Tim Singel

CS-5350

# 5350 Final Report

**Introduction:**

In this final project, we have been tasked with building, serializing, ordering, and coloring several sets of graphs. This will all be done to analyze our runtimes, and hopefully come out with a clear understanding of how it scales compared to input size.

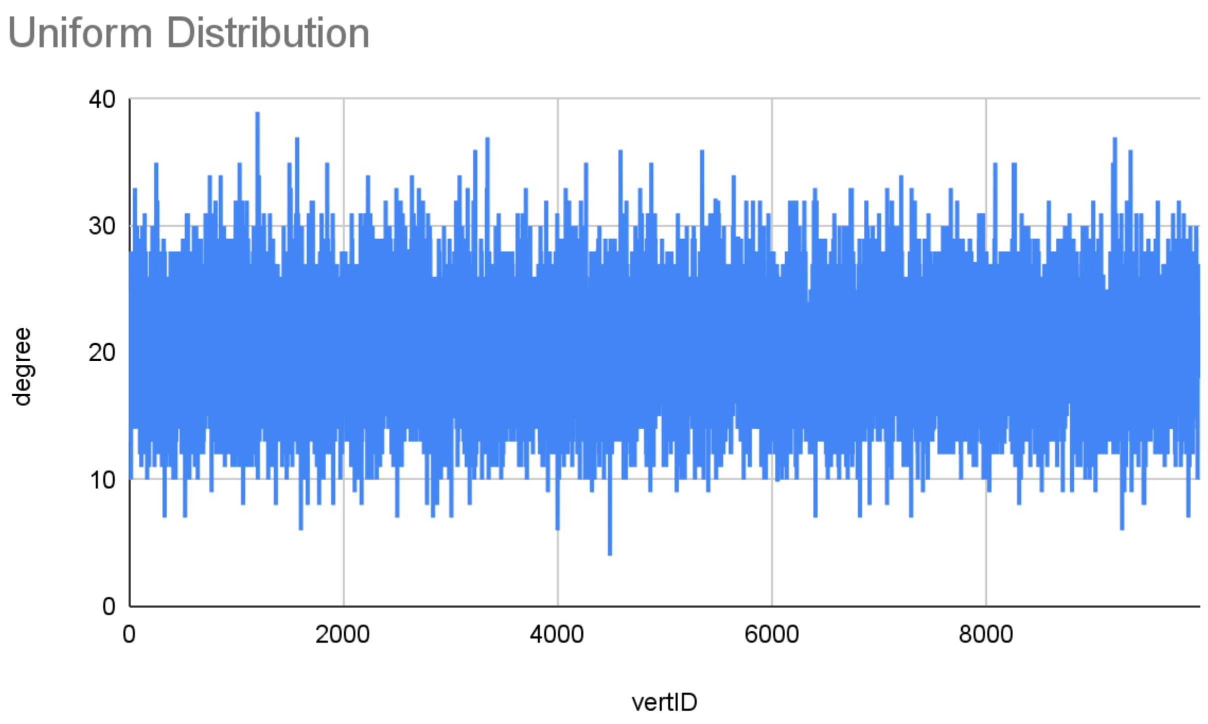
**Environment:**

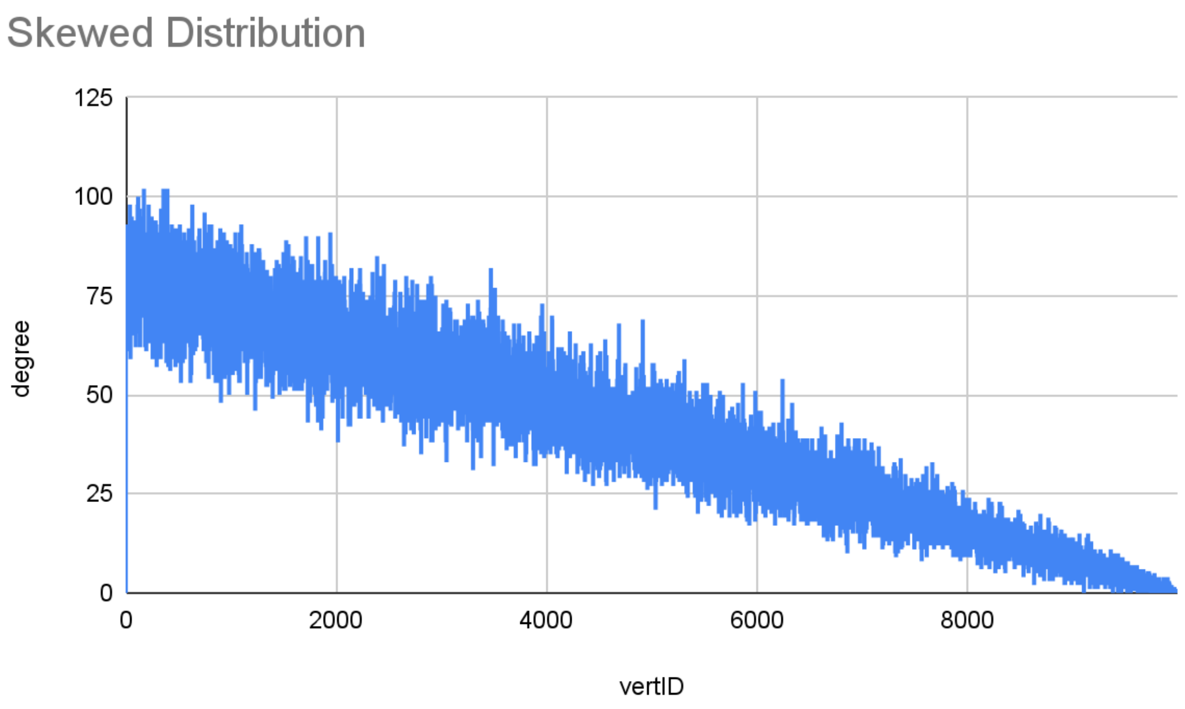
I made the crucial decision of language early on, after spending some serious time thinking about it. I decided on using python, as I am still very new to it myself, and felt I would take the plunge, and try to make my code as fast as it could be in this language.

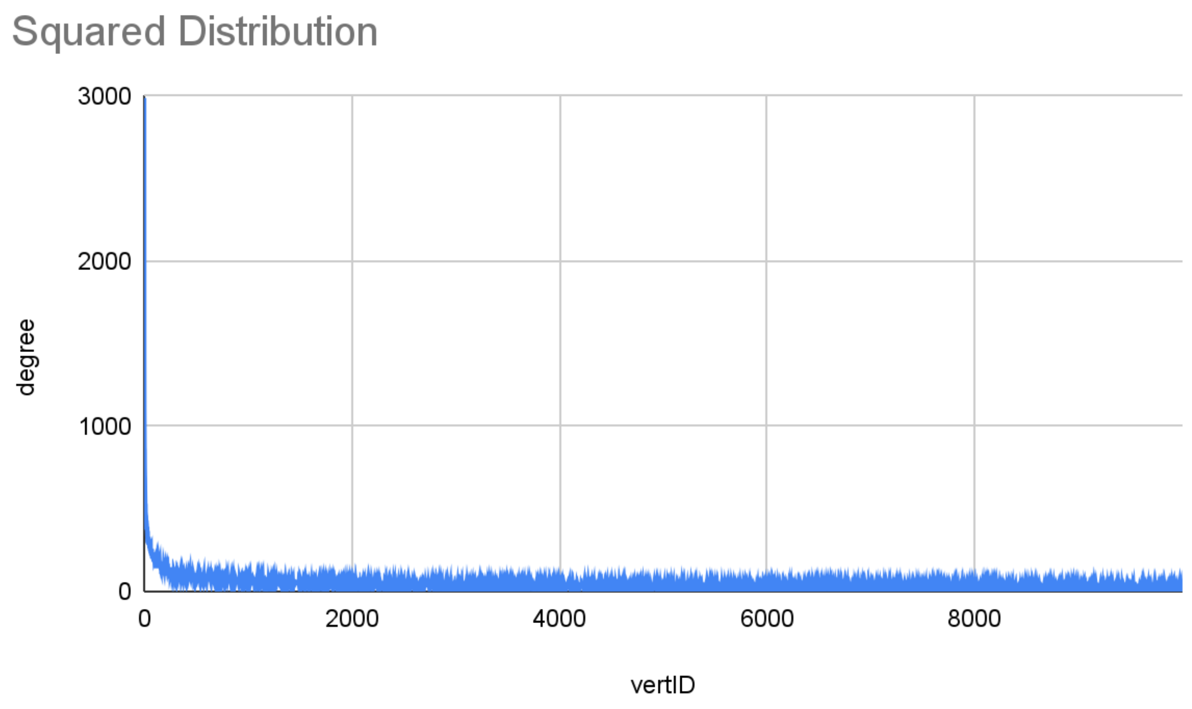
I coded it across several devices, using git to track my process and changes. I made sure that any change I made had no cascading effects, and would ensure any of my code would allow for modularity with any of my coloring and ordering techniques down the line.

