Design and Analysis of Algorithm

Recurrence Equation

(Solving Recurrence using Iteration Methods)

Lecture – 10 and 11

Overview

- A ***recurrence*** is a function is defined in terms of
  - one or more base cases, and
  - itself, with smaller arguments.

Examples:

- $T(n) = \begin{cases} 1 & \text{if } n = 1\,, \\ T(n-1) + 1 & \text{if } n > 1\,. \end{cases}$

  Solution: $T(n) = n$.

Linear Decay

- $T(n) = \begin{cases} 1 & \text{if } n = 1\,, \\ 2T(n/2) + n & \text{if } n \geq 1\,. \end{cases}$

  Solution: $T(n) = n \lg n + n$.

Division

- $T(n) = \begin{cases} 0 & \text{if } n = 2\,, \\ T(\sqrt{n}) + 1 & \text{if } n > 2\,. \end{cases}$

  Solution: $T(n) = \lg \lg n$.

Changing Variable

- $T(n) = \begin{cases} 1 & \text{if } n = 1\,, \\ T(n/3) + T(2n/3) + n & \text{if } n > 1\,. \end{cases}$

  Solution: $T(n) = \Theta(n \lg n)$.

Decision Tree

Overview

- Many technical issues:
  - Floors and ceilings

    [Floors and ceilings can easily be removed and don't affect the solution to the recurrence. They are better left to a discrete math course.]
  - Exact vs. asymptotic functions
  - Boundary conditions

Overview

In algorithm analysis, the recurrence and it's solution are expressed by the help of asymptotic notation.

- Example: $T(n) = 2T(n/2) + \Theta(n)$, with solution $T(n) = \Theta(n \lg n)$.
  - The boundary conditions are usually expressed as $T(n) = O(1)$ for sufficiently small n..
  - But when there is a desire of an exact, rather than an asymptotic, solution, the need is to deal with boundary conditions.
  - In practice, just use asymptotics most of the time, and ignore boundary conditions.

Recursive Function

- Example

$A(n)$

$\{$

$\quad If(n > 1)$

$\quad\quad Return(A(n-1))$

$\}$

The relation is called recurrence relation

The Recurrence relation of given function is written as follows.

$$T(n) = T(n-1) + 1$$

Recursive Function

- To solve the Recurrence relation the following methods are used:

  1. **Iteration method**
  2. Recursion-Tree method
  3. Master Method
  4. Substitution Method

Iteration Method(Example 1)

- In Iteration method the basic idea is to expand the recurrence and express it as a summation of terms dependent only on 'n' (i.e. the number of input) and the initial conditions.

Example 1:

Solve the following recurrence relation by using Iteration method.

$$T(n) = \begin{cases} T(n-1) + 1 & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

Iteration Method (Example 1)

It means $T(n) = T(n-1) + 1 \quad if\ n > 1$

$and\ \ T(n) = 1\ when\ n = 1 -------------- -(1)$

$Put\ n = n - 1\ in\ equation\ 1, we\ get$

$T(n-1) = T(n-2) + 1$

$Put\ the\ value\ of\ T(n-1)\ in\ equation\ 1, we\ get$

$T(n) = T(n-2) + 2 ----------------- -(2)$

Iteration Method (Example 1)

$Put\ n = n - 2\ in\ equation\ 1, we\ get$

$T(n-2) = T(n-3) + 1$

$Put\ the\ value\ of\ T(n-2)\ in\ equation\ 2, we\ get$

$T(n) = T(n-3) + 3 - - - - - - - - - - - - - - - - - - - (3)$

……….

$T(n) = T(n-k) + k - - - - - - - - - - - - - - - - - - - (k)$

Iteration Method (Example 1)

$Let\ T(n-k) = T(1) = 1$

$(As\ per\ the\ base\ condition\ of\ recurrence)$

$So\ n-k = 1$

$\Rightarrow k = n-1$

$Now\ put\ the\ value\ of\ k\ in\ equation\ k$

$T(n) = T\big(n-(n-1)\big) + n - 1$

$T(n) = T(1) + n - 1$

$T(n) = 1 + n - 1 \qquad [\therefore\ T(1) = 1]$

$T(n) = n$

$\therefore\ T(n) = \Theta(n)$

Iteration Method (Example 2)

Example 2:

Solve the following recurrence relation by using Iteration method.

