathrm{Rule}(\dots)}{\Delta \cup \{p_1, \dots, p_n\} \vdash c_1, \dots c_n}\;\mathrm{Rule}(a_1, \dots, a_n)
$$

SMT proof

```
(assume a0 (p a))
(assume a1 (= a b))
(assume a2 (not (p b)))
(step t1 (cl (not (= (p a) (p b))) (not (p a)) (p b)) :rule equiv_pos2)
(step t2 (cl (= (p a) (p b))) :rule cong :premises (a1))
(step t3 (cl (p b)) :rule resolution :premises (t1 t2 a0))
(step t4 (cl) :rule resolution :premises (a2 t3))
```

Carcara

- Carcara is an efficient and independent proof checker and elaborator for Alethe proofs.
- Carcara is written in Rust, a high performance language,
- implements elaboration procedures for a few important rules (ex: infering pivots),
- it remove implicit transformations (ex: reordering clause).

Elaborated proof with Carcara

Make pivot and resolution order explicit

```
(assume a0 (p a))
(assume a1 (= a b))
(assume a2 (not (p b)))
(step t1 (cl (not (= (p a) (p b))) (not (p a)) (p b)) :rule equiv_pos2)
(step t2 (cl (= (p a) (p b))) :rule cong :premises (a1))
(step t3 (cl (p b)) :rule resolution :premises (t1 t2 a0)
    :args ((= (p a) (p b)) false (p a) false))
(step t4 (cl) :rule resolution :premises (a2 t3)
    :args ((p b) false))
```

Corresponding proof tree

$$
\dfrac{
  a_0\ \dfrac{}{\Delta \vdash p(a)}
  \qquad
  t_3'\ \dfrac{
    t_1\ \dfrac{}{\Delta \vdash \neg\,(p(a) = p(b)), \neg p(a), p(b)}\ \text{equiv\_pos2}
    \qquad
    t_2\ \dfrac{}{\Delta \vdash p(a) = p(b)}\ \text{cong}(a_1)
  }{\Delta \vdash \neg p(a), p(b)}\ \text{Resolution}(t_1, t_2)
}{
  t_3\ \ \Delta \vdash p(b)
}\ \text{Resolution}(a_0, t3')
$$

$$
\dfrac{
  t_3\ \ \Delta \vdash p(b)
  \qquad
  a_2\ \dfrac{}{\Delta \vdash \neg\, p(b)}
}{
  t_4\ \ \Delta \vdash \bot
}\ \text{Resolution}(a_2, t_3)
$$

Translate prelude

```
(declare-sort U 0)
(declare-fun a () U)
(declare-fun b () U)
(declare-fun p (U) Bool)
```

$\rightsquigarrow$

```
symbol U : TYPE;
rule U ↪ τ o;

symbol a : U;
symbol b : U;
symbol p : U → Prop;
```

Translate assert/assume

```
(assert (p a))
(assert (= a b))
(assert (not (p b)))
```

```
(assume a0 (p a))
(assume a1 (= a b))
(assume a2 (not (p b)))
```

```
constant symbol a0 : π̇ (p a ∨̇ □);
constant symbol a1 :
    π̇ (a ⟺^c b ∨̇ □);
constant symbol a2 :
    π̇ (¬^c (p b) ∨̇ □);
```

Translation of step t1 and t2

```
(step t1
    (cl (not (= (p a) (p b)))
        (not (p a))
        (p b))
    :rule equiv_pos2)
(step t2 (cl (= (p a) (p b)))
    :rule cong :premises (a1))
...
```

$\rightsquigarrow$

```
opaque symbol pb :
begin
have t1: π̇ (
    ¬ᶜ ((p a) ⟺ᶜ (p b))
    ∨̇ (p a)
    ∨̇ (p b)
    ∨̇ □)
{
    apply equiv_pos2;
};
have t2: π̇ (
    p a ⟺ᶜ p b
    ∨̇ □)
{
    apply ;
    apply cong p ( a1);
};
```

