Elements of Engineering II - Tutorial 4

1 Find the average value of the periodic waveform in the fig 1 and fig 2.

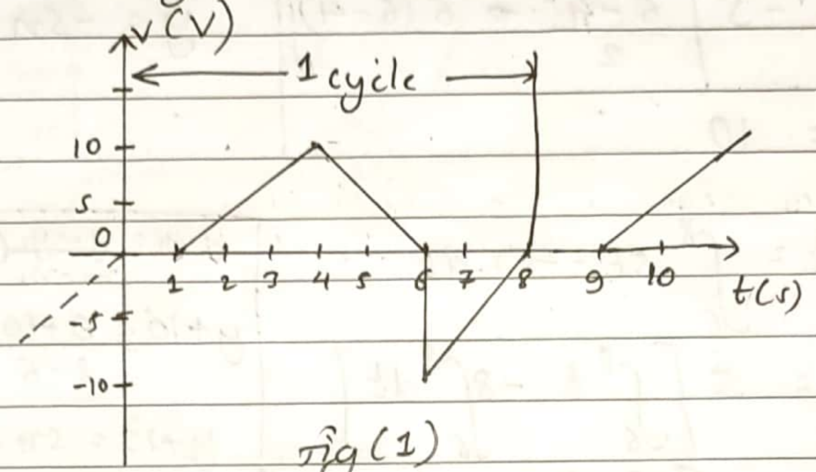

fig (1)

Solution

We know,

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{8}\int_0^8 v(t)\,dt$$

$$V_{avg} = \frac{1}{8}\left[\int_0^1 v(t)dt + \int_1^4 v(t)dt + \int_4^6 v(t)dt + \int_6^8 v(t)dt\right] \quad \text{--- (1)}$$

Now,

i) $\int_0^1 v(t)\,dt = 0$

ii) $\int_1^4 v(t)\,dt = \int_1^4 \frac{10t-10}{3}\,dt$

$= \frac{10}{3}\left[\int_1^4 t\,dt - \int_1^4 dt\right]$

$= \frac{10}{3} \times \left[\frac{4^2-1^2}{2} - 3\right]$

$= 15$ #

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{10-0}{4-1}(x-1)$

$y = \frac{10x - 10}{3}$

iii) $\int_4^6 v(t)\,dt = \int_4^6 -5t+30\,dt$

$= -5\left[\int_4^6 t\,dt - \int_4^6 6\,dt\right]$

$= -5\left[\frac{6^2-4^2}{2} - 6(6-4)\right]$

$= 10$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-10 = \frac{0-10}{6-4}(x-4)$

$y-10 = -5x+20$

$y = -5x+30$

iv) $\int_6^8 v(t)\,dt = \int_6^8 5t-40\,dt$

$= 5\left[\int_6^8 t - 8\int_6^8 dt\right]$

$= 5\left[\frac{8^2-6^2}{2} - 8(8-6)\right]$

$= -10$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y+10 = \frac{0+10}{8-6}(x-6)$

$y+10 = 5x-30$

$y = 5x-40$

Now eqⁿ (i) Becomes

$V_{Avg} = \frac{1}{8}\left[0+15+10-10\right]$

$= \frac{1}{8}\times 15$

$= 1.875\ V$

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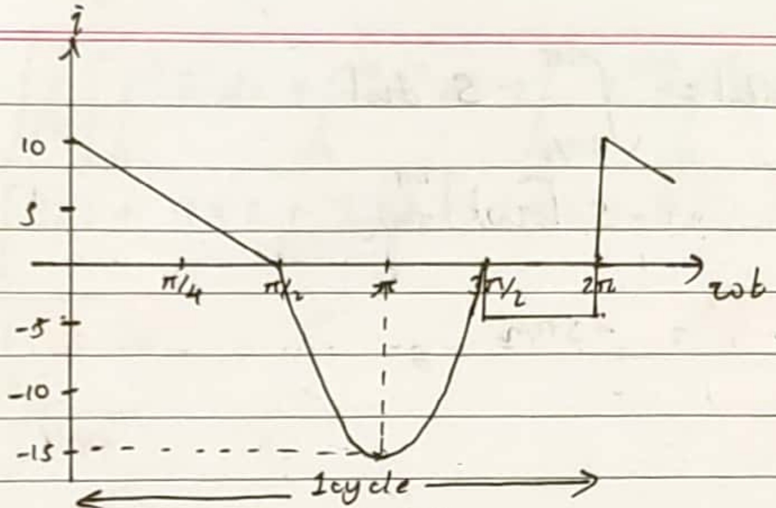


Fig (2)

Solution

We know

$$I_{avg} = \frac{1}{T}\int_0^T i(\omega t)\, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} i(\omega t)\, d\omega t$$

$$I_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi/2} i(\omega t)\, d\omega t + \int_{\pi/2}^{3\pi/2} i(\omega t)\, d\omega t + \int_{3\pi/2}^{2\pi} i(\omega t)\, d\omega t\right] \quad \text{—— (1)}$$

Now,

i) $$\int_0^{\pi/2} i(\omega t)\, d\omega t = \int_0^{\pi/2} \frac{-20\omega t + 10\pi}{\pi}\, d\omega t$$

$$= -10\left[\int_0^{\pi/2} \frac{2\omega t}{\pi} - \frac{\pi}{\pi}\, d\omega t\right]$$

$$= -10\left[\frac{\cancel{2}}{\pi}\frac{\left[(\pi/2)^2 - 0^2\right]}{\cancel{2}} - 1(\pi/2 - 0)\right]$$

$$= 5\pi/2$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 10 = \frac{0 - 10}{\pi/2 - 0}(x - 0)$$

$$y = \frac{-20x + 10\pi}{\pi}$$

ii) $$\int_{\pi/2}^{3\pi/2} i(\omega t)\, d\omega t = \int_{\pi/2}^{3\pi/2} 15\cos\omega t\, d\omega t$$

$$= 15\int_{\pi/2}^{3\pi/2} \cos\omega t\, d\omega t$$

$$= 15\left[\sin\omega t\right]_{\pi/2}^{3\pi/2}$$

$$= \cancel{0.821128}\ -30$$

~~y-y₁~~ sinosuidal wave

$$i(\omega t) = I_0\cos\omega t$$

$$= 15\cos\omega t$$

iii) $\int_{3\pi/2}^{2\pi} i(\omega t)\, d\omega t = \int_{3\pi/2}^{2\pi} -5\, d\omega t$

