Digital Signal Processing Exercise 2

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Exercises for Section 2.1: Discrete-time Fourier Transform (DTFT)

1. Determine the DTFT of
$$x(n) = (0.5)^n u(n)$$
and evaluate it at 501 equispaced points between $[0, \pi]$ and plot its magnitude, angle, real, and imaginary parts.

2. Determine the DTFT of the sequence
$$x(n) = \{1, \underset{\uparrow}{2}, 3, 4, 5\}$$
and evaluate it at 501 equispaced points between $[0, \pi]$.

3. Let
$$x(n) = (0.9e^{j\pi/3})^n, 0 \le n \le 10$$
Evaluate and plot its DTFT at 401 frequencies over two periods between $[-2\pi, 2\pi]$

4. Let
$$x(n) = (0.9)^n, -10 \le n \le 10$$
Evaluate and plot its DTFT at 401 frequencies over two periods between $[-2\pi, 2\pi]$

5. Write an Octave function to compute the DTFT of a finite-duration sequence. The template is provided. Verify your function with the above exercises.

Exercises for Section 2.2: DTFT Properties

1. Let $x(n) = cos(\pi n/2), 0 \le n \le 100$ and $y(n) = e^{j\pi n/4}x(n)$. Verify the frequency shifting property.

2. Let $x(n)$ be a complex-valued random sequence over $-5 \le n \le 10$ with real and imaginary parts uniformly distributed between $[0, 1]$. Verify the conjugation property.

3. Let
$$x(n) = sin(\pi n/2), -5 \le n \le 10$$
Compute the even and odd parts of $x(n)$ and then evaluate their discrete-time Fourier transforms.

Exercises for Section 2.3: Frequency Response

1. Given a system characterized by $h(n) = (0.9)^n u(n)$
   (a) Determine the frequency response $H(e^{j\omega})$. Plot the magnitude and the phase responses.
   (b) Let an input to the system be $x(n) = 0.1u(n)$. Determine the steady-state response $y_{ss}(n)$.

2. An LTI system is specified by the difference equation
$$y(n) = 0.8y(n-1) + x(n)$$
Determine $H(e^{j\omega})$

3. A 3rd-order lowpass filter is described by the difference equation
$$y(n) = 0.0181x(n)+0.0543x(n-1)+0.0543x(n-2)+0.0181x(n-3)+1.76y(n-1)-1.1829y(n-2)+0.2781y(n-3)$$
Plot the magnitude and the phase response of this filter, and verify that it is a lowpass filter.

Exercises for Section 2.4: Sampling & Reconstruction

Let $x_a(t) = e^{-1000|t|}$, using the approximation $e^{-5} \approx 0$, $x_s(t)$ could be approximated by a finite-duration signal over $[-5, 5]$ msec. Its Fourier transform is determined and plotted using *ex09.m*.

1. Sampling: we will sample $x_a(t)$ at 2 different sampling frequencies.
   (a) $F_s = 5000$ samples/sec to obtain $x_1(n)$. Determine and plot $X_1(e^{j\omega})$.
   (b) $F_s = 1000$ samples/sec to obtain $x_2(n)$. Determine and plot $X_2(e^{j\omega})$.
2. Reconstruction
   (a) From the samples $x_1(n)$, reconstruct $x_a(t)$ and comment on the results.
   (b) From the samples $x_2(n)$, reconstruct $x_a(t)$ and comment on the results.