# Design and Analysis of Algorithm

## Dynamic Programming
## (Matrix Chain Multiplication)

**Lecture – 58**

# Overview

- Dynamic Programming is a general algorithm design technique for solving problems defined by recurrences with overlapping subproblems
- Invented by American mathematician “Richard Bellman) in the year 1950s to solve optimization problems and later assimilated by Computer Science.
- “Programming” here means “planning”

# Dynamic Programming

- “Method of solving complex problems by breaking them down into smaller sub-problems, solving each of those sub-problems just once, and storing their solutions.”
- The problem solving approach looks like Divide and conquer approach.(which is not true)

# Dynamic Programming

Difference between Dynamic programming and Divide and Conquer approach.

| Divide & Conquer | Dynamic Programming |
|---|---|
| 1. Partitions a problem into independent smaller sub-problems | 1. Partitions a problem into overlapping sub-problems |
| 2. Doesn't store solutions of sub-problems. (Identical sub-problems may arise - results in the same computations are performed repeatedly.) | 2. Stores solutions of sub-problems: thus avoids calculations of same quantity twice |
| 3. Top down algorithms: which logically progresses from the initial instance down to the smallest sub-instances via intermediate sub-instances. | 3. Bottom up algorithms: in which the smallest sub-problems are explicitly solved first and the results of these used to construct solutions to progressively larger sub-instances |

# Dynamic Programming

Is a Four-step methods

1. Characterize the structure of an optimal solution.
2. Recursively define the value of an optimal solution.
3. Compute the value of an optimal solution, typically in a bottom-up fashion.
4. Construct an optimal solution from computed information.

# Dynamic Programming

Problems:

1. 0/1 Knapsack Problem
2. Floyd-Warshall Algorithm
3. Longest Common Sub-sequence
4. Matrix Chain Multiplication

# Dynamic Programming

**Problem 4: Matrix Chain Multiplication**

Problem:

"Given dimensions $p_0, p_1, p_2, \ldots \ldots, p_n$ corresponding to matrix sequence $\langle A_1, A_2, \ldots \ldots, A_n \rangle$ of $n$ matrices, where for $i = 1, 2, \ldots, n$, matrix $A_i$ has dimension $p_{i-1} \times p_i$, determine the "multiplication sequence" that minimizes the number of scalar multiplications in computing $\langle A_1, A_2, \ldots \ldots, A_n \rangle$."

# Dynamic Programming

**Problem 4: Matrix Chain Multiplication**

Problem:

That is determine how to parenthesize the multiplication.

Example:

$A_1, A_2, A_3, A_4 =$

$$((A_1 A_2)(A_3 A_4))$$
$$(A_1(A_2(A_3 A_4)))$$
$$(A_1((A_2 A_3)A_4))$$
$$((A_1 A_2)(A_3 A_4))$$
$$(((A_1 A_2)A_3)A_4)$$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Given a $p \times q$ matrix A, a $q \times r$ matrix B and a $r \times s$ matrix C, then ABC can be computed in two ways (AB)C and A(BC):

The number of multiplications needed are:

$$mult[(AB)C] = pqr + prs,$$
$$mult[A(BC)] = qrs + pqs.$$

When p = 5, q = 4, r = 6 and s = 2, then

$$mult[(AB)C] = 180,$$
$$mult[A(BC)] = 88.$$

Which is a big difference. Hence the implication is the the multiplication “sequence” (parenthesization) is very important.

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 1: Characterize the structure of an optimal solution

- Decompose thee problem into subproblems:
  - For each pair $1 \leq i \leq j \leq n$, determine the multiplication sequence for $A_{i..j} = A_i, A_{i+1}, \dots \dots, A_j$ that minimize the number of multiplications.
  - Clearly, $A_{i..j}$ is a $p_{i-1} \times p_i$ matrix.
- High-Level Parenthesization for $A_{i..j}$
  - For any optimal multiplication sequence, at the last step you are multiplying two matrices $A_{i..k}$ and $A_{k+1..j}$ for some $k$. That is,

$$A_{i..j} = (A_i \dots A_k)\left(A_{k+1} \dots A_j\right) = A_{i..k}\, A_{k+1..j}$$

  - Example

$$A_{3..6} = ((A_3(A_4A_5))(A_6) = A_{3..5}A_{6..6}.\ (Here\ k = 5.)$$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 1: Characterize the structure of an optimal solution

- Thus, the problem of determining the optimal sequence of multiplications is divided into 2 questions:
  - How do we decide where to split the chain (what is the value of k)? (Search all possible values of k)
  - How do we parenthesize the sub chains $A_{i..k}$ and $A_{k+1..j}$?