$= -5 [\omega t]_{3\pi/2}^{2\pi}$

$= -5\pi/2$

Now

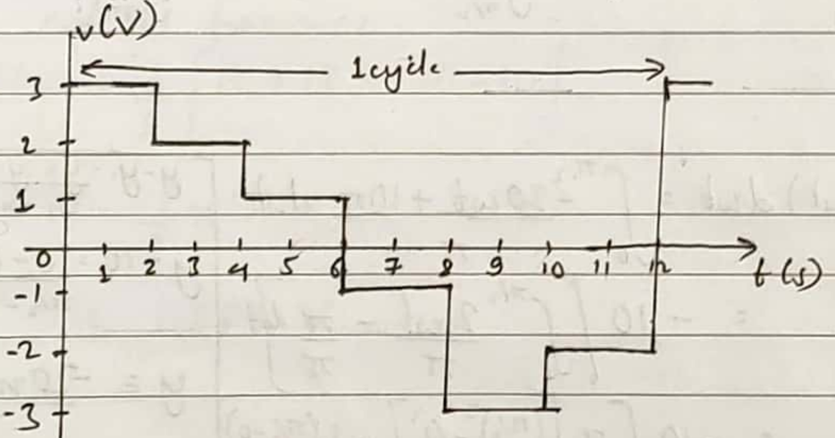
Eqⁿ (i) becomes

$$I_{avg} = \frac{1}{2\pi}\left[5\pi/2 - 30 - 5\pi/2\right]$$

$$= -\frac{30}{2\pi}$$

$$= -4.775 \text{ mA}$$

2) Find the RMS value of the periodic waveform of fig 3



Solution,

We know

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

$$= \sqrt{\frac{1}{12}\left[\int_0^2 v^2(t)dt + \int_2^4 v^2(t)dt + \int_4^6 v^2(t)dt + \int_6^8 v^2(t)dt + \int_8^{10} v^2(t)dt + \int_{10}^{12} v^2(t)dt\right]}$$

$$V_{rms} = \sqrt{\frac{1}{12}\left[\int_0^2 3^2 dt + \int_2^4 2^2 dt + \int_4^6 1^2 dt + \int_6^8 (-1)^2 dt + \int_8^{10} (-3)^2 dt + \int_{10}^{12} (-2)^2 dt\right]}$$

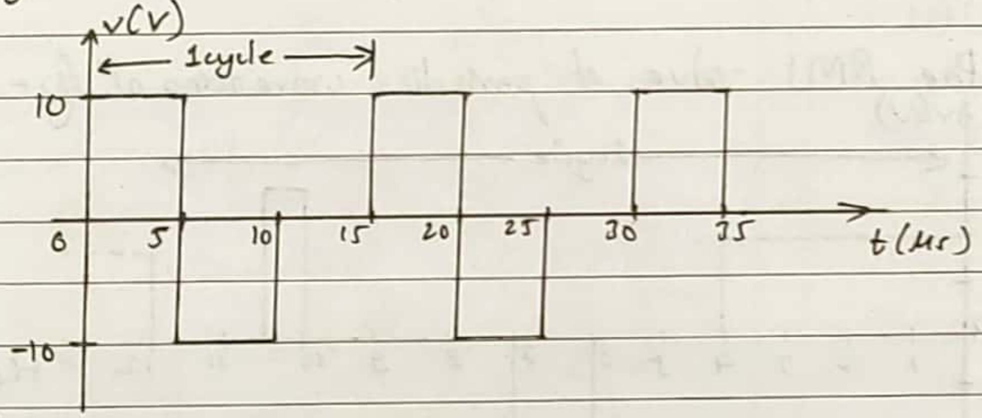
$$= \sqrt{\frac{1}{12}\left[9\times2 + 4\times2 + 1\times2 + 1\times2 + 9\times2 + 4\times2\right]}$$

$$= \sqrt{\frac{1}{12}\times 56}$$

$$= \sqrt{14/3}$$

$$= 2.16\ V$$ #

3 Find the average and RMS value of periodic waveform of fig - 4



Solution

We know

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{15}\left[\int_0^{15} v(t)\,dt\right]$$

$$= \frac{1}{15}\left[\int_0^5 v(t)\,dt + \int_5^{10} v(t)\,dt + \int_{10}^{15} v(t)\,dt\right]$$

$$= \frac{1}{15}\left[\int_0^5 10\,dt + \int_5^{10} -10\,dt + \int_{10}^{15} 0\,dt\right]$$

$$= \frac{1}{15}\left[10\times5 - 10\times5 + 0\right]$$

$$= 0\ V$$

And

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{15}\int_0^{15} v^2(t)\,dt}$$

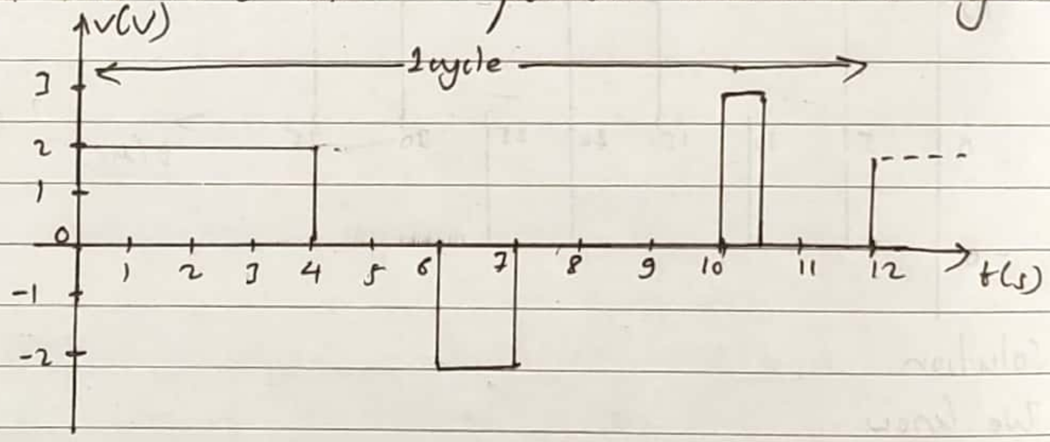
$$= \sqrt{\frac{1}{15}\left[\int_0^5 10^2 dt + \int_5^{10} (-10)^2 dt + \int_{10}^{15} 0^2 dt\right]}$$

