Design and Analysis of Algorithm

Advanced Data Structure
(Red Black Tree)
(Deletion)

LECTURE 32 - 36

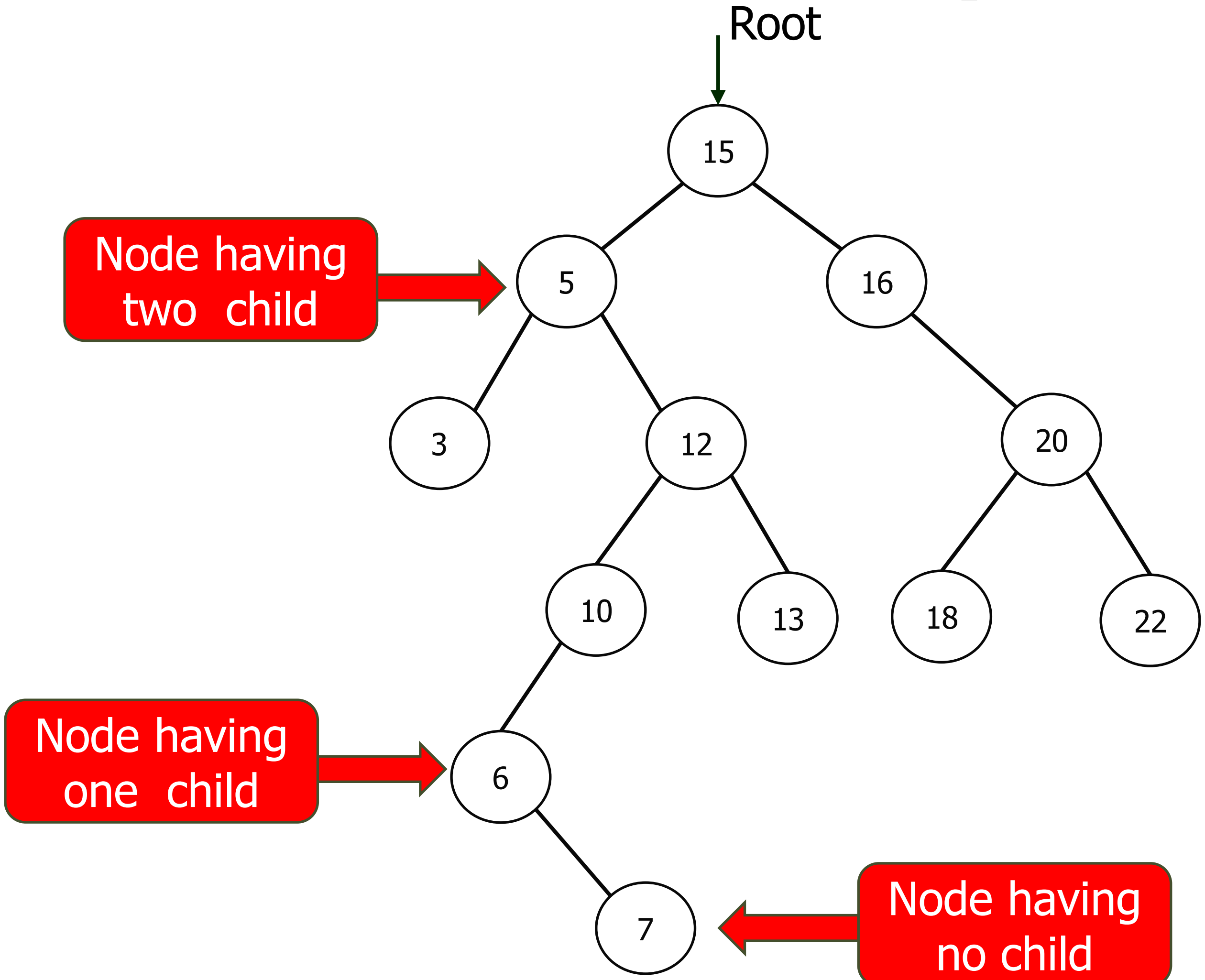
Overview

- A variation of binary search trees.
- Balanced: height is O(lg n), where n is the number of nodes.
- Operations will take O(lg n) time in the worst case.

Red Black Tree (Deletion)

Start by doing regular binary-search-tree deletion:

Red Black Tree (Deletion)



Red Black Tree (Deletion)

RB-DELETE(T, z)

if left[z] = nil[T] or right[z] = nil[T]

then y ← z

else y ← TREE-SUCCESSOR(z)

if left[y] ≠ nil[T]

then x ← left[y]

else x ← right[y]

p[x] ← p[y]

if p[y] = nil[T]

then root[T] ← x

else if y = left[p[y]]

then left[p[y]] ← x

else right[p[y]] ← x

if y ≠ z

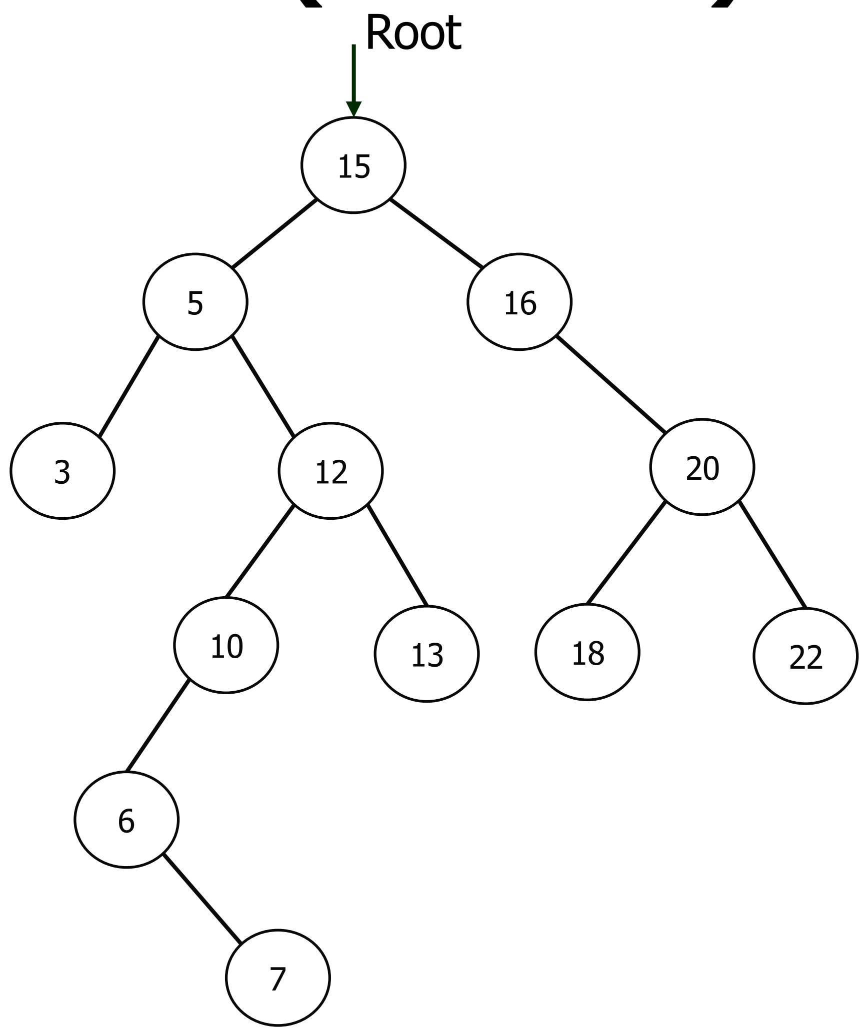
then key[z] ← key[y]

copy $y's$ satellite data into z

if color[y] = BLACK

then RB-DELETE-FIXUP(T, x)

return y



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if p[y] = nil[T ]
      then root[T ] ← x
      else if y = left[p[y]]
                then left[p[y]] ← x
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if y ≠ z
        then key[z] ← key[y]
                 copy y's satellite data into z
if color[y] = BLACK
         then RB-DELETE-FIXUP(T, x)
return y
```

- 'y' is the node that was actually deleted(i.e. Successor of 'z').
- 'x' is either
    - $y's$ sole non-sentinel child before y was deleted, or
    - the sentinel, if y had no children.
- In both cases, p[x] is now the node that was previously $y's$ parent.

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  then x ← left[y]
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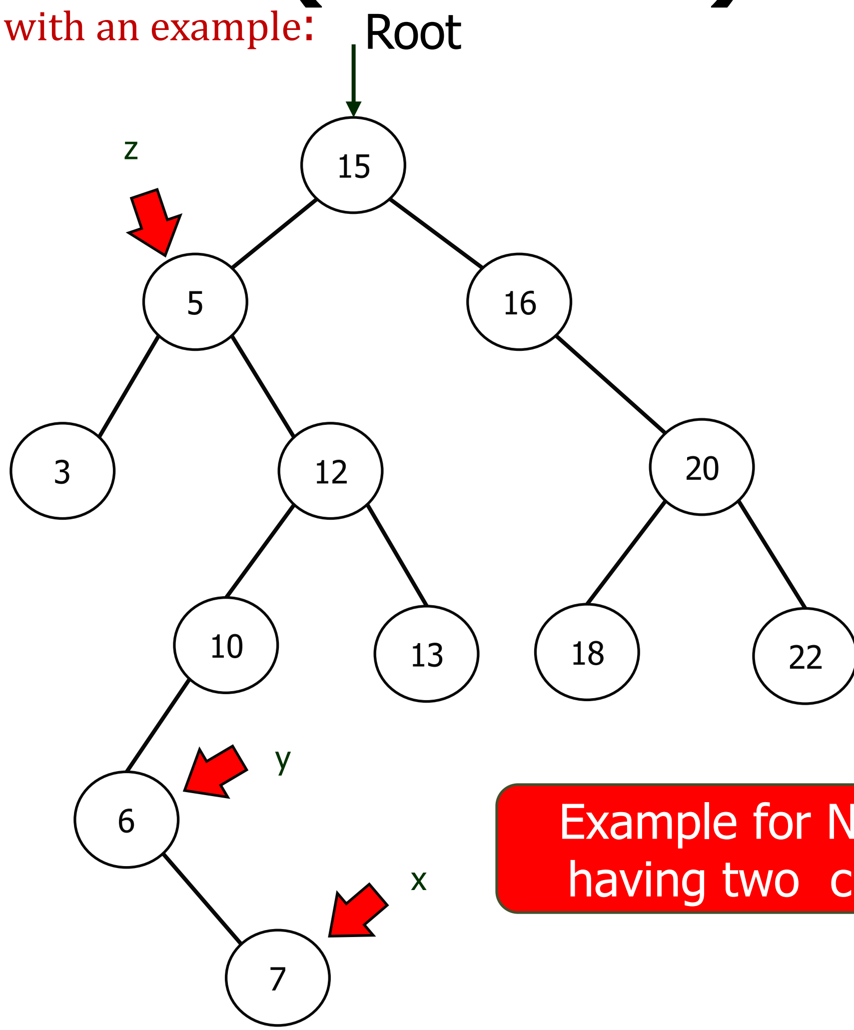
p[x] ← p[y]
if p[y] = nil[T]
  then root[T] ← x
  else if y = left[p[y]]
    then left[p[y]] ← x
    else right[p[y]] ← x
if y ≠ z
  then key[z] ← key[y]
    copy $y's$ satellite data into z
if color[y] = BLACK
  then RB-DELETE-FIXUP(T, x)
return y

Let execute with an example:



Example for Node having two child

Red Black Tree (Deletion)

Now Check

- If y is black, we could have violations of red-black properties:
- Which property might be violated?

P1. Every node is either red or black ?

OK

P2. The root is always black ?

If y is the root and x is red, then the root has become red.

P3. Every leaf (nil[T]) is black ?

OK

P4. If a node is red, then both its children are black ?

Violation if p[y] and x are both red.

Red Black Tree (Deletion)

Now Check

P5. For each node, all paths from the node to descendant leaves contain the same number of black nodes (i.e. bh(x))?

- Any path containing y now has 1 fewer black node.
- Correct by giving x an "**extra black”** and make the node “**double black**".
- Add 1 to count of black nodes on paths containing x.
- Now property 5 is OK, but property 1 is not.
- x is either **doubly black** (if color[x] = BLACK) or red & black (if color[x] = RED).
- The attribute color[x] is still either RED or BLACK. No new values for color attribute.
- In other words, the extra blackness on a node is by virtue of x pointing to the node.

Red Black Tree (Deletion)

Basic Idea for Deletion

Move the extra black up the tree until

- x points to a red & black node ⇒turn it into a black node,
- x points to the root ⇒just remove the extra black, or
- Do certain rotations (i.e. LL or RR) and recoloring the node and finished the deletion by maintain the Red-black tree property.

Red Black Tree (Deletion)

The basic idea for deletion was executed by the help of RB-DELETE-FIXUP function (i.e. Remove the violations by calling RB-DELETE-FIXUP) .

This function is executed until $x \neq root$ and $color[x] = Black$

For this, first find the sibling of 'x' as 'w' and 'w' can not be NIL

There are 8 number of cases:

- 4 of which are symmetric to the other 4 (like RB-INSERT-FIXUP()).
- We look at cases in which 'x' is the left child of it's parents.

Red Black Tree (Deletion)

Case 1: When 'w'(sibling of 'x')is red and w=right[p[x]]

Case 2: When 'w'(sibling of 'x')is black and both the children of 'w' is also black.

Case 3: When 'w'(sibling of 'x')is black and w's left child is red and w's right child is black.

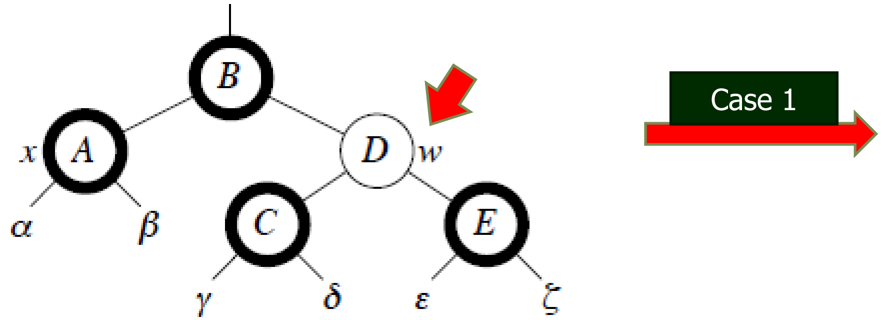
Case 4: When 'w'(sibling of 'x')is black and w's left child is black and w's right child is red.

Red Black Tree (Deletion)

- Case 1:
  - When ‘w’(sibling of ’x’)is red and w=right[p[x]]

  Action to be taken

    - Change the color[w]=Black.
    - Change the color[p[x]]=Red.
    - LEFT_ROTATE(T,p[x])
    - Move w=right[p[x]]
    - Go immediately to case 2, 3, or 4.



[Nodes with bold outline indicate black nodes and light outline indicate red nodes]

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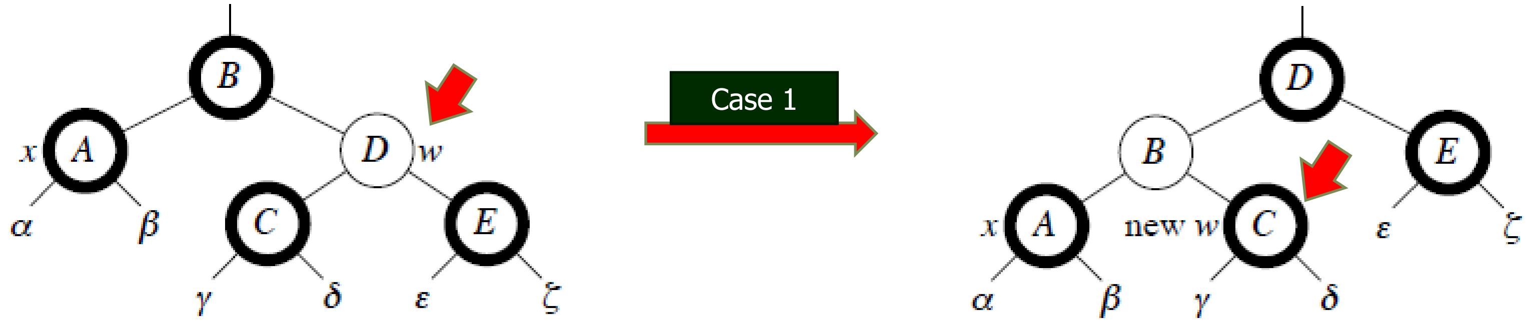

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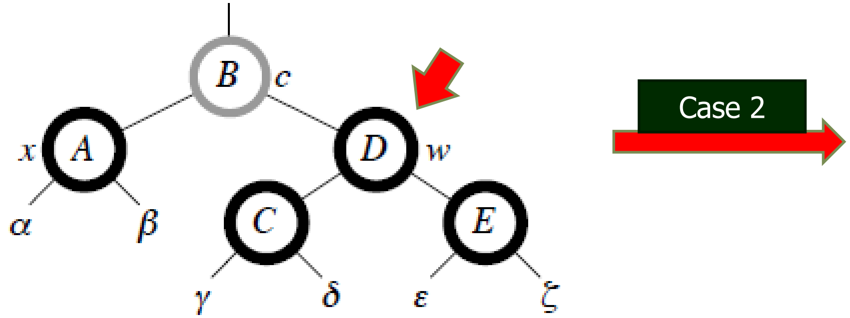
Red Black Tree (Deletion)

- Case 2:
  - *When 'w'(sibling of 'x')is black and both the children of 'w' is also black.*

  Action to be taken
    - Change color[w]=Red.
    - Move x=p[x]



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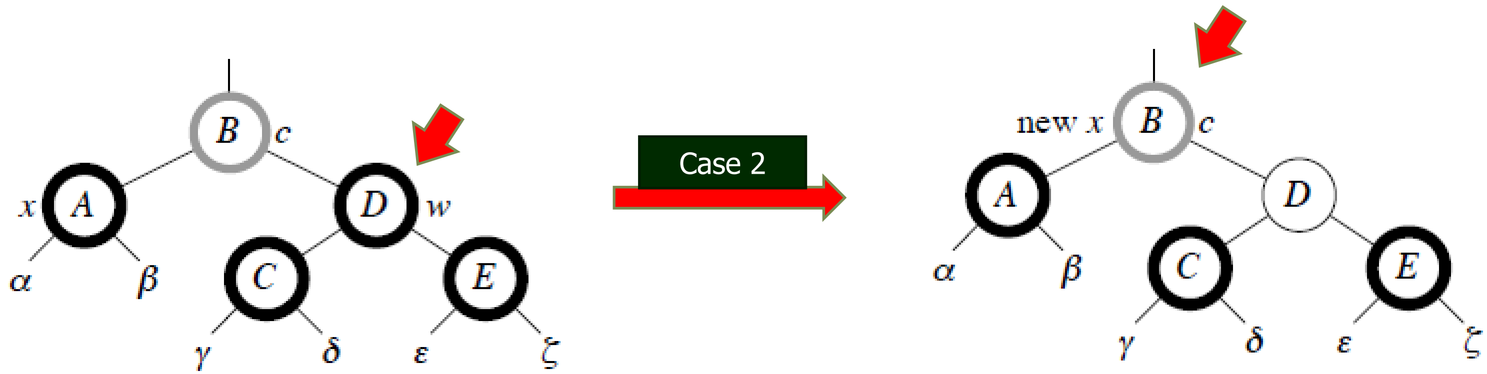

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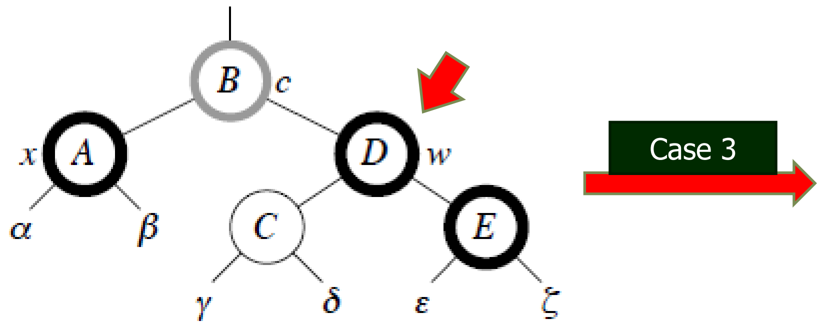
Red Black Tree (Deletion)

- Case 3:
  - *When 'w'(sibling of 'x')is black and w's left child is red and w's right child is black.*

  Action to be taken

    - Color[left[w]]=Black
    - Color[w]=Red
    - RIGHT_ROTATE(T, w)
    - Move w=right[p[x]]



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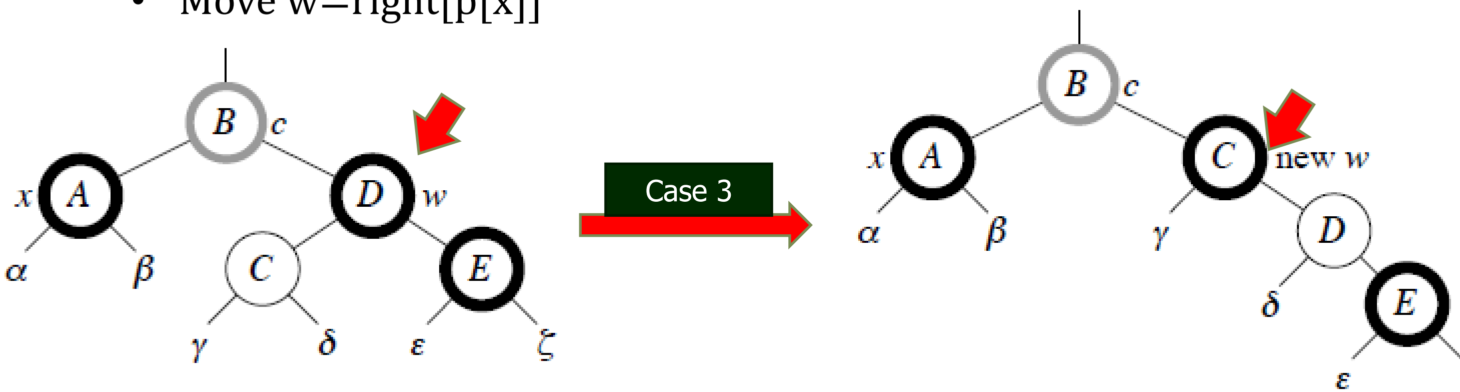

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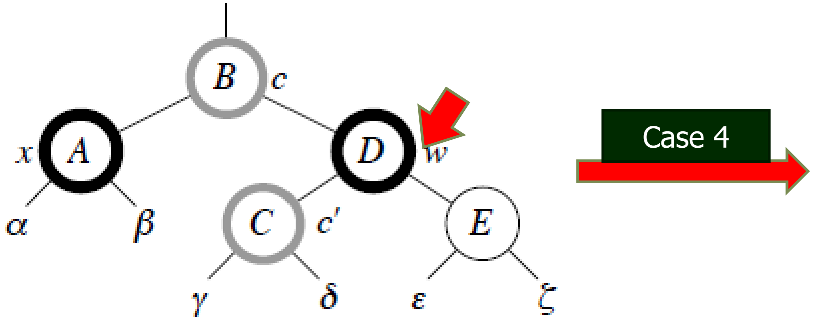
Red Black Tree (Deletion)

- Case 4:
  - *When 'w'(sibling of 'x')is black and w's left child is black and w's right child is red.*

  Action to be taken

  - Color[w]=color[p[x]]
  - Color[p[x]]=Black
  - Color[right[w]]=Black
  - LEFT_ROTATE(T, p[x])
  - x=root[T]



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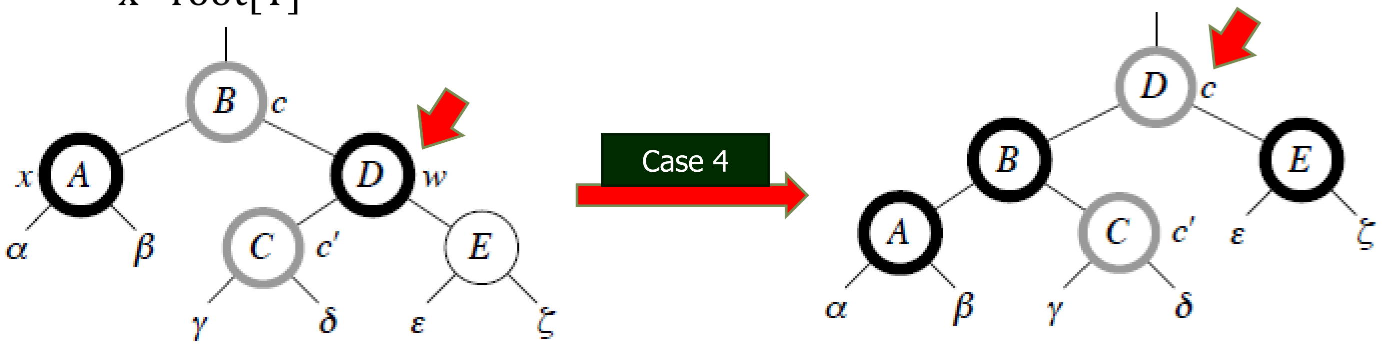

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Red Black Tree (Deletion)

- Case 4:
  - *When 'w'(sibling of 'x')is black and w's left child is black and w's right child is red.*

  Action to be taken
    - Color[w]=color[p[x]]
    - Color[p[x]]=Black
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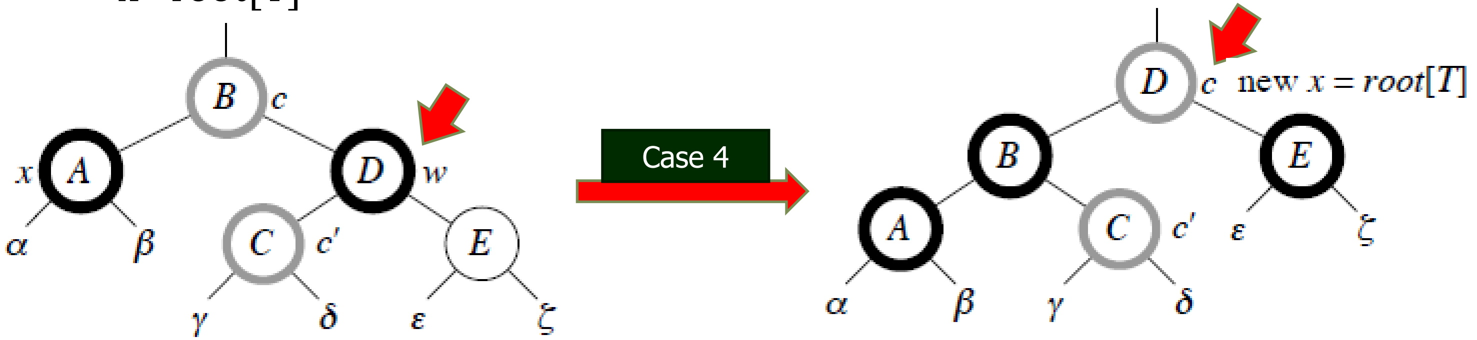
    - LEFT_ROTATE(T, p[x])
    - x=root[T]



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Red Black Tree (Deletion)

RB-DELETE-FIXUP(T, x)

while x ≠ root[T] and color[x] = BLACK

    do if x = left[p[x]]

        then w ← right[p[x]]

          if color[w] = RED

            then Apply Case 1

          if color[left[w]] = BLACK and color[right[w]] = BLACK

            then Apply Case 2

            else if color[right[w]] = BLACK

                then Apply Case 3

              Apply Case 4

      else (same as then clause with “right” and “left” exchanged)

color[x] ← BLACK

Red Black Tree (Deletion)

