Design and Analysis of Algorithm

Branch and Bound
(Travelling salesman Problem)

Lecture – 54

Overview

- The method was first proposed by **Ailsa Land and Alison Doig whilst carrying out research at the London School of Economics sponsored by British Petroleum in 1960** for discrete programming, and has become the most commonly used tool for solving NP-hard optimization problems.
- The name "**branch and bound**" first occurred in the work of **Little et al.** on the traveling salesman problem.

Overview

- Branch and Bound, or B & B, is an algorithm design paradigm that solves combinatorial and discrete optimization problems.
- Many optimization issues, such as crew scheduling, network flow problems, and production planning, cannot be solved in polynomial time.
- Hence, B & B is a paradigm that is widely used to solve such problems.

Overview

- Branch-and-bound algorithm consists of a systematic enumeration of solutions by means of state space search tree.
- The set of solutions is thought of as forming a rooted tree with the full set at the root.
- The algorithm explores branches of this tree, which represent subsets of the solution set.

Overview

- The Branch algorithms incorporate different search techniques to traverse a state space tree. Different search techniques used in B&B are listed below:
  - LC search
  - BFS
  - DFS

Overview

1. LC search (Least Cost Search):
   - It uses a heuristic cost function to compute the bound values at each node. Nodes are added to the list of live nodes as soon as they get generated.
   - The node with the least value of a cost function selected as a next Explored-node.

Overview

2.BFS(Breadth First Search):

- It is also known as a FIFO search.
- It maintains the list of live nodes in first-in-first-out order i.e, in a queue, The live nodes are searched in the FIFO order to make them next Explored-nodes.

Overview

3. DFS (Depth First Search):
   - It is also known as a LIFO search.
   - It maintains the list of live nodes in last-in-first-out order i.e. in a stack.
   - The live nodes are searched in the LIFO

Travelling Salesman Problem

- Problem Statement

  Given a set of cities and distance between every pair of cities, the problem is to find the shortest possible tour that visits every city exactly once and returns to the starting point.

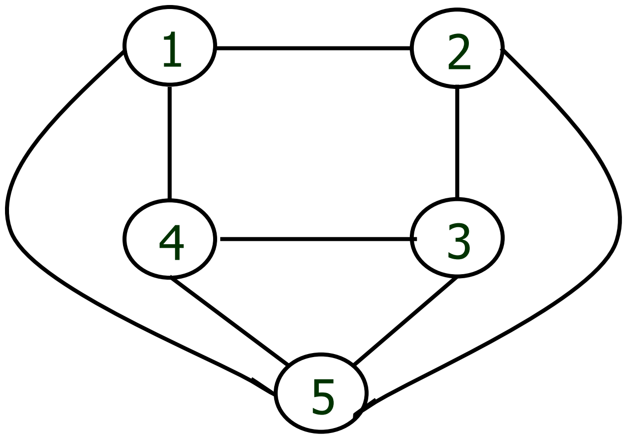
  The cost matrix is defined as:

$$C(i,j) = \begin{cases} w(i,j) & \textit{if there will be direct path between } C_i \textit{ to} C_j \\ \infty & \textit{Otherwise} \end{cases}$$

Note: Use LC method to solve this problem.

Travelling Salesman Problem

Example:



Graph

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & 20 & 30 & 10 & 11 \\
15 & \infty & 16 & 4 & 2 \\
3 & 5 & \infty & 2 & 4 \\
19 & 6 & 18 & \infty & 3 \\
16 & 4 & 7 & 16 & \infty
\end{bmatrix}
\end{array}
$$

Cost Matrix of the Graph

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}
\end{array}$$

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \end{array}$$

Find the minimum value of each row and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced Row)

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & 20 & 30 & 10 & 11 & 10 \\
2 & 15 & \infty & 16 & 4 & 2 & 2 \\
3 & 3 & 5 & \infty & 2 & 4 & 2 \\
4 & 19 & 6 & 18 & \infty & 3 & 3 \\
5 & 16 & 4 & 7 & 16 & \infty & 4
\end{array}
$$

$$\sum row\ min = 21$$

Find the minimum value of each row and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced Row)

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array}
\end{array}$$

$$\sum row\ min = 21$$

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array}
\end{array}$$

Find the minimum value of each row and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced Row)

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array}
\end{array}$$

$$\sum row\ min = 21$$

Find the minimum value of each row and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced Row)

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array}
\end{array}$$

Find the minimum value of each column and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced column)

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$\begin{array}{c|c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} & \begin{matrix} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{matrix}
\end{array}$$

$$\sum row\ min = 21$$

Find the minimum value of each row and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced Row)

$$\begin{array}{c|c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} & \begin{matrix} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{matrix} \\
 & \begin{matrix} 1 & 0 & 3 & 0 & 0 \end{matrix} &
\end{array}$$