$$= \sqrt{\frac{1}{15}\left[100\times 5 + 100\times 5 + 0\right]}$$

$$= \sqrt{\frac{1000}{15}}$$

$$= 8.165\,V$$

4) Find the RMS value of periodic waveform of fig - 5



Sol^n

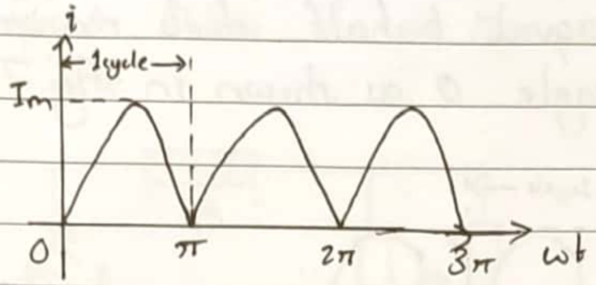
$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{12}\left[\int_0^4 2^2 dt + \int_4^6 0^2 dt + \int_6^7 (-2)^2 dt + \int_7^{10} 0^2 dt + \int_{10}^{10.5} 3^2 dt + \int_{10.5}^{12} 0^2 dt\right]}$$

$$= \sqrt{\frac{1}{12}\left[4\times 4 + 0 + 4\times 1 + 0 + 9\times 0.5 + 0\right]}$$

$$= \sqrt{\frac{1}{12}\times 24.5} = 1.428\,V \;\#$$

5 Find the average & RMS value for the waveform of fig - 6



Solution.

We know.

$$I_{avg} = \frac{1}{T}\int_0^T i(\omega t)\, d\omega t$$

sinusoidal wave eq^n

$\boxed{i(\omega t) = I_0 \sin\omega t}$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t \; d\omega t$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\omega t \; d\omega t$$

$$= \frac{I_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{I_m}{\pi} \times 2 = \frac{2 I_m}{\pi} \; \#$$

And,

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\omega t)\, d\omega t}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\omega t \; d\omega t}$$

$$= \sqrt{\frac{1}{\pi} I_m^2 \int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

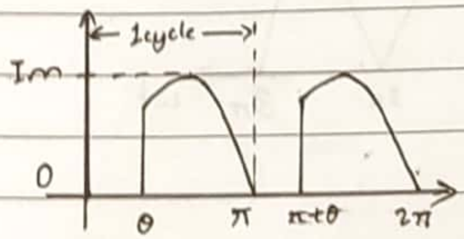
$$= I_m\sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} 1 \, d\omega t - \int_0^{\pi} \cos 2\omega t \; d\omega t\right]}$$

$$= I_m\sqrt{\frac{1}{2\pi}\left[\pi - \left[\frac{\sin 2\omega t}{2}\right]_0^{\pi}\right]}$$

$$= I_m\sqrt{\frac{1}{2\pi}\left[\pi - 0\right]}$$

$$= I_m\sqrt{\frac{1}{2}} = \frac{I_m}{\sqrt{2}} \; \#$$

6 A delayed full wave rectified sinusoidal current wave has an average value equal to half of its maximum value. Find the delayed angle θ as shown in fig 7.



Solution,

According to question

$$I_{avg} = \frac{I_m}{2} \quad \text{— (i)}$$

Also,

$$I_{vag} = \frac{1}{T}\int_0^T i(\omega t)\, d\omega t$$

$$= \frac{1}{\pi}\int_0^\pi i(\theta)\, d\theta$$

$$= \frac{1}{\pi}\left[\int_0^\theta 0\, d\theta + \int_\theta^\pi I_m \sin\theta\, d\theta\right]$$

$$= \frac{1}{\pi}\left[0 + I_m[-\cos\theta]_\theta^\pi\right]$$

$$= \frac{I_m}{\pi}\left[-\cos\pi + \cos\theta\right]$$

$$I_{avg} = \frac{I_m}{\pi}\left[1+\cos\theta\right] \quad \text{— (ii)}$$

From eq^n (i) & (ii), we get

$$\frac{I_m}{2} = \frac{I_m}{\pi}(1+\cos\theta)$$

$$\cancel{\pi = 2} \qquad \frac{\pi}{2} = 1+\cos\theta$$

$$\cos\theta = \pi/2 - 1$$

$$\theta = \cos^{-1}(\pi/2 - 1)$$

$$\therefore \theta = 55.19°$$

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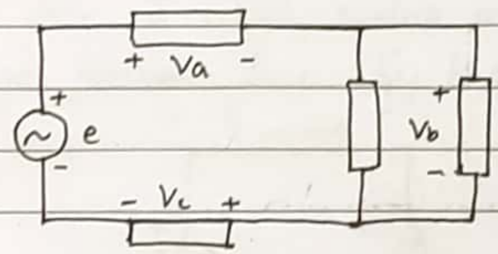
7 Find the sinusoidal expression for voltage V_c of the system shown in fig-8 below if $e = 120 \sin(\omega t + 30^\circ)$, $V_a = 60 \sin \omega t$ and $V_b = 30 \sin \omega t$.



Solution

Given,

$e = 120 \sin(\omega t + 30^\circ)$

$V_a = 60 \sin \omega t$

$V_b = 30 \sin \omega t$

Here

$E_{rms} = \frac{120}{\sqrt{2}} = 84.85$

Then, $e = 84.85 \angle 30$

Similarly $V_a = 42.42 \angle 0^\circ$

$V_b = 21.21 \angle 0^\circ$

Apply KVL, we get

$e - V_a - V_b - V_c = 0$

or, $e - V_a - V_b = V_c$

$V_c = (84.85 \angle 30) - (42.42 \angle 0^\circ) - (21.21 \angle 0^\circ)$

$\therefore V_c = 43.55 \angle 76.92$

$\therefore$ ~~Vc~~ $V_0 = V_{RMS} \times \sqrt{2}$

$= 43.55 \times \sqrt{2}$

$= 61.58$

$\therefore V_c = 61.58 \sin(\omega t + 76.92)$

Elements of Engineering II - Tutorial 5

1) In figure - 1, $e = 100\sin(157t + 30°)$, find the sinusoidal expression for I, the value of the inductance L and the average power loss by the inductor.