```
RB-DELETE-FIXUP(T, x)
while x ≠ root[T ] and color[x] = BLACK
        do if x = left[p[x]]
                then w ← right[p[x]]
                    if color[w] = RED
                        then color[w] ← BLACK                    ✄ Case 1
                                color[p[x]] ← RED                ✄ Case 1
                                LEFT-ROTATE(T, p[x])             ✄ Case 1
                                w ← right[p[x]]                  ✄ Case 1
                    if color[left[w]] = BLACK and color[right[w]] = BLACK
                        then color[w] ← RED                      ✄ Case 2
                                x ← p[x]                         ✄ Case 2
                        else if color[right[w]] = BLACK
                                then color[left[w]] ← BLACK      ✄ Case 3
                                        color[w] ← RED           ✄ Case 3
                                        RIGHT-ROTATE(T, w)       ✄ Case 3
                                        w ← right[p[x]]          ✄ Case 3
                                color[w] ← color[p[x]]           ✄ Case 4
                                color[p[x]] ← BLACK              ✄ Case 4
                                color[right[w]] ← BLACK          ✄ Case 4
                                LEFT-ROTATE(T, p[x])             ✄ Case 4
                                x ← root[T ]                     ✄ Case 4
            else (same as then clause with "right" and "left" exchanged)
    color[x] ← BLACK
```

Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

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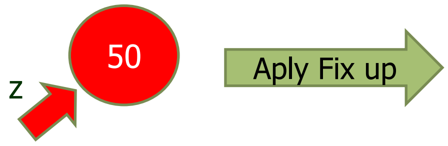
Insert -50



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Red Black Tree

No Fixup required

Insertion (Example 1):

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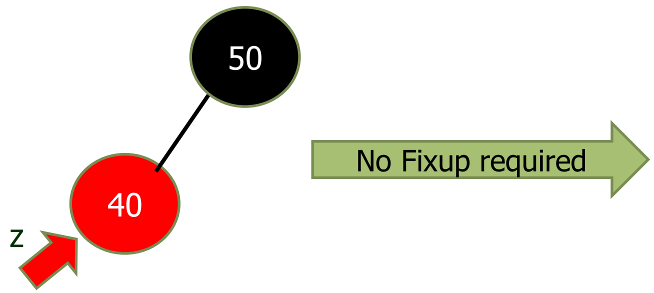
Insert -40



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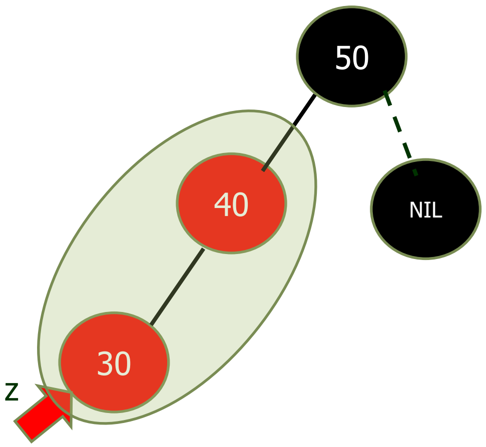
Insert -30



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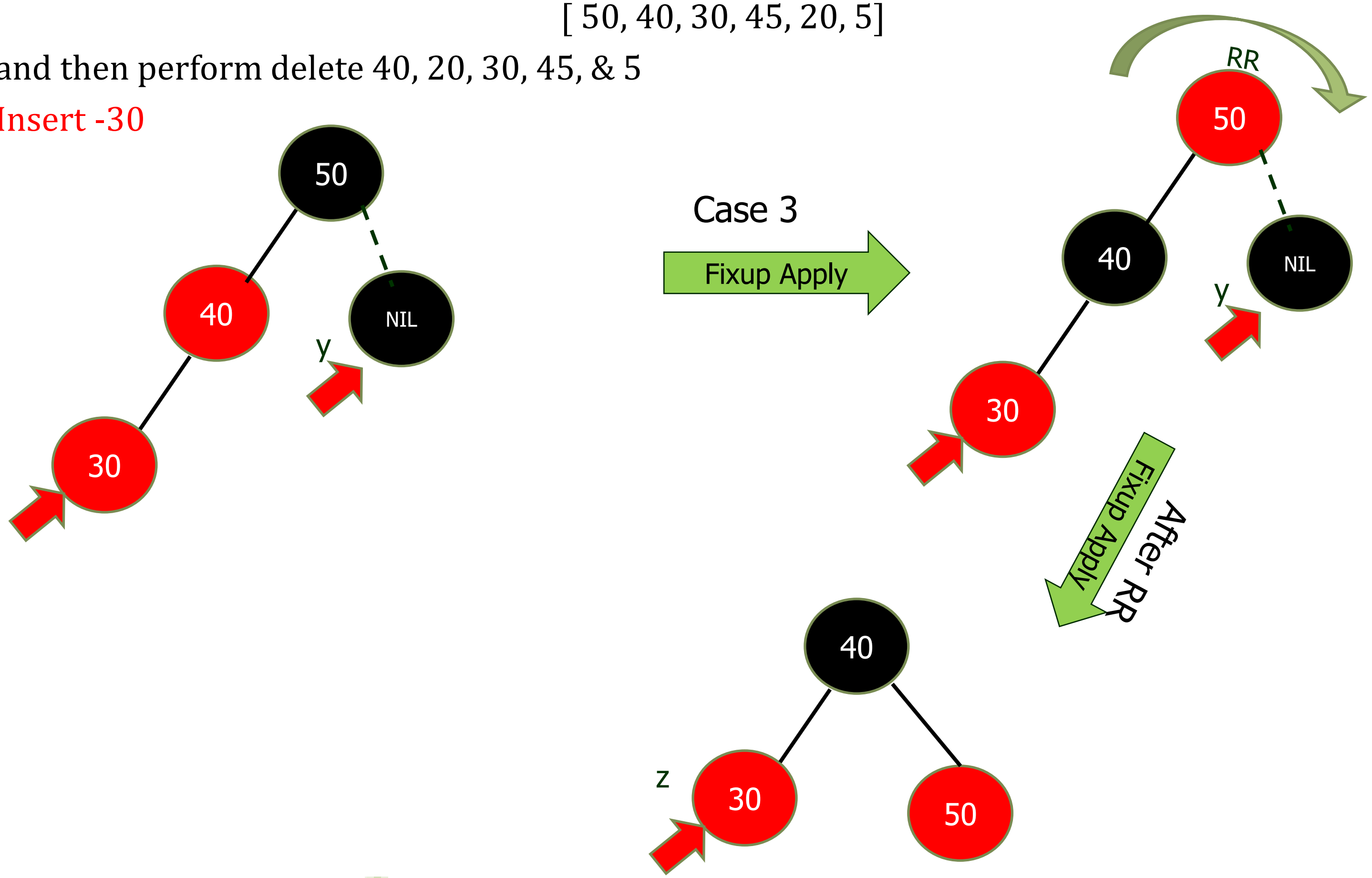

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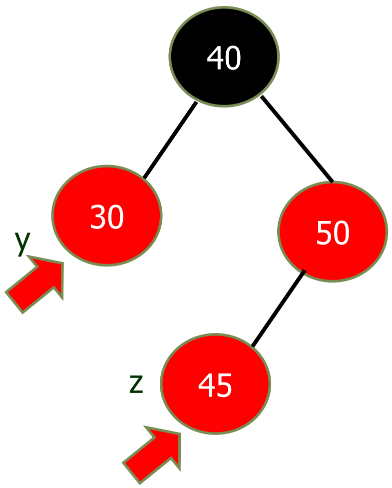
Insert -45



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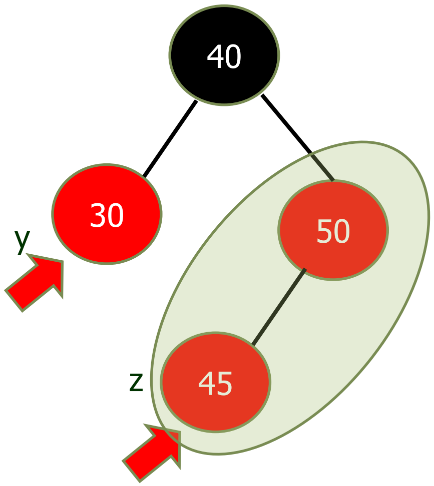

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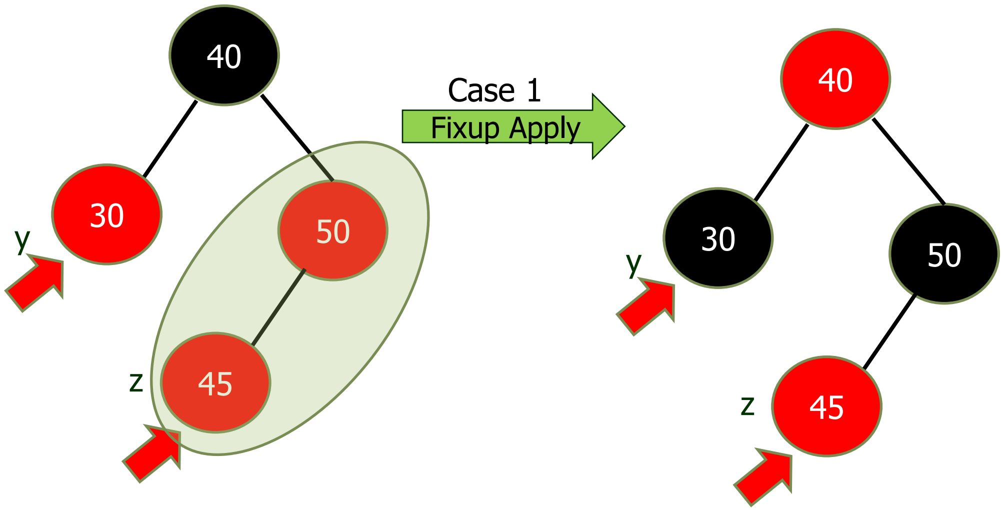

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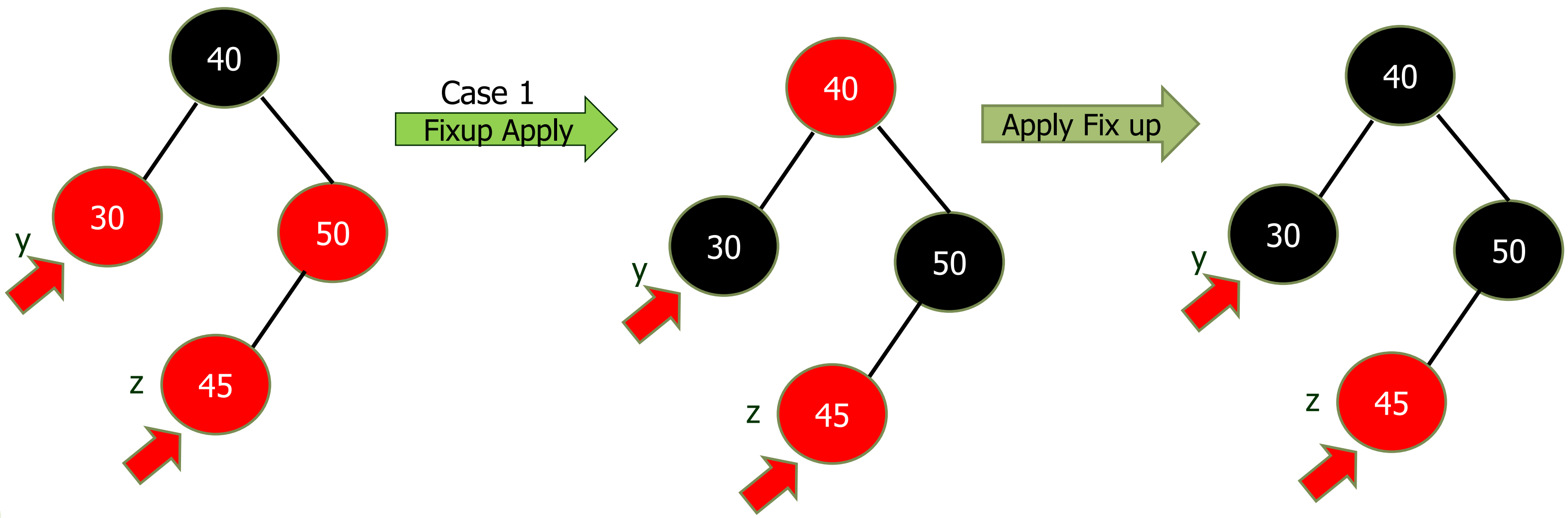

