MATH 2004: Multivariable Calculus Equations

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Chapter 1

Trig Identities

$cosh\, t = \frac{1}{2}(e^t + e^{-t})$

$sinh\, t = \frac{1}{2}(e^t - e^{-t})$

$cos^2 x = \frac{1+cos2x}{2}$

$sin^2 x = \frac{1-cos2x}{2}$

$2sinxcosx = sin2x$

$sinasinb = \frac{cos(a-b)-cos(a+b)}{2}$

$cosacosb = \frac{cos(a+b)+cos(a-b)}{2}$

$sinacosb = \frac{sin(a+b)+sin(a-b)}{2}$

Trig Integral

$\int sin^m x cos^n x dx:$

- If m is odd, let $u = cosx$
- If n is odd, let $u = sinx$
- If both are even, then use half-angle formula

Half-angle Formula

$sin(\frac{\alpha}{2}) = \pm\sqrt{\frac{1-cos\alpha}{2}}$

$cos(\frac{\alpha}{2}) = \pm\sqrt{\frac{1+cos\alpha}{2}}$

Trig Substitution

$\sqrt{a^2-x^2} \to x = asin\theta, \frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sqrt{a^2+x^2} \to x = atan\theta, \frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sqrt{x^2-a^2} \to x = asec\theta, \frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

1.1-1.6

Length (norm,magnitude) $|(a,b,c)| = \sqrt{a^2+b^2+c^2}$

Unit Vector $|\vec{u}| = 1$

1.7-1.10

Dot Product $\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

Orthogonal $\vec{u} \perp \vec{v}$ if $\vec{u}\cdot\vec{v} = 0$

Angle $cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}, 0 \le \theta\pi$

$cos\alpha = \frac{\vec{u}\cdot\vec{i}}{|\vec{u}||\vec{i}|}$

$cos\beta = \frac{\vec{u}\cdot\vec{j}}{|\vec{u}||\vec{j}|}$

$cos\gamma = \frac{\vec{u}\cdot\vec{k}}{|\vec{u}||\vec{k}|}$

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1, \frac{\vec{u}}{|\vec{u}|} = (cos\alpha, cos\beta, cos\gamma)$

Cross Product

$\vec{u}\times\vec{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_3 - u_2v_1)$

Area of Parallelogram $A = |\vec{u}\times\vec{v}|$

Area of Triangle $A = \frac{1}{2}|\vec{u}\times\vec{v}|$

Volume of Parallelopiped $V = |\vec{u}\cdot(\vec{v}\times\vec{w})|$

Chapter 2

2.1-2.5 Lines and Planes

Equation of Line $\vec{r}(t) = P + t\vec{v}$

Line Segment $\vec{r}(t) = (1-t)\vec{P} + t\vec{Q}$

2.6 Rotations/Translations in Plane

Counter-Clockwise

$\begin{pmatrix} x' \\ y' \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xcos\theta - ysin\theta \\ xsin\theta + ycos\theta \end{pmatrix}, A = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

$A^{-1} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$

Translated Origin

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x-h \\ y-k \end{pmatrix}$

2.7-2.8 Parametric Curves

Parabola $(x-x_0)^2 = 4p(y-y_0)$, $y = \frac{1}{4p}(t-x_0)^2 + y_0$

Ellipse $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{a^2} = 1,$

$x = x_0 + a\cos t, y = y_0 + b\sin t$

Hyperbola $\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{a^2} = 1,$

$x = x_0 + a\sec t, y = y_0 + b\tan t, \pi < t < \pi$

General Conic Sections

If $B^2 - 4AC = 0$, either parabola, 2 parallel lines, 1 line, or no curve

If $B^2 - 4AC > 0$, either hyperbola, or 2 intersecting lines

If $B^2 - 4AC < 0$, either ellipse, circle, point, or no curve

Derivatives

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$

2.9 Applications to Area Problems

Area under Parametric Curve

$$A = \int_\alpha^\beta y(t)x'(t)\,dt$$

Area of region R enclosed by C

$$A = |\int_\alpha^\beta y(t)x'(t)\,dt|$$

Arc Length

$$L = \int_\alpha^\beta \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$$

2.11-2.14 Polar Coordinates

Polar $\rightleftharpoons$ Cartesian

$x = rcos\theta, y = rsin\theta, r = \sqrt{x^2+y^2}, tan\theta = \frac{y}{x}$

Derivative

$\frac{dy}{dx} = \frac{r'_\theta sin\theta + rcos\theta}{r'_\theta cos\theta - rsin\theta}$

Area of region bounded by $r = f(\theta)$

$$A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta$$

Area of more general region

$$A = \int_\alpha^\beta \frac{1}{2}(r_o^2 - r_i^2)\,d\theta$$

Arc Length

$$L = \int_\alpha^\beta \sqrt{(r'_\theta)^2 + r^2}\,d\theta$$

Chapter 3

3.2-3.3 Partial Derivatives

$$z_x = \frac{\partial z}{\partial x} := \frac{\partial f}{\partial x} := D_x f := \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$

$$z_y = \frac{\partial z}{\partial y} := \frac{\partial f}{\partial y} := D_y f := \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$

$$f_x(x,y,z)$$

$$= \frac{\partial w}{\partial x} := \frac{\partial f}{\partial x} := D_x f := \lim_{h\to 0}\frac{f(x+h,y,z)-f(x,y,z)}{h}$$

3.5 Directional Derivatives and Gradients

$f_{\vec{u}}(x_0,y_0)$ *or* $D_{\vec{u}}f(x_0,y_0) = (f_x(x_0,y_0)u_1 + f_y(x_0,y_0)u_2)$

$\nabla f(x_0,y_0) = (f_x(x_0,y_0))$

$f_{\vec{u}}(x_0,y_0,z_0) =$

$(f_x(x_0,y_0,z_0)u_1 + f_y(x_0,y_0,z_0)u_2 + f_z(x_0,y_0,z_0)u_3))$

$\nabla f(x_0,y_0,z_0) = (f_x(x_0,y_0,z_0))$