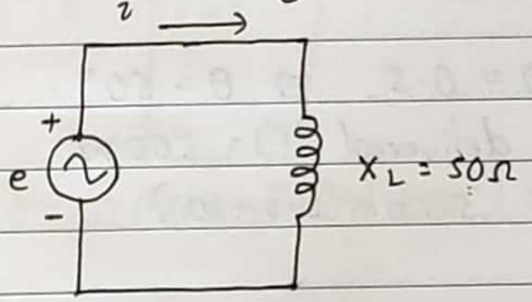
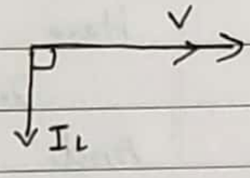


fig-1

Phasor diagram



Solution

Given

$e = 100\sin(157t + 30°)$ — ①

$X_L = 50\Omega$

Comparing eq^n ① with $e = E_m\sin(\omega t + \phi)$

$E_m = 100$, $\omega = 157$, $\phi = 30°$

Then

$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 50\sqrt{2}$

And,

$Z = j\omega L = 50j = 50\angle 90°$

So,

$I = \frac{E_{rms}}{Z} = \frac{50\sqrt{2}\angle 30°}{50\angle 90°}$

$= \sqrt{2}\angle -60°$

Value of L

i) $X_L = \omega L$

$L = \frac{50}{157} = 318.47\,mH$

ii) Average power loss by the inductor

Avg. power $= \frac{I_m}{\sqrt{2}} \times \frac{V_m}{\sqrt{2}}\cos\theta$

$= \frac{2 \times 100}{2}\cos\pi/2$

$= 0\,W$ #

So,

$I_{rms} = \sqrt{2} \Rightarrow I_m = I_{rms} \times \sqrt{2} = \sqrt{2} \times \sqrt{2} = 2$

Therefore,

$\dot{I} = 2\sin(157t + (-60°))$

$\therefore i = 2\sin(157t - 60°)$ (lag)

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2. The power factor of a circuit is 0.5 lagging. The power delivered in watts is 500. If the input voltage is $50 \sin(\omega t + 10^\circ)$, find the sinusoidal expression for the input current.

Solution.

Given

$$\cos\theta = 0.5 \Rightarrow \theta = 60^\circ$$

Power delivered $(P) = 500W$

$$v = 50 \sin(\omega t + 10^\circ)$$

Here,

$$V_m = 50, \quad \phi = 10^\circ$$

And,

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 25\sqrt{2}\ V$$

Now,

$$P = V_{rms} \cdot I_{rms} \cos\theta$$

$$500 = 25\sqrt{2} \times I_{rms} \times 0.5$$

$$\therefore I_{rms} = 20\sqrt{2}\ A$$

And,

$$I_m = I_{rms} \times \sqrt{2}$$

$$= 20\sqrt{2} \times \sqrt{2}$$

$$= 40\ A$$

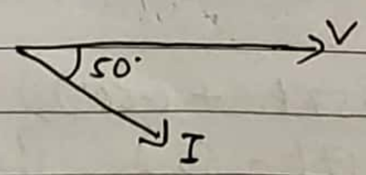
Then,

$$i = I_m(\sin \omega t + \phi)$$

$$i = 40(\sin \omega t + 10 - 60^\circ)$$

$$\therefore i = 40(\sin \omega t - 50^\circ) \text{ (lag)}$$

<u>Phasor diagram</u>



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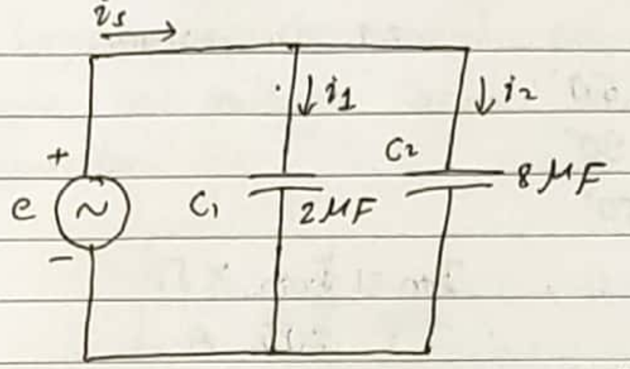
3 For the network of fig-2 and the applied signal, determine i_1, i_2 and i_s



