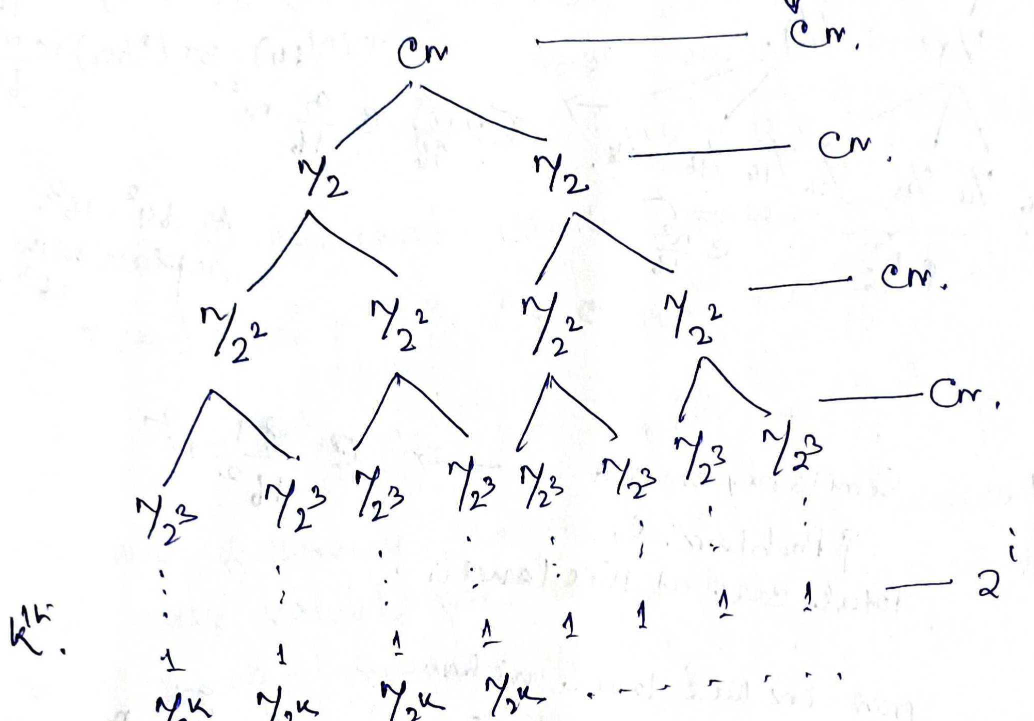
Q1. $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n/2) + cn & \text{if } n > 1. \end{cases}$ Solve by using Recursive Tree Method.

In general

$$T(n) = 2T(n/2) + cn.$$

$$= T(n/2) + T(n/2) + \underbrace{cn}_{\text{combine cost}}.$$

combine cost.



So $\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

So $k = \log n$.

Total cost.

$cn + cn + cn + \dots + cn.$

what is the last number.

So we calculate the height.

Here the height is k.

$\Rightarrow k \cdot cn.$ $\quad \because k = \log n$

$\Rightarrow c\, n \lg n.$

$\Rightarrow O(n \lg n)$

Q2. Solve the following Recurrance with the help of Recursion Tree Method.

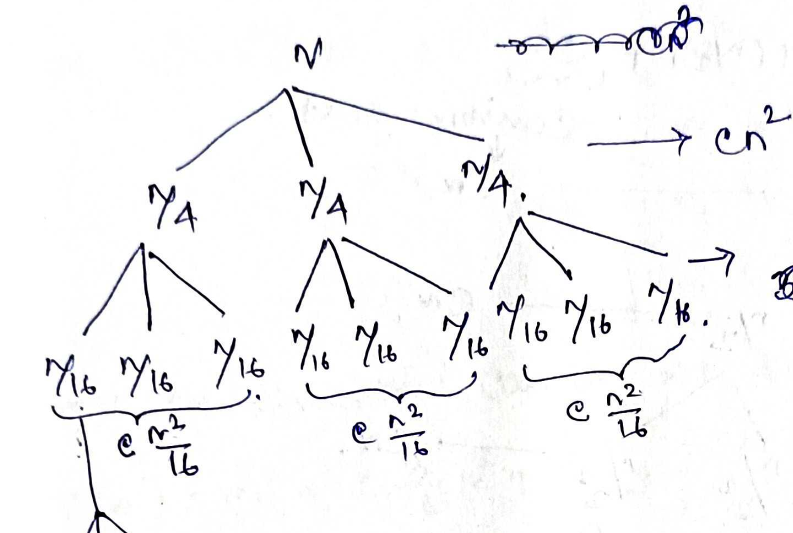
$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 3T(\lfloor n/4 \rfloor) + cn^2 & \text{if } n > 1. \end{cases}$$