Find the minimum value of each column and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced column)

Travelling Salesman Problem

Example:

Step 1: First reduce the cost of above matrix(i.e Reducing matrix)

$$\begin{array}{c|c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

$$\sum row\ min = 21$$

Find the minimum value of each row and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced Row)

$$\begin{array}{c|c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array} \\ & \begin{array}{ccccc} 1 & 0 & 3 & 0 & 0 \end{array} & \end{array}$$

$$\sum col\ min = 4$$

Find the minimum value of each column and then create the resultant matrix by subtracting the minimum value from each element of the same row (i.e Reduced column)

Travelling Salesman Problem

Example:

The final reduced matrix after step 1 is:

$$C1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \end{array}$$

<u>THE COST OF NODE 1</u>

Total cost of reduction of all rows $= \sum row\ min = 21$

Total cost of reduction of all columns $= \sum col\ min = 4$

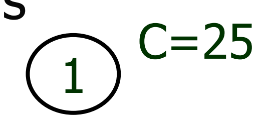
So the reduced cost after step 1 =21+4=25

"Now the matrix is a reduced matrix. It means the matrix contain that one zero in each row and one zero in each column."

Travelling Salesman Problem

Example:

From the step 1, it was observed that the cost of 1st node is 25. Hence the State space tree is



Now, we calculate the cost from node 1 to node 2, node 1 to node 3, node 1 to node 4, and node 1 to node 5.

And check, whether there is a minimum cost path from node 1 to node 2 or node 1 to node 3, node 1 to node 4 or node 1 to node 5 is exists? And find which one is minimum and explore that node again. And show the procedure through **state space tree**.

Let us do it one by one…….

Travelling Salesman Problem

Example:

Step 2 Find the cost from node 1 to node 2.

- Make all the value of row1 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using following formula.

$$C(i,j) + r + \hat{r}$$

Cost from node i to j in reduction matrix

Reduction cost of current matrix

Reduced cost of node i . (calculated previously)

Travelling Salesman Problem

Example:

Step 2 Find the cost from node 1 to node 2.

- Make all the value of row1 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \end{array}$$

Travelling Salesman Problem

Example:

Step 2 Find the cost from node 1 to node 2.

- Make all the value of row1 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$
\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 11 & 2 & 0 \\
0 & \infty & \infty & 0 & 2 \\
15 & \infty & 12 & \infty & 0 \\
11 & \infty & 0 & 12 & \infty
\end{bmatrix} &
\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}
\end{array}
$$

$$\sum row\ min = 0$$

Travelling Salesman Problem

Example:

Step 2 Find the cost from node 1 to node 2.

- Make all the value of row1 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$
\begin{array}{c|c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array} &
\end{array}
$$

$\sum row\ min = 0$ $\sum row\ min = 0$

Travelling Salesman Problem

Example:

Step 2 Find the cost from node 1 to node 2.

- Make all the value of row1 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c|c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} &
\end{array}$$

$\sum row\ min = 0$ $\sum row\ min = 0$

Cost from node 1 to node 2

$\Longrightarrow C(1,2) + r + \hat{r}$

$\Longrightarrow \ 10 + 25 + 0 = 35$

Travelling Salesman Problem

Example:

Step 3 Find the cost from node 1 to node 3.

- Make all the value of row1 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
12 & \infty & \infty & 2 & 0 \\
\infty & 3 & \infty & 0 & 2 \\
15 & 3 & \infty & \infty & 0 \\
11 & 0 & \infty & 12 & \infty
\end{bmatrix}
\end{array}
$$

Travelling Salesman Problem

Example:

Step 3 Find the cost from node 1 to node 3.

- Make all the value of row1 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}
\end{array}$$

$$\sum row\ min = 0$$

Travelling Salesman Problem

Example:

Step 3 Find the cost from node 1 to node 3.

- Make all the value of row1 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 11 & 0 & 0 & 0 & 0 \end{array} &
\end{array}$$

$\sum row\ min = 0$ $\sum row\ min = 11$

Travelling Salesman Problem

Example:

Step 3 Find the cost from node 1 to node 3.

- Make all the value of row1 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 11 & 0 & 0 & 0 & 0 \end{array} &
\end{array}$$

$\sum row\ min = 0$ $\sum row\ min = 11$

Cost from node 1 to node 3

$\Longrightarrow C(1,3) + r + \hat{r}$

$\Longrightarrow\ 17 + 25 + 11 = 53$

Travelling Salesman Problem

Example:

Step 4 Similarly we can find the cost from node 1 to node 4.

