Fundamentals of Artificial Intelligence and Knowledge Representation (Module 3)

Last update: 13 October 2023

Academic Year 2023 – 2024

Alma Mater Studiorum · University of Bologna

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1 Introduction

1.1 Uncertainty

Uncertainty A task is uncertain if we have:

- Partial observations
- Noisy or wrong information
- Uncertain action outcomes
- Complex models

A purely logic approach leads to:

- Risks falsehood: unreasonable conclusion when applied in practice.
- Weak decisions: too many conditions required to make a conclusion.

1.1.1 Handling uncertainty

Default/nonmonotonic logic Works on assumptions. An assumption can be contradicted by an evidence.

Rule-based systems with fudge factors Formulated as premise $\rightarrow_{\text{prob.}}$ effect. Have the following issues:

- Locality: how can the probability account all the evidence.
- Combination: chaining of unrelated concepts.

Probability Assign a probability given the available known evidence.

Note: fuzzy logic handles the degree of truth and not the uncertainty.

Decision theory Defined as:

$$\text{Decision theory} = \text{Utility theory} + \text{Probability theory}$$

where the utility theory depends on one's preferences.

1.1.2 Probability

Sample space Set Ω of all possible worlds.

Event Subset $A \subseteq \Omega$.

Sample point/Possible world/Atomic event Element $\omega \in \Omega$.

Probability space A probability space/model is a function $\mathcal{P}(\cdot) : \Omega \to [0, 1]$ assigned to a sample space such that:

- $0 \leq \mathcal{P}(\omega) \leq 1$
- $\sum_{\omega \in \Omega} \mathcal{P}(\omega) = 1$

- $\mathcal{P}(A) = \sum_{\omega \in A} \mathcal{P}(\omega)$

Random variable A function from an event to some range (e.g. reals, booleans, ...).

Probability distribution For any random variable X:

$$\mathcal{P}(X = x_i) = \sum_{\omega \text{ st } X(\omega) = x_i} \mathcal{P}(\omega)$$

Proposition Event where a random variable has a certain value.

$$a = \{\omega \,|\, A(\omega) = \texttt{true}\}$$

$$\neg a = \{\omega \,|\, A(\omega) = \texttt{false}\}$$

$$(\texttt{Weather} = \texttt{rain}) = \{\omega \,|\, B(\omega) = \texttt{rain}\}$$

Prior probability Prior/unconditional probability of a proposition based on known evidence.

Probability distribution (all) Gives all the probabilities of a random variable.

$$\mathbf{P}(A) = \langle \mathcal{P}(A = a_1), \ldots, \mathcal{P}(A = a_n) \rangle$$

Joint probability distribution The joint probability distribution of a set of random variables gives the probability of all the different combinations of their atomic events.

Note: Every question on a domain can, in theory, be answered using the joint distribution. In practice, it is hard to apply.

Example. $\mathbf{P}(\texttt{Weather}, \texttt{Cavity}) =$

	Weather=sunny	Weather=rain	Weather=cloudy	Weather=snow
Cavity=true	0.144	0.02	0.016	0.02
Cavity=false	0.576	0.08	0.064	0.08

Probability density function The probability density function (PDF) of a random variable X is a function $p : \mathbb{R} \to \mathbb{R}$ such that:

$$\int_{\mathcal{T}_X} p(x) \, dx = 1$$

Uniform distribution

$$p(x) = \text{Unif}[a, b](x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Gaussian (normal) distribution

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$\mathcal{N}(0, 1)$ is the standard gaussian.

Conditional probability Probability of a prior knowledge with new evidence:

$$\mathcal{P}(a|b) = \frac{\mathcal{P}(a \wedge b)}{\mathcal{P}(b)}$$