Combinatorial Decision Making and Optimization

Last update: 23 February 2024

Academic Year 2023 – 2024

Alma Mater Studiorum · University of Bologna

Contents

1 Constraint programming

1.1 Definitions

Constraint satisfaction problem (CSP) Triple $\langle X, D, C \rangle$ where:

- X is the set of decision variables $\{x_1, \ldots, x_n\}$.
- D is the set of domains $\{D_1, \ldots, D_n\}$ for X.
- C is the set of constraints $\{C_1, \ldots, C_m\}$. Each C_i is a relation over the domain of X (i.e. $C_i \subseteq D_j \times \cdots \times D_k$).

Constraint optimization problem (COP) Tuple $\langle X, D, C, f \rangle$ where $\langle X, D, C \rangle$ is a CSP and f is the objective variable to minimize or maximize.

Constraint

    Extensional representation List all allowed combinations.

    Intensional representation Declarative relations between variables.

Symmetry Search states that lead to the same result.

    Variable symmetry A permutation of the assignment order of the variables results in the same feasible or unfeasible solution.

    Value symmetry A permutation of the values in the domain results in the same feasible or unfeasible solution.

    Remark. Variable and value symmetries can be combined resulting in a total of $2n!$ possible symmetries.

1.2 Modeling techniques

Auxiliary variables Add new variables to capture constraints difficult to model or to reduce the search space by collapsing multiple variables into one.

Global constraints Relation between an arbitrary number of variables. It is usually computationally faster than listing multiple constraints.

Implied constraints Semantically redundant constraints with the advantage of pruning the search space earlier.

    Remark. A purely redundant constraint is also an implied constraint but it does not give any computational improvement.

Symmetry breaking constraints Constraints to avoid considering symmetric states. Usually, it is sufficient to fix an ordering of the variables.

    Remark. When introducing symmetry breaking constraints, it might be possible to add new simplifications and implied constraints.

Dual viewpoint Modeling a problem from a different perspective might result in a more efficient search.

> **Example.** Exploiting geometric symmetries.

Combined model Merging two or more models of the same problem by adding channeling constraints to guarantee consistency.

> Combining two models can be useful for obtaining the advantages of both (e.g. one model uses global constraints, while the other handles symmetries).

> **Remark.** When combining multiple models, some constraints might be simplified as one of the models already captures it natively.