Design and Analysis of Algorithm

Dynamic Programming (Matrix Chain Multiplication)



Lecture - 58

Overview

- Dynamic Programming is a general algorithm design technique for solving problems defined by recurrences with overlapping subproblems
- Invented by American mathematician "Richard Bellman) in the year 1950s to solve optimization problems and later assimilated by Computer Science.
- "Programming" here means "planning"

- "Method of solving complex problems by breaking them down into smaller sub-problems, solving each of those sub-problems just once, and storing their solutions."
- The problem solving approach looks like Divide and conquer approach.(which is not true)

Difference between Dynamic programming and Divide and Conquer approach.

| Divide & Conquer | Dynamic Programming |
|---|--|
| Partitions a problem into independent smaller sub-problems | Partitions a problem into overlapping sub-problems |
| Doesn't store solutions of sub- problems. (Identical sub-problems may arise - results in the same computations are performed repeatedly.) | Stores solutions of sub- problems: thus avoids calculations of same quantity twice |
| 3. Top down algorithms: which logically progresses from the initial instance down to the smallest sub-instances via intermediate sub-instances. | 3. Bottom up algorithms: in which the smallest sub-problems are explicitly solved first and the results of these used to construct solutions to progressively larger sub-instances |

Is a Four-step methods

- 1. Characterize the structure of an optimal solution.
- 2. Recursively define the value of an optimal solution.
- 3. Compute the value of an optimal solution, typically in a bottom-up fashion.
- 4. Construct an optimal solution from computed information.

Problems:

- 1. 0/1 Knapsack Problem
- 2. Floyd-Warshall Algorithm
- 3. Longest Common Sub-sequence
- 4. Matrix Chain Multiplication

Problem 4: Matrix Chain Multiplication

Problem:

"Given dimensions $p_0, p_1, p_2, \ldots, p_n$ corresponding to matrix sequence $\langle A_1, A_2, \ldots, A_n \rangle$ of n matrices, where for $i=1,2,\ldots,n$, matrix A_i has dimension $p_{i-1} \times p_i$, determine the "multiplication sequence" that minimizes the number of scalar multiplications in computing $\langle A_1, A_2, \ldots, A_n \rangle$."

Problem 4: Matrix Chain Multiplication

Problem:

That is determine how to parenthesize the multiplication.

Example:

$$A_{1}, A_{2}, A_{3}, A_{4} = ((A_{1}A_{2})(A_{3}A_{4}))$$

$$(A_{1}(A_{2}(A_{3}A_{4})))$$

$$(A_{1}((A_{2}A_{3})A_{4}))$$

$$(((A_{1}A_{2})(A_{3}A_{4}))$$

$$(((A_{1}A_{2})A_{3})A_{4})$$

Problem 4: Matrix Chain Multiplication

Given a $p \times q$ matrix A, a $q \times r$ matrix B and a $r \times s$ matrix C, then ABC can be computed in two ways (AB)C and A(BC):

The number of multiplications needed are:

```
mult[(AB)C] = pqr + prs,

mult[A(BC)] = qrs + pqs.
```

When p = 5, q = 4, r = 6 and s = 2, then

```
mult[(AB)C] = 180,
mult[A(BC)] = 88.
```

Which is a big difference. Hence the implication is the the multiplication "sequence" (parenthesization) is very important.

Problem 4: Matrix Chain Multiplication

Step 1: Characterize the structure of an optimal solution

- Decompose thee problem into subproblems:
 - For each pair $1 \le i \le j \le n$, determine the multiplication sequence for $A_{i...j} = A_i, A_{i+1}, \dots, A_j$ that minimize the number of multiplications.
 - Clearly, $A_{i...i}$ is a $p_{i-1} \times p_i$ matrix.
- High-Level Parenthesization for $A_{i...j}$
 - For any optimal multiplication sequence, at the last step you are multiplying two matrices $A_{i...k}$ and $A_{k+1...j}$ for some k. That is,

$$A_{i...j} = (A_i ... A_k) (A_{k+1} ... A_j) = A_{i...k} A_{k+1...j}$$

Example

$$A_{3..6} = ((A_3(A_4A_5))(A_6)) = A_{3..5}A_{6..6}$$
. (Here $k = 5$.)

Problem 4: Matrix Chain Multiplication

Step 1: Characterize the structure of an optimal solution

- Thus, the problem of determining the optimal sequence of multiplications is divided into 2 questions:
 - How do we decide where to split the chain (what is the value of k)? (Search all possible values of k)
 - How do we parenthesize the sub chains $A_{i..k}$ and $A_{k+1..j}$?

(Problem has optimal substructure property that $A_{i..k}$ and $A_{k+1..j}$ must be optimal so the same procedure can be applied recursively)

Problem 4: Matrix Chain Multiplication

Step 1: Characterize the structure of an optimal solution

- What is Optimal Substructure Property?
 - If final "optimal" solution of $A_{i...j}$ involves splitting into $A_{i...k}$ and $A_{k+1...j}$ at final step then parenthesization of $A_{i...k}$ and $A_{k+1...j}$ in final optimal solution must also be optimal for the subproblems "standing alone":
 - If parenthisization of $A_{i..k}$ was not optimal we could replace it by a better parenthesization and get a cheaper final solution, leading to a contradiction.
 - Similarly, if parenthisization of $A_{k+1...j}$ was not optimal we could replace it by a better parenthesization and get a cheaper final solution, also leading to a contradiction.

Problem 4: Matrix Chain Multiplication

Step 2: Recursively define the value of optimal solution.

• For $1 \le i \le j \le n$, let m[i,j] denote the minimum number of multiplications needed to compute $A_{i...j}$. The optimum cost can be described by the following recursive definition.

$$m[i,j] = \begin{cases} 0 & \text{if } i = j \\ \min_{i \le k < j} \{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j\} & \text{if } i < j \end{cases}$$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

```
MATRIX - CHAIN - ORDER(p)
 1 n \leftarrow length[p] - 1
2 let m[1...n, 1...n] and s[1...n - 1, 2...n] be new tables.
3 \ for \ i \leftarrow 1 \ to \ n
4 m[i,i] \leftarrow 0
5 \ for \ l \leftarrow 2 \ to \ n \triangleright l \ is \ the \ chain \ length.
    for i \leftarrow 1 to n - l + 1
j \leftarrow i + l - 1
8 m[i,j] \leftarrow \infty
9 for k \leftarrow i to j - 1
10
             q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j
11
            if q < m[i,j]
12
                 then m[i,j] \leftarrow q
13
                       s[i,j] \leftarrow k
14 return m and s
```

Lets illustrate the example with the help of an example.