    (Problem has optimal substructure property that $A_{i..k}$ and $A_{k+1..j}$ must be optimal so the same procedure can be applied recursively)

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 1: Characterize the structure of an optimal solution

- What is Optimal Substructure Property?
  - If final "optimal" solution of $A_{i..j}$ involves splitting into $A_{i..k}$ and $A_{k+1..j}$ at final step then parenthesization of $A_{i..k}$ and $A_{k+1..j}$ in final optimal solution must also be optimal for the subproblems "standing alone":
  - If parenthisization of $A_{i..k}$ was not optimal we could replace it by a better parenthesization and get a cheaper final solution, leading to a contradiction.
  - Similarly, if parenthisization of $A_{k+1..j}$ was not optimal we could replace it by a better parenthesization and get a cheaper final solution, also leading to a contradiction.

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 2: Recursively define the value of optimal solution.

- For $1 \leq i \leq j \leq n$, let $m[i,j]$ denote the minimum number of multiplications needed to compute $A_{i..j}$. The optimum cost can be described by the following recursive definition.

$$m[i,j] = \begin{cases} 0 & if\ i = j \\ \min\limits_{i \leq k < j} \{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j\} & if\ i < j \end{cases}$$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

```
MATRIX − CHAIN − ORDER(p)
n ← length[p] − 1
let m[1..n, 1..n] and s[1..n − 1,2..n] be new tables.
for i ← 1 to n
    m[i, i] ← 0
for l ← 2 to n   ▹ l is the chain length.
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i, j] ← ∞
        for k ← i to j − 1
            q ← m[i, k] + m[k + 1, j] + p_{i−1} p_k p_j
            if q < m[i, j]
                then m[i, j] ← q
                     s[i, j] ← k
return m and s
```

Lets illustrate the example with the help of an example.

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

$$\langle 30, 35, 15, 5, 10, 20, 25 \rangle$$

Solution:

Here

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

$$\langle 30, 35, 15, 5, 10, 20, 25 \rangle$$

Solution:

Here

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

| Matrix | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|
| Dimensions | 30 x 35 | 35 x 15 | 15 x 5 | 5 x 10 | 10 x 20 | 20 x 25 |

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | | 0 | | | | |
| 3 | | | 0 | | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | | 0 | | | | |
| 3 | | | 0 | | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

$n \leftarrow length[p] - 1$

$let\ m[1..n, 1..n]\ and\ s[1..n-1, 2..n]\ be\ new\ tables.$

$for\ i \leftarrow 1\ to\ n$

$\quad m[i,i] \leftarrow 0$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | ∞ | | | | |
| 2 | | 0 | | | | |
| 3 | | | 0 | | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

$l = 2\ , i = 1, j = 2, k = 1$

$q =$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | | | | |
| 3 | | | 0 | | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\quad\quad j \leftarrow i + l - 1$
$\quad\quad m[i,j] \leftarrow \infty$
$\quad\quad for\ k \leftarrow i\ to\ j - 1$
$\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_k p_j$
$\quad\quad\quad if\ q < m[i,j]$
$\quad\quad\quad\quad then\ m[i,j] \leftarrow q$
$\quad\quad\quad\quad\quad s[i,j] \leftarrow k$

$l = 2\ , i = 1, j = 2, k = 1$

$q = 0 + 0 + 30 * 35 * 15 = 15750$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | $\infty$ | | | |
| 3 | | | 0 | | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

$l = 2\ , i = 2, j = 3, k = 2$

$q =$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\quad\quad j \leftarrow i + l - 1$
$\quad\quad m[i,j] \leftarrow \infty$
$\quad\quad for\ k \leftarrow i\ to\ j - 1$
$\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$
$\quad\quad\quad if\ q < m[i,j]$
$\quad\quad\quad\quad then\ m[i,j] \leftarrow q$
$\quad\quad\quad\quad\quad s[i,j] \leftarrow k$

$l = 2\ , i = 2, j = 3, k = 2$

$q = 0 + 0 + 30 * 15 * 5 = 2625$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | ∞ | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

$l = 2\ , i = 3, j = 4, k = 3$

$q =$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\qquad j \leftarrow i + l - 1$
$\qquad m[i,j] \leftarrow \infty$
$\qquad for\ k \leftarrow i\ to\ j - 1$
$\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$
$\qquad\quad if\ q < m[i,j]$
$\qquad\qquad then\ m[i,j] \leftarrow q$
$\qquad\qquad\quad s[i,j] \leftarrow k$

$l = 2\ , i = 3, j = 4, k = 3$

$q = 0 + 0 + 15 * 5 * 10 = 750$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | ∞ | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | | |
| 5 | | | | | |

s matrix

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\quad\quad j \leftarrow i + l - 1$
$\quad\quad m[i,j] \leftarrow \infty$
$\quad\quad for\ k \leftarrow i\ to\ j - 1$
$\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$
$\quad\quad\quad if\ q < m[i,j]$
$\quad\quad\quad\quad then\ m[i,j] \leftarrow q$
$\quad\quad\quad\quad\quad s[i,j] \leftarrow k$

$l = 2\ , i = 4, j = 5, k = 4$

$q =$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 2\ , i = 4, j = 5, k = 4$