$T(n) = 3T(n/4) + cn^2$

$T(n/4) = 3T(n/16) + c\frac{n^2}{16}$

$T(n/16) = 3T(n/64) + c\frac{n^2}{16^2}$

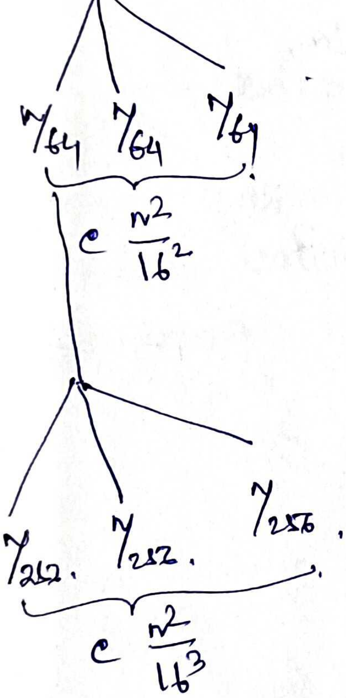
$T(n/64) = 3T(n/256) + c\frac{n^2}{64^2}$



$\rightarrow cn^2$

$\rightarrow c\frac{3}{16}n^2$

As $64^2 = 16^3$. replace with 16^3.



Similarly we get 9 Problem. So Total cost at this level is $\rightarrow c\frac{9}{16^2}n^2$

now in this level we have 27 Sub problem. So the total cost at this level is $\rightarrow c\frac{27}{16^3}n^2$

After carefully visualization it was observed that the following equation is formed to calculate the total cost.

$$T(n) = \underbrace{cn^2}_{\text{1st term}} + \underbrace{c\frac{3}{16}n^2}_{\text{2nd term}} + \underbrace{c\frac{9}{16^2}n^2}_{\text{3rd term}} + \underbrace{c\frac{27}{16^3}n^2}_{\text{4th term.}} + \cdots$$

So to identify the last term we need to calculate the height of the tree. The observation says that the tree is grown in ~~n/4~~ $n = N/4$ order. So at the K^{th} level. The value of n is $\frac{n}{4^k}$. and at last level it should be 1. So we can write that.

$$\frac{n}{4^k} = 1 \Rightarrow n = 4^k$$

Apply log both side.

$$\boxed{K = \log_4 n} \rightarrow \text{Height of the tree.}$$

now the equation can be written as.

$$T(n) = cn^2 + c\frac{3}{16}n^2 + c\frac{9}{16^2}n^2 + c\frac{27}{16^3}n^2 + \dots$$

$$= cn^2\left[1 + \frac{3}{16} + \left(\frac{3}{16}\right)^2 + \left(\frac{3}{16}\right)^3 + \dots + \infty\right]$$

GP.

We assume that the generalized GP series is ∞ series. for easy calculation. Hence by using the formula of ∞ GP series. we can write.

$$\leq cn^2\left[\frac{1}{1 - 3/16}\right]$$

$$\leq cn^2 \frac{1}{13/16}.$$

$$\leq cn^2 \frac{16}{13}.$$

Hence the complexity $O(n^2)$.