- Make all the value of row1 to ∞
- Make all the value of col 4 to ∞
- Make node 4 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i, j) + r + \hat{r}$

$$\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array} &
\end{array}$$

$\sum row\ min = 0$ $\sum row\ min = 0$

Cost from node 1 to node 4

$\Longrightarrow C(1,4) + r + \hat{r}$

$\Longrightarrow \ 0 + 25 + 0 = 25$

Travelling Salesman Problem

Example:

Step 5 Similarly we can find the cost from node 1 to node 5.

- Make all the value of row1 to ∞
- Make all the value of col 5 to ∞
- Make node 5 to 1 is also ∞
- And then apply reduction technique to reduced the matrix.
- Calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c|c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 2 \\ 0 \\ 3 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array} &
\end{array}$$

$\sum row\ min = 5$ $\sum row\ min = 0$

Cost from node 1 to node 5

$\Longrightarrow C(1{,}5) + r + \hat{r}$

$\Longrightarrow \ 1 + 25 + 5 = 31$

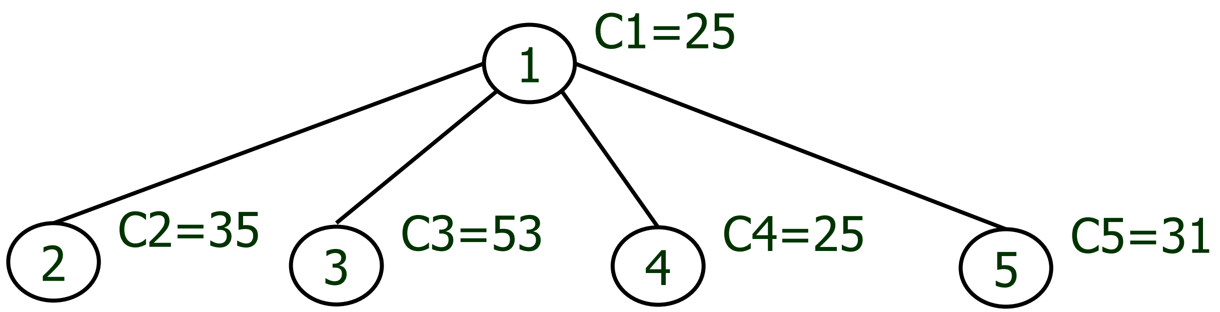
Travelling Salesman Problem

Example:

Hence

- the cost from node 1 to node 2 =35
- the cost from node 1 to node 3 =53
- the cost from node 1 to node 4 =25
- the cost from node 1 to node 5 =31

And the State space tree is given below:



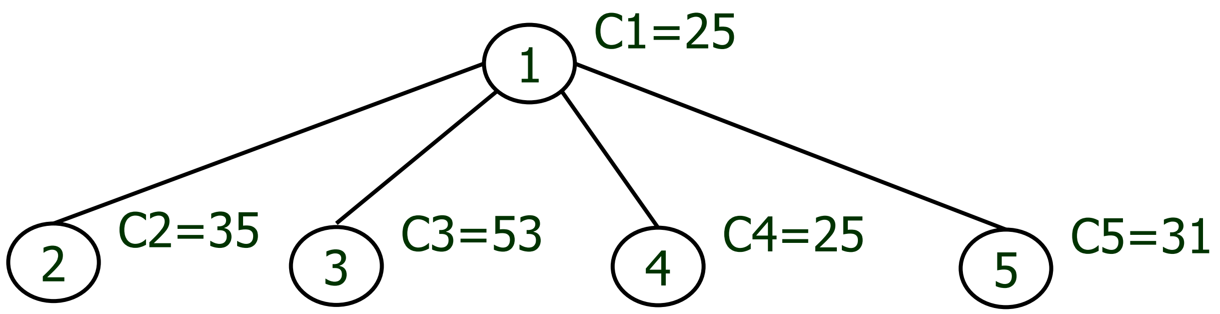
Travelling Salesman Problem

Example:

Hence

- the cost from node 1 to node 2 =35
- the cost from node 1 to node 3 =53
- the cost from node 1 to node 4 =25
- the cost from node 1 to node 5 =31

And the State space tree is given below:



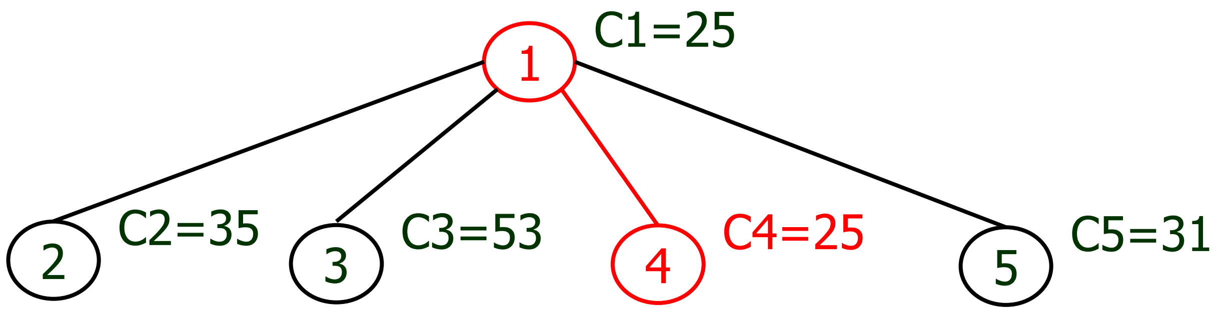
Find Minimum?

Travelling Salesman Problem

Example:

Hence

- the cost from node 1 to node 2 =35
- the cost from node 1 to node 3 =53
- the cost from node 1 to node 4 =25 (Minimum)
- the cost from node 1 to node 5 =31

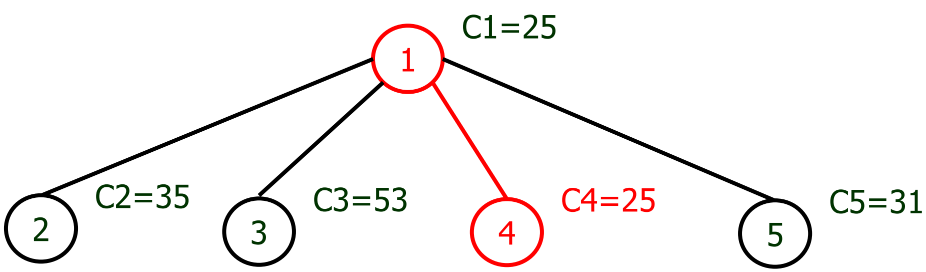


Travelling Salesman Problem

Example:

Hence

- the cost from node 1 to node 2 =35
- the cost from node 1 to node 3 =53
- the cost from node 1 to node 4 =25 (Minimum)
- the cost from node 1 to node 5 =31



$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \end{array}$$

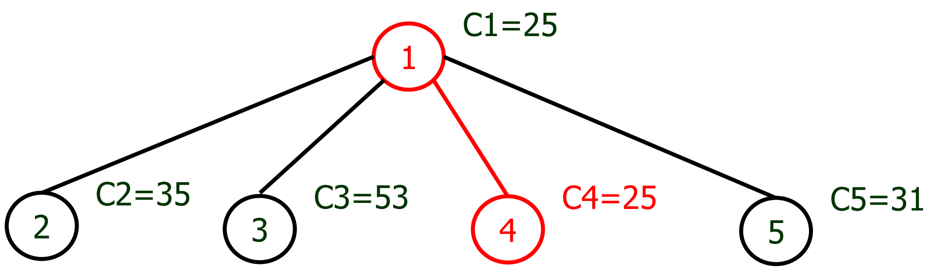
"Hence the reduced matrix obtained from node 1 to node 4 will be treated as reduced matrix for next level of the graph."

Travelling Salesman Problem

Example:

Hence

- the cost from node 1 to node 2 =35
- the cost from node 1 to node 3 =53
- the cost from node 1 to node 4 =25 (Minimum)
- the cost from node 1 to node 5 =31



$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
12 & \infty & 11 & \infty & 0 \\
0 & 3 & \infty & \infty & 2 \\
\infty & 3 & 12 & \infty & 0 \\
11 & 0 & 0 & \infty & \infty
\end{bmatrix}
\end{array}
$$

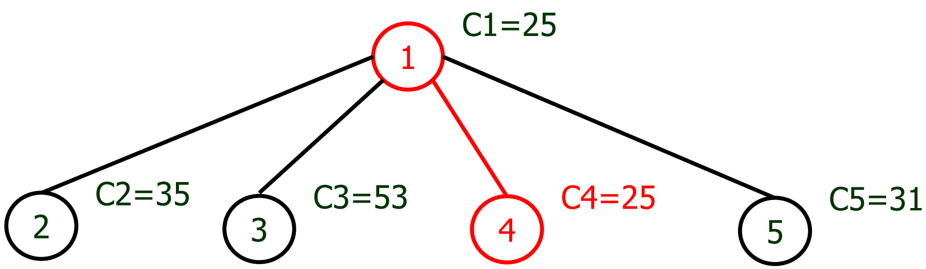
"Hence the reduced matrix obtained from node 1 to node 4 will be treated as reduced matrix for next level of the graph"

Now further find who is the next vertex in next level?(i.e. node 2 or node 3 or node 5)

Hint: Apply the same methodology

Travelling Salesman Problem

Example:



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
12 & \infty & 11 & \infty & 0 \\
0 & 3 & \infty & \infty & 2 \\
\infty & 3 & 12 & \infty & 0 \\
11 & 0 & 0 & \infty & \infty
\end{bmatrix}
\end{array}
$$

Now, we calculate the cost from node 4 to node 2, node 4 to node 3, and node 4 to node 5.
And check, whether there is a minimum cost path from node 4 to node 2 or node 4 to node 3, or node 4 to node 5 is exists? And find which one is minimum and explore that node again. And show the procedure through **state space tree**.
Let us do it one by one……..