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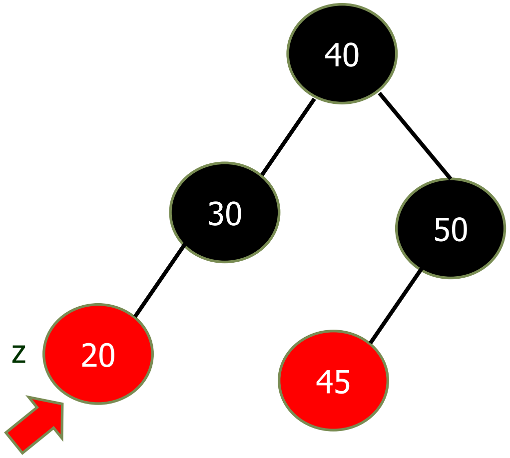

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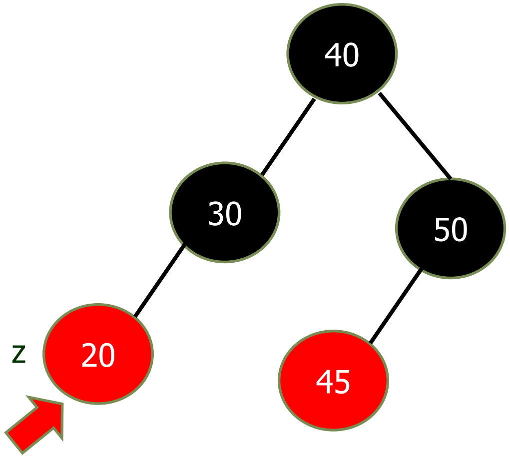

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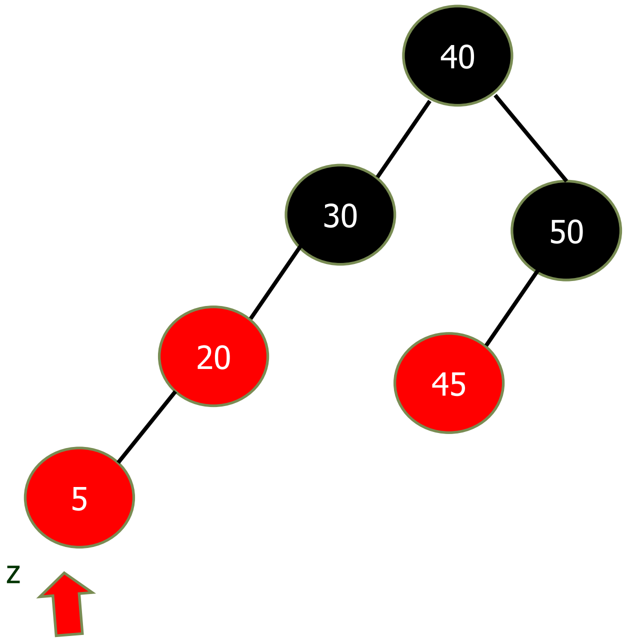
Insert -5



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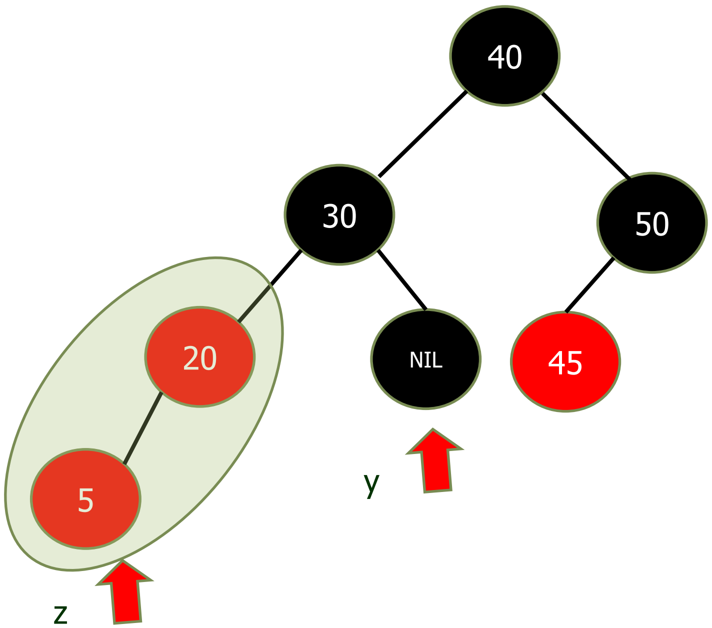

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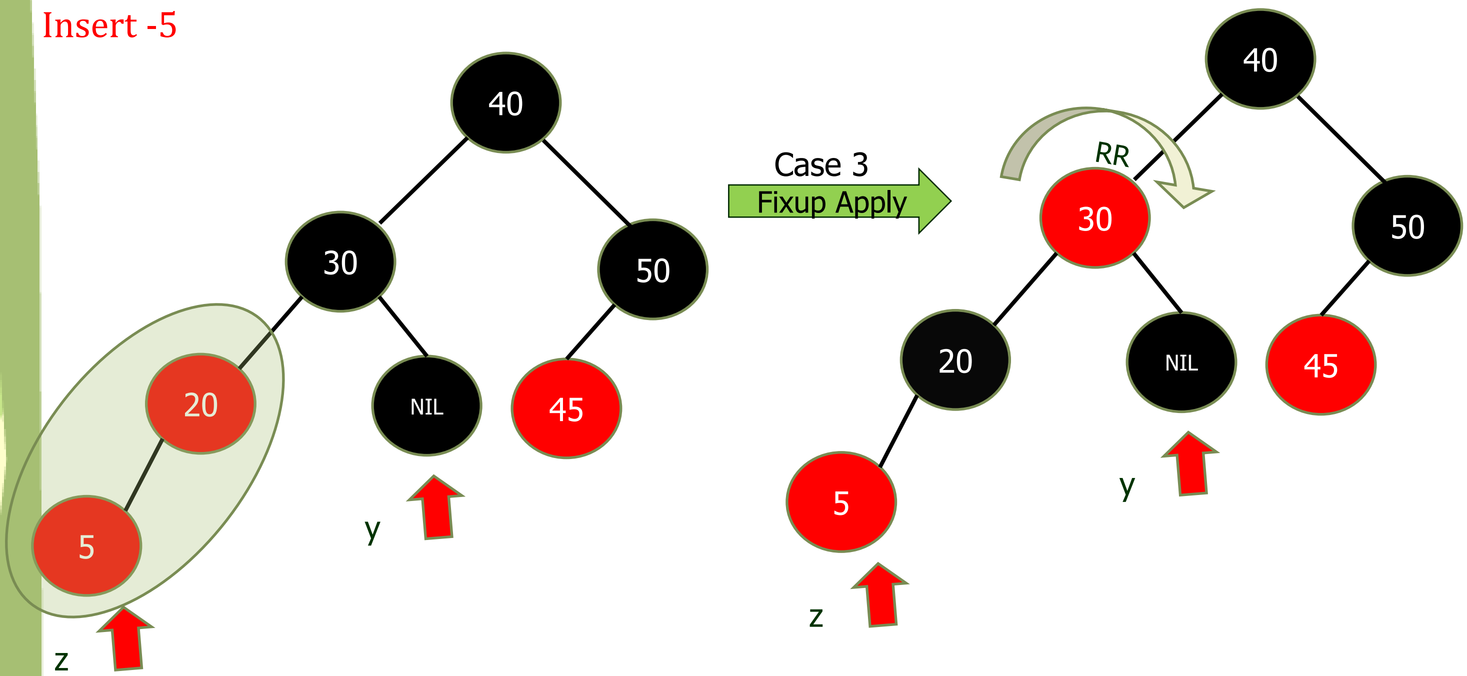

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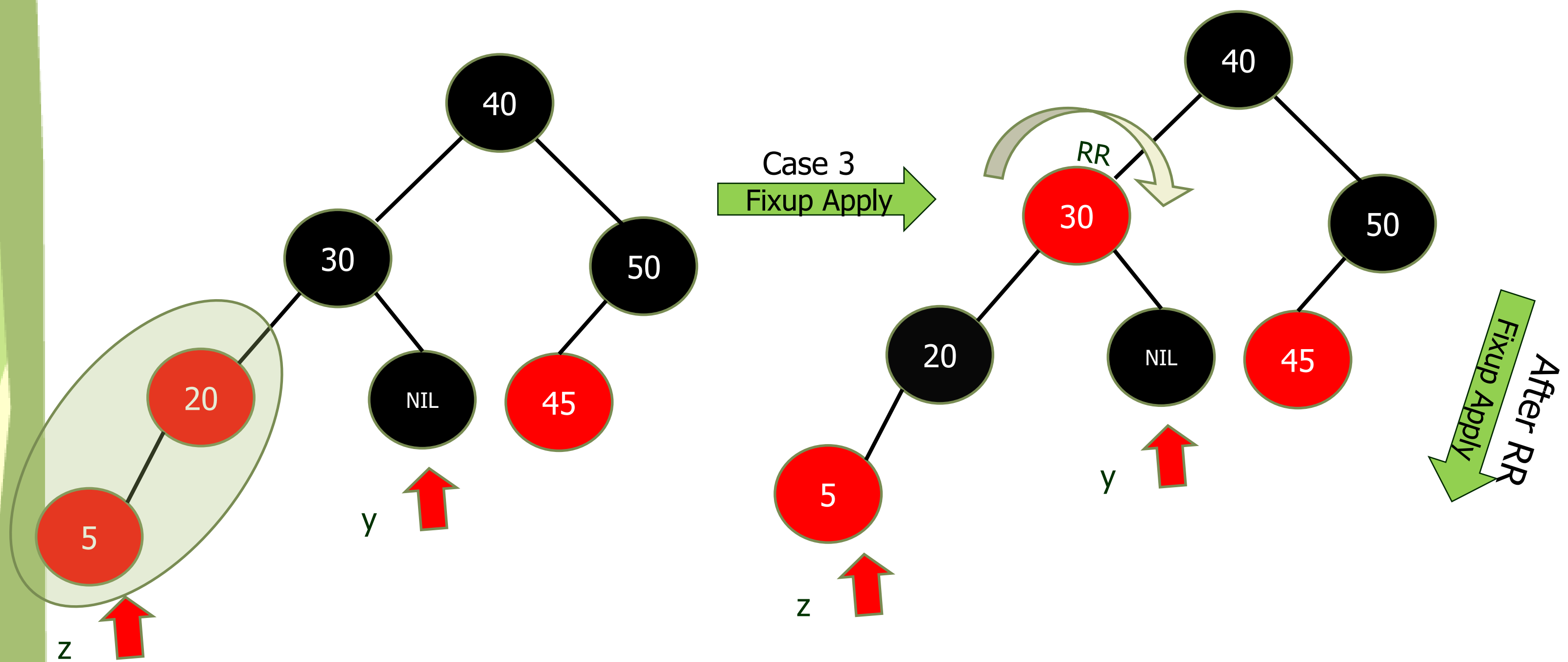

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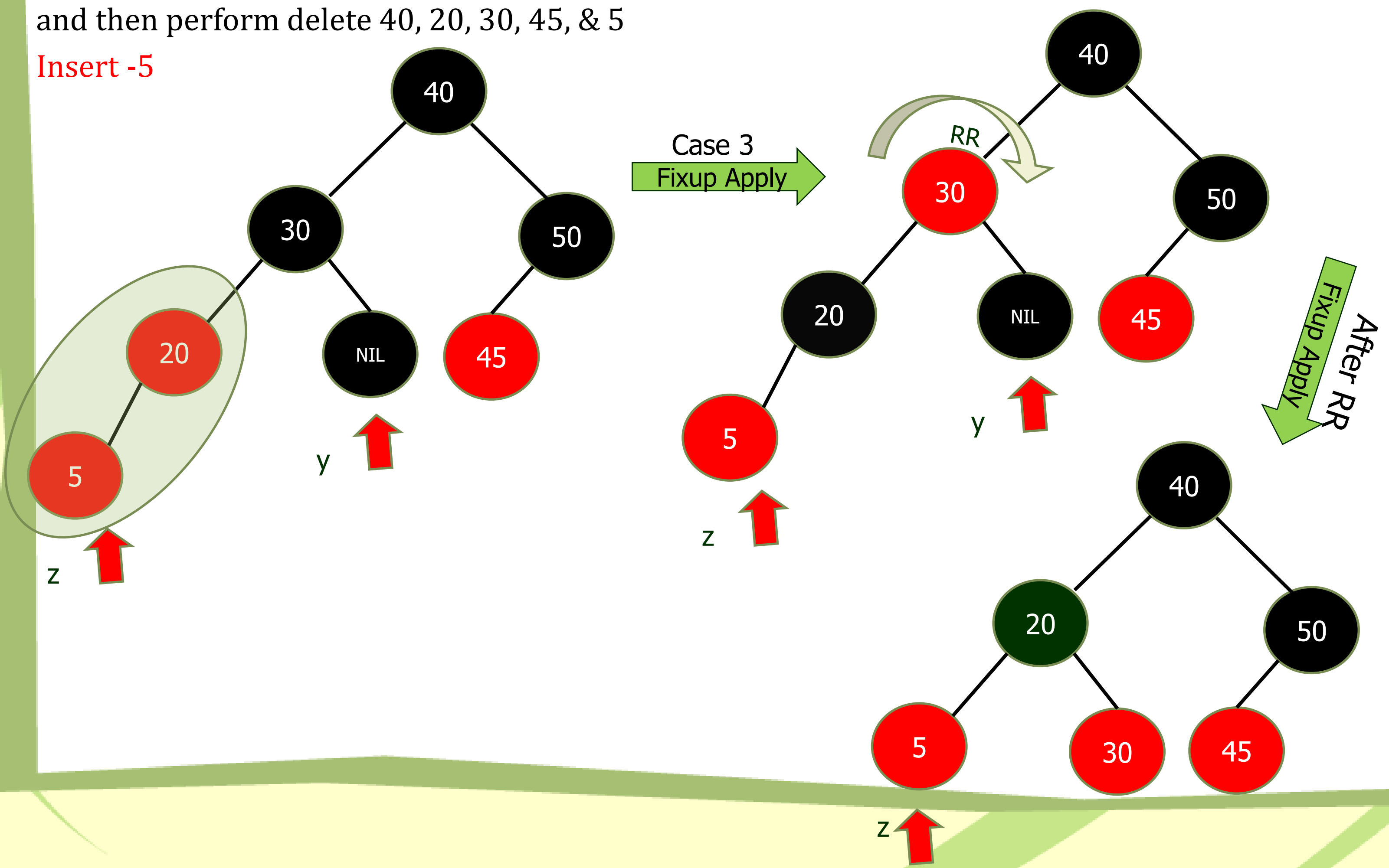

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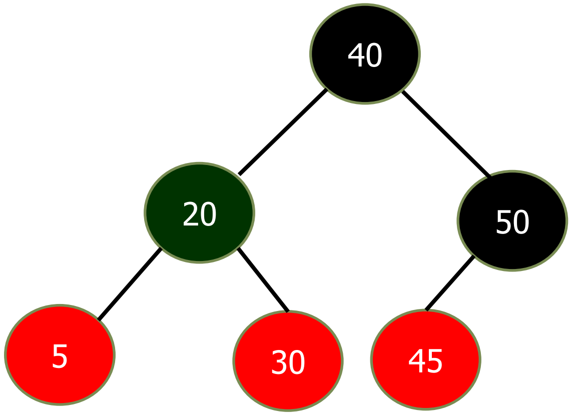

After the Insertion the tree finally looks as shown above.

Now we perform Deletion on RB Tree.

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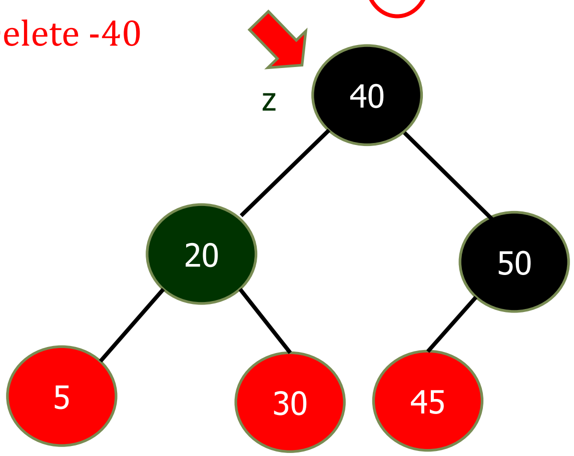
Delete -40



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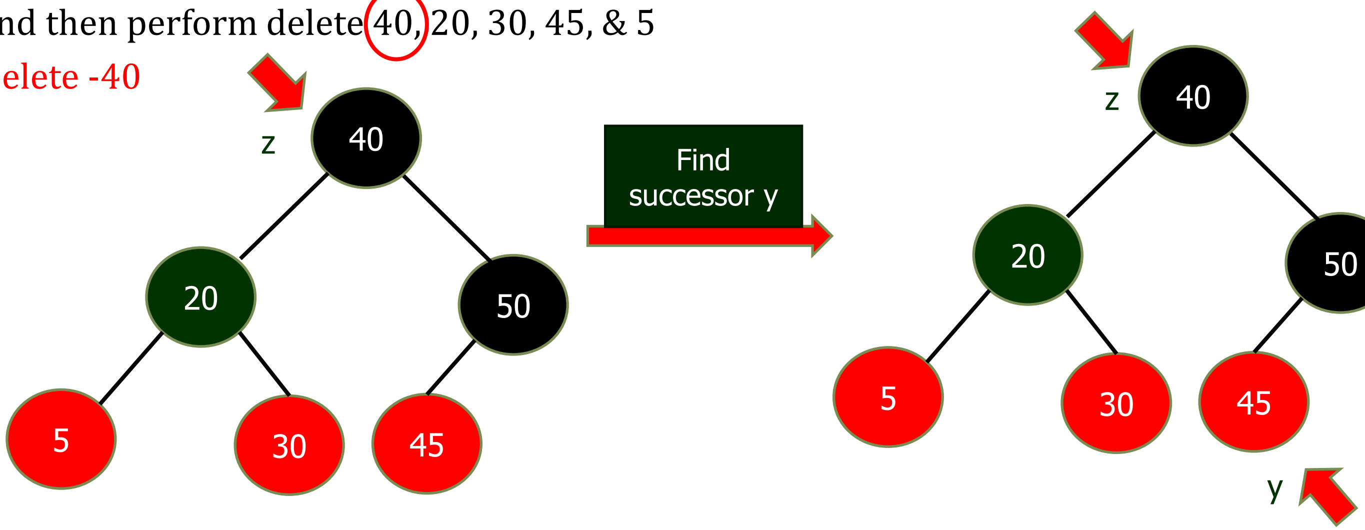

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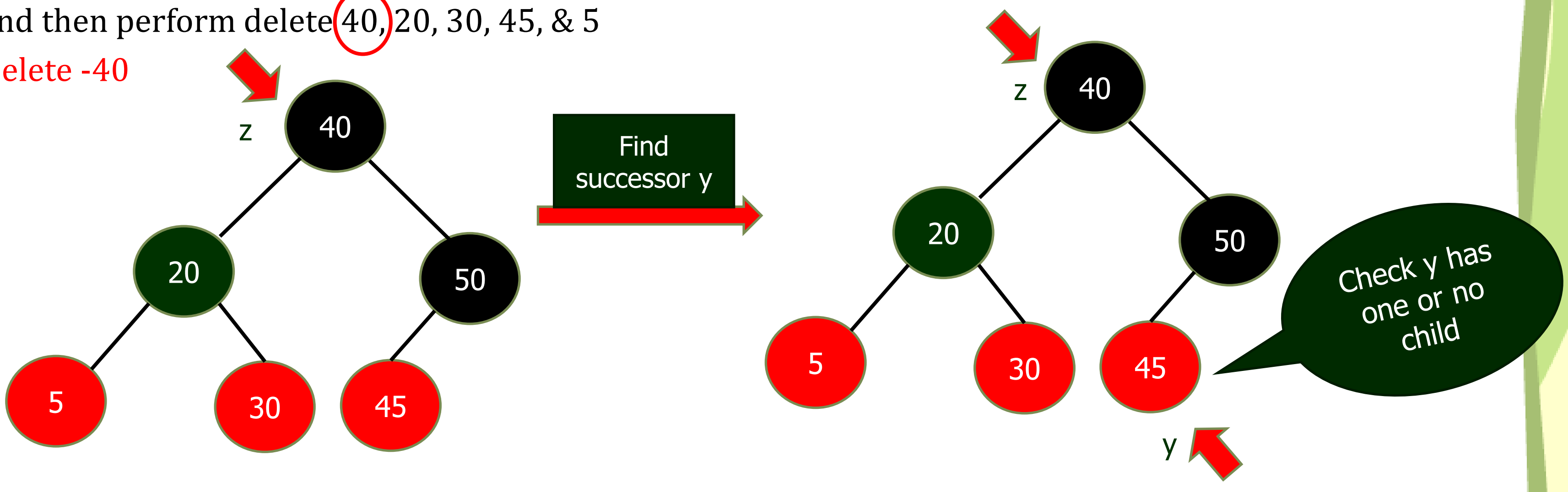

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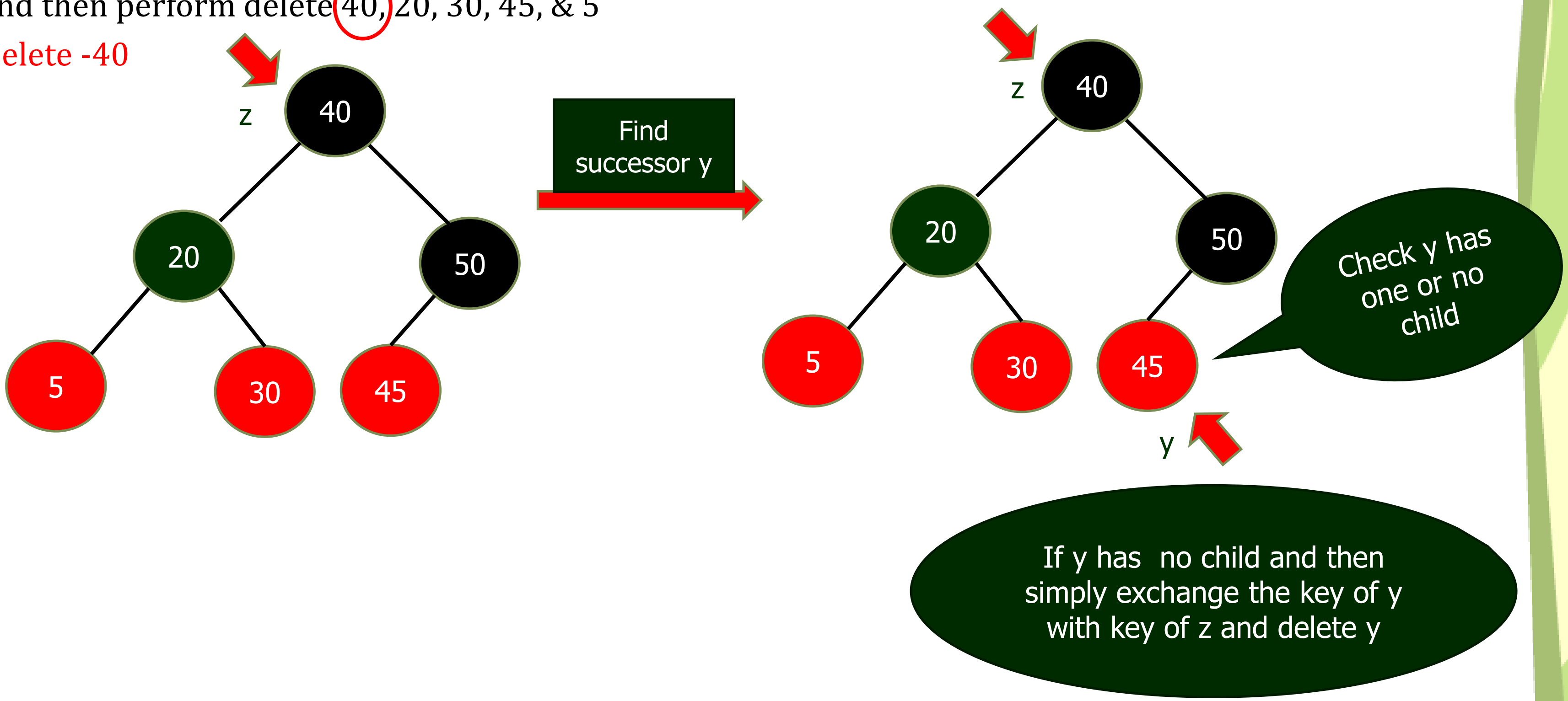
and then perform delete 40, 20, 30, 45, & 5

Delete -40



Red Black Tree

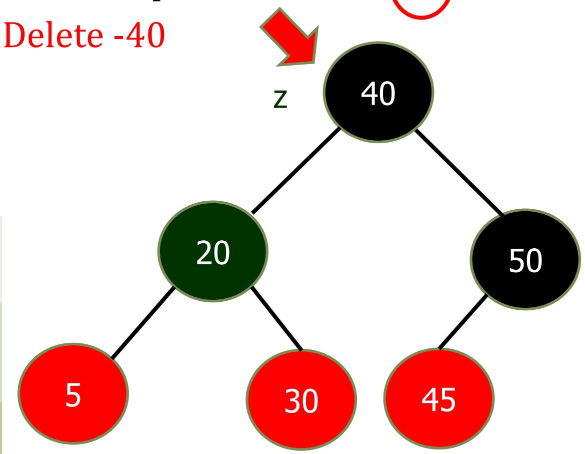
Insertion (Example 1):

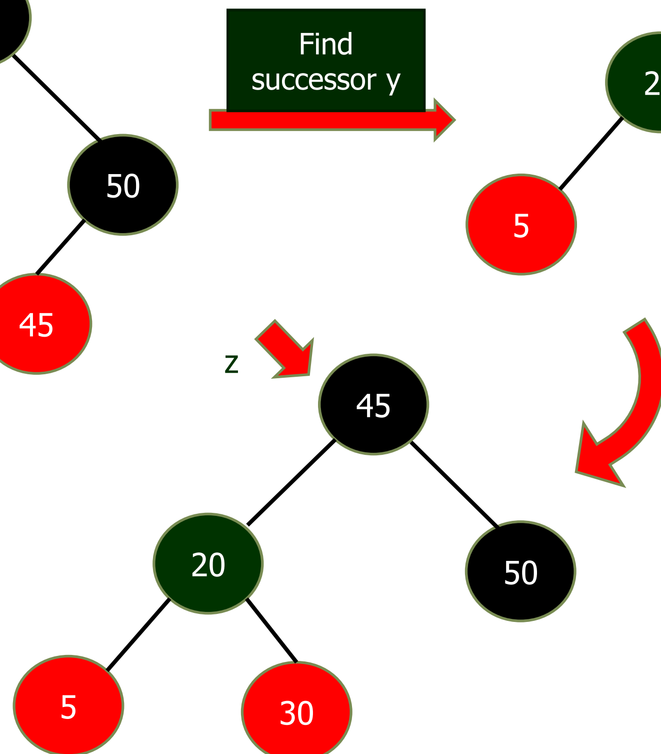
Insert the following elements into an empty RB-Tree.

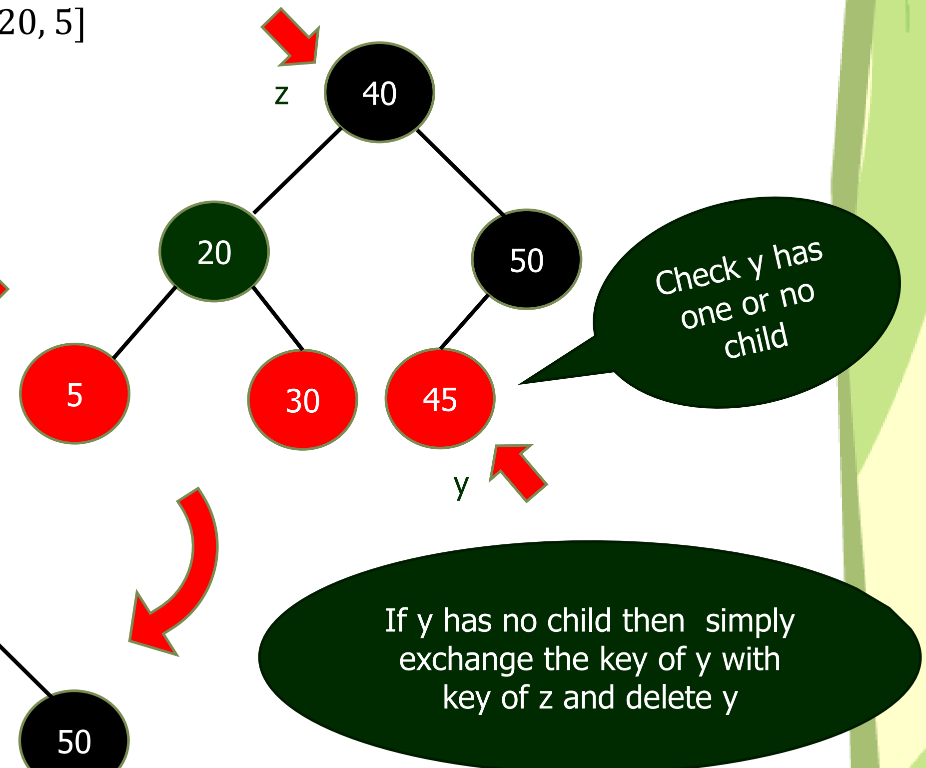
[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -40







Red Black Tree

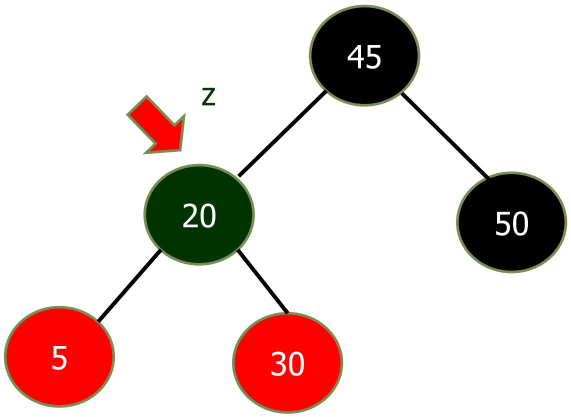
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -20



Red Black Tree

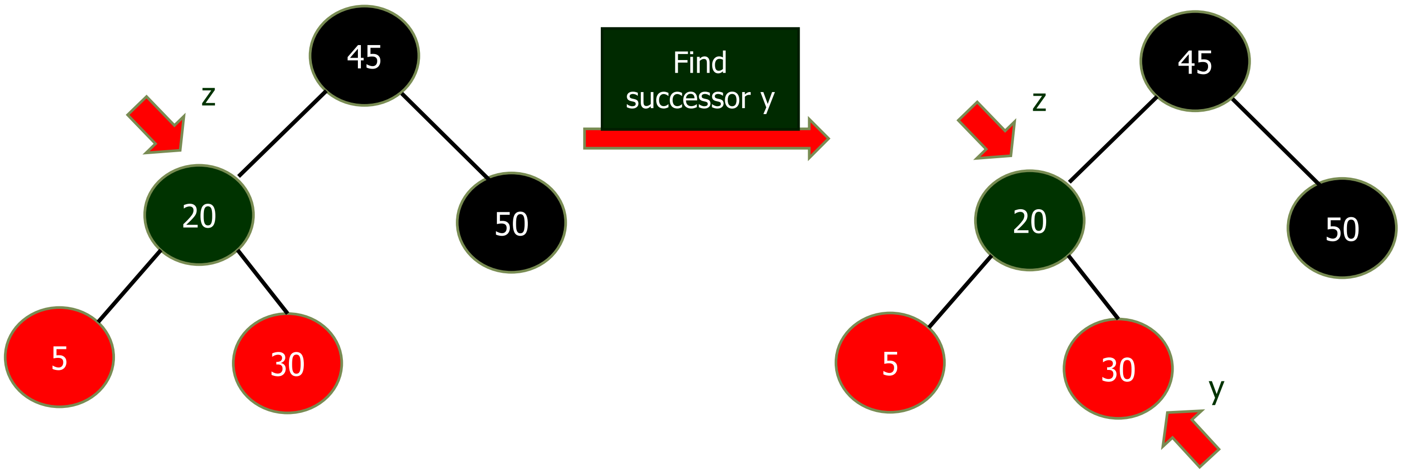
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -20



Red Black Tree

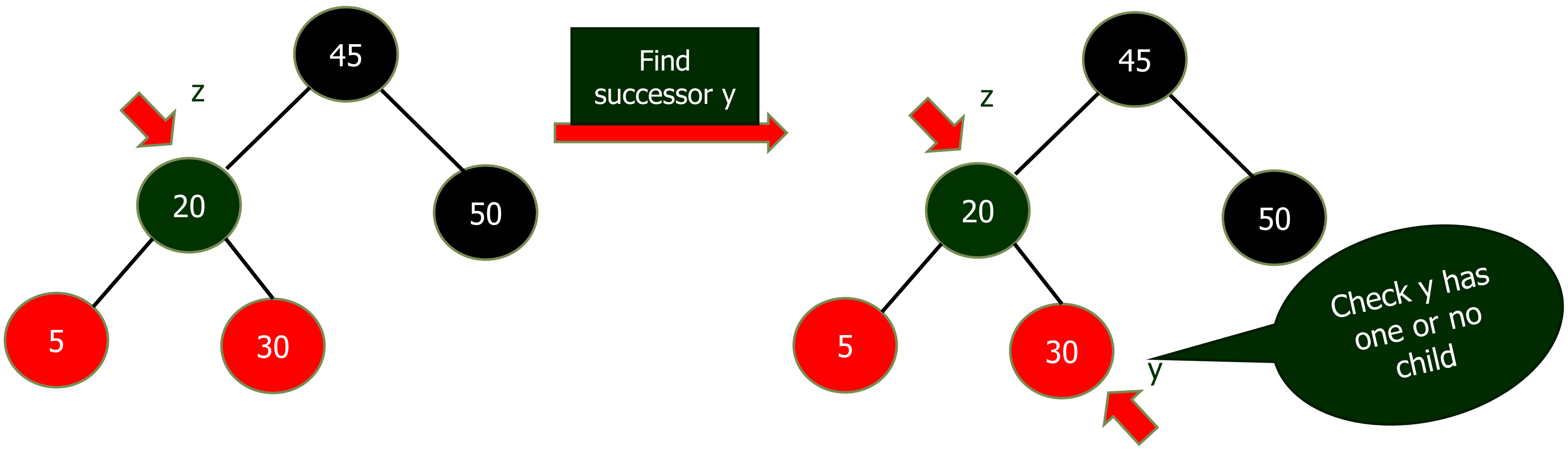
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -20



Red Black Tree

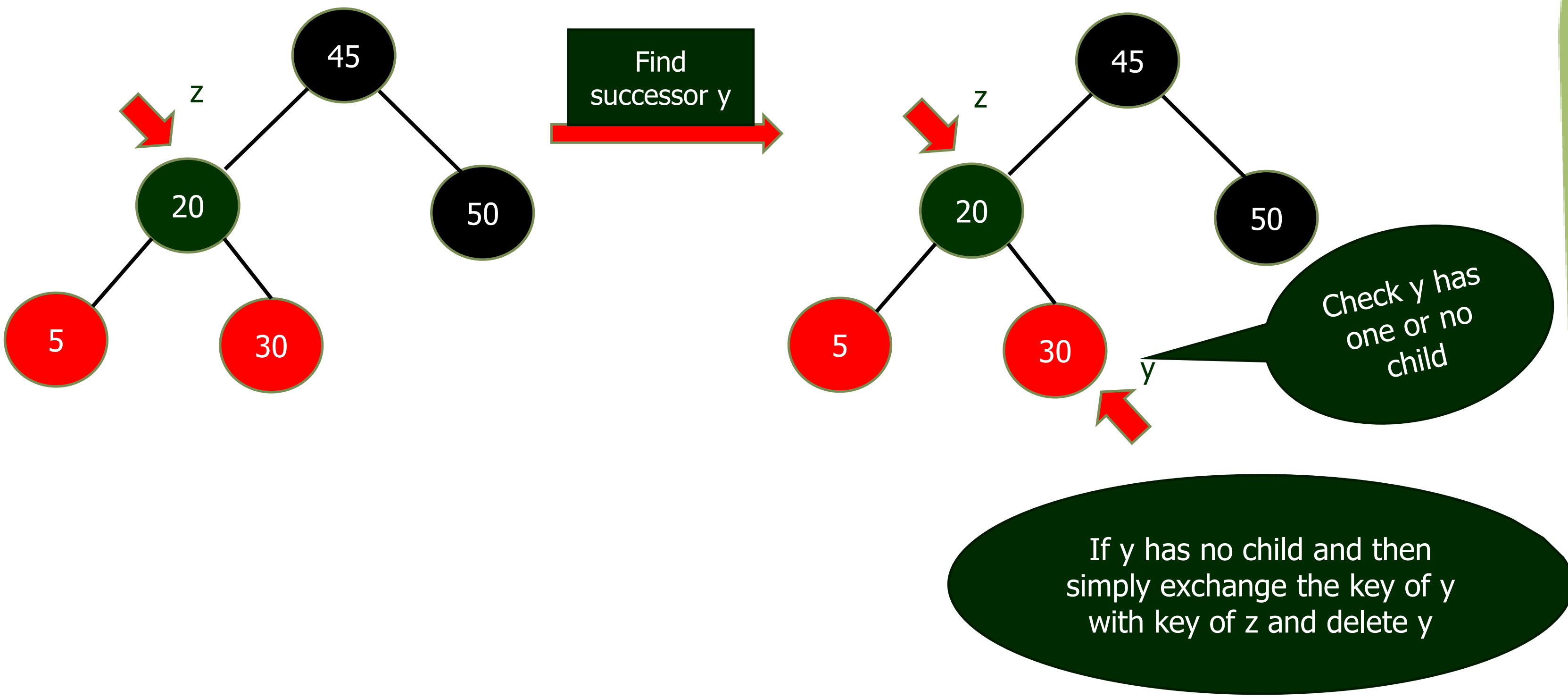
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -20



Red Black Tree

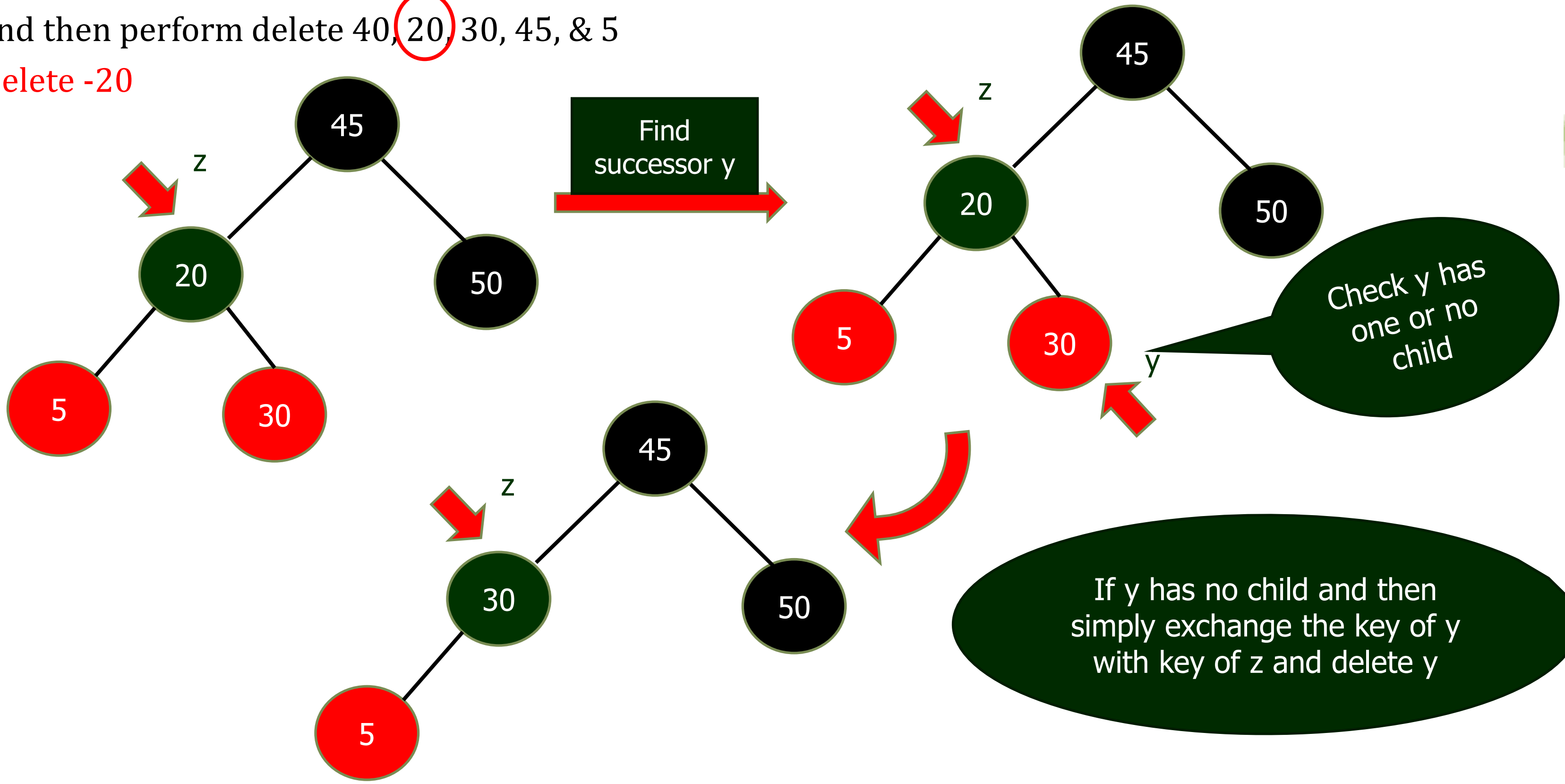
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -20



Red Black Tree

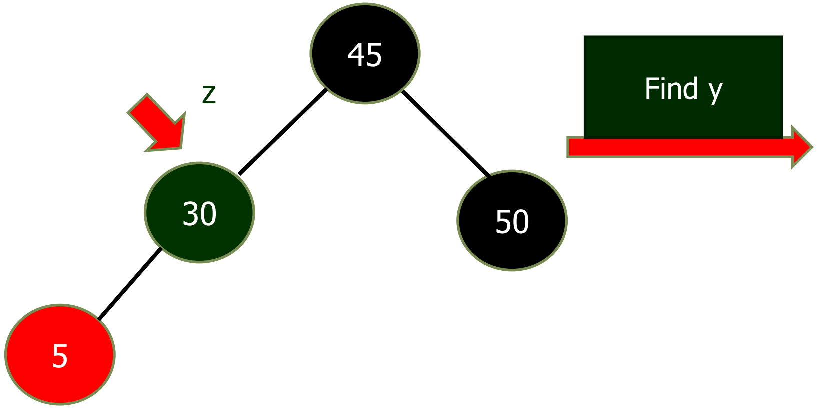
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



Red Black Tree

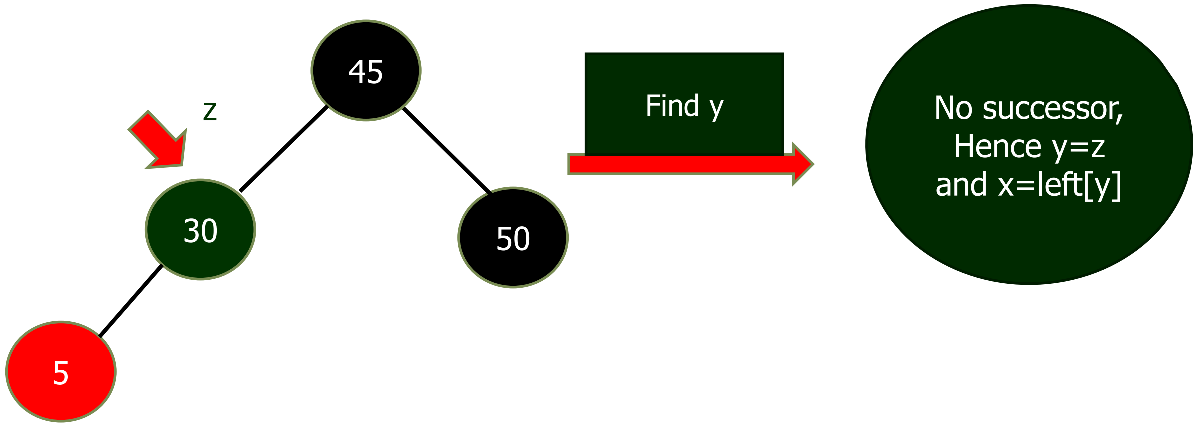
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



Red Black Tree

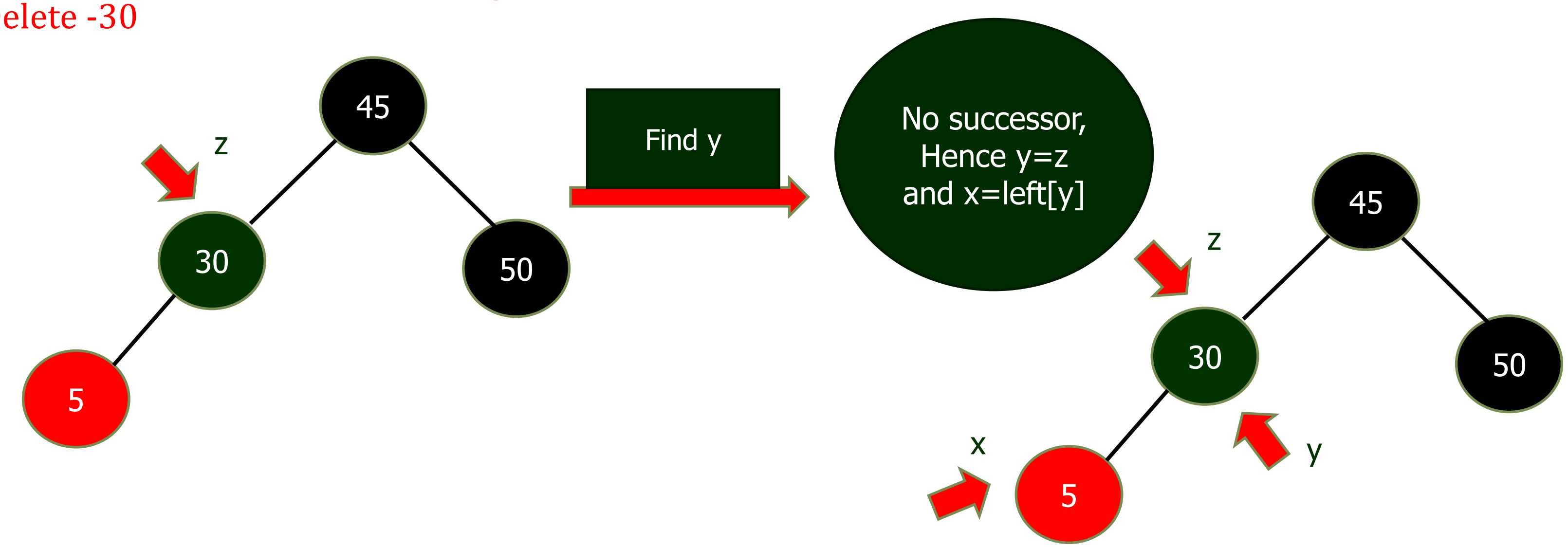
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