**Random Distribution Types:**

On the following pages are the random distributions supported by my graph generation techniques.


Uniform Distribution
40
30
20
10
0
degree
0
2000
4000
6000
8000
vertID


Skewed Distribution
125
100
75
50
25
0
degree
0
2000
4000
6000
8000
vertID

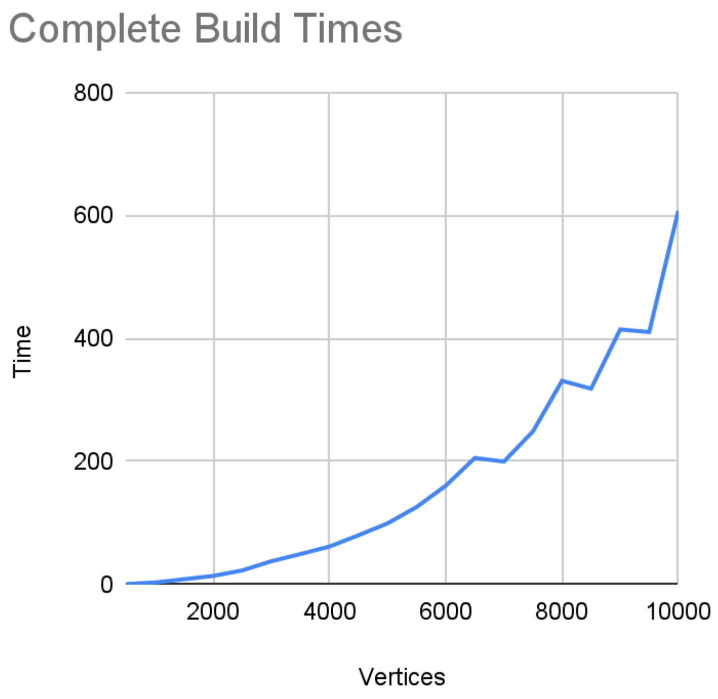


**Graph Generation Runtimes:**

The following tables and graphs were generated using a shell script to run them in order using > to inject it to a csv. I would then loop this with several different V or E values given whichever output I was going for. If I scaled the Vertices count, then Edges is set to 100k in all cases, and if I scaled the Edges, Vertices was set to 10k out of the box.

**Complete Graphs:**

| Vertices | Time |
|---|---|
| 500 | 0.565956831 |
| 1000 | 3.141708374 |
| 1500 | 8.448401451 |
| 2000 | 13.82903528 |
| 2500 | 22.78555989 |
| 3000 | 37.7728467 |
| 3500 | 49.33704376 |
| 4000 | 61.59974337 |
| 4500 | 79.92107153 |
| 5000 | 99.35481358 |
| 5500 | 125.9474072 |
| 6000 | 160.4970536 |
| 6500 | 205.5820093 |
| 7000 | 199.7333093 |
| 7500 | 248.7258615 |
| 8000 | 330.9938192 |
| 8500 | 318.2370021 |
| 9000 | 414.4258409 |
| 9500 | 410.3291762 |
| 10000 | 607.2947783 |

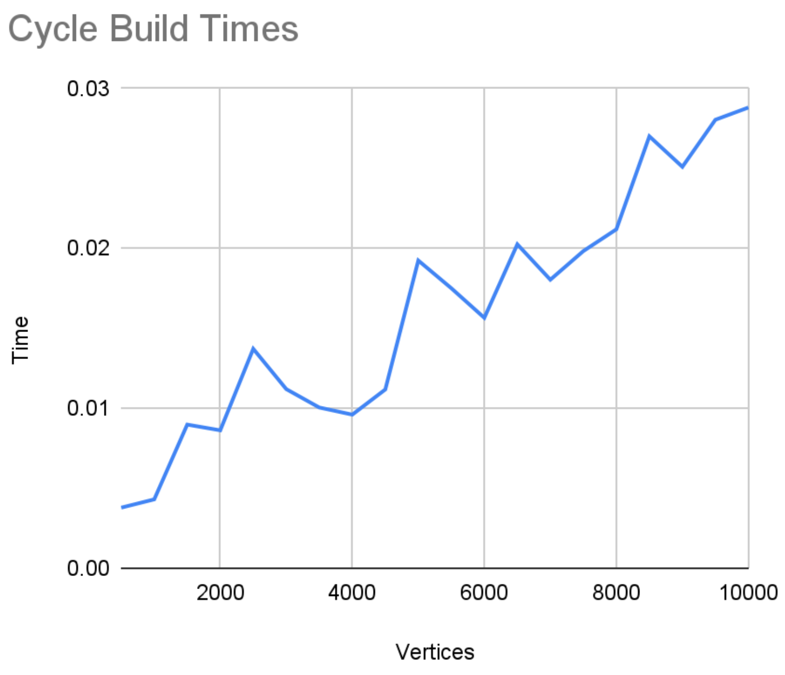


**Analysis:**

Based on my tabled runtimes and the graph, I have a feeling that my complete generation is among the slowest. My sincere guess is O(V^2), as the most extreme cases have extreme runtimes. Even running on the schools server with an intel xeon, 10000 which is around 50 million edges took nearly 10 minutes. I would say this is the best run I got, as at times the server froze up the python instance. This graph shows the change of Vertices by the runtime. I ended up with O(V^2)

**Cycle Graphs:**

| Vertices | Time |
|---|---|
| 500 | 0.003776311874 |
| 1000 | 0.00429391861 |
| 1500 | 0.008969545364 |
| 2000 | 0.008611440659 |
| 2500 | 0.01370668411 |
| 3000 | 0.01118707657 |
| 3500 | 0.01003718376 |
| 4000 | 0.009592533112 |
| 4500 | 0.01116490364 |
| 5000 | 0.01922893524 |
| 5500 | 0.01749515533 |
| 6000 | 0.015645504 |
| 6500 | 0.02024102211 |
| 7000 | 0.0180284977 |
| 7500 | 0.01981878281 |
| 8000 | 0.02117800713 |
| 8500 | 0.02699398994 |
| 9000 | 0.02508664131 |
| 9500 | 0.02802944183 |
| 10000 | 0.02879357338 |