$q = 0 + 0 + 5 * 10 * 20 = 1000$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | $A_1$ | 0 | 2625 | | | |
| 3 | | $A_2$ | 0 | 750 | | |
| 4 | | | $A_3$ | 0 | 1000 | |
| 5 | | | | $A_4$ | 0 | $\infty$ |
| 6 | | | | | $A_5$ | 0 |

$A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | |

s matrix

$$
\begin{array}{l}
\textit{for } l \leftarrow 2 \textit{ to } n \\
\quad \textit{for } i \leftarrow 1 \textit{ to } n - l + 1 \\
\quad\quad j \leftarrow i + l - 1 \\
\quad\quad m[i,j] \leftarrow \infty \\
\quad\quad \textit{for } k \leftarrow i \textit{ to } j - 1 \\
\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\quad\quad\quad \textit{if } q < m[i,j] \\
\quad\quad\quad\quad \textit{then } m[i,j] \leftarrow q \\
\quad\quad\quad\quad\quad s[i,j] \leftarrow k
\end{array}
$$

$l = 2\ , i = 4, j = 5, k = 4$

$q =$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 2\ , i = 4, j = 5, k = 4$

$q = 0 + 0 + 10 * 20 * 25 = 5000$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | ∞ | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

$$
\begin{array}{l}
for\ l \leftarrow 2\ to\ n \\
\quad for\ i \leftarrow 1\ to\ n - l + 1 \\
\qquad j \leftarrow i + l - 1 \\
\qquad m[i,j] \leftarrow \infty \\
\qquad for\ k \leftarrow i\ to\ j - 1 \\
\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\qquad\quad if\ q < m[i,j] \\
\qquad\qquad then\ m[i,j] \leftarrow q \\
\qquad\qquad\quad s[i,j] \leftarrow k
\end{array}
$$

$l = 3\ \ , i = 1, j = 3, k = 1$

$q = 0 + 2625 + 30 * 35 * 5 = 7875$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | ∞ | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

$l = 3\ , i = 1, j = 3, k = 1$

$q = 0 + 2625 + 30 * 35 * 5 = 7875$

$l = 3\ , i = 1, j = 3, k = 2$

$q = 15750 + 0 + 30 * 15 * 5 = 18000$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 3 \ , i = 1, j = 3, k = 1$

$q = 0 + 2625 + 30 * 35 * 5 = 7875$ min

$l = 3 \ , i = 1, j = 3, k = 2$

$q = 15750 + 0 + 30 * 15 * 5 = 18000$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | $\infty$ | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

$l = 3 \ , i = 2, j = 4, k = 2$

$q = 0 + 750 + 35 * 15 * 10 = 6000$

$for\ l \leftarrow 2\ to\ n$

$\quad for\ i \leftarrow 1\ to\ n - l + 1$

$\quad\quad j \leftarrow i + l - 1$

$\quad\quad m[i,j] \leftarrow \infty$

$\quad\quad for\ k \leftarrow i\ to\ j - 1$

$\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_k p_j$

$\quad\quad\quad if\ q < m[i,j]$

$\quad\quad\quad\quad then\ m[i,j] \leftarrow q$

$\quad\quad\quad\quad\quad s[i,j] \leftarrow k$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | ∞ | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

$l = 3\ ,i = 2, j = 4, k = 2$

$q = 0 + 750 + 35 * 15 * 10 = 6000$

$l = 3\ ,i = 2, j = 4, k = 3$

$q = 2625 + 0 + 35 * 5 * 10 = 4375$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

$l = 3\ , i = 2, j = 4, k = 2$

$q = 0 + 750 + 35 * 15 * 10 = 6000$

$l = 3\ , i = 2, j = 4, k = 3$

$q = 2625 + 0 + 35 * 5 * 10 = 4375$ min

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | ∞ | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 3\ , i = 3, j = 5, k = 3$

$q = 0 + 1000 + 15 * 5 * 20 = 2500$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | $\infty$ | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

$l = 3\ , i = 3, j = 5, k = 3$

$q = 0 + 1000 + 15 * 5 * 20 = 2500$

$l = 3\ , i = 3, j = 5, k = 4$

$q = 750 + 0 + 15 * 10 * 20 = 3750$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

$$
\begin{array}{l}
for\ l \leftarrow 2\ to\ n \\
\quad for\ i \leftarrow 1\ to\ n - l + 1 \\
\qquad j \leftarrow i + l - 1 \\
\qquad m[i,j] \leftarrow \infty \\
\qquad for\ k \leftarrow i\ to\ j - 1 \\
\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\qquad\quad if\ q < m[i,j] \\
\qquad\qquad then\ m[i,j] \leftarrow q \\
\qquad\qquad\quad s[i,j] \leftarrow k
\end{array}
$$

min

$l = 3\ , i = 3, j = 5, k = 3$

$q = 0 + 1000 + 15 * 5 * 20 = 2500$

$l = 3\ , i = 3, j = 5, k = 4$

$q = 750 + 0 + 15 * 10 * 20 = 3750$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | ∞ |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 4, j = 6, k = 4$

