Optimized Collatz-like Operation with Large `k`

Problem Summary

We are given three integers: x, y, k. For k operations, do the following repeatedly:

- Increment x by 1.
- While $x \mod y = 0$, divide x by y.

Constraints: $1 \leq x, k \leq 10^9$, $2 \leq y \leq 10^9$.
Simulating all k steps naively takes too long, so an optimized approach is required.

Optimization Strategy

Step 1: Predicting the Next Divisible Point

Suppose at a given point, x is not divisible by y. We want to find the next value of x such that:

$$x + \text{need} \equiv 0 \mod y$$

That is, we want the next multiple of y after x. This can be calculated as:

$$\text{next\_divisible} = y \cdot \left\lceil \frac{x}{y} \right\rceil$$

So the number of additions required to reach this point is:

$$\text{need} = y \cdot \left\lceil \frac{x}{y} \right\rceil - x$$

Case 1: If need $> k$, then we can simply do k additions:

$$\text{final } x = x + k$$

Step 2: When $x \leq y$

In this case, we note that when x becomes 1, the divisions cycle repeat every $y - 1$. So we can reduce k modulo this cycle length:

$$x = 1 + (k \mod (y - 1))$$

We then do divisions if x is divisible by y:

$$\text{while } x \mod y = 0 \Rightarrow x = \frac{x}{y}$$

Step 3: Looping with Reduction

If need $\leq k$, we perform:

$$x = x + \text{need}, \quad k = k - \text{need}$$

Then reduce x by dividing it by y repeatedly:

$$\text{while } x \mod y = 0 \Rightarrow x = \frac{x}{y}$$

Repeat the process until $k = 0$ or we reach one of the above special cases.

Why It Is Fast

- Avoids looping k times.
- Uses ceiling and modulo arithmetic to jump over many redundant steps.
- Applies repeated division only when needed, which takes $\log_y x$ time at most.
- Reduces k drastically by skipping ahead to nearest divisible numbers.