

# **Fundamentals of Artificial Intelligence and Knowledge Representation (Module 1)**

Last update: 24 October 2023

Academic Year 2023 – 2024  
Alma Mater Studiorum · University of Bologna

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	AI systems classification . . . . .	1
1.1.1	Intelligence classification . . . . .	1
1.1.2	Capability classification . . . . .	1
1.1.3	AI approaches . . . . .	1
1.2	Symbolic AI . . . . .	1
1.3	Machine learning . . . . .	2
1.3.1	Training approach . . . . .	2
1.3.2	Tasks . . . . .	2
1.3.3	Neural networks . . . . .	2
1.4	Automated planning . . . . .	3
1.5	Swarm intelligence . . . . .	3
1.6	Decision support systems . . . . .	3
<b>2</b>	<b>Search problems</b>	<b>5</b>
2.1	Search strategies . . . . .	5
2.1.1	Search tree . . . . .	5
2.1.2	Strategies . . . . .	5
2.1.3	Evaluation . . . . .	6
2.2	Non-informed search . . . . .	6
2.2.1	Breadth-first search (BFS) . . . . .	6
2.2.2	Uniform-cost search . . . . .	6
2.2.3	Depth-first search (DFS) . . . . .	7
2.2.4	Depth-limited search . . . . .	7
2.2.5	Iterative deepening . . . . .	7
2.3	Informed search . . . . .	8
2.3.1	Best-first search . . . . .	8
2.4	Graph search . . . . .	10
2.4.1	A* with graphs . . . . .	10
<b>3</b>	<b>Local search</b>	<b>11</b>
3.1	Iterative improvement (hill climbing) . . . . .	11
3.2	Meta heuristics . . . . .	12
3.2.1	Simulated annealing . . . . .	12
3.2.2	Tabu search . . . . .	13
3.2.3	Iterated local search . . . . .	13
3.2.4	Population based (genetic algorithm) . . . . .	14
<b>4</b>	<b>Swarm intelligence</b>	<b>16</b>
4.1	Ant colony optimization (ACO) . . . . .	16
4.1.1	ACO system . . . . .	17
4.2	Artificial bee colony algorithm (ABC) . . . . .	17
4.3	Particle swarm optimization (PSO) . . . . .	18

# 1 Introduction

## 1.1 AI systems classification

### 1.1.1 Intelligence classification

Intelligence is defined as the ability to perceive or infer information and to retain the knowledge for future use.

**Weak AI** aims to build a system that acts as an intelligent system. Weak AI

**Strong AI** aims to build a system that is actually intelligent. Strong AI

### 1.1.2 Capability classification

**General AI** systems able to solve any generalized task. General AI

**Narrow AI** systems able to solve a particular task. Narrow AI

### 1.1.3 AI approaches

**Symbolic AI (top-down)** Symbolic representation of knowledge, understandable by humans. Symbolic AI

**Connectionist approach (bottom up)** Neural networks. Knowledge is encoded and not understandable by humans. Connectionist approach

## 1.2 Symbolic AI

**Deductive reasoning** Conclude something given some premises (general to specific). It is unable to produce new knowledge. Deductive reasoning

**Example.** "All men are mortal" and "Socrates is a man"  $\rightarrow$  "Socrates is mortal"

**Inductive reasoning** A conclusion is derived from an observation (specific to general). Produces new knowledge, but correctness is not guaranteed. Inductive reasoning

**Example.** "Several birds fly"  $\rightarrow$  "All birds fly"

**Abduction reasoning** An explanation of the conclusion is found from known premises. Differently from inductive reasoning, it does not search for a general rule. Produces new knowledge, but correctness is not guaranteed. Abduction reasoning

**Example.** "Socrates is dead" (conclusion) and "All men are mortal" (knowledge)  $\rightarrow$  "Socrates is a man"

**Reasoning by analogy** Principle of similarity (e.g. k-nearest-neighbor algorithm). Reasoning by analogy

**Example.** "Socrates loves philosophy" and Socrates resembles John  $\rightarrow$  "John loves philosophy"

**Constraint reasoning and optimization** Constraints, probability, statistics. Constraint reasoning

## 1.3 Machine learning

### 1.3.1 Training approach

<b>Supervised learning</b> Trained on labeled data (ground truth is known). Suitable for classification and regression tasks.	Supervised learning
<b>Unsupervised learning</b> Trained on unlabeled data (the system makes its own discoveries). Suitable for clustering and data mining.	Unsupervised learning
<b>Semi-supervised learning</b> The system is first trained to synthesize data in an unsupervised manner, followed by a supervised phase.	Semi-supervised learning
<b>Reinforcement learning</b> An agent learns by simulating actions in an environment with rewards and punishments depending on its choices.	Reinforcement learning

### 1.3.2 Tasks

<b>Classification</b> Supervised task that, given the input variables $X$ and the output (discrete) categories $Y$ , aims to approximate a mapping function $f : X \rightarrow Y$ .	Classification
<b>Regression</b> Supervised task that, given the input variables $X$ and the output (continuous) variables $Y$ , aims to approximate a mapping function $f : X \rightarrow Y$ .	Regression
<b>Clustering</b> Unsupervised task that aims to organize objects into groups.	Clustering