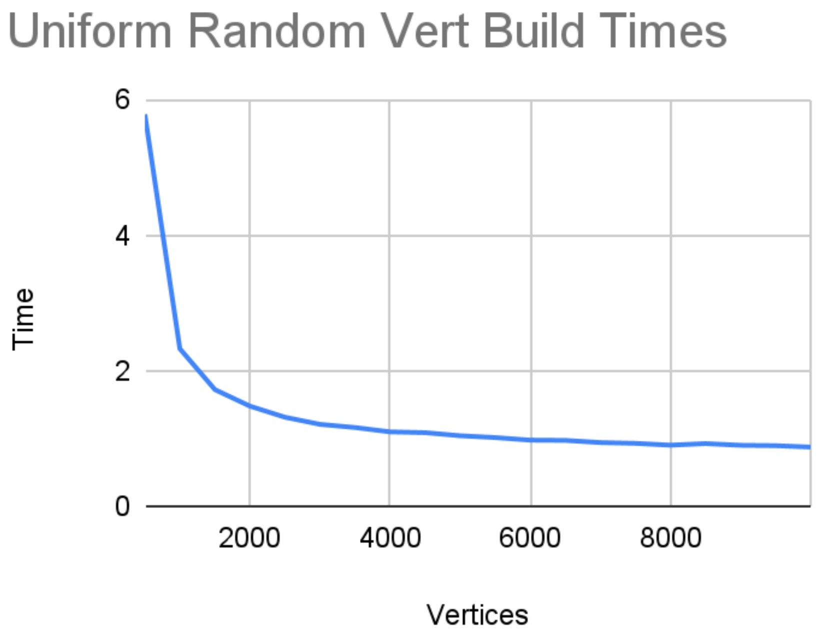


**Analysis:**

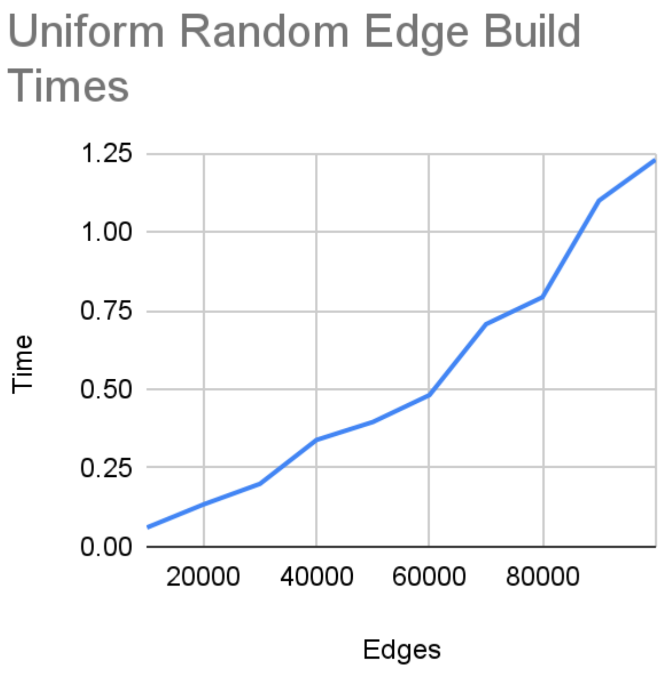
From my graphs and data outputs, the performance gain is clear between this and complete, and I would say that immediately this is O(V) runtime, as it goes to each vertice in order, and adds to cycle in what I would say is about linear time, meaning the main attribute scaling time is the vertices count. The strange up down variance is most likely from variance in the computer itself, not a programmatic error.

**Uniform Random Graphs (Vert/Edge):**

| Vertices | Time |
|---|---|
| 500 | 5.784392118 |
| 1000 | 2.325575113f |
| 1500 | 1.722511053 |
| 2000 | 1.478859663 |
| 2500 | 1.315148115 |
| 3000 | 1.210270882 |
| 3500 | 1.163339376 |
| 4000 | 1.099345684 |
| 4500 | 1.087580919 |
| 5000 | 1.042124271 |
| 5500 | 1.01512599 |
| 6000 | 0.9773306847 |
| 6500 | 0.9734556675 |
| 7000 | 0.9415352345 |
| 7500 | 0.9298424721 |
| 8000 | 0.9031300545 |
| 8500 | 0.9264605045 |
| 9000 | 0.9013941288 |
| 9500 | 0.895537138 |
| 10000 | 0.8748147488 |

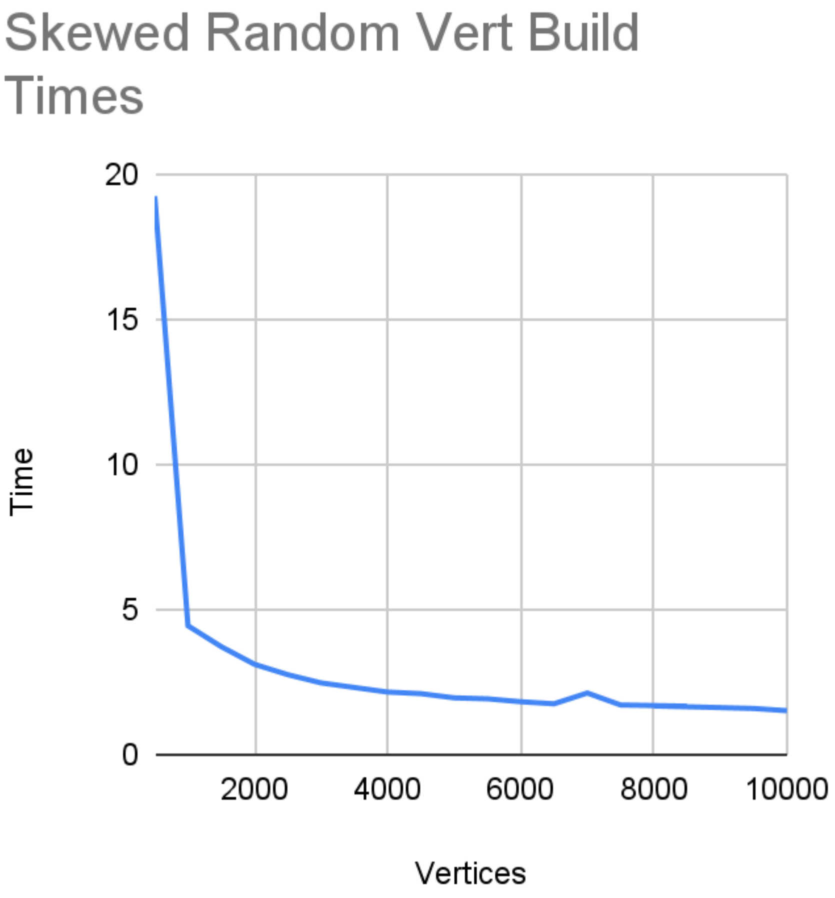


| Edge | Time |
|---|---|
| 10000 | 0.0600066185 |
| 20000 | 0.1334102154 |
| 30000 | 0.1989719868 |
| 40000 | 0.3384768963 |
| 50000 | 0.3957509995 |
| 60000 | 0.4812150002 |
| 70000 | 0.7065689564 |
| 80000 | 0.7925348282 |
| 90000 | 1.099574089 |
| 100000 | 1.229662895 |

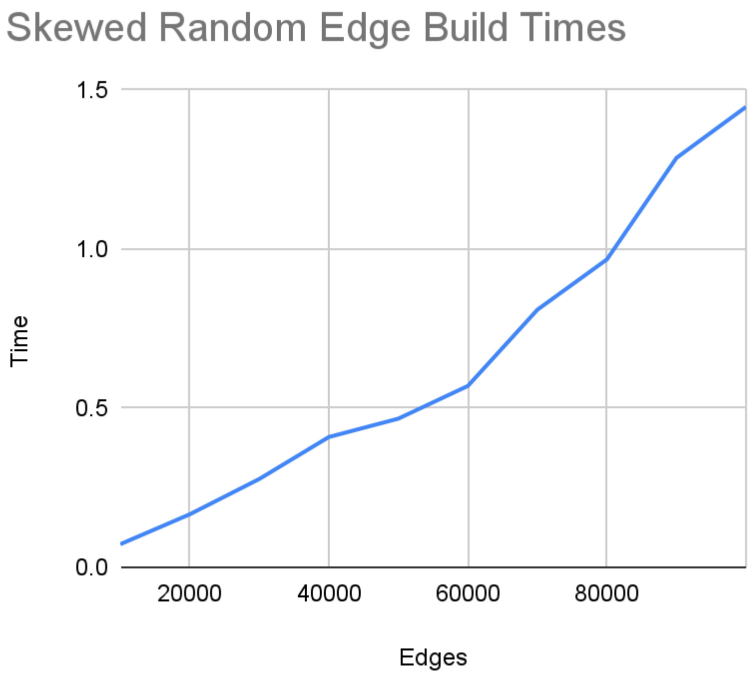


**Skewed Random Graphs:**

| Vertices | Time |
|---|---|
| 10000 | 1.5178895 |
| 9500 | 1.594171047 |
| 9000 | 1.624655008 |
| 8000 | 1.681574821 |
| 8500 | 1.674163818 |
| 7500 | 1.71496892 |
| 7000 | 2.12772274 |
| 6500 | 1.755957127 |
| 6000 | 1.826788425 |
| 5500 | 1.926962852 |
| 5000 | 1.960597754 |
| 4500 | 2.105140209 |
| 4000 | 2.16060257 |
| 3500 | 2.317932129 |
| 3000 | 2.475925922 |
| 2500 | 2.759656906 |
| 2000 | 3.117156506 |
| 1500 | 3.726086378 |
| 1000 | 4.446886778 |
| 500 | 19.26128769 |