Translation of step t3

```
(step t3 (cl (p b))
    :rule resolution
    :premises (t1 t2 a0))
```

$\rightsquigarrow$

```
...
have t3 : π̇ ((p b) ∨̇ □) {
    have t1_t2 : π̇ (
        (¬ᶜ ((p a)))
        ∨̇ (p b) ∨̇ □)
    {
        apply resolution t1 t2;
    };
    have t1_t2_a0 : π̇ ((p b) ∨̇ □)
    {
        apply resolution t1_t2 a0;
    };
    apply t1_t2_a0;
};
```

Translation of step t4

```
(step t4 (cl)
    :rule resolution
    :premises (a2 t3))
```

⇝

```
...
have t4 : π̇ □ {
    have a2_t3 : π̇ □  {
        apply resolution a2 t3;
    };
    apply a2_t3;
};
apply t4;
proofterm;
end;
```

Corresponding proof term

```
resolution_r (p b) _ _
    a2
    (resolution_r (p a) _ _
          (resolution_r (p a ⟺^c p b) _ _
               equiv_pos2
               (∨_i1^c (cong p (π a1)))
          )
          a0
    )
```

Supported rules overview

1. Special rules ✓
   * $\vdash\ \varphi$ `asssume`
   * $\vdash\ \varphi$ $(\texttt{hole}; p_1 \dots p_n)[a_1 \dots a_n]$
   * $\varphi_1 \dots \varphi_n, \psi\ i.\ \vdash\ \neg\varphi_1 \dots \neg\varphi_n \psi$
     $(\texttt{subproof};\ p_1 \dots p_n)$
2. Resolution rules ✓
   * `th_resolution,resolution`
   * `contraction`
3. Introducing tautologies ✓
   * $\vdash\ \neg(\neg\neg\varphi) \vee \varphi$ (`not_not`)
   * $\vdash\ \neg(\varphi_1 \approx \varphi_2) \vee \neg\varphi_1 \vee \varphi_2$ (`equiv_pos2`)
   * $\vdash\ \neg(\varphi_1 \wedge \dots \wedge \varphi_n) \vee \varphi_k$ (`and_pos`)
4. Linear arithmetic ×
   * `lia_generic,la_generic`
   * $\vdash\ t_1 \leq t_2 \vee t_2 \leq t_1$ (`la_totality`)
5. Quantifier handling (WIP)
   * $j.\Delta, x_i \mapsto y_i \vdash\ \varphi \approx \varphi'$
     $i. \vdash \forall x_1 \dots x_n, \varphi \approx \forall y_1 \dots y_n, \varphi'$ (`bind`)
   * `forall_inst`
6. Skolemization (WIP)
   * `sko_ex`
   * `sko_forall`
7. Clausification rules (WIP)
   * **let**
   * `distinct_elim`
8. Simplification rules ×
   * `and_simplify`
   * `bool_simplify`
   * `eq_simplify`
   * `sum_simplify`

Contexts in Alethe

Definition (Proof context)

We denote Δ_c as the Alethe proof context. It is use to reason about bound variable and store previous proved **step**.

$$\text{If} \quad j. \quad x_1 \mapsto y_1, \ldots, x_n \mapsto y_n \quad \vdash \; \varphi\, y_1 \ldots y_n \quad (R, p_1 \ldots p_n)[x_1, \ldots, x_n] \quad \text{proved}$$

Then $j(\varphi\, y_1 \ldots y_n;\; R;\; p_1 \ldots p_n;\; a_1 \ldots a\_n) \in \Delta$
And $x_1 \mapsto y_1, \ldots, x_n \mapsto y_n \in \Delta$

Definition (Definition context)

We denote Δ_{def} as the Alethe definition context. It is use to store user declarations `declare-sort` and `declare-fun`.

