Thermal Analysis of small satellites (from University of Georgia) - also used on their CubeSat

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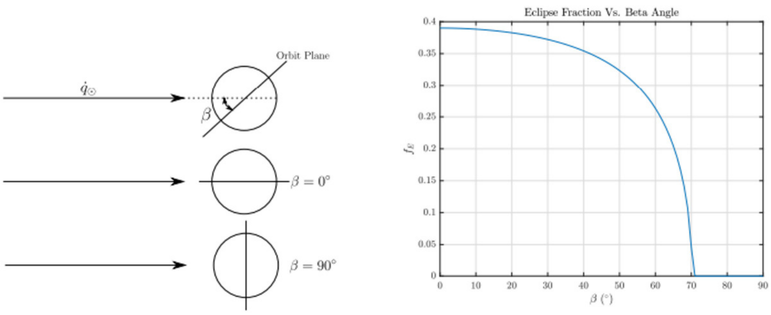
Ideally code this up on Python and try to adapt to Haskell to avoid data damage (also Haskell is faster at performing this type of data processing).
-> then log data and plot using MatPlotLib in Python maybe

Variable definitions:

β	=	Beta angle, superscript star (*) denotes critical beta angle (°)
R	=	Radius of Earth, $\sim$ 6873km
h	=	Altitude (m)
f_E	=	Eclipse fraction (unitless)
a	=	Albedo factor (unitless)
$\dot{q}$	=	Heat flux ($\mathrm{W \cdot m^{-2}}$)
$\dot{Q}$	=	Rate of heat transfer (W)
$\hat{s}$	=	Solar heating rate step multiplier (unitless)
τ	=	Orbital period (s)
$\alpha,\ \varepsilon$	=	Absorptivity, emissivity (unitless)
T	=	Temperature (K, unless otherwise noted)
t	=	Time (s)
σ	=	Stefan-Boltzmann constant, $\sim 5.67 \cdot 10^{-8} \mathrm{W \cdot m^{-2} \cdot K^{-4}}$
A	=	Area ($\mathrm{m^2}$)
F	=	View factor (unitless)
$\odot$	=	Subscript, astronomical sign for Sun
$zen,\ nad$	=	Subscripts for Zenith and Nadir sides
$-/+v$	=	Subscripts for negative and positive velocity pointing sides
N/S	=	Subscripts for North and South pointing sides

Beta angle - the angle between sun to satellite vector and the orbiting plane.

Eclipse fraction is the fraction of the orbit for which the satellite is behind the earth so will not be actively being heated.



(a) Geometry **(b) Eclipse fraction Vs. beta angle**

Fig. 1 Geometric definition of beta angle

For a beta angle of ±90°, the sun-vector "sees" the entire orbit plane as a flat disk. This has the direct consequence that the satellite in orbit will be constantly exposed to sunlight, spending no time in eclipse. As the beta angle decreases the orbit plane will eventually intersect the horizon of the Earth at which point eclipse will be a factor to consider. This angle is defined as the critical beta angle [4]

$$\beta^* = \sin^{-1}\left(\frac{R}{R+h}\right) \tag{1}$$

This critical beta angle will give a piece-wise function describing the fraction of time in an orbit spent in eclipse

$$f_E = \begin{cases} \frac{1}{180°}\cos^{-1}\left(\frac{\sqrt{h^2+2Rh}}{(R+h)\cos(\beta)}\right) & |\beta| < \beta^* \\ 0 & |\beta| \geq \beta^* \end{cases} \tag{2}$$

The eclipse fraction itself governs the governing heat transfer equation on the satellite structure, as it dictates the time for which some heat transfer parameters are active. A plot of Eq. 2 is shown in Figure 1b

The heating rates:

The beta angle affects the heat transfer caused by the sun as well.

Albedo - Sunlight reflected by earth;s surface
IR - Infrafred from earth

B. Heating Rates

Beyond orbital geometry, the beta angle also has an implication on heating rates in orbit. Besides solar radiation, a spacecraft in orbit will also experience heating from Earth infrared (IR), sunlight reflected from

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the surface (Albedo) and heat generated internally. Both IR and Albedo heating rates are influenced by the beta angle. The following values are used in modeling albedo and IR, providing 3.3-σ accuracy values over a 90 minute period [4].

$$a = \begin{cases} 0.14 & \beta < 30° \\ 0.19 & \beta \geq 30° \end{cases} \quad (3)$$

$$\dot{q}_{IR} = \begin{cases} 228\mathrm{W}\cdot\mathrm{m}^{-2} & \beta < 30° \\ 218\mathrm{W}\cdot\mathrm{m}^{-2} & \beta \geq 30° \end{cases} \quad (4)$$

View factors

A view factor is the proportion of radiation which leaves the surface after striking the surface. This will be different for each 6 sides of the CubeSat.
These factors are given by the following piecewise functions.

B. View Factors

Because the six-node model lacks the symmetry assumed in the single-node model, view factors are used to determine which sides are receiving solar radiation at what time. The view factor from one surface to another is defined as [3]

$$F_{1\rightarrow 2} = \frac{\cos(\theta_1)\cos(\theta_2)}{\pi r^2} dA_2 \quad (10)$$

Wherein the assumption can be made that $dA_2 \approx A_s$, because A_1 is the surface area of the Sun, which is much greater. Furthermore, because $\theta_1 << 1$, $\cos(\theta_1) \approx 1$. To retain the solar heating rates as defined in the single-node analysis, any distance dependency is removed.