### 1.3.3 Neural networks

A neuron (**perceptron**) computes a weighted sum of its inputs and passes the result to an activation function to produce the output. Perceptron

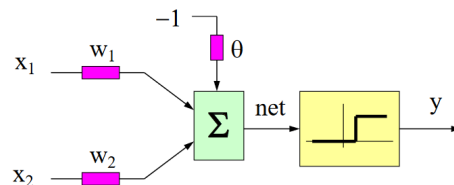


Figure 1.1: Representation of an artificial neuron

A **feed-forward neural network** is composed of multiple layers of neurons, each connected to the next one. The first layer is the input layer, while the last is the output layer. Intermediate layers are hidden layers. Feed-forward neural network

The expressivity of a neural networks increases when more neurons are used:

**Single perceptron** Able to compute a linear separation.

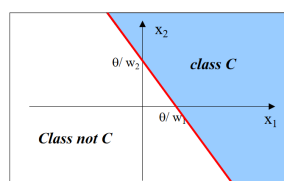


Figure 1.2: Separation performed by one perceptron

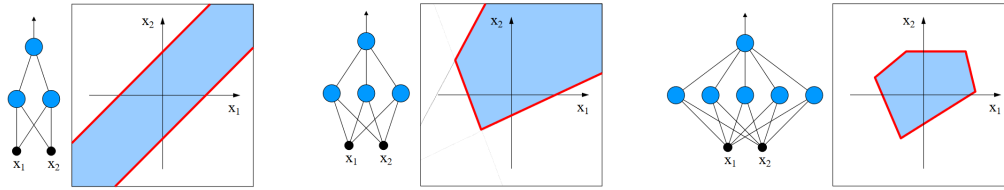


Figure 1.3: Separation performed by a three-layer network

**Three-layer network** Able to separate a convex region ( $n_{\text{edges}} \leq n_{\text{hidden neurons}}$ )

**Four-layer network** Able to separate regions of arbitrary shape.

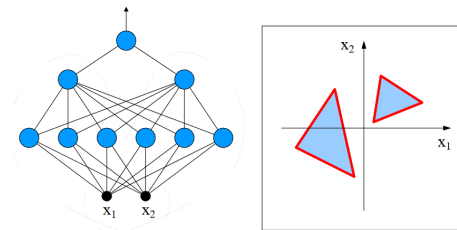


Figure 1.4: Separation performed by a four-layer network

**Theorem 1.3.1** (Universal approximation theorem). A feed-forward network with one hidden layer and a finite number of neurons is able to approximate any continuous function with desired accuracy.

Universal approximation theorem

**Deep learning** Neural network with a large number of layers and neurons. The learning process is hierarchical: the network exploits simple features in the first layers and synthesis more complex concepts while advancing through the layers.

Deep learning

## 1.4 Automated planning

Given an initial state, a set of actions and a goal, **automated planning** aims to find a partially or totally ordered sequence of actions to achieve a goal.

Automated planning

An **automated planner** is an agent that operates in a given domain described by:

- Representation of the initial state
- Representation of a goal
- Formal description of the possible actions (preconditions and effects)

## 1.5 Swarm intelligence

Decentralized and self-organized systems that result in emergent behaviors.

Swarm intelligence

## 1.6 Decision support systems

**Knowledge based system** Use knowledge (and data) to support human decisions. Bottlenecked by knowledge acquisition.

Knowledge based system

*Not required for the exam*

Different levels of decision support exist:

<b>Descriptive analytics</b>	Data are used to describe the system (e.g. dashboards, reports, ...). Human intervention is required.	Descriptive analytics
<b>Diagnostic analytics</b>	Data are used to understand causes (e.g. fault diagnosis) Decisions are made by humans.	Diagnostic analytics
<b>Predictive analytics</b>	Data are used to predict future evolutions of the system. Uses machine learning models or simulators (digital twins)	Predictive analytics
<b>Prescriptive analytics</b>	Make decisions by finding the preferred scenario. Uses optimization systems, combinatorial solvers or logical solvers.	Prescriptive analytics

## 2 Search problems

### 2.1 Search strategies

<b>Solution space</b>	Set of all the possible sequences of actions an agent may apply. Some of these lead to a solution.	Solution space
<b>Search algorithm</b>	Takes a problem as input and returns a sequence of actions that solves the problem (if exists).	Search algorithm

#### 2.1.1 Search tree

<b>Expansion</b>	Starting from a state, apply a successor function and generate a new state.	Expansion
<b>Search strategy</b>	Choose which state to expand. Usually is implemented using a fringe that decides which is the next node to expand.	Search strategy
<b>Search tree</b>	Tree structure to represent the expansion of all states starting from a root (i.e. the representation of the solution space).  Nodes are states and branches are actions. A leaf can be a state to expand, a solution or a dead-end. Algorithm 1 describes a generic tree search algorithm.	Search tree