Definition (Alethe context)

We set $\Delta = \Delta_c \cup \Delta_{def}$

Alethe encoding in Lambdapi

We denote $\Gamma_{\mathcal{A}}$ as the Lambdapi context with Alethe definitions.

Encoding of classical logic in $\Gamma_{\mathcal{A}}$ [1]

- The set of terms `Set: TYPE`
  function symbol `Set →... →Set`,
- the set of propositions `Prop : TYPE`
  predicate symbol `Set →... →Prop`,
- and the classical connectives $\forall^c \mid \exists^c \mid \wedge^c \mid \neg^c \mid \vee^c \mid \Rightarrow^c \mid \iff^c$.
- $\pi^c := \neg\neg p$ the definition of classical proofs of proposition p,
- the axioms of classical natural deduction system $\mathcal{NK}$ and some lemmas.
- quantification on propositions/impredicativity ($e.g. \forall p, p \Rightarrow p$) :

```
symbol o : Set;
rule τo ↪Prop;
```

[1] Classical logic definitions is based on Lambdapi Stdlib.

Alethe encoding in Lambdapi

We encode Alethe rule $\mathcal{R}$ as corresponding **symbol** R.

Clause

Alethe treats clause as a set creating a canonical representation problem. We define clause as list in a Church encoding style to solve it:

```
constant symbol Clause : TYPE;
symbol □ : Clause; // Nil
injective symbol ⩒ : Prop → Clause →  Clause; // Cons x l
sequential symbol ++ : Clause → Clause → Clause;
rule □ ++  $m ↪ $m
with ( $x ⩒  $l) ++  $m ↪ $x ⩒ ( $l ++  $m);

symbol Clause_ind:  Π P: (Clause → Prop), Π l,
π (P □) → (Π x: Prop, Π l: Clause, π (P l) →
π (P (x ⩒ l))) → π (P l);
```

Clause concatenation and canonical form

With clause as disjunction

$$((x_1 \dot{\vee} x_2) \vee x_3) \quad \vee \quad (y_1 \vee y_2 \vee y_3) \rightsquigarrow (((x_1 \vee x_2) \vee x_3) \vee (y_1 \vee y_2 \vee y_3))$$

With clause with type Clause

$$((x_1 \dot{\vee} x_2) \dot{\vee} x_3 \dot{\vee} \Box) \quad {+}{+} \quad (y_1 \dot{\vee} y_2 \dot{\vee} y_3 \dot{\vee} \Box) \rightsquigarrow x_1 \dot{\vee} x_2 \dot{\vee} x_3 \dot{\vee} y_1 \dot{\vee} y_2 \dot{\vee} y_3 \dot{\vee} \Box$$

Proof of clause

Proof of clause $(\mathrm{cl}\ \ \varphi_1, \ldots, \varphi_n)$ with $(\mathcal{R}; p_1 \ldots p_n)[a_1 \ldots a_n]$ in a step such as

$$j. \quad \Delta \quad \vdash (\mathrm{cl}\ \ \varphi_1, \ldots, \varphi_n) \quad (R; p_1 \ldots p_n)[a_1, \ldots, a_n]$$

are encoded as proof:

```
injective symbol π̇ c: TYPE ≔ π (ℰ c);

sequential symbol ℰ: Clause → Prop;
rule ℰ ( $x ∨̇ $y) ↪ $x ∨^c (ℰ $y)
with ℰ □ ↪ ⊥;
```

Alethe rule encoding example

The rule *not_implies*1 in Alethe set of rules

$$\begin{array}{lll} i. & \vdash \neg(\varphi_1 \rightarrow \varphi_2) & (\ldots) \\ j. & \vdash \varphi_1 & (not\_implies1; i) \end{array}$$

is translated into:

```
opaque symbol not_implies_1 [φ1 φ2] : π(¬ᶜ (φ1 ⇒ᶜ φ2)) → π̇ (φ1 ∨̇ □) ≔
begin
    assume φ1 φ2 H;
    apply ∨ᶜ_i1;
    apply ∧ᶜ_e1 (imply_to_and H);
end;
```