$q = 0 + 5000 + 5 * 10 * 25 = 6250$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | $\infty$ |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | | |
| 4 | | | | 4 | |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 3\ , i = 4, j = 6, k = 4$

$q = 0 + 5000 + 5 * 10 * 25 = 6250$

$l = 3\ , i = 4, j = 6, k = 5$

$q = 1000 + 0 + 5 * 20 * 25 = 3500$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 3\ , i = 4, j = 6, k = 4$

$q = 0 + 5000 + 5 * 10 * 25 = 6250$

$l = 3\ , i = 4, j = 6, k = 5$

$q = 1000 + 0 + 5 * 20 * 25 = 3500$

min

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | ∞ | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 1, j = 4$

$k = 1, q = 0 + 4375 + 30 * 35 * 10 = 14875$

$$
\begin{array}{l}
\mathit{for}\ l \leftarrow 2\ \mathit{to}\ n \\
\quad \mathit{for}\ i \leftarrow 1\ \mathit{to}\ n - l + 1 \\
\quad\quad j \leftarrow i + l - 1 \\
\quad\quad m[i,j] \leftarrow \infty \\
\quad\quad \mathit{for}\ k \leftarrow i\ \mathit{to}\ j - 1 \\
\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\quad\quad\quad \mathit{if}\ q < m[i,j] \\
\quad\quad\quad\quad \mathit{then}\ m[i,j] \leftarrow q \\
\quad\quad\quad\quad\quad s[i,j] \leftarrow k
\end{array}
$$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | ∞ | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 1, j = 4$

$k = 1, q = 0 + 4375 + 30 * 35 * 10 = 14875$

$k = 2, q = 15750 + 750 + 30 * 15 * 10 = 18000$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 2 | | 0 | 2625 | 4375 | | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3 \ , i = 1, j = 4$

$k = 1, q = 0 + 4375 + 30 * 35 * 10 = 14875$

$k = 2, q = 15750 + 750 + 30 * 15 * 10 = 18000$

$k = 3, q = 7875 + 0 + 30 * 5 * 10 = 9375$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
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# Dynamic Programming

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### Step 3: Compute optimal cost.

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| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$
\begin{aligned}
&\textit{for } l \leftarrow 2 \textit{ to } n \\
&\quad \textit{for } i \leftarrow 1 \textit{ to } n - l + 1 \\
&\qquad j \leftarrow i + l - 1 \\
&\qquad m[i,j] \leftarrow \infty \\
&\qquad \textit{for } k \leftarrow i \textit{ to } j - 1 \\
&\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_k p_j \\
&\qquad\quad \textit{if } q < m[i,j] \\
&\qquad\qquad \textit{then } m[i,j] \leftarrow q \\
&\qquad\qquad\quad s[i,j] \leftarrow k
\end{aligned}
$$

$l = 3\ , i = 1, j = 4$

$k = 1, q = 0 + 4375 + 30 * 35 * 10 = 14875$

$k = 2, q = 15750 + 750 + 30 * 15 * 10 = 18000$ min

$k = 3, q = 7875 + 0 + 30 * 5 * 10 = 9375$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 2 | | 0 | 2625 | 4375 | ∞ | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 2, j = 5$

$k = 2, q = 0 + 4375 + 35 * 15 * 20 = 10500$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
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| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 2, j = 5$

$k = 2, q = 0 + 4375 + 35 * 15 * 20 = 10500$

$k = 3, q = 2625 + 1000 + 35 * 5 * 20 = 7125$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

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Solution:

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| 1 | 0 | 15750 | 7875 | 9375 | | |
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| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$
\begin{array}{l}
\mathit{for}\ l \leftarrow 2\ \mathit{to}\ n \\
\quad \mathit{for}\ i \leftarrow 1\ \mathit{to}\ n - l + 1 \\
\quad\quad j \leftarrow i + l - 1 \\
\quad\quad m[i,j] \leftarrow \infty \\
\quad\quad \mathit{for}\ k \leftarrow i\ \mathit{to}\ j - 1 \\
\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_k p_j \\
\quad\quad\quad \mathit{if}\ q < m[i,j] \\
\quad\quad\quad\quad \mathit{then}\ m[i,j] \leftarrow q \\
\quad\quad\quad\quad\quad s[i,j] \leftarrow k
\end{array}
$$

$l = 3\ , i = 2, j = 5$

$k = 2, q = 0 + 4375 + 35 * 15 * 20 = 10500$

$k = 3, q = 2625 + 1000 + 35 * 5 * 20 = 7125$

$k = 4, q = 4375 + 0 + 35 * 10 * 20 = 11375$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
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| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 2, j = 5$

