Reguleringsteknik 1

J. Christian Andersen

Kursusuge 5

Plan

- Linearisering → udskudt til kursusuge 6
- Frekvensanalyse
  - Bodeplot – 1. og 2. orden
  - Stabilitet, Stabilitetsmargin
- Grupperegning
  - Frekvensanalyse
- Øvelse → også udskudt til kursusuge 6
  - Dampmaskineregulering
    (over 3 gange)

DTU Electrical Engineering
Department of Electrical Engineering

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  - Stabilitet, Stabilitetsmargin
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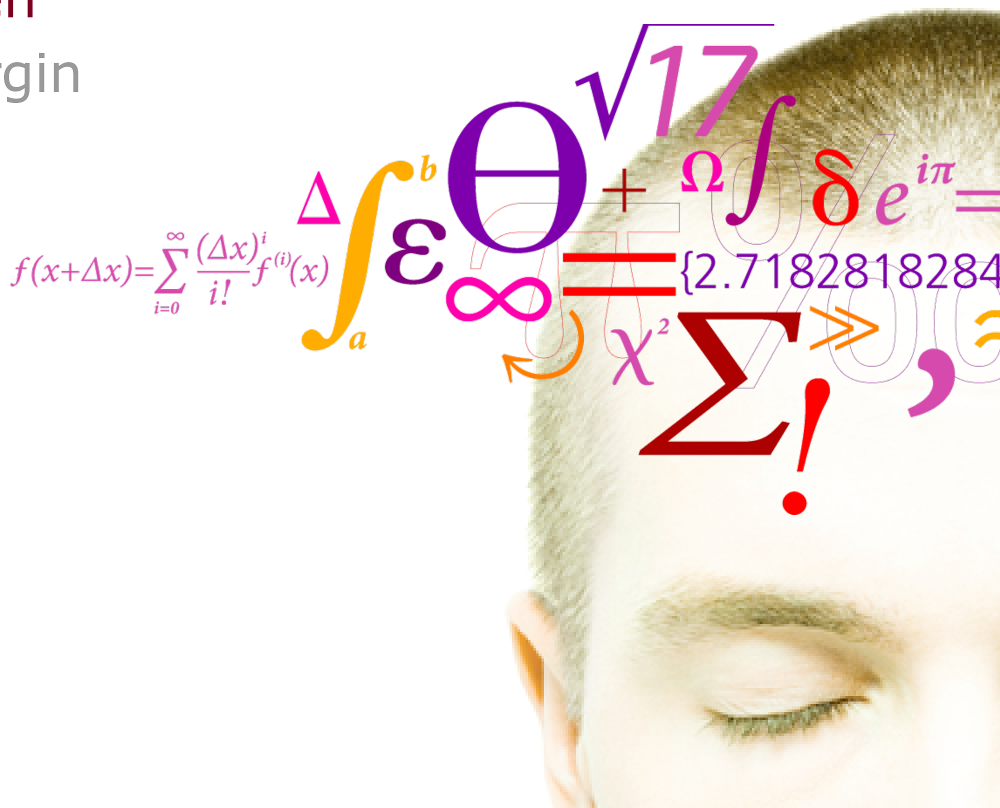
  - Frekvensanalyse
-



Frekvensanalyse

"fysiske frekvenser" i laplace domæne

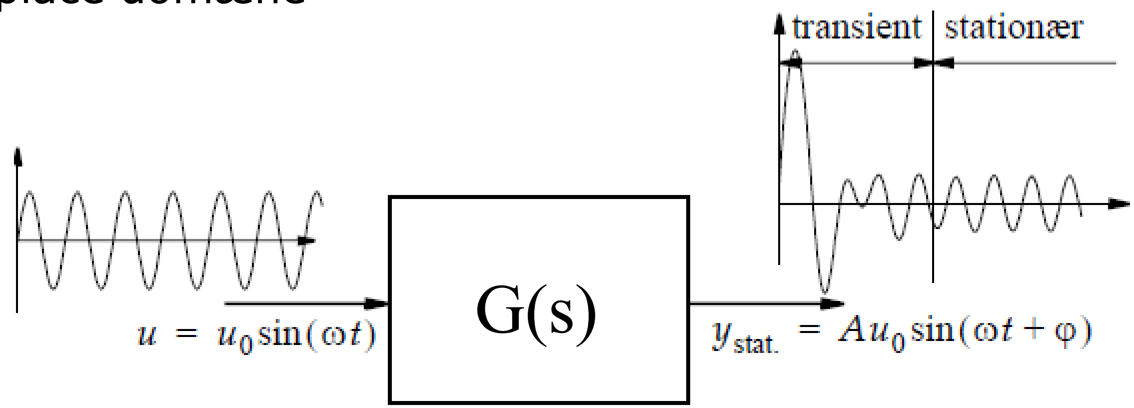
For input sinusformet:

$$u = u_0 \sin(\omega t)$$

Er output sinusformet:

$$y = A \cdot u_0 \sin(\omega t + \phi)$$

(Stationær del)



Amplitude (Magnitude):

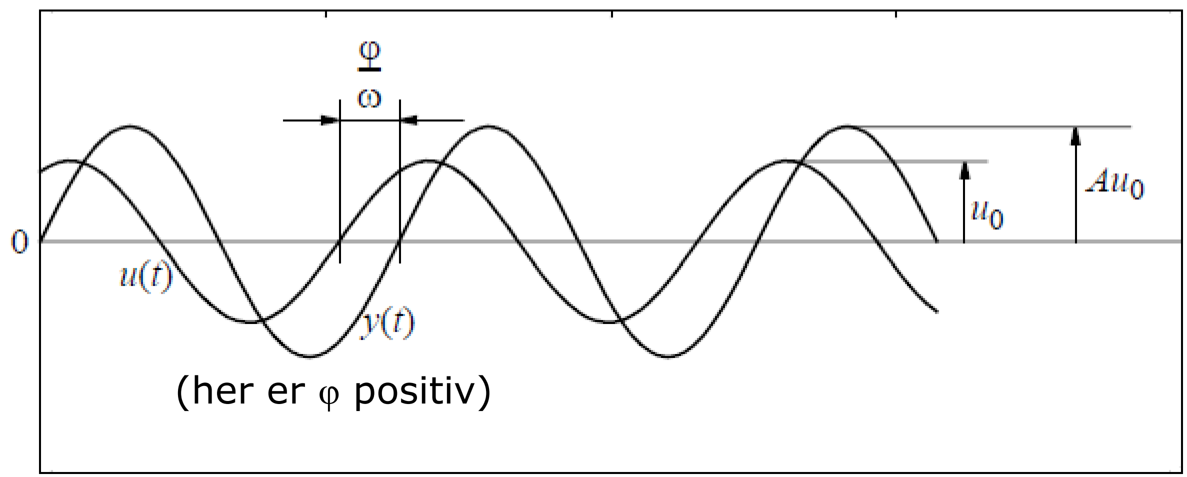
$$A = |G(\omega)|$$

Fase (phi):

$$\phi = argument(G(\omega))$$

hvor:

$$G(\omega) = G(s)|_{s = j \cdot \omega}$$



(er φ positiv er output fase før input)