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + 3n^2 & if\ n > 1 \\ 11 & if\ n = 1 \end{cases}$$

Iteration Method (Example 2)

$It\ means\ T(n) = 2T\left(\frac{n}{2}\right) + 3n^2\ \ if\ n > 1\ \ and\ \ T(n) = 11\ when\ n = 1 - -(1)$

$Put\ n = \frac{n}{2}\ in\ equation\ 1, we\ get$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + 3\ \left(\frac{n}{2}\right)^2$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + 3\ \left(\frac{n}{2}\right)^2$

$Put\ the\ value\ of\ T\left(\frac{n}{2}\right)\ in\ equation\ 1, we\ get$

$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + 3\ \left(\frac{n}{2}\right)^2\right] + 3n^2$

$T(n) = 2^2T\left(\frac{n}{2^2}\right) + 2.3\frac{n^2}{4} + 3n^2$

$T(n) = 2^2T\left(\frac{n}{2^2}\right) + 3\frac{n^2}{2} + 3n^2\ \ ----------------(2)$

Iteration Method (Example 2)

$$Put\ n = \frac{n}{4}\ in\ equation\ 1, we\ get$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + 3\ \left(\frac{n}{4}\right)^2$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{2^3}\right) + 3\ \left(\frac{n}{4}\right)^2$$

$$Put\ the\ value\ of\ T\left(\frac{n}{4}\right)\ in\ equation\ 2, we\ get$$

$$T(n) = 2^2\left[2T\left(\frac{n}{8}\right) + 3\frac{n^2}{16}\right] + 3\frac{n^2}{2} + 3n^2$$

$$T(n) = 2^2\left[2T\left(\frac{n}{2^3}\right) + 3\ \left(\frac{n}{4}\right)^2\right] + 3\frac{n^2}{2} + 3n^2$$

$$T(n) = 2^3T\left(\frac{n}{2^3}\right) + 4.3\frac{n^2}{16} + 3\frac{n^2}{2} + 3n^2$$

Iteration Method (Example 2)

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3\frac{n^2}{2^2} + 3\frac{n^2}{2} + 3n^2 \text{ ---------------- } -(3)$$

$\ldots\ldots.$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + \cdots + \cdots + \cdots + 3\frac{n^2}{2^2} + 3\frac{n^2}{2} + 3n^2 \text{ ---------- } -(i^{th}\ term)$$

$and\ the\ series\ terminate\ when\ \dfrac{n}{2^i} = 1$

$\Rightarrow n = \ 2^i$

$Taking \log both\ side$

$\Rightarrow \log_2 n = i \log_2 2$

$\Rightarrow i = \ \log_2 n \quad (because\ \log_2 2 = 1)$

Iteration Method (Example 2)

$Hence\ we\ can\ write\ the\ i^{th}\ term\ as\ follows$

$$\Rightarrow T(n) = 3n^2 + 3\frac{n^2}{2} + 3\frac{n^2}{2^2} + \cdots + \cdots + \cdots + \ 2^i T\left(\frac{n}{2^i}\right)$$

$$\Rightarrow T(n) = 3n^2 + 3\frac{n^2}{2} + 3\frac{n^2}{2^2} + \cdots + \cdots + \cdots + \ 2^{\log_2 n} T(1)$$

$$\Rightarrow T(n) = 3n^2 + 3\frac{n^2}{2} + 3\frac{n^2}{2^2} + \cdots + \cdots + \cdots + \ 2^{\log_2 n}.11$$

$$\Rightarrow T(n) = 3n^2 + 3\frac{n^2}{2} + 3\frac{n^2}{2^2} + \cdots + \cdots + \cdots + \ n^{\log_2 2}.11 \qquad [\ As\ \log_2 2 = 1]$$

$$\Rightarrow T(n) = 3n^2 + 3\frac{n^2}{2} + 3\frac{n^2}{2^2} + \cdots + \cdots + \cdots + \ n.11$$

$$\Rightarrow T(n) = \left[3n^2 + 3\frac{n^2}{2} + 3\frac{n^2}{2^2} + \cdots + \cdots + \cdots\right] + 11.n$$

$$\Rightarrow T(n) = 3n^2\left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \cdots + \cdots\right] + 11.n$$