$k = 2, q = 0 + 4375 + 35 * 15 * 20 = 10500$

$k = 3, q = 2625 + 1000 + 35 * 5 * 20 = 7125$ mi

$k = 4, q = 4375 + 0 + 35 * 10 * 20 = 11375$

```
for l ← 2 to n
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# Dynamic Programming

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| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | ∞ |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 3, j = 6$

$k = 3, q = 0 + 3500 + 15 * 5 * 25 = 5375$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
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# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

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| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3\ , i = 3, j = 6$

$k = 3, q = 0 + 3500 + 15 * 5 * 25 = 5375$

$k = 4, q = 750 + 5000 + 15 * 10 * 25 = 9500$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i, j] ← ∞
        for k ← i to j − 1
            q ← m[i, k] + m[k + 1, j] + p_{i−1} p_k p_j
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# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

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|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | ∞ |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
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            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

$l = 3\ , i = 3, j = 6$

$k = 3, q = 0 + 3500 + 15 * 5 * 25 = 5375$

$k = 4, q = 750 + 5000 + 15 * 10 * 25 = 9500$

$k = 5, q = 2500 + 0 + 15 * 20 * 25 = 10000$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
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Solution:

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|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | | |
| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 3 \ , i = 3, j = 6$

$k = 3, q = 0 + 3500 + 15 * 5 * 25 = 5375$ min

$k = 4, q = 750 + 5000 + 15 * 10 * 25 = 9500$

$k = 5, q = 2500 + 0 + 15 * 20 * 25 = 10000$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
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```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

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Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | ∞ | |
| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 5$

$k = 1, q = 0 + 7125 + 30 * 35 * 20 = 28125$

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\qquad j \leftarrow i + l - 1$
$\qquad m[i,j] \leftarrow \infty$
$\qquad for\ k \leftarrow i\ to\ j - 1$
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$\qquad\quad if\ q < m[i,j]$
$\qquad\qquad then\ m[i,j] \leftarrow q$
$\qquad\qquad\quad s[i,j] \leftarrow k$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

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Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
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| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 5$

$k = 1, q = 0 + 7125 + 30 * 35 * 20 = 28125$

$k = 2, q = 15750 + 2500 + 30 * 15 * 20 = 27250$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
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# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
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| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 5$

$k = 1, q = 0 + 7125 + 30 * 35 * 20 = 28125$

$k = 2, q = 15750 + 2500 + 30 * 15 * 20 = 27250$

$k = 3, q = 7875 + 1000 + 30 * 5 * 20 = 11875$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
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```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | ∞ | |
| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 5$

$k = 1, q = 0 + 7125 + 30 * 35 * 20 = 28125$

$k = 2, q = 15750 + 2500 + 30 * 15 * 20 = 27250$

$k = 3, q = 7875 + 1000 + 30 * 5 * 20 = 11875$

$k = 4, q = 9375 + 0 + 30 * 10 * 20 = 15375$

$for\ l \leftarrow 2\ to\ n$

$\quad for\ i \leftarrow 1\ to\ n - l + 1$

$\quad\quad j \leftarrow i + l - 1$

$\quad\quad m[i,j] \leftarrow \infty$

$\quad\quad for\ k \leftarrow i\ to\ j - 1$

$\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$

$\quad\quad\quad if\ q < m[i,j]$

$\quad\quad\quad\quad then\ m[i,j] \leftarrow q$

$\quad\quad\quad\quad\quad s[i,j] \leftarrow k$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 | | 0 | 2625 | 4375 | 7125 | |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 5$

$k = 1, q = 0 + 7125 + 30 * 35 * 20 = 28125$

$k = 2, q = 15750 + 2500 + 30 * 15 * 20 =$ 27[illegible]0

$k = 3, q = 7875 + 1000 + 30 * 5 * 20 = 11875$ min

$k = 4, q = 9375 + 0 + 30 * 10 * 20 = 15375$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 | | 0 | 2625 | 4375 | 7125 | $\infty$ |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 2, j = 6$

$k = 2, q = 0 + 5375 + 35 * 15 * 25 = 15375$

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\quad\quad j \leftarrow i + l - 1$
$\quad\quad m[i,j] \leftarrow \infty$
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$\quad\quad\quad if\ q < m[i,j]$
$\quad\quad\quad\quad then\ m[i,j] \leftarrow q$
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# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 | | 0 | 2625 | 4375 | 7125 | ∞ |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4 \ , i = 2, j = 6$

$k = 2, q = 0 + 5375 + 35 * 15 * 25 = 15375$

$k = 3, q = 2625 + 3500 + 35 * 5 * 25 = 10500$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                     s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 | | 0 | 2625 | 4375 | 7125 | ∞ |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 2, j = 6$

$k = 2, q = 0 + 5375 + 35 * 15 * 25 = 15375$

$k = 3, q = 2625 + 3500 + 35 * 5 * 25 = 10500$

$k = 4, q = 4375 + 5000 + 35 * 10 * 25 = 18125$

$for\ l \leftarrow 2\ to\ n$

$\quad for\ i \leftarrow 1\ to\ n - l + 1$

$\quad\quad j \leftarrow i + l - 1$

$\quad\quad m[i,j] \leftarrow \infty$

$\quad\quad for\ k \leftarrow i\ to\ j - 1$

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# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

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|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 | | 0 | 2625 | 4375 | 7125 | ∞ |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ \ , i = 2, j = 6$

$k = 2, q = 0 + 5375 + 35 * 15 * 25 = 15375$

$k = 3, q = 2625 + 3500 + 35 * 5 * 25 = 10500$

$k = 4, q = 4375 + 5000 + 35 * 10 * 25 = 18125$

$k = 5, q = 7125 + 0 + 35 * 20 * 25 = 24625$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | |
| 2 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$
\begin{array}{l}
for\ l \leftarrow 2\ to\ n \\
\quad for\ i \leftarrow 1\ to\ n - l + 1 \\
\quad\quad j \leftarrow i + l - 1 \\
\quad\quad m[i,j] \leftarrow \infty \\
\quad\quad for\ k \leftarrow i\ to\ j - 1 \\
\quad\quad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\quad\quad\quad if\ q < m[i,j] \\
\quad\quad\quad\quad then\ m[i,j] \leftarrow q \\
\quad\quad\quad\quad\quad s[i,j] \leftarrow k
\end{array}
$$

$l = 4\ , i = 2, j = 6$

$k = 2, q = 0 + 5375 + 35 * 15 * 25 = 15375$

$k = 3, q = 2625 + 3500 + 35 * 5 * 25 = 10500$ min

$k = 4, q = 4375 + 5000 + 35 * 10 * 25 = 18125$

$k = 5, q = 7125 + 0 + 35 * 20 * 25 = 24625$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | ∞ |
| 2 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4 \;, i = 1, j = 6$

$k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
                then m[i,j] ← q
                    s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
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Solution:

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| 2 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 6$

$k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$

$k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375$

$for\ l \leftarrow 2\ to\ n$
$\quad for\ i \leftarrow 1\ to\ n - l + 1$
$\qquad j \leftarrow i + l - 1$
$\qquad m[i,j] \leftarrow \infty$
$\qquad for\ k \leftarrow i\ to\ j - 1$
$\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$
$\qquad\quad if\ q < m[i,j]$
$\qquad\qquad then\ m[i,j] \leftarrow q$
$\qquad\qquad\quad s[i,j] \leftarrow k$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

### Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | ∞ |
| 2 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4 \ , i = 1, j = 6$

$k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$

$k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375$

$k = 3, q = 7875 + 3500 + 30 * 5 * 25 = 15125$

