Discrete-time Signals & Systems

TRAN Hoang Tung

Information and Communication Technology (ICT) Department
University of Science and Technology of Hanoi (USTH)

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1 Discrete-time Signals

2 Discrete-time Systems

3 Convolution

4 Difference Equations

1 Discrete-time Signals
- Fundamental Signals
- Operations
- Some Useful Results

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Digital Signal Processing

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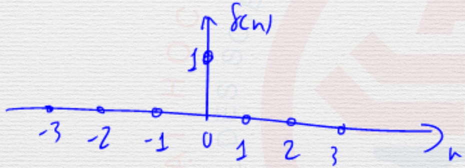
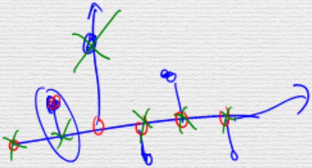
Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$





Delta Signal

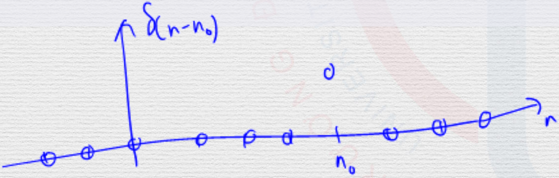
Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

Shifted

Delta Signal

$$\delta(n - n_0) = \begin{cases} 1 & \text{if } n = n_0 \\ 0 & \text{if } n \neq n_0. \end{cases}$$



Digital Signal Processing

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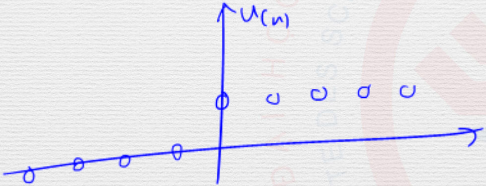
Unit Step Signal

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$



Unit Step Signal

Unit Step Signal

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$

Unit Step Signal

$$u(n - n_0) = \begin{cases} 1 & \text{if } n \geq n_0 \\ 0 & \text{if } n < n_0. \end{cases}$$

Exponential Signal

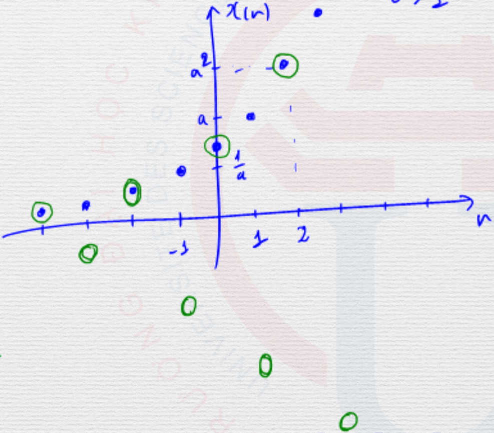
Real-valued Exponential Signal

$n \in \mathbb{Z}$

$$x(n) = a^n$$

$a^0 = 1$

$a > 1$



* $1 > a > 0$
* $a = 1$
* $-1 < a < 0$
* $a = -1$
* $\boxed{a < -1}$

$(-5)^2 = 5^2$

Exponential Signal

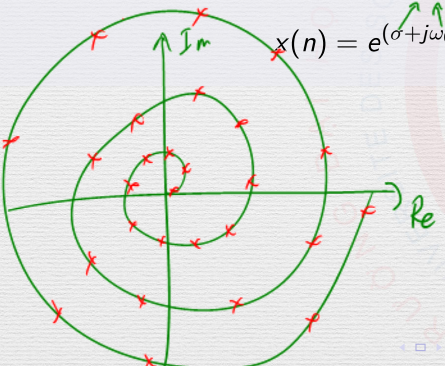
Real-valued Exponential Signal

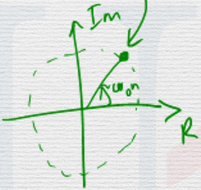
$$x(n) = a^n$$

Complex-valued Exponential Signal

Real

$$x(n) = e^{(\sigma + j\omega_0)n} = e^{\sigma n} \cdot e^{j\omega_0 n}$$





Exponential Signal

Real-valued Exponential Signal

$$x(n) = a^n$$

Complex-valued Exponential Signal

$$x(n) = e^{(\sigma + j\omega_0)n}$$

Periodic Signal

$$x(n) = x(n + N)$$

$x(n) = x(n+5)$

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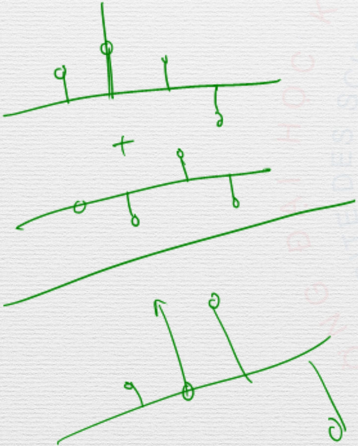
Operations (1)

Signal Addition

$$\{x1(n)\} + \{x2(n)\} = \{x1(n) + x2(n)\}$$



Operations (1)

Signal Addition

$$\{x1(n)\} + \{x2(n)\} = \{x1(n) + x2(n)\}$$

Signal Multiplication

$$\{x1(n)\}.\{x2(n)\} = \{x1(n)x2(n)\}$$

Operations (1)