Iteration Method (Example 2)

$As\ we\ know\ that\ \ Sum\ of\ \ infinite\ Geometric\ series\ is$

$$= a\ +\ ar\ +\ ar^2\ + \ldots + ar^{(n-1)} = \sum_{i=0}^{\infty} ar^i = a\left(\frac{1}{1-r}\right) = \frac{a}{1-r}$$

$Hence,$

$$\Rightarrow T(n) \leq 3n^2\left[\frac{1}{1-\frac{1}{2}}\right] + 11\,n$$

$$\Rightarrow T(n) \leq 3n^2.2 + 11\,n$$

$$\Rightarrow T(n) \leq 6n^2 + 11\,n$$

$$Hence\ T(n) = O(n^2)$$

Iteration Method (Example 3)

Example 3:

Solve the following recurrence relation by using Iteration method.

$$T(n) = \begin{cases} 8T\left(\frac{n}{2}\right) + n^2 & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

Iteration Method (Example 3)

$$It\ means\ T(n) = 8T\left(\frac{n}{2}\right) + n^2\ \ if\ n > 1\ \ and\ \ T(n) = 1\ when\ n = 1 ---- (1)$$

$$Put\ n = \frac{n}{2}\ in\ equation\ 1, we\ get$$

$$T\left(\frac{n}{2}\right) = 8T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$$

$$Put\ the\ value\ of\ T\left(\frac{n}{2}\right)\ in\ equation\ 1, we\ get$$

$$T(n) = 8\left[8T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2\right] + n^2$$

$$T(n) = 8^2T\left(\frac{n}{4}\right) + 8\frac{n^2}{4} + n^2 ---------------- (2)$$

$$Put\ n = \frac{n}{4}\ in\ equation\ 1, we\ get$$

$$T\left(\frac{n}{4}\right) = 8T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2$$

Iteration Method (Example 3)

Put the value of $T\left(\frac{n}{4}\right)$ *in equation* 2, *we get*

$$T(n) = 8^2\left[8T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2\right] + 8\frac{n^2}{4} + n^2$$

$$T(n) = 8^3T\left(\frac{n}{8}\right) + 8^2\frac{n^2}{4^2} + 8\frac{n^2}{4} + n^2 ---------------(3)$$

… …

$T(n)$

$$= 8^kT\left(\frac{n}{2^k}\right) + 8^{k-1}\frac{n^2}{4^{k-1}} + \cdots + \cdots + \cdots + 8^2\frac{n^2}{4^2} + 8\frac{n^2}{4} + n^2 ----(k^{th}\ term)$$

$$T(n) = 8^kT\left(\frac{n}{2^k}\right) + n^2\left[\frac{8^{k-1}}{4^{k-1}} + \frac{8^{k-2}}{4^{k-2}} \ldots + \cdots + \cdots + \frac{8^2}{4^2} + \frac{8}{4} + 1\right]$$

$$T(n) = 8^kT\left(\frac{n}{2^k}\right) + n^2[2^{k-1} + 2^{k-2} \ldots + \cdots + \cdots + 2^2 + 2 + 1] -----(4)$$

Iteration Method (Example 3)

$Put\ the\ value\ of\ T\left(\frac{n}{4}\right)\ in\ equation\ 2, we\ get$

$$T(n) = 8^2\left[8T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2\right] + 8\frac{n^2}{4} + n^2$$

$$T(n) = 8^3T\left(\frac{n}{8}\right) + 8^2\frac{n^2}{4^2} + 8\frac{n^2}{4} + n^2 --------------(3)$$