The incident angle is a function of time, as the satellite progresses through the orbit, and must be piece-wise as some sides can be out of sunlight under certain conditions. A correction can be made to the view factors to account for the beta angle. As the beta angle increases, more of either the North or South faces becomes visible as the spacecraft progresses through an orbit. A geometric derivation of view factors leads to the following set of piece-wise equations

$$F_{zen} = \begin{cases} \cos\left(\frac{2\pi}{\tau}t\right)\cos(\beta) & \frac{\tau}{4} > t > \frac{3\tau}{4} \\ 0 & \text{else} \end{cases} \quad (11)$$

$$F_{-v} = \begin{cases} \sin\left(\frac{2\pi}{\tau}t\right)\cos(\beta) & t < \frac{\tau}{2}(1 - f_E) \\ 0 & \text{else} \end{cases} \quad (12)$$

$$F_{+v} = \begin{cases} -\sin\left(\frac{2\pi}{\tau}t\right)\cos(\beta) & t > \frac{\tau}{2}(1 + f_E) \\ 0 & \text{else} \end{cases} \quad (13)$$

$$F_{nad} = \begin{cases} -\cos\left(\frac{2\pi}{\tau}t\right)\cos(\beta) & \begin{cases} \frac{\tau}{4} < t < \frac{\tau}{2}(1 - f_E) \\ \frac{\tau}{2}(1 + f_E) < t < \frac{3\tau}{4} \end{cases} \\ 0 & \text{else} \end{cases} \quad (14)$$

$$F_{N/S} = \begin{cases} \sin(\beta) & \frac{\tau}{2}(1 - f_E) > t > \frac{\tau}{2}(1 + f_E) \\ 0 & \text{else} \end{cases} \quad (15)$$

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Heat transfer equations

C. Heat Transfer Model

As in the single-node analysis, a governing energy balance can be set up. In this case, it will involve a set of six equations, each modeling the energy balance for each of the six sides of the box. This must involve a conduction model between each of the six sides. This can be captured using a thermal contact conductance coefficient k, and the interface area between the relevant sides. Contact conductance is well documented in literature, where it is controlled for contact pressure and surface roughness. To create a conservative model, a low value of 400 $\mathrm{Wm^{-2}K^{-1}}$ based on [4] is chosen for this analysis. The set of heat transfer equations is as follows:

$$\dot{Q}_{zen} = F_{zen}A_{zen}\dot{q}_{\odot}\alpha + k\sum_{i=1}^{4} A_i(T_i - T_{zen}) - \sigma\varepsilon_{zen}A_{zen}T_{zen}^4 \tag{16}$$

$$\dot{Q}_{-v} = F_{-v}A_{-v}\dot{q}_{\odot}\alpha + k\sum_{i=1}^{4} A_i(T_i - T_{-v}) - \sigma\varepsilon_{-v}A_{-v}T_{-v}^4 \tag{17}$$

$$\dot{Q}_{+v} = F_{+v}A_{+v}\dot{q}_{\odot}\alpha + k\sum_{i=1}^{4} A_i(T_i - T_{+v}) - \sigma\varepsilon_{+v}A_{+v}T_{+v}^4 \tag{18}$$

$$\dot{Q}_{nad} = (F_{nad} + a)A_{zen}\dot{q}_{\odot}\alpha + \dot{q}_{IR}A_{nad} + k\sum_{i=1}^{4} A_i(T_i - T_{nad}) - \sigma\varepsilon_{nad}A_{nad}T_{nad}^4 \tag{19}$$

$$\dot{Q}_{N} = F_{N}A_{N}\dot{q}_{\odot}\alpha + k\sum_{i=1}^{4} A_i(T_i - T_{N}) - \sigma\varepsilon_{N}A_{N}T_{N}^4 \tag{20}$$

$$\dot{Q}_{S} = k\sum_{i=1}^{4} A_i(T_i - T_{zen}) - \sigma\varepsilon_{S}A_{S}T_{S}^4 \tag{21}$$

Final equation

good initial estimate.

To solve this model for temperature, it can be discretely integrated using the specific heat formula

$$\dot{Q} = c_p m\frac{dT}{dt} \approx c_p m\frac{\Delta T}{\Delta t} \tag{8}$$

which requires an assumption of material for the specific heat. Since the structure of the satellite is primarily Aluminium 6061-T6, this is the value assumed for the specific heat. The mass in this equation is taken to be the maximum mass of a 3U CubeSat (4 kg).

For smaller time-steps the approximation in Eq. 8 becomes more accurate, as the change in temperature over time approaches the exact derivative as the time-step approaches zero. This equation can be solved for temperature, noting that the total heat rate is a function of time.

$$T_{i+1} = T_i + \frac{\Delta t}{c_p m}\dot{Q}_i \tag{9}$$

Use euler's method for numeric integration in order to solve this differential equation, maybe look into alternate ways to numerically solve this as well...