Design and Analysis of Algorithm

Greedy Methods
(Activity Selection Algorithm, Task Scheduling, Huffman Coding)

Lecture – 59-61

Overview

- A greedy algorithm always makes the choice that looks best at the moment. (i.e. it makes a locally optimal choice in the hope that this choice will lead to a globally optimal solution).
- The objective of this section is to explores optimization problems that are solvable by greedy algorithms.

Greedy Algorithm

- In mathematics, computer science and economics, an optimization problem is the problem of finding the best solution from all feasible solutions.
- Algorithms for optimization problems typically go through a sequence of steps, with a set of choices at each step.
- Many optimization problems can be solved using a greedy approach.
- Greedy algorithms are simple and straightforward.

Greedy Algorithm

- A greedy algorithm always makes the choice that looks best at the moment.
- That is, it makes a locally optimal choice in the hope that this choice will lead to a globally optimal solution.
- Greedy algorithms do not always yield optimal solutions, but for many problems they do.
- This algorithms are easy to invent, easy to implement and most of the time provides best and optimized solution.

Greedy Algorithm

- Application of Greedy Algorithm:
  - A simple but nontrivial problem, the activity-selection problem, for which a greedy algorithm efficiently computes a solution.
  - In combinatorics,(a branch of mathematics), a ‘matroid’ is a structure that abstracts and generalizes the notion of linear independence in vector spaces. Greedy algorithm always produces an optimal solution for such problems. Scheduling unit-time tasks with deadlines and penalties is an example of such problem.

Greedy Algorithm

- Application of Greedy Algorithm:
  - An important application of greedy techniques is the design of data-compression codes (i.e. Huffman code) .
  - The greedy method is quite powerful and works well for a wide range of problems. They are:
    - Minimum-spanning-tree algorithms

      (Example: Prims and Kruskal algorithm)
    - Single Source Shortest Path.

      (Example: Dijkstra's and Bellman ford algorithm)

Greedy Algorithm

- Application of Greedy Algorithm:
  - A problem exhibits optimal substructure if an optimal solution to the problem contains within it optimal solutions to subproblems.
  - This property is a key ingredient of assessing the applicability of dynamic programming as well as greedy algorithms.
  - The subtleties between the above two techniques are illustrated with the help of two variants of a classical optimization problem known as knapsack problem. These variants are:
    - 0-1 knapsack problem (Dynamic Programming)
    - Fractional knapsack problem (Greedy Algorithm)

Greedy Algorithm

- **Problem 1: An activity-selection problem**
  - Is a problem of scheduling several competing activities that require exclusive use of a common resource, with a goal of selecting a maximum-size set of mutually compatible activities.
  - Let us take a set $S = \{a_1, a_2, a_3, \ldots, a_n\}$ on n proposed activities that wish to use a resource (i.e. lecture hall) which can serve only one activity at a time.
  - Assume that the activities are sorted in monotonically increasing order of finish time:

$$f_1 \leq f_2 \leq f_3 \leq \cdots \leq f_{n-1} \leq f_n$$

Greedy Algorithm

- **Problem 1: An activity-selection problem**

  - Each activity a_i has a start time s_i and a finish time f_i , where $0 \leq s_i < f_i < \infty$.
  - Activities a_i and a_j are compatible if the intervals $[s_i, f_i]$ and $[s_j, f_j]$ do not overlap. (i.e. $[s_j \geq f_i]$).
  - In the activity-selection problem, the main goal is to select a maximum-size subset of mutually compatible activities.

Greedy Algorithm

- **Problem 1: An activity-selection problem**

  Example 1: Given 10 activities with their start and finish time compute a schedule where the largest number of activities take place.

Activity	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
s_i	1	2	3	4	7	8	9	9	11	12
f_i	3	5	4	7	10	9	11	13	12	14

Greedy Algorithm

- **Problem 1: An activity-selection problem**

  Solution:

  First arraigning the following activities in increasing order on their finishing

Activity	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
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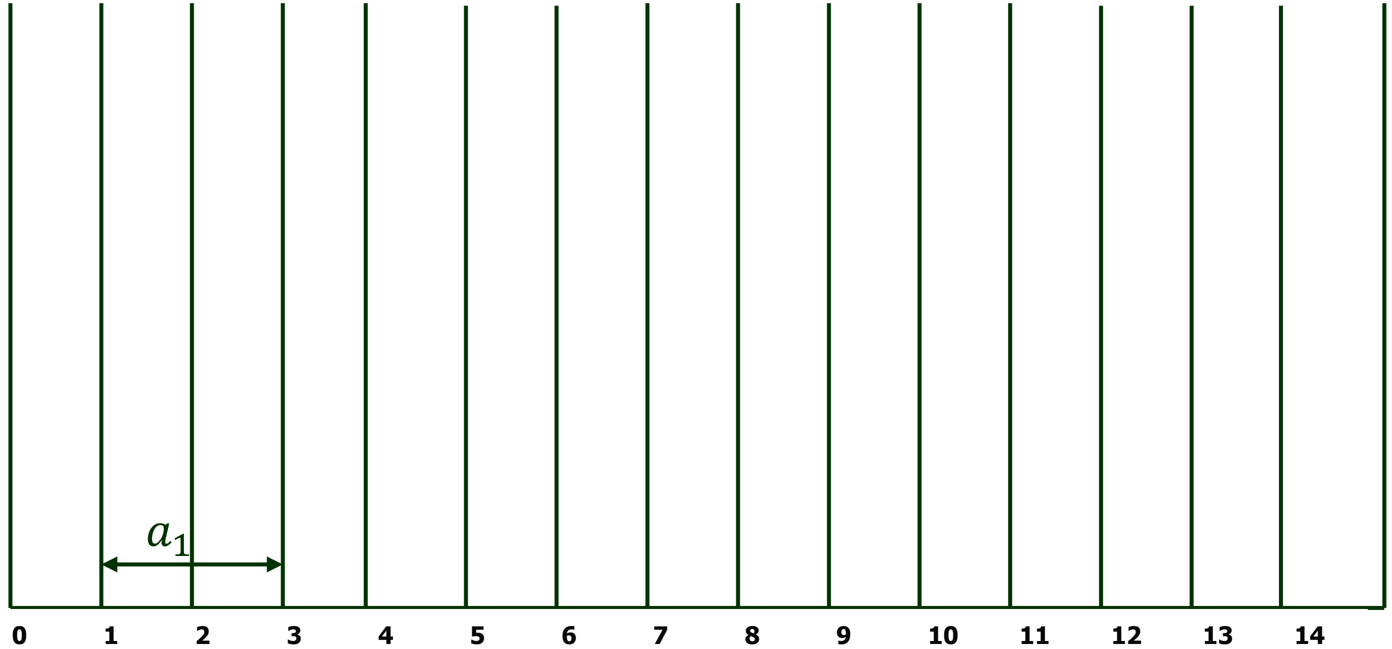

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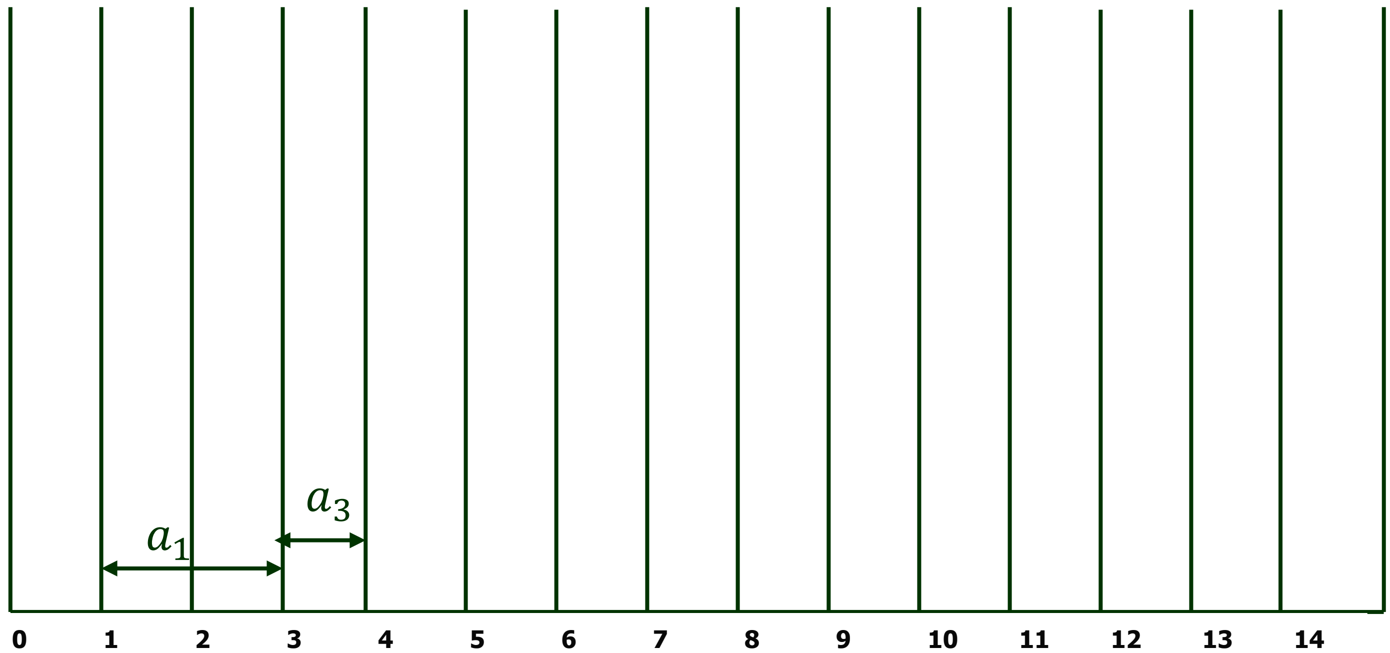

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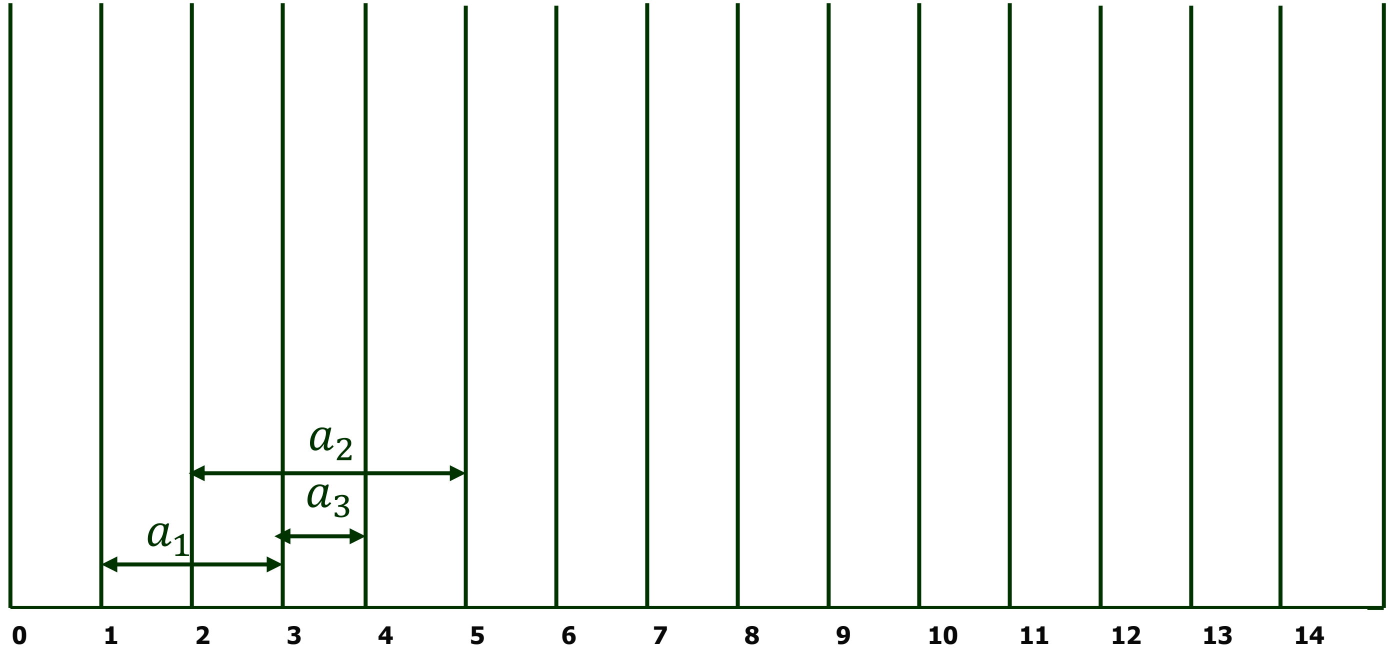

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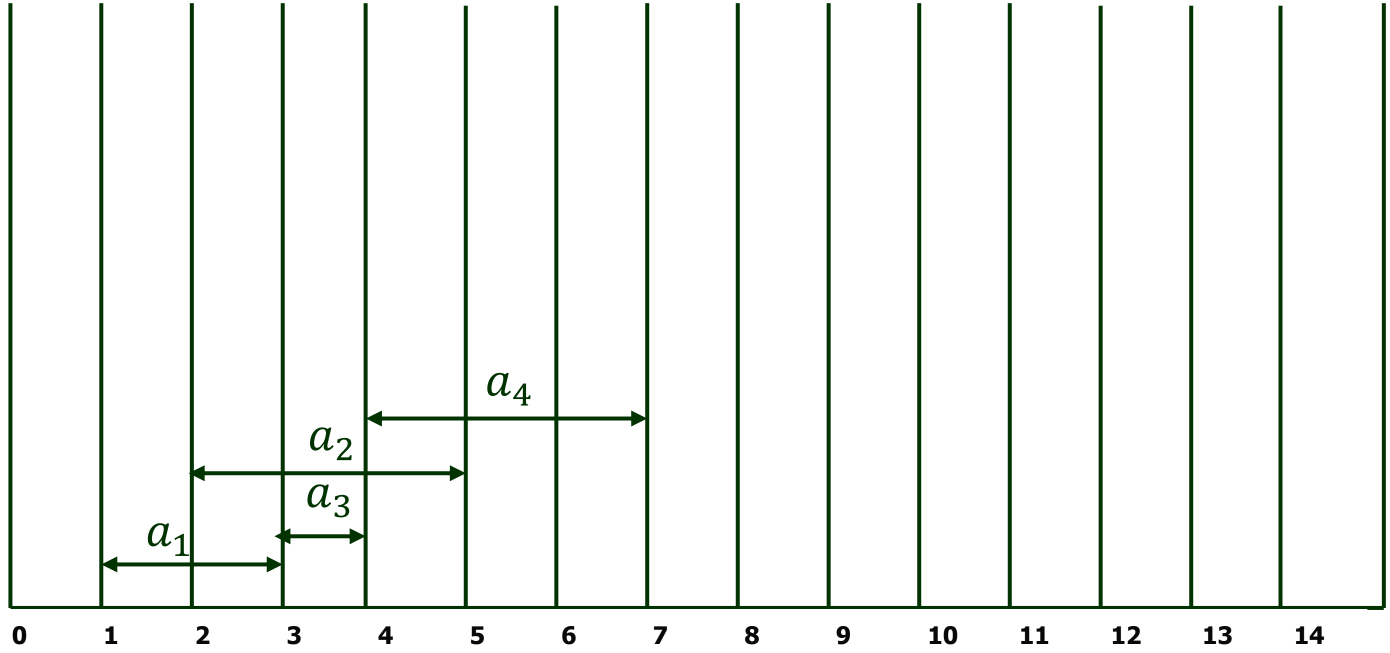

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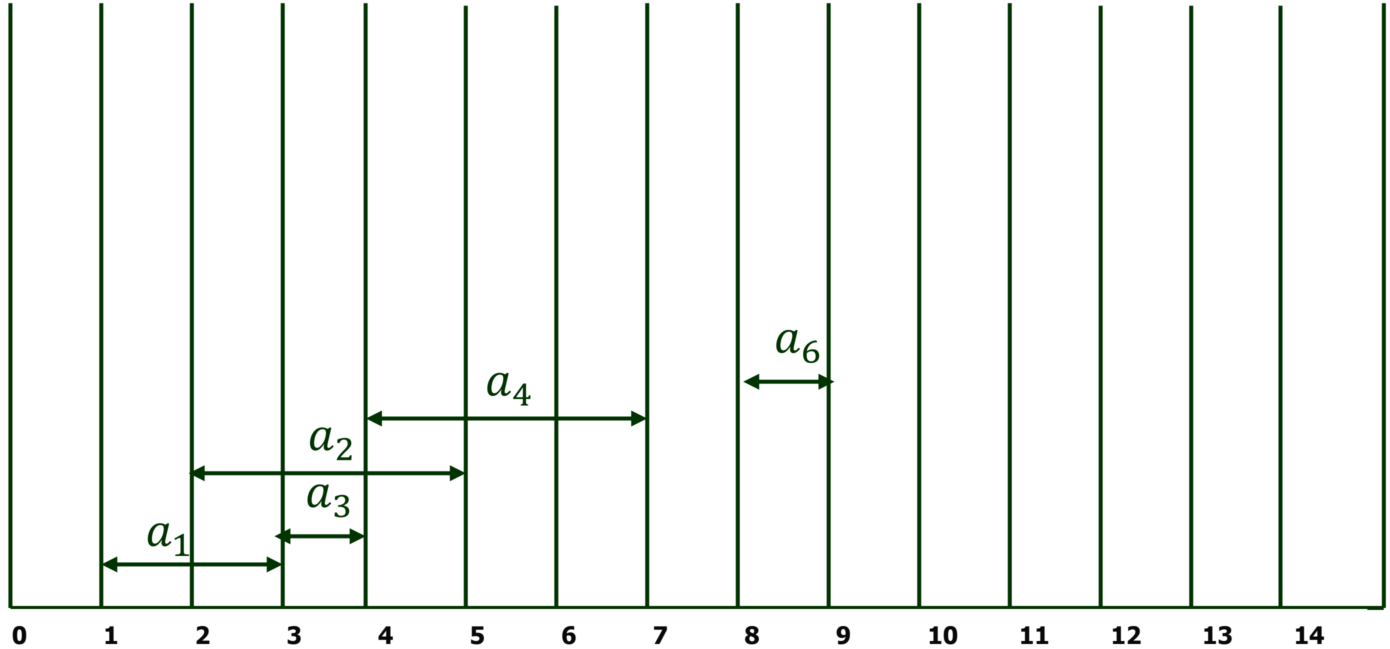