… …

$$T(n) = 8^kT\left(\frac{n}{2^k}\right) + 8^{k-1}\frac{n^2}{4^{k-1}} + \cdots + \cdots + 8^2\frac{n^2}{4^2} + 8\frac{n^2}{4} + n^2 ----(k^{th}\ term)$$

$$T(n) = 8^kT\left(\frac{n}{2^k}\right) + n^2\left[\frac{8^{k-1}}{4^{k-1}} + \frac{8^{k-2}}{4^{k-2}} \ldots + \cdots + \cdots + \frac{8^2}{4^2} + \frac{8}{4} + 1\right]$$

$$T(n) = 8^kT\left(\frac{n}{2^k}\right) + n^2[2^{k-1} + 2^{k-2} \ldots + \cdots + \cdots + 2^2 + 2 + 1] -----(4)$$

Iteration Method (Example 3)

$$T(n) = 8^k T\left(\frac{n}{2^k}\right) + \; n^2[2^{k-1} + 2^{k-2} \ldots + \cdots + \cdots + 2^2 + 2 + 1] \;---- (4)$$

$$T(n) = 8^k T\left(\frac{n}{2^k}\right) + n^2[1 + 2 + 2^2 + \cdots + \cdots + 2^{k-2} + 2^{k-1}] \;---- \; (5)$$

$$\text{and the series terminate when } \frac{n}{2^k} = 1$$

$$\Rightarrow n = \;\; 2^k$$

$$\text{Taking } \log \text{ both side}$$

$$\Rightarrow \log_2 n = k \log_2 2$$

$$\Rightarrow k = \;\; \log_2 n \qquad (\text{because } \; \log_2 2 = 1)$$

$$\text{Now, apply the value of } k = \log_2 n \text{ and } \frac{n}{2^k} = 1 \text{ in equation } 5$$

Iteration Method (Example 3)

$T(n) = 8^{\log_2 n} T(1) + n^2\left[1 + 2 + 2^2 + \cdots + \cdots + 2^{\log_2 n - 2} + 2^{\log_2 n - 1}\right] -$ (6)

$$= n^{\log_2 8}.1 + n^2\left[1 + 2 + 2^2 + \cdots + \cdots + 2^{\log_2 n - 2} + 2^{\log_2 n - 1}\right]$$

$$= n^3 + n^2\left[1 \underbrace{+ 2 + 2^2 + \cdots + \cdots + 2^{\log_2 n - 2} + 2^{\log_2 n - 1}}\right]$$

Is a G.P Series, but in this case no need of evaluation. Because the highest order polynomial is n^3. So no need to calculate n^2.

$$= n^3$$

Hence the complexity is $T(n) = n^3$

Iteration Method (Example 3)

$$T(n) = 8^{\log_2 n}T(1) + n^2\left[1 + 2 + 2^2 + \cdots + 2^{\log_2 n-2} + 2^{\log_2 n-1}\right] - (6)$$

$$= n^{\log_2 8}.1 + n^2\left[1 + 2 + 2^2 + \cdots + \cdots + 2^{\log_2 n-2} + 2^{\log_2 n-1}\right]$$

$$= n^3 + n^2\left[1 + 2 + 2^2 + \cdots + \cdots + 2^{\log_2 n-2} + 2^{\log_2 n-1}\right]$$

Is a G.P Series, but in this case no need of evaluation. Because the highest order polynomial is n^3. So no need to calculate n^2.

$$= n^3$$

Hence the complexity is $T(n) = \boldsymbol{O}(n^3)$

$Sum\ of\ finite\ Geometric\ Progression\ series\ is$

$$= a + ar + ar^2 + \ldots + ar^n = \sum_{i=0}^{n} ar^i = a\left(\frac{r^{n+1}-1}{r-1}\right)$$

Iteration Method (Example 4)

Example 4:

Solve the following recurrence relation by using Iteration method.

$$T(n) = \begin{cases} 7T\left(\frac{n}{2}\right) + n^2 & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