```
for l ← 2 to n
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i,j] ← ∞
        for k ← i to j − 1
            q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
            if q < m[i,j]
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                    s[i,j] ← k
```

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

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| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 15750 | 7875 | 9375 | 11875 | ∞ |
| 2 | | 0 | 2625 | 4375 | 7125 | 10500 |
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| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$l = 4\ , i = 1, j = 6$

$k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$

$k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375$

$k = 3, q = 7875 + 3500 + 30 * 5 * 25 = 15125$

$k = 4, q = 9375 + 5000 + 30 * 10 * 25 = 21875$

$$
\begin{array}{l}
for\ l \leftarrow 2\ to\ n \\
\quad for\ i \leftarrow 1\ to\ n - l + 1 \\
\qquad j \leftarrow i + l - 1 \\
\qquad m[i,j] \leftarrow \infty \\
\qquad for\ k \leftarrow i\ to\ j - 1 \\
\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\qquad\quad if\ q < m[i,j] \\
\qquad\qquad then\ m[i,j] \leftarrow q \\
\qquad\qquad\quad s[i,j] \leftarrow k
\end{array}
$$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

**Step 3:** Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
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| 1 | 0 | 15750 | 7875 | 9375 | 11875 | ∞ |
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| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

```
for l ← 2 to n
  for i ← 1 to n − l + 1
      j ← i + l − 1
      m[i,j] ← ∞
      for k ← i to j − 1
          q ← m[i,k] + m[k + 1,j] + p_{i−1} p_k p_j
          if q < m[i,j]
              then m[i,j] ← q
                   s[i,j] ← k
```

$l = 4\ , i = 1, j = 6$

$k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$

$k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375$

$k = 3, q = 7875 + 3500 + 30 * 5 * 25 = 15125$

$k = 4, q = 9375 + 5000 + 30 * 10 * 25 = 21875$

$k = 5, q = 11875 + 0 + 30 * 20 * 25 = 26875$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
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| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

Solution:

| | 1 | 2 | 3 | 4 | 5 | 6 |
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| 1 | 0 | 15750 | 7875 | 9375 | 11875 | 15125 |
| 2 | | 0 | 2625 | 4375 | 7125 | 10500 |
| 3 | | | 0 | 750 | 2500 | 5375 |
| 4 | | | | 0 | 1000 | 3500 |
| 5 | | | | | 0 | 5000 |
| 6 | | | | | | 0 |

$A_1$ $A_2$ $A_3$ $A_4$ $A_5$ $A_6$

m matrix

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

$$
\begin{array}{l}
\mathit{for}\ l \leftarrow 2\ \mathit{to}\ n \\
\quad \mathit{for}\ i \leftarrow 1\ \mathit{to}\ n - l + 1 \\
\qquad j \leftarrow i + l - 1 \\
\qquad m[i,j] \leftarrow \infty \\
\qquad \mathit{for}\ k \leftarrow i\ \mathit{to}\ j - 1 \\
\qquad\quad q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j \\
\qquad\quad \mathit{if}\ q < m[i,j] \\
\qquad\qquad \mathit{then}\ m[i,j] \leftarrow q \\
\qquad\qquad\quad s[i,j] \leftarrow k
\end{array}
$$

$l = 4\ , i = 1, j = 6$

$k = 1, q = 0 + 10500 + 30 * 35 * 25 = 36750$

$k = 2, q = 15750 + 5375 + 30 * 15 * 25 = 32375$

$k = 3, q = 7875 + 3500 + 30 * 5 * 25 = 15125$ mi

$k = 4, q = 9375 + 5000 + 30 * 10 * 25 = 21875$

$k = 5, q = 11875 + 0 + 30 * 20 * 25 = 26875$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 3: Compute optimal cost.