Clause conversion lemma

Lemma (Clause equivalence with disjunctions)

Given a b Clause we have the equivalence:
$\forall a\ b : Clause.\ [\![a\ \ ++\ b]\!] \quad \Longleftrightarrow^{c} \quad [\![a]\!] \vee^{c} [\![a]\!]$

Clause resolution rule translation

Lemma (Resolution)

Given a b: Clause and a privot x: Prop, then a premise of $(x \dot{\vee} a) \in \Gamma_{\mathcal{A}}$ *and a premise of* $(\neg^c x \dot{\vee} b) \in \Gamma_{\mathcal{A}}$ *implies a clause* $a + +b$.

Lambdapi encoding:

```
opaque symbol resolution x a b : π̇ (x ∨̇ a) →π̇ (¬ᶜx ∨̇ b) →π̇ (a ++ b) ≔
```

Translation functions

The embedding uses four functions:

- $\mathcal{F}$ which translates first order formulas to $\Gamma_{\mathcal{A}}$-propositions,
- $\mathcal{S}$ which translates SMTLib sort from theory to $\Gamma_{\mathcal{A}}$-type,
- $\mathcal{T}$ which translates first order individual terms to $\Gamma_{\mathcal{A}}$-terms,
- $\mathcal{C}(\Gamma, c_1 \dots c_n)$ which translates a non-empty set of commands $c_1 \dots c_n$ to typing goal $\Gamma \vdash M : N$ and a term of type $M : N$ being proof term.

Function $\mathcal{F}$

translates first order formulas to $\Gamma_{\mathcal{A}}$-propositions

Definition ($\mathcal{F}$)

The definition of $\mathcal{F}$(f) is as follows.

- if f = $cl\ x_1 \ldots x_n$, then $\mathcal{F}_\Delta(cl\ x_1 \ldots x_n) = x_1 \dot{\vee} \cdots \dot{\vee}\ x_n \dot{\vee} \square$,
- if f = $a_1 \wedge \cdots \wedge a_n$, then $\mathcal{F}_\Delta(a_1 \wedge \cdots \wedge a_2) = a_1 \wedge^c \cdots \wedge^c a_2 \wedge^c \top$,
- if f = $a_1 \vee \cdots \vee a_n$, then $\mathcal{F}_\Delta(a_1 \vee \cdots \vee a_2) = a_1 \vee^c \cdots \vee^c a_2 \vee^c \perp$,
- if f = $a \approx b$ and $a\ b \in$ **Bool**, then $\mathcal{F}_\Delta(a \approx b) = a \iff^c b$,
- if f = $a \approx b$ and $a\ b \notin$ **Bool**, then $\mathcal{F}_\Delta(a \approx b) = (a = b)$,
- otherwise we are in the case $\mathcal{F}_\Delta(f) = f$ with all connector changes for their Corresponding classical connector $\star^c$.

Why this $\mathcal{F}$ definitions for conjunctions and disjunctions ?

N-ary rules are proved with "reflexivity" proof. For example

$$i. \quad \Delta \quad \vdash \neg(\varphi_1 \wedge \cdots \wedge \varphi_n), \varphi_k \quad (\mathit{and\_pos})$$

with $1 \leq k \leq n$

where we have the reflexivity proof:

```
sequential symbol In_∧ᶜ: Prop → Prop → 𝔹;
rule In_∧ᶜ $x ( $h ∧ᶜ $tl) ↪ (eq $x $h) Bool.or (In_∧ᶜ $x $tl)
with In_∧ᶜ $x ⊤ ↪ false;

symbol and_pos [φ1‥φn φk]:
    π ((In_∧ᶜ φk φ1‥φn) = true) → π̇ (¬ᶜ φ1‥φn ∨̇ φk ∨̇ □);
```

Function $\mathcal{S}$

translates SMTLib sort from theory to $\Gamma_{\mathcal{A}}$-type

The definition of $\mathcal{S}$(t) is as follows.