Red Black Tree

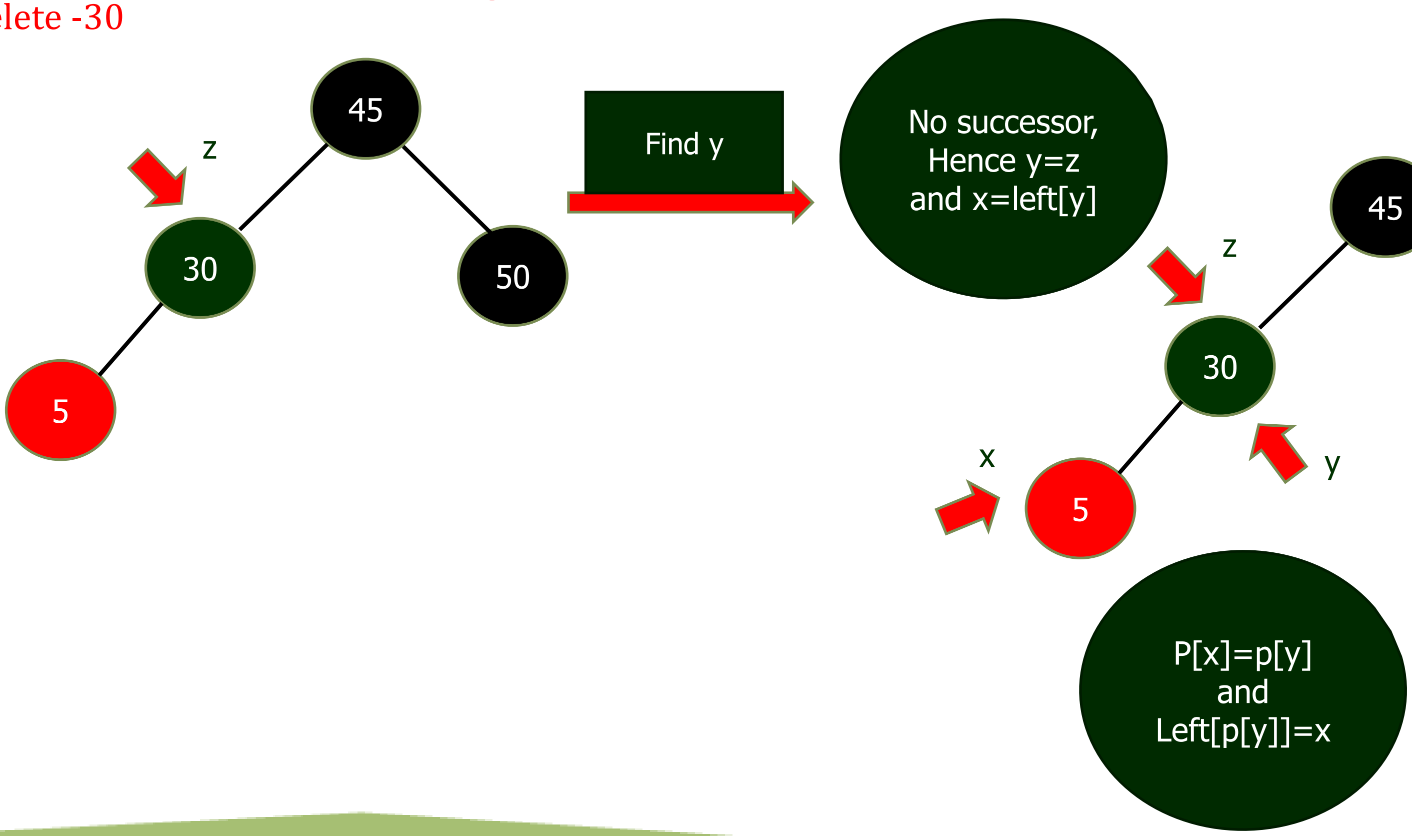
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



Red Black Tree

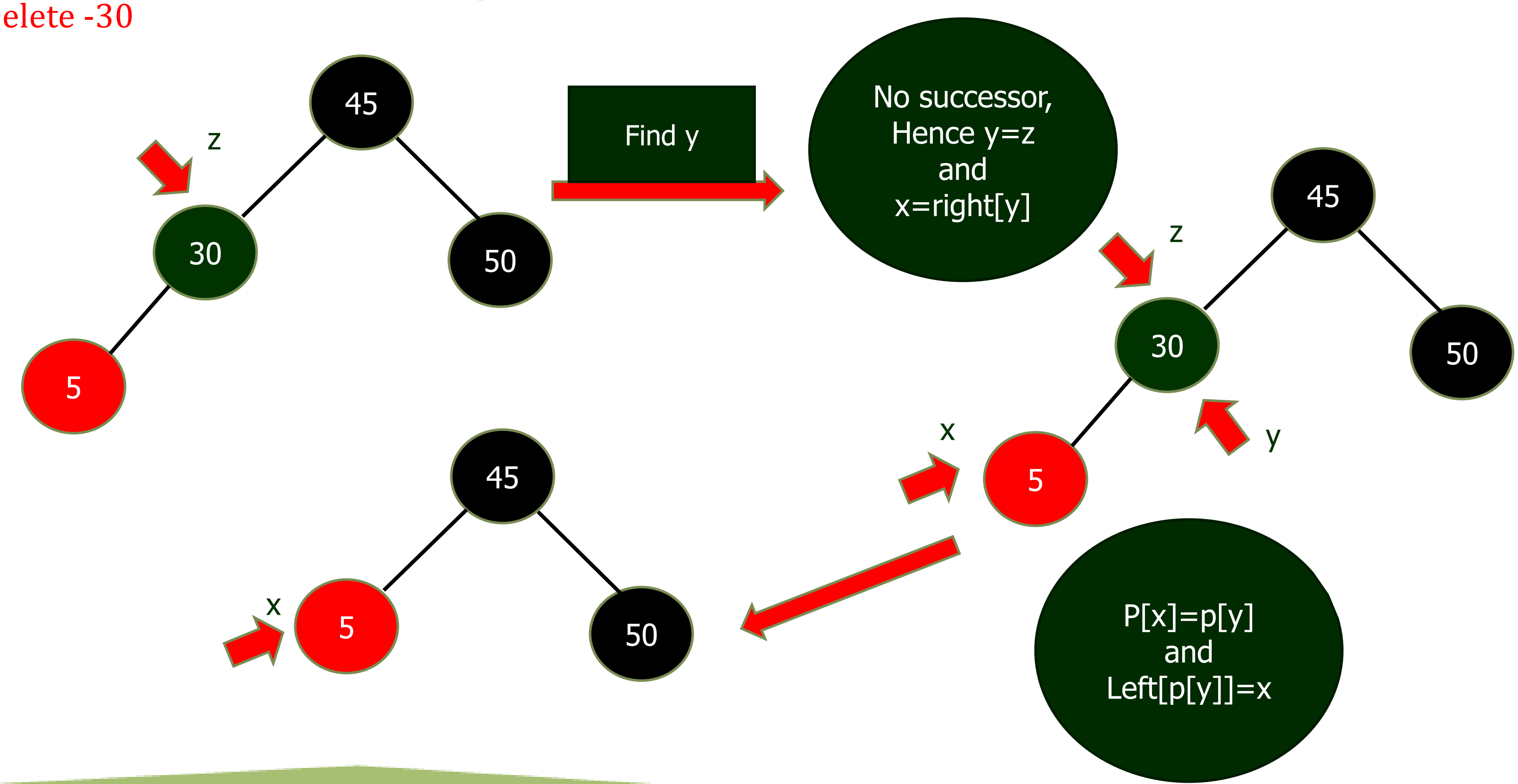
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



Red Black Tree

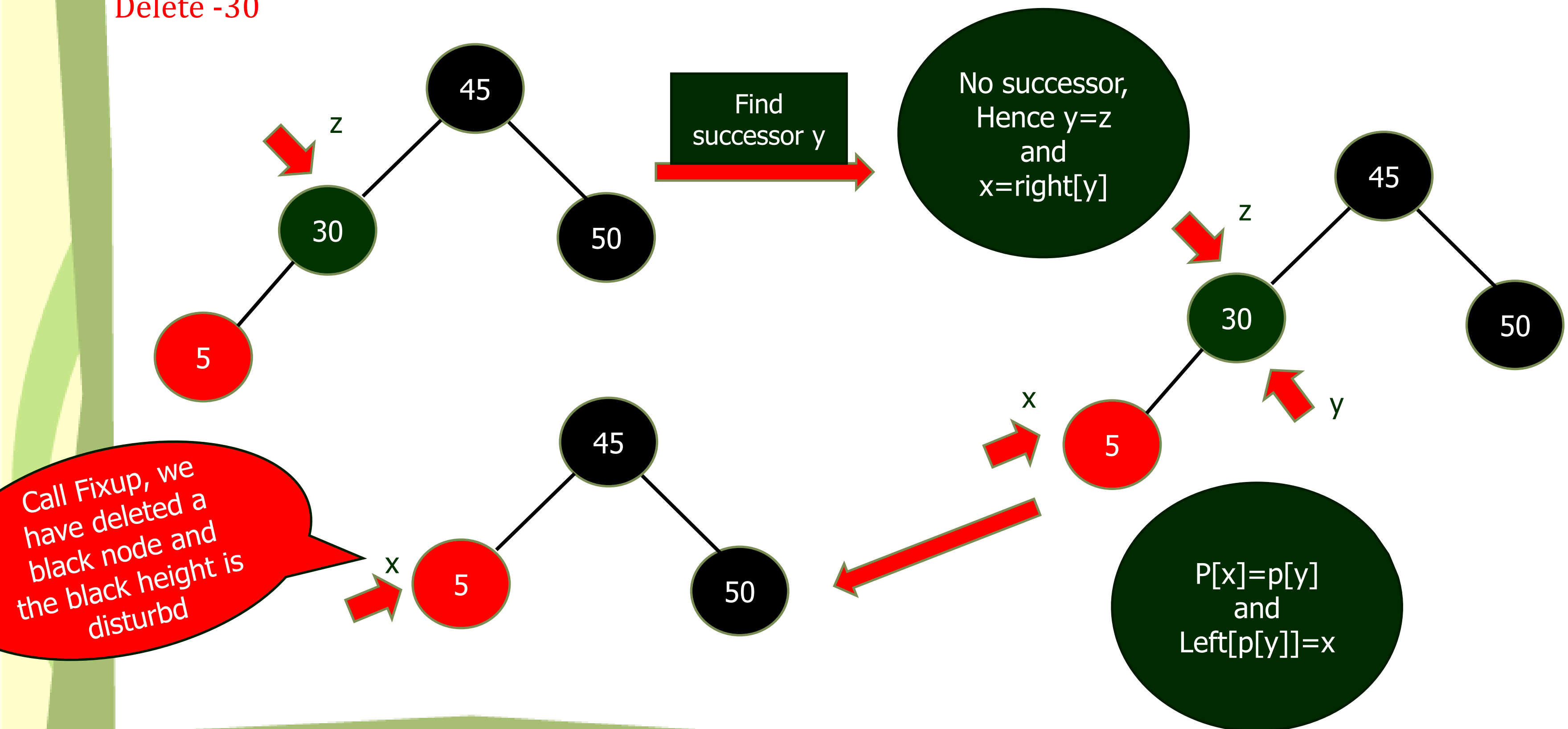
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



Red Black Tree

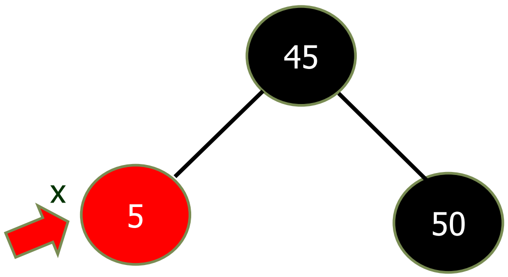
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



It was found that, we are unable to enter inside the while loop, so execute the last line of RB-Delete-Fixup,(i.e. color[x]=Black)

while x ≠ root[T] and color[x] = BLACK

Red Black Tree

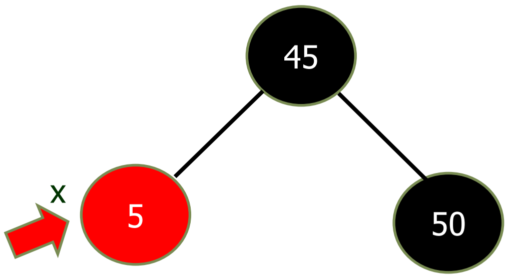
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -30



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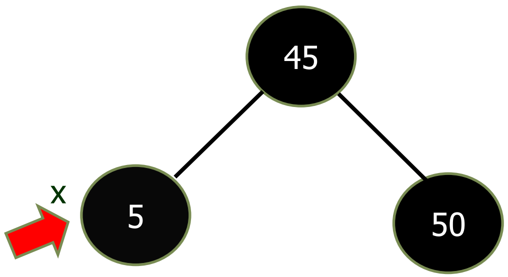


Red Black Tree

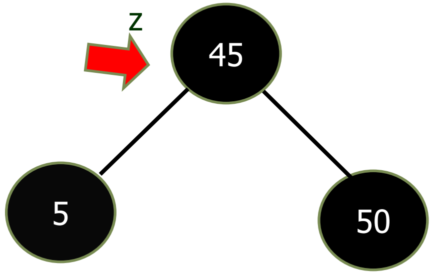
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

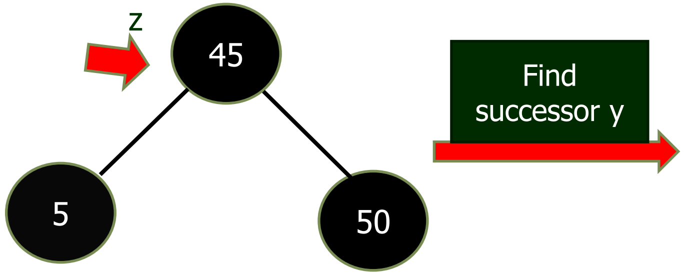
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

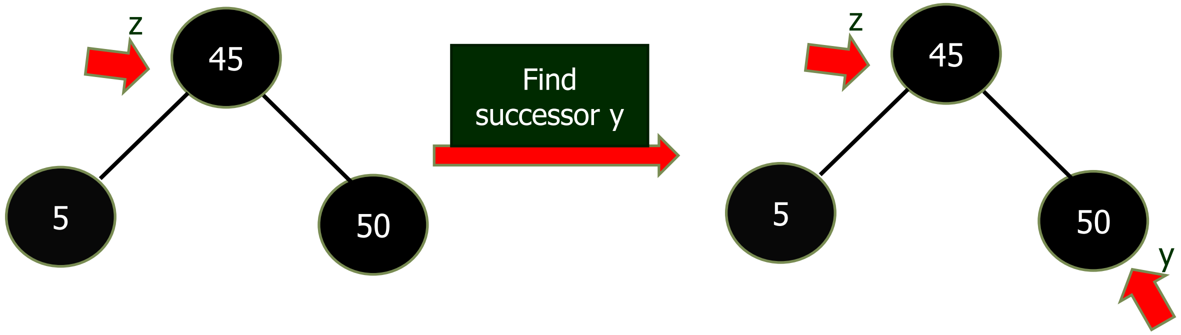
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

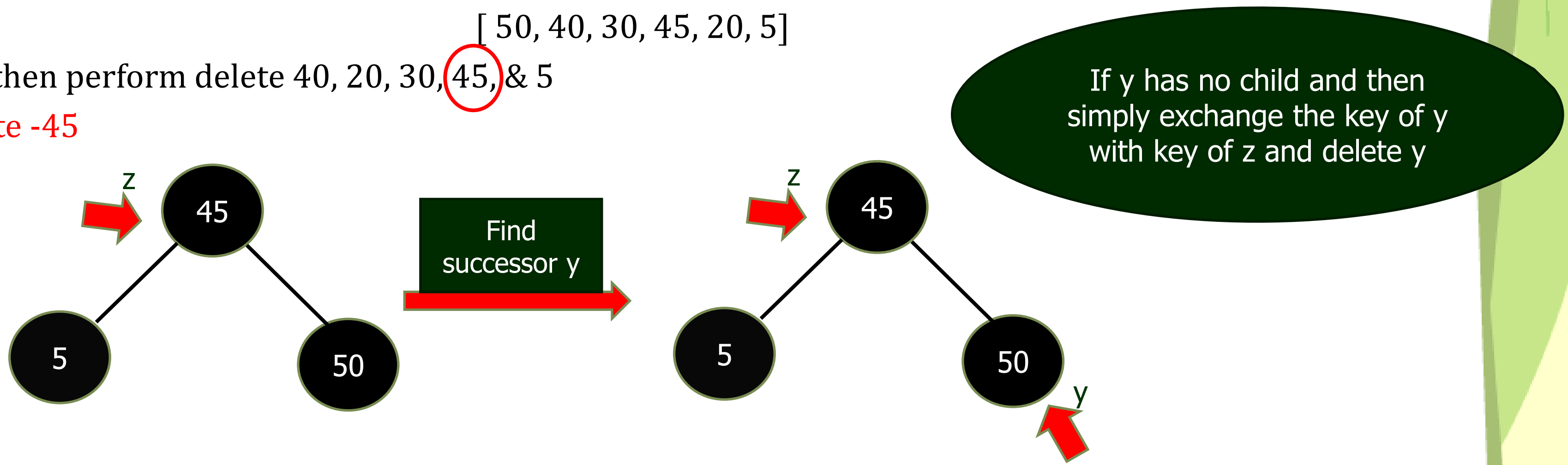
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

Insertion (Example 1):

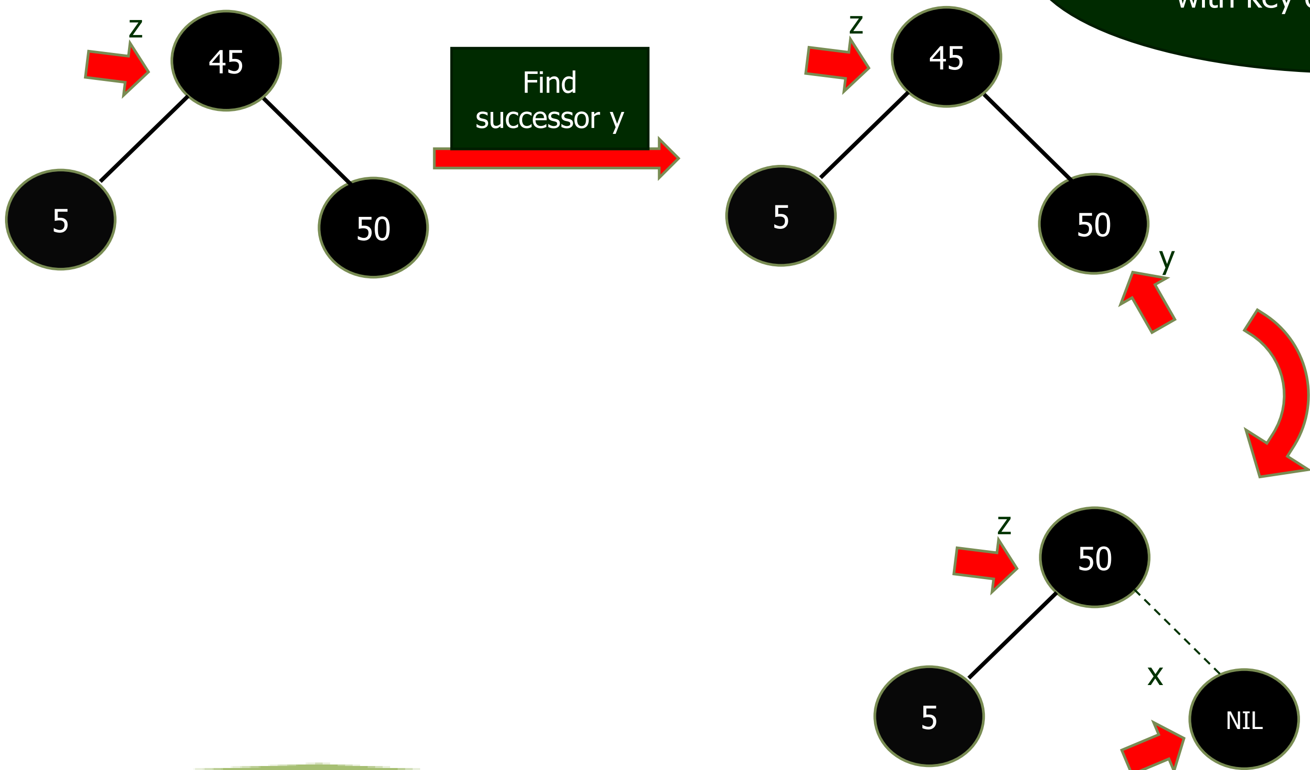
Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45

If y has no child and then simply exchange the key of y with key of z and delete y



Red Black Tree

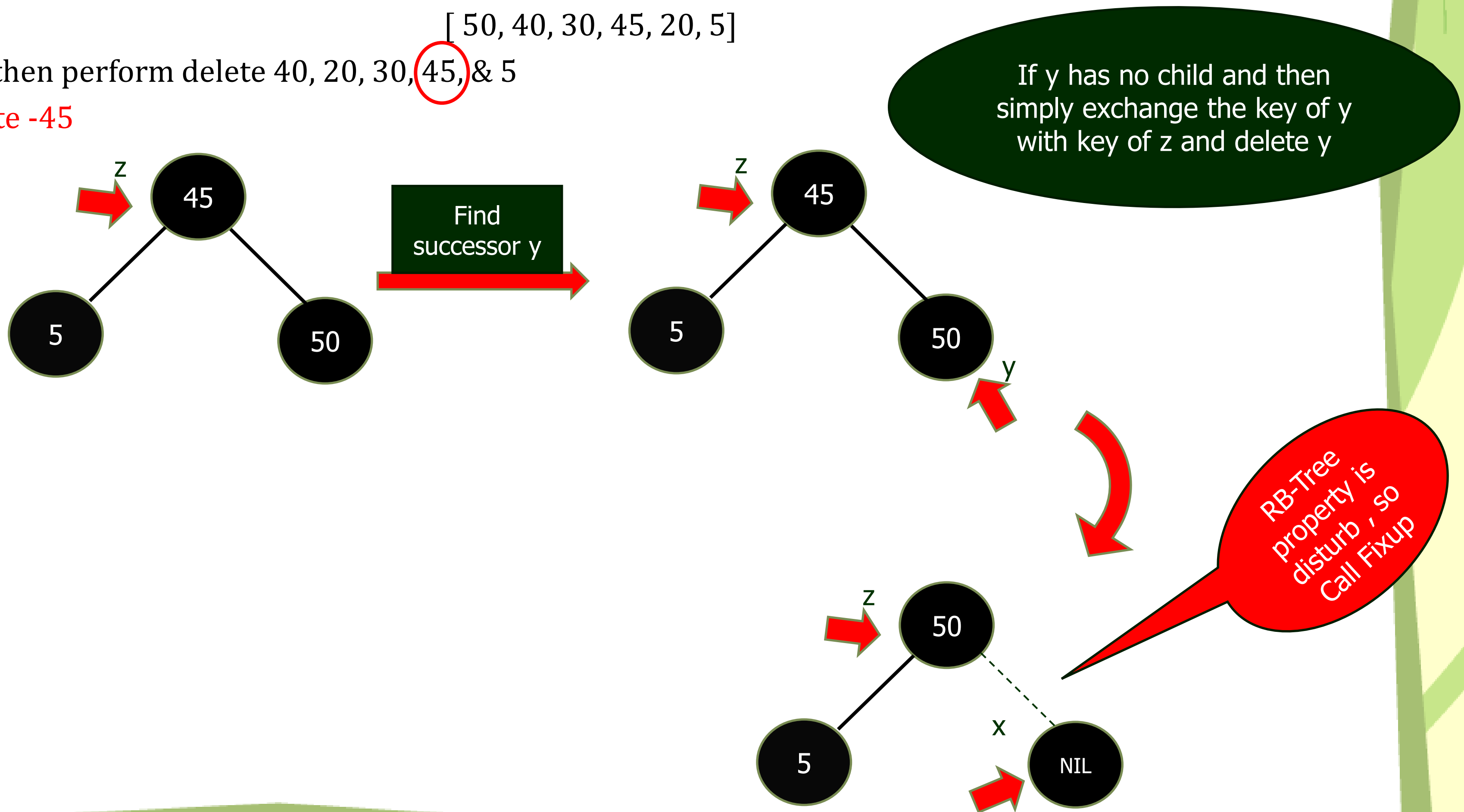
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

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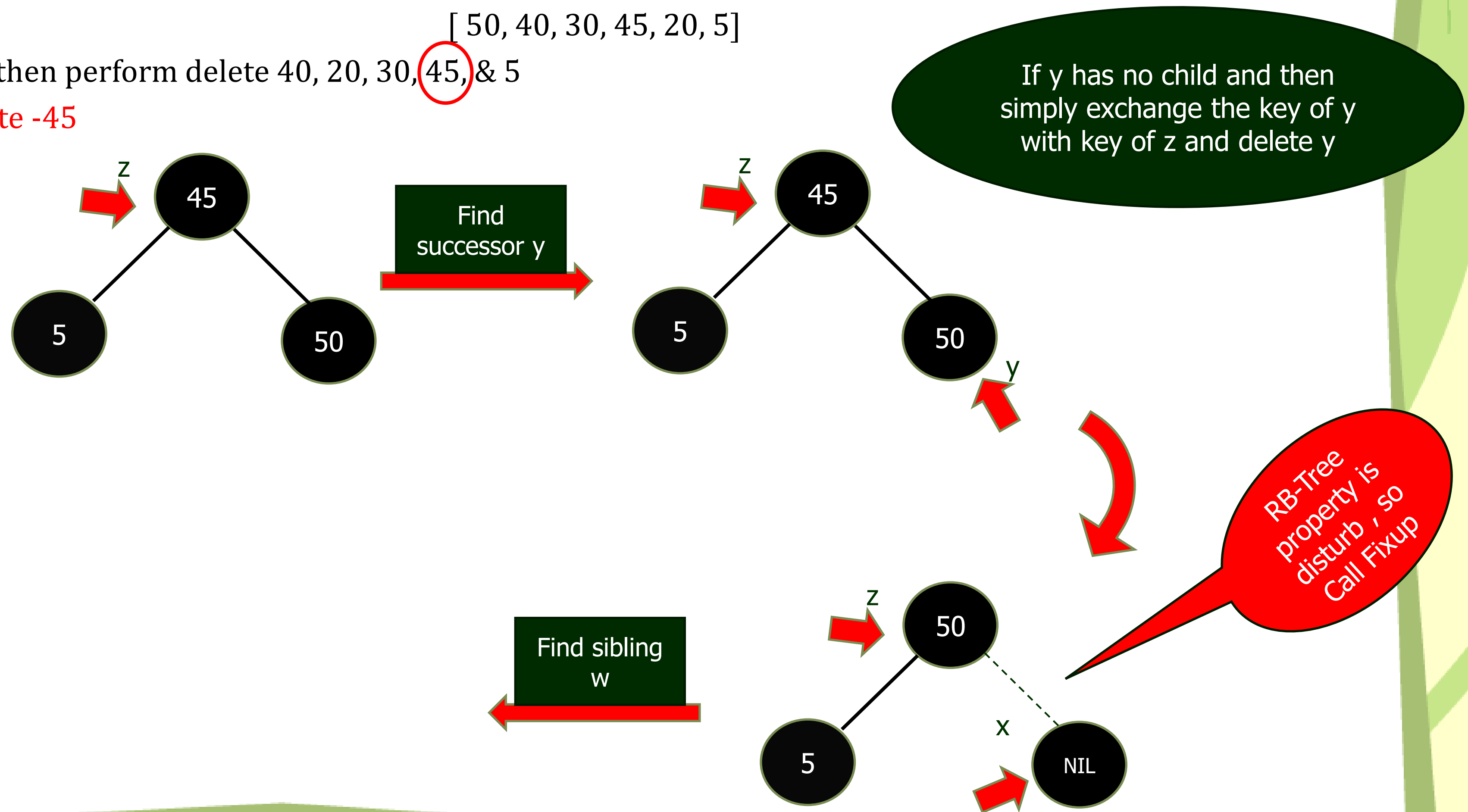
[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



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and then perform delete 40, 20, 30, 45, & 5

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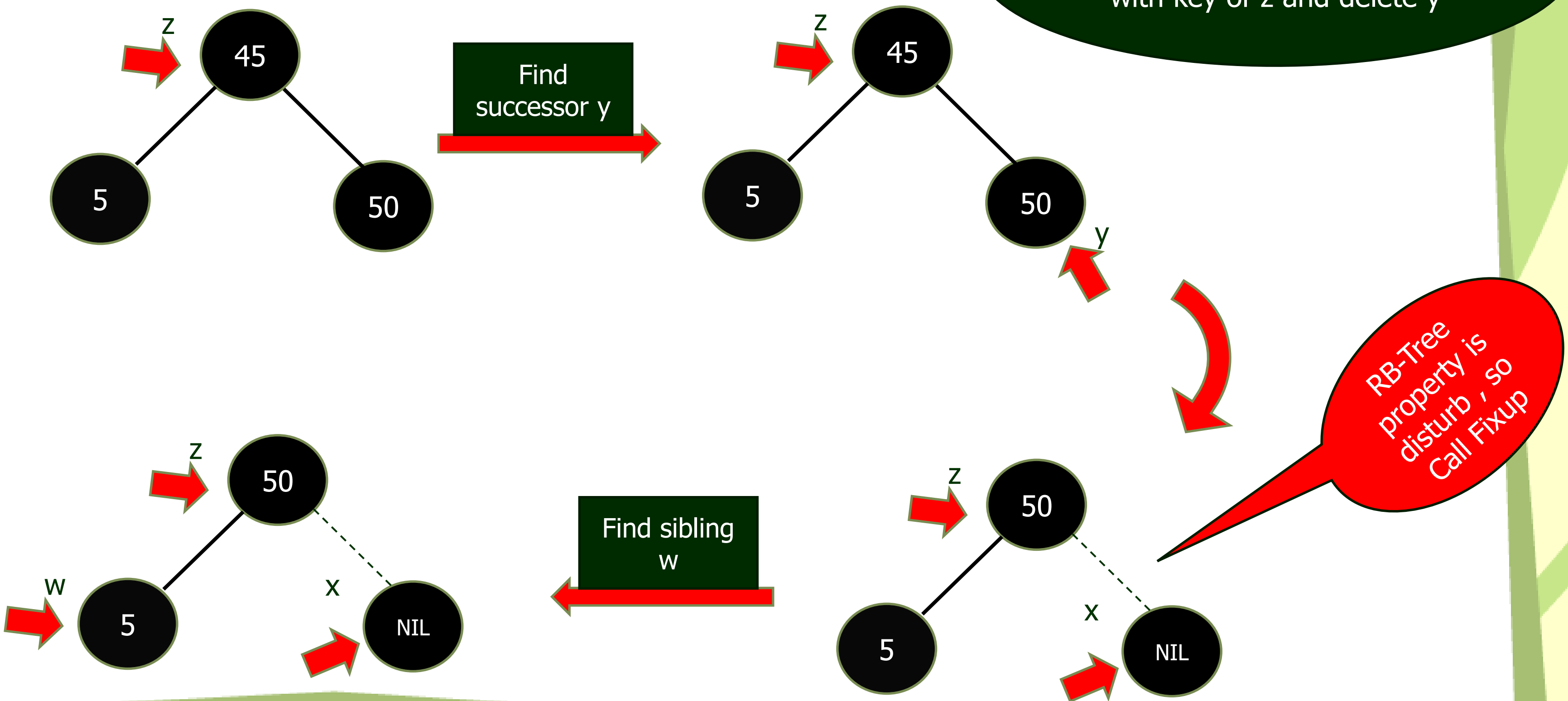


Red Black Tree

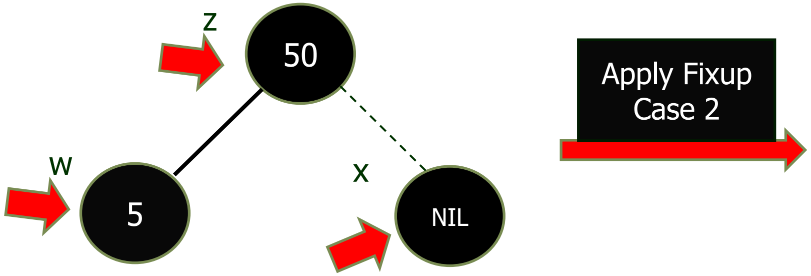
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

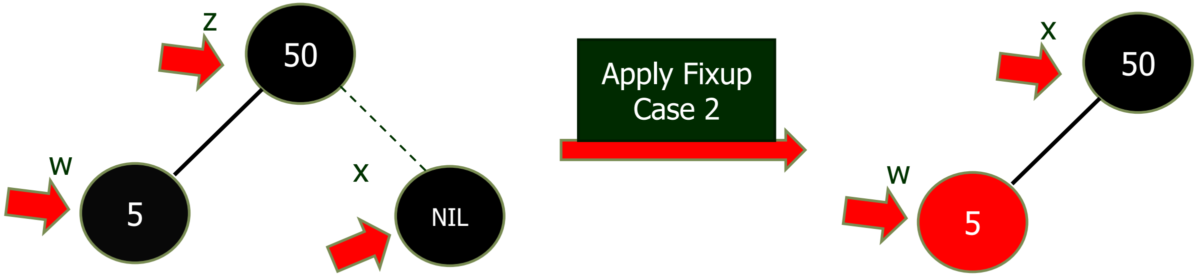
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -45



Red Black Tree

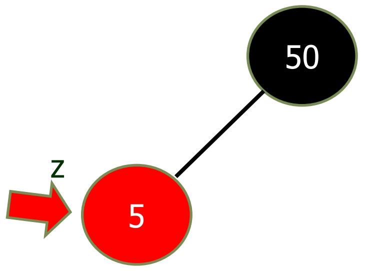
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -5



Red Black Tree

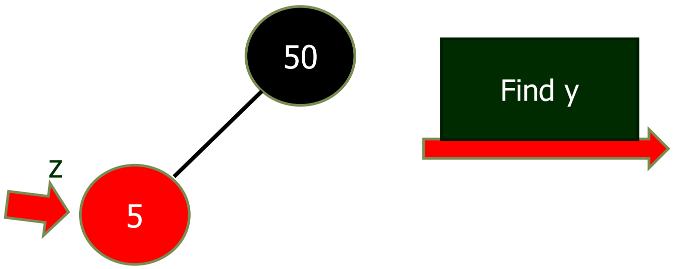
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -5



Red Black Tree

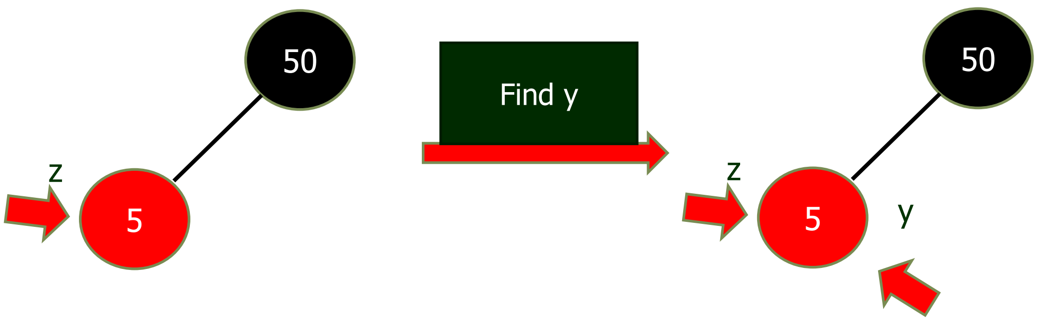
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -5



Red Black Tree

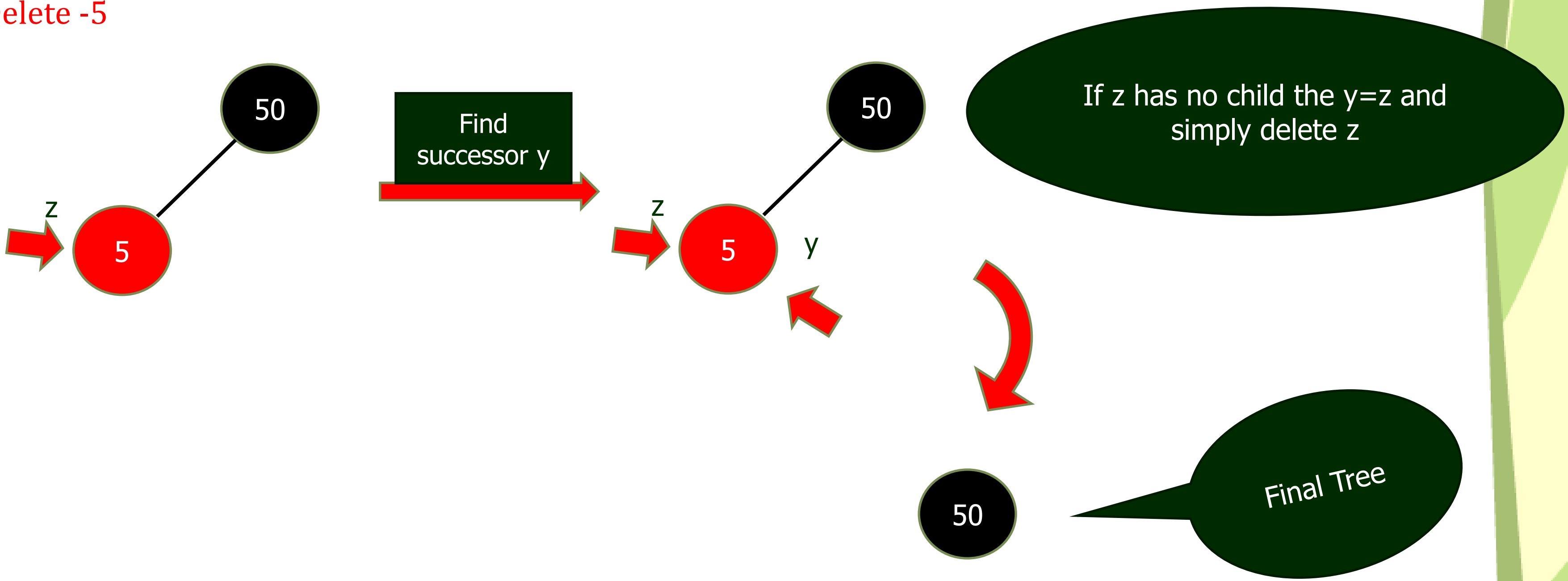
Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

and then perform delete 40, 20, 30, 45, & 5

Delete -5

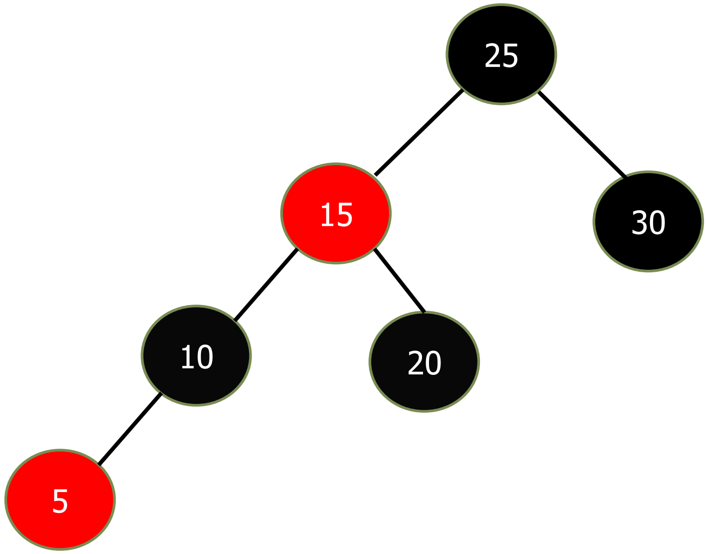


Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>



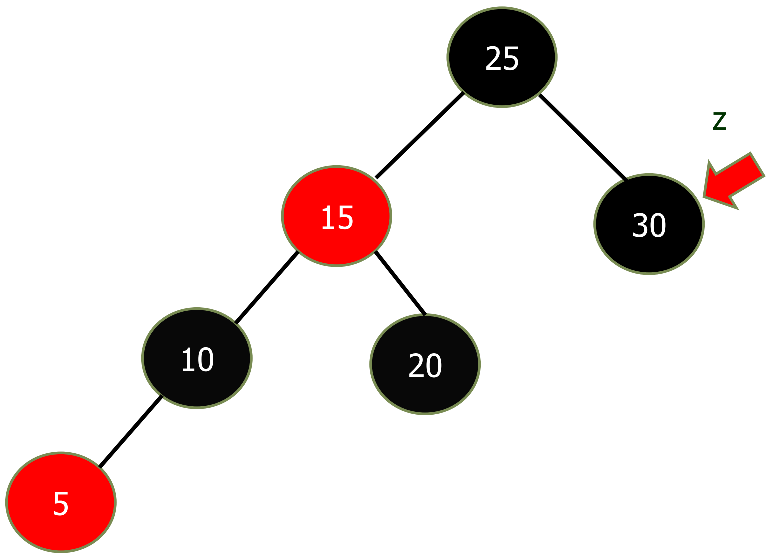
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -30



Red Black Tree

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With the help of following Red-black tree perform the deletion operation on the given keys respectively.

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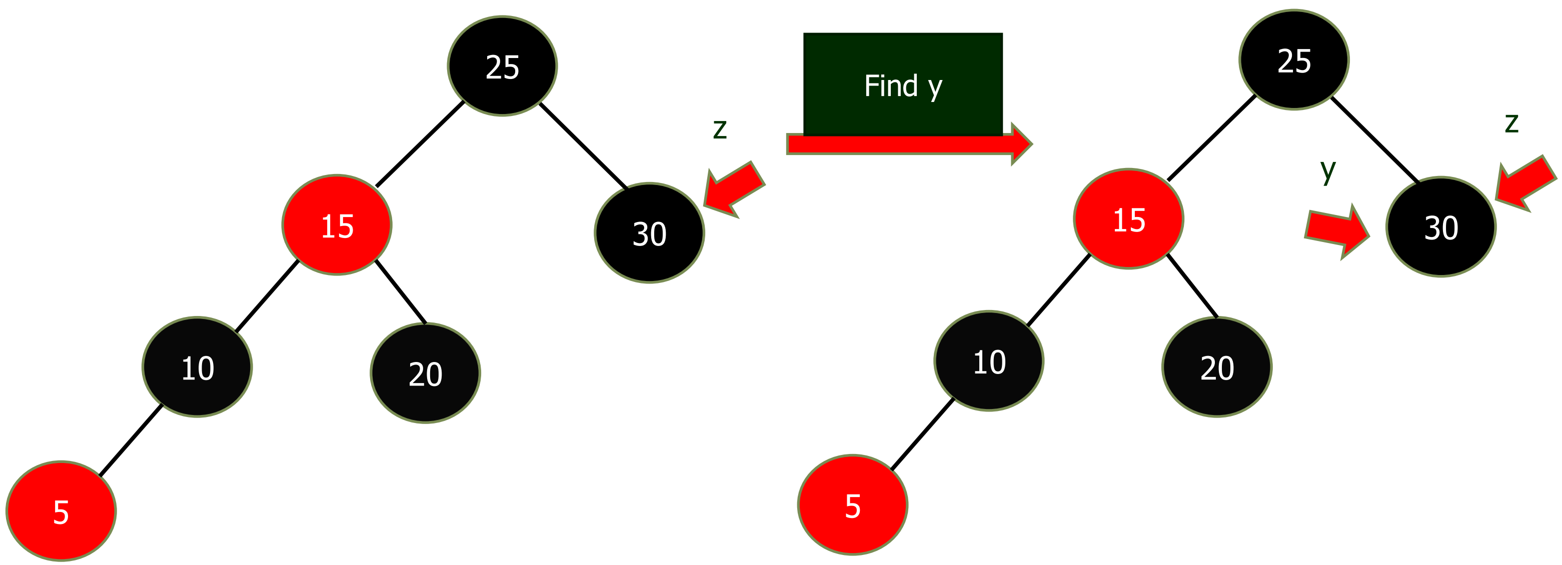
Delete -30



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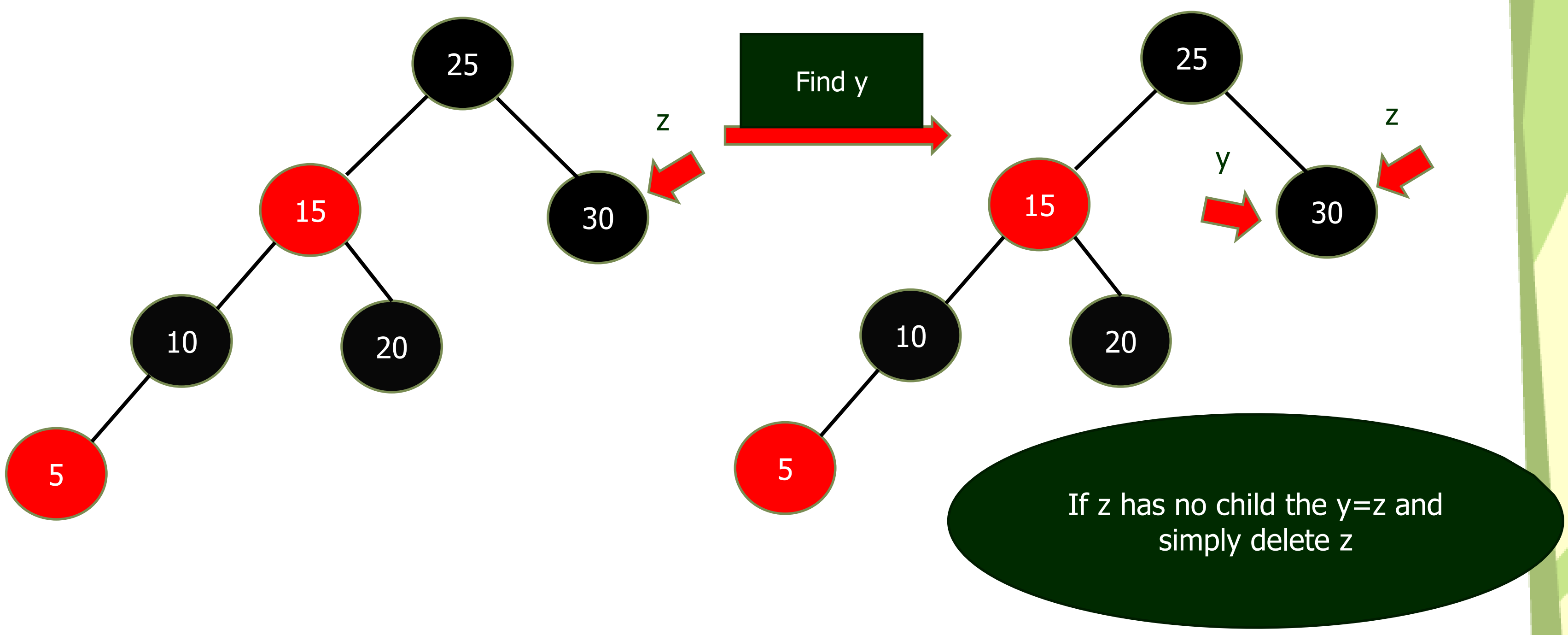

Red Black Tree

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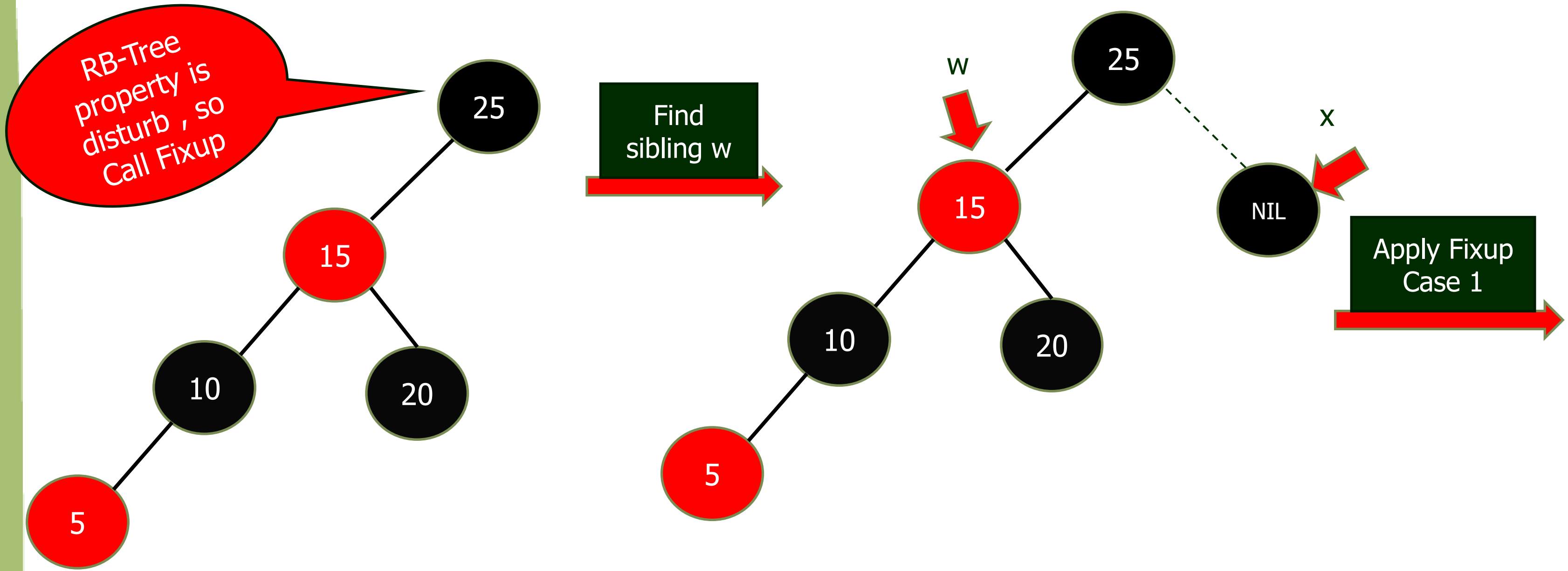
Delete -30



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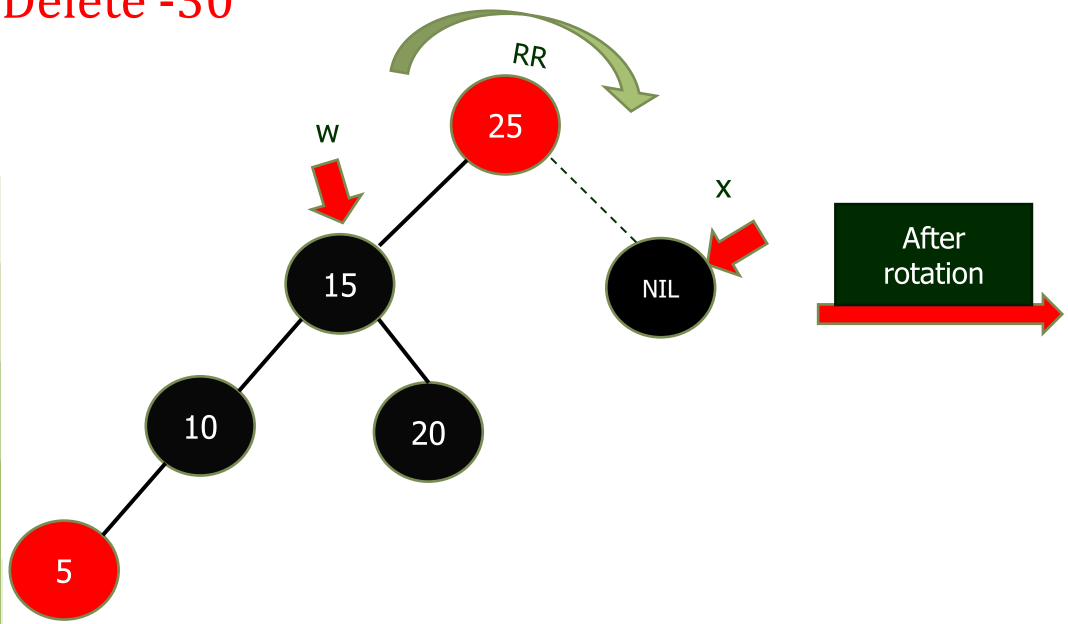

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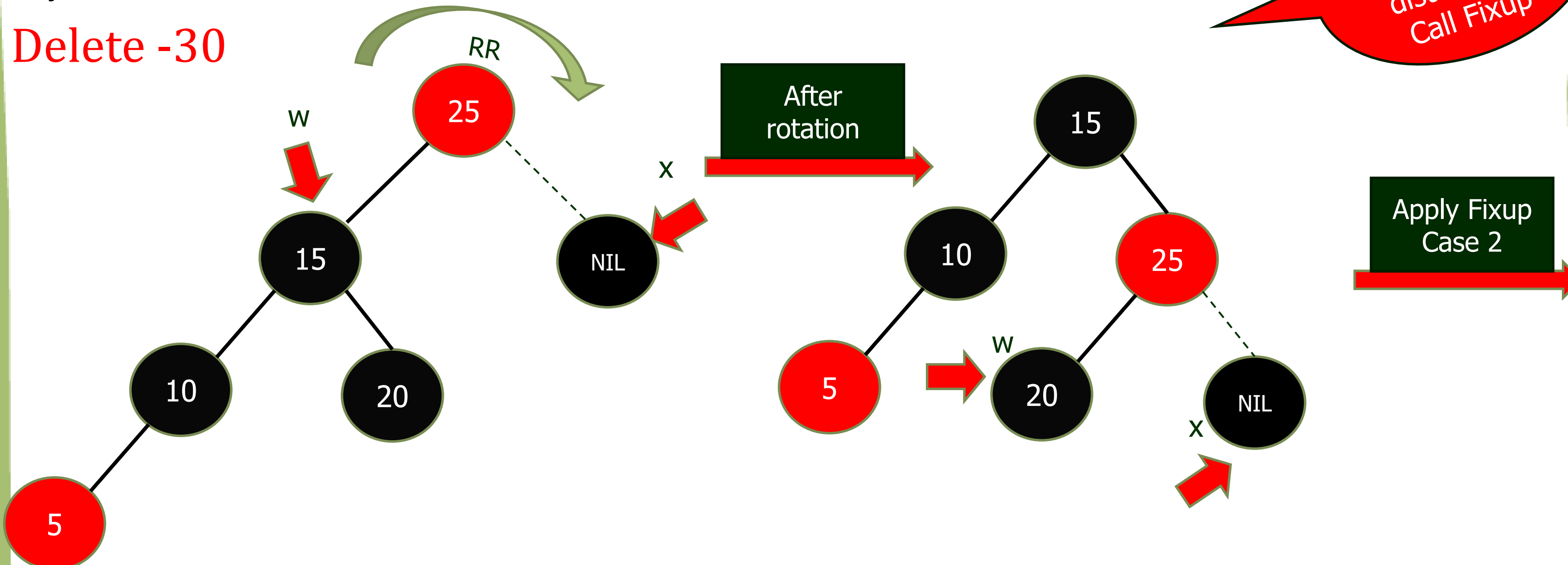


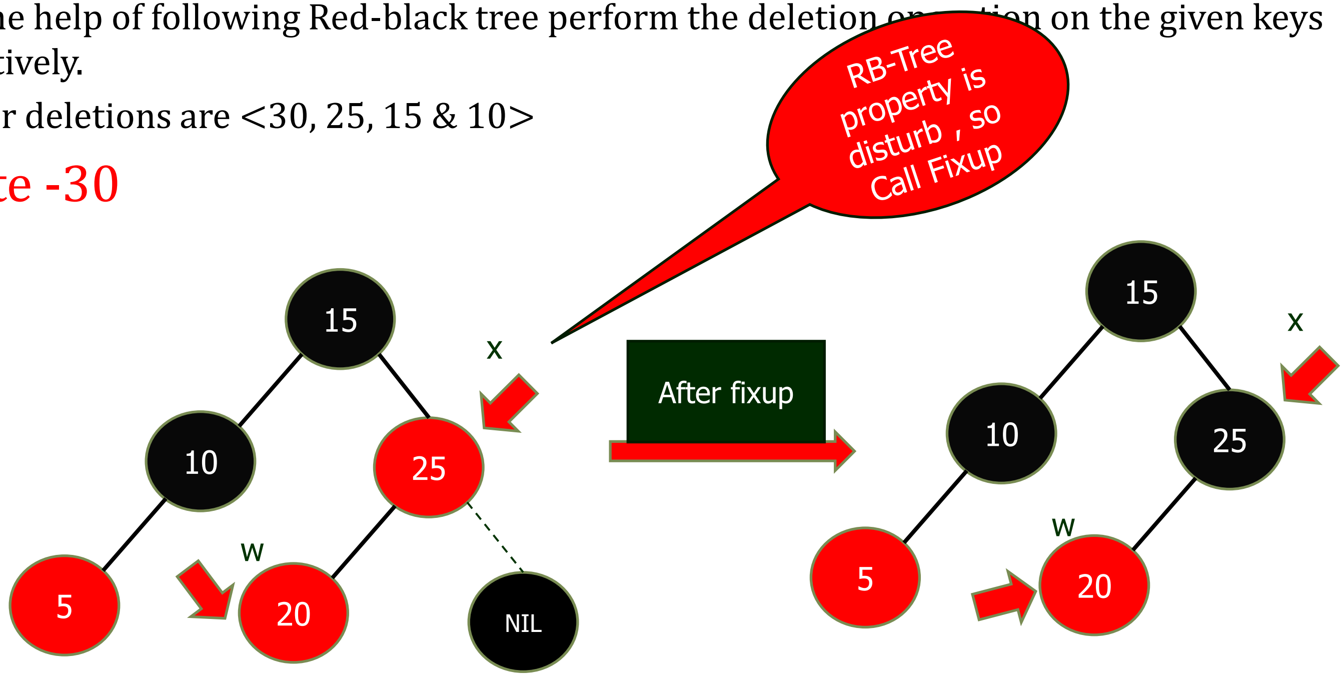
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -30



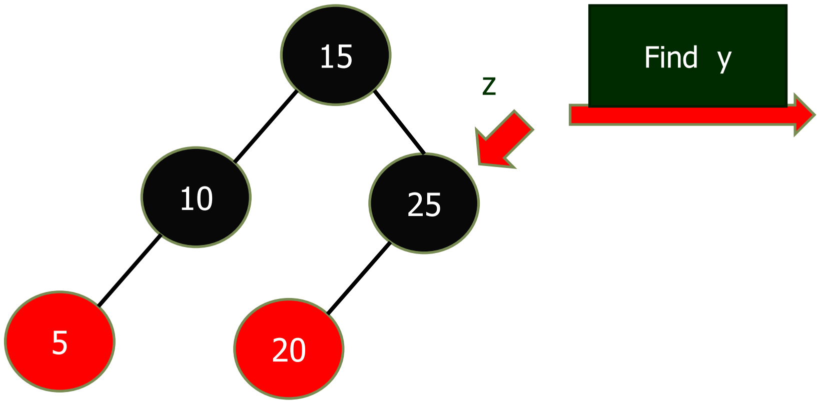
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -25



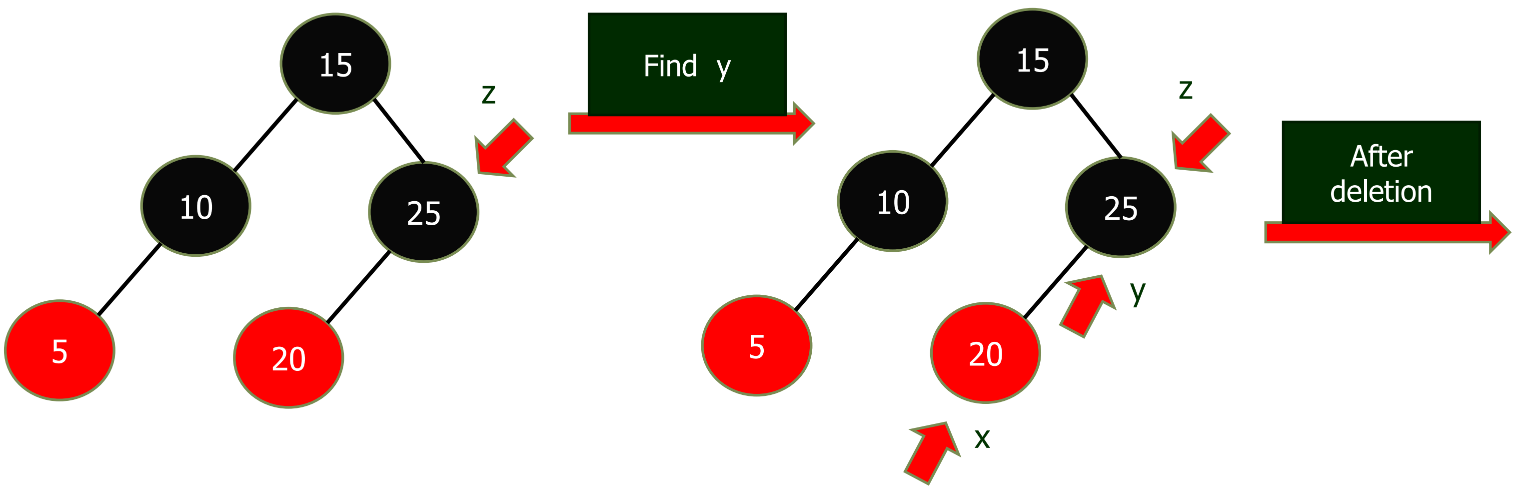
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -25



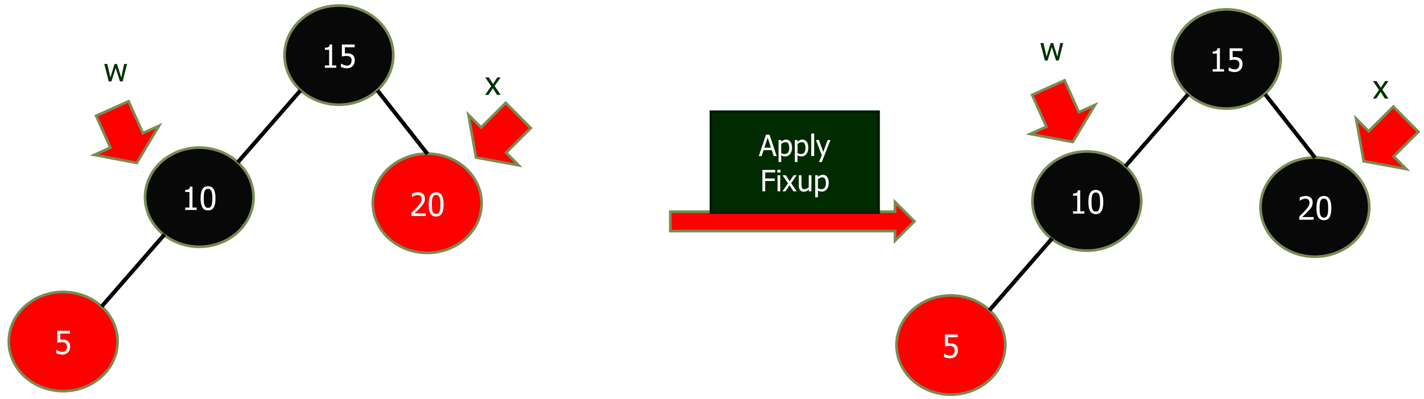
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -25



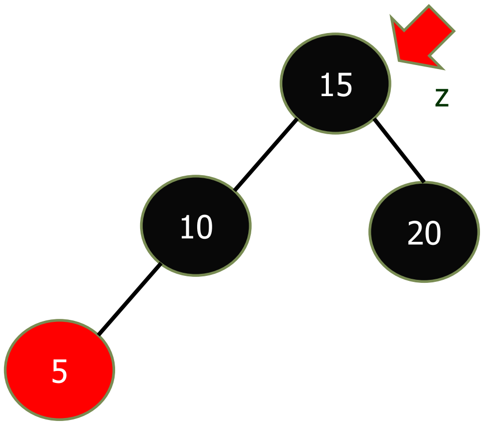
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



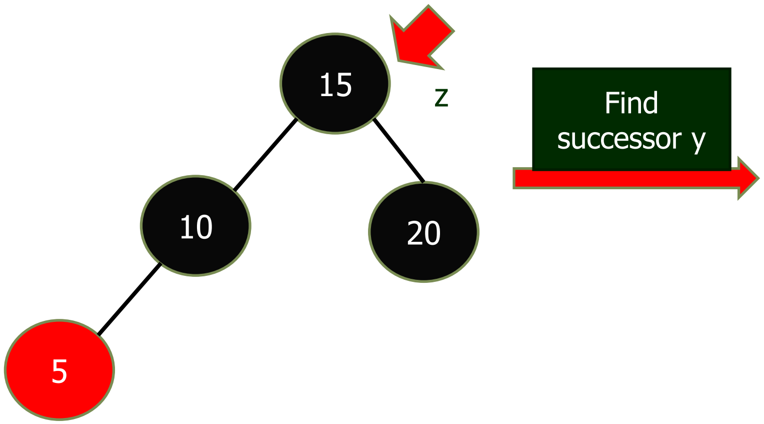
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



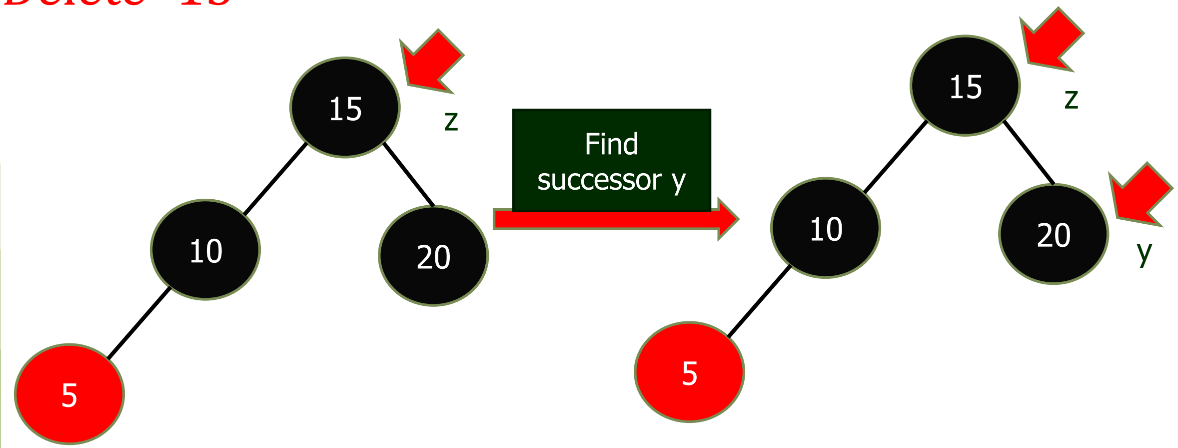
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



Red Black Tree

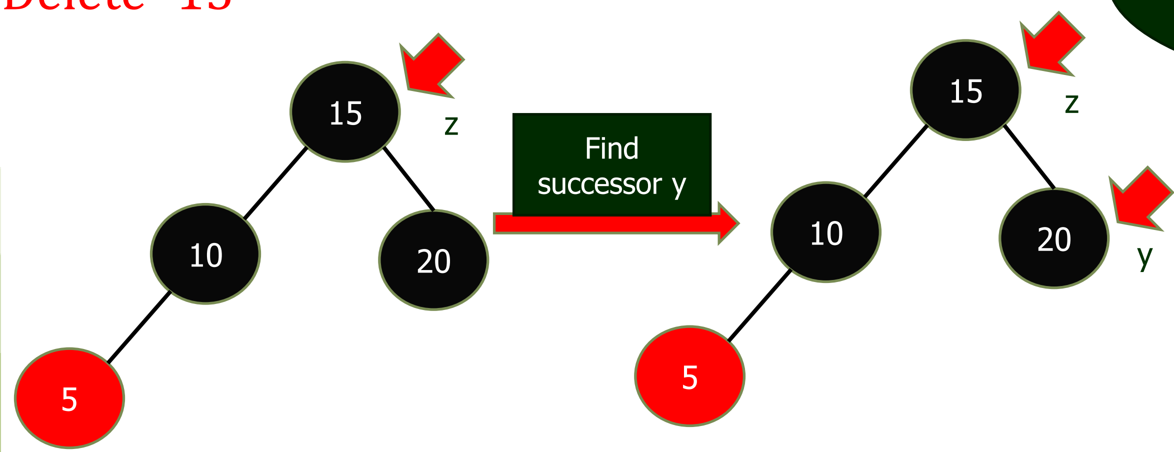
Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15

If y has no child and then simply exchange the key of y with key of z and delete y



Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15

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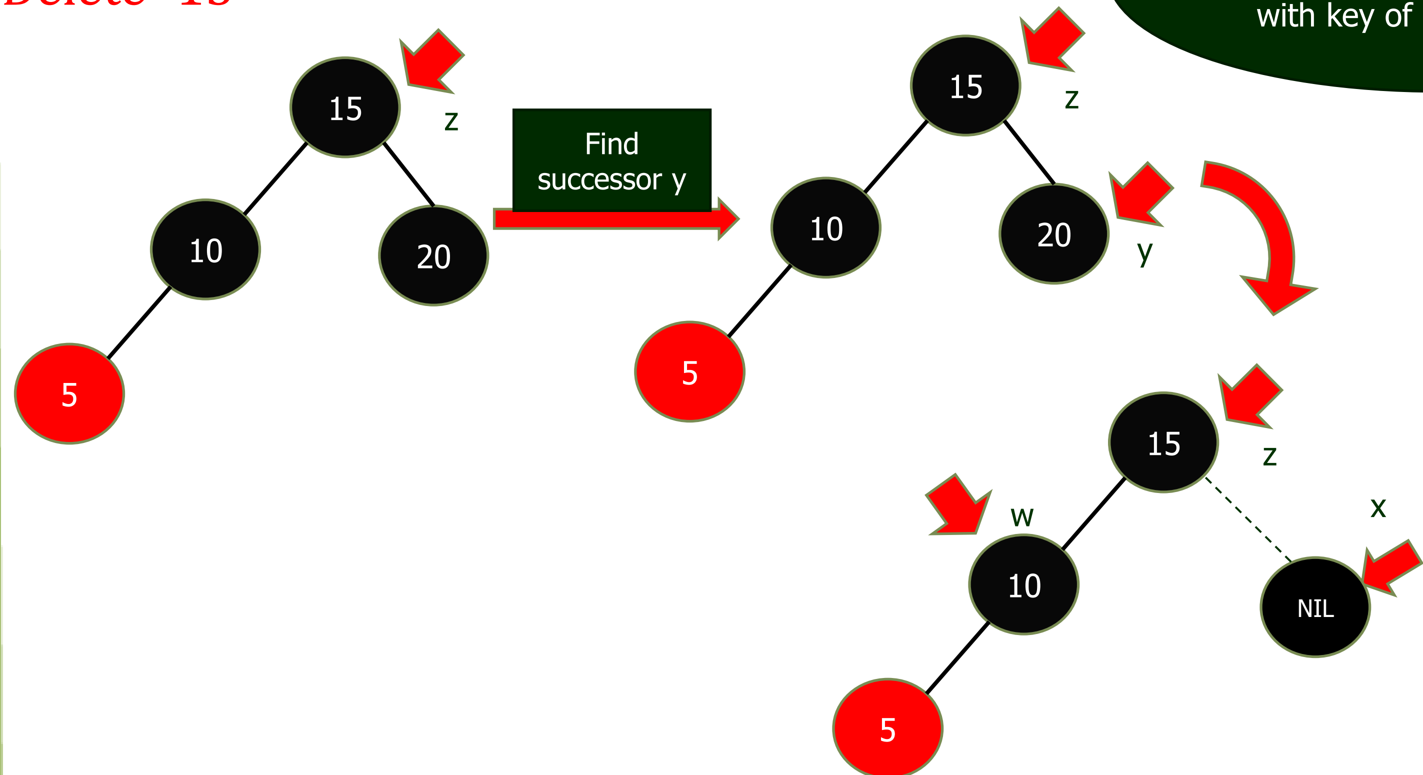


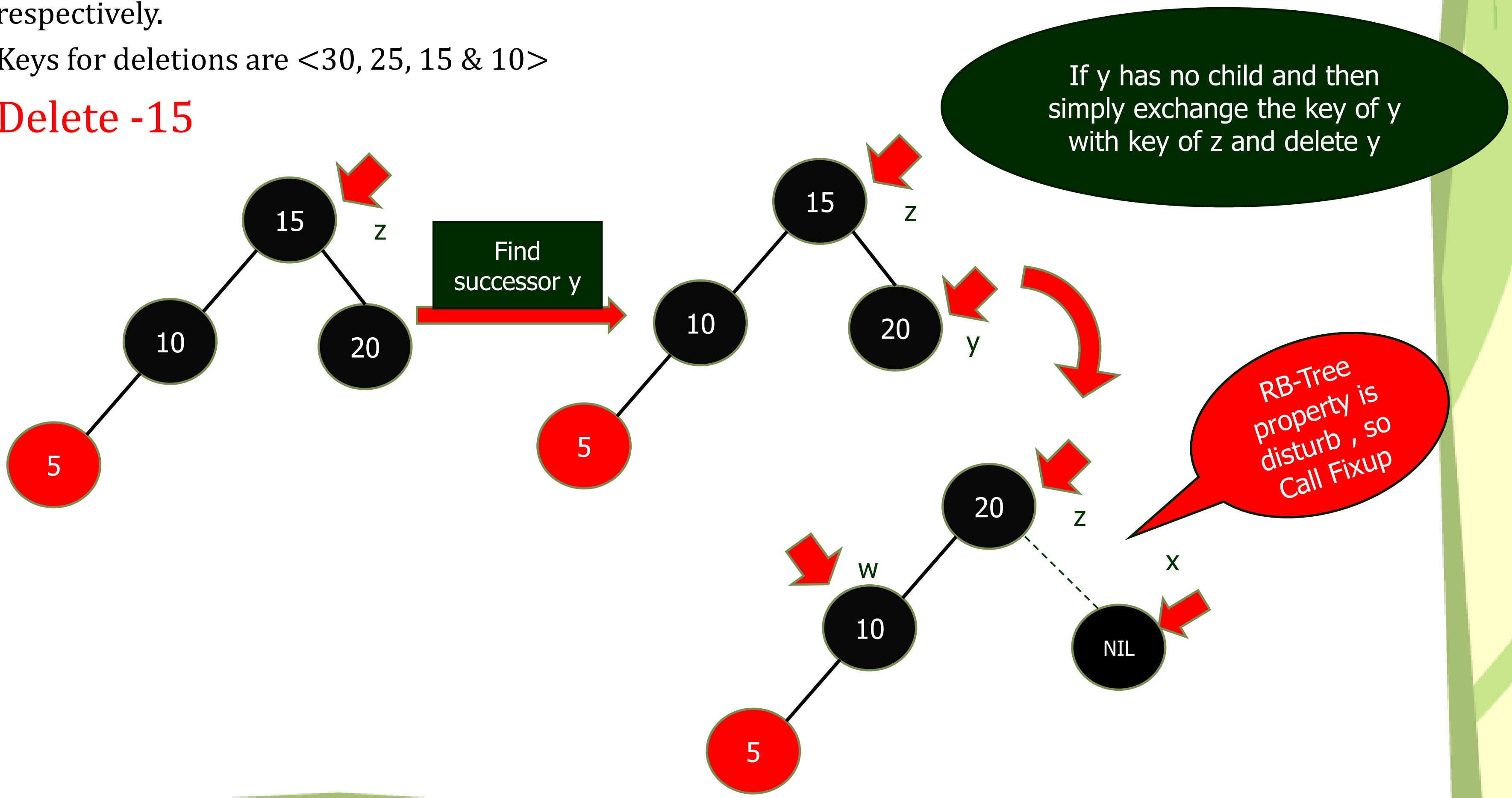
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



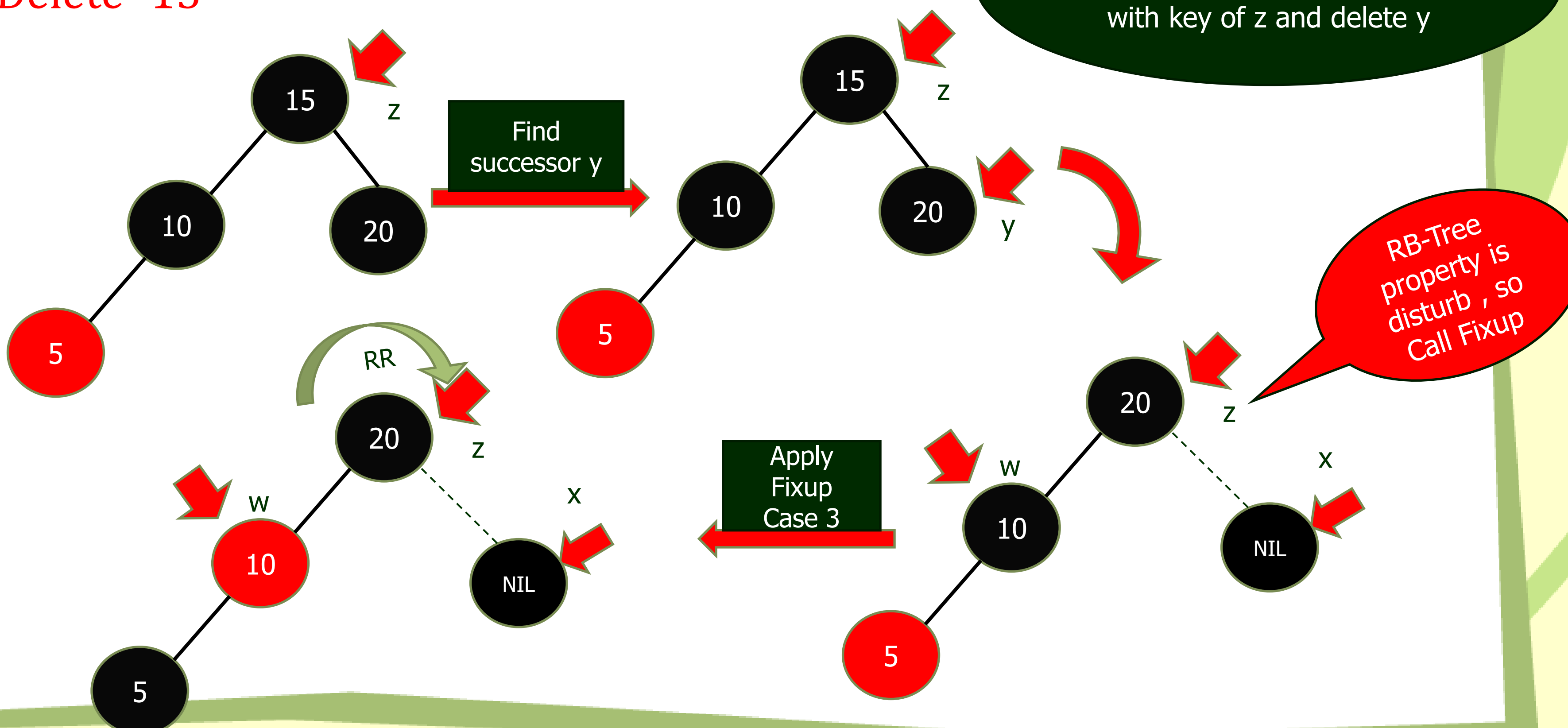
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



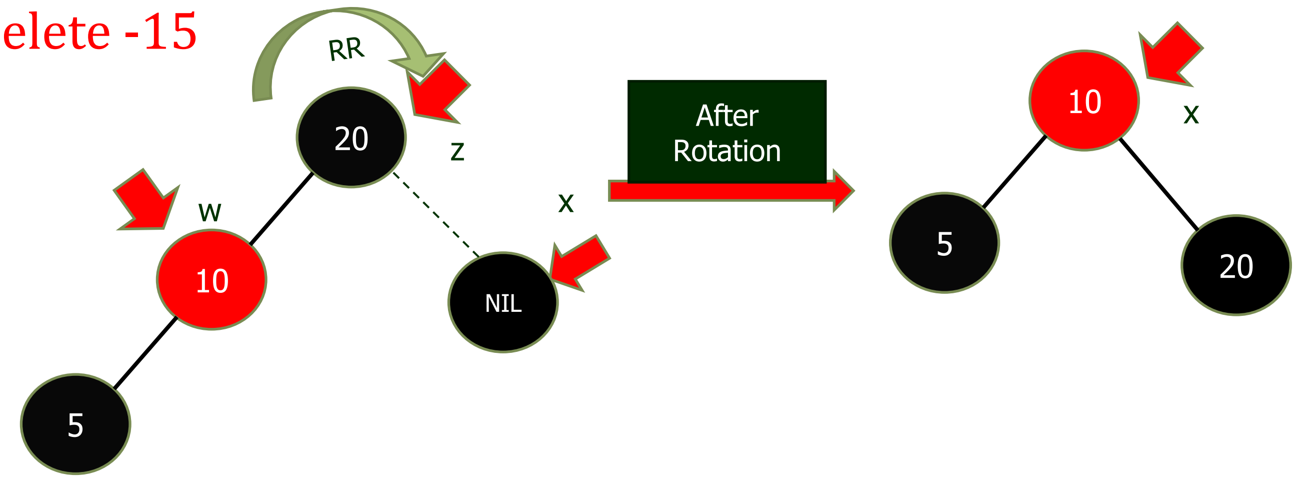
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



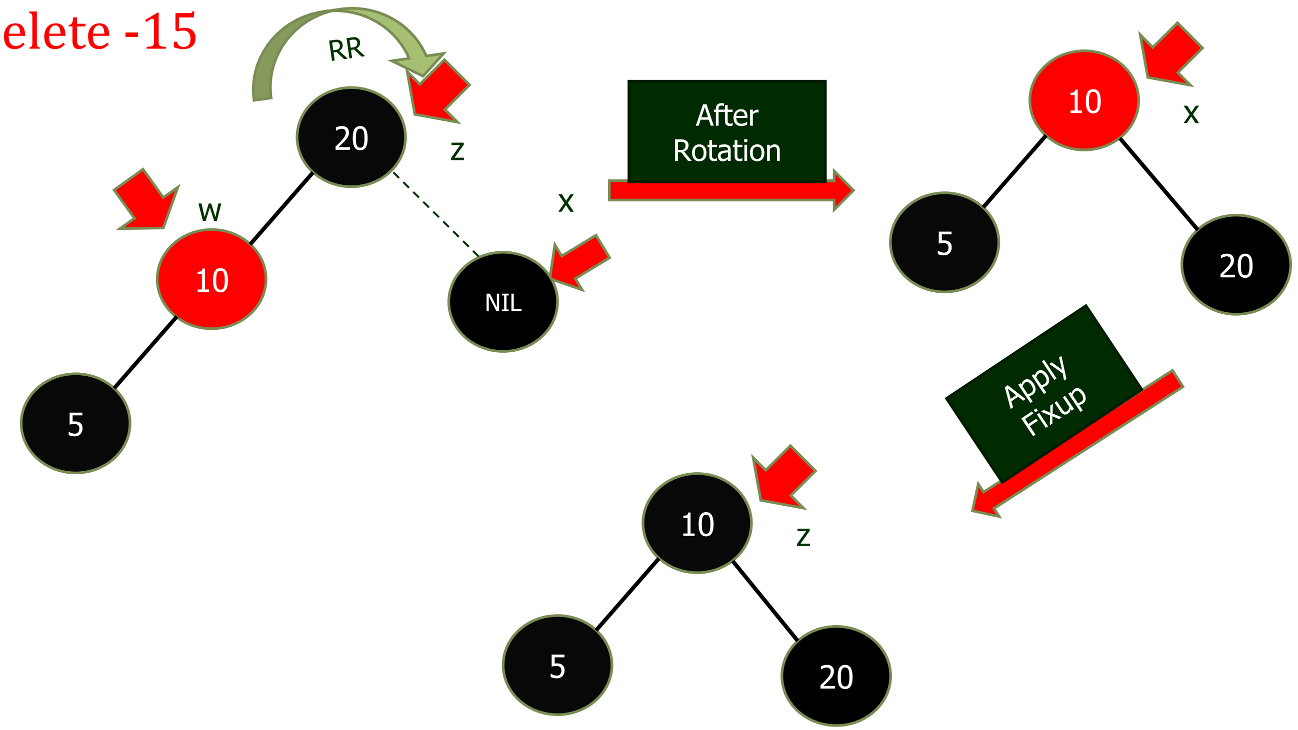
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -15



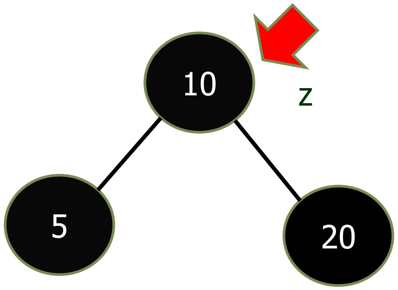