$e = \sqrt{2}\,100 \sin(10^4 t + 60^\circ)$

Solution,

Given,

$$e = \sqrt{2}\,100 \sin(10^4 t + 60^\circ)$$

Here

$$E_m = 100\sqrt{2} \quad , \quad \omega = 10^4 \quad , \quad \phi = 60^\circ$$

And

$$E_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100\,V$$

Then,

$$Z_1 = -j\frac{1}{X_{C_1}} = -j\frac{1}{\omega C_1}$$

$$= -j\frac{1}{10^4 \times 2\times 10^{-6}}$$

$$= -j\,50\,\Omega = (50\angle -90^\circ)$$

And

$$Z_2 = -j\frac{1}{X_{C_2}} = -j\frac{1}{\omega C_2}$$

$$= -j\frac{1}{10^4 \times 8\times 10^{-6}}$$

$$= -j\,12.5\,\Omega = (12.5\angle -90^\circ)$$

Hence

a) $I_1 = \dfrac{\varepsilon}{Z_1}$

$= \dfrac{100\angle 60^\circ}{50\angle -90^\circ}$

$= 2\angle 150^\circ$

$\therefore I_{rms} = 2$, $I_m = I_{rm} \times \sqrt{2}$

$= 2\sqrt{2}\ A$

$i_1 = 2\sqrt{2}\sin(\omega t + 60^\circ + 150^\circ)$

$\therefore i_1 = 2\sqrt{2}\sin(10^4 t + 210^\circ)$

b) $I_2 = \dfrac{\varepsilon}{Z_2}$

$= \dfrac{100\angle 60^\circ}{12.5\angle -90^\circ}$

$= \cancel{8\sqrt{}}\ 8\angle 150^\circ$

$I_{rms} = 8$, $I_m = I_{rms} \times \sqrt{2}$

$= 8 \times \sqrt{2} = 8\sqrt{2}\ A$

$i_2 = 8\sqrt{2}\sin(\omega t + 60^\circ + 150^\circ)$

$\therefore i_2 = 8\sqrt{2}\sin(\cancel{\omega}\, 10^4 t + 210^\circ)$

c) $i_s = i_1 + i_2$

$= (2\cancel{\sqrt{2}}\angle 150^\circ) + (8\cancel{\sqrt{2}}\angle 150)$

$= 10\cancel{\sqrt{2}}\angle 150^\circ$ Then, $I_m = I_{rms} \times \sqrt{2}$

$= 10\sqrt{2}\ A$

Then,

$\therefore i_s = 10\sqrt{2}\sin(10^4 t + 210^\circ)$ #

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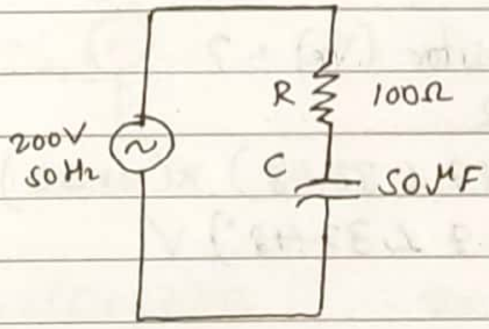
4 A series circuit has a resistor of $100\,\Omega$ and capacitor of $50\,\mu F$. The circuit is fed by a 200V, 50Hz supply. Find (a) Impedance (b) current (c) power factor (d) phase angle (e) voltage across resistor (f) voltage across capacitor.



Solution

$V_{rms} = 200V$ frequency $(f) = 50\,Hz$

$R = 100\,\Omega$ $C = 50\,\mu F$

Now,

1) $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\times\pi\times50\times50\times10^{-6}}$

$= 63.66\,\Omega$

a) $Z = R - jX_c$

$= (100 - 63.66j)\,\Omega$

$= (118.54 \angle -32.48^\circ)\,\Omega$

b) $I = \frac{V}{Z}$

$= \frac{200\angle 0^\circ}{118.54\angle -32.48^\circ}$

$= (1.687 \angle 32.48^\circ)\,A$

$I_{rms} = 1.687\,A$ & $I_m = 1.687 \times \sqrt{2}$

$= 2.37\,A$

c) Power factor (P.f) = $\cos\theta$

$= \cos 32.48$

$= 0.8435$ (leading)

d) Phase ange (θ) = 32.48°

e, Voltage across resistor (V_R) = ?

$V_R = I R$

$= (1.687 \angle 32.48) \times (100 \angle 0)$

$= (168.7 \angle 32.48^\circ)$ V

f) $V_{rms(R)} = 168.7$ V

$V_{R(max)} = 168.7 \times \sqrt{2}$

$= 238.57$ V

f) Voltage across capacitor

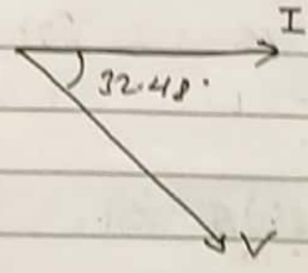
$V_c = I \times Z$

$= (1.687 \angle 32.48) \times (63.66 \angle -90^\circ)$

$= (107.39 \angle -57.52^\circ)$ V

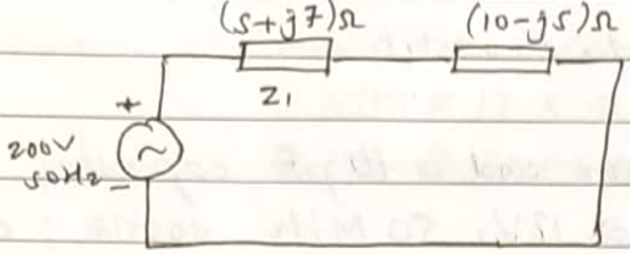
$V_{rms(c)} = 107.39$ V

Phasor diagram



5 Two impedances of $(5+j7)\,\Omega$ and $(10-j5)\,\Omega$ are connected in series across a 200V, 50 Hz supply. Find the current, active power, apparent power and power factor.

Solution



Sol^n

Given

$Z_1 = (5+j7)\,\Omega$ $\quad Z_2 = (10-j5)\,\Omega$

$V_{rms} = 200V$ $\quad$ frequency $(f) = 50\,Hz$

Now,

$$Z_{eq} = Z_1 + Z_2$$
$$= (5+j7) + (10-j5)$$
$$= (15+2j)\,\Omega$$
$$= (15.132 \angle 7.59^\circ)\,\Omega$$

i) Current $(I) = \dfrac{\hat{V}}{Z_{eq}} = \dfrac{200 \angle 0}{15.132 \angle 7.59^\circ}$

$$I_{rms} = (13.27 \angle -7.59^\circ)$$
$$\therefore I_{rms} = 13.27\,A$$

ii) Active power

$$(P) = I_{rms} \times V_{rms} \times \cos\phi$$
$$= 13.27 \times 200 \times \cos(0-(-7.59^\circ))$$
$$= 2618.85\,W$$

iii) Apparent power $= I_{rms} \times V_{rms}$

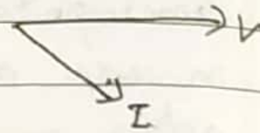
$$= 13.27 \times 200$$
$$= 2642\,V$$

iv) Power factor (P.f) = $\cos\phi$

$= \cos 7.59°$

$= 0.991$ (lagging)

Phasor diagram



6) A 200Ω resistance and a 10 pF capacitor are connected in series across a 12V, 50 MHz source; calculate the (a) Impedance (b) current (c) Power factor (d) active power. Draw the phasor diagram of the circuit.