- if t = **Bool**, then $\mathcal{S}(\mathbf{Bool}) = \mathit{Prop}$,
- if $t \neq$ **Bool**, then $\mathcal{S}(t) = \tau\ o$,
- if $t = f(a_1 \ldots a_n)$ and $\mathit{codomain}(f)$ = **Bool**, then $\mathcal{S}(f(a_1 \ldots a_n)) = f : \mathcal{S}(a_1) \to \cdots \to \mathcal{S}(a_1) \to \mathbf{Prop}$,
- if $t = f(a_1 \ldots a_n)$ and $\mathit{codomain}(f) \neq$ **Bool**, then $\mathcal{S}(f(a_1 \ldots a_n)) = f : \mathcal{S}(a_1) \to \cdots \to \mathcal{S}(a_1) \to \mathbf{Set}$,

Function $\mathcal{T}(t)$

Definition

The definition of $\mathcal{T}(t)$ is a direct shallow embedding of t in corresponding term t in $\Gamma_{\mathcal{A}}$ (variables, functions, constants).

Function $\mathcal{C}(\Gamma, -)$

translates Alethe commands

Definition

The function $\mathcal{C}(\Gamma, i.\ \Delta\ \vdash \varphi \quad (R; p_1 \ldots p_n)[a_1 \ldots a_n]) \rightarrow \Gamma'$ translates, in a given context Γ, a step i into a judgement (typing goal) $\Gamma \vdash i : \mathcal{F}(\varphi)$ with a term M along satisfying the goal. It returns a new context Γ' with $i \in \Gamma'$. The definition of $\mathcal{C}$ is defined recursively on R.

Notation

The notation $tac(A)[\Gamma \vdash \varphi]$ means applying the Lambdapi tactic tac (with argument A) to the judgement $\Gamma \vdash \varphi$ and making the judgements (subgoals) generated by the tactic be the premises of the rule.

Example 1: *equiv_pos2* case

We translate a step i using the alethe rule *equiv_pos2*:

$$i. \qquad \vdash \neg(\varphi_1 \approx \varphi_2), \neg\varphi_1, \varphi_2 \qquad\qquad (\mathit{equiv\_pos2})$$

with given a context Γ as

$$\mathcal{C}(\Gamma,\ i.\ \vdash \neg(\varphi_1 \approx \varphi_2), \neg\varphi_1, \varphi_2\ (\mathit{equiv\_pos2})) = \mathit{apply}(\mathit{equiv\_pos2})[\Gamma \vdash i : \mathcal{F}(\neg(\varphi_1 \approx \varphi_2), \neg\varphi_1, \varphi_2)]$$

Example 2: *cong* case

We translate a step k using the alethe rule *cong*

$$\begin{array}{llll} i. & \Delta & \vdash t_1 \approx u_1 & (\dots) \\ & \vdots & & \\ j. & \Delta & \vdash t_n \approx u_n & (\dots) \\ k. & \Delta & \vdash f\ t_1 \dots t_n \approx f\ u_1 \dots u_n & (cong; p_1 \dots p_n) \end{array}$$

as:

if codomain(f) $\in$ **Bool**, then $\mathcal{C}(\Gamma,\ i.\ \Delta\ \ \vdash f\ t_1 \dots t_n \approx f\ u_1 \dots u_n\ (cong; p_1 \dots p_n))$

$= \ cong2_f(f\ p_1 \dots cong2_f(f\ p_{n-1}\ p_n))[\Gamma \vdash i : \mathcal{F}(f\ t_1 \dots t_n \approx f\ u_1 u_n)]$

otherwise $f\_equal_n(f\ p_1\ \ \dots\ \ p_n)[\Gamma \vdash i : \mathcal{F}(f\ t_1 \dots t_n \approx f\ u_1 u_n)]$,

with $\mathcal{C}(\Gamma \backslash \{i\},\ i.\ \Delta\ \ \vdash t_1 \approx u_1\ (\dots)) \in \Gamma$ and

