Digital Signal Processing   Exercise 3: z-Transform

TRAN Hoang Tung

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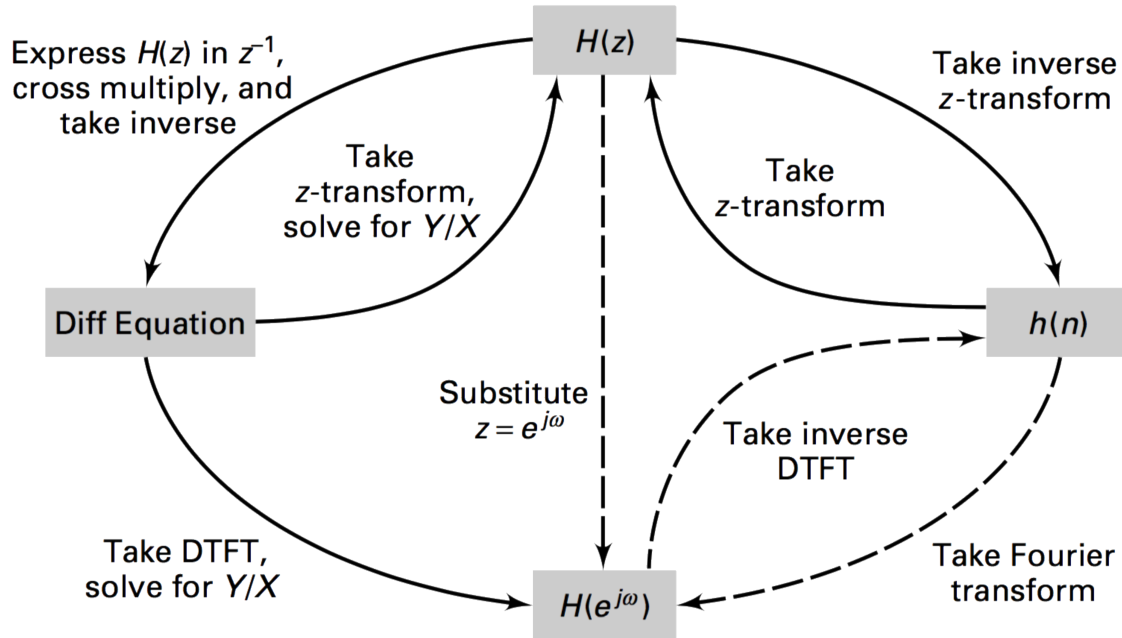
Sequence	*Transform*	*ROC*
$\delta(n)$	1	$\forall z$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-b^n u(-n-1)$	$\frac{1}{1-bz^{-1}}$	$\|z\| < \|b\|$
$[a^n \sin\omega_0 n]\, u(n)$	$\frac{(a\sin\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
$[a^n \cos\omega_0 n]\, u(n)$	$\frac{1-(a\cos\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-nb^n u(-n-1)$	$\frac{bz^{-1}}{(1-bz^{-1})^2}$	$\|z\| < \|b\|$

1. Given that
$$H(z) = \frac{z+1}{z^2 - 0.9z + 0.81}$$
is a causal system, find
   (a) its frequency response representation
   (b) its difference equation representation
   (c) its impulse response (Octave function: *filter*)

2. Given a causal system
$$y(n) = 0.9y(n-1) + x(n)$$
   (a) Determine $H(z)$ and sketch its pole-zero plot (Octave function: *zplane*)
   (b) Plot $H(e^{j\omega})$ and $\angle H(e^{j\omega})$
   (c) Determine the impulse response $h(n)$

3. Determine the z-transform of the following sequences. Indicate the region of convergence (ROC) for each sequence and verify the z-transform expression using Octave
   (a) $x_1(n) = [3, 2, \underset{\uparrow}{1}, -2, -3]$
   (b) $x_2(n) = (0.8)^n u(n-2)$
   (c) $x_3(n) = [(0.5)^n + (-0.8)^n]u(n)$
   (d) $x_4(n) = (n+1)3^n u(n)$


$H(z)$
Express $H(z)$ in z^{-1}, cross multiply, and take inverse
Take inverse z-transform
Take z-transform, solve for Y/X
Take z-transform
Diff Equation
$h(n)$
Substitute $z = e^{j\omega}$
Take inverse DTFT
Take DTFT, solve for Y/X
Take Fourier transform
$H(e^{j\omega})$