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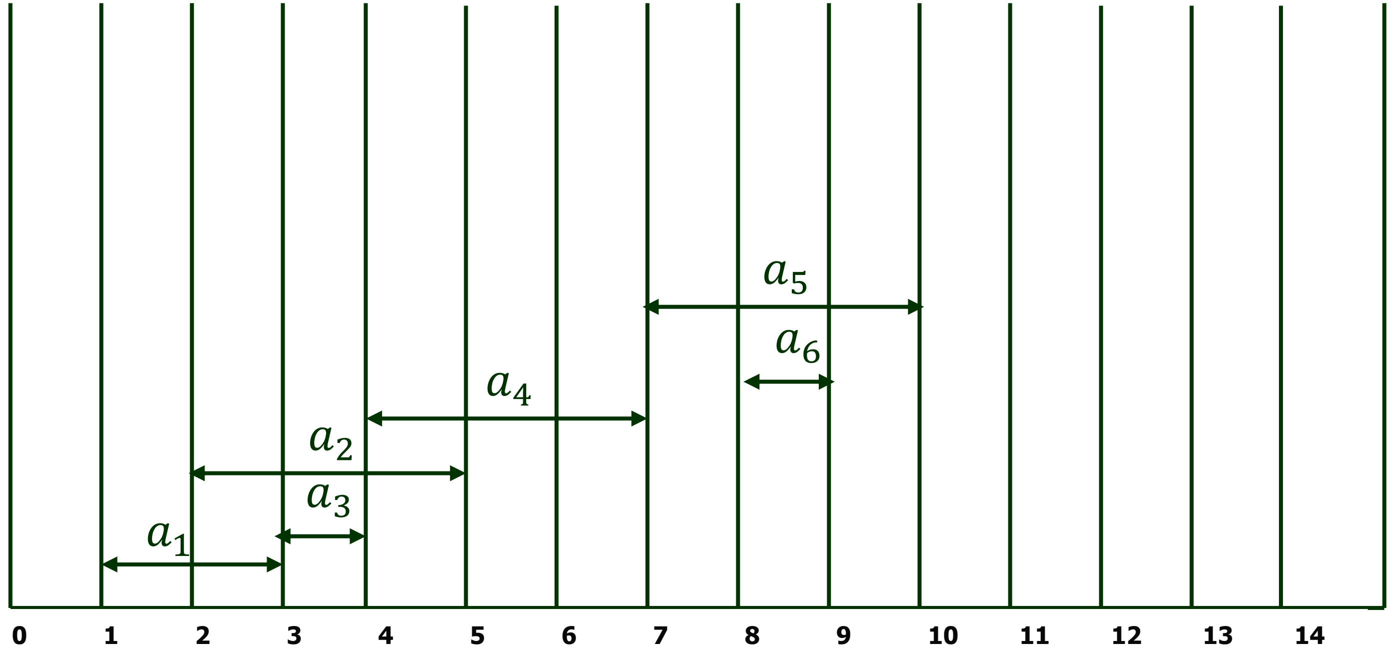

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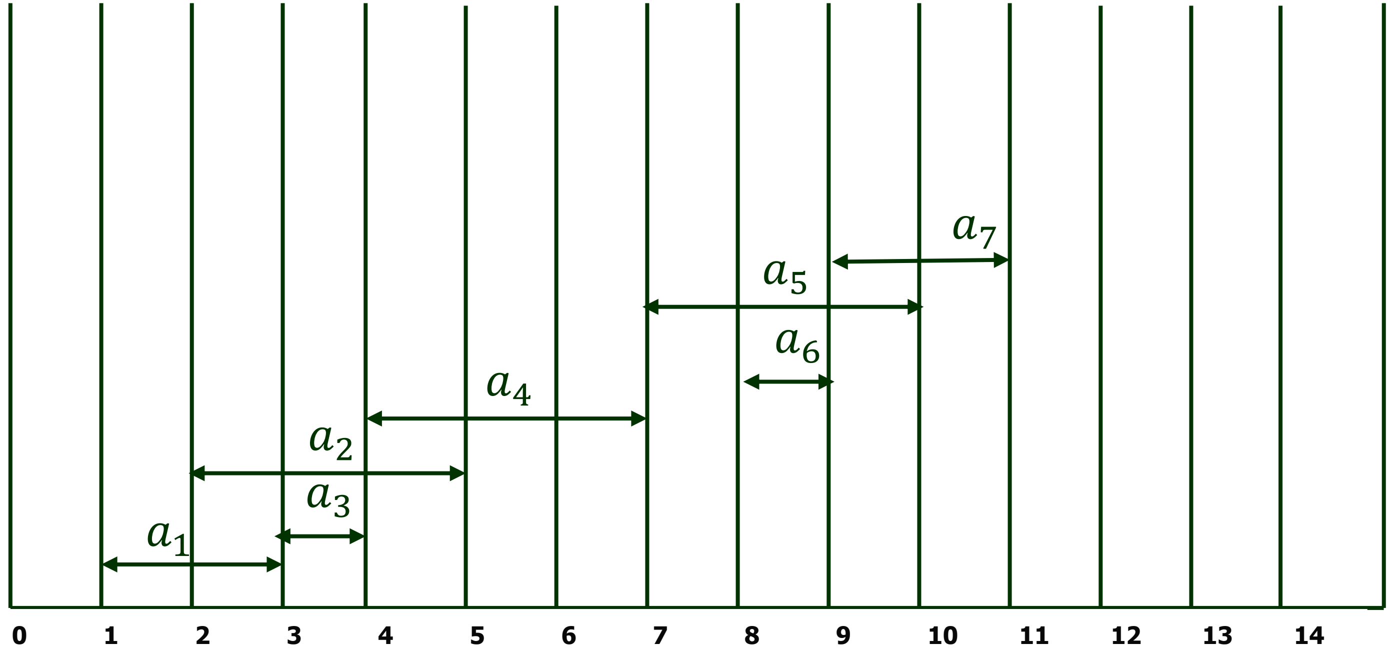

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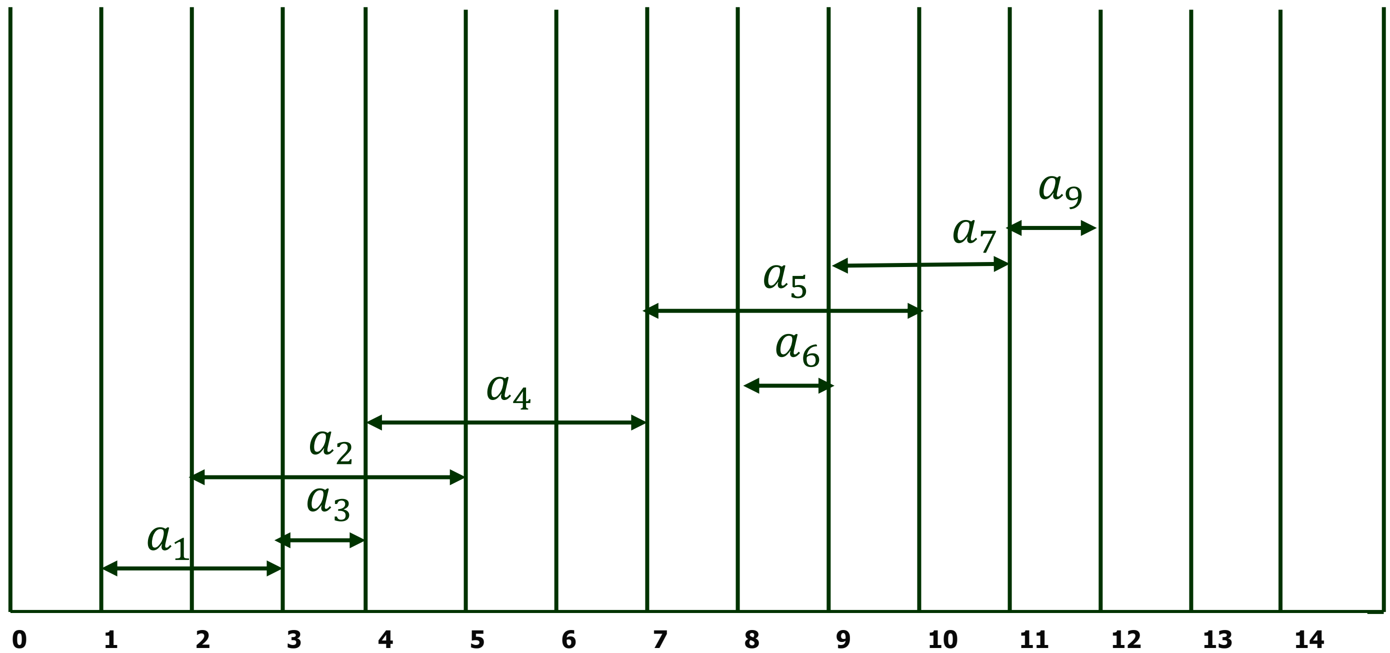

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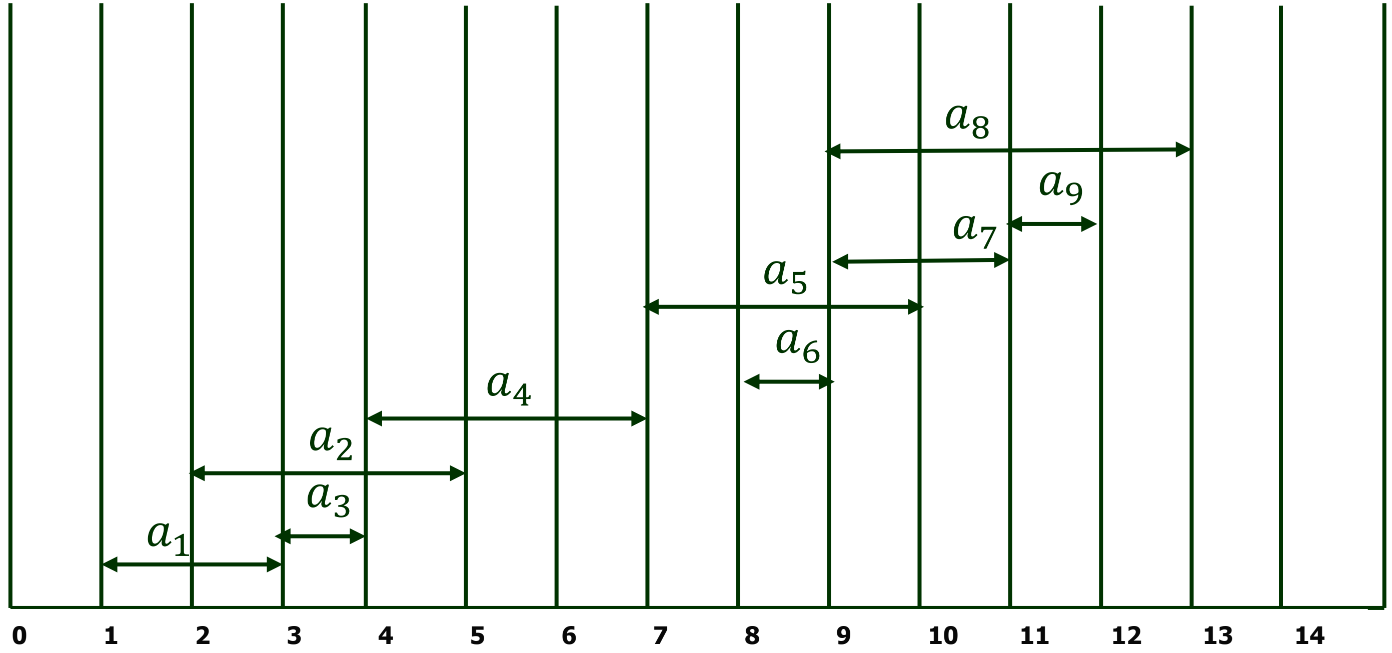

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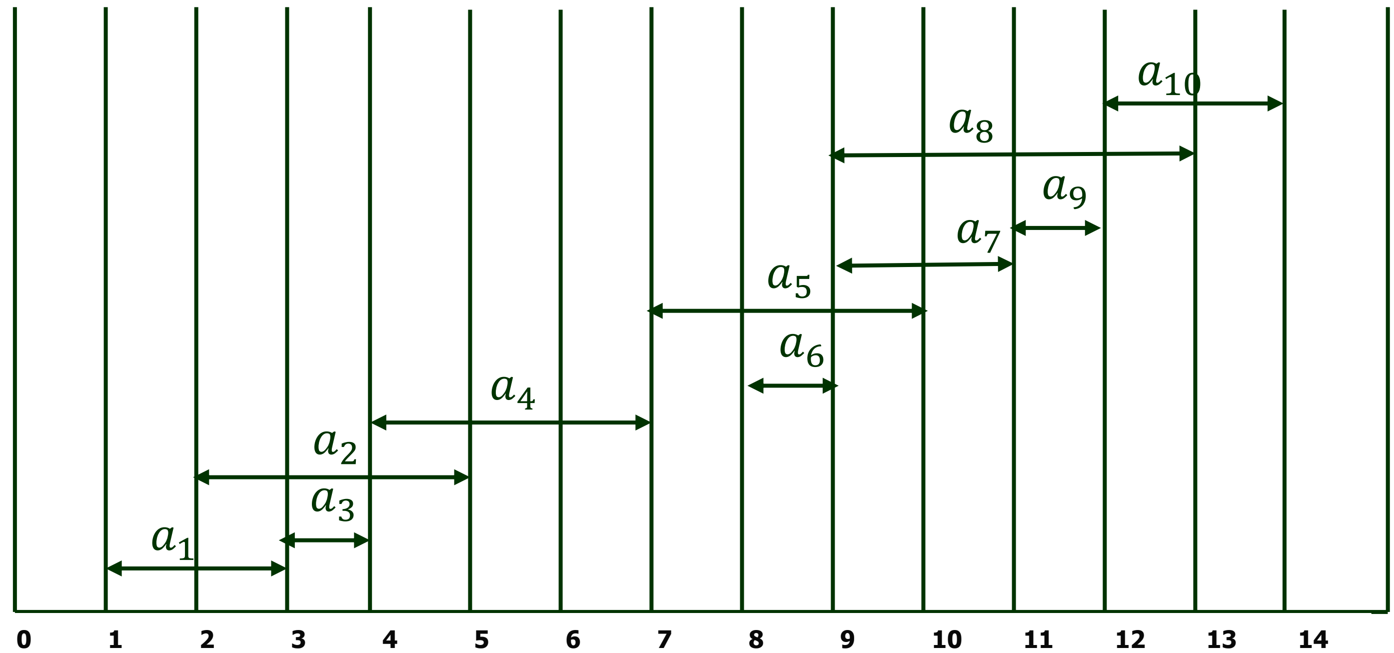

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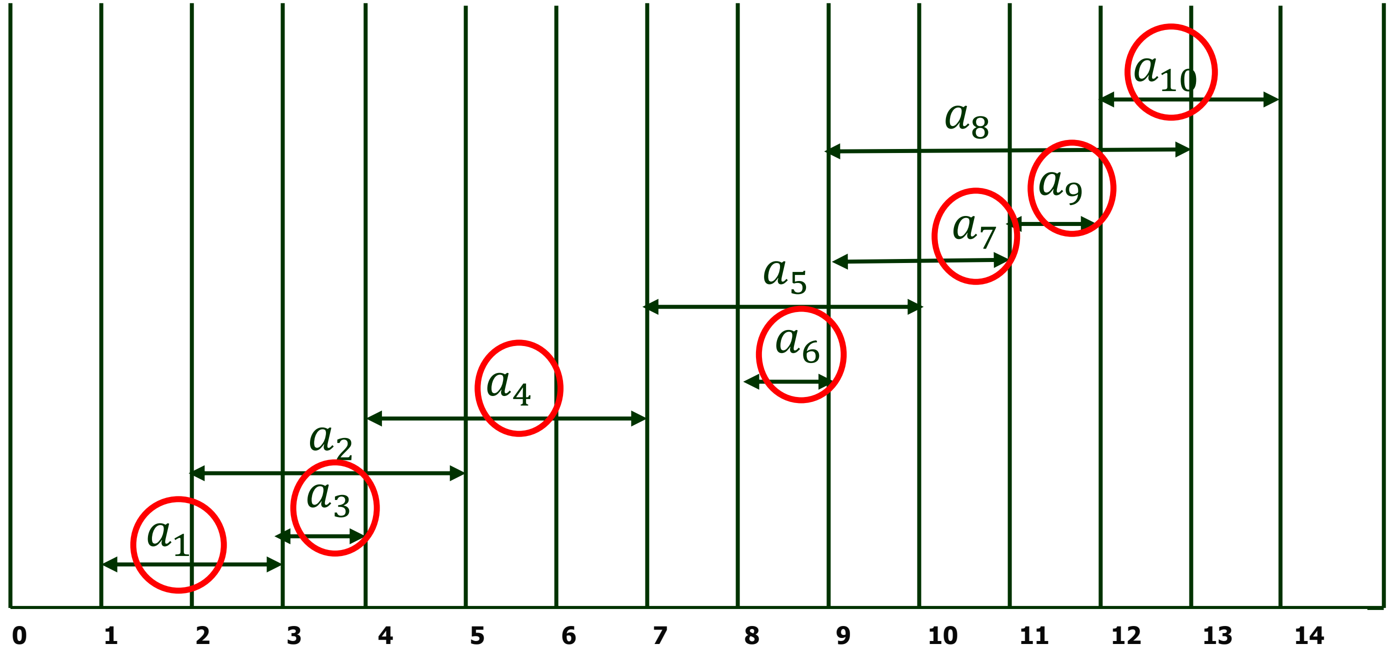