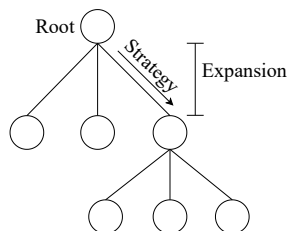


Figure 2.1: Search tree

Each node contains:

- The state
- The parent node
- The action that led to this node
- The depth of the node
- The cost of the path from the root to this node

#### 2.1.2 Strategies

<b>Non-informed strategy</b>	Domain knowledge not available. Usually does an exhaustive search.	Non-informed strategy
<b>Informed strategy</b>	Use domain knowledge by using heuristics.	Informed strategy

---

**Algorithm 1** Tree search

---

```
def treeSearch(problem, fringe):
    fringe.push(problem.initial_state)
    # Get a node in the fringe and expand it if it is not a solution
    while fringe.notEmpty():
        node = fringe.pop()
        if problem.isGoal(node.state):
            return node.solution
        fringe.pushAll(expand(node, problem))
    return FAILURE

def expand(node, problem):
    successors = set()
    # List all neighboring nodes
    for action, result in problem.successor(node.state):
        s = new Node(
            parent=node, action=action, state=result, depth=node.depth+1,
            cost=node.cost + problem.pathCost(node, s, action)
        )
        successors.add(s)
    return successors
```

---

### 2.1.3 Evaluation

**Completeness** if the strategy is guaranteed to find a solution (when exists).

Completeness

**Time complexity** time needed to complete the search.

Time complexity

**Space complexity** memory needed to complete the search.

Space complexity

**Optimality** if the strategy finds the best solution (when more solutions are possible).

Optimality

## 2.2 Non-informed search

### 2.2.1 Breadth-first search (BFS)

Always expands the less deep node. The fringe is implemented as a queue (FIFO).

Breadth-first search

<b>Completeness</b>	Yes
<b>Optimality</b>	Only with uniform cost (i.e. all edges have same cost)
<b>Time and space complexity</b>	$O(b^d)$ , where the solution depth is $d$ and the branching factor is $b$ (i.e. each non-leaf node has $b$ children)

The exponential space complexity makes BFS impractical for large problems.

### 2.2.2 Uniform-cost search

Same as BFS, but always expands the node with the lowest cumulative cost.

Uniform-cost search

<b>Completeness</b>	Yes
<b>Optimality</b>	Yes
<b>Time and space complexity</b>	$O(b^d)$ , with solution depth $d$ and branching factor $b$



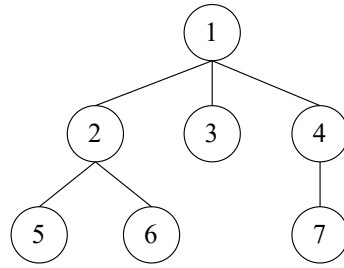


Figure 2.2: BFS visit order

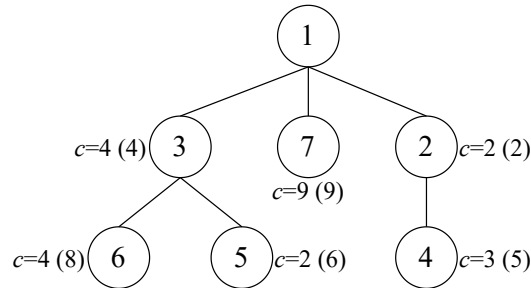


Figure 2.3: Uniform-cost search visit order.  $(n)$  is the cumulative cost

### 2.2.3 Depth-first search (DFS)

Always expands the deepest node. The fringe is implemented as a stack (LIFO).

Depth-first search

<b>Completeness</b>	No (loops)
<b>Optimality</b>	No
<b>Time complexity</b>	$O(b^m)$ , with maximum depth $m$ and branching factor $b$
<b>Space complexity</b>	$O(b \cdot m)$ , with maximum depth $m$ and branching factor $b$

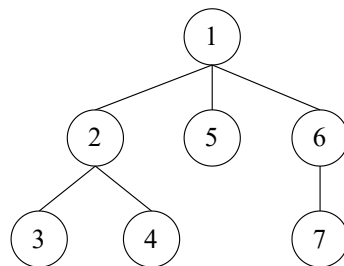


Figure 2.4: DFS visit order

### 2.2.4 Depth-limited search

Same as DFS, but introduces a maximum depth. A node at the maximum depth will not be explored further.

Depth-limited search

This allows to avoid infinite branches (i.e. loops).

### 2.2.5 Iterative deepening

Iterative deepening

Runs a depth-limited search by trying all possible depth limits. It is important to note that each iteration is executed from scratch (i.e. a new execution of depth-limited search).

---

**Algorithm 2** Iterative deepening

---

```
def iterativeDeepening(G):
    for c in range(G.max_depth):
        sol = depthLimitedSearch(G, c)
        if sol is not FAILURE:
            return sol
    return FAILURE
```

---

Both advantages of DFS and BFS are combined.