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

(30, 35, 15, 5, 10, 20, 25)

Solution:

Here

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

(30, 35, 15, 5, 10, 20, 25)

Solution:

Here

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

| Matrix | A_1 | A_2 | A_3 | A_4 | A_5 | A_6 |
|------------|---------|---------|--------|--------|---------|---------|
| Dimensions | 30 x 35 | 35 x 15 | 15 x 5 | 5 x 10 | 10 x 20 | 20 x 25 |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|----|------|----|----|---|
| 1 | 0 | | | | | |
| 2 ⁴ / ₂ | | 0 | | | | |
| 3 | 3 | | 0 | | | |
| 4 | | Z3 | | 0 | | |
| 5 | | | A | | 0 | |
| 6 | | | | 35 | | 0 |
| | | m | matı | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| | | | p_3 | | | _ |
|----|----|----|-------|----|----|----|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|------|----|----|---|
| 1 | 0 | | | | | |
| 1 2 ⁷ / ₂ 3 | | 0 | | | | |
| 3 | 3 | | 0 | | | |
| 4 | | 3 | | 0 | | |
| 5 | | | A | | 0 | |
| 6 | | | | 75 | | 0 |
| | | m | matı | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

$$1 n \leftarrow length[p] - 1$$

2 let
$$m[1..n, 1..n]$$
 and $s[1..n - 1, 2..n]$ be new tables.

$$3 for i \leftarrow 1 to n$$

4
$$m[i,i] \leftarrow 0$$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

11

13

| _ | 1 | 2 | 3 | 4 | 5 | 6 |
|----|-----|----|------|---|---|---|
| 1 | 0 | 8 | | | | |
| 27 |) | 0 | | | | |
| 3 | (3) | | 0 | | | |
| 4 | | 33 | | 0 | | |
| 5 | | | A | | 0 | |
| 6 | | | | 3 | | 0 |
| | | m | mati | | 7 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

m matrix \circ 5 for $l \leftarrow 2$ to n6 for $i \leftarrow 1$ to n - l + 17 $j \leftarrow i + l - 1$ 8 $m[i,j] \leftarrow \infty$ 9 for $k \leftarrow i$ to j - 110 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_i$

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

if q < m[i,j]

$$l = 2$$
, $i = 1, j = 2, k = 1$
 $q =$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p ₁ | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-----------------------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| _ | 1 | 2 | 3 | 4 | 5 | 6 |
|----|--------------|-------|------|----|----|---|
| 1 | 0 | 15750 | | | | |
| 23 | $)_{\frown}$ | 0 | | | | |
| 3 | (3) | | 0 | | | |
| 4 | | 73 | | 0 | | |
| 5 | | | A | | 0 | |
| 6 | | | | 75 | | 0 |
| | | m | matı | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

$$l = 2$$
 , $i = 1, j = 2, k = 1$
 $q = 0 + 0 + 30 * 35 * 15 = 15750$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|----|----|---|
| 1 | 0 | 15750 | | | | |
| 2 ⁴ / ₂ | | 0 | 8 | | | |
| 3 | (3) |) | 0 | | | |
| 4 | | (3) | | 0 | | |
| 5 | | | A | | 0 | |
| 6 | | | | 75 | | 0 |
| | | m | matı | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|----|----|---|
| 1 | 0 | 15750 | | | | |
| 2 ⁷ / ₂ | | 0 | 2625 | | | |
| 3 | (3) |) | 0 | | | |
| 4 | | (3) | | 0 | | |
| 5 | | | A | | 0 | |
| 6 | | | | 75 | | 0 |
| | | m | mati | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

$$5 for l \leftarrow 2 to n$$

$$6 for i \leftarrow 1 to n - l + 1$$

$$7 j \leftarrow i + l - 1$$

$$8 m[i,j] \leftarrow \infty$$

$$9 for k \leftarrow i to j - 1$$

$$10 q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$$

$$11 if q < m[i,j]$$

$$12 then m[i,j] \leftarrow q$$

$$13 s[i,j] \leftarrow k$$

$$l = 2$$
, $i = 2, j = 3, k = 2$
 $q = 0 + 0 + 30 * 15 * 5 = 2625$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|----|----|---|
| 1 | 0 | 15750 | | | | |
| 2 ⁷ / ₂ | | 0 | 2625 | | | |
| 3 | 3 | | 0 | 8 | | |
| 4 | | (3) |) | 0 | | |
| 5 | | | (A | | 0 | |
| 6 | | | | 3, | | 0 |
| | | m | matı | | 76 | - |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|-----|----|---|
| 1 | 0 | 15750 | | | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | | | |
| 3 | 3 | | 0 | 750 | | |
| 4 | | (3) |) | 0 | | |
| 5 | | | (A | | 0 | |
| 6 | | | | Z | | 0 |
| | | m | mati | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

l = 2, i = 3, j = 4, k = 3

q = 0 + 0 + 15 * 5 * 10 = 750

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|------|----|---|
| 1 | 0 | 15750 | | | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | | | |
| 3 | 3 | | 0 | 750 | | |
| 4 | | 3 | | 0 | 8 | |
| 5 | | | (A |) | 0 | |
| 6 | | | | (3/2 | | 0 |
| | | m | mati | | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | | |
| 5 | | | | | |

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|-----|------|---|
| 1 | 0 | 15750 | | | | |
| 2 ⁷ / ₂ | | 0 | 2625 | | | |
| 3 | 3 | | 0 | 750 | | |
| 4 | | 3 | | 0 | 1000 | |
| 5 | | | A A | | 0 | |
| 6 | | | | 75 | | 0 |
| | | m | mati | rix | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | · | |

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

$$l = 2$$
, $i = 4$, $j = 5$, $k = 4$
 $q = 0 + 0 + 5 * 10 * 20 = 1000$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|-------|------|-----|------|----------|
| 1 | 0 | 15750 | | | | |
| 1 2 ⁷ / ₂ 3 | _ | 0 | 2625 | | | |
| 3 | 3 | | 0 | 750 | | |
| 4 | | 3 | | 0 | 1000 | |
| 5 | | | A | | 0 | ∞ |
| 6 | | | | 75 |) | 0 |
| | | m | mati | | (75) | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | |

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

$$l = 2$$
, $i = 4$, $j = 5$, $k = 4$
 $q =$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|-----|------|------|
| 1 | 0 | 15750 | | | | |
| 2 ⁴ / ₂ | _ | 0 | 2625 | | | |
| 3 | 3 | | 0 | 750 | | |
| 4 | | 3 | | 0 | 1000 | |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 75 |) | 0 |
| | |) | | | | |

| | _ 2 | 3 | 4 | 5 | 6 |
|---|-----|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

$$5 for l \leftarrow 2 to n$$

$$6 for i \leftarrow 1 to n - l + 1$$

$$7 j \leftarrow i + l - 1$$

$$8 m[i,j] \leftarrow \infty$$

$$9 for k \leftarrow i to j - 1$$

$$10 q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$$

$$11 if q < m[i,j]$$

$$12 then m[i,j] \leftarrow q$$

$$13 s[i,j] \leftarrow k$$

$$l = 2$$
, $i = 4$, $j = 5$, $k = 4$
 $q = 0 + 0 + 10 * 20 * 25 = 5000$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|--------------|-------|------|-----|------|------|
| 1 | 0 | 15750 | 8 | | | |
| 27 | $)_{\frown}$ | 0 | 2625 | | | |
| 3 | 13 | | 0 | 750 | | |
| 4 | | 33 | | 0 | 1000 | |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | | | • | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

m matrix

s matrix

q = 0 + 2625 + 30 * 35 * 5 = 7875

l = 3, i = 1, j = 3, k = 1

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|------|-----|------|------|
| 1 | 0 | 15750 | 8 | | | |
| 33 | | 0 | 2625 | | | |
| 3 | T. | | 0 | 750 | | |
| 4 | | (3) |) | 0 | 1000 | |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | | | | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

m matrix

 $s[i, j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p ₁ | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-----------------------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|------|-----|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 53 |) | 0 | 2625 | | | |
| 3 | 13 | | 0 | 750 | | |
| 4 | | 33 |) | 0 | 1000 | |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 7 | | 0 |
| | | | _ | | ~ | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

m matrix

 $s[i, j] \leftarrow k$

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
1 $q = 0 + 2625 + 30 * 35 * 5 = 7875$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
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1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
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1 $l = 3$, $l = 1, j = 3, k = 2$
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1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
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1 $l = 3$, $l = 1, j = 3, k = 2$
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1 $l = 3$, $l = 1, j = 3, k = 2$
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1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
1 $l = 3$, $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
3 $l = 1, j = 3, k = 2$
4 $l = 1, j = 3, k = 2$
2 $l = 1, j = 3, k = 2$
3 $l = 1, j = 3, k = 2$
4 $l = 1, j = 1, j = 3, k = 2$
4 $l = 1, j = 1, j = 3, k = 2$
4 $l = 1, j = 1, j = 3, k = 2$
4 $l = 1, j = 1, j = 3, k = 2$
4 l

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|-----|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 2 ⁷ / ₂ | | 0 | 2625 | 8 | | |
| 3 | (3) |) | 0 | 750 | | |
| 4 | | 3 | | 0 | 1000 | |
| 5 | | | Z Z |) | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | m | mati | | 70 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

q = 0 + 750 + 35 * 15 * 10 = 6000

l = 3, i = 2, j = 4, k = 2

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|-----|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 27 | | 0 | 2625 | 8 | | |
| 2 ⁴ / ₂ | (3) | | 0 | 750 | | |
| 4 | | Z3 |) | 0 | 1000 | |
| 5 | | | (A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

m matrix

 $s[i, j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|-------------------|------|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 2 ⁷ / ₂ | | 0 | 2625 | 4375 | | |
| 3 | (3) | | 0 | 750 | | |
| 4 | | Z3 | $)$ $\overline{}$ | 0 | 1000 | |
| 5 | | | (A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | | | | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

m matrix

then $m[i, j] \leftarrow q$

 $s[i, j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|---|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 27 | _ | 0 | 2625 | 4375 | | |
| 3 | 3 | | 0 | 750 | 8 | |
| 4 | | (3) | | 0 | 1000 | |
| 5 | | | (A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | m | mati | rix | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

$$5 \text{ for } l \leftarrow 2 \text{ to } n$$

 $6 \text{ for } i \leftarrow 1 \text{ to } n - l + 1$
 $7 \text{ } j \leftarrow i + l - 1$
 $l = 3, j = 5, k = 3$
 $q = 0 + 1000 + 15 * 5 * 20 = 2500$

10
$$q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$$

11 $if \ q < m[i,j]$

for $k \leftarrow i$ to j-1

 $m[i,j] \leftarrow \infty$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|----|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 2 ⁴ / ₂ | | 0 | 2625 | 4375 | | |
| 3 | 43 | | 0 | 750 | ∞ | |
| 4 | | (3) | | 0 | 1000 | |
| 5 | | | A A | | 0 | 5000 |
| 6 | | | | (32 | | 0 |
| m matrix 7 | | | | | | |

then $m[i, j] \leftarrow q$

 $s[i, j] \leftarrow k$

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$

$$l = 3, i = 3, j = 5, k = 3$$

$$l = 3, i = 3, j = 5, k = 4$$

$$q = 750 + 0 + 15 * 10 * 20 = 3750$$

$$l = 3$$
, $l = 3$, $j = 5$, $k = 5$
 $q = 0 + 1000 + 15 * 5 * 20 = 2500$
 $l = 3$, $i = 3$, $j = 5$, $k = 4$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | 4375 | | |
| 3 | 3 | | 0 | 750 | 2500 | |
| 4 | | (3) |) | 0 | 1000 | |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | m | mati | rix | Z | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