Q3. Solve the following Recurrence by using Recurrence Tree Method.

$$T(N) = \begin{cases} 1 & \text{if } N = 1 \\ 5T(N/5) + cN & \text{if } N > 1. \end{cases}$$

$T(N) = 5T(N/5) + cN.$

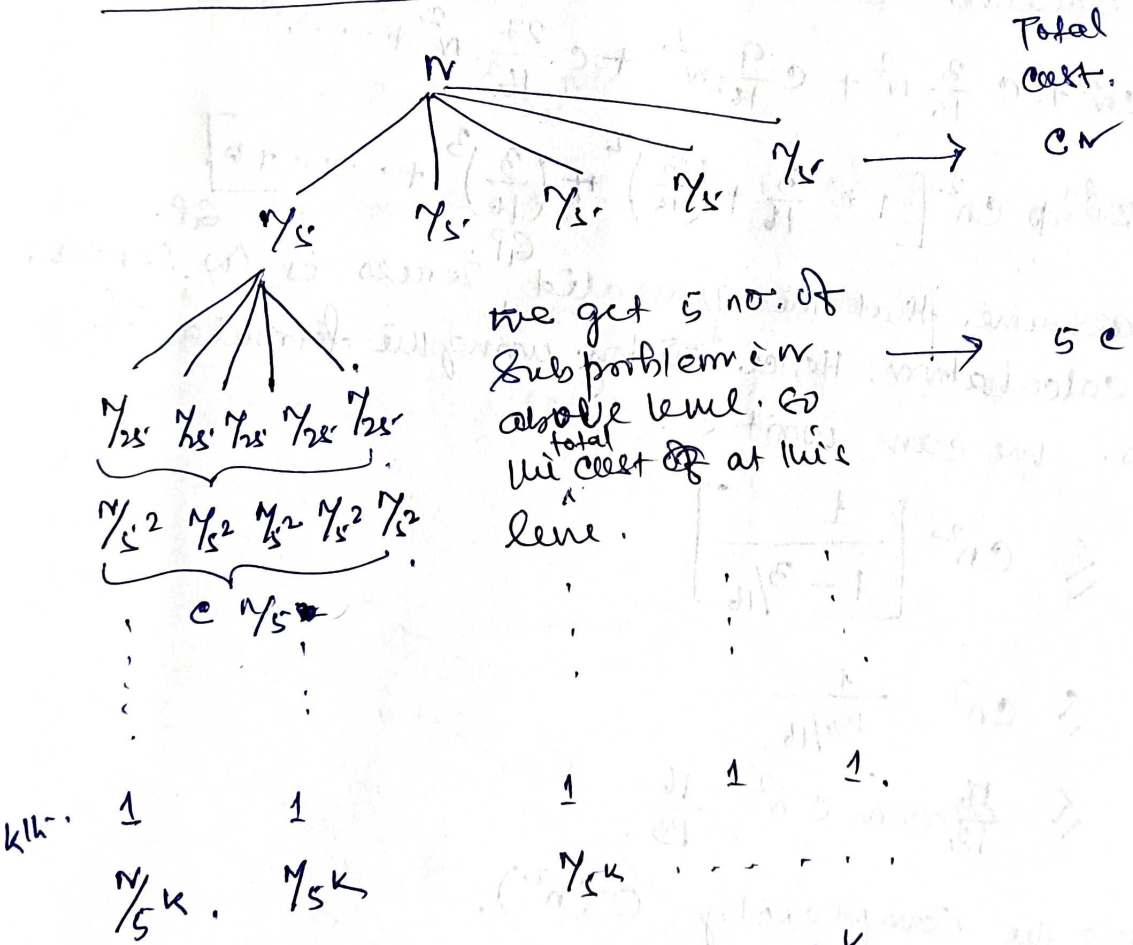
$T(N/5) = 5T(N/25) + cN/5$

In general. $T(N) = 5T(N/5) + cN.$

$$= \underbrace{T(N/5) + T(N/5) + T(N/5) + T(N/5) + T(N/5)}_{\text{Combine cost of these subproblems}} + cN$$

is cN

First draw the Recursion Tree.



So $\frac{N}{5^k} = 1 \Rightarrow N = 5^k$

$\boxed{k = \log_5 N}$ height of the tree.

now the total cost in each level.

$\Rightarrow \underbrace{cn + cn + cn + \cdots\cdots + cn}$.

The main purpose is, how many term. $\longrightarrow$ what is the last level number. So we calculate the height.