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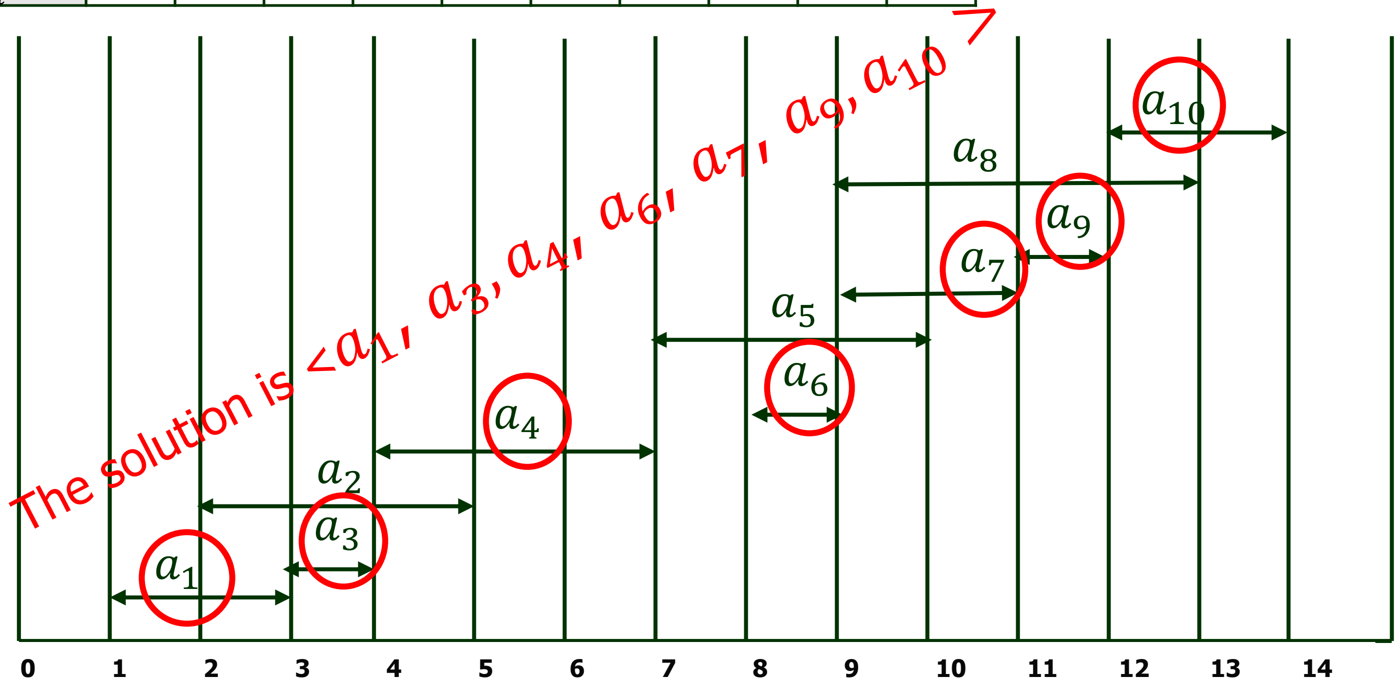

Greedy Algorithm

- **Problem 1: An activity-selection problem**

  Example 2: Find the optimal set in the given activity selection problem.

Activity	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
s_i	1	2	3	4	7	8	9	9	11	12
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Greedy Algorithm

- **Problem 1: An activity-selection problem**

Solution:

First arraigning the following activities in increasing order on their finishing

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Activity	a_2	a_3	a_1	a_4	a_5	a_6	a_8	a_7	a_9	a_{10}
s_i	2	3	1	4	5	6	8	7	9	10
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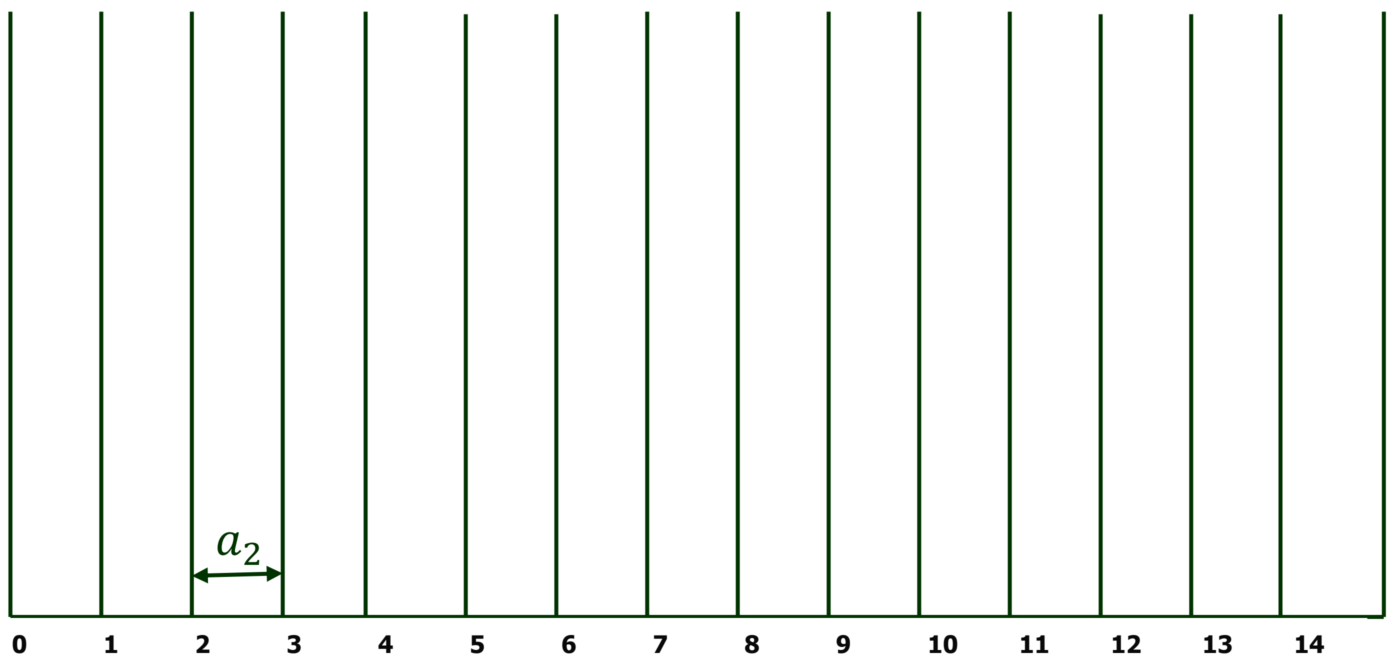

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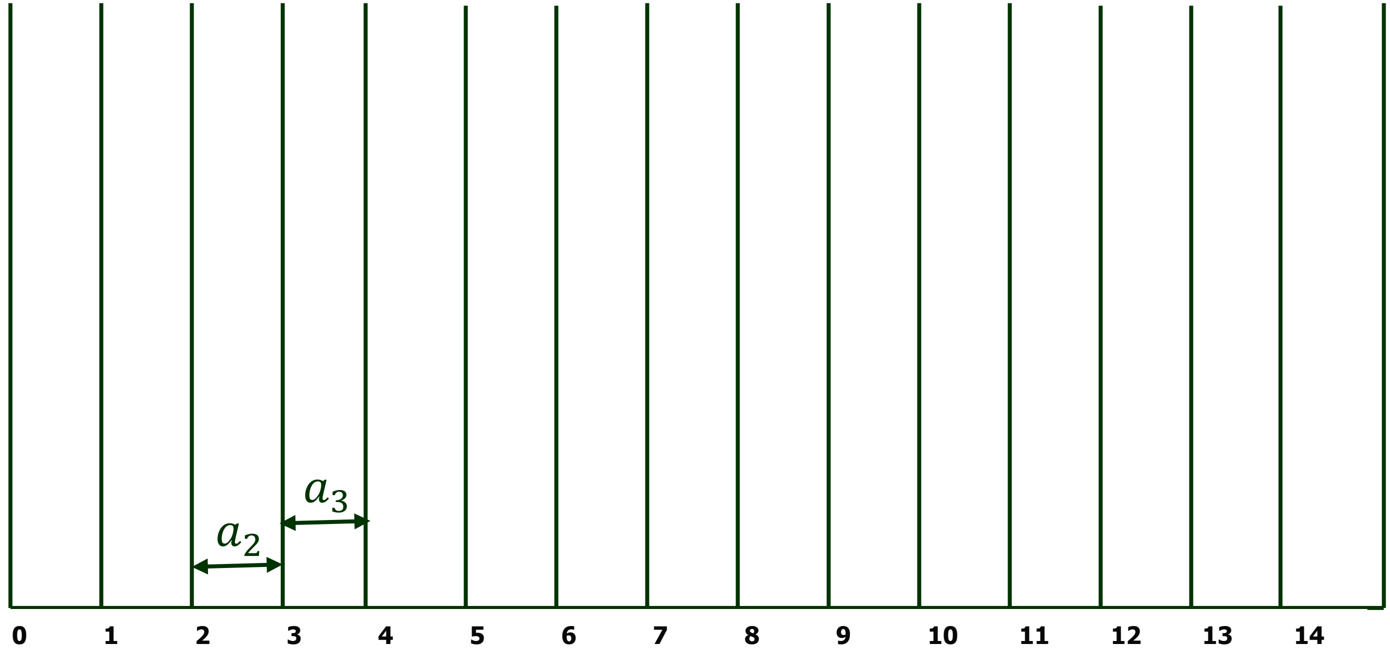

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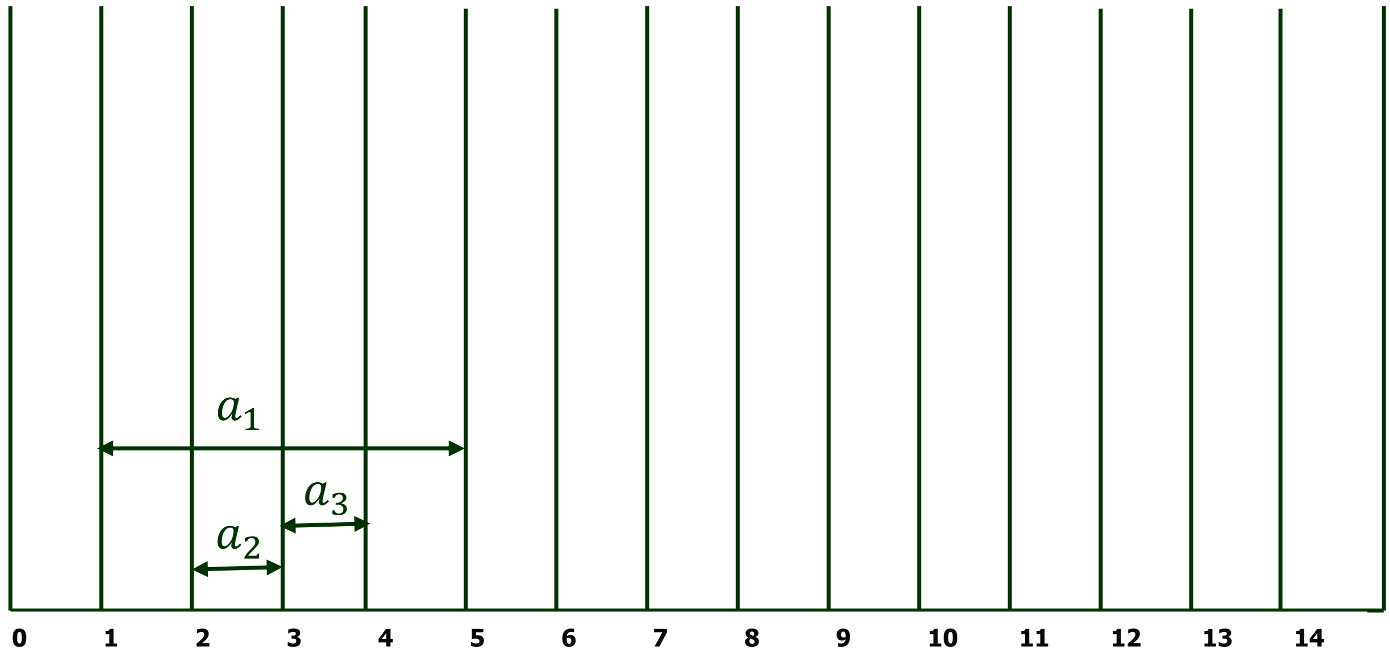

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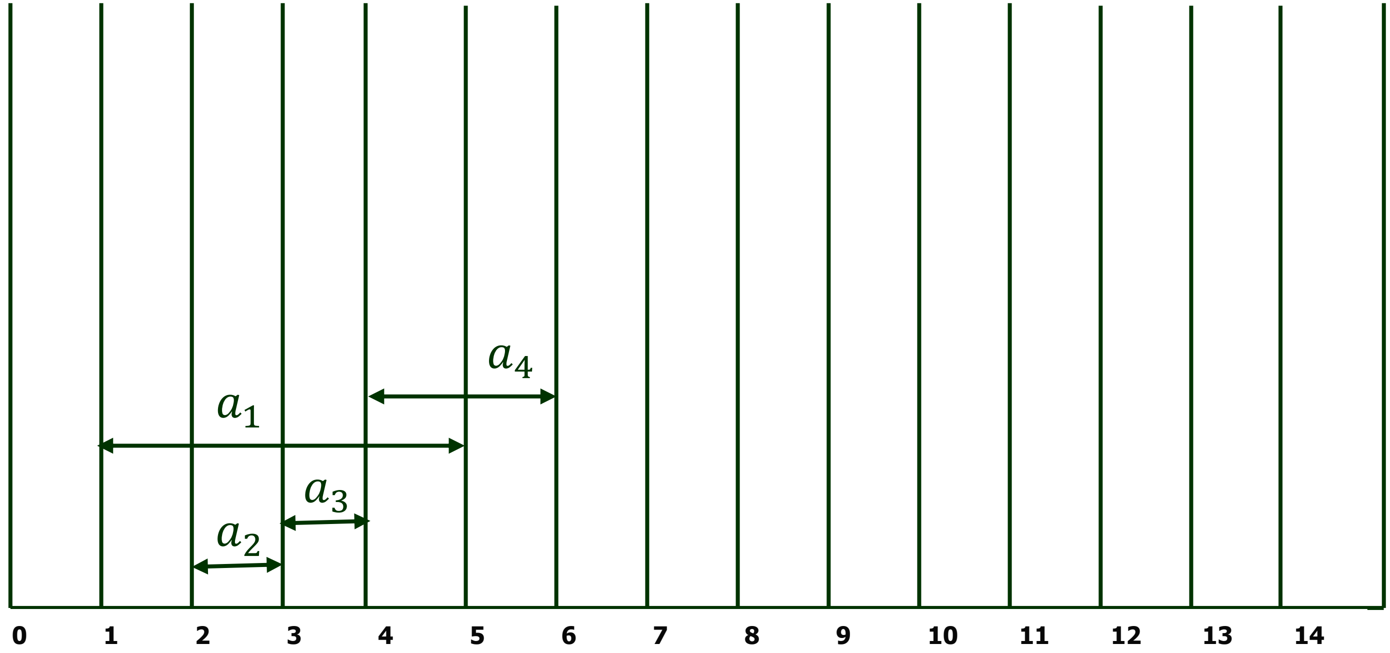

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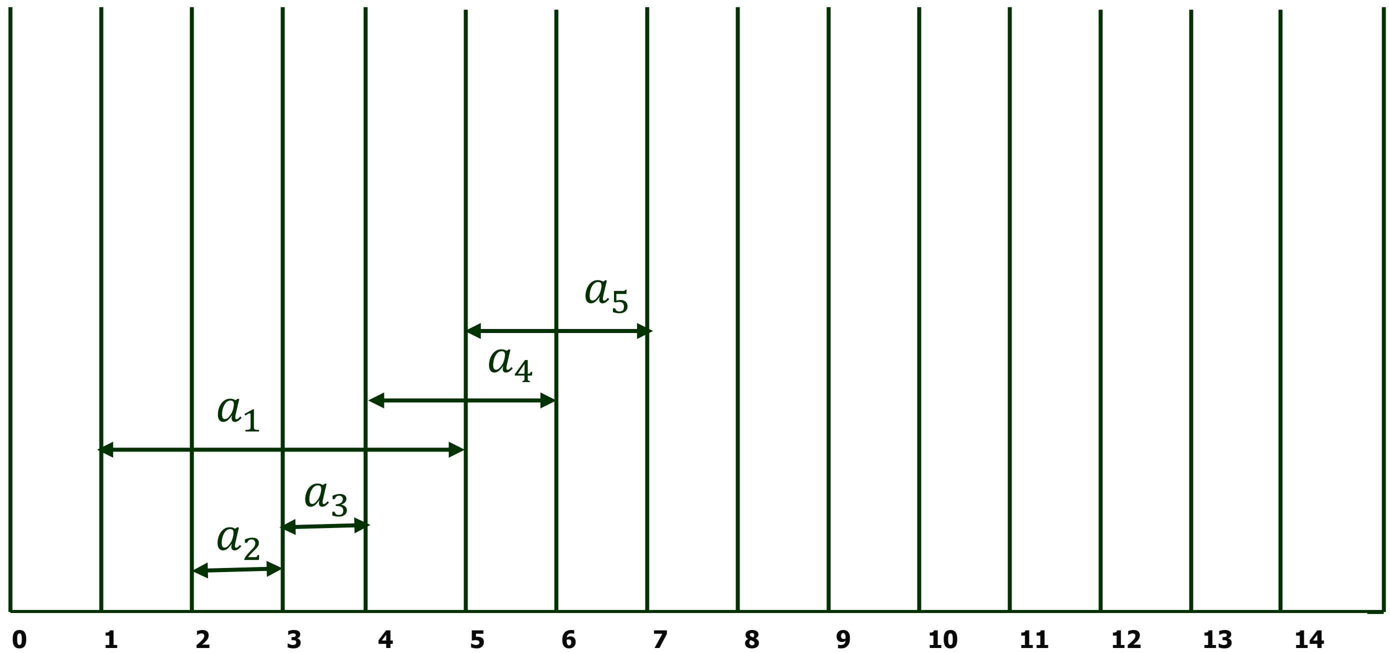