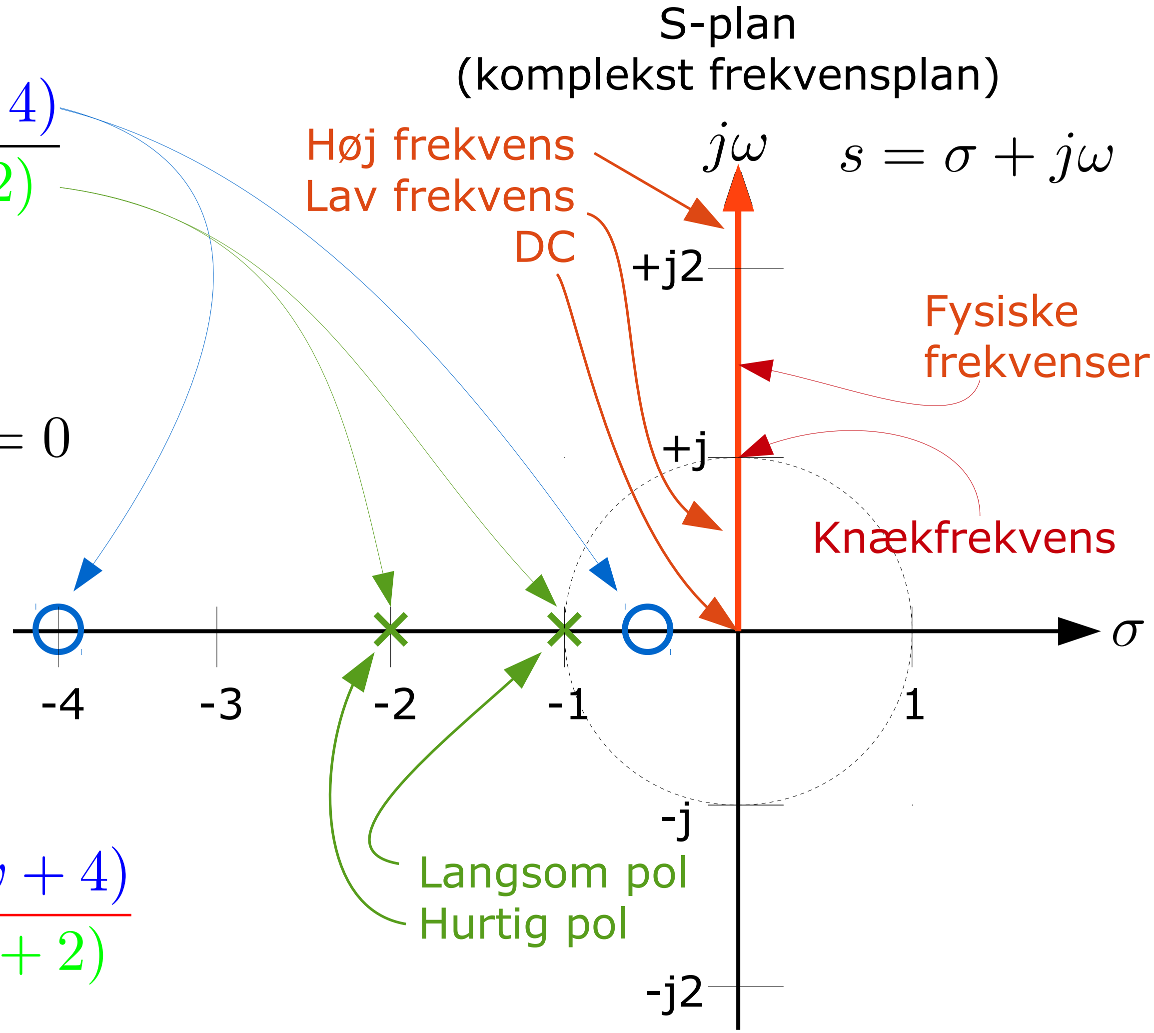
Tid

Poler og nulpunkter

$$G(s) = \frac{(s+0.5)(s+4)}{(s+1)(s+2)}$$

× pol: $|G(s)| = \infty$

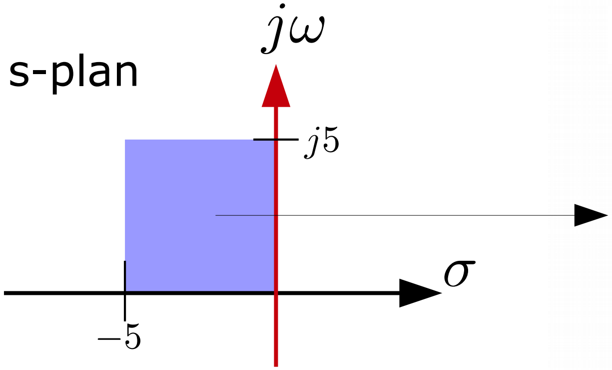
○ nulpunkt: $|G(s)| = 0$



$$s = \sigma + j\omega$$

$$G(\omega) = \frac{(j\omega+0.5)(j\omega+4)}{(j\omega+1)(j\omega+2)}$$