```
MATRIX − CHAIN − ORDER(p)
n ← length[p] − 1
let m[1..n, 1..n] and s[1..n − 1,2..n] be new tables.
for i ← 1 to n
    m[i, i] ← 0
for l ← 2 to n   ▹ l is the chain length.
    for i ← 1 to n − l + 1
        j ← i + l − 1
        m[i, j] ← ∞
        for k ← i to j − 1
            q ← m[i, k] + m[k + 1, j] + p_{i−1} p_k p_j
            if q < m[i, j]
                then m[i, j] ← q
                     s[i, j] ← k
return m and s
```

A simple inspection of the nested loop structure of MATRIX-CHAIN-ORDER yields a running time of $O(n^3)$ for the algorithm. The loops are nested three deep, and each loop index $(l, i, and\ k)$ takes on at most $n - 1$ values.

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

| $p_0$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|---|---|---|---|---|---|---|
| 30 | 35 | 15 | 5 | 10 | 20 | 25 |

$PRINT-OPTIMAL-PARENS(s,i,j)$

$if\ i = j$

$\quad then\quad print\ "A_i"$

$\quad else\quad print\ "("$

$\qquad\qquad PRINT-OPTIMAL-PARENS(s,i,s[i,j])$

$\qquad\qquad PRINT-OPTIMAL-PARENS(s,s[i,j]+1,j)$

$\qquad\qquad print\ ")"$

Lets see, how in the discussed example the call $PRINT-OPTIMAL-PARENS(s,1,6)$ prints the parenthesization $((A1\ (A2\ A3))\ ((A4\ A5)A6))$.

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

$PRINT - OPTIMAL - PARENS(s, i, j)$

$if\ i = j$

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$\qquad\qquad PRINT - OPTIMAL - PARENS(s, i, s[i, j])$

$\qquad\qquad PRINT - OPTIMAL - PARENS(s, s[i, j] + 1, j)$

$\qquad\qquad print\ ")"$

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

# Dynamic Programming

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$\qquad\qquad PRINT-OPTIMAL-PARENS(s,s[i,j]\ +\ 1,j)$

$\qquad\qquad print\ ")"$

**POP(S,1,6)**

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

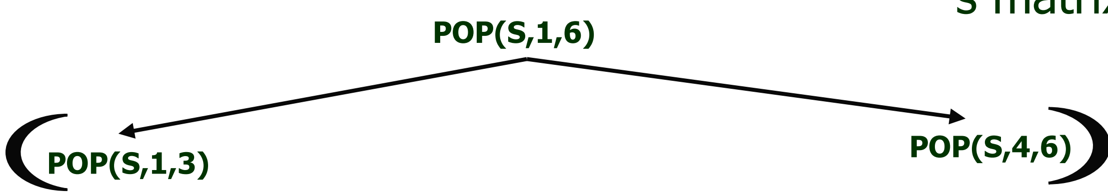
Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

```
PRINT − OPTIMAL − PARENS(s, i, j)
if i = j
  then  print "A_i"
  else  print "("
        PRINT − OPTIMAL − PARENS(s, i, s[i, j])
        PRINT − OPTIMAL − PARENS(s, s[i, j] + 1, j)
        print ")"
```

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix



# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

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```
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if i = j
   then print "A_i"
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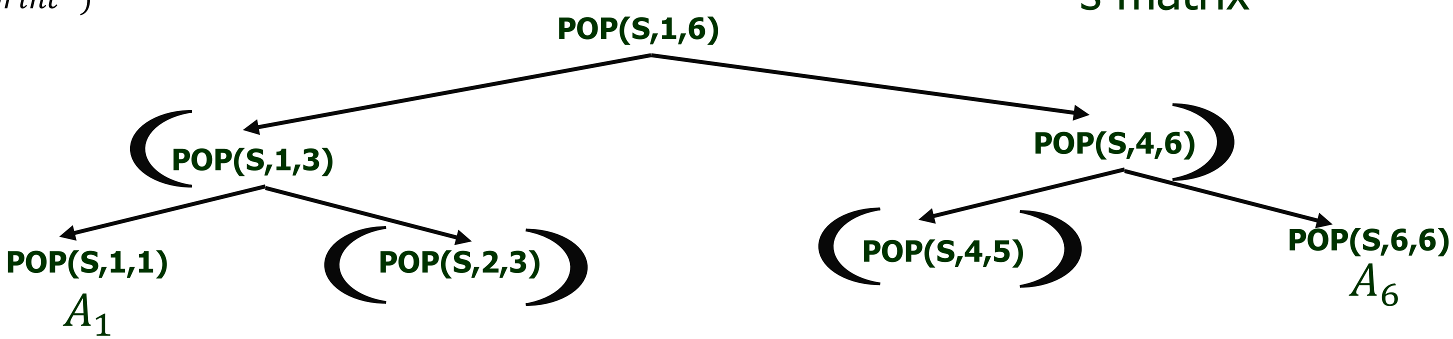
        PRINT − OPTIMAL − PARENS(s, i, s[i, j])
        PRINT − OPTIMAL − PARENS(s, s[i, j] + 1, j)
        print ")"
```