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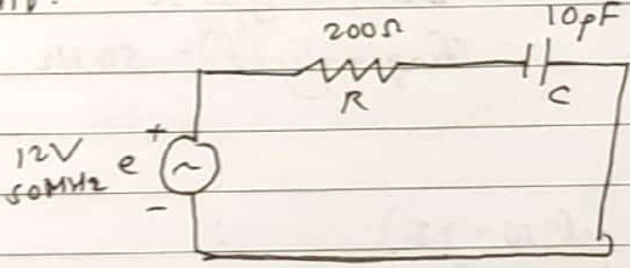


Solution,

$V_{rms} = 12V$ frequency (f) = 50 MHz

$R = 200\Omega$

Then,

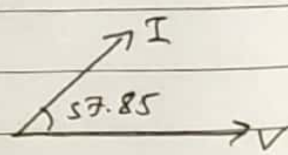
a) $Z = R - j/\omega C$

$= (200 - j/2\pi \times 50 \times 10^{6} \times 10 \times 10^{-12})\Omega$

$= (200 - 318.30j)\Omega$

$= (375.92 \angle -57.85°)\Omega$

Phasor diagram



b) $I_{rms} = \frac{V_{rms}}{Z}$

$= \frac{12\angle 0°}{375.92\angle -57.85}$

$= 0.0319 \angle 57.85°$

$\therefore I_{rms} = 0.0319$ A * $I_m = 0.0319 \times \sqrt{2}$

$= 0.045$ A

c) Power factor (p.f) = $\cos\phi$

$= \cos 57.85^\circ$

$= \cancel{0.576}$ (leading)

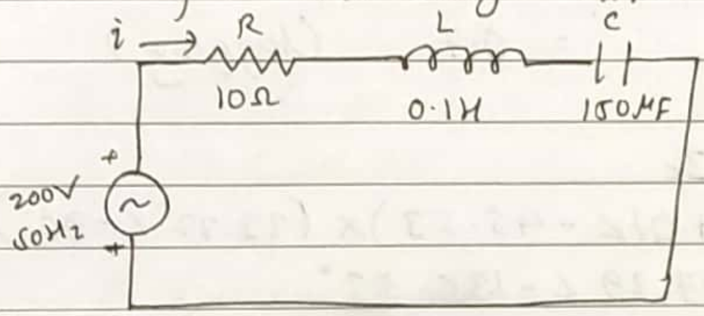
$= 0.532$

d) Active Power $= I_{rms} \times V_{rms} \times \cos\phi$

$= 0.0319 \times 12 \times 0.532$

$= \cancel{0.02\ 0.205\,W\ 0.0}\ 0.2036\,W$

7) A coil of $R = 10\Omega$ and $L = 0.1H$ is connected in series with a capacitor of $150\mu F$ across 200V, 50 Hz supply. Find X_L, X_C, Z, I, p.f and voltage across the capacitor.



Solution

$V_{rms} = 200V$ frequency (f) = 50 Hz

$R = 10\Omega$ $L = 0.1H$ $C = 150\mu F$

Now,

i) $X_L = \omega L$

$= 2\pi \times 50 \times 0.1$

$= 31.41\Omega$

ii) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$

$= 21.22\Omega$

iii) $Z = R + X_L + X_C$

$= 10 + 31.41j - 21.22j$

$= 10 + 10.19j = 14.27 \angle 45.53^\circ$

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iv) $I = \dfrac{V}{Z}$

$= \dfrac{200\angle 0^\circ}{14.27\angle 45.53}$

$= (14.01 \angle -45.53)\,A$

$\therefore I = 14.01\,A$

Then

v) Power factor (p.f) $= \cos\phi$

$= \cos(0-(-45.53))$

$= \cos 45.53$

$= 0.7$ (lagging)

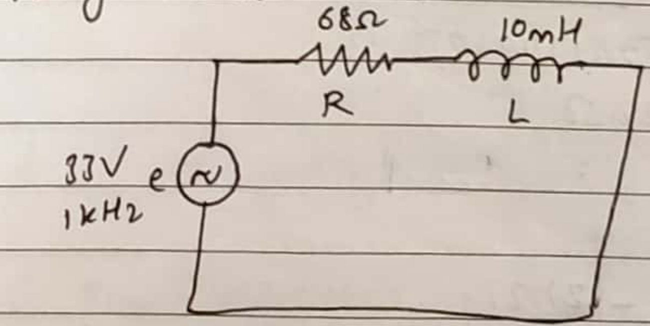
vi) $V_c = I Z_c$

$= (14.01\angle -45.53) \times (21.22 \angle -90^\circ)$

$= 297.29 \angle -135.53^\circ$

$\therefore V_c = 297.29\ V$

8 A series circuit consisting of 68Ω resistance and 10mH inductance is excited by a 33V, 1kHz source. Find the (a) current (b) voltage across resistor (c) voltage across inductor (d) phase angle between voltage and current. Draw the phasor diagram for the voltages and current



Solution

Here

$R = 68\Omega$

$L = 10mH = 10\times10^{-3} H$

frequency $(f) = 1kHz = 10^3 Hz$

$V_{rms} = 33V$

Then,

$X_L = \omega L$

$= 2\pi \times 10^3 \times 10\times10^{-3}$

$= 20\pi \ \Omega = 62.83\Omega$

And

$Z = R + X_L$

$= 68 + 62.83j$

$= 92.58 \angle 42.73^\circ$

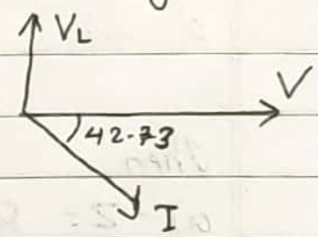
Then

a) $I = \frac{V}{Z} = \frac{33\angle0^\circ}{92.58\angle42.73}$

$= 0.356 \angle -42.73^\circ$

$\therefore I = 0.356 A$

Phasor diagram



b) $V_R = I \times R$

$= 0.356 \times 68$

$= 24.20 V$

c) $V_L = I \times X_L$

$= 0.356 \times 62.83$

$= 22.36 V$

d) Phase angle $(\phi) = 42.73^\circ$

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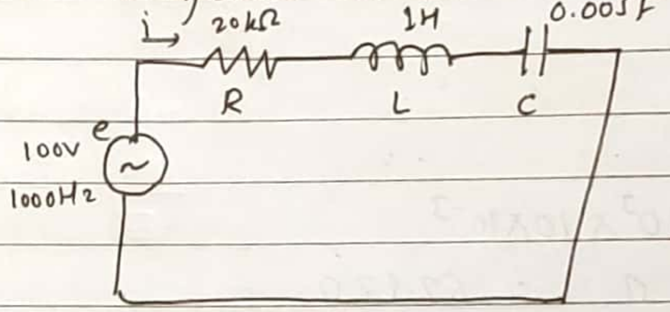
9, A series RLC circuit has $R = 20k\Omega$, $L = 1H$ and $C = 0.005F$. If the circuit is connected to a 100V, 1000 Hz supply, calculate impedance, current, phase angle between voltage & current, p.f and the power dissipated in the circuit