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



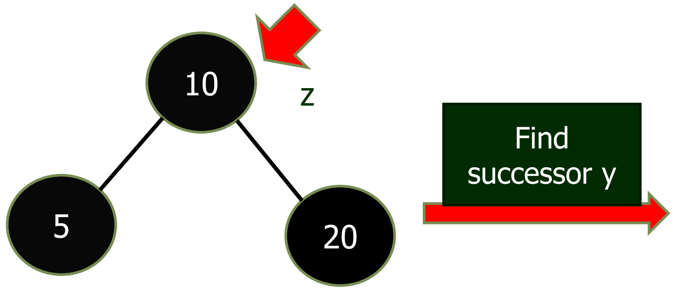
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



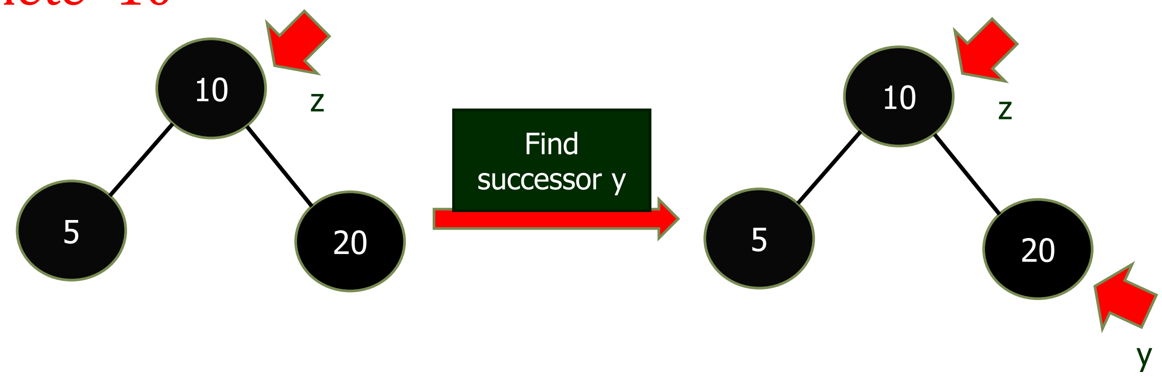
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



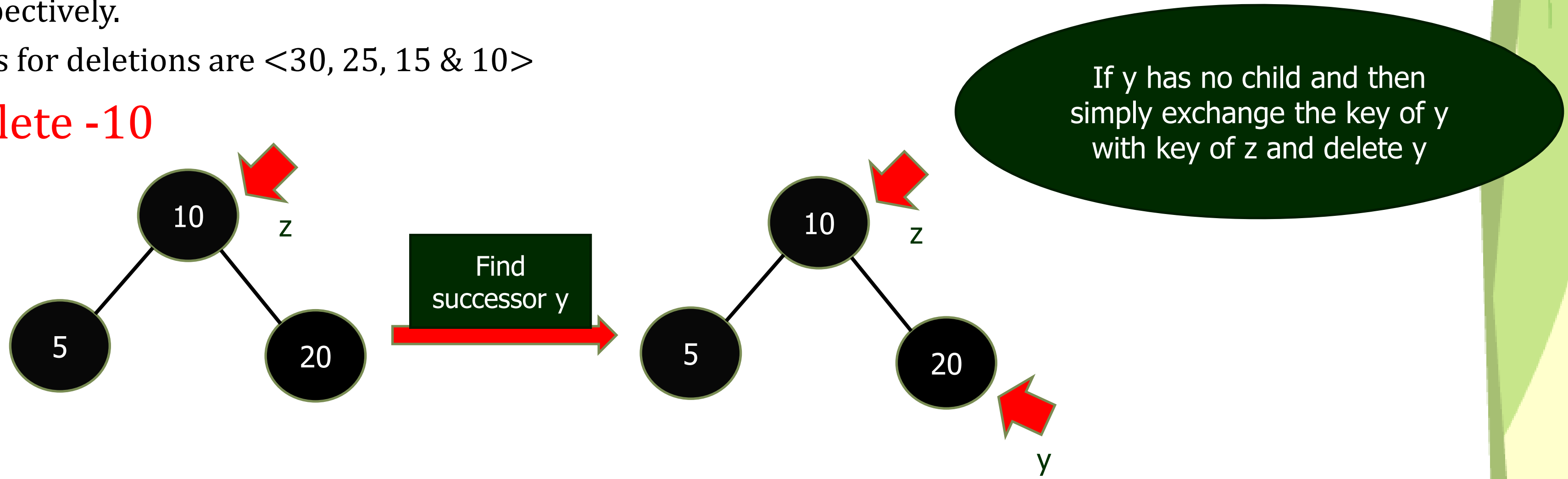
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



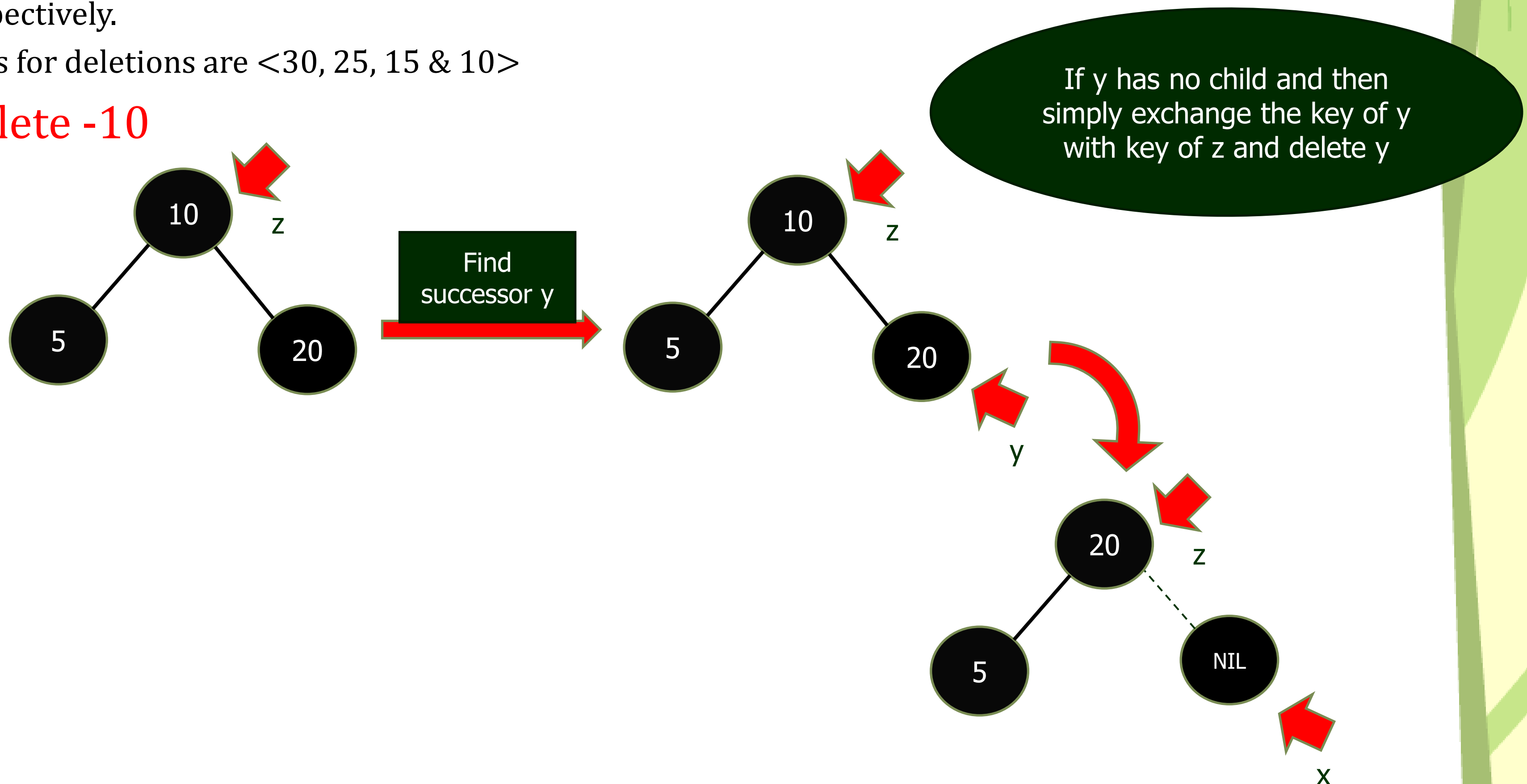
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



Red Black Tree

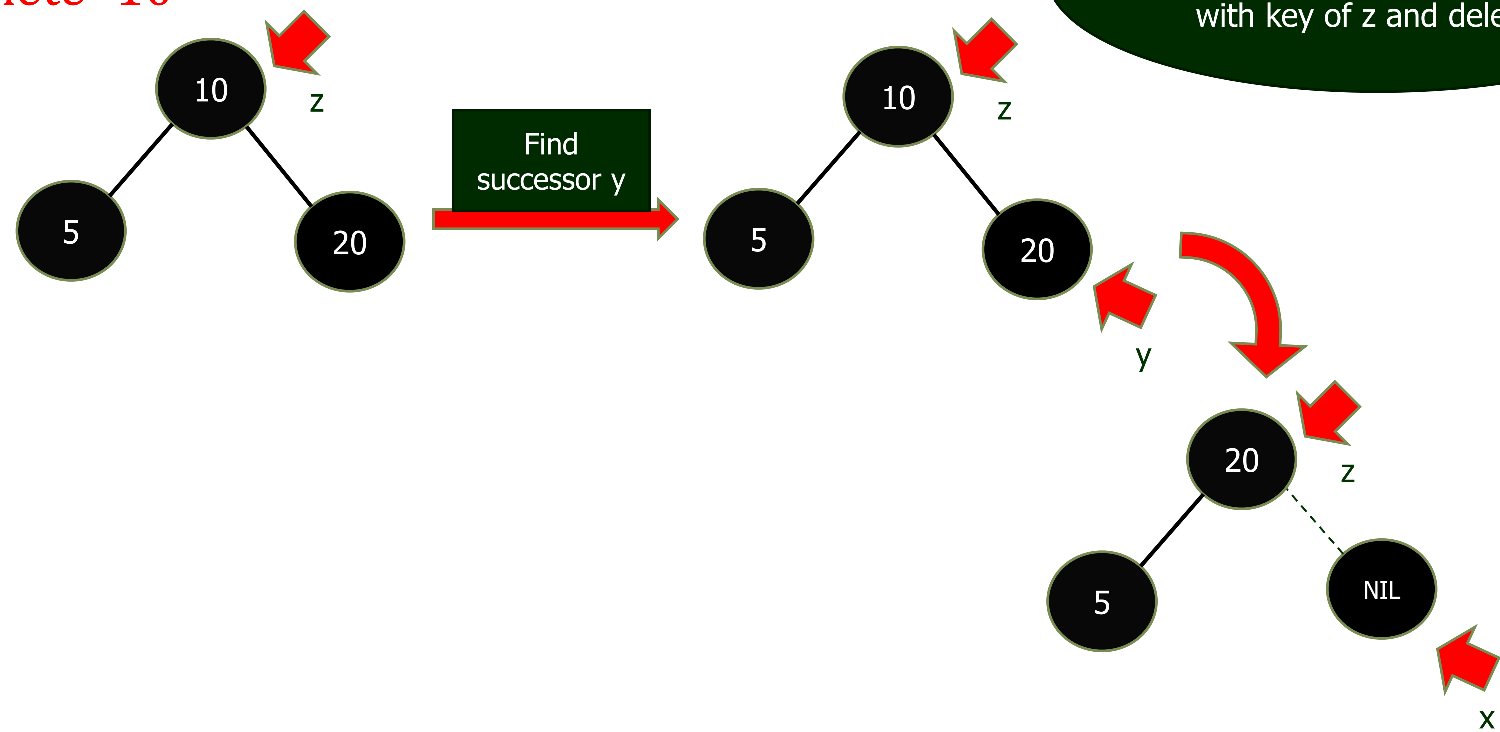
Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10

If y has no child and then simply exchange the key of y with key of z and delete y



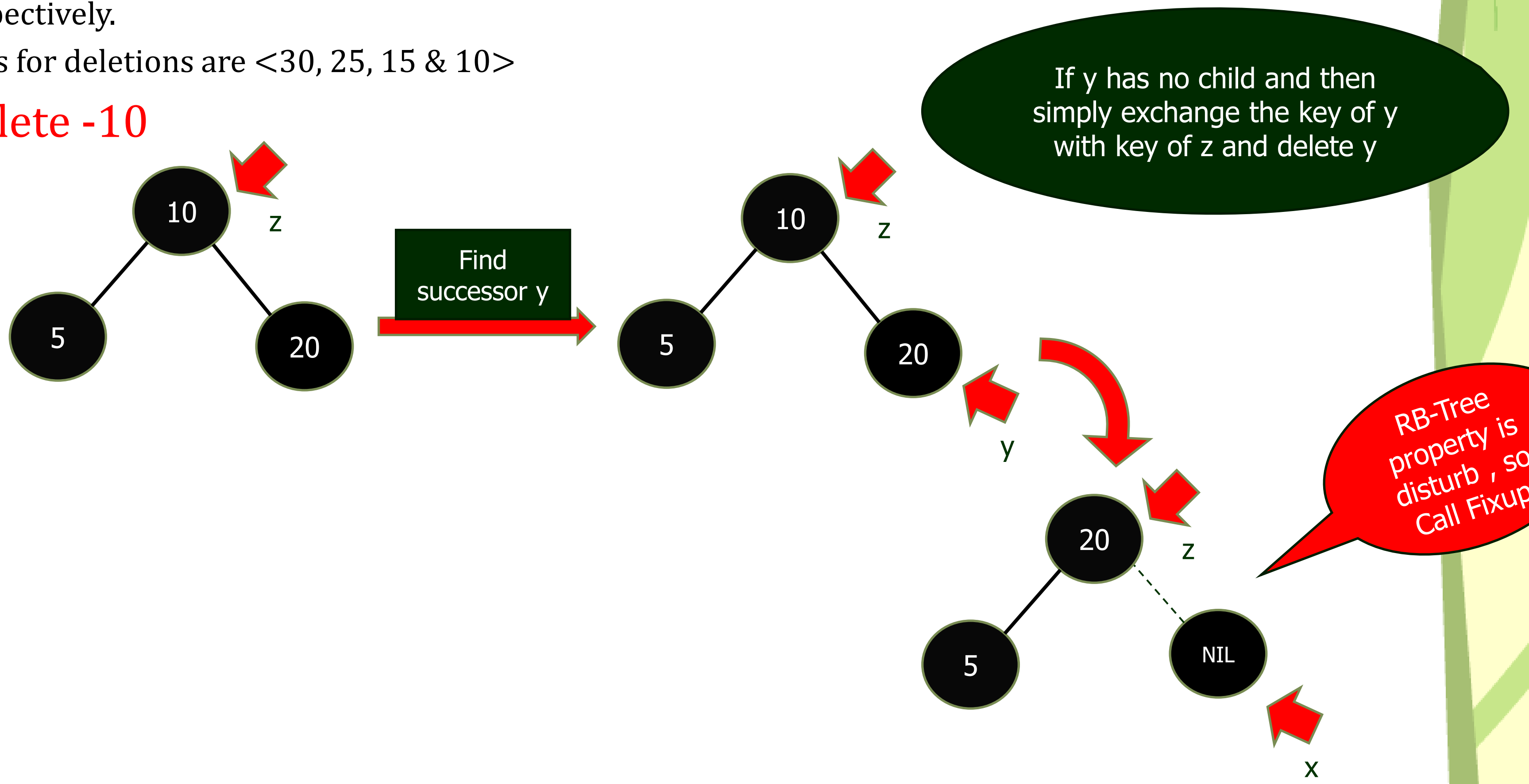
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



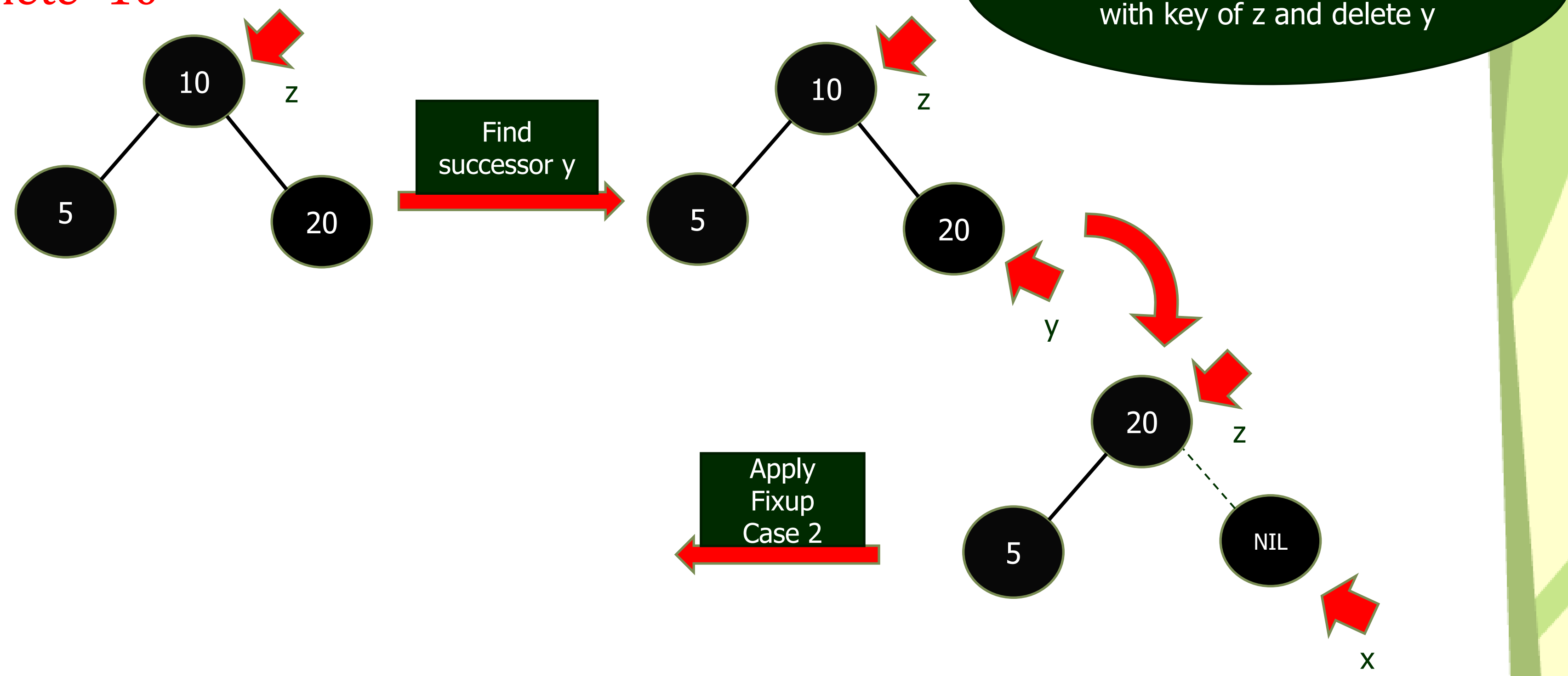
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



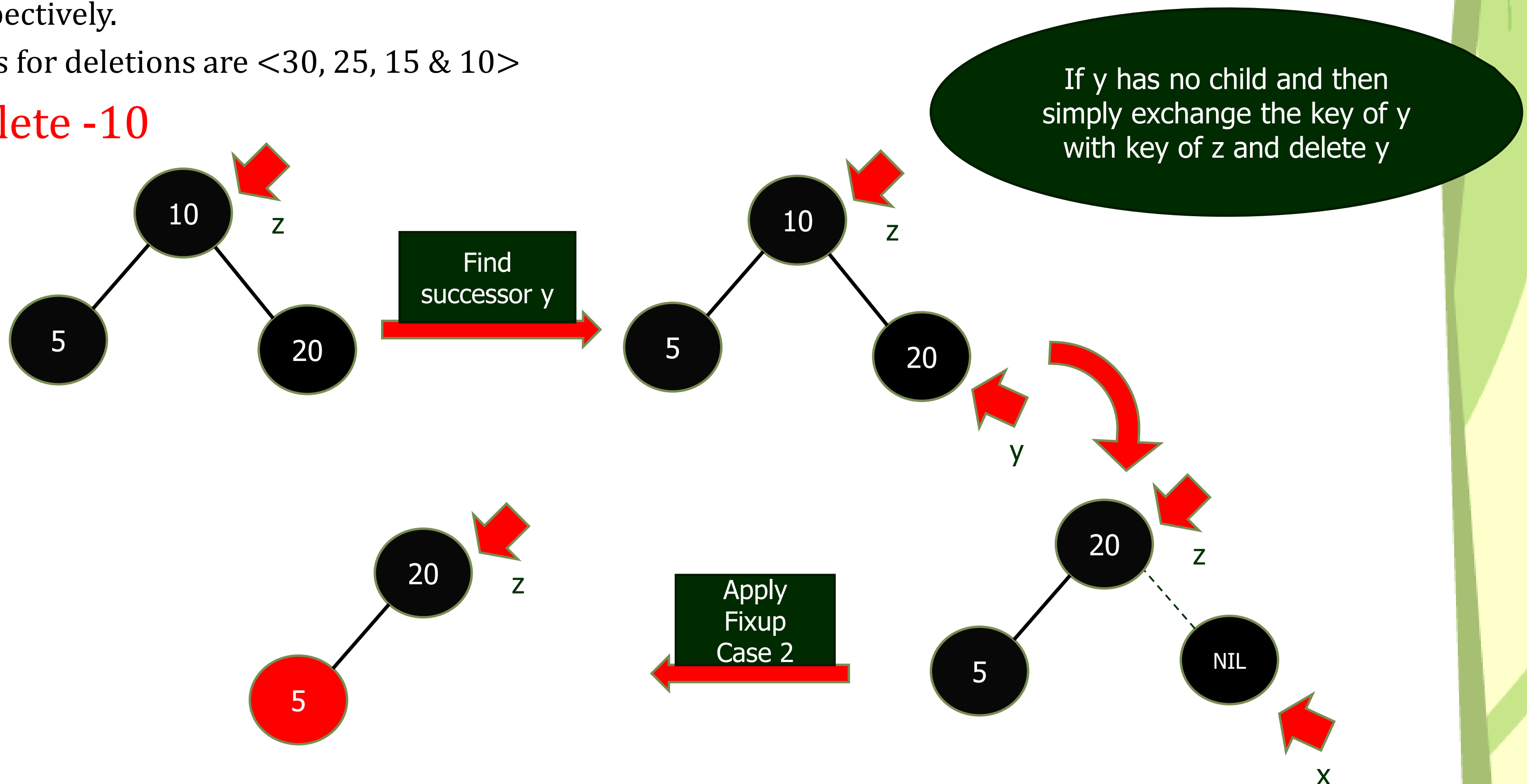
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10



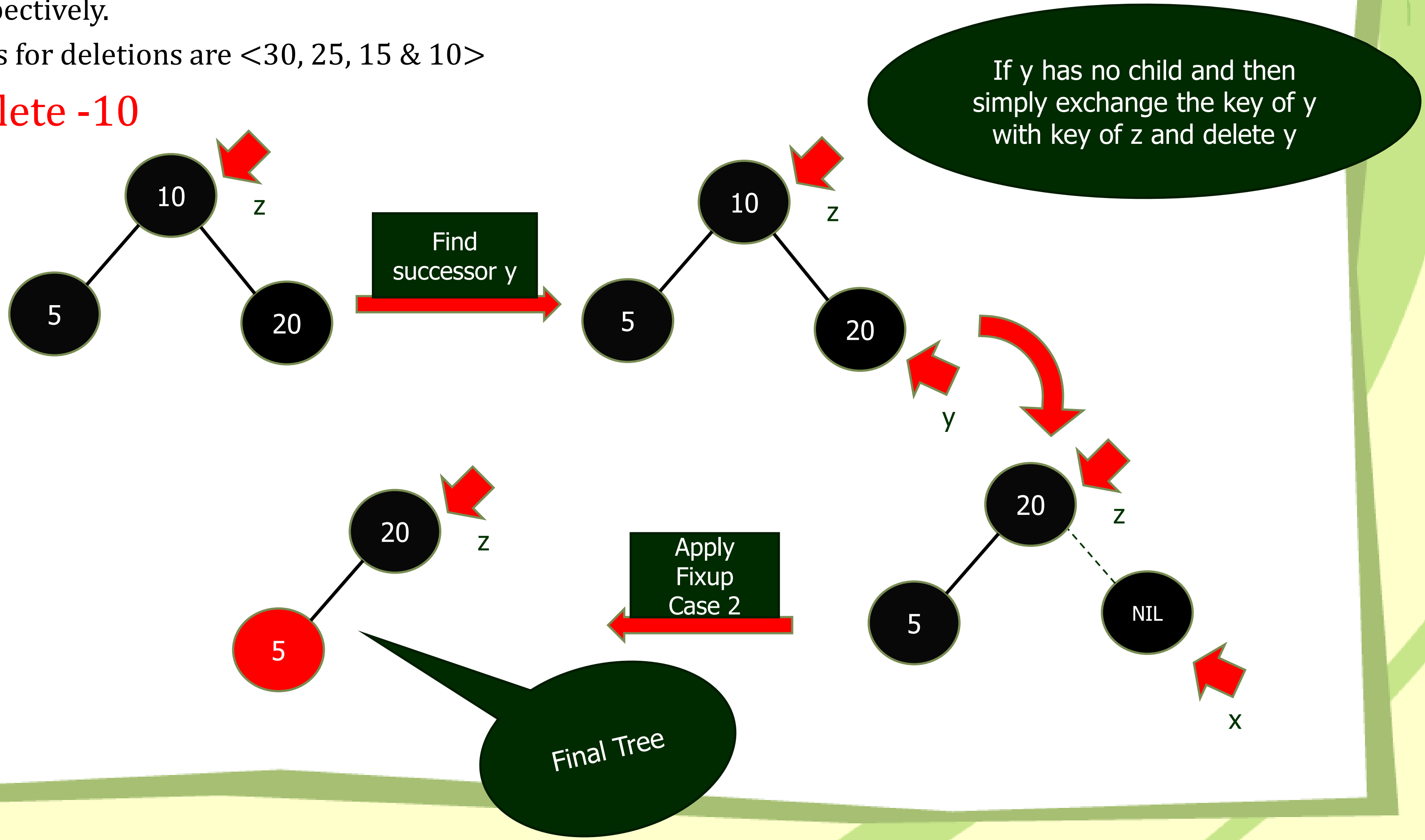
Red Black Tree

Insertion (Example 2):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <30, 25, 15 & 10>

Delete -10

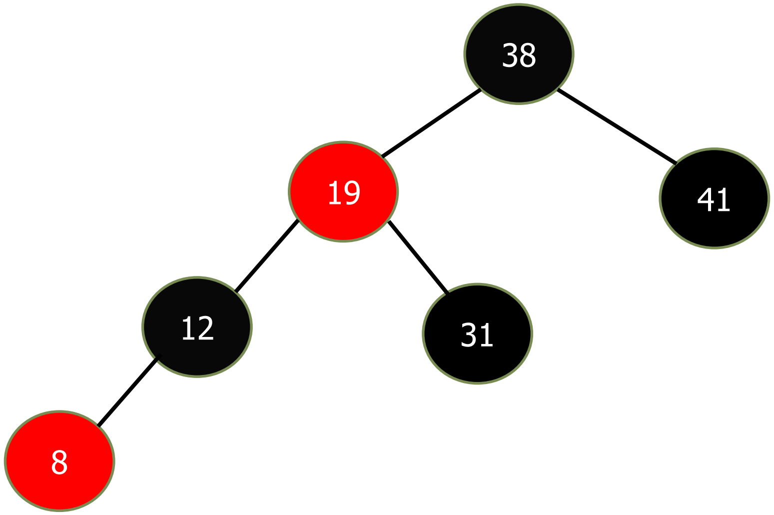


Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>



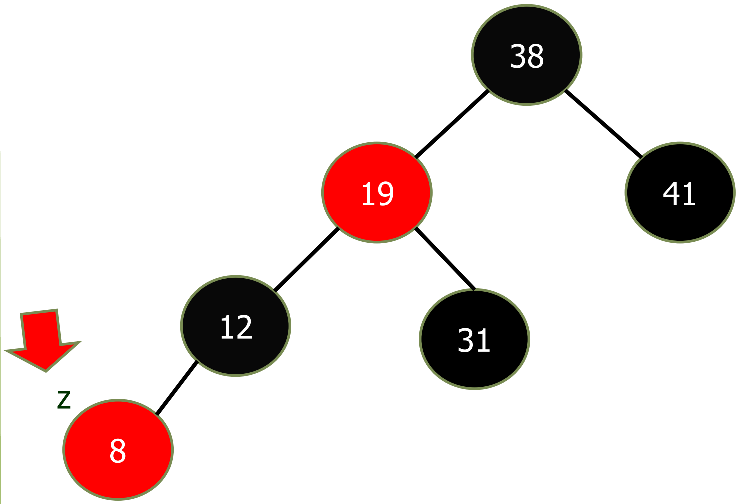
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -8



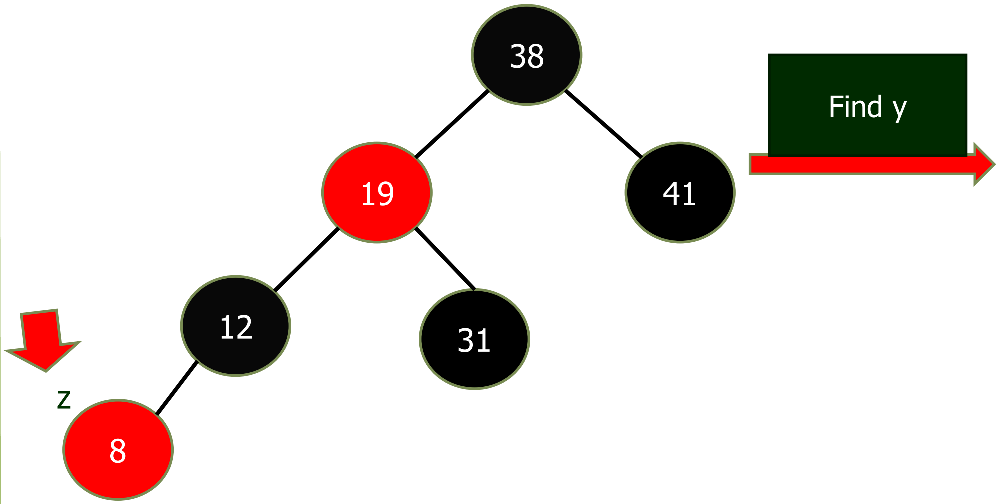
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -8



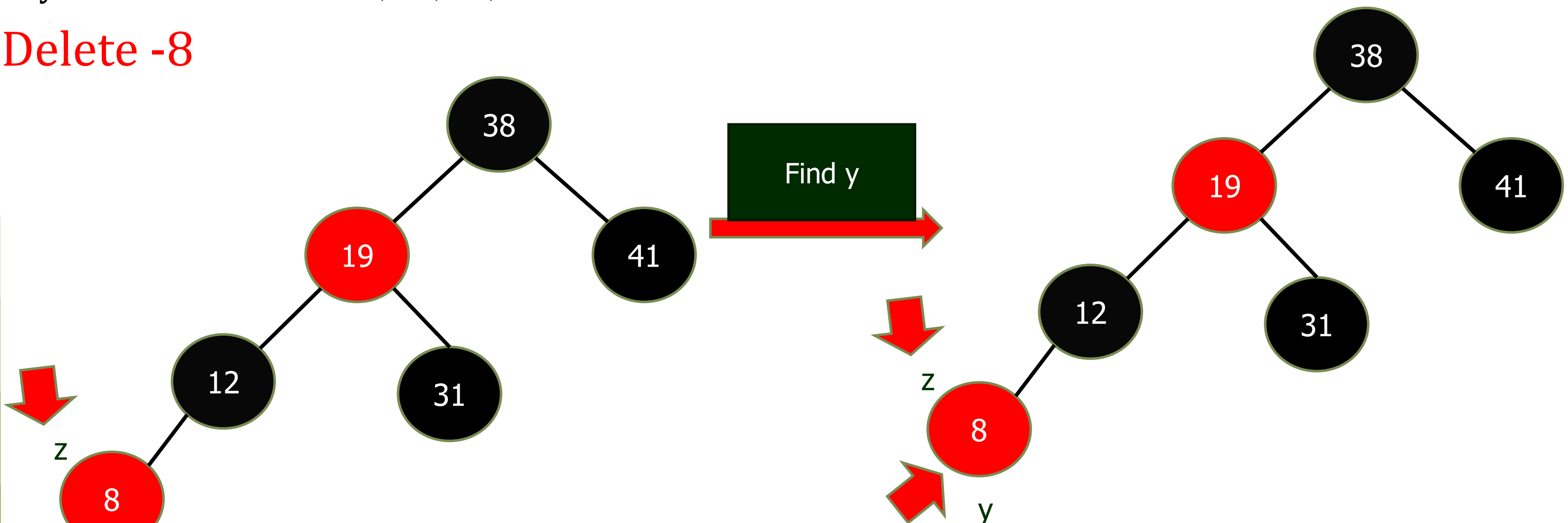
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -8



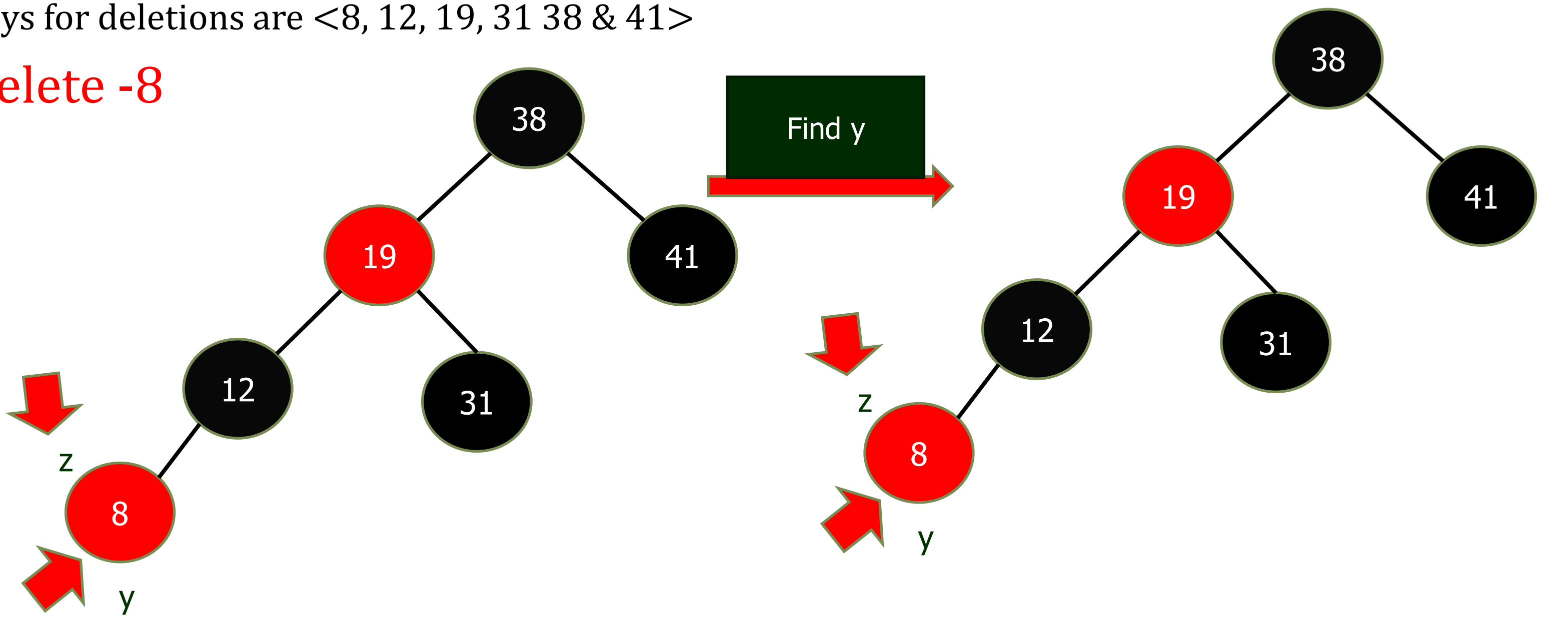
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -8



Red Black Tree

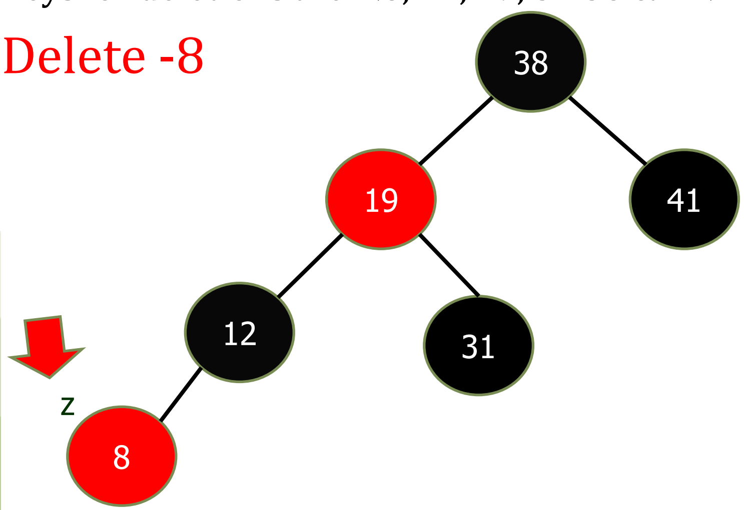
If z has no child the y=z and simply delete z

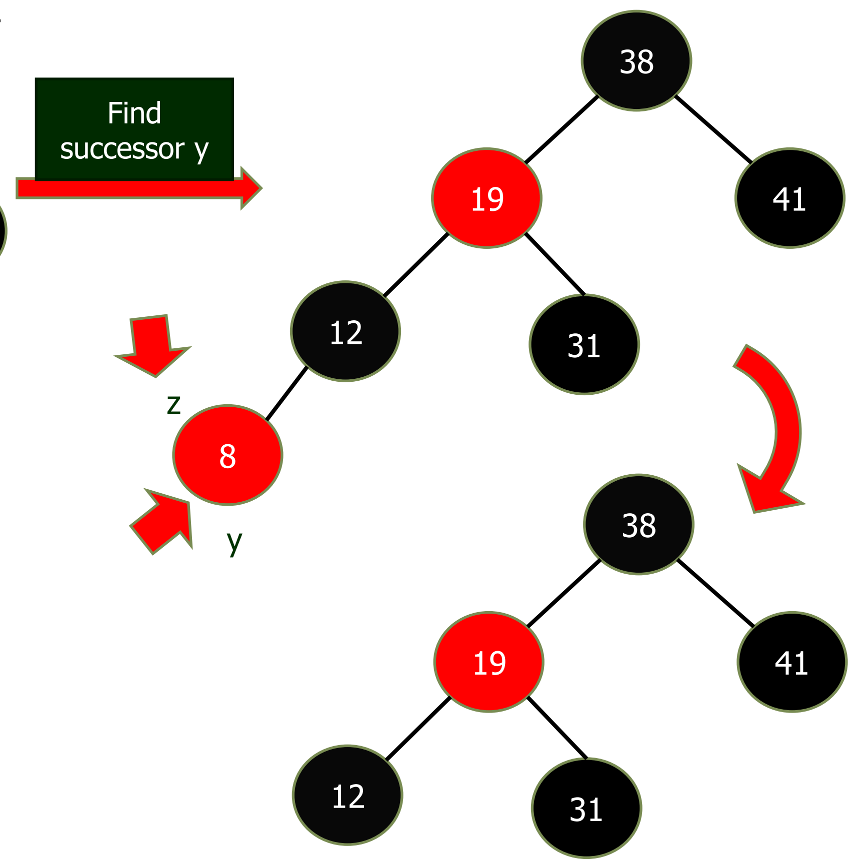
Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -8





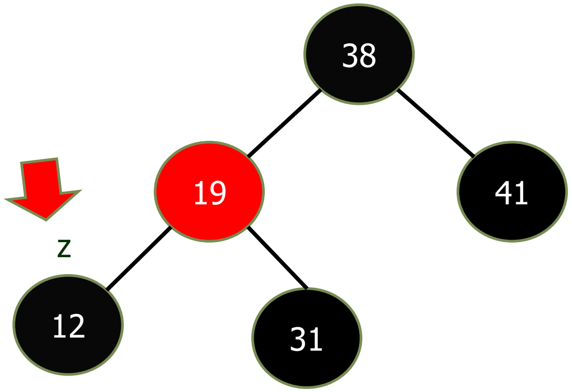
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12



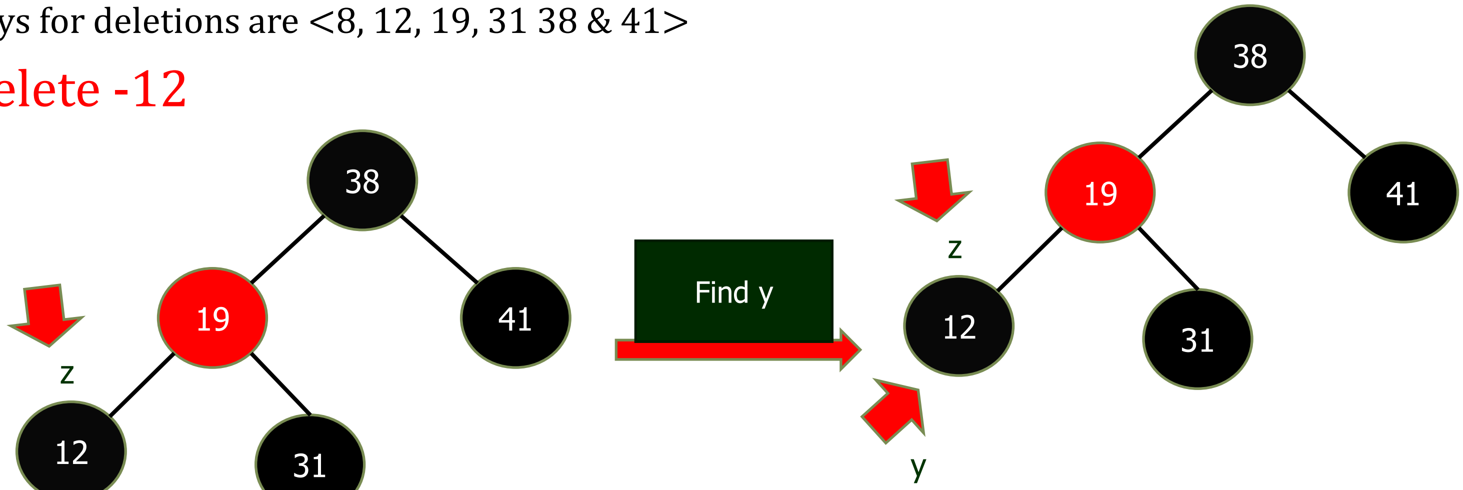
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12



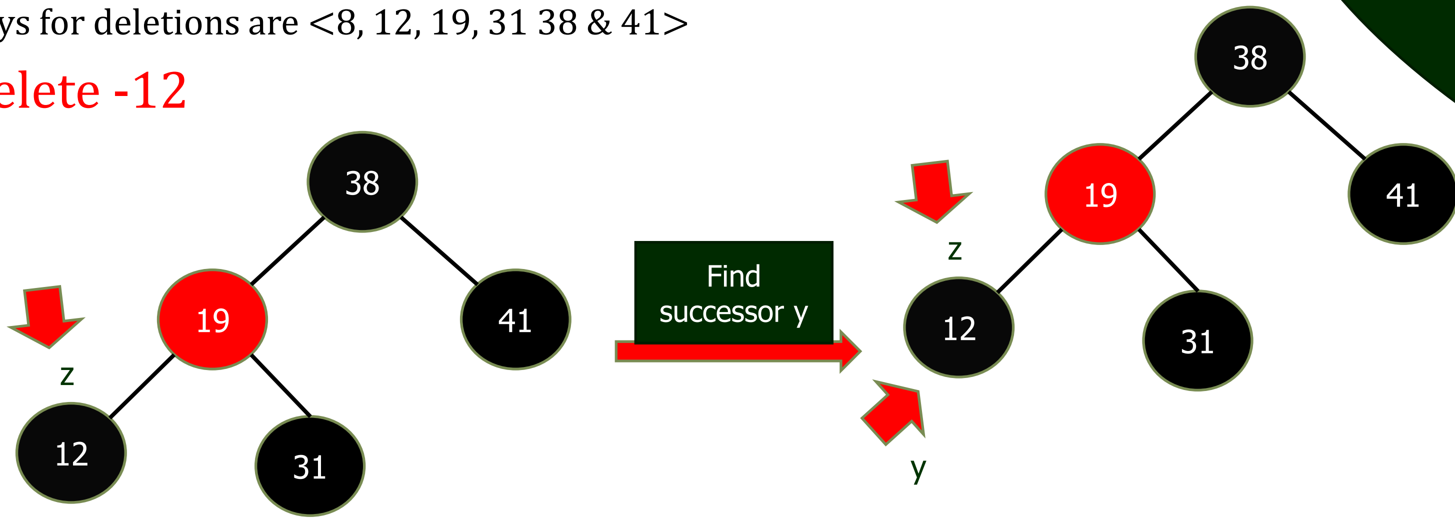
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the give respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12



If z has no child the y=z and simply delete z

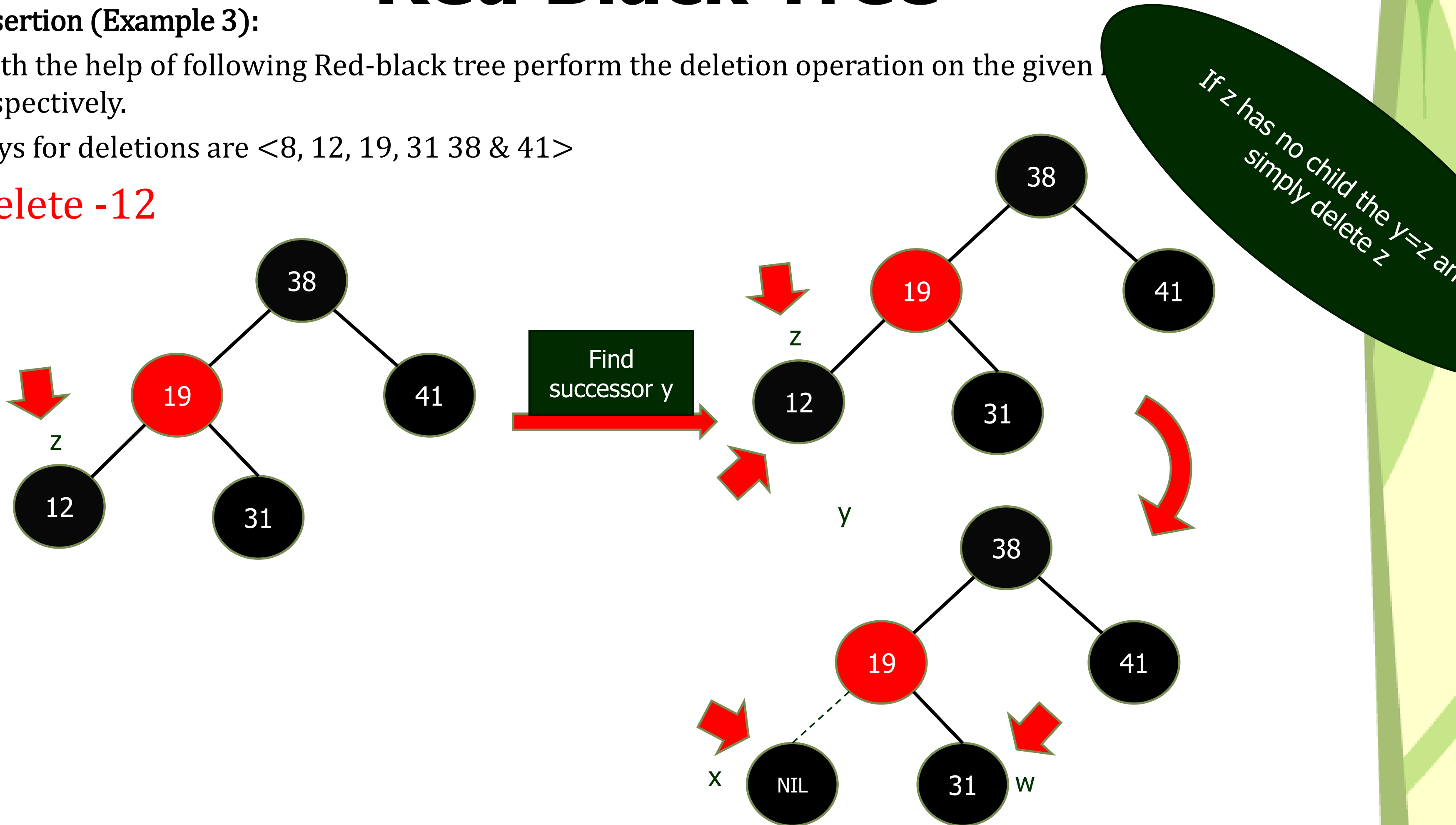
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given k
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12



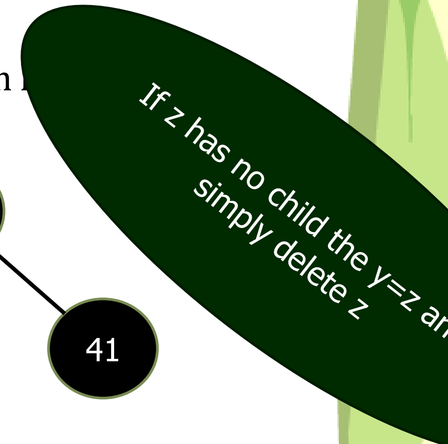
Red Black Tree

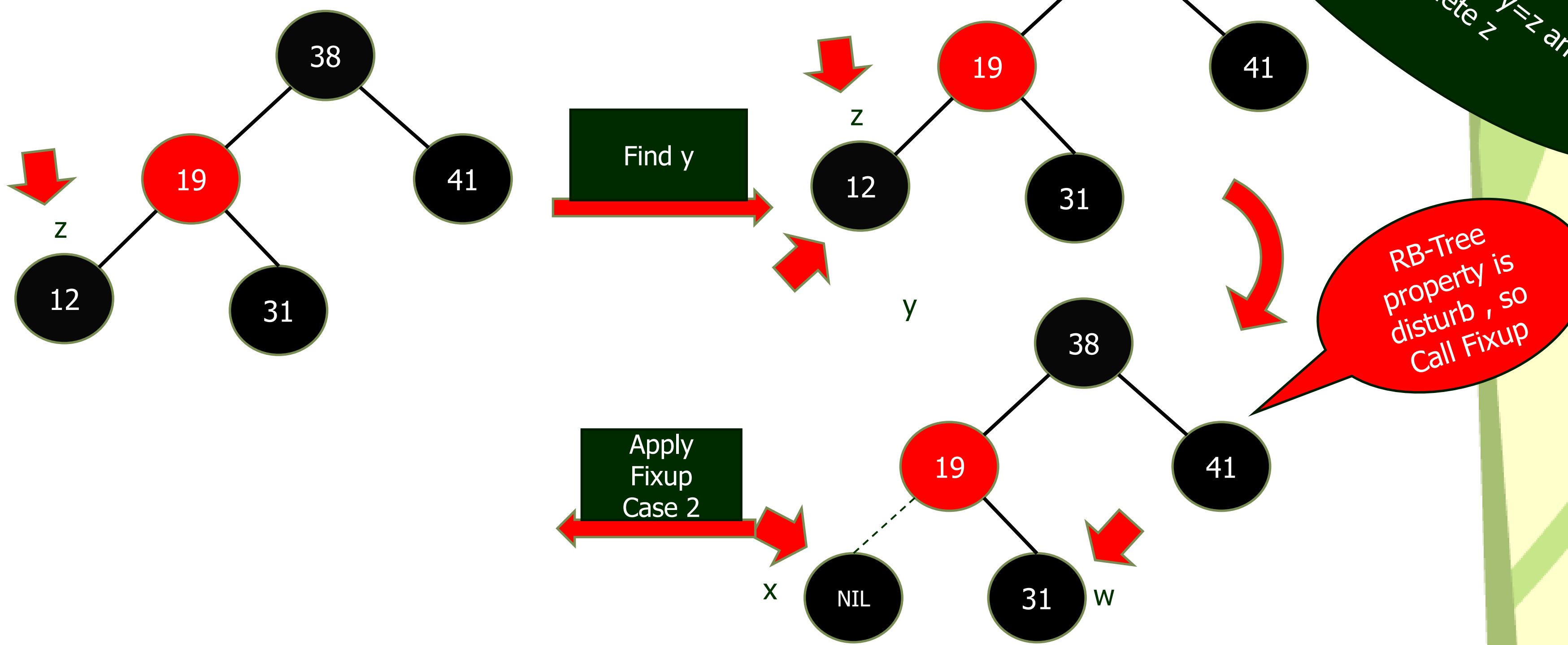
Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12





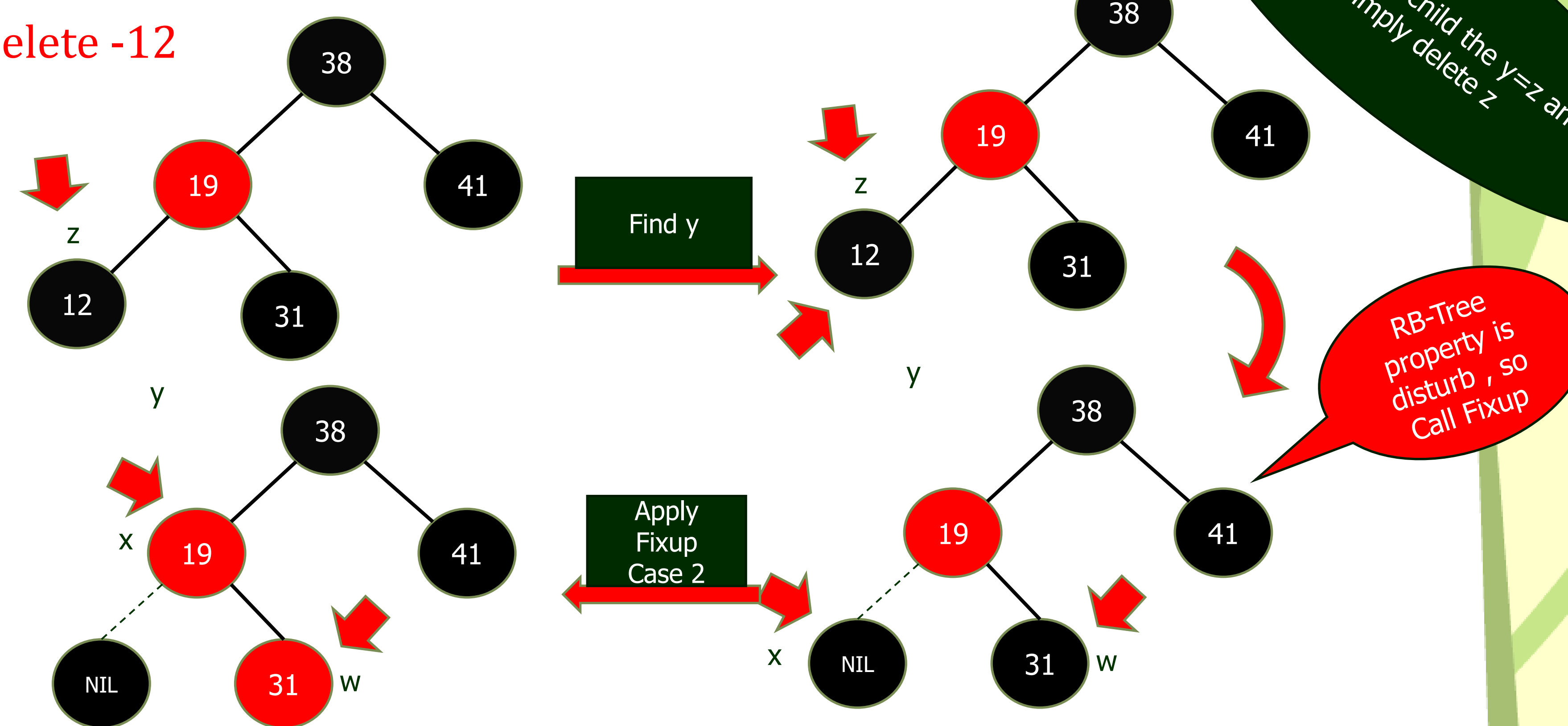
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12



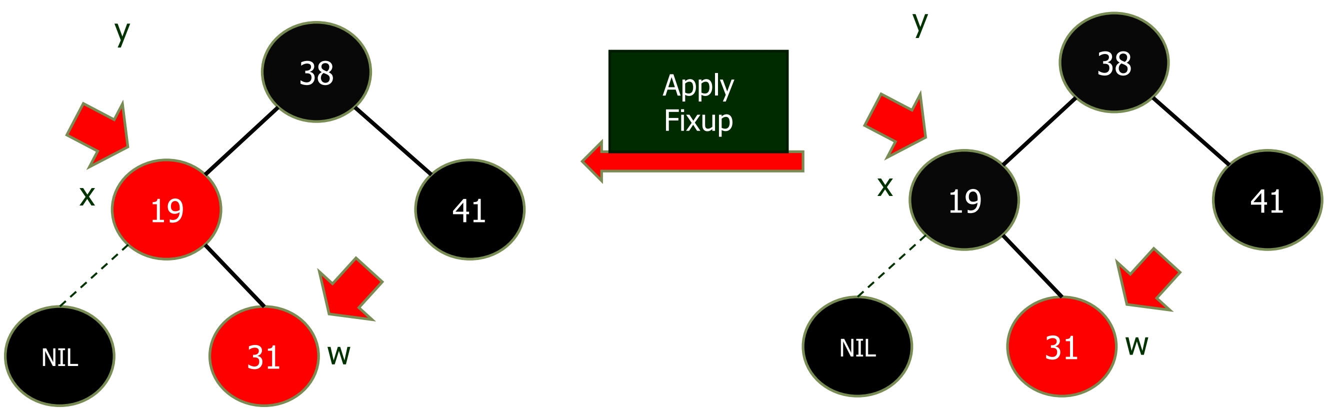
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -12



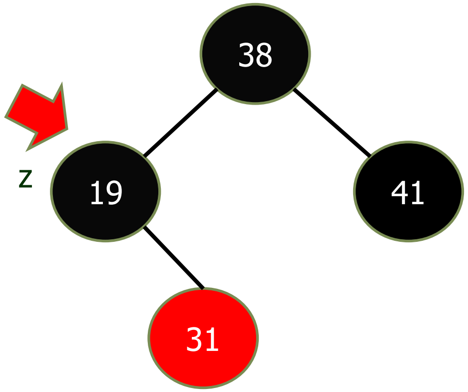
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -19



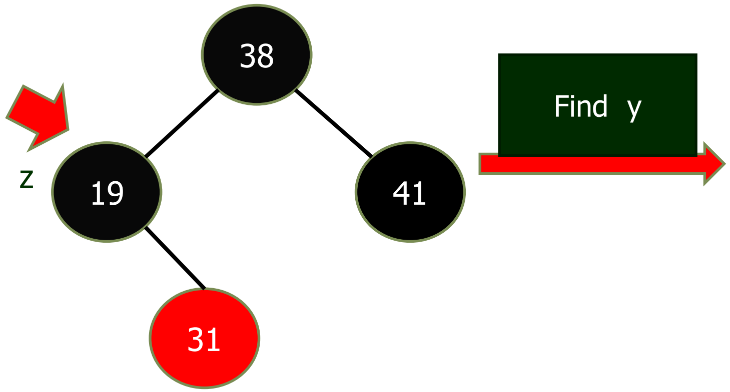
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -19



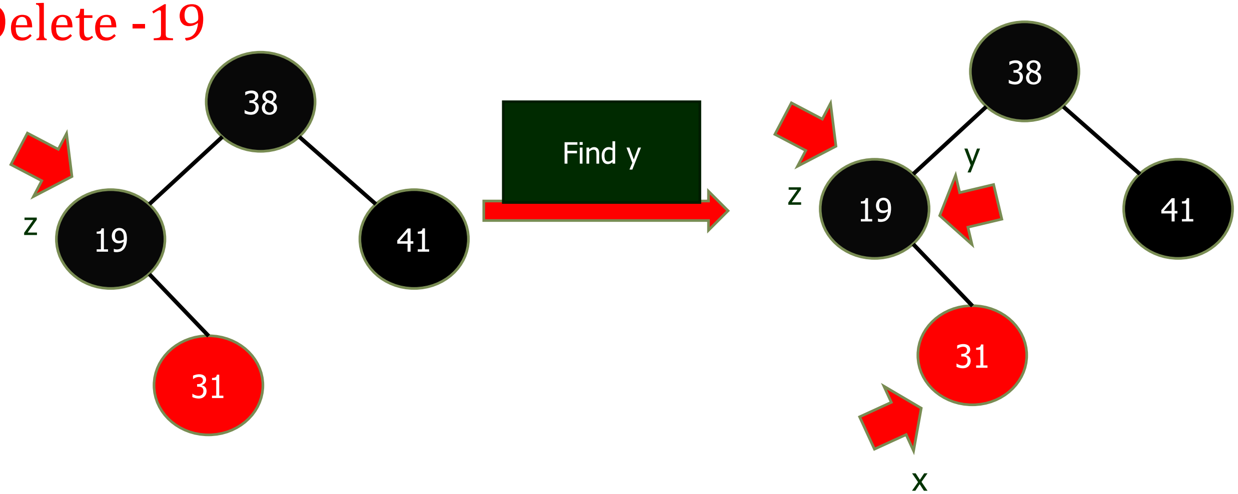
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -19



If z has one child then y=z and apply BST Deletion

Red Black Tree

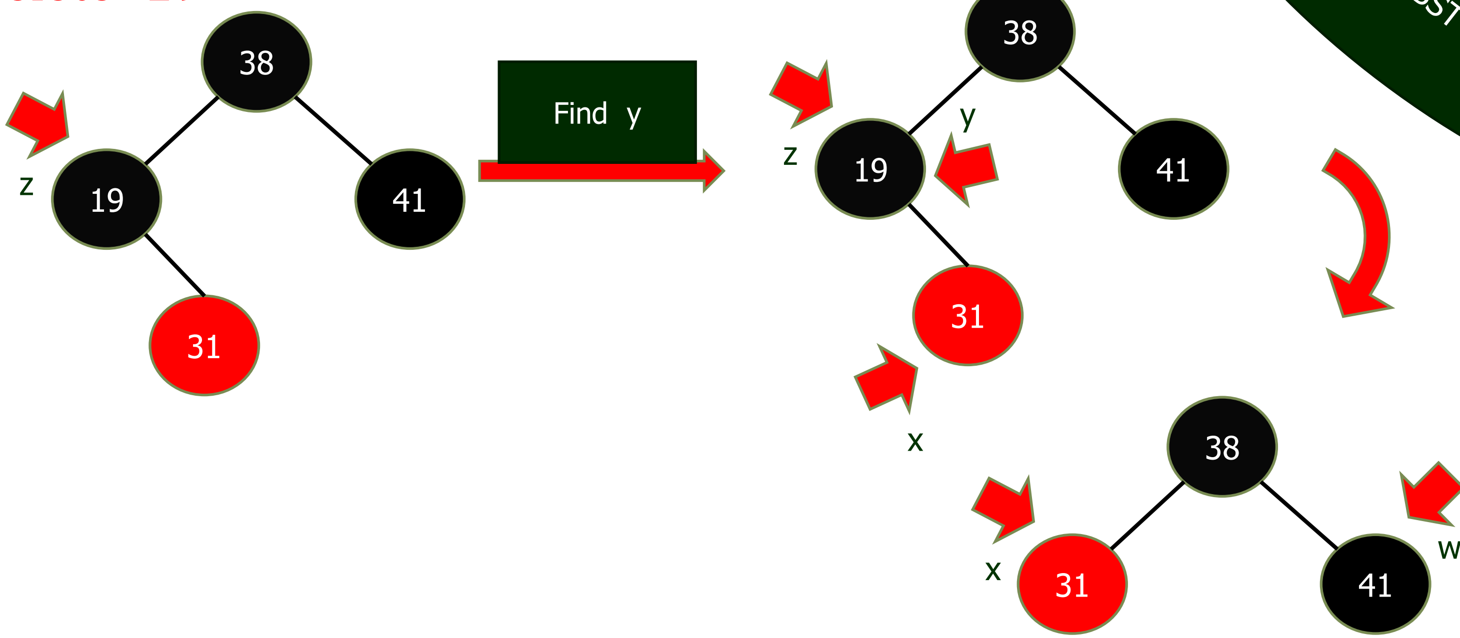
Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

If z has one child then y=z and apply BST Deletion

Delete -19



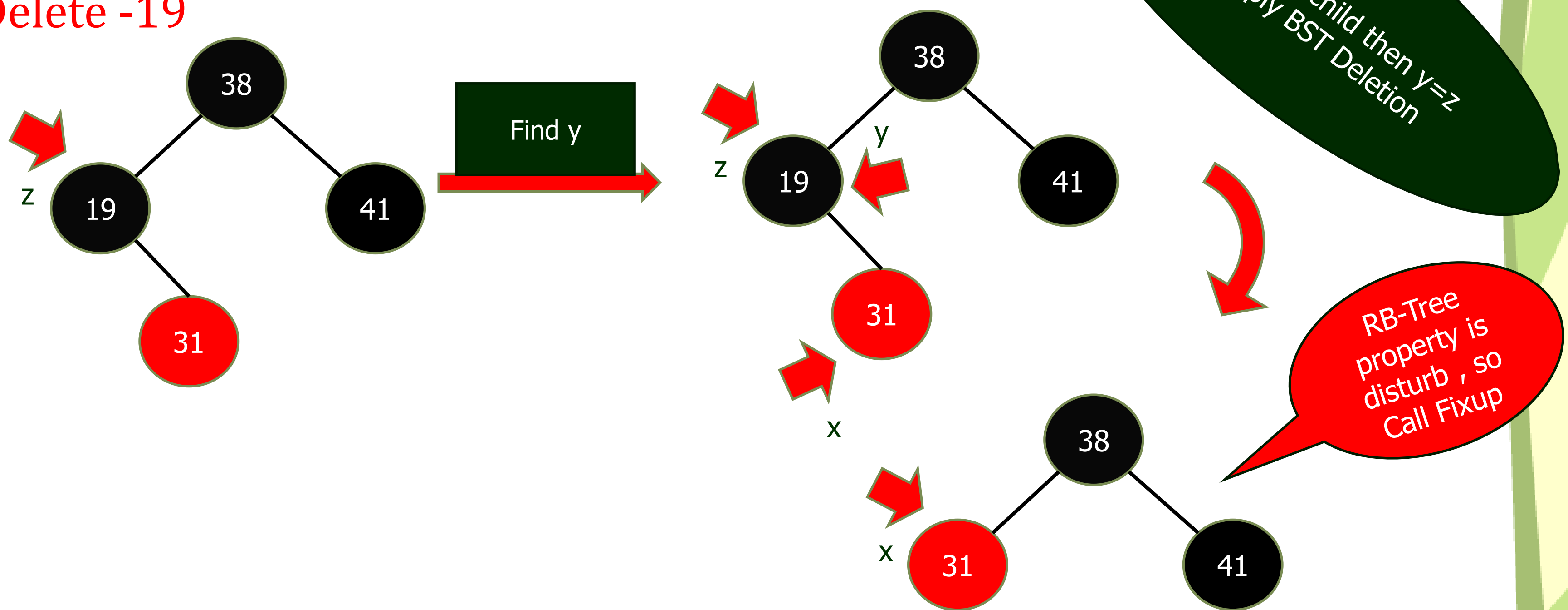
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -19



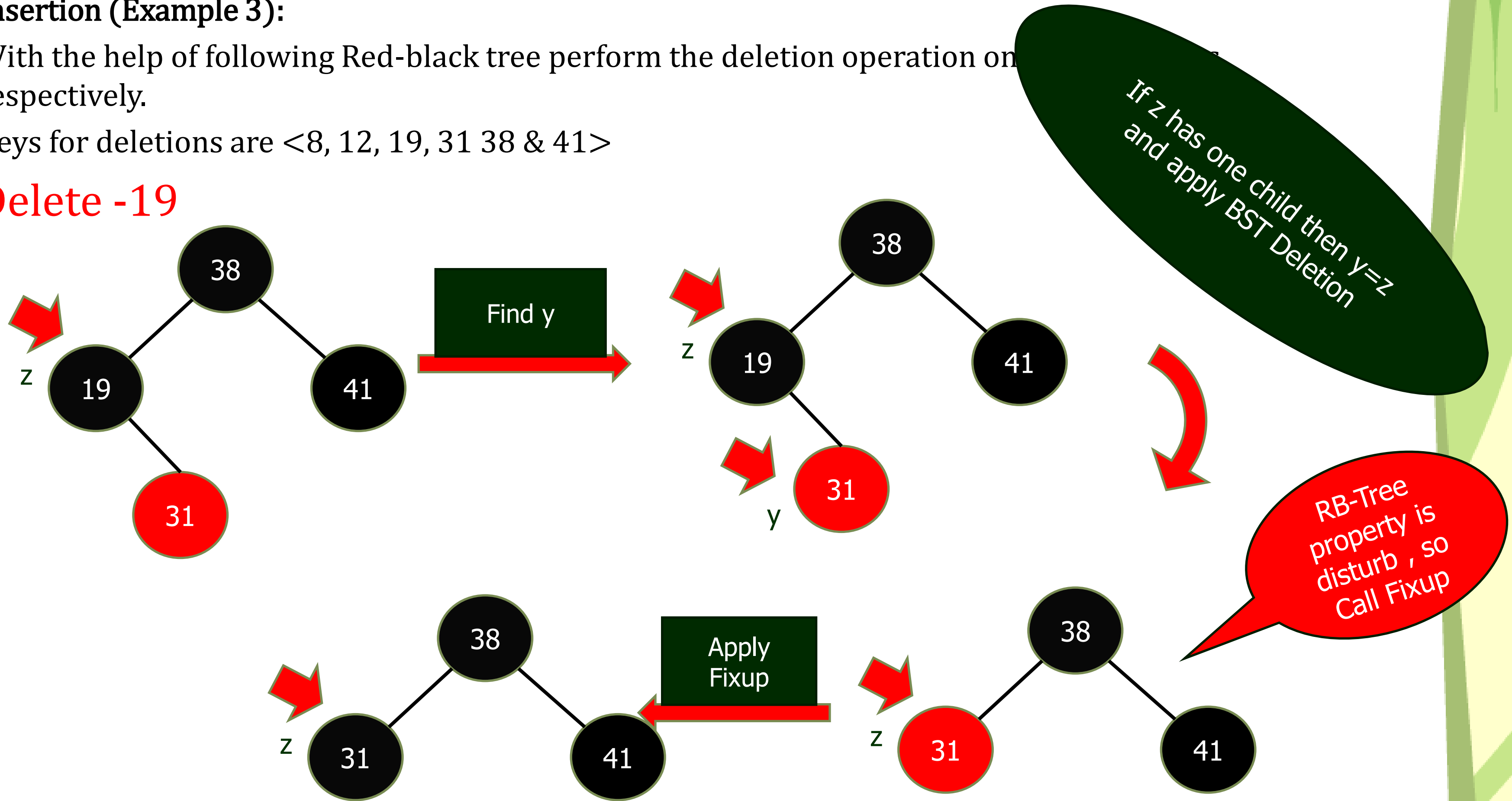
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -19



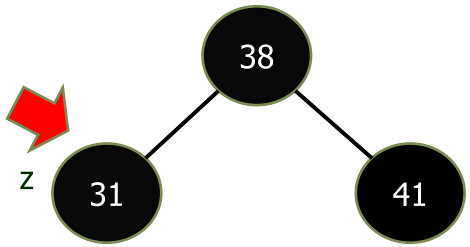
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31



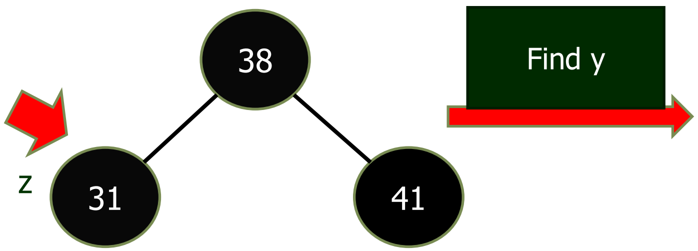
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31



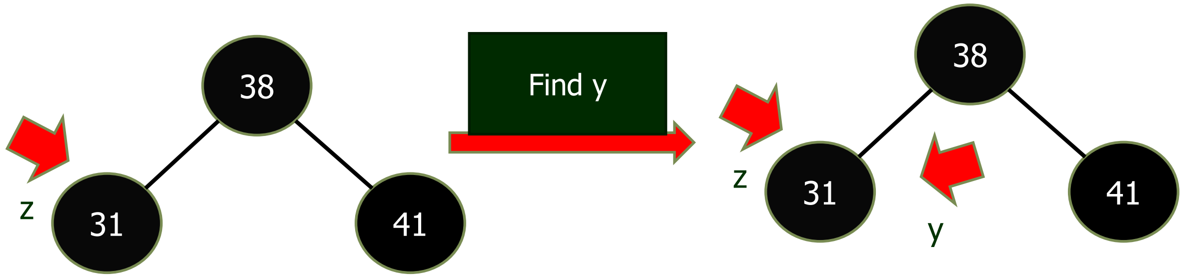
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31



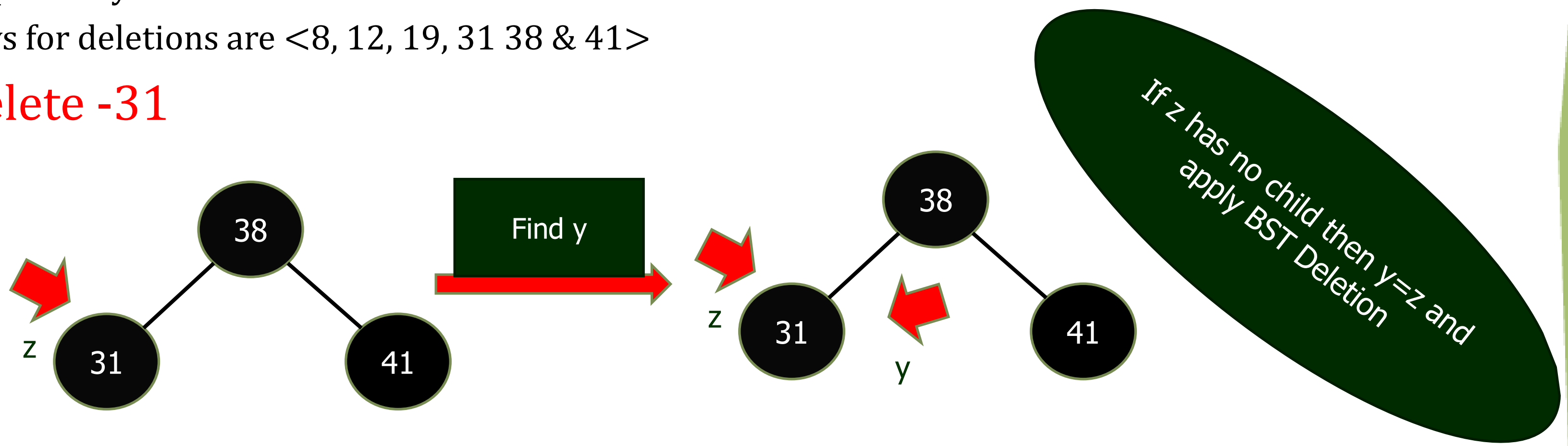
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31



Red Black Tree

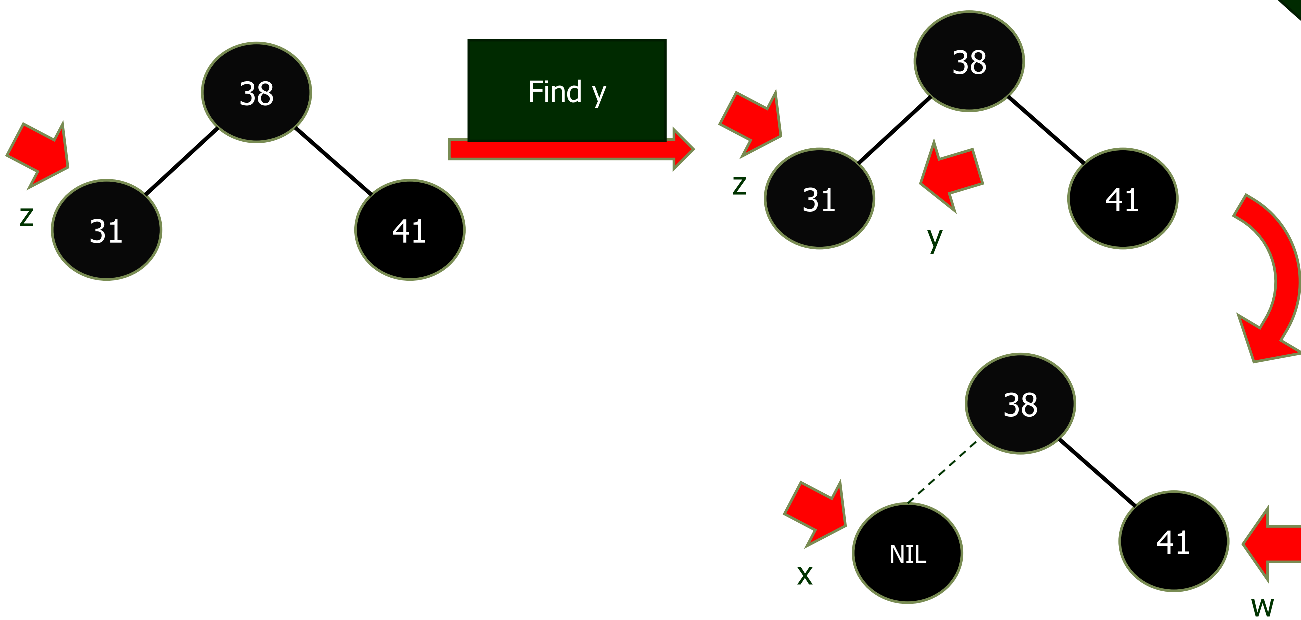
Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31

If z has no child then y=z and apply BST Deletion



Red Black Tree

Insertion (Example 3):

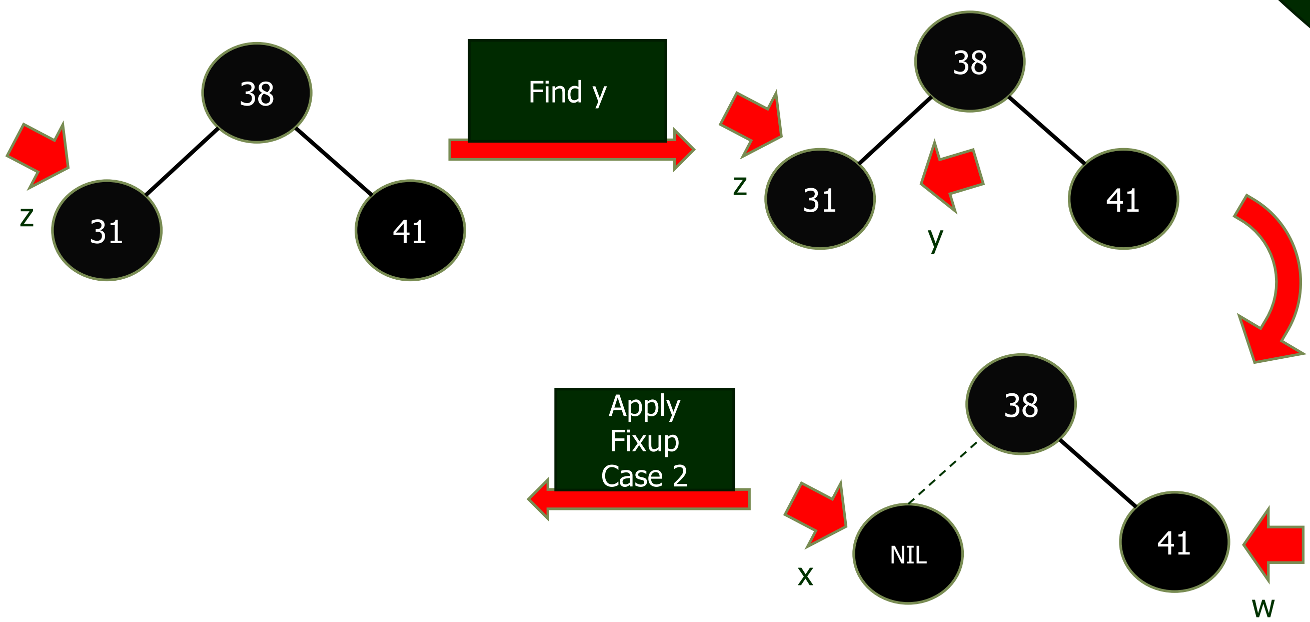
With the help of following Red-black tree perform the deletion operation on

respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31

If z has no child then y=z and apply BST Deletion



Red Black Tree

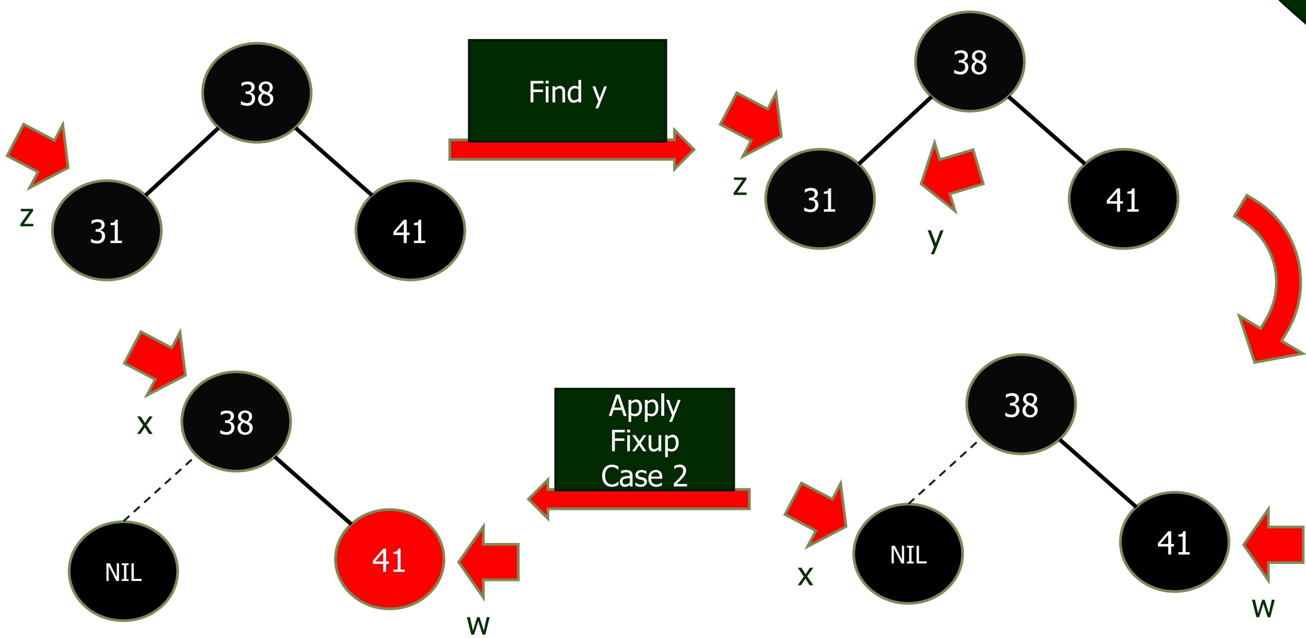
Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on
respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -31

If z has no child then y=z and apply BST Deletion



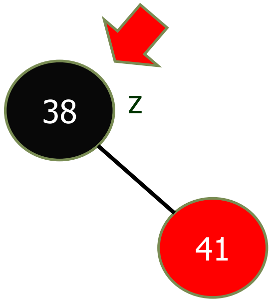
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -38



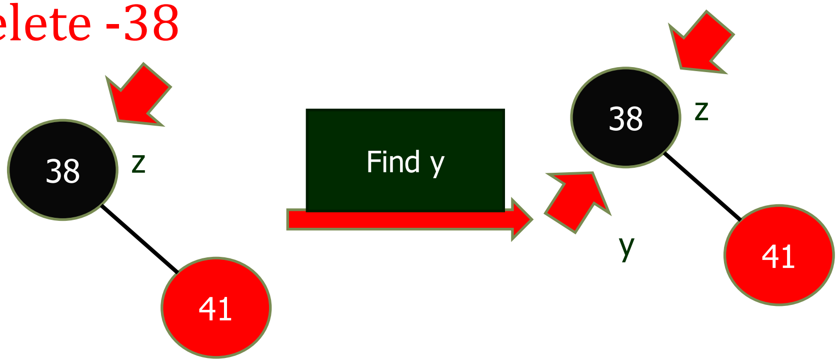
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -38



If z has no child then y=z and apply BST Deletion

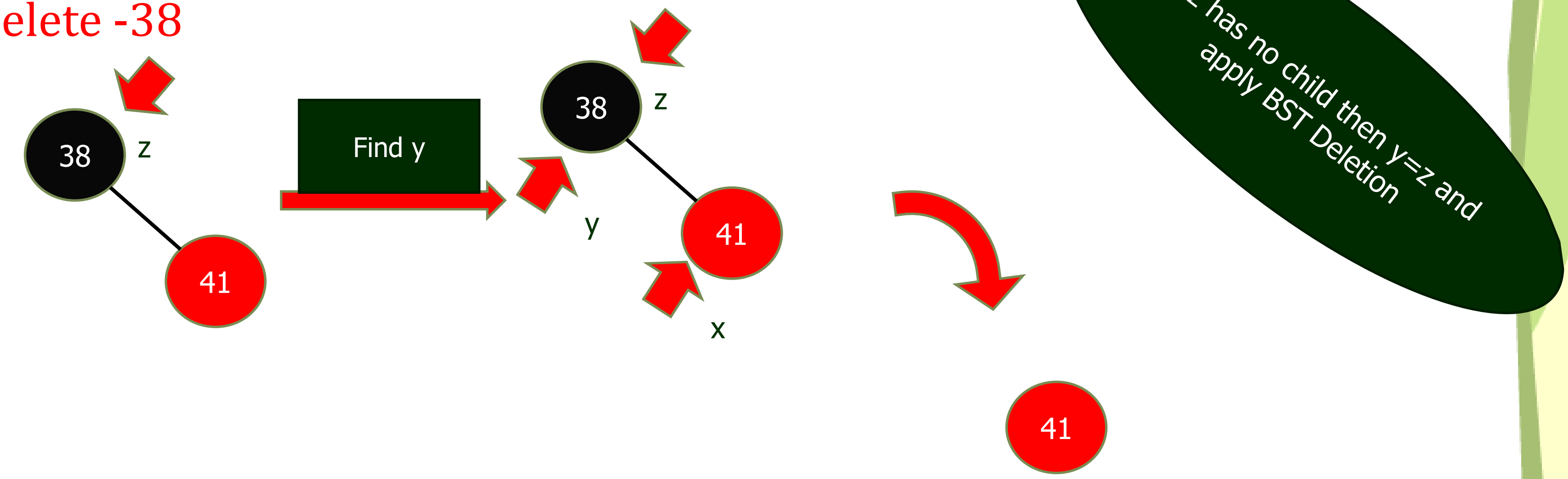
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -38



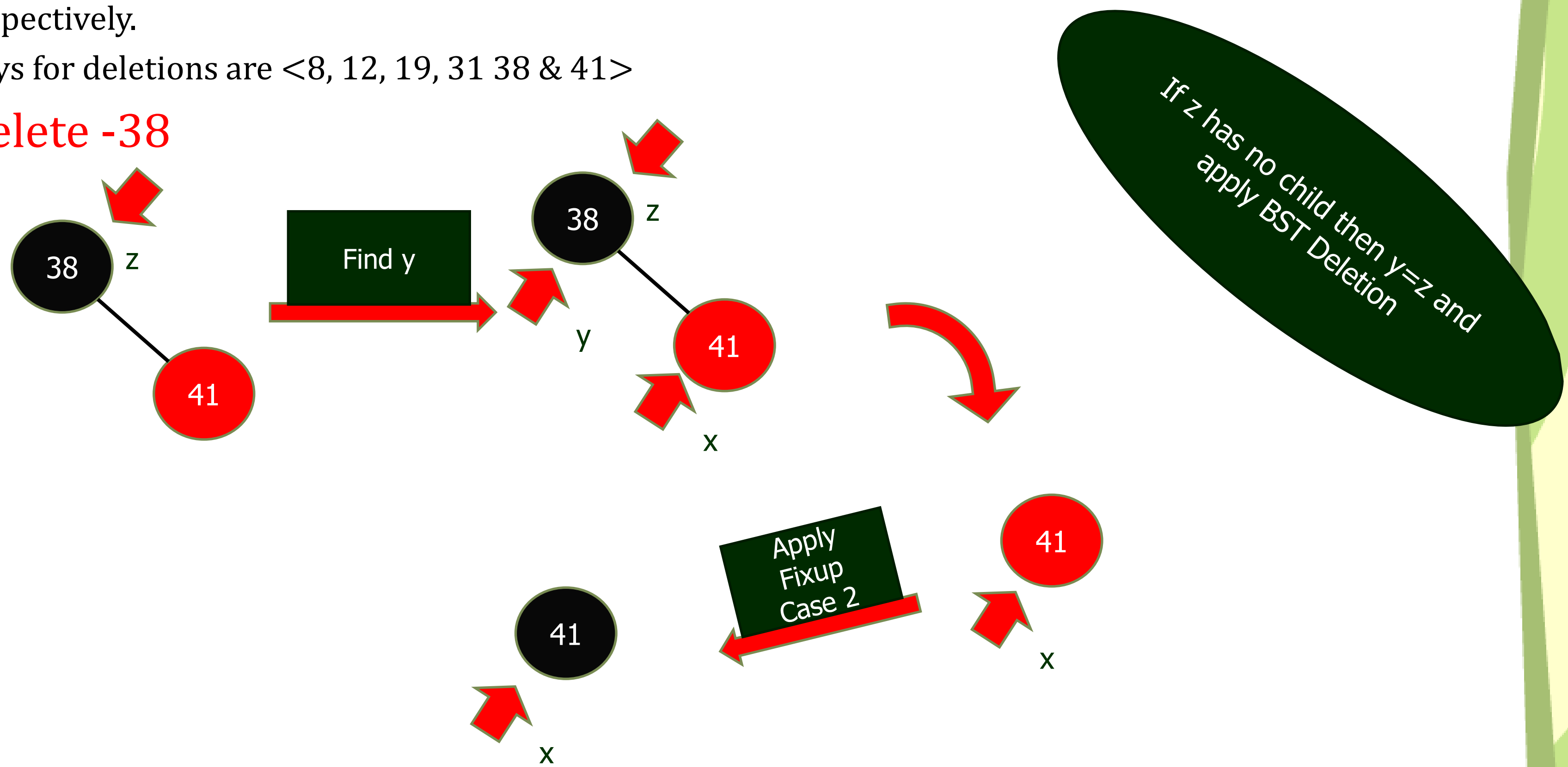
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -38