$\vdots$

with $\mathcal{C}(\Gamma \backslash \{j\},\ j.\ \Delta\ \ \vdash t_n \approx u_n\ (\dots)) \in \Gamma$

Soundness argument

Theorem (soundness)

We define the soundness as for a Alethe context Δ and a first order formula φ in a step $i.\ \Delta \vdash \varphi\ (\mathcal{R}; p_1 \dots p_n)[a_1 \dots a_n]$ proved by rule $\mathcal{R}$, the translation $\mathcal{C}(\Gamma, i.\Delta \vdash_{\mathit{FOL}} \varphi(\mathcal{R}; p_1 \dots p_n)[a_1 \dots a_n])$ give a term $i : \tau$ such that $\Gamma_{\mathcal{A}} \vdash i : \tau$.

Proof.

(proof intuition) By induction on $\mathcal{R}$. $\square$

Evaluation with TLA+ example

Stephan Merz. TLA+ Case Study: A Resource Allocator.

- Use set theory only,
- no skolemization,
- 25 proofs obligations,
- size of the Alethe proofs varies between 4 and 288 steps.

Evaluation results

Proofs obligations

- 16 on the 25 proofs obligations passed,
- some proofs obligations do not passed due to rules not supported yet.

Bug founds

- 2 importants bugs in Carcara elaboration process found,
- 1 bug found in CVC5 with `and_not` rule,
- and a related bug found in the new TLAPS SMT encoding.

Issues for reconstructing simplification rules

Transformation case is implicit, and multiple transformations in a step are possible.

Rules example

$j.\quad \Delta \quad \vdash \varphi_1 \vee \cdots \vee \varphi_n \quad$ (**or_simplify**)

- $\bot \vee \cdots \vee \bot \Rightarrow \bot$
- $\varphi_1 \vee \cdots \vee \varphi_n \Rightarrow \varphi_1 \vee \cdots \vee \varphi_{n'}$ where all $\bot$ literals removed
- $\varphi_1 \vee \cdots \vee \top \vee \cdots \vee \varphi_n \Rightarrow \top$
- ...

$j.\quad \Delta \quad \vdash (\varphi_1 \approx \varphi_2) \approx \psi \quad$ (**equiv_simplify**)

- $(\neg\varphi_1 \approx \neg\varphi_2) \Rightarrow \varphi_1 \approx \varphi_2$
- $(\varphi \approx \varphi) \Rightarrow \top$
- $(\varphi \approx \bot) \Rightarrow \bot$

...

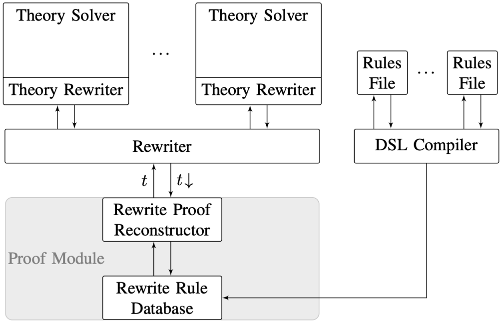
$j.\quad \Delta \quad \vdash \varphi \approx \psi \quad$ (**bool_simplify**)

- $\neg(\varphi_1 \rightarrow \neg\varphi_2) \Rightarrow \varphi_1 \wedge \neg\varphi_2$
- $\varphi_1 \rightarrow (\varphi_2 \rightarrow \varphi_3) \Rightarrow (\varphi_1 \wedge \varphi_2) \rightarrow \varphi_3$
- $\neg(\varphi_1 \rightarrow \neg\varphi_2) \Rightarrow \varphi_1 \wedge \neg\varphi_2$
- ...

