

Problem: Alice and Bob take turns removing coins from their wallet. They can optionally swap wallets before removing 1 coin per turn. The player who cannot remove a coin loses.

Observation: Each move removes exactly one coin. Therefore, the total number of valid moves is:

$$T = a + b$$

where a is Alice's starting coins and b is Bob's.

Turns: The game starts with Alice. The players alternate turns: Turn 1 : Alice

Turn 2 : Bob

Turn 3 : Alice

\vdots

Turn T : ?

Winning Condition: The player who makes the last move (i.e., performs the T -th move) wins.

- If $T = a + b$ is odd, then Alice plays the last move. So, **Alice wins**.
- If $T = a + b$ is even, then Bob plays the last move. So, **Bob wins**.

$$Winner = \{ Alice, if(a + b) \bmod 2 = 1 Bob, if(a + b) \bmod 2 = 0 \}$$