<b>Completeness</b>	Yes
<b>Optimality</b>	Only with uniform cost
<b>Time complexity</b>	$O(b^d)$ , with solution depth $d$ and branching factor $b$
<b>Space complexity</b>	$O(b \cdot d)$ , with solution depth $d$ and branching factor $b$

## 2.3 Informed search

Informed search uses evaluation functions (heuristics) to reduce the search space and estimate the effort needed to reach the final goal.

Informed search

### 2.3.1 Best-first search

Uses heuristics to compute the desirability of the nodes (i.e. how close they are to the goal). The fringe is ordered according to the estimated scores.

Best-first search

**Greedy search / Hill climbing** The heuristic only evaluates nodes individually and does not consider the path to the root (i.e. expands the node that currently seems closer to the goal).

Greedy search / Hill climbing

<b>Completeness</b>	No (loops)
<b>Optimality</b>	No
<b>Time and space complexity</b>	$O(b^d)$ , with solution depth $d$ and branching factor $b$

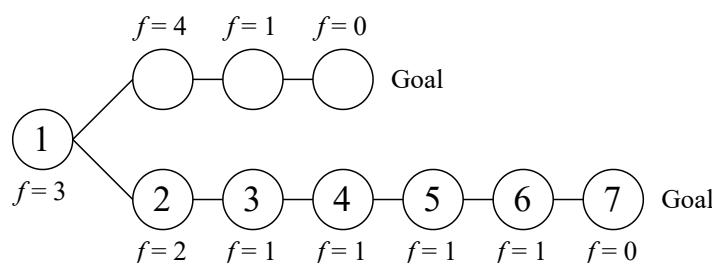


Figure 2.5: Hill climbing visit order

**A\*** The heuristic also considers the cumulative cost needed to reach a node from the root. The score associated to a node  $n$  is:

$$f(n) = g(n) + h'(n)$$

where  $g$  is the depth of the node and  $h'$  is the heuristic that computes the distance to the goal.

**Optimistic/Feasible heuristic** Given  $t(n)$  that computes the true distance of a node  $n$  to the goal. An heuristic  $h'(n)$  is optimistic (i.e. feasible) if:

$$h'(n) \leq t(n)$$

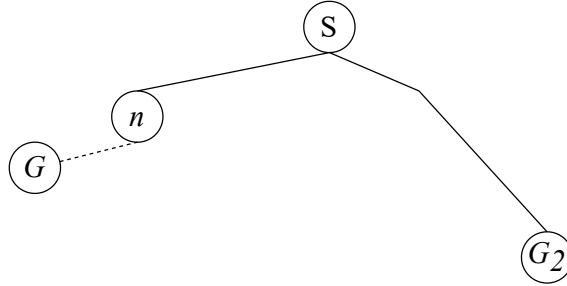
Optimistic/Feasible heuristic

In other words,  $h'$  is optimistic if it always underestimates the distance to the goal.

**Theorem 2.3.1.** If the heuristic used by A\* is optimistic  $\Rightarrow$  A\* is optimal

*Proof.* Consider a scenario where the queue contains:

- A node  $n$  whose child is the optimal solution
- A sub-optimal solution  $G_2$



We want to prove that A\* will always expand  $n$ .

Given an optimistic heuristic  $f(n) = g(n) + h'(n)$  and the true distance of a node  $n$  to the goal  $t(n)$ , we have that:

$$f(G_2) = g(G_2) + h'(G_2) = g(G_2), \text{ as } G_2 \text{ is a solution: } h'(G_2) = 0$$

$$f(G) = g(G) + h'(G) = g(G), \text{ as } G \text{ is a solution: } h'(G) = 0$$

Moreover,  $g(G_2) > g(G)$  as  $G_2$  is suboptimal. Therefore,  $f(G_2) > f(G)$ .

Furthermore, as  $h'$  is feasible, we have that:

$$\begin{aligned} h'(n) \leq t(n) &\iff g(n) + h'(n) \leq g(n) + t(n) = g(G) = f(G) \\ &\iff f(n) \leq f(G) \end{aligned}$$

In the end, we have that  $f(G_2) > f(G) \geq f(n)$ . So we can conclude that A\* will never expand  $G_2$  as:

$$f(G_2) > f(n)$$

□

<b>Completeness</b>	Yes
<b>Optimality</b>	Only if the heuristic is optimistic
<b>Time and space complexity</b>	$O(b^d)$ , with solution depth $d$ and branching factor $b$

In generally, it is better to use heuristics with large values (i.e. heuristics that don't underestimate too much).

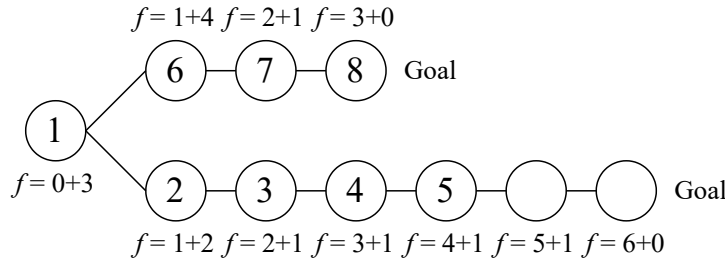


Figure 2.6: A\* visit order

## 2.4 Graph search

Differently from a tree search, searching in a graph requires to keep track of the explored nodes.