 $5 for l \leftarrow 2 to n$ $6 for i \leftarrow 1 to n - l + 1$ $7 j \leftarrow i + l - 1$ $8 m[i,j] \leftarrow \infty$ $9 for k \leftarrow i to j - 1$ $10 q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$ 11 if q < m[i,j] $12 then m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

$$l = 3$$
, $i = 3$, $j = 5$, $k = 3$
 $q = 0 + 1000 + 15 * 5 * 20 = 2500$

s matrix

$$\begin{array}{ll} lifting (i,j) \\ pr(k) \leftarrow i \ to \ j-1 \\ q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \end{array} \quad \begin{array}{ll} l=3, \ i=3, \ j=5, \ k=4 \\ q=750+0+15*10*20=3750 \end{array}$$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|-------------------|------|----------|
| 1 | 0 | 15750 | 7875 | | | |
| 2 ⁴ / ₂ | _ | 0 | 2625 | 4375 | | |
| 3 | 3 | | 0 | 750 | 2500 | |
| 4 | | 3 | | 0 | 1000 | ∞ |
| 5 | | | A | $)$ $\overline{}$ | 0 | 5000 |
| 6 | | | | (3 | | 0 |
| 7 | 2.4 | m | mati | rix | 76 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

$$l = 3$$
, $i = 4$, $j = 6$, $k = 4$
 $q = 0 + 5000 + 5 * 10 * 25 = 6250$

$$5 for l \leftarrow 2 to n$$

$$6 for i \leftarrow 1 to n - l + 1$$

$$7 j \leftarrow i + l - 1$$

$$8 m[i,j] \leftarrow \infty$$

$$9 for k \leftarrow i to j - 1$$

$$10 q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$$

$$11 if q < m[i,j]$$

$$12 then m[i,j] \leftarrow q$$

$$13 s[i,j] \leftarrow k$$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|-------------------------------|----------|-------|------|------|------|----------|--|
| 1 | 0 | 15750 | 7875 | | | | |
| 2 ⁴ / ₂ | _ | 0 | 2625 | 4375 | | | |
| 3 | 3 | | 0 | 750 | 2500 | | |
| 4 | | 33 | | 0 | 1000 | ∞ | |
| 5 | | | (ZX | | 0 | 5000 | |
| 6 | | | | Z | | 0 | |
| 1 . | m matrix | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

5 for
$$l \leftarrow 2$$
 to n
6 for $i \leftarrow 1$ to $n - l + 1$
7 $j \leftarrow i + l - 1$
8 $m[i,j] \leftarrow \infty$
9 for $k \leftarrow i$ to $j - 1$
10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$

then $m[i, j] \leftarrow q$

 $s[i, j] \leftarrow k$

$$l = 3$$
, $i = 4$, $j = 6$, $k = 4$
 $q = 0 + 5000 + 5 * 10 * 25 = 6250$
 $l = 3$, $i = 4$, $j = 6$, $k = 5$
 $q = 1000 + 0 + 5 * 20 * 25 = 3500$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

11

13

if q < m[i,j]

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|-------------------------------|----------|-------|------|------|------|------|--|
| 1 | 0 | 15750 | 7875 | | | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | 4375 | | | |
| 3 | 3 | | 0 | 750 | 2500 | | |
| 4 | | 3 | | 0 | 1000 | 3500 | |
| 5 | | | A | | 0 | 5000 | |
| 6 | | | | Z | | 0 | |
| | m matrix | | | | | | |

| | _ 2 | 3 | 4 | 5 | 6 |
|---|-----|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$5 \text{ for } l \leftarrow 2 \text{ to } n$$

 $6 \text{ for } i \leftarrow 1 \text{ to } n - l + 1$
 $7 \text{ } j \leftarrow i + l - 1$
 $l = 3, i = 4, j = 6, k = 4$
 $q = 0 + 5000 + 5 * 10 * 25 = 6250$

8
$$m[i,j] \leftarrow \infty$$

9 $for k \leftarrow i to j - 1$
10 $q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$ $l = 3, i = 4, j = 6, k = 5$
 $q = 1000 + 0 + 5 * 20 * 25 = 3500$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | 8 | | |
| 27 |) | 0 | 2625 | 4375 | | |
| 3 | 12 | | 0 | 750 | 2500 | |
| 4 | | 3 | | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | | | | 7 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$l = 3$$
, $i = 1, j = 4$
 $k = 1, q = 0 + 4375 + 30 * 35 * 10 = 14875$

9 for
$$k \leftarrow i$$
 to $j - 1$
10 $q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$
11 if $q < m[i,j]$
12 then $m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|---|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | 8 | | |
| 33 | | 0 | 2625 | 4375 | | |
| 3 | 3 | | 0 | 750 | 2500 | |
| 4 | | 13 | | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | | | | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$j \in \{for \ l \in 2 \ to \ n \}$$
 $j \in \{i + l = 1, j = 4, j \in \{i + l = 1, j = 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4, j \in 4, j \in \{i + l = 1, j = 4, j \in 4,$

9
$$for k \leftarrow i to j - 1$$

10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
11 $if q < m[i,j]$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|-----------|------|------|------|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 27 | | 0 | 2625 | 4375 | | |
| 3 | 13 | | 0 | 750 | 2500 | |
| 4 | | 3 | \rangle | 0 | 1000 | 3500 |
| 5 | | | 1 7 x | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | | _ | | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$\begin{cases} 5 & \text{for } l \leftarrow 2 & \text{to } n \\ 6 & \text{for } i \leftarrow 1 & \text{to } n - l + 1 \\ 7 & j \leftarrow i + l - 1 \\ 8 & m[i,j] \leftarrow \infty \end{cases}$$

$$k = 1, q = 0 + 4375 + 30 * 35 * 10 = 14875$$

$$k = 2, q = 15750 + 750 + 30 * 15 * 10 = 18000$$

$$k = 3, q = 7875 + 0 + 30 * 5 * 10 = 9375$$

$$for k \leftarrow i & \text{to } j - 1$$

$$q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$$

$$\begin{array}{ccc}
11 & if & q < m[i, j] \\
12 & then & m[i, j] \leftarrow q
\end{array}$$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|-----------|------|------|------|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 27 | | 0 | 2625 | 4375 | | |
| 3 | 13 | | 0 | 750 | 2500 | |
| 4 | | 3 | \rangle | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | | | | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$j \in \{c, c\}$$
 $j \in \{c, c\}$ $j \in \{c,$

10
$$q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$$

11 $if \ q < m[i,j]$
12 $then \ m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|---|-------|-------|-------|-------|-------|-------|
| 30 |) | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|------------|-----|-------|------|------|------|------|--|
| 1 | 0 | 15750 | 7875 | 9375 | | | |
| 27 | | 0 | 2625 | 4375 | 8 | | |
| 3 | 135 |) | 0 | 750 | 2500 | | |
| 4 | | (3) | | 0 | 1000 | 3500 | |
| 5 | | | Z | | 0 | 5000 | |
| 6 | | | | 7 | | 0 | |
| m matrix 3 | | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix l = 3, i = 2, i = 5

