99. Draw a line from each of the five functions in the center to the best big-Ω value on the left, and the best big-O value on the right.

$\Omega(1/n)$		$O(1/n)$
$\Omega(1)$		$O(1)$
$\Omega(\log\log n)$		$O(\log\log n)$
$\Omega(\log n)$		$O(\log n)$
$\Omega(\log^2 n)$		$O(\log^2 n)$
$\Omega(\sqrt[3]{n})$		$O(\sqrt[3]{n})$
$\Omega(n/\log n)$	$1/(\log n)$	$O(n/\log n)$
$\Omega(n)$	$7n^5 - 3n + 2$	$O(n)$
$\Omega(n^{1.00001})$	$(n^2+n)/(\log^2 n + \log n)$	$O(n^{1.00001})$
$\Omega(n^2/\log^2 n)$	$2^{\log^2 n}$	$O(n^2/\log^2 n)$
$\Omega(n^2/\log n)$	3^n	$O(n^2/\log n)$
$\Omega(n^2)$		$O(n^2)$
$\cancel{\Omega(n^{3/2})}$		$\cancel{O(n^{3/2})}$
$\Omega(2^n)$		$O(2^n)$
$\Omega(5^n)$		$O(5^n)$
$\Omega(n^n)$		$O(n^n)$
$\Omega(n^{n^2})$		$O(n^{n^2})$

$\Omega(n^{3/2})$ $O(n^{3/2})$

$1/(\log n)$ 은 $\Omega(1/n)$, $O(1)$

$7n^5-3n+2$ 은 n^5만 넘음, $\Omega(n^2)$, $O(2^n)$

$(n^2+n)/(\log^2 n+\log n)$ 은 $n^2/\log^2 n$, $n^2/\log n$ 에 걸침, $\Omega(n^2/\log^2 n)$, $O(n^2/\log n)$

$2^{\log^2 n}$ 는 2^n보다 느리고, n보다 빠르다. $\Omega(2^n)$ $O(5^n)$

3^n 은 $\Omega(2^n)$, $O(5^n)$

For each of the following pairs of functions $f(n)$ and $g(n)$, find $c \in \mathbb{R}^+$ such that $f(n) \le c \cdot g(n)$ for all $n > 1$.

163. $f(n) = n^2 + n$, $g(n) = n^2$.

$f(2) = 6 \quad g(2) = 4 \qquad \frac{f(2)}{g(2)} = \frac{3}{2}$

$f(3) = 12 \quad g(3) = 9 \qquad \frac{f(3)}{g(3)} = \frac{4}{3}$

$f(4) = 20 \quad g(4) = 16 \qquad \frac{f(4)}{g(4)} = \frac{5}{4} \qquad \frac{f(n)}{g(n)} = 1 + \frac{1}{n}$

$c = \left\lceil \frac{3}{2} \right\rceil = 2$

164. $f(n) = 2\sqrt{n} + 1$, $g(n) = n + n^2$.

$f(2) = 2\sqrt{2} + 1 \quad g(2) = 6 \qquad \frac{f(2)}{g(2)} = \frac{6}{3{,}8xx} \qquad \frac{f(n)}{g(n)} = 1 + \frac{1}{n}$

$f(3) = 2\sqrt{3} + 1 \quad g(3) = 12$

$f(4) = 4 + 1 = 5 \quad g(4) = 20 \qquad \frac{f(4)}{g(4)} = 4 \qquad c = \left\lceil 1 + \frac{1}{n} \right\rceil = 2$

165. $f(n) = n^2 + n + 1$, $g(n) = 2n^3$.

$f(2) = 7 \quad g(2) = 16 \qquad \frac{f(2)}{g(2)} = \frac{7}{16} \qquad \frac{f(n)}{g(n)} = 2\left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}\right)$

$f(3) = 13 \quad g(3) = 54 \qquad \frac{f(3)}{g(3)} = \frac{13}{54}$

$c = \left\lceil \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} \right\rceil = 1$

166. $f(n) = n\sqrt{n} + n^2$, $g(n) = n^2$.

$f(2) = 2\sqrt{2} + 4 \qquad \frac{f(2)}{g(2)} = 1 + \frac{\sqrt{2}}{2} \qquad \frac{f(n)}{g(n)} = n + \frac{\sqrt{n}}{n}$

$g(2) = 4$

$f(3) = 3\sqrt{3} + 9 \qquad \frac{f(3)}{g(3)} = 1 + \frac{\sqrt{3}}{3}$

$g(3) = 9 \qquad c = \left\lceil 1 + \frac{\sqrt{n}}{n} \right\rceil = 2$

2

167. $f(n) = 12n + 3,\ g(n) = 2n - 1.$

$f(2) = 27$ $\quad \dfrac{f(2)}{g(2)} = 9$ $\quad \dfrac{f(n)}{g(n)} = \dfrac{12n+3}{2n-1}$ $\quad \lim_{n\to\infty} \dfrac{12n+3}{2n-1} = 6$

$g(2) = 3$

$f(3) = 39$ $\quad \dfrac{f(3)}{g(3)} = \dfrac{39}{5}$ $\quad c = 6$

$g(3) = 5$

168. $f(n) = n^2 - n + 1,\ g(n) = n^2/2.$

$f(2) = 3$ $\quad g(2) = 2$ $\quad \dfrac{f(2)}{g(2)} = \dfrac{3}{2}$ $\quad \lim_{n\to\infty} \dfrac{f(n)}{g(n)} = 2$ $\quad c = 2$

$f(3) = 7$ $\quad g(3) = \dfrac{9}{2}$ $\quad \dfrac{f(3)}{g(3)} = \dfrac{14}{9}$

169. $f(n) = 5n + 1,\ g(n) = (n^2 - 6n)/2.$

$f(2) = 11$ $\quad g(2) = -4$ $\quad n^2 - 6n > 0$ 이므로 $n > 6$

$n = 7$ 일때부터

$f(7) = 36$ $\quad g(7) = \dfrac{7}{2}$ $\quad \dfrac{f(7)}{g(7)} = \dfrac{72}{7} = 10.x$ $\quad c = \lceil 10.x \rceil = 11$

$f(8) = 41$ $\quad g(8) = 8$ $\quad \dfrac{f(8)}{g(8)} = \dfrac{41}{8} = 5.125$ $\quad n = 7$

$f(9) = 46$ $\quad g(9) = \dfrac{27}{2}$ $\quad \dfrac{f(9)}{g(9)} = \dfrac{92}{27} = 3.x$

170. $f(n) = 5\lfloor\sqrt{n}\rfloor - 1,\ g(n) = n - \lceil\sqrt{n}\rceil.$

$g(2) = 2 - \lceil 1.414\ldots \rceil = 0$ $\quad n = 3$ 부터 가능

$g(3) = 3 - \lceil 1.732x \rceil = 1$

$f(3) = 4$ $\quad \dfrac{f(3)}{g(3)} = 4$ $\quad f(5) = 9$ $\quad \dfrac{f(5)}{g(5)} = 4.5$ $\quad c = \lceil 4.5 \rceil = 5$

$g(5) = 2$

$f(4) = 9$ $\quad \dfrac{f(4)}{g(4)} = 4.5$ $\quad f(6) = 9$ $\quad \dfrac{f(6)}{g(6)} = 3$

$g(4) = 2$ $\quad g(6) = 3$

$f(7) = 14$ $\quad g(7) = 6$ $\quad \dfrac{f(7)}{g(6)} = \dfrac{14}{6} = 2.xx$

$\boxed{n = 3 \quad c = 5}$