Graph search

---

### Algorithm 3 Graph search

---

```
def graphSearch(problem, fringe):
    closed = set()
    fringe.push(problem.initial_state)
    # Get a node in the fringe and
    # expand it if it is not a solution and is not closed
    while fringe.notEmpty():
        node = fringe.pop()
        if problem.isGoal(node.state):
            return node.solution
        if node.state not in closed:
            closed.add(node.state)
            fringe.pushAll(expand(node, problem))
    return FAILURE
```

---

### 2.4.1 A\* with graphs

The algorithm keeps track of closed and open nodes. The heuristic  $g(n)$  evaluates the minimum distance from the root to the node  $n$ .

A\* with graphs

**Consistent heuristic (monotone)** An heuristic is consistent if for each  $n$ , for any successor  $n'$  of  $n$  (i.e. nodes reachable from  $n$  by making an action) holds that:

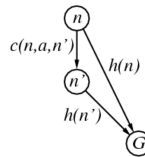
Consistent heuristic (monotone)

$$\begin{cases} h(n) = 0 & \text{if the corresponding status is the goal} \\ h(n) \leq c(n, a, n') + h(n') & \text{otherwise} \end{cases}$$

where  $c(n, a, n')$  is the cost to reach  $n'$  from  $n$  by taking the action  $a$ .

In other words,  $f$  never decreases along a path. In fact:

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$



**Theorem 2.4.1.** If  $h$  is a consistent heuristic, A\* on graphs is optimal.

## 3 Local search

**Local search** Starting from an initial state, iteratively improves it by making local moves in a neighborhood. Local search

Useful when the path to reach the solution is not important (i.e. no optimality).

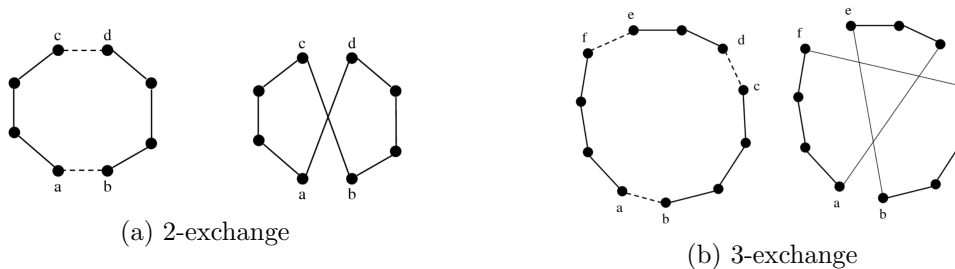
**Neighborhood** Given a set of states  $\mathcal{S}$ , a neighborhood is a function: Neighborhood

$$\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$$

In other words, for each  $s \in \mathcal{S}$ ,  $\mathcal{N}(s) \subseteq \mathcal{S}$ .

**Example** (Travelling salesman problem). Problem: find an Hamiltonian tour of minimum cost in an undirected graph.

A possible neighborhood of a state applies the  $k$ -exchange that guarantees to maintain an Hamiltonian tour.



**Local optima** Given an evaluation function  $f$ , a local optima (maximization case) is a state  $s$  such that:

$$\forall s' \in \mathcal{N}(s) : f(s) \geq f(s')$$

**Global optima** Given an evaluation function  $f$ , a global optima (maximization case) is a state  $s_{\text{opt}}$  such that:

$$\forall s \in \mathcal{S} : f(s_{\text{opt}}) \geq f(s)$$

Note: a larger neighborhood usually allows to obtain better solutions.

**Plateau** Flat area of the evaluation function.

**Ridges** Higher area of the evaluation function that is not directly reachable.

### 3.1 Iterative improvement (hill climbing)

Algorithm that only performs moves that improve the current solution.

It does not keep track of the explored states (i.e. may return in a previously visited state) and stops after reaching a local optima.

Iterative improvement (hill climbing)

---

**Algorithm 4** Iterative improvement
 

---

```

def iterativeImprovement(problem):
    s = problem.initial_state
    while noImprovement():
        s = bestOf(problem.neighborhood(s))
  
```

---

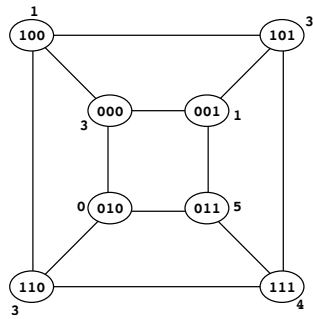
## 3.2 Meta heuristics

Methods that aim to improve the final solution. Can be seen as a search process over graphs:

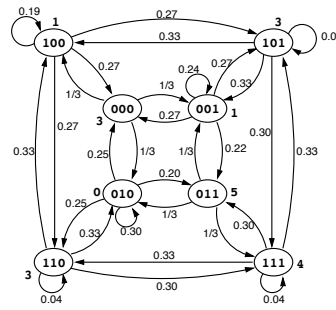
Meta heuristics

**Neighborhood graph** The search space topology.

**Search graph** The explored space.



(a) Neighborhood graph



(b) Search graph. Edges are probabilities

Meta heuristics finds a balance between:

**Intensification** Look for moves near the neighborhood.

Intensification

**Diversification** Look for moves somewhere else.

Diversification

Different termination criteria can be used with meta heuristics:

- Time constraints.
- Iterations limit.
- Absence of improving moves (stagnation).

