Digital Signal Processing Exercise Set 1

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Exercises for Section 1.1: Signals

1. Generate and plot each of the following sequences over the indicated interval
   (a) $x(n) = 2\delta(n+2) - \delta(n-4)$, with $-5 \leq n \leq 5$
   (b) $x(n) = n[u(n) - u(n-10)] + 10e^{-0.3(n-10)}[u(n-10) - u(n-20)], 0 \leq n \leq 20$
   (c) $x(n) = cos(0.04\pi n) + 0.2w(n), 0 \leq n \leq 50$, where $w(n)$ is a Gaussian random sequence with zero mean and unit variance
   (d) $x(n) = e^{(-0.1+j0.3)n}, -10 \leq n \leq 10$ (plot its magnitude, phase, the real part, and the imaginary part)
2. Let $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5, 6, 7, 6, 5, 4, 3, 2, 1\}$. Determine and plot
   (a) $x_1(n) = 2x(n-5) - 3x(n+4)$
   (b) $x_2(n) = x(3-n) + x(n)x(n-2)$
3. Let $x(n) = u(n) - u(n-10)$. Decompose $x(n)$ into even and odd components.

Exercises for Section 1.2: Systems

1. Determine whether the following linear systems are time-invariant
   (a) $y(n) = L[x(n)] = 10sin(0.1\pi n)x(n)$
   (b) $y(n) = L[x(n)] = x(n+1) - x(1-n)$
   (c) $y(n) = L[x(n)] = \frac{1}{4}x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

Exercises for Section 1.3: Convolution

1. Let the rectangular pulse $x(n) = u(n) - u(n-10)$ be an input to an LTI system with impulse response
   $$h(n) = 0.9^n u(n)$$
   Determine and plot the output y(n)
2. Determine the convolution $y(n) = x(n) * h(n)$ of the following two sequences
   $$x(n) = [3, 11, 7, \underset{\uparrow}{0}, -1, 4, 2], -3 \leq n \leq 3$$
   $$h(n) = [2, \underset{\uparrow}{3}, 0, -5, 2, 1], -1 \leq n \leq 4$$
3. Let $x(n) = [3, 11, 7, \underset{\uparrow}{0}, -1, 4, 2]$ and $y(n) = x(n-2) + w(n)$ where $w(n)$ is Gaussian sequence with mean 0 and variance 1. Compute the crosscorrelation between $y(n)$ and $x(n)$.

Exercises for Section 1.4: Difference Equations

1. Given the following difference equation
   $$y(n) - y(n-1) + 0.9y(n-2) = x(n)$$
   (a) Calculate and plot the impulse response $h(n)$ at $n = -20, \ldots, 100$
   (b) Calculate and plot the unit step response $h(n)$ at $n = -20, \ldots, 100$
   (c) Is the system specified by $h(n)$ stable?
2. Redo exercise 1, section 1.3 using difference equation.