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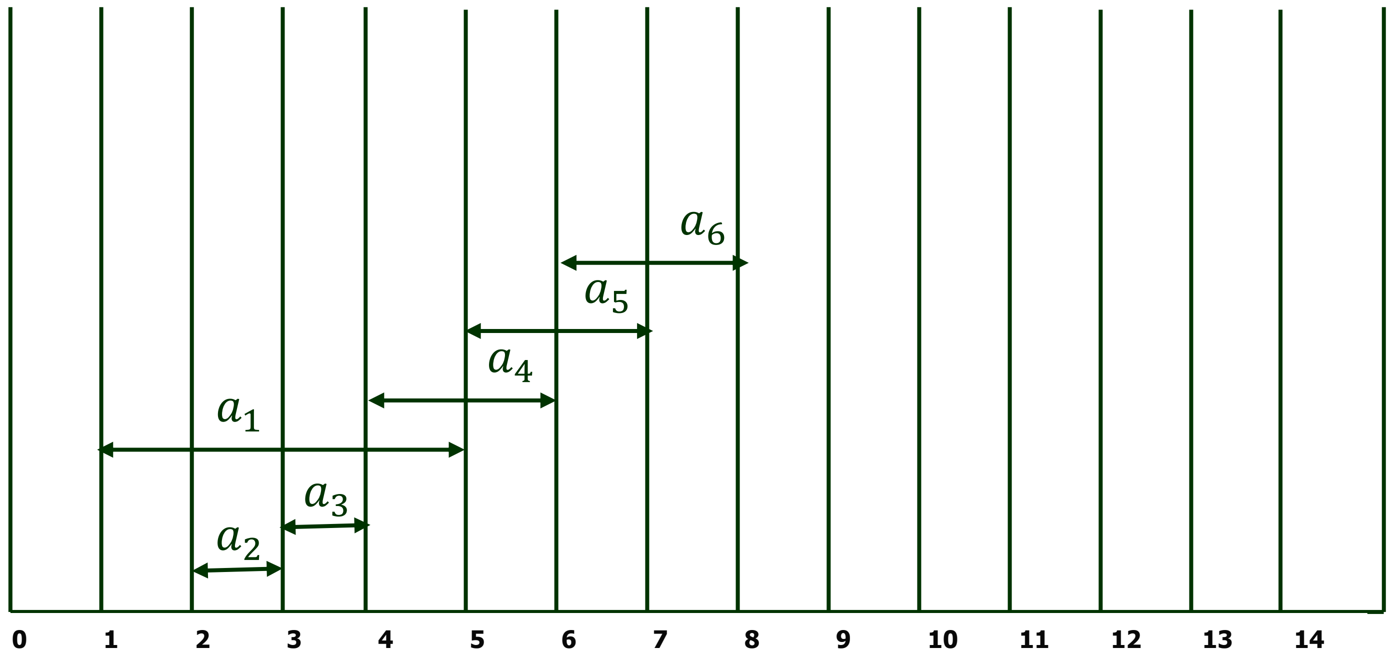

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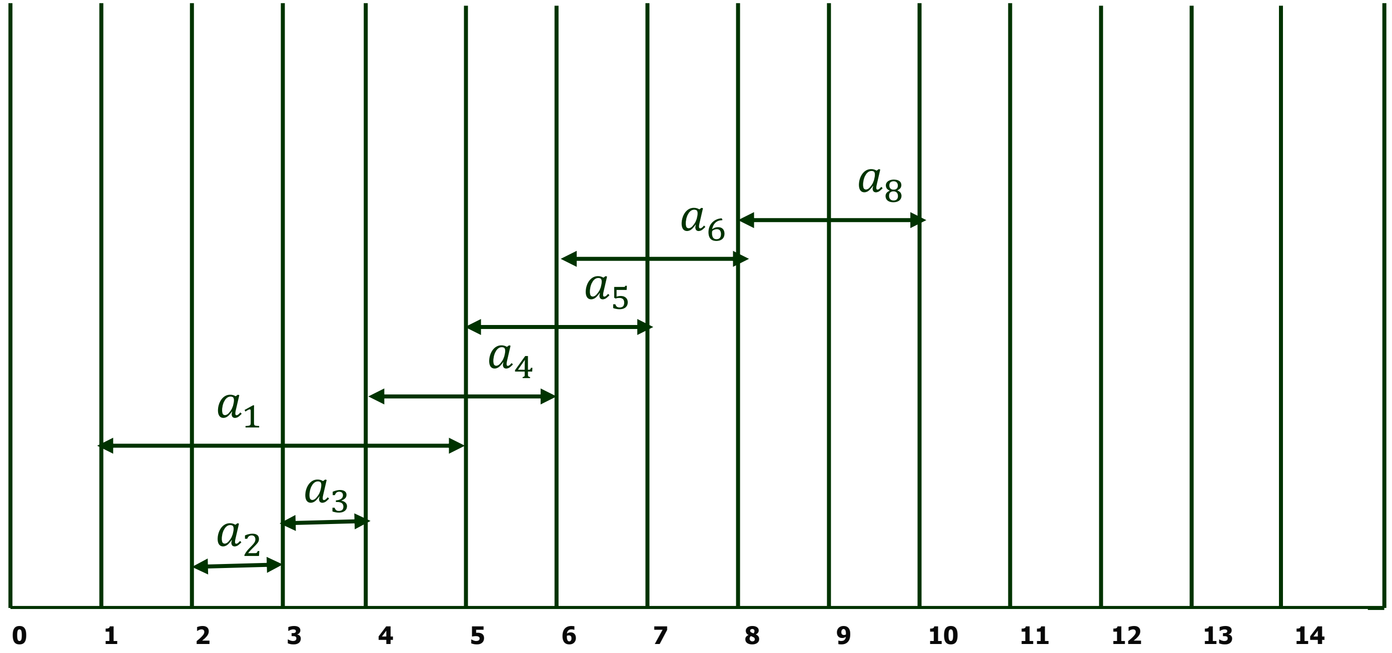

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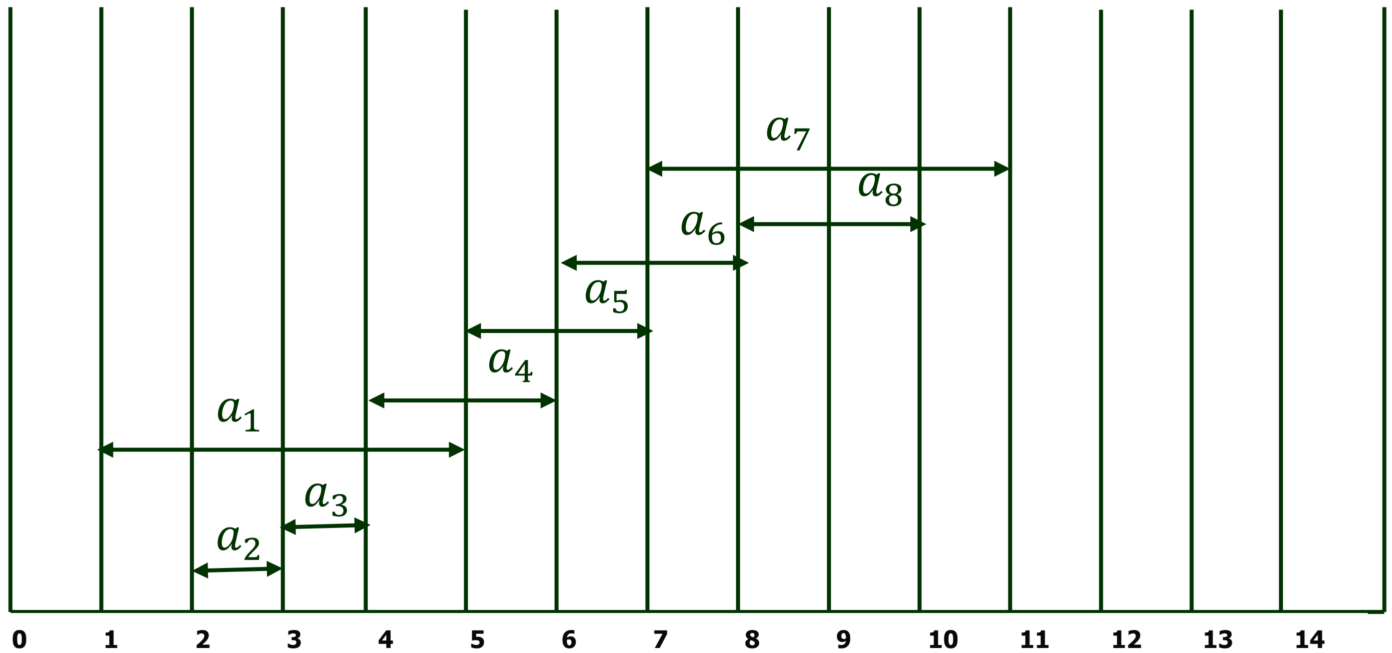

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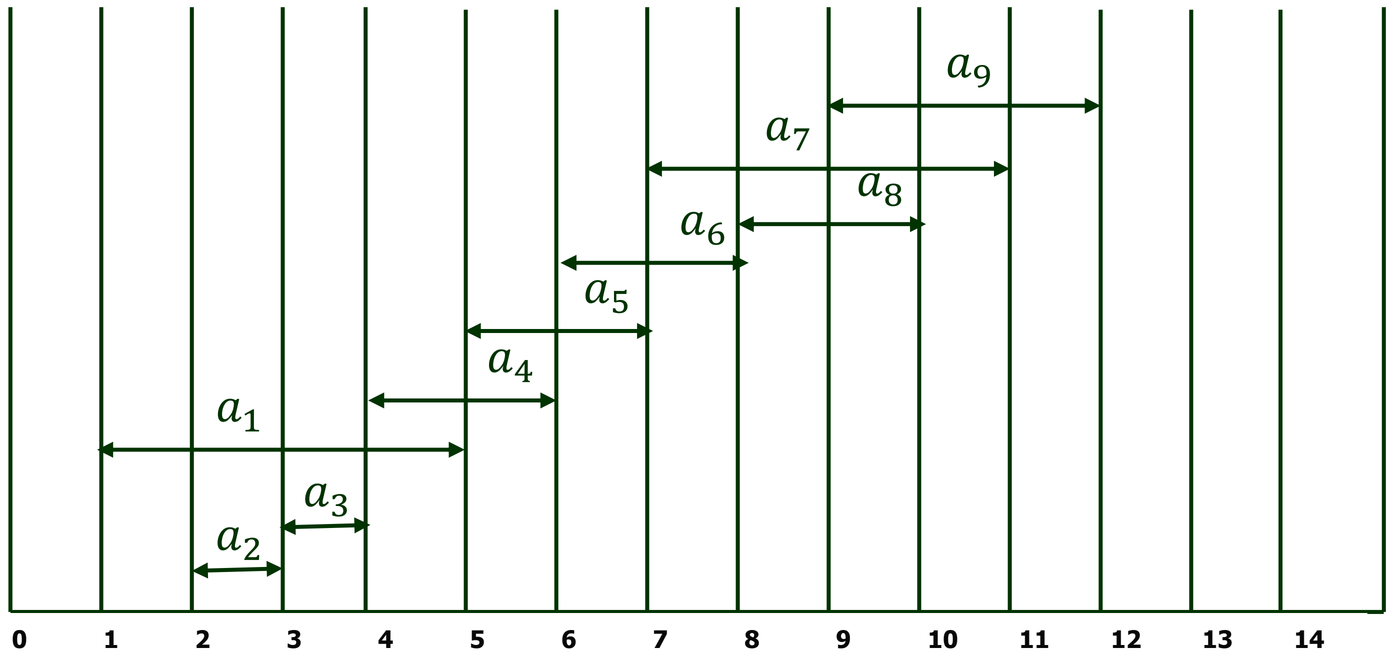

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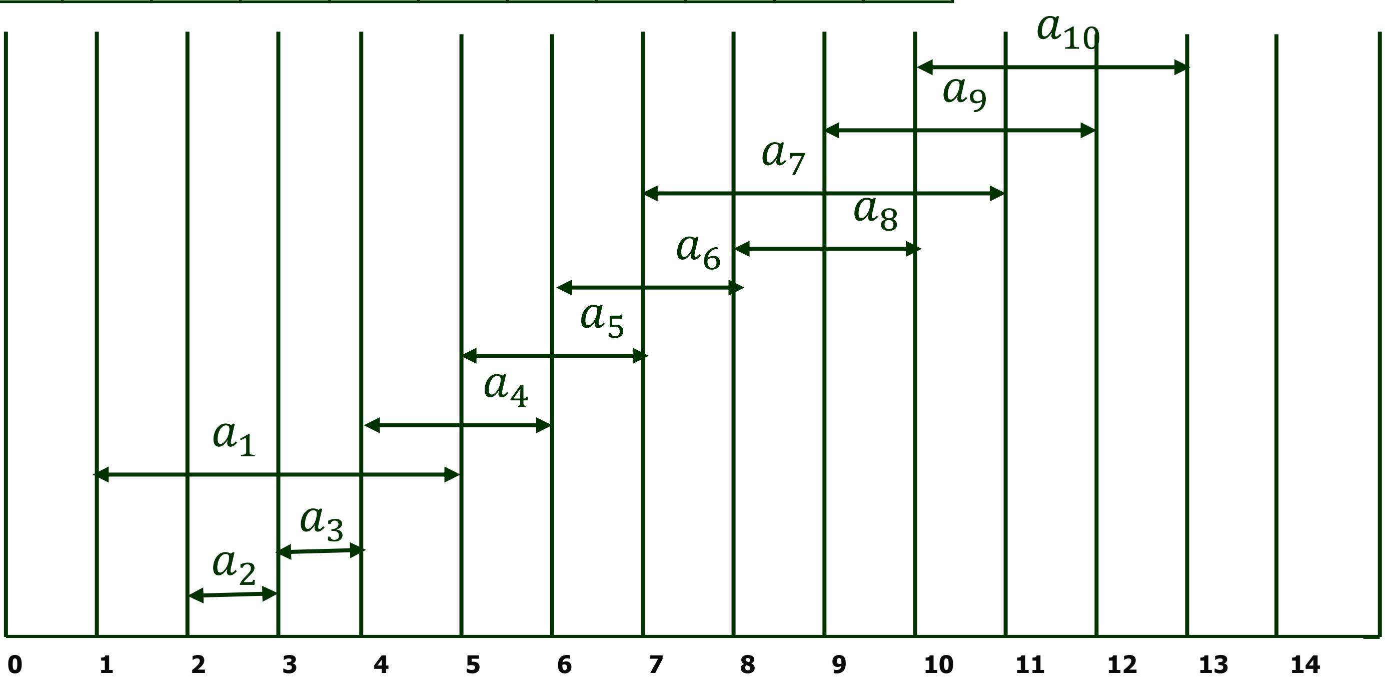

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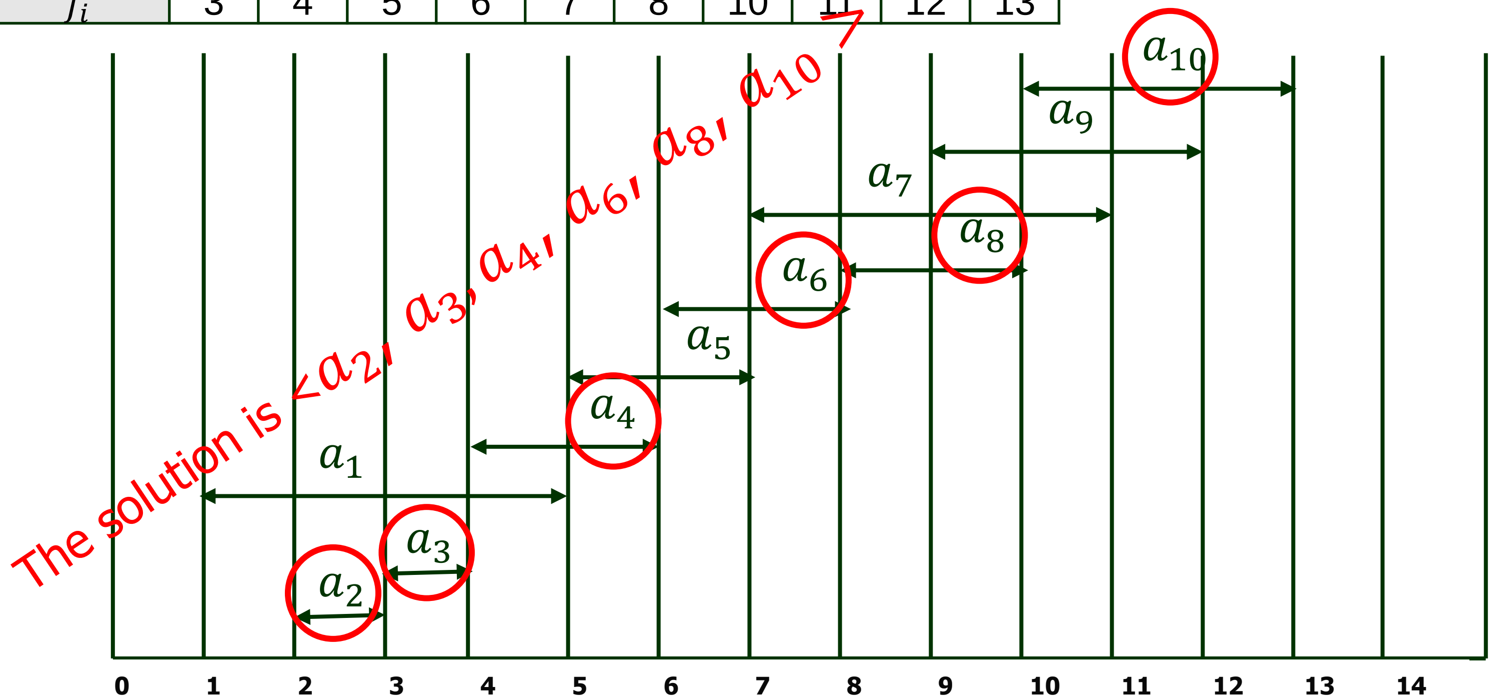

Greedy Algorithm

- **Problem 1: An activity-selection problem**

  A recursive greedy algorithm

  The initial call is Recursive-Activity-Selector(s,f,0,n)

  Recursive-Activity-Selector(s,f,k,n)

  1. m=k+1
  2. while $(m \leq n) and\ (s[m] \geq f[k])$
  3. m=m+1
  4. If ($(m \leq n)$
  5. return$\{a_m\}$ ∪ Recursive-Activity-Selector(s,f,m,n)
  6. else return 0

Greedy Algorithm

- **Problem 1: An activity-selection problem**

A recursive greedy algorithm

Recursive-Activity-Selector(s,f,k,n)

1. m=k+1
2. while $(m \leq n) and\ (s[m] \geq f[k])$
3. m=m+1
4. If $((m \leq n)$
5. return$\{a_m\} \cup$ Recursive-Activity-Selector(s,f,m,n)
6. else return 0

Complexity without sorting is $\Theta(n)$.