$(i.e. Strassion\ Algorithm)$

Iteration Method (Example 4)

$$It\ means\ T(n) = 7T\left(\frac{n}{2}\right) + n^2 \quad if\ n > 1\ \ and\ \ T(n) = 1\ when\ n = 1 ---- (1)$$

$$Put\ n = \frac{n}{2}\ in\ equation\ 1, we\ get$$

$$T\left(\frac{n}{2}\right) = 7T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$$

$$Put\ the\ value\ of\ T\left(\frac{n}{2}\right)\ in\ equation\ 1, we\ get$$

$$T(n) = 7\left[7T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2\right] + n^2$$

$$T(n) = 7^2T\left(\frac{n}{4}\right) + 7\frac{n^2}{4} + n^2 ---------------- (2)$$

$$Put\ n = \frac{n}{4}\ in\ equation\ 1, we\ get$$

$$T\left(\frac{n}{4}\right) = 7T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2$$

Iteration Method (Example 4)

Put the value of $T\left(\frac{n}{4}\right)$ *in equation* 2, *we get*

$$T(n) = 7^2\left[7T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2\right] + 7\frac{n^2}{4} + n^2$$

$$T(n) = 7^3T\left(\frac{n}{8}\right) + 7^2\frac{n^2}{4^2} + 7\frac{n^2}{4} + n^2 ---------------(3)$$

… …

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + 7^{k-1}\frac{n^2}{4^{k-1}} + \cdots + \cdots + \cdots + 7^2\frac{n^2}{4^2} + 7\frac{n^2}{4} + n^2 ---(k^{th}\ term)$$

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + n^2\left[\frac{7^{k-1}}{4^{k-1}} + \frac{7^{k-2}}{4^{k-2}} \ldots + \cdots + \cdots + \frac{7^2}{4^2} + \frac{7}{4} + 1\right]$$

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + n^2\left[\sum_{i=0}^{k-1}\left(\frac{7}{4}\right)^i\right] -----(4)$$

Iteration Method (Example 4)

$Put\ the\ value\ of\ T\left(\frac{n}{4}\right)\ in\ equation\ 2, we\ get$

$$T(n) = 7^2\left[7T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2\right] + 7\frac{n^2}{4} + n^2$$

$$T(n) = 7^3T\left(\frac{n}{8}\right) + 7^2\frac{n^2}{4^2} + 7\frac{n^2}{4} + n^2 ---------------(3)$$

… …

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + 7^{k-1}\frac{n^2}{4^{k-1}} + \cdots + \cdots + \cdots + 7^2\frac{n^2}{4^2} + 7\frac{n^2}{4} + n^2 ---(k^{th}\ term)$$

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + n^2\left[\frac{7^{k-1}}{4^{k-1}} + \frac{7^{k-2}}{4^{k-2}} \ldots + \cdots + \cdots + \frac{7^2}{4^2} + \frac{7}{4} + 1\right]$$

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + n^2\left[\sum_{i=0}^{k-1}\left(\frac{7}{4}\right)^i\right] -----(4)$$

Iteration Method (Example 4)

$$T(n) = 7^k T\left(\frac{n}{2^k}\right) + n^2 \left[\sum_{i=0}^{k-1} \left(\frac{7}{4}\right)^i\right] - - - - - (4)$$

$$and\ the\ series\ terminate\ when\ \frac{n}{2^k} = 1$$

$$\Rightarrow n = 2^k$$

$$Taking \log both\ side$$

$$\Rightarrow \log_2 n = k \log_2 2$$

$$\Rightarrow k = \log_2 n \quad (because\ \log_2 2 = 1)$$

$$Now, apply\ the\ value\ of\ k = \log_2 n\ \ and\ \frac{n}{2^k} = 1\ \ in\ equation\ 4$$

Iteration Method (Example 4)

$$T(n) = 7^{\log_2 n}T(1) + n^2\left[\sum_{i=0}^{\log_2 n-1}\left(\frac{7}{4}\right)^i\right] -----(5)$$

$$= n^{\log_2 7}.1 + n^2\left[\sum_{i=0}^{\log_2 n-1}\left(\frac{7}{4}\right)^i\right]$$

$$= n^{\log_2 7}.1 + n^2\left[\sum_{i=0}^{\log_2 n-1}\left(\frac{7}{4}\right)^i\right]$$

$$= n^{2.8} + n^2\underbrace{\left[\sum_{i=0}^{\log_2 n-1}\left(\frac{7}{4}\right)^i\right]}$$

Is a G.P Series, but in this case no need of evaluation. Because the highest order polynomial is n^3. So no need to calculate n^2.