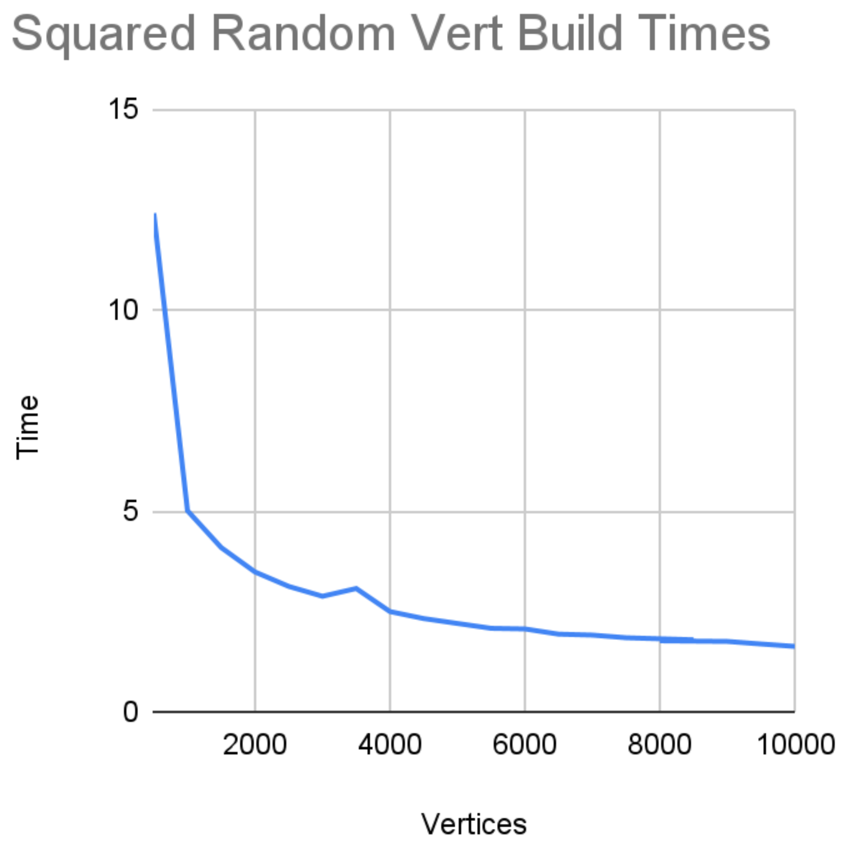


| Edges | Time |
|---|---|
| 10000 | 0.07255530357 |
| 20000 | 0.1665816307 |
| 30000 | 0.2775835991 |
| 40000 | 0.4093325138 |
| 50000 | 0.4670917988 |
| 60000 | 0.5698983669 |
| 70000 | 0.8094027042 |
| 80000 | 0.9674723148 |
| 90000 | 1.286313295 |
| 100000 | 1.446612835 |

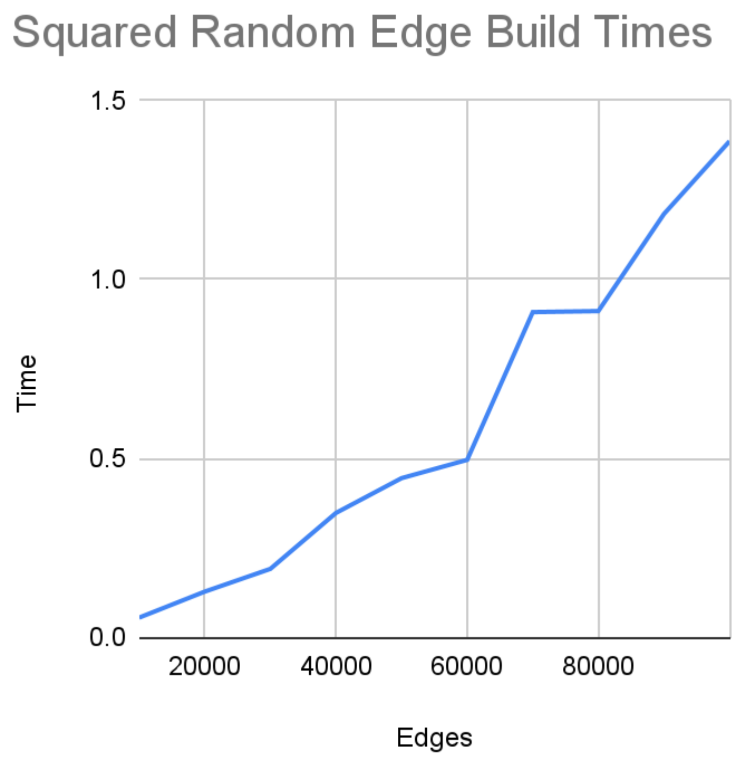


**Squared Random Graphs:**

| Vertices | Time |
|---|---|
| 10000 | 1.633201361 |
| 9500 | 1.693156242 |
| 9000 | 1.756319761 |
| 8000 | 1.772512197 |
| 8500 | 1.79771328 |
| 7500 | 1.848349094 |
| 7000 | 1.91458869 |
| 6500 | 1.937674046 |
| 6000 | 2.067599773 |
| 5500 | 2.08323288 |
| 5000 | 2.204091072 |
| 4500 | 2.32565093 |
| 4000 | 2.499406338 |
| 3500 | 3.073340178 |
| 3000 | 2.880060911 |
| 2500 | 3.125814438 |
| 2000 | 3.485306501 |
| 1500 | 4.09101367 |
| 1000 | 5.008760929 |
| 500 | 12.40857577 |



| Edges | Time |
|---|---|
| 10000 | 0.05501270294 |
| 20000 | 0.1276676655 |
| 30000 | 0.191190958 |
| 40000 | 0.3474667072 |
| 50000 | 0.4446377754 |
| 60000 | 0.4957227707 |
| 70000 | 0.9085805416 |
| 80000 | 0.9114742279 |
| 90000 | 1.182811499 |
| 100000 | 1.386120319 |



**Analysis:**

From my analysis and limited understanding of random generation complexity, my guess is that all of these random techniques will follow an O(E/V^2), as the edges are very impactful to the performance given there are little vertices, but given there is a sufficient vertex count, the runtime should scale down even with the same high edge counts. This equation may not be exact, but from my guess, E should be the heavy time sink, and V should in some way affect the weight of E given there's enough Vertices to spread them across

## Ordering Techniques:

### Smallest Last Vertex Ordering:

The small last vertex order(or SLVO) is created by using a degreeList which sorts references to the vertice objects in an adjacency list in order of degree. This is fed in and parsed, adding each vertex to the ordering, and adjusting the degrees of its connected partners. At each removal and addition to the ordering, the vertex saves its degree when removed, and goes on.

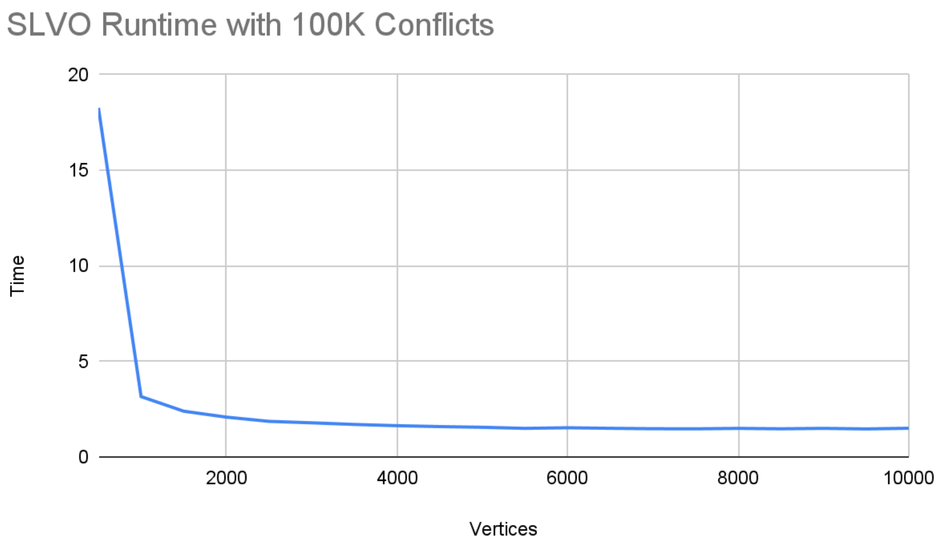
**Test Walkthrough on Small Set:**

The example below starts with 5 vertices and 10 edges, and for each step it prints the vertices info, and if it has been removed or not. At the end I demonstrate what the SLVO coloring would be for the same graph.

