Mathematical Explanation of Optimized Approach

Problem Statement

Given three integer points x_1, x_2, x_3 on the X-axis (where $1 \leq x_i \leq 10$), choose an integer point a such that the total distance from each of the points to a is minimized. Define this total distance as:

$$f(a) = |x_1 - a| + |x_2 - a| + |x_3 - a|$$

We aim to find the value of a that minimizes $f(a)$.

Mathematical Insight

The function $f(a)$ is a sum of absolute differences. A well-known property in mathematics is:

> The value of a that minimizes the sum of absolute differences from a set of numbers is the **median** of that set.

Why the Median Minimizes Total Absolute Distance

Let x_1, x_2, x_3 be sorted such that:

$$x_{\min} \leq x_{\text{mid}} \leq x_{\max}$$

Then the sum of distances from any point a is:

$$f(a) = |x_{\min} - a| + |x_{\text{mid}} - a| + |x_{\max} - a|$$

We analyze the behavior of $f(a)$:

- If $a < x_{\min}$: all differences are increasing.
- If $a > x_{\max}$: again, all differences are increasing.
- The total distance $f(a)$ is minimized when $a = x_{\text{mid}}$, the median.

Example

Let $x_1 = 1, x_2 = 2, x_3 = 3$. The median is 2.

$$f(2) = |1 - 2| + |2 - 2| + |3 - 2| = 1 + 0 + 1 = 2$$

Any other choice of a will result in a higher value.

C Code: Finding the Median Using Sorting

To implement this efficiently in C, we sort the three numbers and select the middle one:

Code Snippet

```
void sort(int* a, int* b, int* c) {
    if (*a > *b) { int t = *a; *a = *b; *b = t; }
    if (*b > *c) { int t = *b; *b = *c; *c = t; }
    if (*a > *b) { int t = *a; *a = *b; *b = t; }
}
```

Explanation

This function uses basic pairwise comparisons and swaps to sort the three integers in non-decreasing order:

- First, compare `*a` and `*b`; if out of order, swap them.
- Then, compare `*b` and `*c`; if out of order, swap them.
- Finally, compare `*a` and `*b` again in case the second swap disturbed the earlier order.

After this process:

$$*a \leq *b \leq *c$$

Thus, the median is stored in `*b`.

Conclusion

Therefore, for any three integers x_1, x_2, x_3, the minimum value of:

$$f(a) = |x_1 - a| + |x_2 - a| + |x_3 - a|$$

is achieved when a is the **median** of x_1, x_2, x_3. We compute it efficiently using the sorting function shown above. The final result is:

$$\boxed{f(a) = |x_1 - \text{med}| + |x_2 - \text{med}| + |x_3 - \text{med}|}$$

where $\text{med} = \text{median}(x_1, x_2, x_3)$.