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Signal Addition

$$\{x1(n)\} + \{x2(n)\} = \{x1(n) + x2(n)\}$$

Signal Multiplication

$$\{x1(n)\}.\{x2(n)\} = \{x1(n)x2(n)\}$$

Signal Scaling

$$\alpha\{x(n)\} = \{\alpha x(n)\}$$

x(a.n)

Operations (2)

Signal Shifting

$$y(n) = \{x(n-k)\}$$

Operations (2)

Signal Shifting

$$y(n) = \{x(n-k)\}$$

Signal Folding

$$y(n) = \{x(-n)\}$$

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Signal Energy

$$E_x = \sum_{-\infty}^{+\infty} |x(n)|^2$$

Operations (3)

Signal Energy

$$E_x = \sum_{-\infty}^{+\infty} |x(n)|^2$$

Signal Power of a periodic $x(n)$

$$P_x = \frac{1}{N} \sum_{0}^{N-1} |x(n)|^2$$

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Some Useful Results

Delta

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)$$

Some Useful Results

Delta

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)$$

Even and odd synthesis

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

Some Useful Results

The Geometric Series

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

$$\alpha^0 + \alpha^1 + \alpha^2 + \ldots + \alpha^{N-1} = \frac{1-\alpha^N}{(1-\alpha)}$$

Some Useful Results

The Geometric Series

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

Correlations

$$r_{x,y}(l) = \sum_{n=-\infty}^{+\infty} x(n)y(n-l)$$

$$r_{x,x}(l) = \sum_{n=-\infty}^{+\infty} x(n)x(n-l)$$

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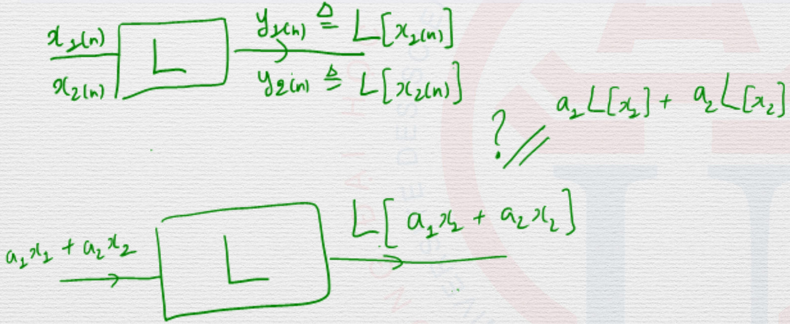
Linear Systems (1)

Linearity

A discrete system $L[\cdot]$ is linear iff

$$L[a_1 x_1(n) + a_2 x_2(n)] = a_1 L[x_1(n)] + a_2 L[x_2(n)]$$



Linear Systems (1)

Linearity

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Time-invariant

$$y(n) = L[x(n)] \rightarrow L[x(n-k)] = y(n-k)$$

Linear Systems (1)

Linearity

A discrete system $L[\cdot]$ is linear iff

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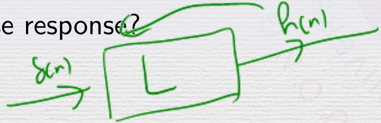
Time-invariant

$$y(n) = L[x(n)] \rightarrow L[x(n-k)] = y(n-k)$$

Linear Time-invariant

Impulse response?



Stability - BIBO

$$|x(n)| < \infty \to |y(n)| < \infty$$

$$\Leftrightarrow \sum_{n=-\infty}^{+\infty} |h(n)| < \infty$$

Linear Systems (2)

Stability - BIBO

$$|x(n)| < \infty \to |y(n)| < \infty$$

$$\Leftrightarrow \sum_{n=-\infty}^{+\infty} |h(n)| < \infty$$

Causality

$$h(n) = 0, n < 0$$

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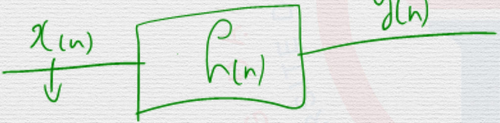
$y(n) = x(n) * h(n)$



$\sum x(k) \cdot \delta(n-k)$

$\downarrow$

$h(n-k)$

Convolution Sum

Convolution

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k) \cdot \delta(n-k)$$

$$y(n) = \sum x(k) \cdot h(n-k)$$

Convolution Sum

Convolution

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

Correlations

$$r_{x,h}(n) = \sum_{k=-\infty}^{+\infty} x(k)h(k-n)$$

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Difference Equations

need examples

An LTI discrete system can also be described by a linear constant coefficient difference equation of the form

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{m=0}^{M} b_m x(n-m)$$

or

$$y(n) = \sum_{m=0}^{M} b_m x(n-m) - \sum_{k=1}^{N} a_k y(n-k)$$

$3y(n) - 5y(n-1) = x(n) + 11 \cdot x(n-1) - x(n+1)$

Function *filter* to solve difference equation

$$y = filter(b, a, x)$$

where

$$b = [b_0, b_1, \ldots, b_M]; a = [a_0, a_1, \ldots, a_N]$$

Octave Implementation

Function *filter* to solve difference equation

$$y = filter(b, a, x)$$

where

$$b = [b_0, b_1, \ldots, b_M]; a = [a_0, a_1, \ldots, a_N]$$

Function *impz* to compute and plot impulse response

$$h = impz(b, a, n)$$

Digital Filters

There are two types of filters:

1. FIR filter (finite-duration impulse response): non-recursive or moving average (MA)
2. IIR filter (infinite-duration impulse response): autoregressive moving average (ARMA)