Sec. 1.11, Combinators: Determine if each of the following are combinators or not

1. $\lambda x.xxx$
2. $\lambda xy.zx$
3. $\lambda xyz.xy(zx)$
4. $\lambda xyz.xy(zxy)$
5. $\lambda xy.xy(zxy)$

Answer:

Combinators simply have free variables, so 2 and 5.

Sec. 1.11, Normal form or diverge?: Determine if each of the following can be reduced to a normal form or if they diverge.

1. $\lambda x.xxx$
2. $(\lambda z.zz)(\lambda y.yy)$
3. $(\lambda x.xxx)z$

Answer:

1. $\lambda x.xxx$ is in normal form, so it's convergent.
2. $(\lambda z.zz)(\lambda y.yy)$ diverges, since after a beta reduction it returns to itself.
3. $(\lambda x.xxx)z$ evaluates to zzz, so it's convergent.

Beta reduce: Evaluate (that is, beta reduce) each of the following expressions to normal form. We *strongly* recommend writing out the steps on paper with a pencil or pen.

1. $(\lambda abc.cba)zz(\lambda wv.w)$
2. $(\lambda x.\lambda y.xyy)(\lambda a.a)b$
3. $(\lambda y.y)(\lambda x.xx)(\lambda z.zq)$
4. $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$ (Hint: alpha equivalence.)
5. $(\lambda x.\lambda y.xyy)(\lambda y.y)y$
6. $(\lambda a.aa)(\lambda b.ba)c$
7. $(\lambda xyz.xz(yz))(\lambda x.z)(\lambda x.a)$

Answer:

1:

$(\lambda abc.cba)zz(\lambda wv.w)$
$(\lambda bc.cbz)z(\lambda wv.w)$
$(\lambda c.czz)(\lambda wv.w)$
$(\lambda wv.w)zz$
z.

2:

$(\lambda x.\lambda y.xyy)(\lambda a.a)b$
$(\lambda y.(\lambda a.a)yy)b$
$(\lambda a.a)bb$
bb.

3:

$(\lambda y.y)(\lambda x.xx)(\lambda z.za)$
$(\lambda x.xx)(\lambda z.za)$
$(\lambda z.za)(\lambda z.za)$
$(\lambda z.za)a$
$qq.$

4:

$(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$
$(\lambda y.y)(\lambda x.xx)(\lambda z.zy)$
$\ldots$
$yy.$

5:

$(\lambda x.\lambda y.xyy)(\lambda y.y)y$
$(\lambda y.(\lambda y.y)yy)y$
$(\lambda y.y)yy$
$yy.$

6:

$(\lambda a.aa)(\lambda b.ba)c$
$(\lambda b.ba)(\lambda b.ba)c$
$(\lambda b.ba)ac$
$aac.$

7:

$(\lambda xyz.xz(yz))(\lambda x.z)(\lambda x.a)$
$(\lambda xyq.xq(yq))(\lambda x.z)(\lambda x.a)$
$(\lambda yq.(\lambda x.z)q(yq))(\lambda x.a)$
$\lambda q.(\lambda x.z)q((\lambda x.a)q)$
$\lambda q.z((\lambda x.a)q)$
$\lambda q.za.$