Travelling Salesman Problem

Example:

Step 6 Find the cost from node 4 to node 2.

- Make all the value of row 4 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow\ C(i,j) + r + \hat{r}$

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \end{array}$$

Travelling Salesman Problem

Example:

Step 6 Find the cost from node 4 to node 2.

- Make all the value of row 4 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using → $C(i,j) + r + \hat{r}$

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \end{array}$$

Apply reduction technique and reduced the matrix

Travelling Salesman Problem

Example:

Step 6 Find the cost from node 4 to node 2.

- Make all the value of row 4 to ∞
- Make all the value of col 2 to ∞
- Make node 2 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow\ C(i,j) + r + \hat{r}$

$$
\begin{array}{c|c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array} &
\end{array}
$$

$\sum row\ min = 0$ $\sum row\ min = 0$

Cost from node 4 to node 2

$\Longrightarrow C(4,2) + r + \hat{r}$

$\Longrightarrow\ 3 + 25 + 0 = 28$

Travelling Salesman Problem

Example:

Step 7 Find the cost from node 4 to node 3.

- Make all the value of row 4 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 3 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow\ C(i,j) + r + \hat{r}$

$$
\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
12 & \infty & \infty & \infty & 0 \\
\infty & 3 & \infty & \infty & 2 \\
\infty & \infty & \infty & \infty & \infty \\
11 & 0 & \infty & \infty & \infty
\end{bmatrix} &
\begin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 11 & 0 & 0 & 0 & 0 \end{array} &
\end{array}
$$

$\sum row\ min = 2$ $\quad \sum row\ min = 3$

Cost from node 4 to node 3

$\Longrightarrow C(4,3) + r + \hat{r}$

$\Longrightarrow\ 12 + 25 + 13 = 50$

Travelling Salesman Problem

Example:

Step 8 Find the cost from node 4 to node 5.

- Make all the value of row 4 to ∞
- Make all the value of col 5 to ∞
- Make node 5 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 5 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow\ C(i,j) + r + \hat{r}$

$$
\begin{array}{c|c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 11 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array} &
\end{array}
$$

$\sum row\ min = 11$ $\quad$ $\sum row\ min = 0$

Cost from node 4 to node 3

$\Longrightarrow C(4,5) + r + \hat{r}$

$\Longrightarrow\ 0 + 25 + 11 = 36$

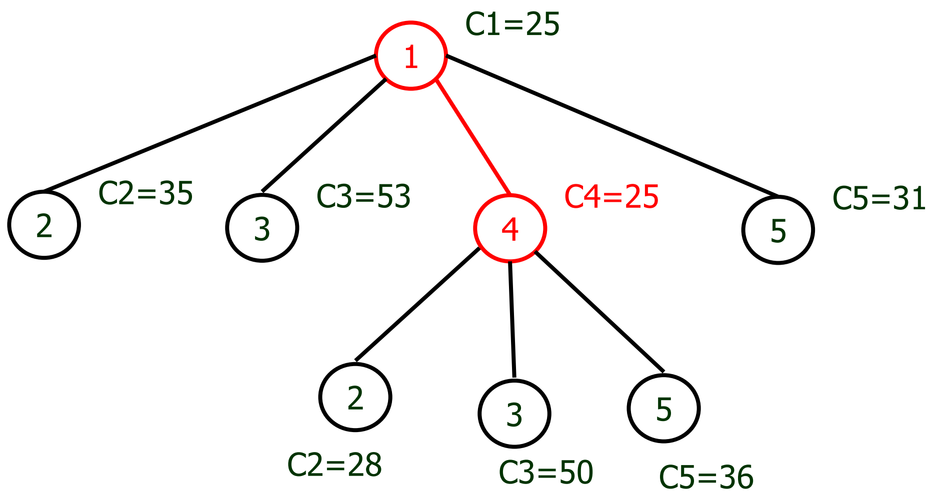
Travelling Salesman Problem

Example:

Hence the cost from

- node 4 to node 2 =28
- node 4 to node 3 =50
- node 4 to node 5 =36

And the State space tree is grown as:



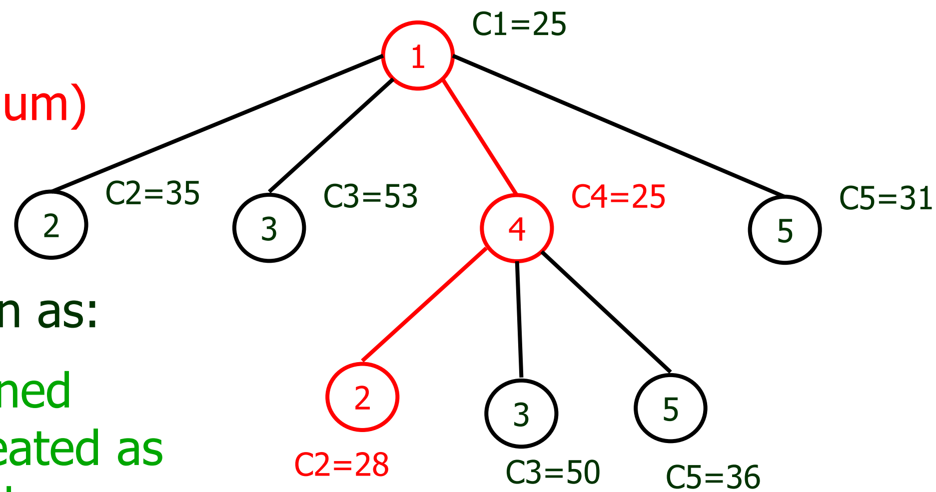
Travelling Salesman Problem

Example:

Hence the cost from

- node 4 to node 2 =28 (Minimum)
- node 4 to node 3 =50
- node 4 to node 5 =36

And the State space tree is grown as:



"Hence the reduced matrix obtained from node 4 to node 2 will be treated as reduced matrix for next level of the graph"

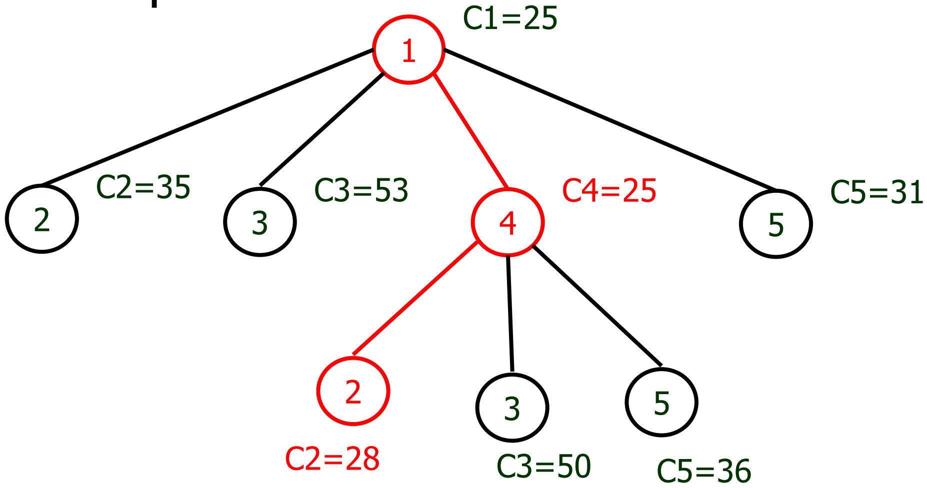
$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \end{array}$$

Now further find who is the next vertex in next level?(i.e. node 3 or node 5)

Hint: Apply the same methodology

Travelling Salesman Problem

Example:



$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}
\end{array}
$$

Now, we calculate the cost from node 2 to node 3, and node 2 to node 5. And check, whether there is a minimum cost path from node 2 to node 3 or node 2 to node 5, is exists? And find which one is minimum and explore that node again. And show the procedure through **state space tree**.

Let us do it one by one……..

Travelling Salesman Problem

Example:

Step 9 Find the cost from node 2 to node 3.

- Make all the value of row 2 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞ as the path is $1 \to 4 \to 2 \to 3 \to 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\to$ $C(i,j) + r + \hat{r}$

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} \end{array}$$

Travelling Salesman Problem

Example:

Step 9 Find the cost from node 2 to node 3.

- Make all the value of row 2 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞ as the path is $1 \to 4 \to 2 \to 3 \to 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\to\ C(i,j) + r + \hat{r}$

$$
\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ 11 \end{array} \\
 & \begin{array}{ccccc} 11 & 0 & 0 & 0 & 2 \end{array} &
\end{array}
$$

$\sum row\ min = 13$ $\quad$ $\sum row\ min = 13$

Cost from node 4 to node 3

$\Longrightarrow C(2,3) + r + \hat{r}$

$\Longrightarrow\ 11 + 28 + 26 = 65$

Travelling Salesman Problem

Example:

Step 10 Find the cost from node 2 to node 5.

