LAB 9: Non-linear Regression

Shinhoo Kang

April 14, 2017

Non-linear regression

Consider the following nonlinear function

$$y = a_1 x e^{a_2 x}.$$

Non-linear regression

Consider the following nonlinear function

$$y = a_1 x e^{a_2 x}.$$

Estimate the parameters, a_1 and a_2

such that this expression fits a set of x,y data as closely as possible.

$$(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n),$$

Non-linear regression

Consider the following nonlinear function

$$y = a_1 x e^{a_2 x}.$$

Estimate the parameters, a_1 and a_2

such that this expression fits a set of x,y data as closely as possible.

$$(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n),$$

Q. How can we know whether our estimation is good or not?

Optimization

Define the error as follows:

$$Sr = \sum_{i}^{n} r_i^2 = \sum_{i}^{n} (y_i - y)^2 = \sum_{i}^{n} (y_i - a_1 x_i e^{a_2 x_i})^2$$

Optimization

Define the error as follows:

$$Sr = \sum_{i}^{n} r_i^2 = \sum_{i}^{n} (y_i - y)^2 = \sum_{i}^{n} \left(y_i - a_1 x_i e^{a_2 x_i}\right)^2$$

Determine the parameters, a_1 and a_2

that minimize the error Sr.

$$\min_{a_1, a_2} Sr$$

Optimization

Define the error as follows:

$$Sr = \sum_{i}^{n} r_i^2 = \sum_{i}^{n} (y_i - y)^2 = \sum_{i}^{n} (y_i - a_1 x_i e^{a_2 x_i})^2$$

Determine the parameters, a_1 and a_2

that minimize the error Sr.

$$\min_{a_1, a_2} Sr$$

Regression problem $\Rightarrow$ Optimization problem

Optimization

Define the error as follows:

$$Sr = \sum_i^n r_i^2 = \sum_i^n (y_i - y)^2 = \sum_i^n \left(y_i - a_1 x_i e^{a_2 x_i}\right)^2$$

Determine the parameters, a_1 and a_2

that minimize the error Sr.

$$\min_{a_1, a_2} Sr$$

Regression problem $\Rightarrow$ Optimization problem
Q. How can we find the a_1 and a_2?
What is the condition for minimizer?

Optimization

A necessary condition for minimizer

$$\frac{\partial Sr}{\partial a_1} = 0,$$
$$\frac{\partial Sr}{\partial a_2} = 0.$$

Optimization

A necessary condition for minimizer

$$\frac{\partial Sr}{\partial a_1} = 0,$$
$$\frac{\partial Sr}{\partial a_2} = 0.$$

Determine the parameters, a_1 and a_2

such that

$$\nabla Sr(a_1, a_2) = 0$$

Optimization

A necessary condition for minimizer

$$\frac{\partial Sr}{\partial a_1} = 0,$$
$$\frac{\partial Sr}{\partial a_2} = 0.$$

Determine the parameters, a_1 and a_2

such that

$$\nabla Sr(a_1, a_2) = 0$$

Q. What kind of method can we use?

Gradient Descent method

Today we consider Gradient Descent method.

Update

$$x^{(k+1)} = x^{(k)} - \alpha \nabla f(x^{(k)}),$$
$$a^{(k+1)} = a^{(k)} - \alpha \nabla Sr(a^{(k)}).$$

Gradient Descent method

Today we consider Gradient Descent method.

Update

$$x^{(k+1)} = x^{(k)} - \alpha \nabla f(x^{(k)}),$$
$$a^{(k+1)} = a^{(k)} - \alpha \nabla Sr(a^{(k)}).$$

Ensure: Given initial guess $a^{(0)}$, compute $\nabla Sr(a^{(0)})$.

1: **while** not converged, $||\nabla Sr|| > tol$ **do**
2: Compute alpha α using backtracking algorithm.
3: Update $a^{(k+1)}$.
4: **end while**

backtracking algorithm

if $Sr(a^{(k+1)}) <= (Sr(a^{(k)} - 0.001\alpha \nabla Sr(a^{(k)})^T Sr(a^{(k)})$ **then**
    break
else
    $\alpha = \alpha/2$
end if