### 3.2.1 Simulated annealing

Occasionally allow moves that worsen the current solution. The probability of this to happen is:

Simulated annealing

$$\frac{e^{f(s)-f(s')}}{T}$$

where  $s$  is the current state,  $s'$  is the next move and  $T$  is the temperature. The temperature is updated at each iteration and can be:

**Logarithmic**  $T_{k+1} = \Gamma / \log(k + k_0)$

**Geometric**  $T_{k+1} = \alpha T_k$ , where  $\alpha \in ]0, 1[$

**Non-monotonic**  $T$  is alternatively decreased (intensification) and increased (diversification).

---

**Algorithm 5** Meta heuristics – Simulated annealing

---

```
def simulatedAnnealing(problem, T0):
    s = problem.initial_state
    T, k = T0, 0
    while not terminationConditions():
        s_next = randomOf(problem.neighborhood(s))
        if (problem.f(s_next) > problem.f(s) or
            downhillWithProbability(eproblem.f(s) - problem.f(s_next) / T)):
            s = s_next
        k += 1
        update(T, k)
```

---

### 3.2.2 Tabu search

Keep track of the last  $n$  explored solutions in a tabu list and forbid them. Allows to escape from local optima and cycles.

Tabu search

Since keeping track of the visited solutions is inefficient, moves can be stored instead but, with this approach, some still not visited solutions may be cut off. **Aspiration criteria** can be used to allow forbidden moves in the tabu list to be evaluated.

---

**Algorithm 6** Meta heuristics – Tabu search

---

```
def tabuSearch(problem, T0):
    s = problem.initial_state
    tabu_list = [] # limited to n elements
    T, k = T0, 0
    while not terminationConditions():
        allowed_s = {s' ∈ N(s) : s' ∉ tabu_list or aspiration condition satisfied}
        s = bestOf(allowed_s)
        updateTabuListAndAspirationConditions()
        k += 1
        update(T, k)
```

---

### 3.2.3 Iterated local search

Based on two steps:

Iterated local search

**Subsidiary local search steps** Efficiently reach a local optima (intensification).

**Perturbation steps** Escape from a local optima (diversification).

In addition, an acceptance criterion controls the two steps.

---

**Algorithm 7** Meta heuristics – Iterated local search

---

```
def tabuSearch(problem):
    s = localSearch(problem.initial_state)
    while not terminationConditions():
        s_perturbation = perturbation(s, history)
        s_local = localSearch(s_perturbation)
        s = acceptanceCriterion(s, s_local, history)
```

---

### 3.2.4 Population based (genetic algorithm)

Population based meta heuristics are built on the following concepts:

Population based  
(genetic algorithm)

**Adaptation** Organisms are suited to their environment.

**Inheritance** Offspring resemble their parents.

**Natural selection** Fit organisms have many offspring, others become extinct.

Biology	Artificial intelligence
Individuals	Possible solution
Fitness	Quality
Environment	Problem

Table 3.1: Biological evolution metaphors

The following terminology will be used:

**Population** Set of individuals (solutions).

**Genotypes** Individuals of a population.

**Genes** Units of chromosomes.

**Alleles** Domain of values of a gene.

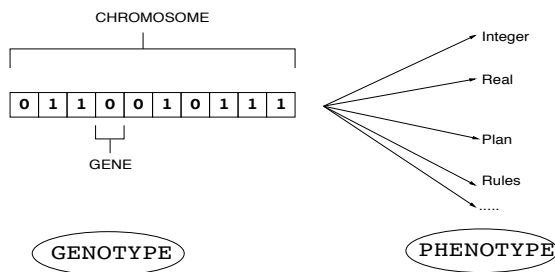
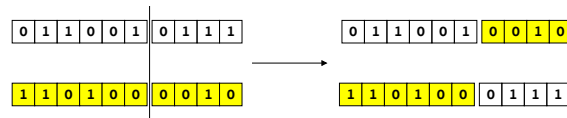


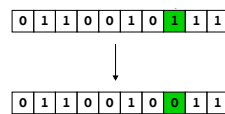
Figure 3.3

Genetic operators are:

**Recombination/Crossover** Cross-combination of two chromosomes.



**Mutation** Random modification of genes.



**Proportional selection** Probability of a individual to be chosen as parent of the next offspring. Depends on the fitness.



**Generational replacement** Create the new generation. Possible approaches are:

- Completely replace the old generation with the new one.
- Keep the best  $n$  individual from the new and old population.