 $5 \ for \ l \leftarrow 2 \ to \ n$ $l = 3 \ , i = 2, j = 5$ $6 \ for \ i \leftarrow 1 \ to \ n - l + 1$ k = 2, q = 0 + 4375 + 35 * 15 * 20 = 10500 $7 \ j \leftarrow i + l - 1$ $8 \ m[i,j] \leftarrow \infty$ $9 \ for \ k \leftarrow i \ to \ j - 1$ $10 \ q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$ $11 \ if \ q < m[i,j]$ $12 \ then \ m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|-----------------------|----|-------|------|------|------|------|--|
| 1 | 0 | 15750 | 7875 | 9375 | | | |
| 27 | | 0 | 2625 | 4375 | ∞ | | |
| 3 | 13 | | 0 | 750 | 2500 | | |
| 4 | | 33 | | 0 | 1000 | 3500 | |
| 5 | | | (A | | 0 | 5000 | |
| 6 | | | | Z |) | 0 | |
| m matrix $\sqrt[7]{}$ | | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$l = 3$$
, $i = 2$, $j = 5$
 $k = 2$, $q = 0 + 4375 + 35 * 15 * 20 = 10500$
 $k = 3$, $q = 2625 + 1000 + 35 * 5 * 20 = 7125$

$$5 \ for \ l \leftarrow 2 \ to \ n$$
 $6 \ for \ i \leftarrow 1 \ to \ n - l + 1$
 $k = 7$
 $j \leftarrow i + l - 1$
 $k = 8$
 $m[i,j] \leftarrow \infty$
 $9 \ for \ k \leftarrow i \ to \ j - 1$
 $10 \ q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$
 $11 \ if \ q < m[i,j]$
 $12 \ then \ m[i,j] \leftarrow q$
 $13 \ s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 | | | | |
|----|------------|-------|------|------|----------|------|--|--|--|--|
| 1 | 0 | 15750 | 7875 | 9375 | | | | | | |
| 27 | | 0 | 2625 | 4375 | ∞ | | | | | |
| 3 | 13 | | 0 | 750 | 2500 | | | | | |
| 4 | | 3 | | 0 | 1000 | 3500 | | | | |
| 5 | | | Z | | 0 | 5000 | | | | |
| 6 | | | | (75. |) | 0 | | | | |
| | m matriy 7 | | | | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

| | m matrix | 6 | s matrix |
|------------|------------------------------------|------------|---|
| 5 <i>f</i> | for $l \leftarrow 2$ to n | | l = 3, i = 2, j = 5 |
| , | $for i \leftarrow 1 to n - l + 1$ | | k = 2, q = 0 + 4375 + 35 * 15 * 20 = 10500 |
| 7 | $j \leftarrow i + l - 1$ | | k = 3, q = 2625 + 1000 + 35 * 5 * 20 = 7125 |
| 8 | $m[i,j] \leftarrow \infty$ | | k = 4, q = 4375 + 0 + 35 * 10 * 20 = 11375 |
| 9 | $for k \leftarrow i to j - 1$ | | |
| 10 | $q \leftarrow m[i,k] + m[k+1,j] -$ | $+ p_{i-}$ | $a_1p_kp_j$ |
| 11 | if q < m[i,j] | | |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 27 | | 0 | 2625 | 4375 | 7125 | |
| 3 | 13 | | 0 | 750 | 2500 | |
| 4 | | 3 |) | 0 | 1000 | 3500 |
| 5 | | | Z | | 0 | 5000 |
| 6 | | | | 7, | | 0 |
| | | | | | 7 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

mir

m matrix
$$5 \text{ for } l \leftarrow 2 \text{ to } n$$
 $5 \text{ for } i \leftarrow 2 \text{ to } n$ $1 = 3$, $i = 2, j = 5$ $1 = 3$, $i = 3$

10
$$q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$$

11 $if \ q < m[i,j]$
12 $then \ m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---|-------|------|------|------|----------|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | 4375 | 7125 | |
| 3 | 3 | | 0 | 750 | 2500 | ∞ |
| 4 | | (3) |)_ | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| _ | _ | m | mat | rix | 70 |) |

 $s[i, j] \leftarrow k$

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|-------------------------------|----------|-------|------|------|------|----------|--|
| 1 | 0 | 15750 | 7875 | 9375 | | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | 4375 | 7125 | | |
| 3 | 3 | | 0 | 750 | 2500 | ∞ | |
| 4 | | 13 | | 0 | 1000 | 3500 | |
| 5 | | | A |) | 0 | 5000 | |
| 6 | | | | (3 | | 0 | |
| | m matrix | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix $\begin{cases} 5 & \text{for } l \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{to } n - l + 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{fo$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p ₆ |
|-------|-------|-------|-------|-------|-------|----------------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 | | |
|-------------------------------|--------------|-------|------|------|------|------|--|--|
| 1 | 0 | 15750 | 7875 | 9375 | | | | |
| 2 ⁷ / ₂ | _ | 0 | 2625 | 4375 | 7125 | | | |
| 3 | 3 | | 0 | 750 | 2500 | 8 | | |
| 4 | | (3) | | 0 | 1000 | 3500 | | |
| 5 | | | Z | | 0 | 5000 | | |
| 6 | | | | Z | | 0 | | |
| | m matrix (%) | | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|----------|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | <u> </u> | 5 |

m matrix s = 3, i = 3, j = 66 for $i \leftarrow 1$ to n - l + 17 $j \leftarrow i + l - 1$ 8 $m[i,j] \leftarrow \infty$ 9 for $k \leftarrow i$ to j - 110 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$ s matrix l = 3, i = 3,j = 6 k = 3, q = 0 + 3500 + 15 * 5 * 25 = 5375 k = 4, q = 750 + 5000 + 15 * 10 * 25 = 9500 k = 5, q = 2500 + 0 + 15 * 20 * 25 = 10000