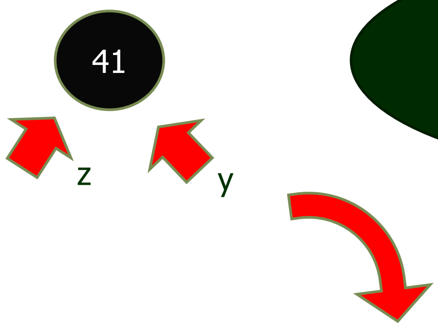
Red Black Tree

Insertion (Example 3):

With the help of following Red-black tree perform the deletion operation on the given keys respectively.

Keys for deletions are <8, 12, 19, 31 38 & 41>

Delete -41



If z has no child then y=z and apply BST Deletion

Tree is Empty

Red Black Tree

Lemma

A red-black tree with n internal nodes has $\boldsymbol{height} \leq \mathbf{2}\lg(\boldsymbol{n} + \mathbf{1})$.

Proof:

The Proof is based on two number of claims:

- Claim 1: Any node with height h has black-height $\geq$ h/2.
- Claim 2: The subtree rooted at any node x contains $\geq 2^{bh(x)} - 1$ internal nodes.

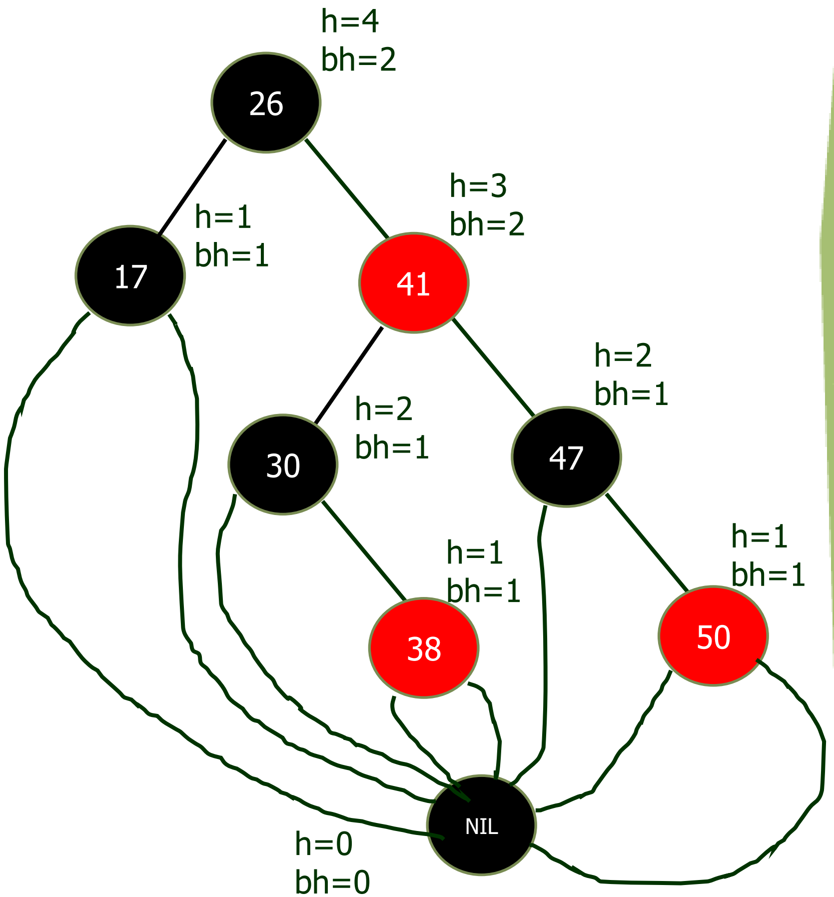
Red Black Tree

- Claim 1: Any node with height h has black-height ≥ h/2.

According to property 4(i.e. If a node is red, then both its children are black) , at least half of the nodes on any simple path from root to leaf node, excluding root node must be black.

Hence, any node(x) with height h has bh(x) ≥ h/2 (i.e. h/2 number of black node.)

■proved



Red Black Tree

Claim 2:

The subtree rooted at any node x contains at least $2^{bh(x)} - 1$ internal nodes (i.e. the nodes who bearing keys only).

Proof by induction method on height of x.

Base case:

When x is a leaf node, h(x)=0 and bh(x)=0.

Hence the internal node of x is $= 2^{bh(x)} - 1$

$= 2^0 - 1$

=1-1=0

Red Black Tree

<u>Induction hypothesis:</u>

Consider a node x that has a positive height and is an internal node with two children of left and right.

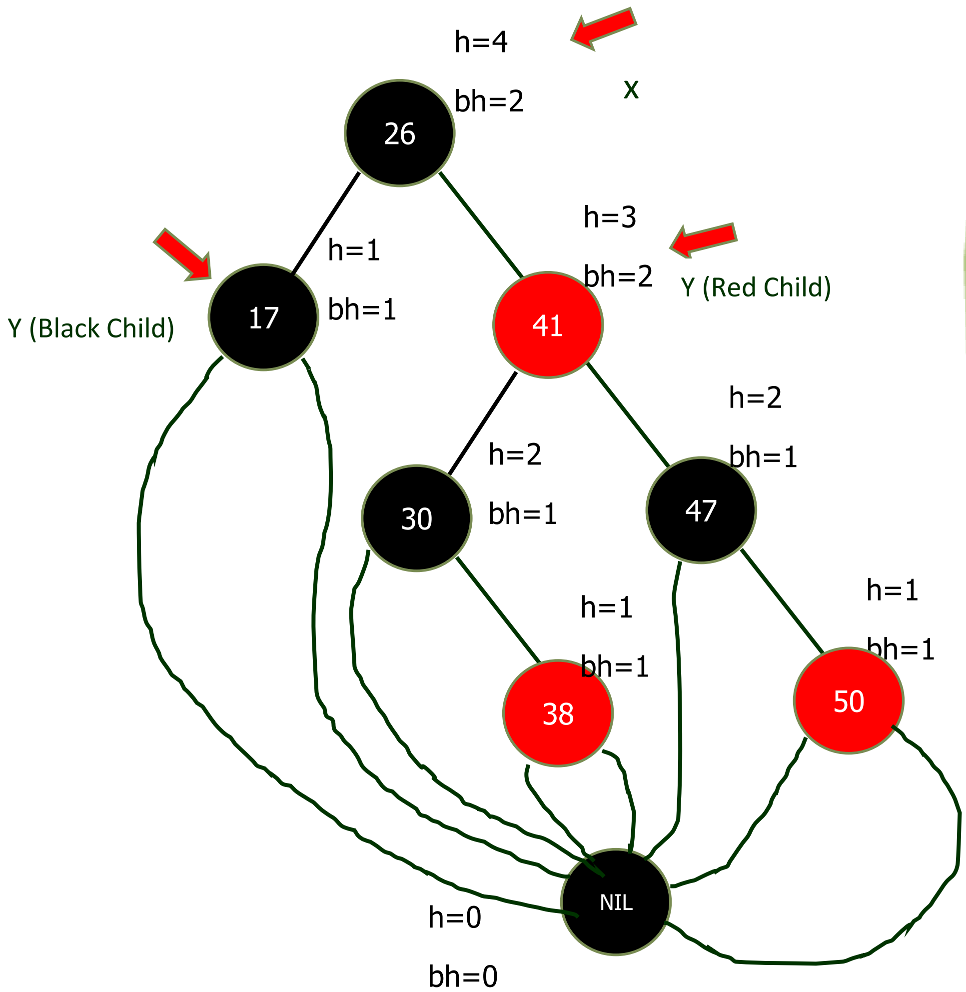
Each children has a black height of either bh(x) or bh(x)-1 depending on weather it's color is red or black respectively.

For Example :

Let bh(x)=b,

Then bh(y)=b if y is a red child of x

bh(y)=b-1 if y is a black child of x.



Red Black Tree

Since ,

$$h\big(child(x)\big) < h(x)$$

By using the inductive hypothesis, we can conclude that each child has at least $2^{bh(x)-1} - 1$ internal node. Thus , the subtree rooted at x contain at least

$$(2^{bh(x)-1}-1) + (2^{bh(x)-1}-1) + 1 = 2^{bh(x)} - 1 \text{ internal nodes.}$$

Which proves the claim 2.

So as per the definition of claim 1 , the bh(root) must be at least h/2 .

Hence $\Rightarrow n \geq 2^{bh(x)} - 1$

$\Rightarrow n \geq 2^{h/2} - 1$ (as bh(x)=h/2) [claim 1]

$\Rightarrow n+1 \geq 2^{h/2}$

Apply log both side

$\Rightarrow \log(n+1) \geq \log 2^{h/2}$

$\Rightarrow \log(n+1) \geq h/2 \log 2$

$\Rightarrow \log(n+1) \geq h/2$

$\Rightarrow h \leq 2\log(n+1)$ ■ Proved

Thank u