**Example** (Real-valued genetic operators). Solution  $x \in [a, b]$  with  $a, b \in \mathbb{R}$ .

**Mutation** Random perturbation:  $x \rightarrow x \pm \delta$ , as long as  $x \pm \delta \in [a, b]$ .

**Crossover** Linear combination:  $x = \lambda_1 y_1 + \lambda_2 y_2$ , as long as  $x \in [a, b]$ .

**Example** (Permutation genetic operators). Solution  $x = (x_1, \dots, x_n)$  is a permutation of  $(1, \dots, n)$ .

**Mutation** Random exchange of two elements at index  $i$  and  $j$ , with  $i \neq j$ .

**Crossover** Crossover avoiding repetitions.

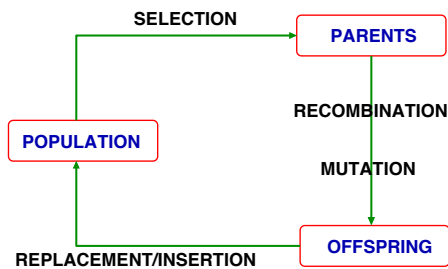


Figure 3.4: Evolutionary cycle

---

#### Algorithm 8 Meta heuristics – Genetic algorithm

---

```
def geneticAlgorithm(problem):
    population = problem.initPopulation()
    evaluate(population)
    while not terminationConditions():
        offspring = []
        while not offspringComplete(offspring):
            p1, p2 = selectParents(population)
            new_individual = crossover(p1, p2)
            new_individual = mutation(new_individual)
            offspring.append(new_individual)
        population = offspring
        evaluate(population)
```

---

## 4 Swarm intelligence

**Swarm intelligence** Group of locally-interacting agents that shows an emergent behavior without a centralized control system. Swarm intelligence

A swarm intelligent system has the following features:

- Individuals are simple and have limited capabilities.
- Individuals are not aware of the global view.
- Individuals have local direct or indirect communication patterns.
- The computation is distributed and not centralized.
- The system works even if some individuals "break" (robustness).
- The system adapts to changes.

Agents interact between each other and obtain positive and negative feedbacks.

**Stigmergy** Form of indirect communication where an agent modifies the environment and the others react to it. Stigmergy

### 4.1 Ant colony optimization (ACO)

Ants release pheromones while walking from the nest to the food. They also tend to prefer paths marked with the highest pheromone concentration.

**Ant colony optimization** Probabilistic parametrized model that builds the solution incrementally. A problem is solved by making stochastic steps in a fully connected graph (construction graph)  $G = (C, L)$  where: Ant colony optimization (ACO)

- The vertexes  $C$  are the solution components.
- The edges  $L$  are connections.
- The paths on  $G$  are states.

Additional constraints may be added if needed.

**Example** (Travelling salesman). The construction graph can be defined as:

- Nodes are cities.
- Edges are connections between cities.
- A solution is an Hamiltonian path in the graph.
- Constraints to avoid sub-cycles (i.e. avoid visiting a city multiple times).

**Pheromone** Value associated to each node and each edge to estimate the quality of the solution. Pheromone

**Heuristic values** Value associated to each node and each edge to represent the prior background knowledge. Heuristic values

### 4.1.1 ACO system

**Transition rule** Ants build a path on the construction graph based on a transition rule that uses pheromones and heuristics. The probability of choosing the node  $j$  starting from  $i$  is parametrized on  $\alpha$  (pheromones) and  $\beta$  (heuristics), and is defined as: ACO system

$$p_{\alpha,\beta}(i, j) = \begin{cases} \frac{(\tau_{ij})^\alpha \cdot (\eta_{ij})^\beta}{\sum_{k \in \text{feasible\_nodes}} (\tau_{ik})^\alpha \cdot (\eta_{ik})^\beta} & \text{if } j \text{ consistent} \\ 0 & \text{otherwise} \end{cases}$$

where  $\tau_{ij}$  is the pheromone trail from  $i$  to  $j$  and  $\eta_{ij} = \frac{1}{d_{ij}}$  is the heuristic ( $d_{ij}$  is the distance).

**Pheromone update** After each step, the pheromone trail is updated depending on an evaporation factor  $\rho \in [0, 1]$ :

$$\tau_{ij} = (1 - \rho)\tau_{ij} + \sum_{k=1}^{n_{\text{ants}}} \Delta\tau_{ij}^{(k)}$$

$\tau_{ij}^{(k)}$  of the  $k$ -th ant is defined as:

$$\tau_{ij}^{(k)} = \begin{cases} \frac{1}{L_k} & \text{if ant } k \text{ used the arch } (i, j) \\ 0 & \text{otherwise} \end{cases}$$

where  $L_k$  is the length of the path followed by the  $k$ -th ant.

For the best ant, the update also affects all the components on the path crossed by the ant.

**Daemon actions** Centralized actions performed on each solution built by the ants. These actions cannot be performed by single ants and are useful to improve the solution using global information. In practice, a local search can be applied to push the result towards a better solution.