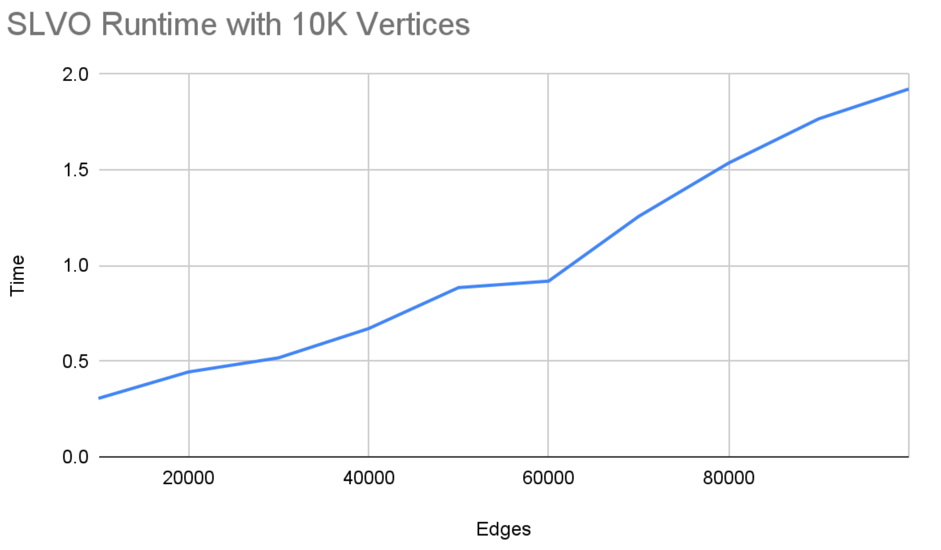
```
VertID: 0, Degree: 4, Removed: False }->3->4->2->1
VertID: 1, Degree: 4, Removed: False }->4->3->0->2
VertID: 2, Degree: 4, Removed: False }->0->4->3->1
VertID: 3, Degree: 4, Removed: False }->0->4->1->2
VertID: 4, Degree: 4, Removed: False }->0->1->3->2
Step #:  1
VertID: 0, Degree: 3, Removed: False }->3->4->2->1
VertID: 1, Degree: 3, Removed: False }->4->3->0->2
VertID: 2, Degree: 3, Removed: False }->0->4->3->1
VertID: 3, Degree: 3, Removed: False }->0->4->1->2
VertID: 4, Degree: 4, Removed: True }->0->1->3->2
Step #:  2
VertID: 0, Degree: 2, Removed: False }->3->4->2->1
VertID: 1, Degree: 2, Removed: False }->4->3->0->2
VertID: 2, Degree: 3, Removed: True }->0->4->3->1
VertID: 3, Degree: 2, Removed: False }->0->4->1->2
VertID: 4, Degree: 4, Removed: True }->0->1->3->2
Step #:  3
VertID: 0, Degree: 1, Removed: False }->3->4->2->1
VertID: 1, Degree: 2, Removed: True }->4->3->0->2
VertID: 2, Degree: 3, Removed: True }->0->4->3->1
VertID: 3, Degree: 1, Removed: False }->0->4->1->2
VertID: 4, Degree: 4, Removed: True }->0->1->3->2
Step #:  4
VertID: 0, Degree: 1, Removed: True }->3->4->2->1
VertID: 1, Degree: 2, Removed: True }->4->3->0->2
VertID: 2, Degree: 3, Removed: True }->0->4->3->1
VertID: 3, Degree: 0, Removed: False }->0->4->1->2
VertID: 4, Degree: 4, Removed: True }->0->1->3->2
Step #:  5
VertID: 0, Degree: 1, Removed: True }->3->4->2->1
VertID: 1, Degree: 2, Removed: True }->4->3->0->2
VertID: 2, Degree: 3, Removed: True }->0->4->3->1
VertID: 3, Degree: 0, Removed: True }->0->4->1->2
VertID: 4, Degree: 4, Removed: True }->0->1->3->2
Vert#:  0  Color:  3
Vert#:  1  Color:  2
Vert#:  2  Color:  1
Vert#:  3  Color:  4
Vert#:  4  Color:  0
```

**SLVO Runtime Examination:**

| Vertices | Time |
|---|---|
| 500 | 18.26828384 |
| 1000 | 3.159342527 |
| 1500 | 2.393642187 |
| 2000 | 2.08405304 |
| 2500 | 1.86239171 |
| 3000 | 1.786045551 |
| 3500 | 1.701081514 |
| 4000 | 1.636190891 |
| 4500 | 1.58955574 |
| 5000 | 1.552648783 |
| 5500 | 1.496366739 |
| 6000 | 1.527740002 |
| 6500 | 1.499153614 |
| 7000 | 1.477698326 |
| 7500 | 1.472055435 |
| 8000 | 1.495309591 |
| 8500 | 1.4739573 |
| 9000 | 1.496223927 |
| 9500 | 1.467093468 |
| 10000 | 1.501807451 |



| Edges | Time |
|---|---|
| 10000 | 0.3057024479 |
| 20000 | 0.4438335896 |
| 30000 | 0.5176608562 |
| 40000 | 0.6704156399 |
| 50000 | 0.8849031925 |
| 60000 | 0.9186694622 |
| 70000 | 1.256967545 |
| 80000 | 1.536068678 |
| 90000 | 1.767205954 |
| 100000 | 1.922997475 |



**SLVO Analysis:**

From the document, I know the algorithm if done correctly is in O(V+E), and I am fairly certain that my graphs and tables support this. The second graph with scaling Edge counts shows a somewhat O(n) style graph, and the program only really has issues when trying to add edges that are near the max conflict for a certain number of vertices. The edge increase is clear to me, as when you scale that alone in a fairly large graph, its runtime scales linearly. SLVO is unique from my other orderings, as it does more work when removing vertices, going to each neighbor to update the conditions there. This I believe is O(V+E) because of the linear increase shown in my second table and graph.

**Smallest Original Vertex Ordering:**

This as mentioned in the document is a subset of SLVO, and operates in a similar way at first. It uses the degree list to know the order of the vertices based on degree, but differently to SLVO when removing a vertice, it does not go and change the degrees of neighboring nodes, and instead is essentially the degree list transitioned to a 1 dimensional array, going through every list at each degree and pushing them. This makes it rather fast as it is more of a parse. Still keeps a rather low amount of colors, but not exactly the same as SLVO

**Smallest Original Inverted Ordering:**

I originally intended on flipping SLVO ordering, but when I got around to it, it was extremely slow in a cascading sense. My guess is that it has much more difficulty working with the high degree nodes first, and will then have difficulty with the low degree that should have been easy. I instead made an invert of the Smallest Original, as all I had to ensure was that the parser could be reversed. It is still rather slow, as the cascading effect still happens when coloring high degree nodes first, but it is a fun twist that I came up with for my 4th coloring method.

**Uniform Random Vertex Ordering:**

This was the simplest for me to implement, given the fantastic python libraries that exist. All that you want here is to make a deep copy of your adjacency list object, holding your Vertices, and then shuffle the contents and simply return that as your ordering. It had the most variance in runtime, and at times it could be pretty quick. For me this seems to be efficient with its copy, as it does not parse in any way, so my guess is that I have it randomizing about as efficiently as possible in my environment. This will often produce high color counts in general, but at times it can get lucky and produce near the optimized amount.