Iteration Method (Example 4)

$$= n^{2.8} + n^2 \underbrace{\left[\sum_{i=0}^{\log_2 n - 1} \left(\frac{7}{4}\right)^i \right]}$$

Is a G.P Series, but in this case no need of evaluation. Because the highest order polynomial is $n^{2.8}$. So no need to calculate n^2.

$$= n^{2.8}$$

Hence the complexity is $T(n) = n^{2.8}$

Iteration Method (Example 4)

$$= n^{2.8} + n^2 \underbrace{\left[\sum_{i=0}^{\log_2 n - 1} \left(\frac{7}{4}\right)^i\right]}$$

Is a G.P Series, but in this case no need of evaluation. Because the highest order polynomial is $n^{2.8}$. So no need to calculate n^2.

$$= n^{2.8}$$

Hence the complexity is $T(n) = \boldsymbol{O}(n^{2.8})$

Sum of finite Geometric Progression series is

$$= a + ar + ar^2 + \ldots + ar^n = \sum_{i=0}^{n} ar^i = a\left(\frac{r^{n+1} - 1}{r - 1}\right)$$

Iteration Method (Example 5)

Example 5:

Solve the following recurrence relation by using Iteration method.

$$T(n) = \begin{cases} T(n-1) + \log n & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

Iteration Method (Example 5)

$It\ means\ T(n) = T(n-1) + \log n \quad if\ n > 1\ \ and\ \ T(n) = 1\ when\ n = 1 - -(1)$

$Put\ n = n - 1\ in\ equation\ 1, we\ get$

$$T(n-1) = T(n-2) + \log(n-1)$$

$Put\ the\ value\ of\ T(n-1)\ in\ equation\ 1, we\ get$

$$T(n) = T(n-2) + \log(n-1) + \log n$$
$$= T(n-3) + \log(n-2) + \log(n-1) + \log n$$
$$= \mathrm{T}(\mathrm{n}-4) + \log\,(n-3) + \log(n-2) + \log(n-1) + \log n$$

....

....

Hence the k^{th}order is :

$$\mathrm{T(n)} = \mathrm{T(n-k)} + \log\big(\mathrm{n} - (\mathrm{k}-1)\big) + \cdots + \log(n-2) + \log(n-1) + \log n$$
$$\mathrm{T(n)} = \mathrm{T(n-k)} + \log(\mathrm{n} - k + 1) + \cdots + \log(n-2) + \log(n-1) + \log n$$

Iteration Method (Example 5)

Hence the k^{th}order is :

$$T(n) = T(n-k) + \log(n-k+1) + \cdots + \log(n-2) + \log(n-1) + \log n$$

As per the assumption $n - k = 1$

So $k = n - 1$

The k^{th}order can be written as:

$$T(n) = T(1) + \log(n-n+1+1) + \cdots + \log(n-2) + \log(n-1) + \log n$$

$$= 1 + \log(2) + \cdots + \log(n-2) + \log(n-1) + \log n$$

$$= 1 + \log(2) + \log(3) + \log(4) \ldots + \log(n-2) + \log(n-1) + \log n$$

$$= 1 + \log(2.3.4.5 \quad \ldots\ldots\ldots\ldots \quad .n)$$

$$= 1 + \log(n!)$$

$\boldsymbol{Hence\ the\ complexity\ is} : \boldsymbol{O(log n!)}$

Iteration Method (Practice)

$$Q1. T(n) = \begin{cases} T\left(\frac{7n}{10}\right) + n & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

$$Q2. T(n) = \begin{cases} T(n-1) + (n-1) & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

$$Q3. T(n) = \begin{cases} T(n-1) + n^2 & if\ n > 1 \\ 1 & if\ n = 1 \end{cases}$$

Thank u