- Make all the value of row 2 to ∞
- Make all the value of col 5 to ∞
- Make node 5 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow$ $C(i,j) + r + \hat{r}$

$$\begin{array}{c c c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} &
\end{array}$$

$\sum row\ min = 0$ $\sum row\ min = 0$

Cost from node 4 to node 3

$\Longrightarrow C(2,5) + r + \hat{r}$

$\Longrightarrow \ 0 + 28 + 0 = 28$

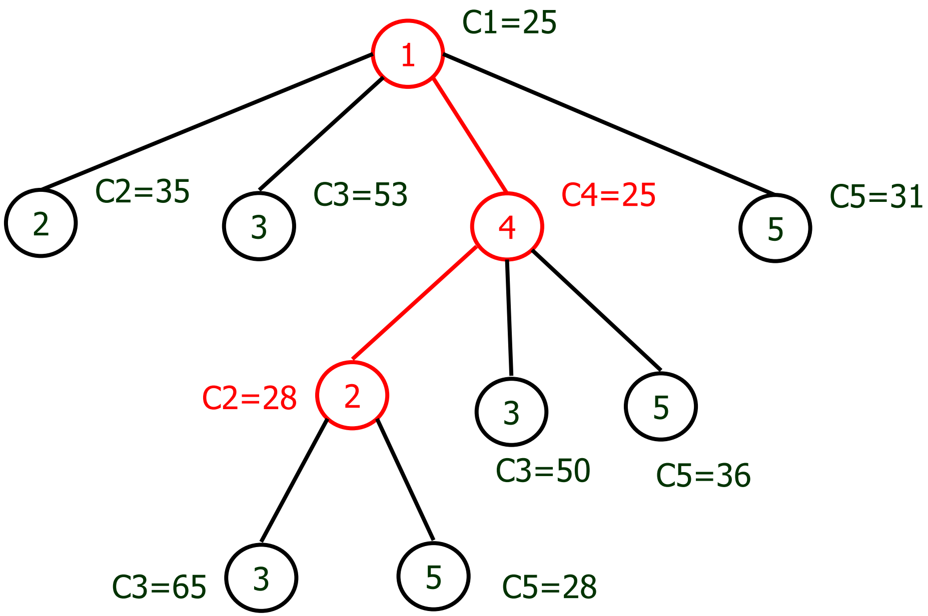
Travelling Salesman Problem

Example:

Hence the cost from

- node 2 to node 3 =65
- node 2 to node 5 =28

And the State space tree is grown as:



Travelling Salesman Problem

Example:

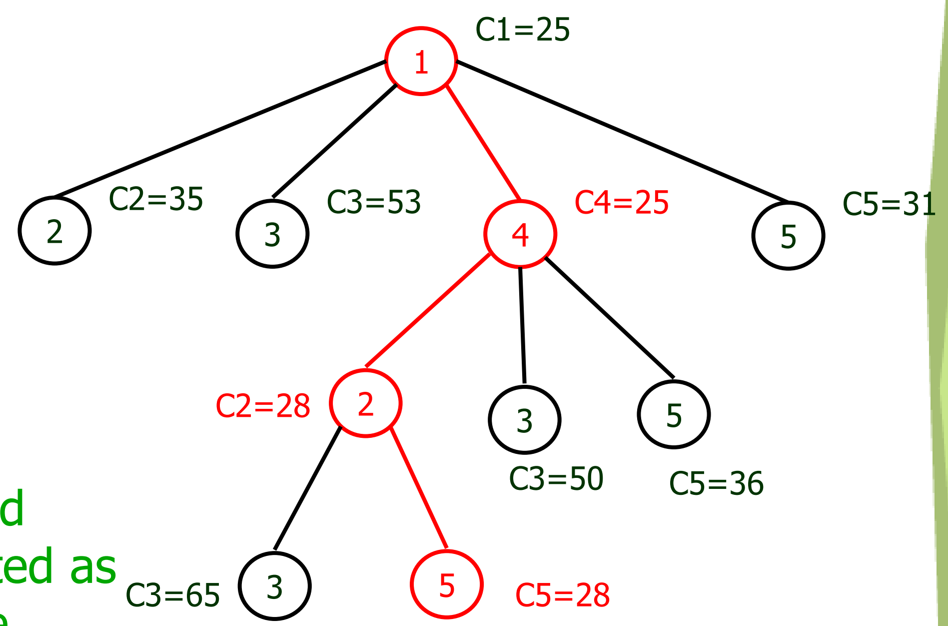
Hence the cost from

- node 2 to node 3 =65
- node 2 to node 5 =28 (Minimum)

And the State space tree is grown as:

"Hence the reduced matrix obtained from node 2 to node 5 will be treated as reduced matrix for next level of the graph"

Now further find who is the next vertex in next level?(i.e. node 3)
Hint: Apply the same methodology



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 0 & \infty & \infty
\end{bmatrix}
\end{array}
$$

Travelling Salesman Problem

Example:

Step 11 Find the cost from node 5 to node 3.