**Coloring Technique:**

All of the ordering techniques are fed into the greedy coloring algorithm as an input. Then the greedy algorithm parses the ordered vertices, examining every edge for used colors nearby, then choosing from the nodes that were not found in neighboring nodes. It is rather fast, and allows for the order to express itself in the colors chosen.

**Vertex Ordering Capabilities:**

**Complete Graphs:**

When being colored, complete are slow, but for good reason, as in any case where you color a complete graph, you will end with the same amount of colors as your vertices count. All the orderings produce different stuff, but it has no effect, as in coloring it's all about distinguishing from your neighbor, but if every single node is your neighbor, every single node will have to be different from each other.

**Cycle Graphs:**

When testing the color counts on Cycle graphs, I found that there were obvious patterns presenting themselves across the board.
When coloring graphs with even vertices count, I found that my non random ordering techniques only colored with 2 colors for all even cycles. But my Uniform Random Ordering almost always produced 3 colors for an even vertex count. My assumption is that it could produce 2, but it would be random producing one of the other 3 orders.
When coloring the odd vertices count, all methods of ordering always had 3 colors.

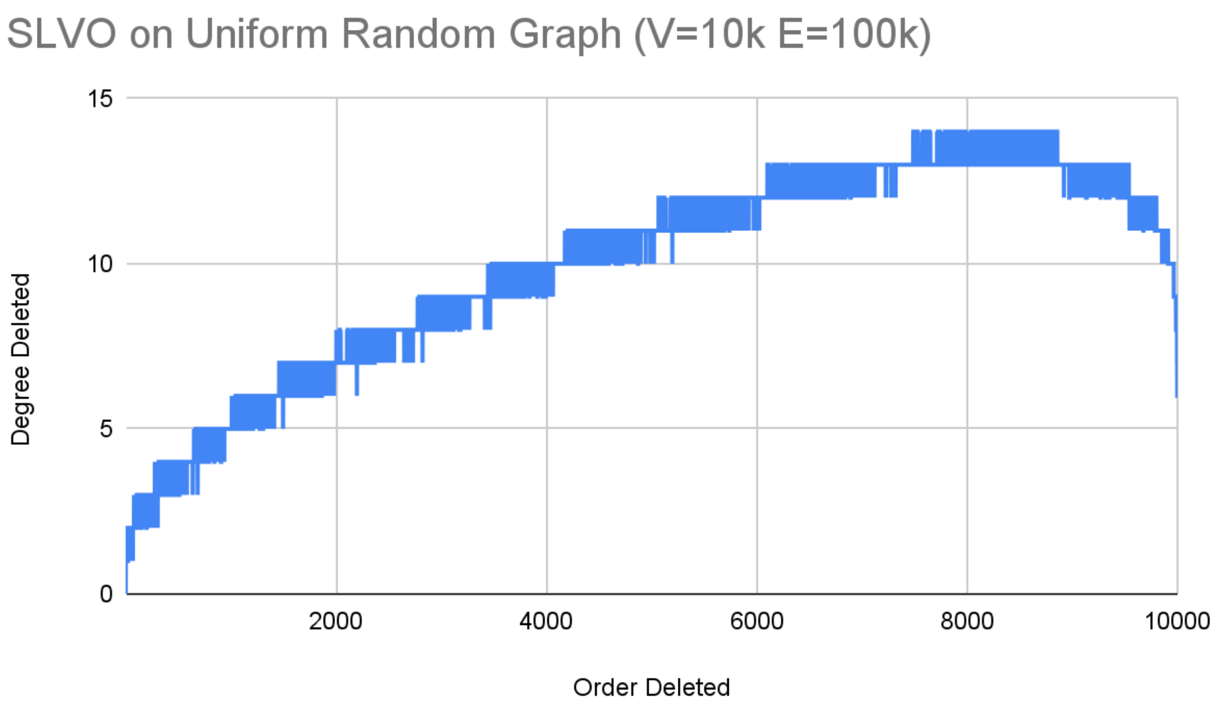
**Random Graphs:**

I started this part by serializing a file for all 3 random distributions with 10k vertices and 100k edges. I then tested all the coloring techniques on each of the graphs, so that I could compare the color counts given for one particular graph.

**Uniform Graph:**

Smallest Last Vertex Ordering:

Number of colors: 11



Max Degree Deleted: 14

Terminal Clique Size: 2

Smallest Original Vertex Ordering:

Number of Colors: 11

Smallest Original Vertex Ordering Inverted:

Number of Colors: 13

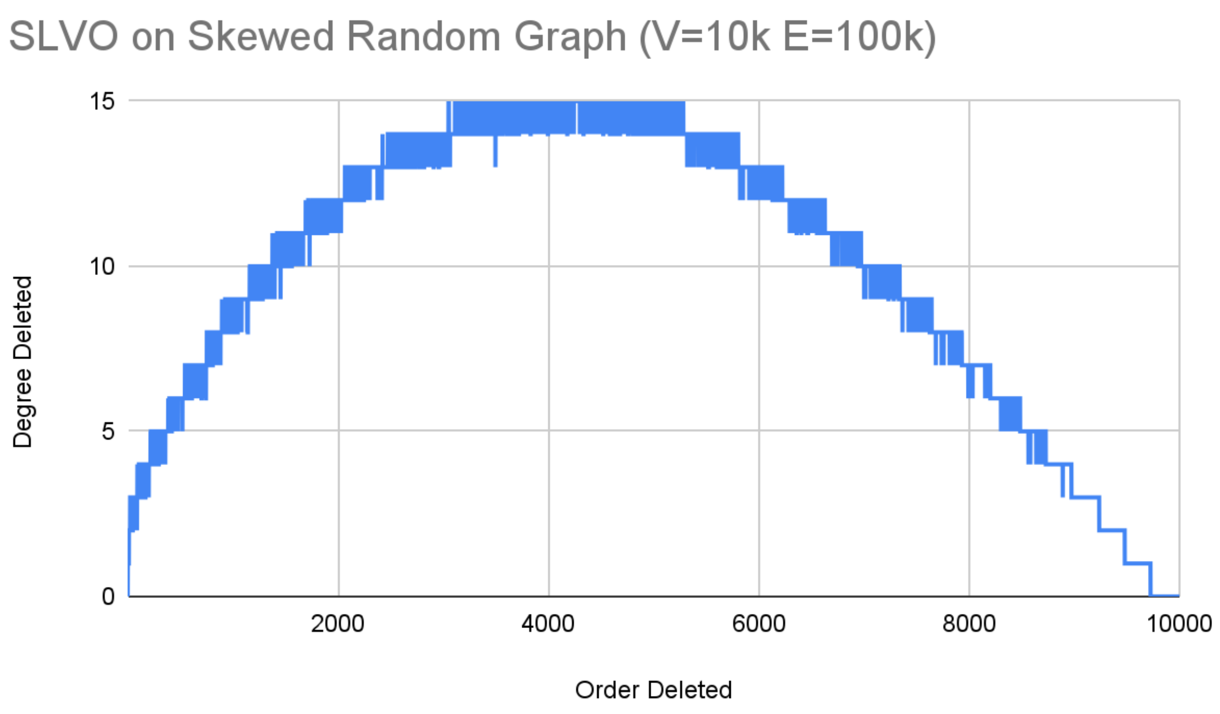
Uniform Random Ordering:

Number of Colors: 12

**Skewed Graph:**

Smallest Last Vertex Ordering:

Number of Colors: 11



Max Degree Deleted: 15

Terminal Clique Size: 3

Smallest Original Vertex Ordering:

Number of Colors: 11

Smallest Original Vertex Ordering Inverted:

Number of Colors: 17

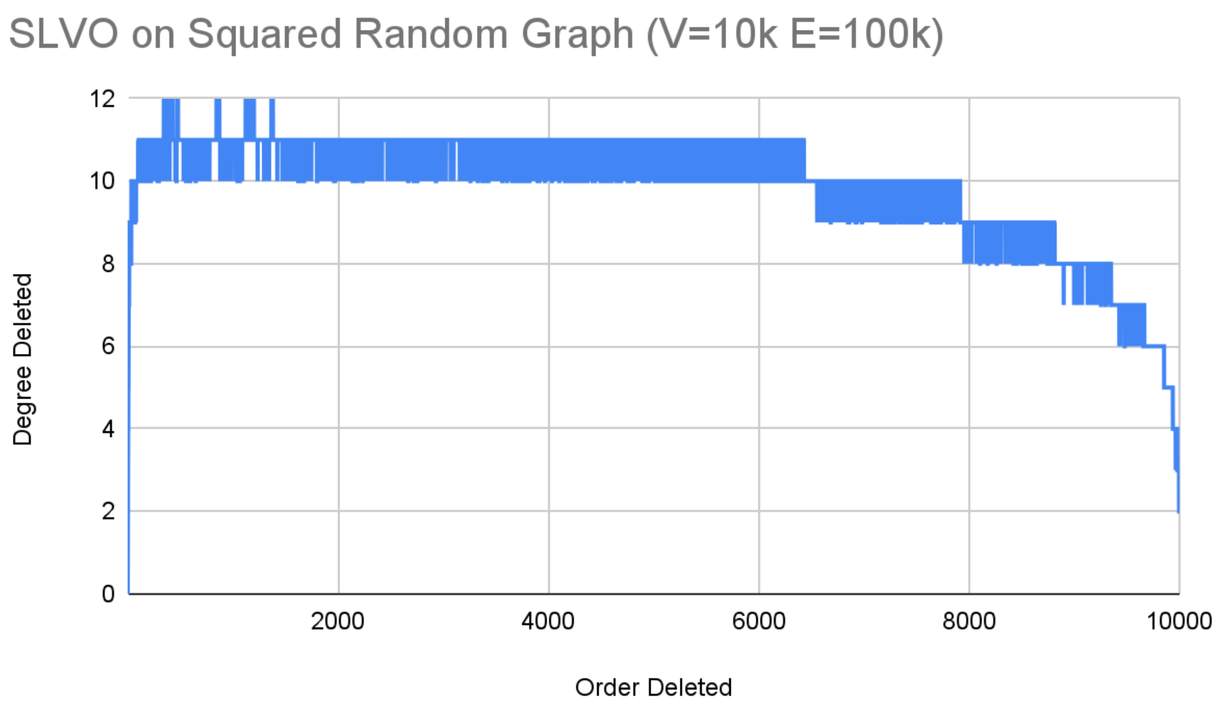
Uniform Random Ordering:

Number of Colors: 14

**Squared Graph:**

Smallest Last Vertex Ordering:

Number of Colors: 11



Max Degree Deleted: 12

Terminal Clique Size: 5

Smallest Original Vertex Ordering:

Number of Colors: 11

Smallest Original Vertex Ordering Inverted:

Number of Colors: 26

Uniform Random Ordering:

Number of Colors: 19

**Ordering Results:**

Smallest Last Vertex Ordering: Clearly the best, and if this was truly a tool I was selling, this would be the default. It clearly outperforms the others(except Smallest Original at times), and orders it with a more complicated technique, to allow later decisions to be more informed than the other orderings. It does have 1 tiny drawback, and that is complexity, as removing a vertex requires more memory reading that just ordering them randomly. The max degree deleted also bounds the colorset, because if you think about it, at any given point in SLVO, a node has at most max degree # neighbors, meaning it can only see that many colors as restricted at any given point.

Smallest Original Vertex Ordering: This is my runner up to SLVO, as they both produced the same color set on the graphs I created above. It should also be faster than SLVO, but my guess is its a scaling issue for much larger graphs. I do not see the color count matching SLVO for all possible graphs, but I cannot prove that. In concept SLVO should be more optimal because of its choice to edit connected nodes.

Smallest Original Vertex Ordering INVERTED: This is my personal coloring method, and by far the worst. In every case I have tested above or during development, this always produced the highest color count. In a sense if SOVO is an optimized order, this is the intentionally unoptimized version. This is mainly here to show how important ordering is when you are coloring a graph. If this was a sales pitch, I would only use this as an example of order's importance.

Uniform Random Vertex Ordering: This was a surprise hit in my opinion, and did not result in the worst color set by a long shot. It does obviously produce a less optimized set, but I feel that it isn't sabotage. The inverted ordering always performed worse than this, showing me that greedy coloring alone can overcome a somewhat unoptimized ordering, but not an intentionally bad one. Its main benefit is its speed, as it simply just shuffles your adjacency list and gives the original and the shuffle to the greedy coloring.

**Conclusion:**

I would say that if this was a product for a ~10000 node system, that Smallest Last Vertex Ordering and Smallest Original Vertex Ordering would be the best options that a user could make. Smallest Last Vertex Ordering is obviously the best, but given you do want a little bit less read/write, Smallest Original takes a very simplified approach. The random is not unusable, but I'd advise against it, as it could produce (in theory) the absolute worst order for a graph. Finally I would say to not use the inverted Smallest Original method, as it is the worst and least effective method for doing this kind of work. If you want the optimal output, use SLVO, but if you are concerned about sheer processing power, Smallest Originals simple parse approach will help lower the complexity of the program overall.

## Code Dump

I recommend anybody interested read this on the github below

https://github.com/NotTimm/5350Final

**Main.py**: this is one file that had seen several changes, and I used it mostly as a driver to set up the list for a given part, not a structure for all the parts in one.

```python
import pickle, sys, time
import adjList, degList, coloring, ordering

def distGraph(path, adj):
    # with open(path) as file:
    print("vertID, degree")
    for vert in adj.vertices:
        print(str(vert.id) + ', ' + str(vert.degree))
        # file.write(vert.id,',',vert.degree,'\n')

startTime = time.time()
sys.setrecursionlimit(10000)
# adj = adjList.AdjacencyList(10000)
V = int(sys.argv[1])
adj = adjList.AdjacencyList.deserialize('graphDumps/squared.adj')
# adj.completeBuild()
# adj.cycleBuild()
# adj.randomUniformBuild(100000)
# adj.randomPersonalBuild(200000)
# adj.randomSkewedBuild(40000)
deg = degList.degreeList(adj)
# adj.printList()
```

```
if V == 1:
    coloring.UROColoring(adj)
elif V == 2:
    coloring.SOVOFColoring(adj,deg)
elif V == 3:
    coloring.SLVOColoing(adj,deg)
else:
    coloring.SOVOColoring(adj,deg)
temp = []
for i in adj.vertices:
    temp.append(i.color)
print(max(temp)+1)
# adj.printList()
# adj1.printList()
# deg.printList()
```