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p ₆ |
|-------|-------|-------|-------|-------|-------|----------------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|---|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 27 | _ | 0 | 2625 | 4375 | 7125 | |
| 3 | 3 | | 0 | 750 | 2500 | 5375 |
| 4 | | (3) | | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| _ | _ | m | mat | rix | 76 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix s = 3, i = 3, j = 6 $for i \leftarrow 1 to n - l + 1$ k = 3, q = 0 + 3500 + 15 * 5 * 25 = 5375 k = 4, q = 750 + 5000 + 15 * 10 * 25 = 9500 miliar matrix <math>miliar matrix matrix matrix <math>miliar matrix matrix matrix matrix <math>miliar matrix matrix matrix matrix matrix <math>miliar matrix <math>miliar matrix matr

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|----------|----|-------|------|------|------|------|--|
| 1 | 0 | 15750 | 7875 | 9375 | ∞ | | |
| 27 | | 0 | 2625 | 4375 | 7125 | | |
| 3 | 13 | | 0 | 750 | 2500 | 5375 | |
| 4 | | 3 | | 0 | 1000 | 3500 | |
| 5 | | | Z Z | | 0 | 5000 | |
| 6 | | | | 75 | | 0 | |
| m matrix | | | | | | | |

 $s[i, j] \leftarrow k$

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix l = 4, i = 1, j = 5 $5 for l \leftarrow 2 to n$ k = 1, q = 0 + 7125 + 30 * 35 * 20 = 28125for $i \leftarrow 1$ to n - l + 1 $j \leftarrow i + l - 1$ $m[i,j] \leftarrow \infty$ for $k \leftarrow i \text{ to } j - 1$ 10 $q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_i$ 11 if q < m[i,j]then $m[i,j] \leftarrow q$ 13

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 | | |
|----------|---|-------|------|------|------|------|--|--|
| 1_ | 0 | 15750 | 7875 | 9375 | 8 | | | |
| 27 | | 0 | 2625 | 4375 | 7125 | | | |
| 3 | 3 |) | 0 | 750 | 2500 | 5375 | | |
| 4 | | 3 | | 0 | 1000 | 3500 | | |
| 5 | | | Z Z | | 0 | 5000 | | |
| 6 | | | | Z | | 0 | | |
| m matrix | | | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix $\begin{cases} 5 & \text{for } l \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 5 & \text{for } l \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 1 & \text{extremath} \\ 1 & \text{for } i \leftarrow 1 & \text{to } n - l + 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } i \leftarrow 1 & \text{for } i \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text$

10
$$q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$$

11 $if \ q < m[i,j]$
12 $then \ m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|----------|----|-------|------|------|------|------|--|
| 1 | 0 | 15750 | 7875 | 9375 | ∞ | | |
| 27 | | 0 | 2625 | 4375 | 7125 | | |
| 3 | 13 | | 0 | 750 | 2500 | 5375 | |
| 4 | | 33 |)_ | 0 | 1000 | 3500 | |
| 5 | | | (A | | 0 | 5000 | |
| 6 | | | | Z | | 0 | |
| m matrix | | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix $\begin{cases} for \ l \leftarrow 2 \ to \ n \end{cases}$ $\begin{cases} for \ l \leftarrow 2 \ to \ n \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ n - l + 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i \leftarrow 1 \end{cases}$ $\begin{cases} for \ i \leftarrow 1 \ to \ i$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|------|------|------|------|
| 1 | 0 | 15750 | 7875 | 9375 | ∞ | |
| 27 | | 0 | 2625 | 4375 | 7125 | |
| 3 | 13 | | 0 | 750 | 2500 | 5375 |
| 4 | | 3 | | 0 | 1000 | 3500 |
| 5 | | | A A |) | 0 | 5000 |
| 6 | | | | (35 | | 0 |
| | | m | mati | | 1 7 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix ightarrow ightarr

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|--------|------|-------|------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 27 | | 0 | 2625 | 4375 | 7125 | |
| 3 | 13 | | 0 | 750 | 2500 | 5375 |
| 4 | | 33 | \sim | 0 | 1000 | 3500 |
| 5 | | | Z | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| | | m | mat | rix | 1 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix $\begin{cases} 5 & \text{for } l \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 1 & \text{for } l \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 1 & \text{for } l \leftarrow 2 & \text{to } n \end{cases}$ $\begin{cases} 1 & \text{for } l \leftarrow 1 & \text{for } l \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } l \leftarrow 1 & \text{for } l \leftarrow 1 \end{cases}$ $\begin{cases} 1 & \text{for } l \leftarrow 1 \end{cases}$ \begin{cases}

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|---------|-------|------|------|-------|----------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 ⁷ / ₂ | | 0 | 2625 | 4375 | 7125 | ∞ |
| | 13 | | 0 | 750 | 2500 | 5375 |
| 4 | | (3) | | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 75 | | 0 |
| 1. | 2 4 5 4 | m | mat | rix | 76 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$s = 1$$
 for $l \leftarrow 2$ to n $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ $l = 4$, $l = 2$, $l = 6$ l

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p ₁ | p_2 | p_3 | p_4 | p_5 | p ₆ |
|-------|-----------------------|-------|-------|-------|-------|----------------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|----|-------|------|------|-------|----------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 ⁷ / ₂ | | 0 | 2625 | 4375 | 7125 | ∞ |
| 3 | 13 | | 0 | 750 | 2500 | 5375 |
| 4 | | 33 | | 0 | 1000 | 3500 |
| 5 | | | (ZX | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | m | mat | rix | 70 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$s$$
 matrix $l = 4$, $i = 2$, $j = 6$

6 for $i \leftarrow 1$ to $n - l + 1$ $k = 2$, $q = 0 + 5375 + 35 * 15 * 25 = 15375$