It was observed that in each level the cost is same and we have k number of levels. So the total cost is =

$T(n) = k\,cn.$ $\qquad \because k = \log_5 n$

$= c\,n \log_5 n$

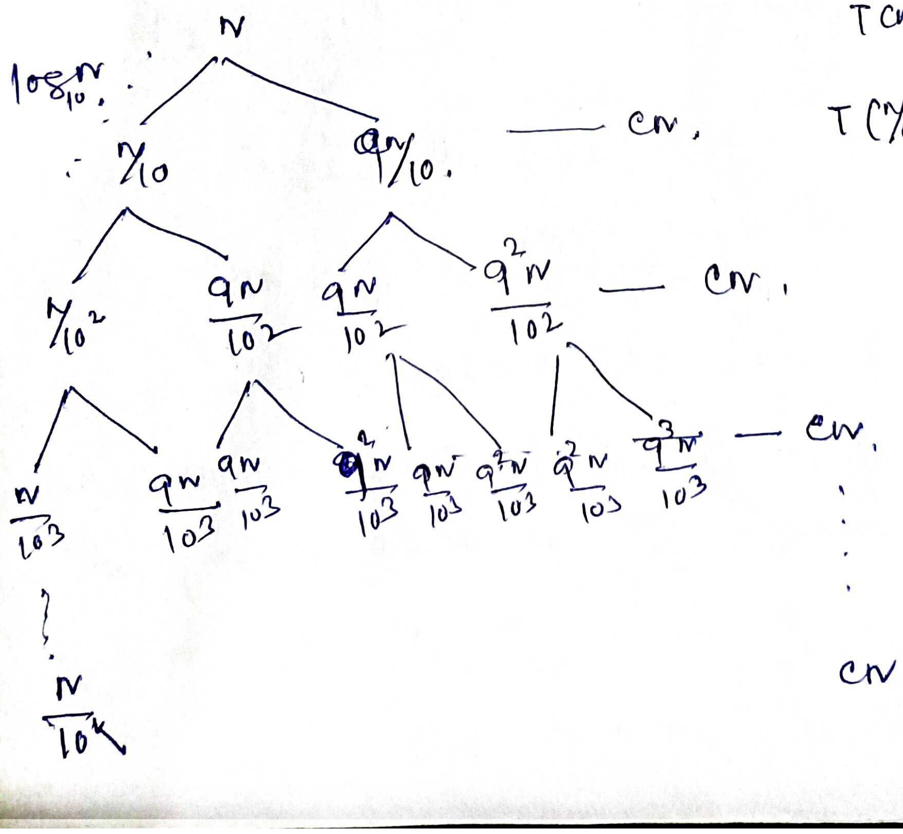
Hence the complexity is $O(n \log_5 n)$.

Q.4 Solve the following Recurrance.

$$T(n) = T(n/10) + T(9n/10) + cn$$

$T(n) = T(n/10) + T(9n/10) + cn$

$T(n/10) = T(n/10^2) + T(9n/10^2) + c\left(\frac{n}{10}\right)$



$$n = \frac{n}{10/9} \rightarrow \frac{n}{(10/9)^2} - \frac{n}{(10/9)^2} \cdots \frac{n}{(10/9)^k}$$

observation says that $10 > 10/9$.

So height $\frac{n}{(10/9)^k} = 1$

$$\log n = k \log 10/9.$$

$$\boxed{k = \log_{10/9} n} \leftarrow \text{Height.}$$

So complexity $\rightarrow$

$$T(n) = k\,cn, \text{ or } cn\,k$$

$$= cn \log_{10/9} n.$$

Hence the complexity $O(cn \log_{10/9} n)$.

Q7.

Q5. Solve the following recurrence $T(n) = T(n/2) + T(n/4) + T(n/8) + cn$ by using recursion tree method.