**adjList.py**

```
import random, math, pickle, struct
class Vertice:
    def __init__(self, degree, id):
        self.id = id
        self.degree = degree
        self.edges = None
        self.next = None
        self.last = None
        self.removed = False
        self.color = None

class Edge:
    def __init__(self, destination, next):
        self.destination = destination
        self.next = next

class AdjacencyList:
    def __init__(self, vertCount):
        self.vertices = []
        for i in range(vertCount):
            self.vertices.append(Vertice(0,i))
```

```
def checkIDs(self, vert1, vert2):
    if vert1 < 0 or vert1 >= len(self.vertices):
        print("ERROR out of bounds")
        exit(69)
    elif vert2 < 0 or vert2 >= len(self.vertices):
        print("ERROR out of bounds")
        exit(69)

def edgeExists(self, vert1, vert2):
    self.checkIDs(vert1, vert2)
    curEdge = Edge(None, self.vertices[vert1].edges)
    while curEdge := curEdge.next:
        if curEdge.destination == vert2:
            return True
    return False

def addEdge(self, vert1, vert2):
    self.checkIDs(vert1, vert2)
    edge1 = Edge(vert2, self.vertices[vert1].edges)
    edge2 = Edge(vert1, self.vertices[vert2].edges)

    self.vertices[vert1].edges = edge1
    self.vertices[vert1].degree += 1

    self.vertices[vert2].edges = edge2
    self.vertices[vert2].degree += 1

### Build Scenarios Below ###

def completeBuild(self):
    vertI = -1
    while (vertI := vertI+1) < len(self.vertices)-1:
        vertO = vertI
        while (vertO := vertO+1) < len(self.vertices):
            self.addEdge(vertI, vertO)

def cycleBuild(self):
    vertI = -1
```

```
        while (vertI := vertI+1) < len(self.vertices)-1:
            self.addEdge(vertI, vertI+1)
        self.addEdge(0, len(self.vertices)-1)

    def randomUniformBuild(graph, conflicts):
        if conflicts > len(graph.vertices) *
(len(graph.vertices)-1)/2:
            print("ERROR too many conflicts")
            exit(420)
        i = -1
        while (i := i+1) < conflicts:
            vert1 = 0
            vert2 = 0

            while vert1 == vert2 or graph.edgeExists(vert1, vert2):
                vert1 = random.randint(0, len(graph.vertices)-1)
                vert2 = random.randint(0, len(graph.vertices)-1)
            graph.addEdge(vert1, vert2)

    def randomSkewedBuild(graph, conflicts):
        if conflicts > len(graph.vertices) *
(len(graph.vertices)-1)/2:
            print("ERROR too many conflicts")
            exit(420)
        i = -1
        while (i := i+1) < conflicts:
            vert1 = 0
            vert2 = 0

            while vert1 == vert2 or graph.edgeExists(vert1, vert2):
                vert1 =
len(graph.vertices)-1-int(math.sqrt(4.0*2.0*float(random.randint(0,(l
en(graph.vertices)-1)*len(graph.vertices)/2))+1.0)/2.0-.5)
                vert2 =
len(graph.vertices)-1-int(math.sqrt(4.0*2.0*float(random.randint(0,(l
en(graph.vertices)-1)*len(graph.vertices)/2))+1.0)/2.0-.5)
            graph.addEdge(vert1, vert2)

    def randomPersonalBuild(graph, conflicts):
        if conflicts > len(graph.vertices) *
(len(graph.vertices)-1)/2:
            print("ERROR too many conflicts")
            exit(420)
        i = -1
        while (i := i+1) < conflicts:
            vert1 = 0
            vert2 = 0

            while vert1 == vert2 or graph.edgeExists(vert1, vert2):
                vert1 = int(math.pow(random.random(), 2) *
float(len(graph.vertices)))
                vert2 = int(math.pow(random.random(), 2) *
float(len(graph.vertices)))
                # vert1 = int(random.betavariate(.5,.5) *
len(graph.vertices))
                # vert2 = int(random.betavariate(.5,.5) *
len(graph.vertices))
            graph.addEdge(vert1, vert2)

    ### Print Function ###

    def printList(self):
        # print("Vert #: ", len(self.vertices))
        for index, i in enumerate(self.vertices):
            print("VertID: ", index, ", Degree: ", i.degree, " Color:
", i.color, ", Removed: ", i.removed, " }", sep = "", end ="")
            cur = Edge(None, i.edges)
            while cur := cur.next:
                print("->", cur.destination, sep = "", end = "")
            print()

    def removeEdge(self, vert1, vert2):
        vertTemp = self.vertices[vert1].edges
        while vertTemp.destination != vert2:
            vertTemp = vertTemp.next
        vertTemp.removed = True
        vertTemp = self.vertices[vert2].edges
        while vertTemp.destination != vert1:
            vertTemp = vertTemp.next
        vertTemp.removed = True

    def save(self, path):
        with open(path, 'wb') as file:
            pickle.dump(self, file)

    def serialize(self, path):
        with open(path, 'wb') as file:
            file.write(struct.pack('<L', len(self.vertices)))
            for vert in self.vertices:
                cur = vert.edges
                while cur != None:
                    if cur.destination > vert.id:
                        file.write(struct.pack('<L',
cur.destination))
                    cur = cur.next
                file.write(struct.pack('<L', 0))

    def deserialize( path):
        with open(path, 'rb') as file:
            size = struct.unpack('<L', file.read(4))[0]
            temp = AdjacencyList(size)
            i = -1
            while (i := i+1) < size:
                while(True):
                    dest = struct.unpack('<L', file.read(4))[0]
                    if dest == 0:
                        break
                    temp.addEdge(i, dest)
            return temp


# if __name__ == '__main__':
#     lltest = LinkedList()
#     lltest.head = Node('poo')
#     temp = lltest.head
#     for i in range(10):
#         new = Node(i)
#         temp.next = new
#         temp = temp.next
#     lltest.printList()
```

**degList.py**

```
import adjList

class listVal:
    def __init__(self, value, next):
        self.last = None
        self.vert = value
        self.next = next

class degreeList:
    def __init__(self, adj):
        self.degrees = [None] * len(adj.vertices)
        n = -1
        while((n := n+1) < len(adj.vertices)):
            self.insert(adj.vertices[n], adj.vertices[n].degree)

    def insert(self, value, degree):
        if value.last != None or value.next != None:
            exit(69)
        # print(degree)
        temp = self.degrees[degree]
        value.last = None
        value.next = temp
        if temp != None:
            temp.last = value
        self.degrees[degree] = value

    def findSmallest(self):
        for degree in self.degrees:
            if degree != None:
                return degree
        return None

    def removeVert(self, adj, vertice):
        vert = vertice
        vert.removed = True
        curTemp = vert.edges
        while curTemp != None:
            dest = adj.vertices[curTemp.destination]
            if(not dest.removed):
                self.remove(dest)
                dest.degree -= 1
                self.insert(dest, dest.degree)
            curTemp = curTemp.next

    def remove(self, vert):
        if vert.last != None:
            vert.last.next = vert.next
        else:
            self.degrees[vert.degree] = vert.next
        if vert.next != None:
            vert.next.last = vert.last
        vert.last = None
        vert.next = None

    def printList(self):
        i = -1
        while (i := i+1) < len(self.degrees):
            print('['+str(i)+'°]:', end='')
            cur = self.degrees[i]
            while cur != None:
                if cur != None:
                    print(', ',end='')
                print(cur.id,end='')
                cur = cur.next
            print()
```

**Ordering.py**

```
import adjList, degList
import random

def smallestLastVertOrder(adj, deg):
    removed = -1
    out = [None] * len(adj.vertices)
    while (removed := removed+1) < len(adj.vertices):
        # print('Step #: ', removed+1)
        smallest = deg.findSmallest()
        deg.removeVert(adj, smallest)
        deg.remove(smallest)
        smallest.removed = True
        out[len(adj.vertices)-removed-1] = smallest
        # adj.printList()
    return out

def smallestOriginalVertOrder(adj, deg):
    out = [None] * len(adj.vertices)
    i = 1
    for obj in deg.degrees:
        copy = obj
        while copy != None:
            out[len(adj.vertices)-i] = copy
            copy = copy.next
            i += 1
    return out

def smallestOriginalVertOrderFlipped(adj, deg): # My personal final
order for the coloring
    out = []
    for obj in deg.degrees:
        copy = obj
        while copy != None:
            out.append(copy)
            copy = copy.next
    return out

def uniformRandomVertOrder(adj):
    copyAdj = adj.vertices.copy()
    random.shuffle(copyAdj)
    return copyAdj

def SLVOTerminalClique(order):
    size = 0
    for vert in order:
        if vert.color == size:
            size += 1
        else:
            break
    return size
```

**Coloring.py**

```
import adjList, degList, ordering

def greedyColoring(adj, order): # can be given an adj list in both
for random greedy, or an adj plus its slvo for slvo coloring
    for vert in order:
        available = [True] * len(order)
        cur = vert.edges
        while cur != None:
            if adj.vertices[cur.destination].color != None:
                available[adj.vertices[cur.destination].color] =
False
            cur = cur.next
        for i in range(len(available)):
            if available[i]:
                vert.color = i
                break

def SLVOColoing(adj, deg):
    slvo = ordering.smallestLastVertOrder(adj, deg)
    greedyColoring(adj, slvo)
    # print(ordering.SLVOTerminalClique(slvo))
    # o = 1
    # l = []
    # for i in slvo:
    #     l.append(i.degree)
```

```
        # print(i.degree,', ', o, sep='')
        # o += 1
    # print(max(l))

def SOVOColoring(adj, deg):
    sovo = ordering.smallestOriginalVertOrder(adj, deg)
    greedyColoring(adj, sovo)

def UROColoring(adj):
    uro = ordering.uniformRandomVertOrder(adj)
    greedyColoring(adj, uro)

def SOVOFColoring(adj, deg):
    sovoFlipped = ordering.smallestOriginalVertOrderFlipped(adj, deg)
    greedyColoring(adj, sovoFlipped)
```