7 $j \leftarrow i + l - 1$ $k = 3$, $q = 2625 + 3500 + 35 * 5 * 25 = 10500$

8 $m[i,j] \leftarrow \infty$

9 for $k \leftarrow i$ to $j - 1$

10 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$

11 if $q < m[i,j]$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

if q < m[i,j]

then $m[i, j] \leftarrow q$

 $s[i, j] \leftarrow k$

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|-------------------------------|----------|-------|------|------|-------|----------|--|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | | |
| 2 ⁷ / ₂ | | 0 | 2625 | 4375 | 7125 | ∞ | |
| 3 | 13 | | 9 | 750 | 2500 | 5375 | |
| 4 | | 3 | | 0 | 1000 | 3500 | |
| 5 | | | A | | 0 | 5000 | |
| 6 | | | | 3 | | 0 | |
| | m matrix | | | | | | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$\begin{cases} 5 \text{ for } l \leftarrow 2 \text{ to } n \end{cases}$$
 $\begin{cases} 1 = 4, i = 2, j = 6 \end{cases}$ $\begin{cases} 6 \text{ for } i \leftarrow 1 \text{ to } n - l + 1 \end{cases}$ $\begin{cases} k = 2, q = 0 + 5375 + 35 * 15 * 25 = 15375 \end{cases}$ $\begin{cases} k = 3, q = 2625 + 3500 + 35 * 5 * 25 = 10500 \end{cases}$ $\begin{cases} m[i,j] \leftarrow \infty \end{cases}$ $\begin{cases} m[i,j] \leftarrow \infty \end{cases}$ $\begin{cases} k = 4, q = 4375 + 5000 + 35 * 10 * 25 = 18125 \end{cases}$ $\begin{cases} for k \leftarrow i \text{ to } j - 1 \end{cases}$ $\begin{cases} q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \end{cases}$ 11 $\begin{cases} if q < m[i,j] \end{cases}$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p ₁ | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-----------------------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|------|-------|----------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 ⁷ / ₂ | | 0 | 2625 | 4375 | 7125 | ∞ |
| 3 | (3) | | 0 | 750 | 2500 | 5375 |
| 4 | | 73 | | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 3 |) | 0 |
| | | m | mat | rix | (8 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix s matrix l = 4, i = 2, j = 66 for $i \leftarrow 1$ to n - l + 1 k = 2, q = 0 + 5375 + 35 * 15 * 25 = 153757 $j \leftarrow i + l - 1$ k = 3, q = 2625 + 3500 + 35 * 5 * 25 = 105008 $m[i,j] \leftarrow \infty$ k = 4, q = 4375 + 5000 + 35 * 10 * 25 = 181259 for $k \leftarrow i$ to j - 1 k = 5, q = 7125 + 0 + 35 * 20 * 25 = 2462510 $q \leftarrow m[i,k] + m[k + 1,j] + p_{i-1}p_kp_j$ 11 if q < m[i,j]

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------------|-----|-------|------|------|-------|-------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 ⁴ / ₂ | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | (3) | | 0 | 750 | 2500 | 5375 |
| 4 | | 73 |) | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | m | mat | rix | 75 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix s = 1 for $l \leftarrow 2$ to n l = 4, l = 2, l = 6 l = 2, l = 6 l = 2, l = 6 l = 4, l = 2, l = 6 l = 2, l = 6 l = 2, l = 6 l = 2, l

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| _1 | 2 | 3 | 4 | 5 | 6 |
|-------|-------|------|------|-------|-------|
| 1 0 | 15750 | 7875 | 9375 | 11875 | 8 |
| 27 | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | | 9 | 750 | 2500 | 5375 |
| 4 | 3 | | 0 | 1000 | 3500 |
| 5 | | Z | | 0 | 5000 |
| 6 | | | Z | | 0 |
| 1 2 . | m | mat | rix | 76 |) |

 $s[i, j] \leftarrow k$

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----|-------|------|------|-------|-------|
| 1 | 9 | 15750 | 7875 | 9375 | 11875 | 8 |
| 53 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | 45 |) | 0 | 750 | 2500 | 5375 |
| 4 | | 13 | | Q | 1000 | 3500 |
| 5 | | | Z | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | m | mat | rix | 76 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$j$$
 $j \leftarrow 2 \text{ to } n$ $k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$ $k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375$ $m[i,j] \leftarrow \infty$ $j \leftarrow i \text{ to } j - 1$

10
$$q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$$

11 $if \ q < m[i,j]$
12 $then \ m[i,j] \leftarrow q$
13 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 |
|-------|-------|-------|-------|-------|-------|-------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|---|-------|------|------|-------|----------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | ∞ |
| 27 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | 3 | | 0 | 750 | 2500 | 5375 |
| 4 | | 33 | | 0 | 1000 | 3500 |
| 5 | | | 1 Z | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | m | mat | rix | 76 |) |

then $m[i,j] \leftarrow q$

 $s[i, j] \leftarrow k$

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$\begin{cases} 5 & \text{for } l \leftarrow 2 & \text{to } n \\ 6 & \text{for } i \leftarrow 1 & \text{to } n - l + 1 \\ 7 & \text{j} \leftarrow i + l - 1 \\ 8 & m[i,j] \leftarrow \infty \\ 9 & \text{for } k \leftarrow i & \text{to } j - 1 \\ 10 & q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\ 11 & \text{if } q < m[i,j] \end{cases}$$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p ₆ |
|-------|-------|-------|-------|-------|-------|-----------------------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|----------|-------|------|------|-------|-------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | 8 |
| 27 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | 3 | | 0 | 750 | 2500 | 5375 |
| 4 | | 33 | | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | 73 | | 0 |
| | | m | mat | rix | 76 |) |
| 7 | \sim . | | | | | _ |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

m matrix
$$\begin{cases} 5 \text{ for } l \leftarrow 2 \text{ to } n \end{cases}$$
 $\begin{cases} 1 = 4, i = 1, j = 6 \end{cases}$ $\begin{cases} 1 = 4, i = 1, j = 6, i = 1, j = 6 \end{cases}$ $\begin{cases} 1 = 4, i = 1, j = 1, j = 6, i = 1, j = 1, j = 6 \end{cases}$ $\begin{cases} 1 = 4, i = 1, j = 1, j$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p ₆ |
|-------|-------|-------|-------|-------|-------|-----------------------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

12

13

| | 1 | 2 | 3 | 4 | 5 | 6 |
|----|---|-------|------|------|-------|-------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | 8 |
| 27 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | 3 | | 0 | 750 | 2500 | 5375 |
| 4 | | 3 | | 0 | 1000 | 3500 |
| 5 | | | Z Z | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| | | 100 | | ui. | 79 | |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

then
$$m[i,j] \leftarrow q$$

 $s[i,j] \leftarrow k$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

 $s[i,i] \leftarrow k$

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of

dimension is:

| p_0 | p_1 | p_2 | p_3 | p_4 | p_5 | p ₆ |
|-------|-------|-------|-------|-------|-------|----------------|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