So

$$T(n) = T(n/2) + T(n/4) + T(n/8) + cn.$$

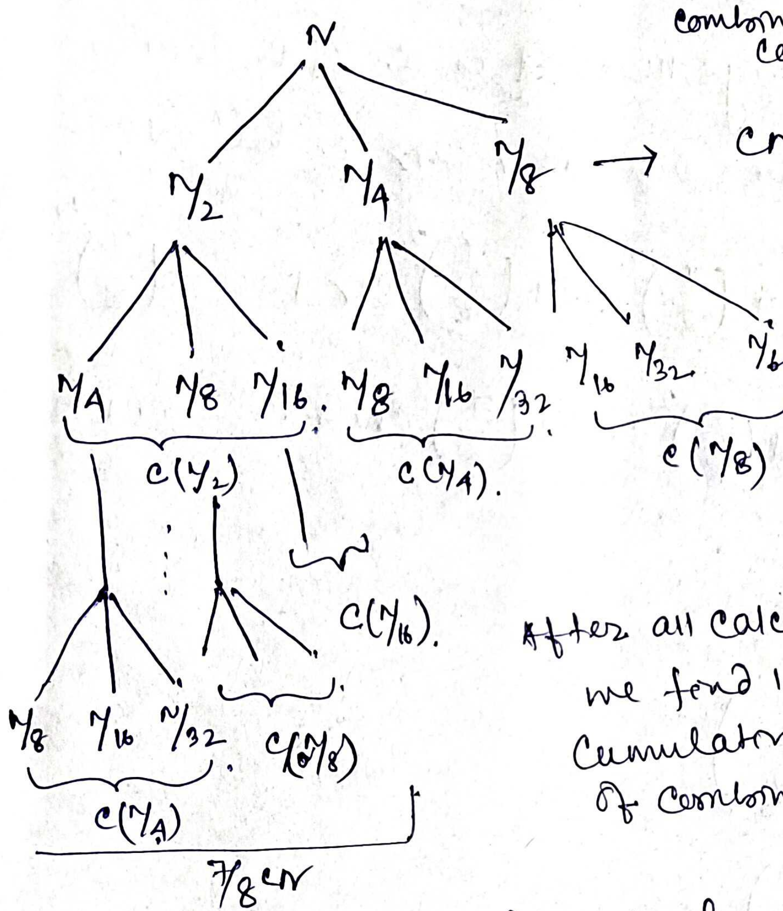
$T(n) = T(n/2) + T(n/4) + T(n/8) + cn$

$T(n/2) = T(n/4) + T(n/8) + T(n/16) + cn/2$

$T(n/4) = T(n/8) + T(n/16) + T(n/32) + cn/4$

$T(n/8) = T(n/16) + T(n/32) + T(n/64) + cn/8$

combine cost.



$\rightarrow cn.$

$\rightarrow \frac{7}{8} cn.$

After all calculation we find the cumulative sum of combine costs $\rightarrow \left(\frac{7}{8}\right)^2 cn$

Hence the equation formed from the tree is

$$T(n) = cn + \frac{7}{8} cn + \left(\frac{7}{8}\right)^2 cn + \left(\frac{7}{8}\right)^3 cn + \cdots \left(\frac{7}{8}\right)^i cn.$$

Now we calculate the value of i (i.e height of the tree).

The tree is grow in three way i.e $\frac{n}{2}$, $\frac{n}{4}$, and $n/8$.

The longest path is $n/2$. Hence the tree grow,

$$n, n/2, n/4, n/8 \cdots n/2^i = 1.$$

$$n/2^i = 1 \Rightarrow n = 2^i \Rightarrow i = \log_2 n.$$

Hence the height of the tree is $\log_2 n$.

now apply the on the equation.

$$T(n) = cn + \left(\frac{7}{8}\right)cn + \left(\frac{7}{8}\right)^2 cn + \left(\frac{7}{8}\right)^3 cn + \cdots\cdots + \left(\frac{7}{8}\right)^{\log_2 n} cn,$$

for simplicity let us take the series is ∞ series. & the observation says that it is a GP series

Hence we write the equation as below.

$$T(n) = cn + \left(\frac{7}{8}\right)cn + \left(\frac{7}{8}\right)^2 cn + \left(\frac{7}{8}\right)^3 cn + \cdots + \left(\frac{7}{8}\right)^{\log_2 n} cn,$$

$$T(n) \leq cn + \left(\frac{7}{8}\right)cn + \left(\frac{7}{8}\right)^2 cn + \left(\frac{7}{8}\right)^3 cn + \cdots\cdots + \infty$$

$$T(n) \leq \sum_{i=0}^{\infty} \left(\frac{7}{8}\right)^i cn$$

$$T(n) \leq cn\left(\frac{1}{1-7/8}\right)$$

$$T(n) \leq cn\,(8/1)$$

$$T(n) \leq 8cn.$$

Hence the complexity is $O(n)$