---

#### Algorithm 9 ACO system

---

```
def acoSystem(problem, α, β):
    initPheromones()
    while not terminationConditions():
        antBasedSolutionConstruction(α, β)
        pheromonesUpdate()
        daemonActions() # Optional
```

---

## 4.2 Artificial bee colony algorithm (ABC)

System where the position of nectar sources represents the solutions and the quantity of nectar sources represents the fitness of the solution. Artificial bees can be:

Artificial bee colony algorithm (ABC)

**Employed** Bee associated to a specific nectar source (intensification) (i.e. it represents a solution).

**Onlooker** Bee that is observing the employed bees and is choosing its nectar source.

**Scout** Bee that discovers new food sources (diversification).

The algorithm has the following phases:

**Initialization** The initial nectar source of each bee is determined randomly. Each solution (nectar source) is a vector  $\mathbf{x}_m \in \mathbb{R}^n$  and each of its component is initialized constrained to a lower ( $l_i$ ) and upper ( $u_i$ ) bound:

$$\mathbf{x}_m[i] = l_i + \text{rand}(0, 1) \cdot (u_i - l_i)$$

**Employed bees** Starting from their assigned nectar source, employed bees look in their neighborhood for a new food source with more fitness (more nectar). The fitness (for minimization problems) of a food source  $\mathbf{x}_m$  is determined as:

$$\text{fit}(\mathbf{x}_m) = \begin{cases} \frac{1}{1+\text{obj}(\mathbf{x}_m)} & \text{if } \mathbf{x}_m \geq 0 \\ 1 + |\text{obj}(\mathbf{x}_m)| & \text{if } \mathbf{x}_m < 0 \end{cases}$$

where  $\text{obj}$  is the objective function.

**Onlooker bees** Onlooker bees stochastically choose their food source. Each food source  $\mathbf{x}_m$  has a probability associated to it defined as:

$$p_m = \frac{\text{fit}(\mathbf{x}_m)}{\sum_{i=1}^{n_{\text{bees}}} \text{fit}(\mathbf{x}_i)}$$

This provides a positive feedback as more promising solutions have a higher probability to be chosen.

**Scout bees** Scout bees choose a nectar source randomly.

An employed bee that cannot improve its solution after a given number of attempts (gets fired and) becomes a scout (negative feedback).

---

#### Algorithm 10 ABC algorithm

---

```
def abcAlgorithm(problem):
    initPhase()
    sol = None
    while not terminationConditions():
        employedBeesPhase()
        onlookerBeesPhase()
        scoutBeesPhase()
        sol = getCurrentBest()
```

---

### 4.3 Particle swarm optimization (PSO)

In a bird flock, the movement of the individuals tend to:

- Follow the neighbors.
- Stay in the flock.
- Avoid collisions.

Particle swarm  
optimization (PSO)

However, a model based on these rules does not have a common objective. PSO introduces as common objective the search of food. Each individual that finds food can:

- Move away from the flock and reach the food.
- Stay in the flock.

Following the movement rules, the entire flock will gradually move towards promising areas.

Applied to optimization problems, the bird flock metaphor can be interpreted as:

**Bird** Agent that represents a possible solution that is progressively improved (exploration).

**Social interaction** Exploiting the knowledge of other agents to move towards a global solution (exploitation).

**Neighborhood** Individuals are affected by the actions of others close to them and are part of one or more sub-groups.

Note that sub-groups are not necessarily defined by physical proximity.

Given a cost function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  to minimize (gradient is not known), PSO initializes a swarm of particles (agents) whose movement is guided by the best known position. Each particle is described by:

- Its position  $\mathbf{x}_i \in \mathbb{R}^n$  in the search space.
- A velocity  $\mathbf{v}_i \in \mathbb{R}^n$  that controls the movement of the particle.
- The best solution  $\mathbf{p}_i$  it has found so far.

---

#### Algorithm 11 PSO algorithm

---

```
def pco(f, n_particles, l, u,  $\omega$ ,  $\varphi_p$ ,  $\varphi_g$ ):
    particles = [Particle()] * n_particles
    global_best = None
    for particle in particles:
        particle.value = randomUniform(l, u) # Search space bounds
        particle.vel = randomUniform(-|u-l|, |u-l|)
        particle.best = particle.value
        if f(particles.best) < f(g): g = particles.best

    while not terminationConditions():
        for particle in particles:
             $r_p$ ,  $r_g$  = randomUniform(0, 1), randomUniform(0, 1)
             $\mathbf{x}_i$ ,  $\mathbf{p}_i$ ,  $\mathbf{v}_i$  = particle.value, particle.best, particle.vel
            g = global_best
            particle.vel =  $\omega * \mathbf{v}_i + \varphi_p * r_p * (\mathbf{p}_i - \mathbf{x}_i) + \varphi_g * r_g * (\mathbf{g} - \mathbf{x}_i)$ 
            particle.value = particle.value + particle.vel
            if f(particle.value) < f(particle.best):
                particle.best = particle.value
            if f(particle.best) < f(g): g = particle.best
```

---