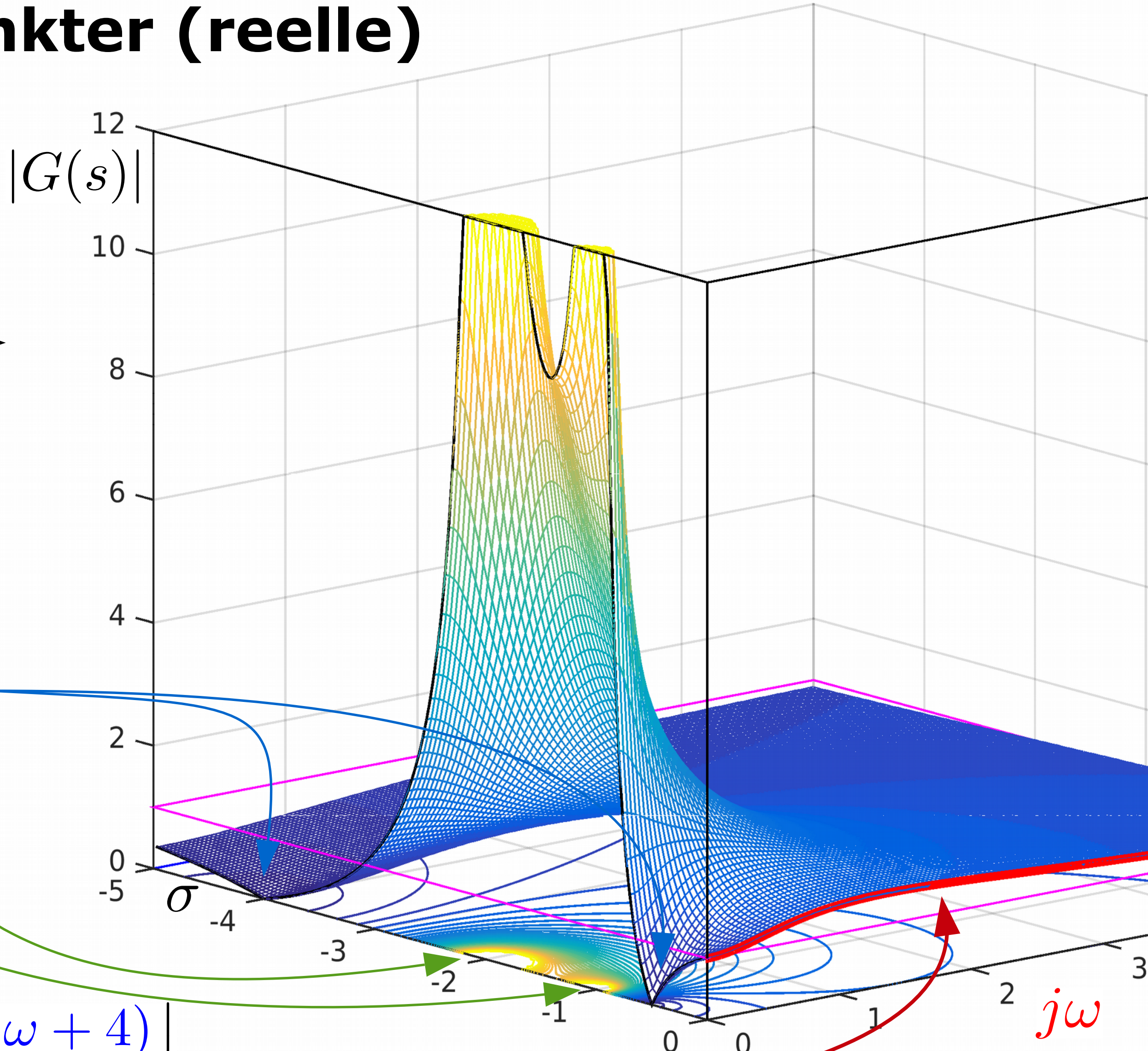
Poler og nulpunkter (reelle)



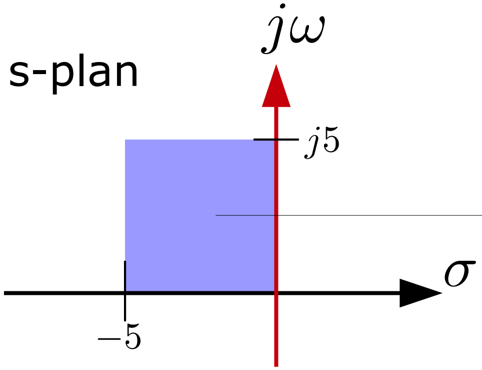
$$s = \sigma + j\omega$$

$$G(s) = \frac{(s+0.5)(s+4)}{(s+1)(s+2)}$$

$$|G(j\omega)| = \left| \frac{(j\omega+0.5)(j\omega+4)}{(j\omega+1)(j\omega+2)} \right|$$