Complexity with sorting is $O(n \lg n) + \Theta(n)$.

Greedy Algorithm

- **Problem 1: An activity-selection problem**

A non recursive greedy algorithm

The procedure GREEDY-ACTIVITY-SELECTOR is an iterative version of the procedure RECURSIVE-ACTIVITY-SELECTOR.

Greedy-Activity-Selector (s, f)

1. n=s.length
2. A=$\{a_1\}$
3. k=1
4. for m=2 to n
5. If $(s[m] \geq f[k])$
6. A=A∪ $\{a_m\}$
7. k=m
8. Return A

Greedy Algorithm

- **Problem 1: An activity-selection problem**

  A non recursive greedy algorithm

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  4. for m=2 to n
  5.   If ($s[m] \geq f[k]$)
  6.   A=A∪ $\{a_m\}$
  7.   k=m
  8. Return A

Like the recursive version, Greedy-Activity-Selector schedules a set of n activities in $\Theta(n)$ time,

Greedy Algorithm

- **Problem 2: Task Scheduling problem**
  - An interesting problem that are solving using matroids is the problem of optimally scheduling unit-time tasks on a single processor, where each task has a deadline, along with a penalty paid if the task misses its deadline.
  - This problem looks complicated, when it was solved in a surprisingly simple manner by casting it as a matroid and using a greedy algorithm.

Greedy Algorithm

- **Problem 2: Task Scheduling problem**
  - A unit-time task is a job, such as a program to be run on a computer, that requires exactly one unit of time to complete.
  - Given a finite set S of unit-time tasks, a schedule for S is a permutation of S specifying the order in which to perform these tasks.
  - The first task in the schedule begins at time 0 and finishes at time 1, the second task begins at time 1 and finishes at time 2, and so on..

Greedy Algorithm

- **Problem 2: Task Scheduling problem**
  - The problem of scheduling unit-time tasks with deadlines and penalties for a single processor has the following inputs:
    - A set $S = \{a_1, a_2, a_3, \ldots\ldots., a_n\}$ of n unit-time tasks;
    - A set of n integer deadlines $d_1, d_2, d_3, \ldots\ldots., d_n$ such that each d_i satisfies $1 \le d_i \le n$ and task a_i is supposed to finish by time d_i.
    - A set of n nonnegative weights or penalties $w_1, w_2, w_3, \ldots\ldots., w_n$, such that a penalty of w_i is incurred if task a_i is not finished by time d_i, and incurred no penalty if a task finishes by its deadline.

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Example 1: Find an optimal schedule from the following table, where the tasks with penalties(weight) and deadlines are given.

Task	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Example 1:

  Solution: As per the greedy algorithm, first sort the tasks in descending order of their penalties. So that minimum penalties will be charged.

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

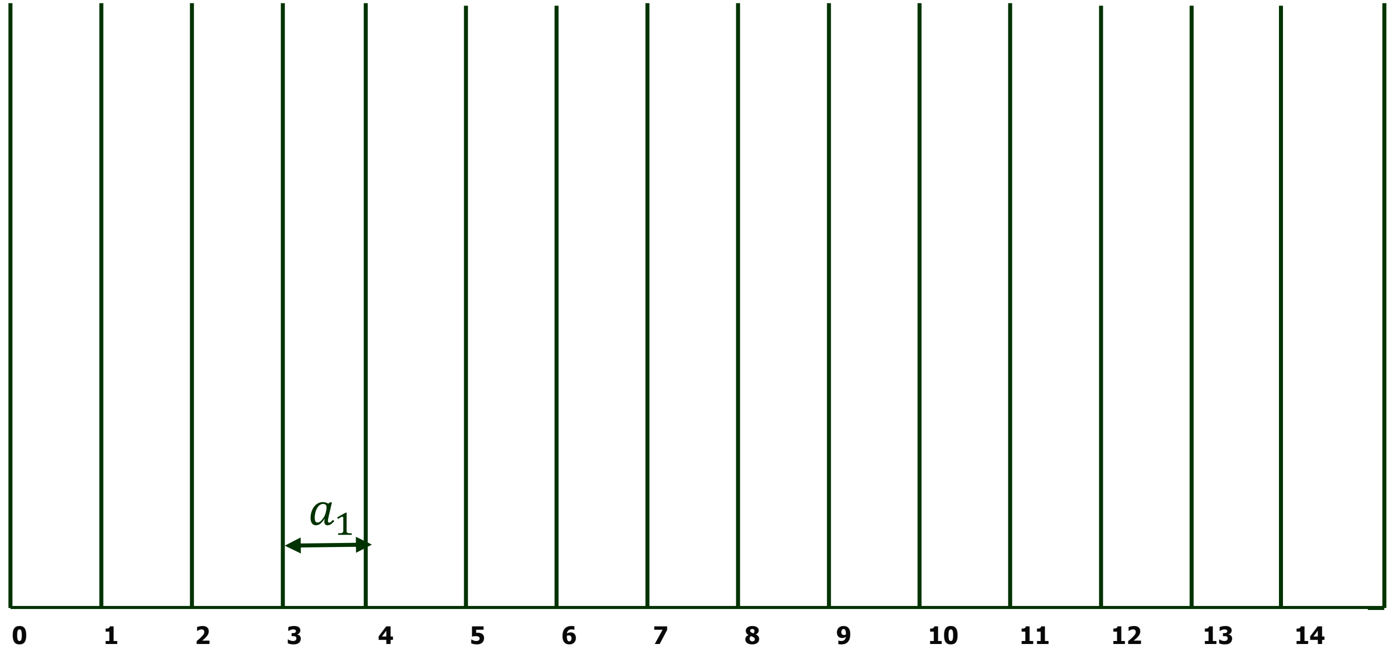
(Note: In this problem the tasks are already sorted)