- Make all the value of row 5 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow\ C(i,j) + r + \hat{r}$

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty
\end{bmatrix}
\end{array}
$$

Travelling Salesman Problem

Example:

Step 11 Find the cost from node 5 to node 3.

- Make all the value of row 5 to ∞
- Make all the value of col 3 to ∞
- Make node 3 to 1 is also ∞ as the path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1$.
- Then apply reduction technique to reduced the matrix.
- And calculate the cost by using $\rightarrow$ $C(i,j) + r + \hat{r}$

$$
\begin{array}{c c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array} &
\end{array}
$$

$\sum row\ min = 0$ $\sum row\ min = 0$

Cost from node 4 to node 3

$\Longrightarrow C(5,3) + r + \hat{r}$

$\Longrightarrow 0 + 28 + 0 = 28$

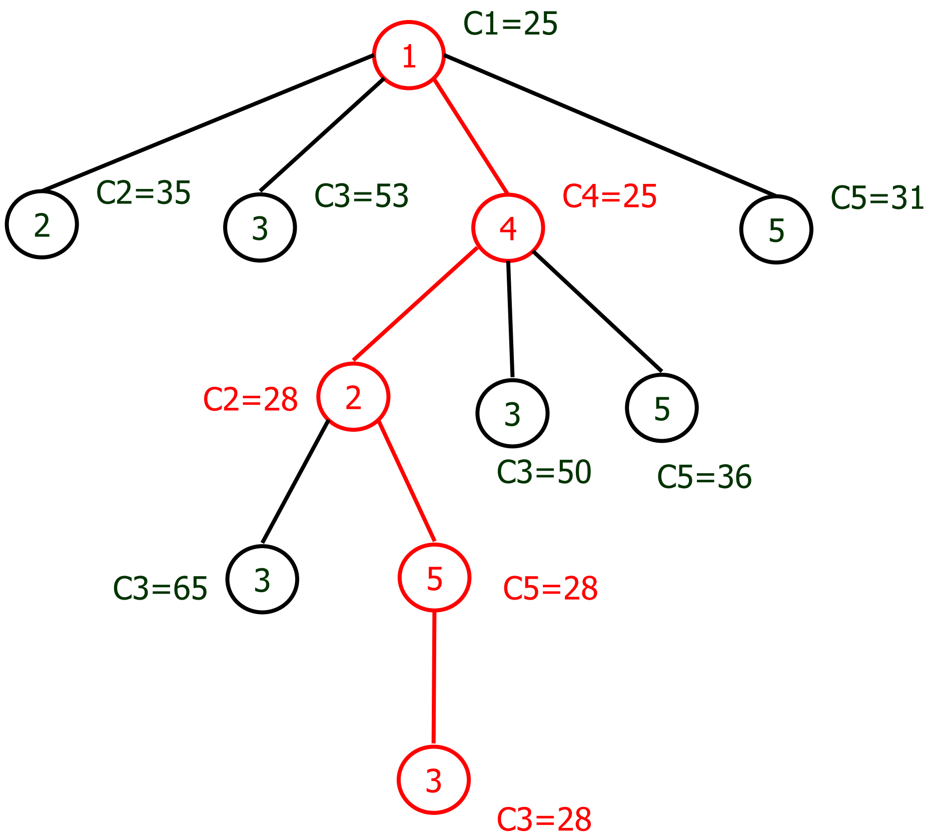
Travelling Salesman Problem

Example:

Hence the cost from

- node 5 to node 3 =28

And the final State space tree is grown as:



Hence the final Path is :

$$1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3$$

(25) (25) (28) (28) (28) $\leftarrow$ *Cost at each node*

Travelling Salesman Problem

Time Complexity:

As there are v(n-1)! Paths the complexity of TSP by using Branch and bound technique is $\Omega(2^n)$ and in worst case if the number of cities are 'n' then the complexity is $O(n^n)$

Suppose we have 'n' nodes (i.e. cities), then there is a need of generating all the permutation of (n-1) nodes. Hence the time complexity is $O(n-1)!$. Which is equal to $O(2^{n-1})$. So the final time complexity is $O(2^n . n^2)$

Thank u