Solution

Here

$V_{rms} = 100V$ $\quad\quad$ $R = 20k\Omega = 20000\Omega$

$f = 1000Hz$ $\quad\quad$ $L = 1H$

$C = 0.005F$

Now,

$X_L = j\omega L$
$= j\, 2 \times \pi \times 1000 \times 1$
$= 2000\pi j\ \Omega$

$X_C = \frac{1}{j\omega C}$
$= -j \frac{1}{2\times\pi\times1000\times0.005}$
$= -0.0318 j\ \Omega$

Then

a) $Z = R + L + C$
$= 20000 + 2000\pi j - 0.0318 j$
$= (20000 + 6283.15 j)\ \Omega$
$= 20963.73 \angle 17.44°$

b) $I = \frac{V}{Z} = \frac{100\angle 0}{20963.7\angle 17.44} = 4.77 \times 10^{-3} A \angle -17.44°$

$= 4.77 mA \angle -17.44°$

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phase angle $\phi = 17.44°$

And

c) Power factor (p.f) $= \cos\phi$

$= \cos 17.44$

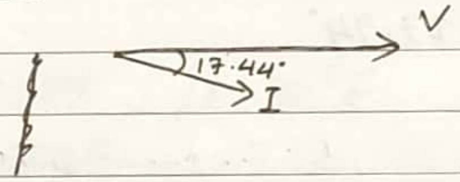
$= 0.954$ (lagging)

d. $P = V_{rms} \cdot I_{rms} \cdot \cos\phi$

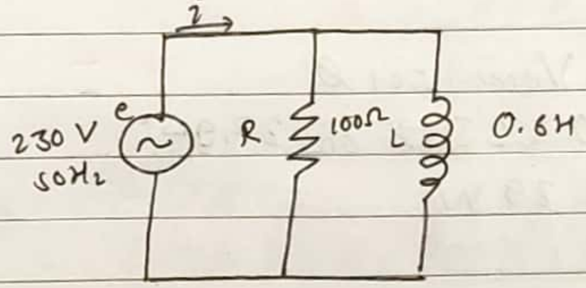
$= 100 \times 4.77 \times 10^{-3} \times 0.954$

$= 0.455 \text{ W}$

Phasor diagram



10) A 100 Ω resistance and 0.6 H inductance are connected in parallel across a 230V, 50 Hz supply. Find the current, phase angle, impedance and power dissipated in the circuit.



Solution

Given

$R = 100\Omega$ $L = 0.6H$

$V_{rms} = 230V$ $f = 50Hz$

And,

$X_L = j\omega L$

$= j\, 2\pi \times 50 \times 0.6$

$= 60\pi j \ \Omega$

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$$Z = R \parallel X_L$$
$$= \frac{R \times X_L}{R + X_L}$$
$$= \frac{100 \times 60\pi j}{100 + 60\pi j}$$
$$= 88.33 \angle 27.94^\circ$$

a) $I = \frac{V}{Z}$

$$= \frac{230 \angle 0^\circ}{81.33 \angle 27.94^\circ}$$
$$= 2.60 \angle -27.94^\circ$$

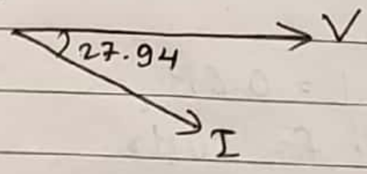
$\therefore I_{rms} = 2.60\ A$

b) Phase angle $(\phi) = 27.94^\circ$

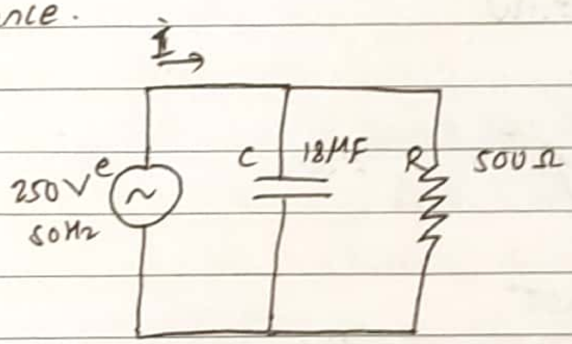
c) Impedance $(Z) = 88.33 \angle 27.94^\circ$
$= 88.33\ \Omega$

d) Power $(P) = I_{rms} \cdot V_{rms} \cdot \cos\phi$
$= 2.60 \times 230 \times \cos 27.94^\circ$
$= 528.29\ W$

Phasor diagram



11) A 18 μF capacitor is connected in parallel with 500Ω resistor. The circuit is connected to a 250V, 50 Hz source. Find the line current, phase angle, power dissipated in the circuit and the total circuit impedance.



Sol^n

Given

$C = 18\mu F = 18 \times 10^{-6} F$

$R = 500\Omega$

$V_{rms} = 250 V$

$f = 50 Hz$

Then,

$X_c = \frac{1}{j\omega C} = \frac{-j}{2\pi \times 50 \times 18 \times 10^{-6}}$

$= -176.83j \ \Omega$

$= 176.83 \angle -90^\circ$

d, $Z_{eq} = X_C // R$

$= \frac{(176.83 \angle -90) \times (500 \angle 0)}{(176.83 \angle -90^\circ) + (500 \angle 0)}$