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix



# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

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PRINT − OPTIMAL − PARENS(s, i, j)
if i = j
   then print "A_i"
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        PRINT − OPTIMAL − PARENS(s, i, s[i, j])
        PRINT − OPTIMAL − PARENS(s, s[i, j] + 1, j)
        print ")"
```
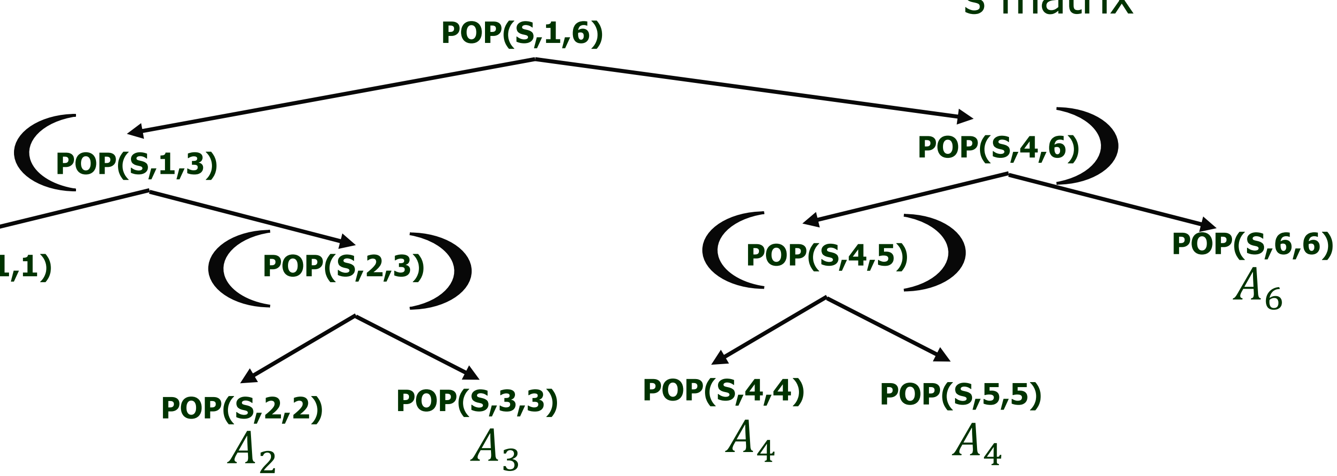

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix




$((A1\ (A2\ A3))\ ((A4\ A5)\ A6))$

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Step 4: Construct / print the optimal solution.

Example 1: Find an optimal parenthesization of a matrix chain product whose sequence of dimension is :

```
PRINT-OPTIMAL-PARENS(s, i, j)
if i = j
   then print "A_i"
   else print "("
        PRINT-OPTIMAL-PARENS(s, i, s[i, j])
        PRINT-OPTIMAL-PARENS(s, s[i, j] + 1, j)
        print ")"
```

| | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 | 3 |
| 2 | | 2 | 3 | 3 | 3 |
| 3 | | | 3 | 3 | 3 |
| 4 | | | | 4 | 5 |
| 5 | | | | | 5 |

s matrix

POP(s, 1, 6), s[1, 6] = 3, (A1A2A3)(A4A5A6)
POP(s, 1, 3), s[1, 3] = 1, ((A1)(A2A3))(A4A5A6)
POP(s, 4, 6), s[4, 6] = 5, ((A1)(A2A3))((A4A5)(A6))
POP(s, 2, 3), s[2, 3] = 2, ((A1)((A2)(A3)))((A4A5)(A6))
POP(s, 4, 5), s[4, 5] = 4, ((A1)((A2)(A3)))(((A4)(A5))(A6))
Hence the product is computed as follows

(A1(A2A3))((A4A5)A6).

# Dynamic Programming

## Problem 4: Matrix Chain Multiplication

Example 2: Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$

Example 3: Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 4, 6, 2, 7 \rangle$

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Self practice

# Thank u