13

| _ | 1 | 2 | 3 | 4 | 5 | 6 |
|--------|---|-------|------|------|-------|-------|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | 15125 |
| 33 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | 3 | | 0 | 750 | 2500 | 5375 |
| 4 | | 33 |) | 0 | 1000 | 3500 |
| 5 | | | A | | 0 | 5000 |
| 6 | | | | Z | | 0 |
| 2 to n | | m | mati | rix | 76 |) |

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix l = 4, i = 1, i = 65 for $l \leftarrow 2$ to n for $i \leftarrow 1$ to n - l + 1k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750 $j \leftarrow i + l - 1$ k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375 min $m[i,j] \leftarrow \infty$ k = 3, q = 7875 + 3500 + 30 * 5 * 25 = 15125for $k \leftarrow i$ to j - 1 $q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j^k = 4, q = 9375 + 5000 + 30 * 10 * 25 = 21875$ 10 k = 5, q = 11875 + 0 + 30 * 20 * 25 = 2687511 if q < m[i, j]12 then $m[i,j] \leftarrow q$

Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

```
MATRIX - CHAIN - ORDER(p)
 1 n \leftarrow length[p] - 1
2 let m[1...n, 1...n] and s[1...n - 1, 2...n] be new tables.
3 \ for \ i \leftarrow 1 \ to \ n
4 m[i,i] \leftarrow 0
5 for l \leftarrow 2 to n \triangleright l is the chain length.
    for i \leftarrow 1 to n - l + 1
     j \leftarrow i + l - 1
8 m[i,j] \leftarrow \infty
9 for k \leftarrow i to j - 1
10
             q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j
11
            if q < m[i,j]
12
                 then m[i,j] \leftarrow q
13
                       s[i,i] \leftarrow k
14 return m and s
```

A simple inspection of the nested loop structure of MATRIX-CHAIN-ORDER yields a running time of $O(n^3)$ for the algorithm. The loops are nested three deep, and each loop index (l, i, and k) takes on at most n-1 values.

Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose

sequence of dimension is:

| p_0 | | | | | | |
|-------|----|----|---|----|----|----|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

```
PRINT - OPTIMAL - PARENS(s, i, j)

1 if i = j

2 then print "A<sub>i</sub>"

3 else print "("

4 PRINT - OPTIMAL - PARENS(s, i, s[i, j])

5 PRINT - OPTIMAL - PARENS(s, s[i, j] + 1, j)

6 print ")"
```

Lets see, how in the discussed example the call PRINT - OPTIMAL - PARENS(s, 1, 6) prints the parenthesization ((A1 (A2 A3)) ((A4 A5)A6)).

Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose 3

sequence of dimension is:

| | _ | | | • | | | |
|--|----------|---|---|------|-----|---|---|
| PRINT - OPTIMAL - PARENS(s, i, j) | 1 [| 1 | 1 | 3 | 3 | 3 | |
| $ \begin{array}{ll} 1 \ if \ i = j \\ 2 \ then \ print "A_i" \end{array} $ | 2 | | 2 | 3 | 3 | 3 | |
| 3 else print "(" | 3 | | | 3 | 3 | 3 | |
| 4 $PRINT - OPTIMAL - PARENS(s, i, s[i, j])$ | 4 | | | | 4 | 5 | |
| 5 $PRINT - OPTIMAL - PARENS(s, s[i, j] + 1, j)$ | 5 | | C | mati | riv | 5 | |
| 6 | s matrix | | | | | | _ |

Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose

sequence of dimension is:

```
6
PRINT - OPTIMAL - PARENS(s, i, j)
1 if i = j
   then print "A_i"
                                                                                                   3
   else print "("
        PRINT - OPTIMAL - PARENS(s, i, s[i, j])
5
         PRINT - OPTIMAL - PARENS(s, s[i, j] + 1, j)
                                                                                                   5
                                                                                  s matrix
         print")"
                                               POP(S,1,6)
```

Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose

Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

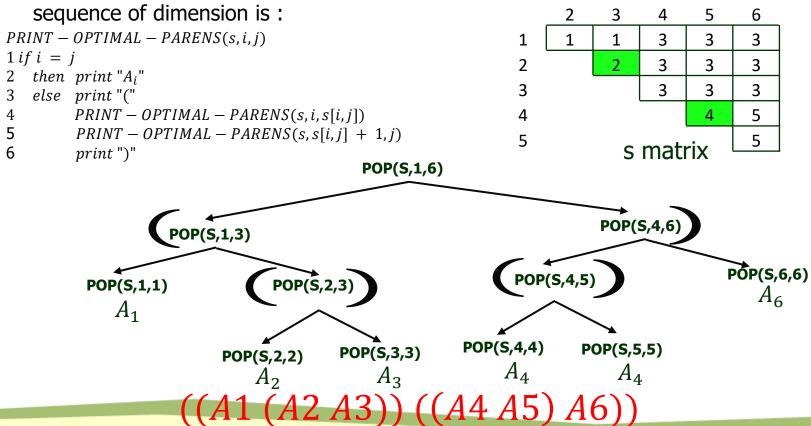
Example 1: Find an optimal parenthesization of a matrix chain product whose

sequence of dimension is: PRINT - OPTIMAL - PARENS(s, i, j)1 if i = jthen print " A_i " else print "(" PRINT - OPTIMAL - PARENS(s, i, s[i, j])5 PRINT - OPTIMAL - PARENS(s, s[i, j] + 1, j)5 s matrix print")" POP(S,1,6) POP(S,4,6) POP(S,1,3) POP(S,6,6) POP(S,4,5) POP(S,2,3) POP(S,1,1)

Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose



Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose

sequence of dimension is:

```
POP(s, 1, 6), s[1, 6] = 3, (A1A2A3)(A4A5A6)

POP(s, 1, 3), s[1, 3] = 1, ((A1)(A2A3))(A4A5A6)

POP(s, 4, 6), s[4, 6] = 5, ((A1)(A2A3))((A4A5)(A6))

POP(s, 2, 3), s[2, 3] = 2, ((A1)((A2)(A3)))((A4A5)(A6))

POP(s, 4, 5), s[4, 5] = 4, ((A1)((A2)(A3)))(((A4)(A5))(A6))

Hence the product is computed as follows
```

(A1(A2A3))((A4A5)A6).

Problem 4: Matrix Chain Multiplication

Example 2: Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$

Example 3: Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 4, 6, 2, 7 \rangle$

ı

Self practice