$= (166.71 \angle -70.52^\circ) \ \Omega$

$= 166.71 \ \Omega$

a) $I = \frac{V}{Z} = \frac{(250 \angle 0)}{(166.71 \angle -70.52)}$

$= (1.49 \angle 70.52) \ A$

$\therefore \ I_{rms} = 1.49 \ A$

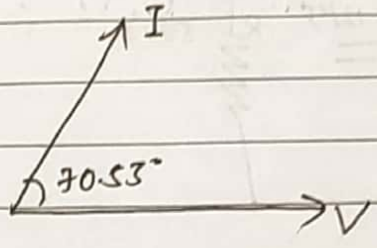
b) phase angle (ϕ) = $70.53°$ (leading)

c) Power (P) = $I_{rms} \cdot V_{rms} \cdot \cos\phi$

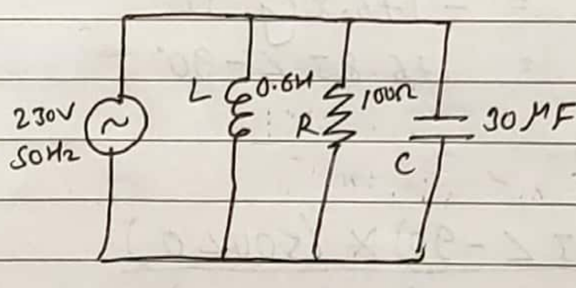
$= 1.49 \times 250 \times \cos 70.53°$

$= 124.97$ W

Phasor diagram



12) An inductance of 0.6H, a resistor of 100Ω and a capacitor of 30 μF are connected in parallel across a 230V, 50Hz supply. Find the line current circuit phase angle, power dissipated in the circuit and total circuit impedance. Also draw the phasor diagram for voltages and currents in the circuit.



Solution

Given,

$V_{rms} = 230V$ $\quad$ $f = 50Hz$

$R = 100\Omega$ $\quad$ $C = 30\mu F = 30 \times 10^{-6} F$

$L = 0.6H$

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Now,

$X_L = j\omega L$

$= j \times 2\pi \times 50 \times 0.6$

$= 60\pi j \ \Omega = (188.49 \angle 90^\circ)$

$X_C = \dfrac{1}{j\omega C} = \dfrac{1}{j \cdot 2\times\pi\times 50\times 30\times 10^{-6}}$

$= -j\,106.1 \ \Omega$

$= (106.1 \angle -90^\circ)$

Then

d) $Z_{eq} = R // X_L // X_C$

$= \dfrac{R \times X_L \times X_C}{R \times X_L + X_L \times X_C + X_C \times R}$

$= \dfrac{(100\angle 0^\circ) \times (188.49\angle 90^\circ) \times (106.1\angle -90^\circ)}{(100\angle 0) \times (188.49\angle 90) + (188.49\angle 90) \times (106.1\angle -90^\circ) + (106.1\angle -90^\circ) \times (100\angle 0^\circ)}$

$= \dfrac{(1999878.9 \angle 0^\circ)\ \Omega}{\cancel{Z_{eq}} = (18849\angle 90) + (19998.789\angle 0^\circ) + (10610\angle -90)}$

$= \dfrac{1999878.9 \angle 0}{21629.44 \angle 22.39}$

$= (92.46 \angle -22.39^\circ)$

$\therefore Z_{eq} = 92.46 \ \Omega$

a) $I = \dfrac{V}{Z_{eq}} = \dfrac{230 \angle 0}{92.46\angle -22.39}$

$= (2.48 \angle 22.39)\ A$

$\therefore I_{rms} = 2.48 A$

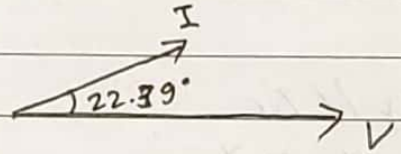
b) Phase angle (ϕ) = 22.39° (leading)

c) Power (P) = $I_{rms} \times V_{rms} \times \cos\phi$

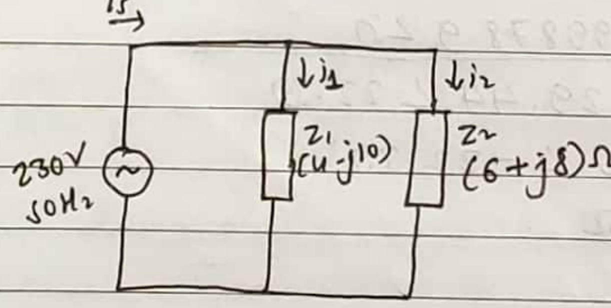
$= 2.48 \times 230 \times \cos 22.39°$

$= 527.39\,W$

Phasor diagram



13 An impedance of $(4-j10)\Omega$ is connected in parallel with an impedance $(6+j8)$. The circuit is fed from a 230V, 50 Hz supply. Find the current through each branch, total circuit current, total impedance p.f, active power, reactive power and apparent power. Also draw the phasor diagram for voltage and currents in the circuit.



Sol^n

Given

$Z_1 = (4-j10)\Omega = (2\sqrt{29} \angle -68.19°)\Omega$

$Z_2 = (6+j8)\Omega = (10 \angle 53.13)\Omega$

$V_{rms} = 230\,V$

$f = 50\,Hz$

a) Current through each branch

$$I_1 = \frac{V}{Z_1}$$

$$= \frac{230 \angle 0^\circ}{(2\sqrt{29} \angle -68.19^\circ)}$$

$$= (21.35 \angle 68.19)A$$

$$I_2 = \frac{V}{Z_2}$$

$$= \frac{230 \angle 0}{10 \angle 53.13}$$

$$= (23 \angle -53.13)A$$

b) Total current $I_s = I_1 + I_2$

$$= (21.35 \angle 68.19) + (23 \angle -53.13)$$

$$= (21.77 \angle 3.74^\circ)A$$

c) $$Z = \frac{V}{I_s}$$

$$= \frac{(230 \angle 0)}{(21.77 \angle 3.74^\circ)}$$

$$= (10.56 \angle -3.74^\circ)\Omega$$

d) $$p.f = \cos\phi$$

$$= \cos 3.74$$

$$= 0.9978 \text{ (leading)}$$

e) Active power $(P) = V_{rms} \times I_{rms} \times \cos\phi$

$$= 230 \times 21.77 \times 0.997$$

$$= 4996.08\ W$$

f) reactive power $= I_{rms} \times V_{rms} \times \sin\phi$

$= 21.77 \times 230 \times \sin 3.74$

$= 328.26$ VAR

g) apparent power $= I_{rms} \times V_{rms}$

$= 21.77 \times 230$

$= 5007.1$ VA

Phasor diagram