$j.\quad \Delta \quad \vdash \varphi_1 \bowtie \varphi_n \approx \psi \quad$ (**comp_simplify**)

- $t < t \Rightarrow \bot$
- $t_1 < t_2 \Rightarrow \neg(t_2 \leq t_1)$
- $t_1 \leq t_2 \Rightarrow t_2 \leq t_1$
- ...

Reconstruction with RARE



[RARE] Schurr, HJ., et.al. Reliable Reconstruction of Fine-grained Proofs in a Proof Assistant. CADE 2021. Springer, Cham.

Checking linear arithmetic steps

- The `la_generic` rule models linear arithmetic reasoning
- For example, consider this `la_generic` step:

```
(step t1
    (cl (<= (- x) 1) (<= (+ (* 2 x) (* (- 3) y)) 2) (<= y (- 1)))
    :rule la_generic :args (2 1 3))
```

- It introduces the following tautology:

$$(-x \leq 1) \vee (2x - 3y \leq 2) \vee (y \leq -1)$$

Checking linear arithmetic steps

```
(step t1
    (cl (<= (- x) 1) (<= (+ (* 2 x) (* (- 3) y)) 2) (<= y (- 1)))
    :rule la_generic :args (2 1 3))
```

- Checking that this clause is true is equivalent to proving that its negation, the following three inequalites, are contradictory
- Since `la_generic` steps provide the needed coefficients as arguments, checking them is simple
- Computing $2 \cdot (a) + 1 \cdot (b) + 3 \cdot (c)$, we get $0 > 1$, so the step must be true

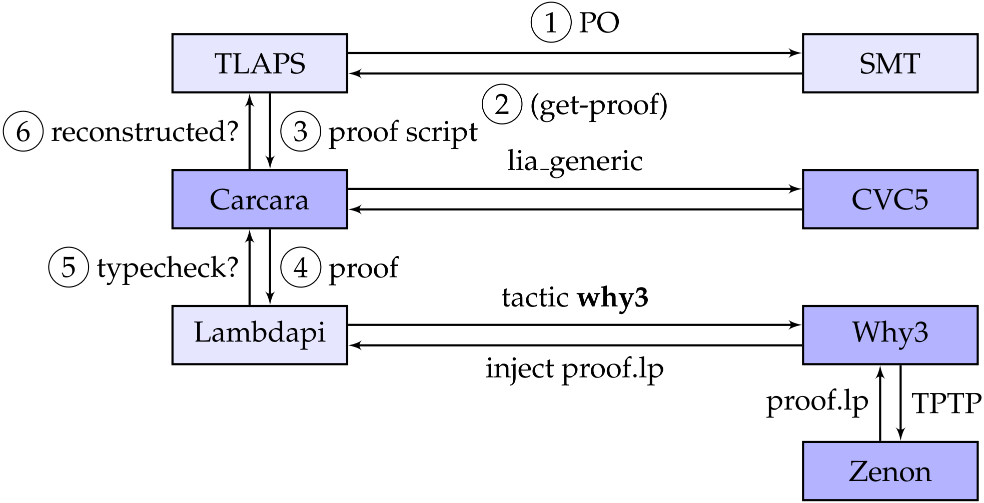
Checking linear arithmetic steps

- The `lia_generic` rule is very similar to `la_generic`, but it does not provide the coefficients as arguments:

```
(step t1
    (cl (<= (- x) 1) (<= (+ (* 2 x) (* (- 3) y)) 2) (<= y (- 1)))
    :rule lia_generic)
```

- In this case, the checker would need to search for the coefficients, which is an NP-hard problem
- Instead, occurences of this rule are not checked, and are considered holes by Carcara

Proposal for reconstructing arithmetic steps



Translation through an example

Formalisation overview

Evaluation

Blocking points

Future perspectives

Future perspectives

- Finish to validate *Allocator.tla*,
- add support for arithmetic steps,
- support for simplification steps,
- connect lambdapi TLA$^+$ encoding (*tla-lambdapi*) with TLA$^+$ SMT encoding,
- link Event-B encoding with tla-lambdapi (shared set theory library).