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

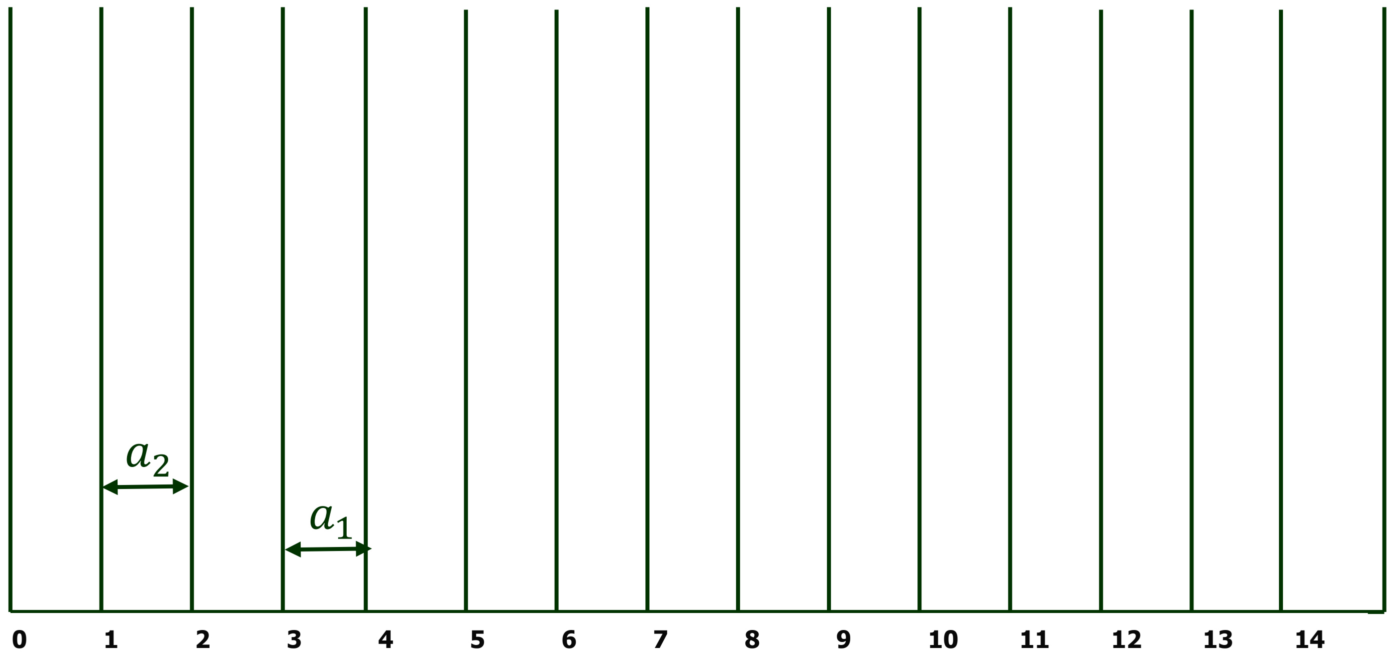


Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

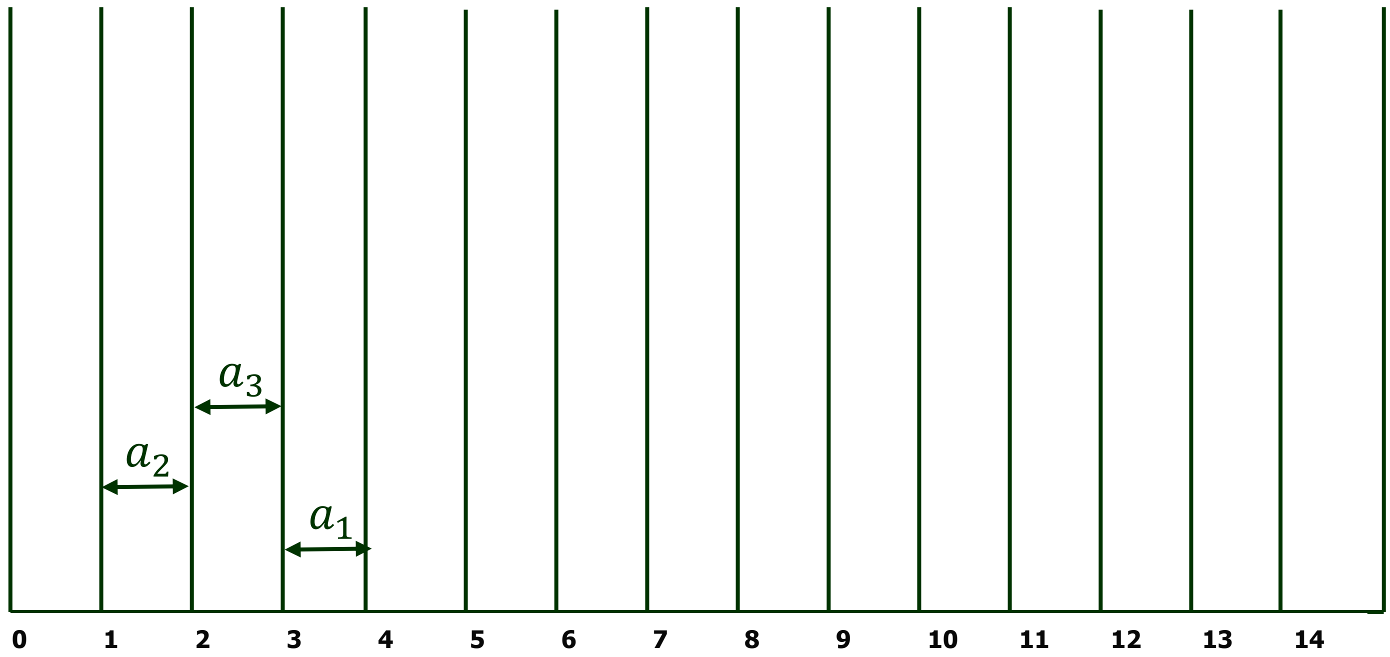


Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

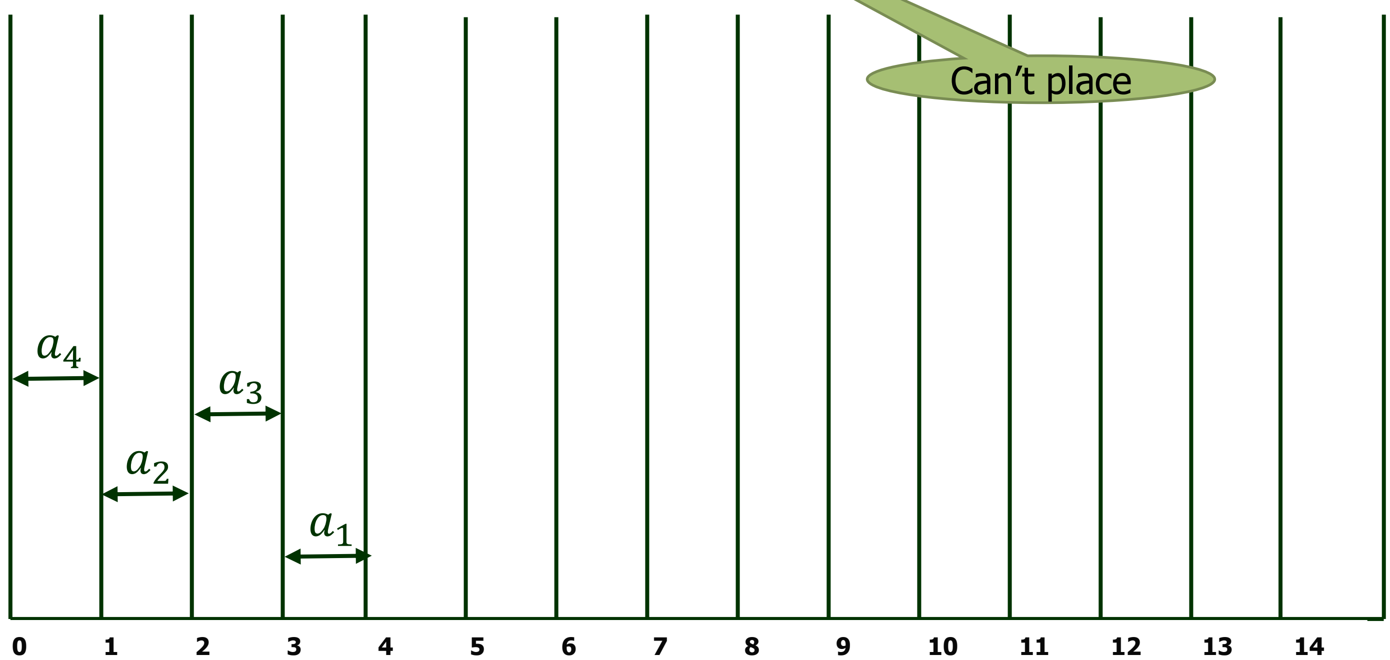


Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

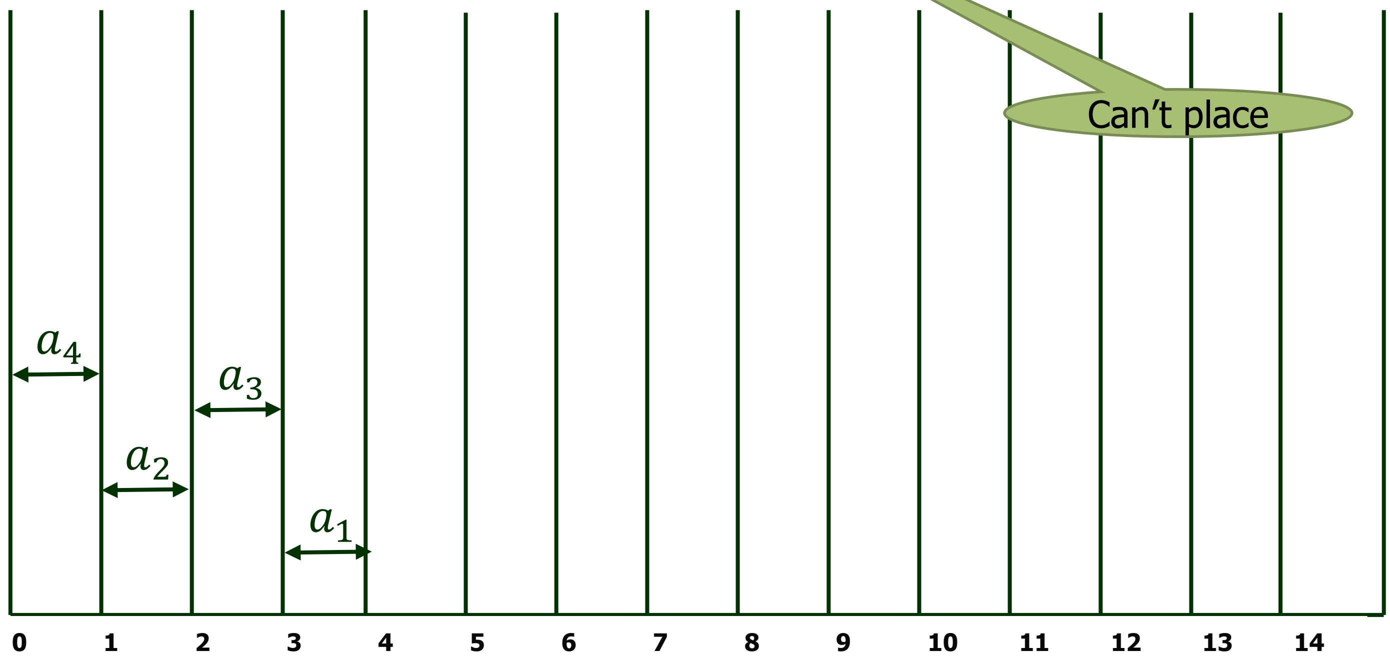


Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

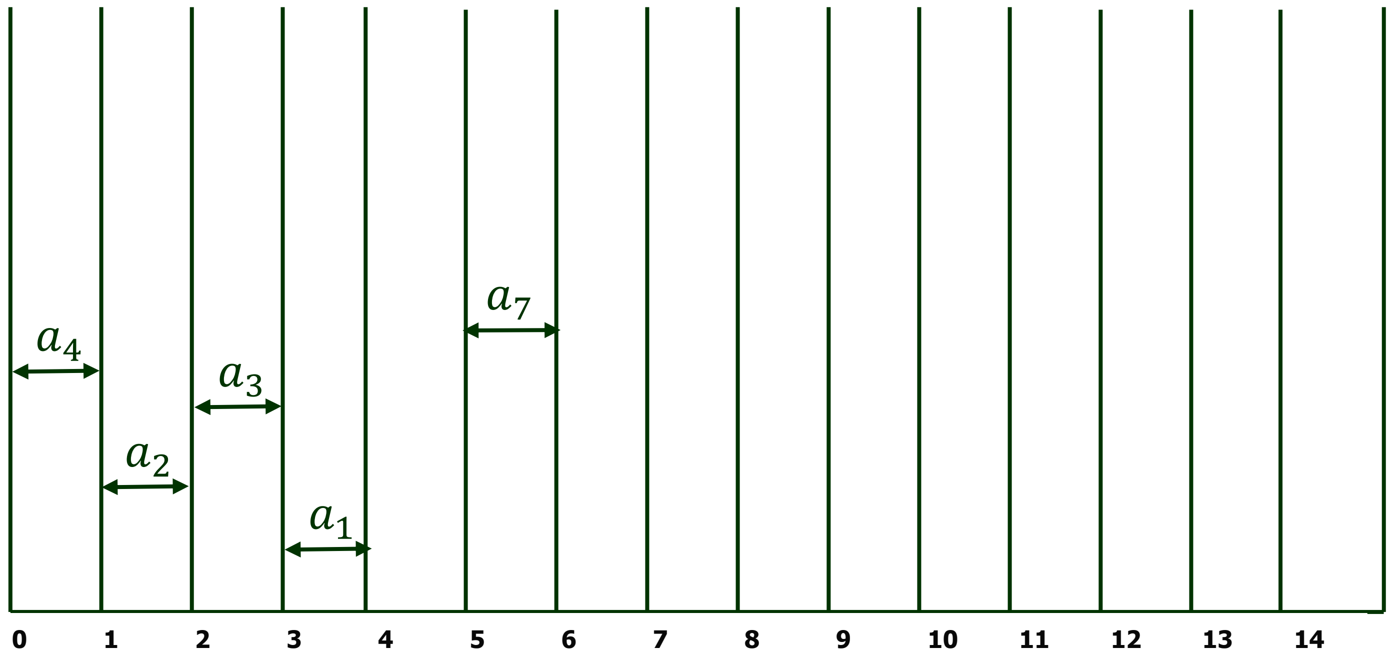


Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10

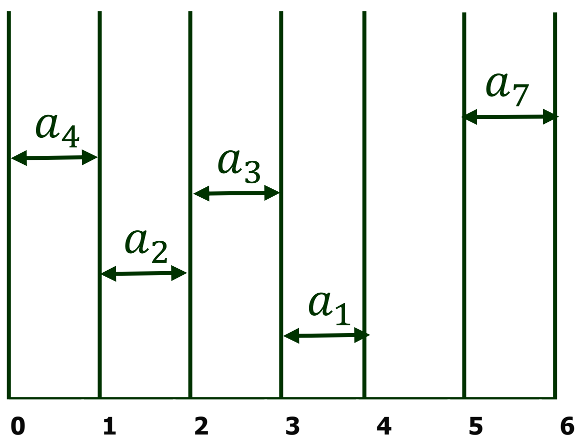


Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Tasks→	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	4	2	4	3	1	4	6
p_i	70	60	50	40	30	20	10



Gantt Chart

a_4	a_2	a_3	a_1		a_7

The Maximum deadline limit =6

So total loss is p[a_5]+p[a_6] = 30 +20= 50

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Example 2: Find an optimal schedule from the following table, where the tasks with penalties(weight) and deadlines are given.

Task	a_1	a_2	a_3	a_4
d_i	2	1	2	1
p_i	100	10	15	27

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Example 2:

  Solution: As per the greedy algorithm, first sort the tasks in descending order of their penalties. So that minimum penalties will be charged.

Task	a_1	a_2	a_3	a_4
d_i	2	1	2	1
p_i	100	10	15	27

After Sorting

Task	a_1	a_4	a_3	a_2
d_i	2	1	2	1
p_i	100	27	15	10

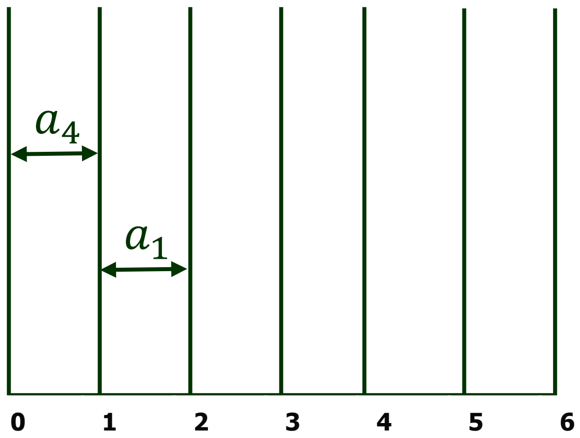
Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Task	a_1	a_4	a_3	a_2
d_i	2	1	2	1
p_i	100	27	15	10

Can't place



Gantt Chart

a_4	a_1

The Maximum deadline limit =2

So total loss is p[a_3]+p[a_2] = 15 +10= 25

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Example 3: Find an optimal schedule from the following table, where the tasks with penalties(weight) and deadlines are given.

Task	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	1	3	3	4	1	2	1
p_i	3	5	18	20	6	1	38

Greedy Algorithm

- **Problem 2: Task Scheduling problem**

  Example 3:

  Solution: As per the greedy algorithm, first sort the tasks in descending order of their penalties. So that minimum penalties will be charged.

Task	a_1	a_2	a_3	a_4	a_5	a_6	a_7
d_i	1	3	3	4	1	2	1
p_i	3	5	18	20	6	1	38

Task	a_7	a_4	a_3	a_5	a_2	a_1	a_6
d_i	1	4	3	1	3	1	2
p_i	38	20	18	6	5	3	1

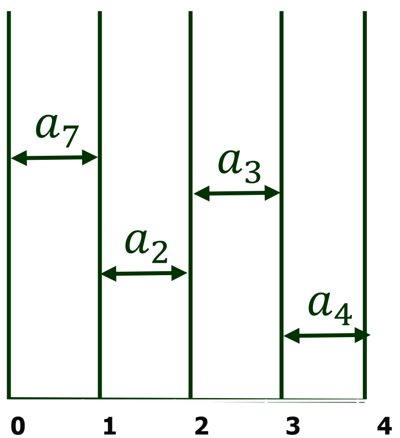
Greedy Algorithm

- **Problem 2: Task Scheduling problem**

Solution:

Task	a_7	a_4	a_3	a_5	a_2	a_1	a_6
d_i	1	4	3	1	3	1	2
p_i	38	20	18	6	5	3	1

Can't place



Gantt Chart

a_7	a_2	a_3	a_4

The Maximum deadline limit =4,

So total loss is $p[a_5]+p[a_1]+p[a_6] = 6+3+1 = 10$

Greedy Algorithm

- **Problem 3: Huffman Coding (A solution to encoding problem)**

Problem:

Suppose we have a 58,000 characters data file that we wish to store compactly. The characters occurred with frequencies on that file is given below:

Character	a	e	i	o	u	s	t
Frequency (in thousand)	10	15	12	3	4	13	1
Fixed length codeword (3-bit)	000	001	010	011	100	101	110

It was observed that if we assigned 3-bit to each character, we required 1,74,000 bit to encode the file.

Greedy Algorithm

- **Problem 3: Huffman Coding**
  - Huffman Coding is a famous Greedy Algorithm.
  - It is used for the lossless compression of data.
  - It uses variable length encoding.
  - It assigns variable length code to all the characters.
  - The code length of a character depends on how frequently it occurs in the given text.
  - The character which occurs most frequently gets the smallest code.
  - The character which occurs least frequently gets the largest code.
  - It is also known as Huffman Encoding

Greedy Algorithm

- **Problem 3: Huffman Coding**
  - Huffman Coding implements a rule known as a prefix rule.
  - This is to prevent the ambiguities while decoding.
  - It ensures that the code assigned to any character is not a prefix of the code assigned to any other character.

Greedy Algorithm

- **Problem 3: Huffman Coding**
  - **Major Steps in Huffman Coding-**
  - There are two major steps in Huffman Coding-
    1. Building a Huffman Tree from the input characters.
    2. Assigning code to the characters by traversing the Huffman Tree.

Greedy Algorithm

- **Problem 3: Huffman Coding**

1. Building a Huffman Tree from the input characters.

The steps involved in the construction of Huffman Tree are as follows-

Step-01:

- Create a leaf node for each character of the text.
- Leaf node of a character contains the occurring frequency of that character.

Step-02:

- Arrange all the nodes in increasing order of their frequency value.

Greedy Algorithm

- **Problem 3: Huffman Coding**

1. Building a Huffman Tree from the input characters.

Step-03:

- Considering the first two nodes having minimum frequency,
- Create a new internal node.
- The frequency of this new node is the sum of frequency of those two nodes.
- Make the first node as a left child and the other node as a right child of the newly created node.

Step-04:

- Keep repeating Step-02 and Step-03 until all the nodes form a single tree.

The tree finally obtained is the desired Huffman Tree.

Greedy Algorithm

- **Problem 3: Huffman Coding**

  1. Building a Huffman Tree from the input characters.

  Time Complexity-

  The time complexity analysis of Huffman Coding is as follows-

  - extractMin( ) is called 2 x (n-1) times if there are n nodes.
  - As extractMin( ) calls minHeapify( ), it takes O(logn) time.

  Thus, Overall time complexity of Huffman Coding becomes O(nlogn).

  [Note: Here, n is the number of unique characters in the given text.]

Greedy Algorithm

- **Problem 3: Huffman Coding**
  2. Assigning code to the characters by traversing the Huffman Tree.
     - Assign weight to all the edges of the constructed Huffman Tree.
     - Let us assign weight '0' to the left edges and weight '1' to the right edges.

Greedy Algorithm

- **Problem 3: Huffman Coding**
  2. Assigning code to the characters by traversing the Huffman Tree.
     - Assign weight to all the edges of the constructed Huffman Tree.
     - Let us assign weight '0' to the left edges and weight '1' to the right edges.
- Rule
  - If you assign weight '0' to the left edges, then assign weight '1' to the right edges.
  - If you assign weight '1' to the left edges, then assign weight '0' to the right edges.
  - Any of the above two conventions may be followed.
  - But follow the same convention at the time of decoding that is adopted at the time of encoding.

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1-

A file contains the following characters with the frequencies as shown. If Huffman Coding is used for data compression, determine-

- Huffman Code for each character
- Average code length
- Length of Huffman encoded message (in bits)

Characters	Frequencies
a	10
e	15
i	12
o	3
u	4
s	13
t	1

Greedy Algorithm

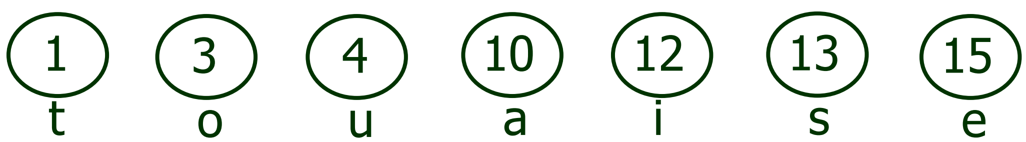
- **Problem 3: Huffman Coding**

  Example 1-

  Solution

  - First let us construct the Huffman Tree.
  - Huffman Tree is constructed in the following steps-

Step 1:



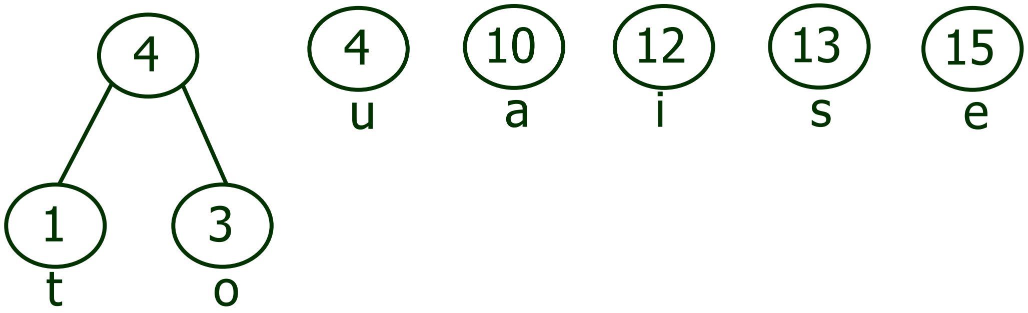
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 2:



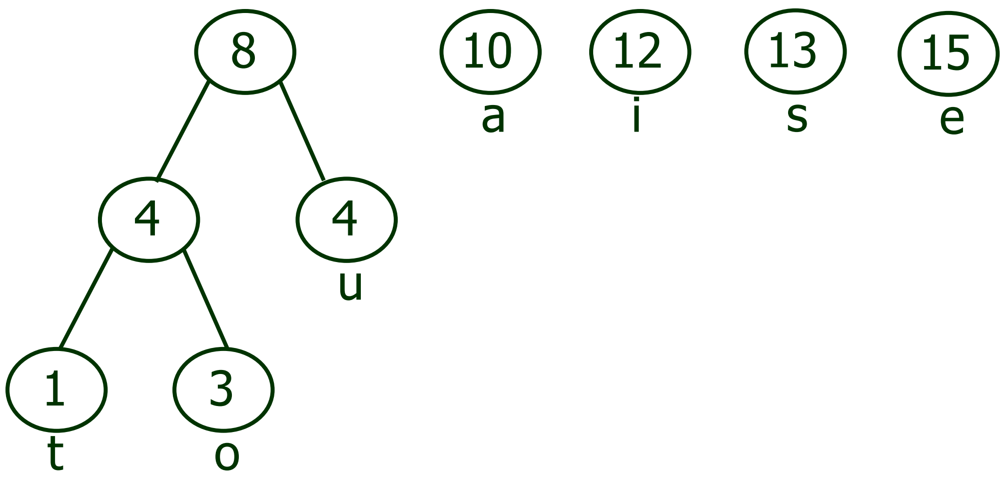
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 3:



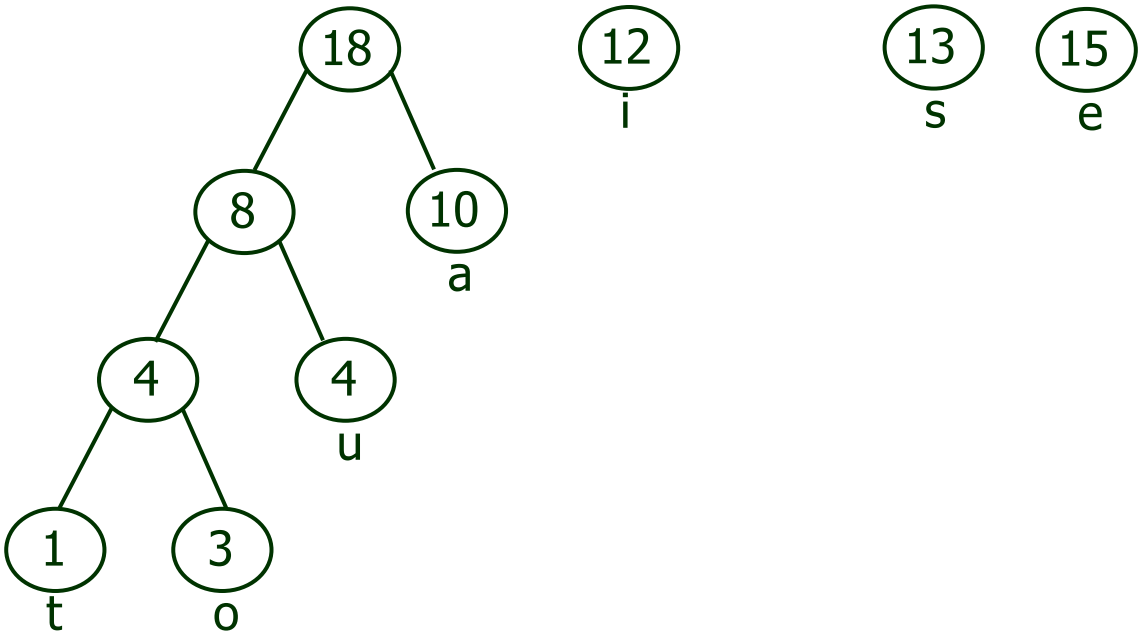
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 4:



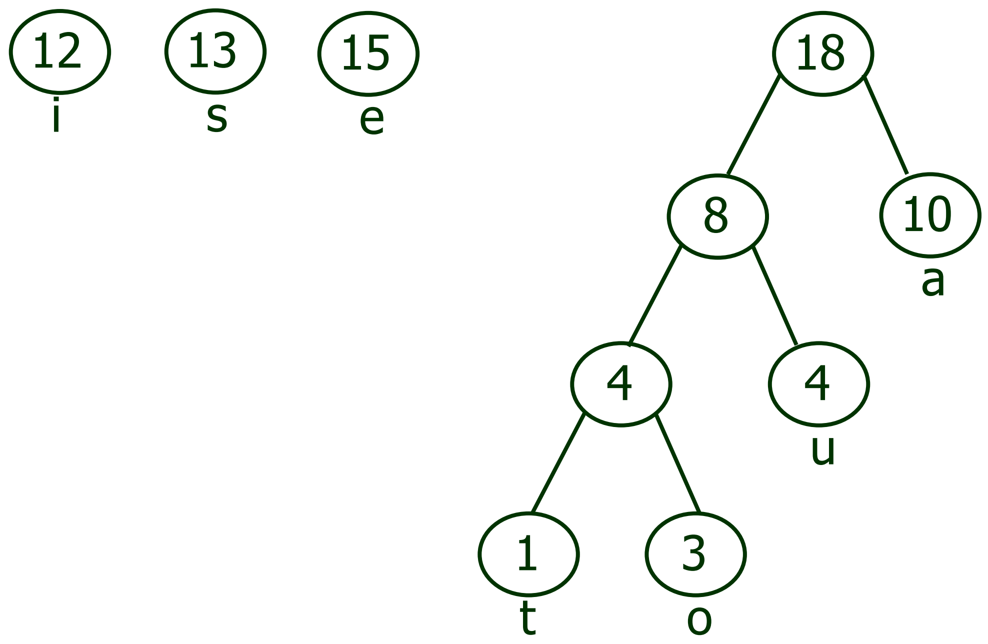
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 6:



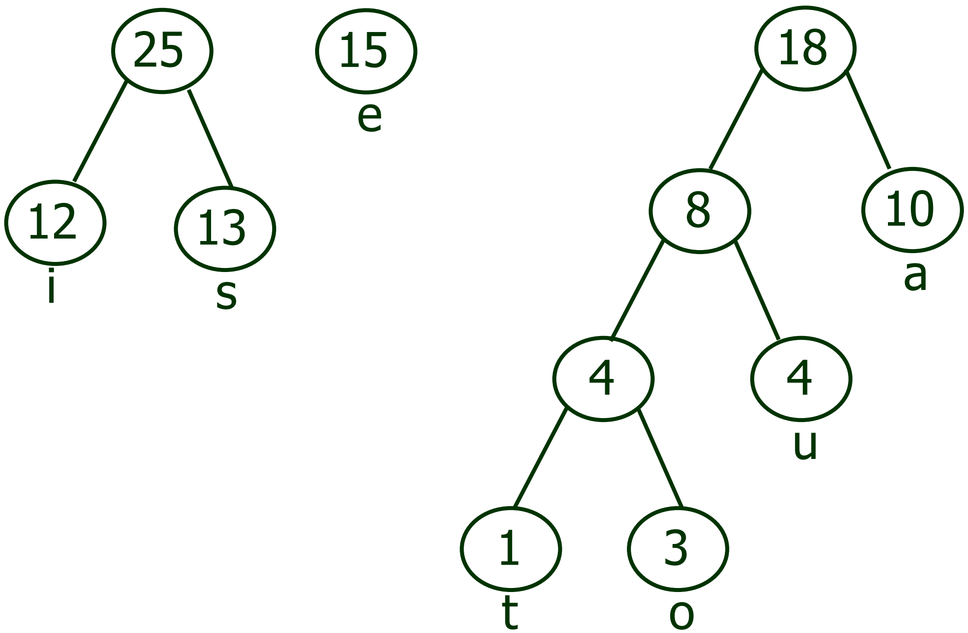
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 7:



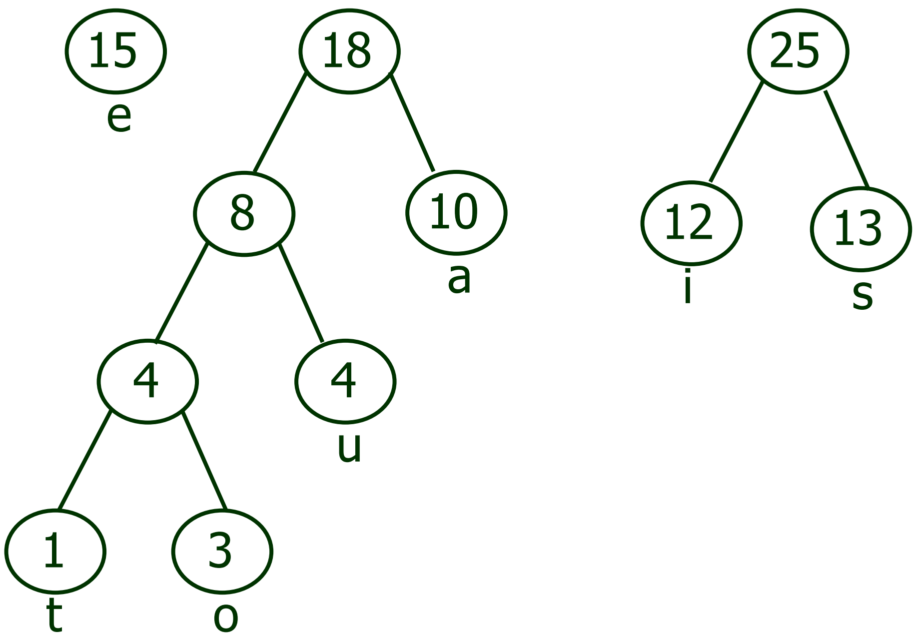
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 8:



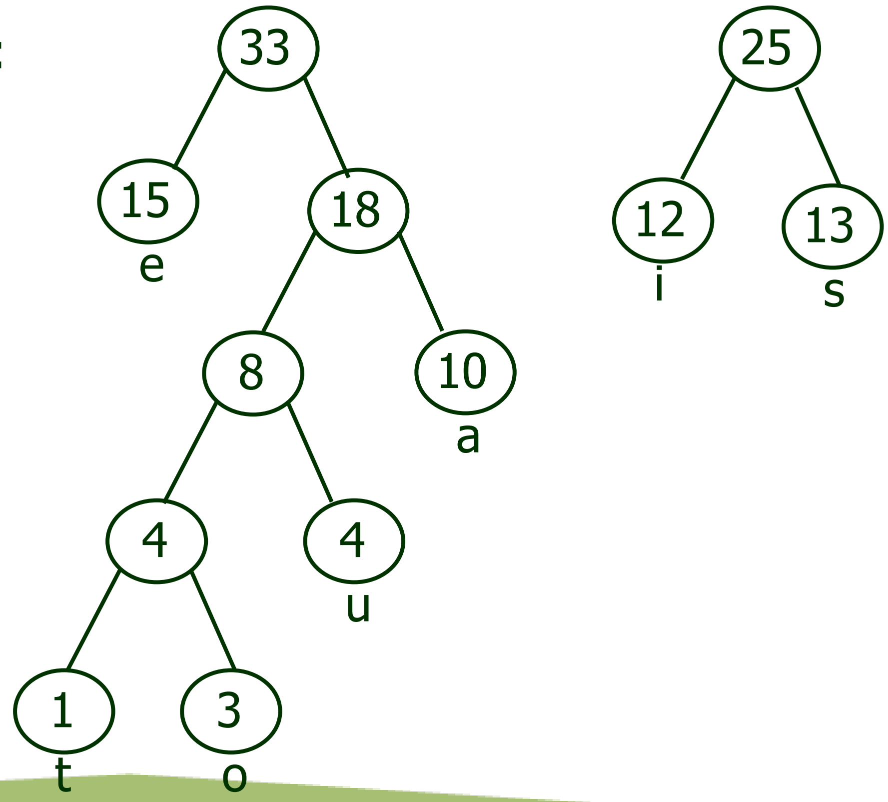
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 9:



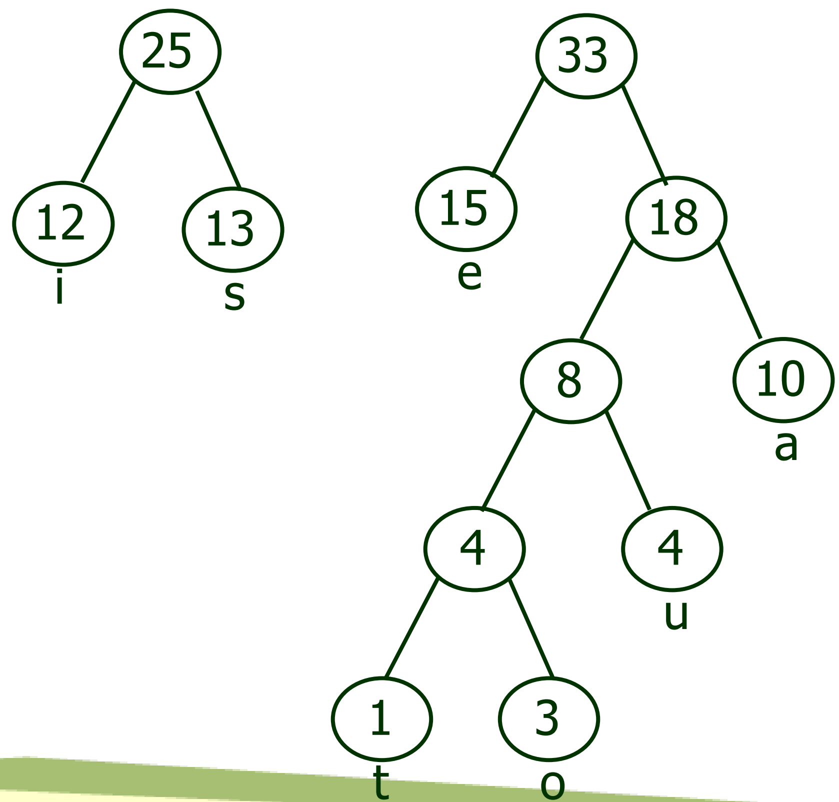
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 10:



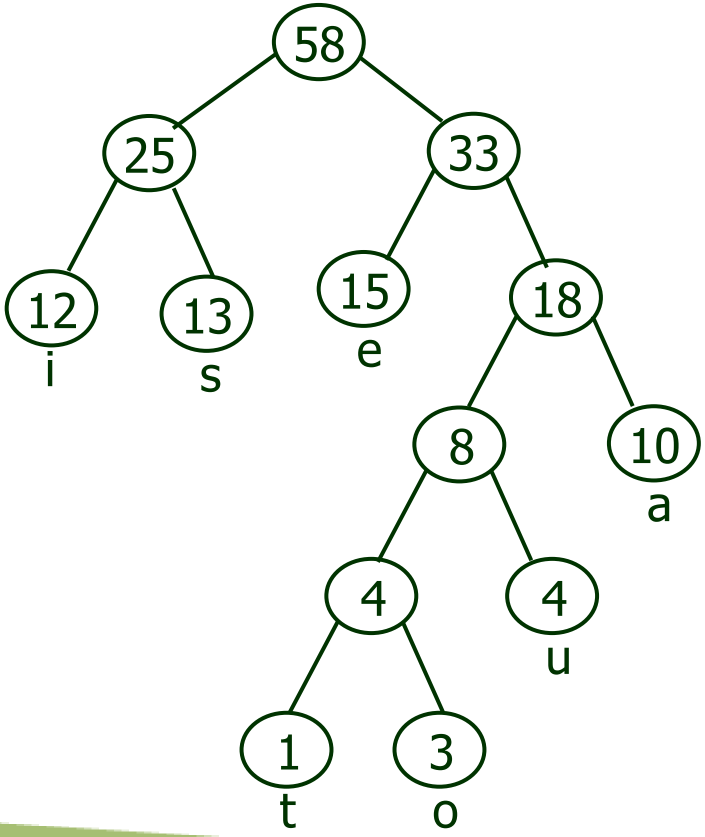
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 10:



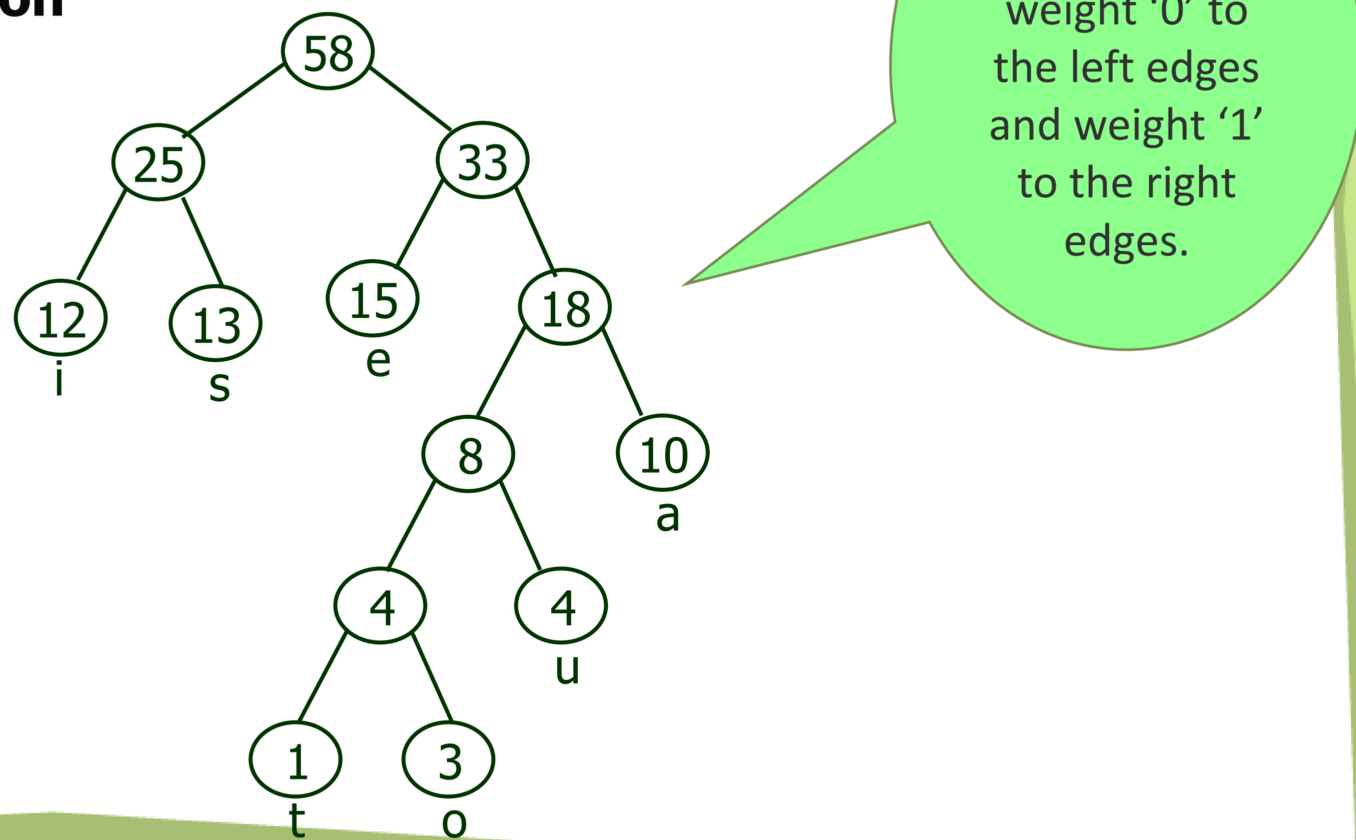
Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1-

Solution

Step 10:



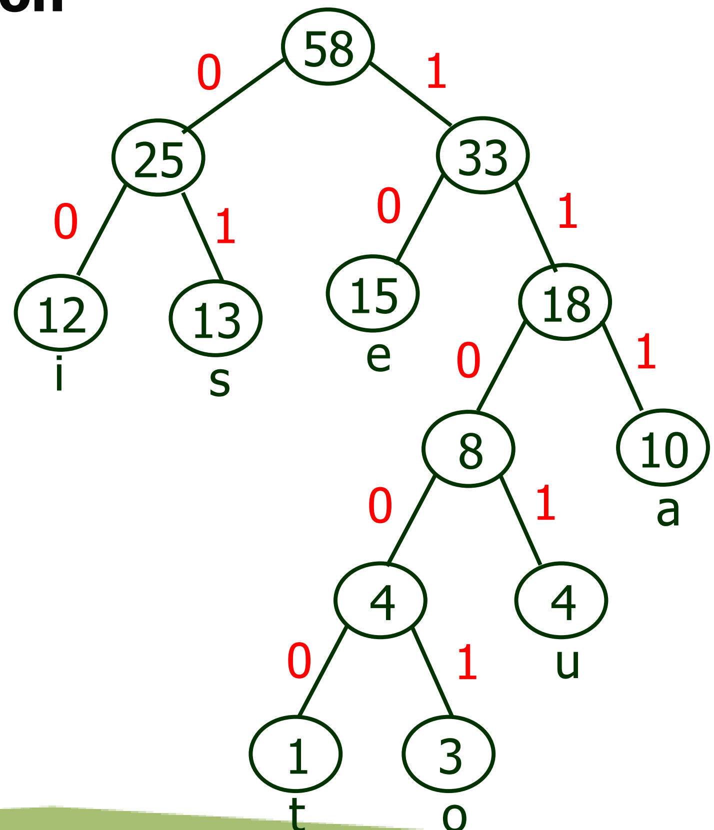
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 10:



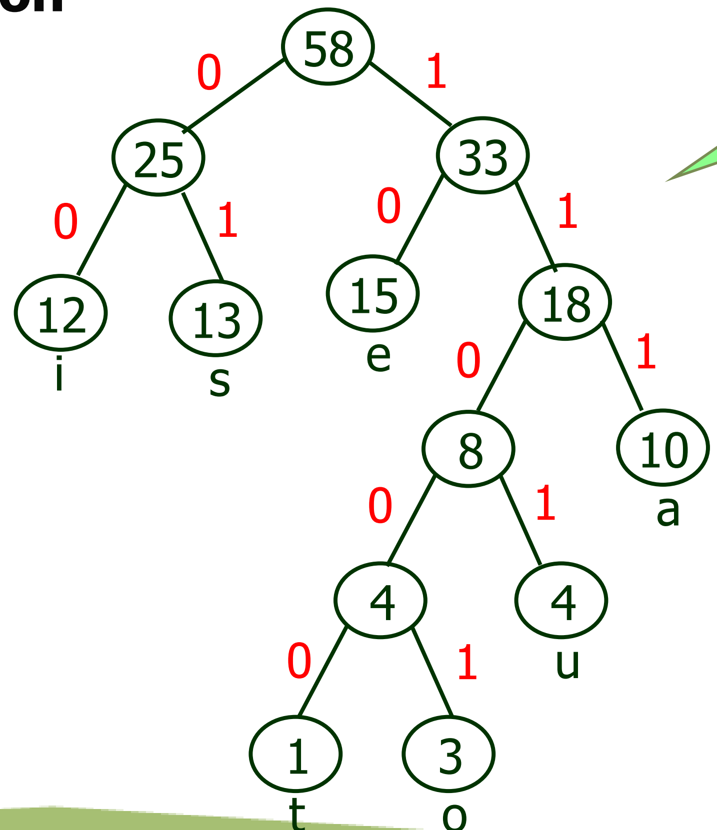
Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 1-

  Solution

Step 10:

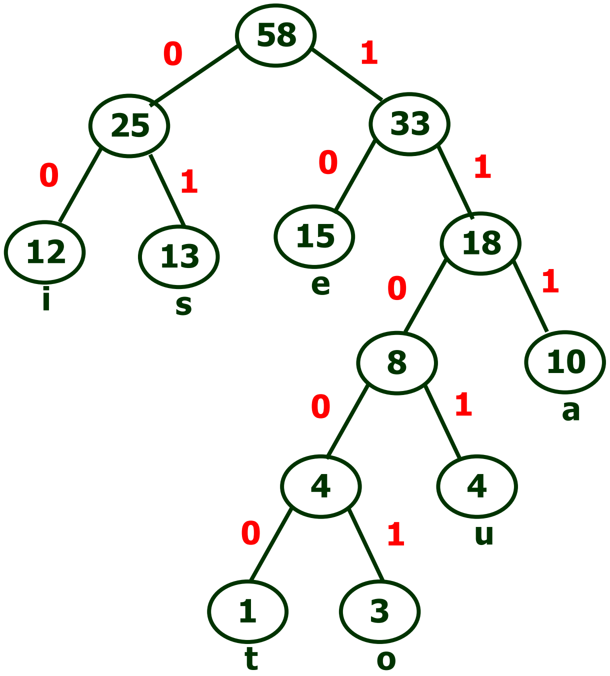


Now, find which number take how many bit?

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1-Solution



Huffman Tree

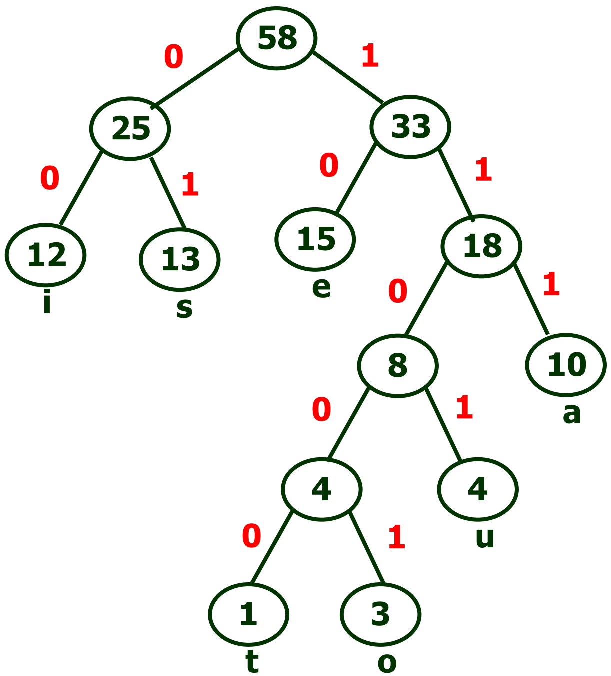
Question 1: Huffman Code for each character.

Characters	Frequencies	Huffman Code
a	10	111
e	15	10
i	12	00
o	3	11001
u	4	1101
s	13	01
t	1	11000

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1-Solution



Huffman Tree

Question 1: Huffman Code for each character.

Characters	Frequencies	Huffman Code
a	10	111
e	15	10
i	12	00
o	3	11001
u	4	1101
s	13	01
t	1	11000

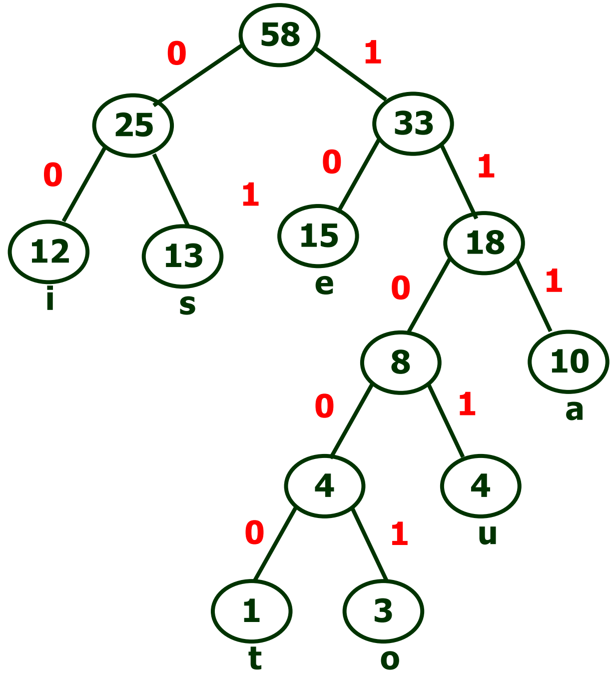
Observation:

- Characters occurring less frequently in the text are assigned the larger code.
- Characters occurring more frequently in the text are assigned the smaller code.

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1- Solution



Huffman Tree

Question 2: Average code length

.

Characters	Frequencies	Huffman Code
a	10	111
e	15	10
i	12	00
o	3	11001
u	4	1101
s	13	01
t	1	11000

Average code length

$= \sum (\boldsymbol{frequency_i} \ \boldsymbol{X} \ \boldsymbol{code\ length_i}) / \sum (\boldsymbol{frequency_i})$

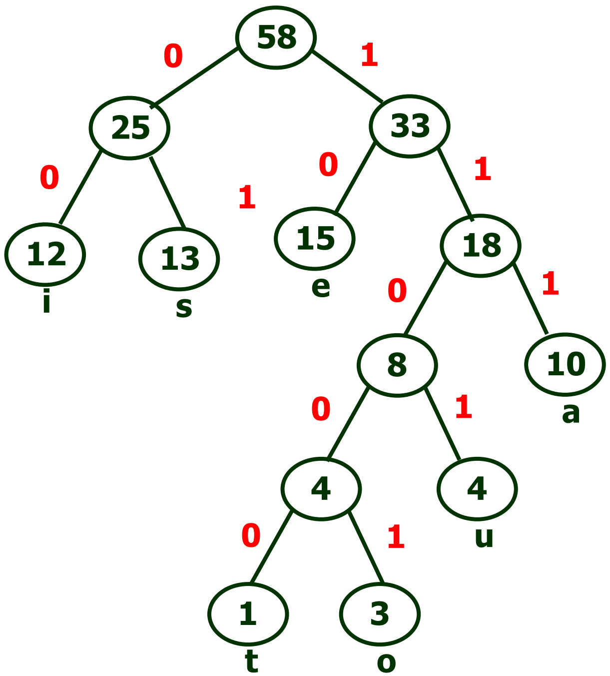
$= \{ (10\ x\ 3) + (15\ x\ 2) + (12\ x\ 2) + (3\ x\ 5) + (4\ x\ 4) + (13\ x\ 2) + (1\ x\ 5) \} / (10 + 15 + 12 + 3 + 4 + 13 + 1)$

$= 2.52$

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1- Solution



Huffman Tree

Question 3: Length of Huffman encoded message (in bits)

.

Characters	Frequencies	Huffman Code
a	10	111
e	15	10
i	12	00
o	3	11001
u	4	1101
s	13	01
t	1	11000

Total number of bits in Huffman encoded message

= Total number of characters in the message x Average code length per character

= 58 x 2.52

= 146.16

≅ 147 bits

Greedy Algorithm

- **Problem 3: Huffman Coding (Algorithm)**

```
HUFFMAN(C)
n ← |C|
Q ← C
for i 1 to n - 1
    do allocate a new node z
        left[z] ← x ← EXTRACT-MIN (Q)
        right[z] ← y ← EXTRACT-MIN (Q)
        f [z] ← f [x] + f [y]
        INSERT(Q, z)
return EXTRACT-MIN(Q)
```

Greedy Algorithm

- **Problem 3: Huffman Coding (Algorithm)**

```
HUFFMAN(C)
n ← |C|
Q ← C
for i 1 to n - 1
    do allocate a new node z
        left[z] ← x ← EXTRACT-MIN (Q)
        right[z] ← y ← EXTRACT-MIN (Q)
        f [z] ← f [x] + f [y]
        INSERT(Q, z)
return EXTRACT-MIN(Q)
```

The total running time of HUFFMAN on a set of n characters is O (n lg n).

Greedy Algorithm

- **Problem 3: Huffman Coding**

  Example 2-

  A file contains the following characters with the frequencies as shown. If Huffman Coding is used for data compression, determine-

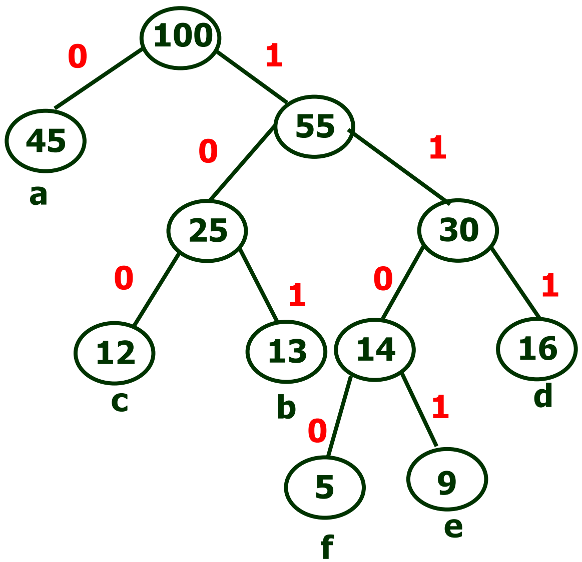
  - Huffman Code for each character
  - Average code length
  - Length of Huffman encoded message (in bits)

Characters	Frequencies
a	45
b	13
c	12
d	16
e	9
f	5

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 2-Solution



Huffman Tree

Question 1: Huffman Code for each character.

Characters	Frequencies	Huffman Code
a	45	0
b	13	101
c	12	100
d	16	111
e	9	1101
f	5	1100

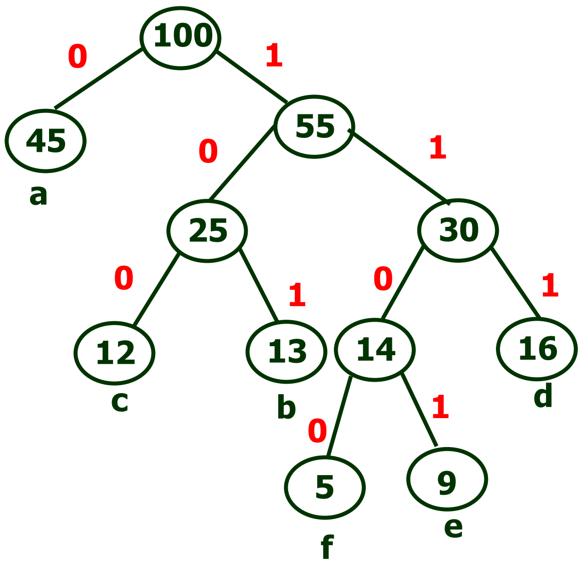
Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1- Solution

Question 1: Huffman Code for each character.

Characters	Frequencies	Huffman Code
a	45	0
b	13	101
c	12	100
d	16	111
e	9	1101
f	5	1100



Huffman Tree

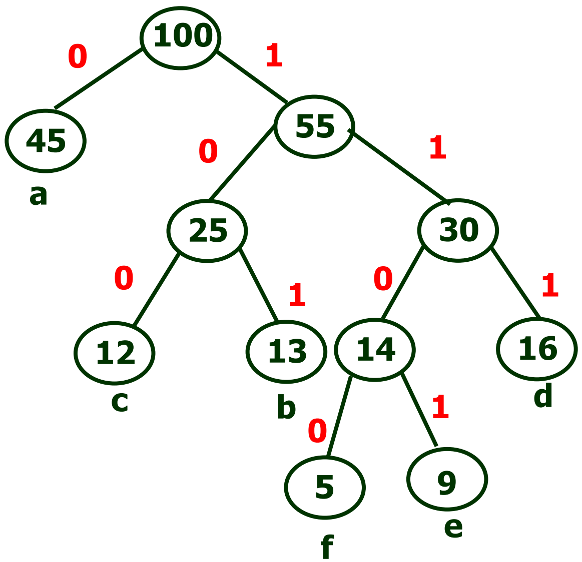
Observation:

- Characters occurring less frequently in the text are assigned the larger code.
- Characters occurring more frequently in the text are assigned the smaller code.

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1- Solution



Huffman Tree

Question 2: Average code length

.

Characters	Frequencies	Huffman Code
a	45	0
b	13	101
c	12	100
d	16	111
e	9	1101
f	5	1100

Average code length

$= \sum (\boldsymbol{frequency_i \; x \; code \; length_i}) / \sum (\boldsymbol{frequency_i})$

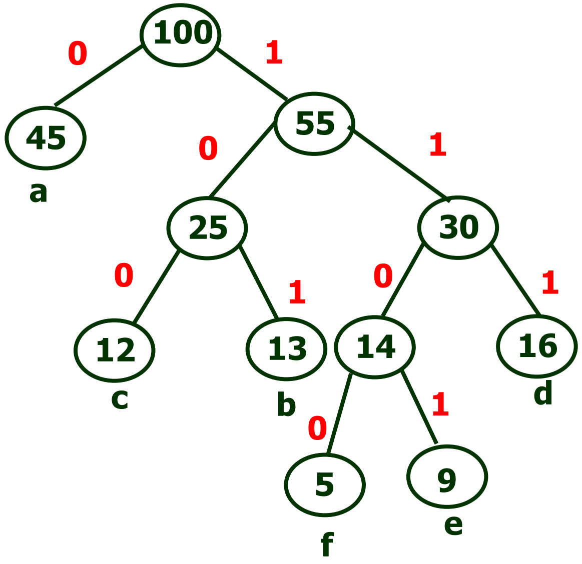
$= \{ (45 \; x \; 1) + (13 \; x \; 3) + (12 \; x \; 3) + (16 \; x \; 3) + (9 \; x \; 4) + (5 \; x \; 4)\} / (45 + 13 + 12 + 16 + 9 + 5)$

$= 2.24$

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 1- Solution



Huffman Tree

Question 3: Length of Huffman encoded message (in bits)

.

Characters	Frequencies	Huffman Code
a	45	0
b	13	101
c	12	100
d	16	111
e	9	1101
f	5	1100

Total number of bits in Huffman encoded message

= Total number of characters in the message x Average code length per character

= 100 x 2.24

= 224 bits

Greedy Algorithm

- **Problem 3: Huffman Coding**

Example 3- (Practice yourself)

A file contains the following characters with the frequencies as shown. If Huffman Coding is used for data compression, determine-

- Huffman Code for each character
- Average code length
- Length of Huffman encoded message (in bits)

Characters	a	b	c	d	e	f	g	h
Frequencies	1	1	2	3	5	8	13	21

Thank u