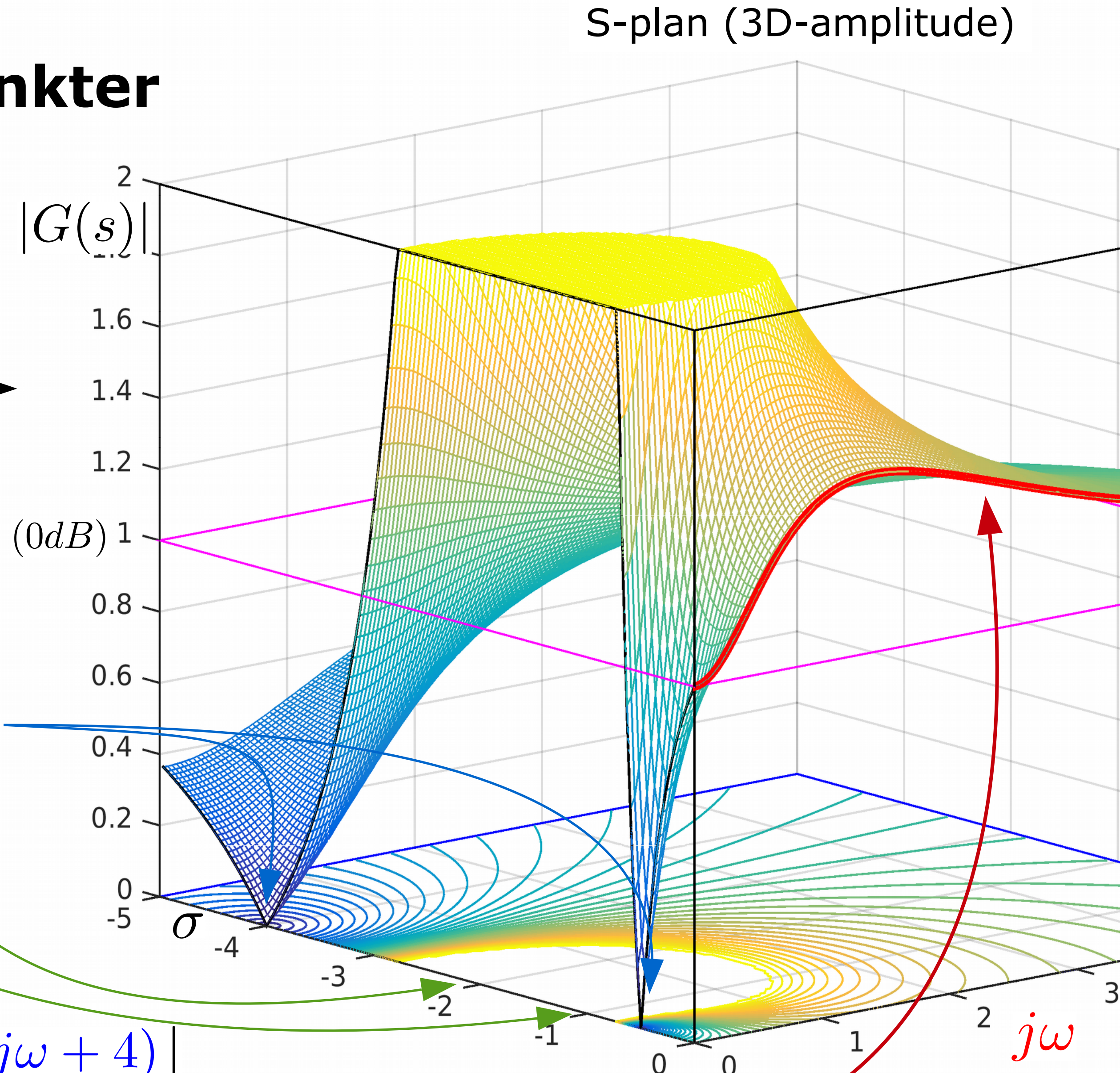
Poler og nulpunkter



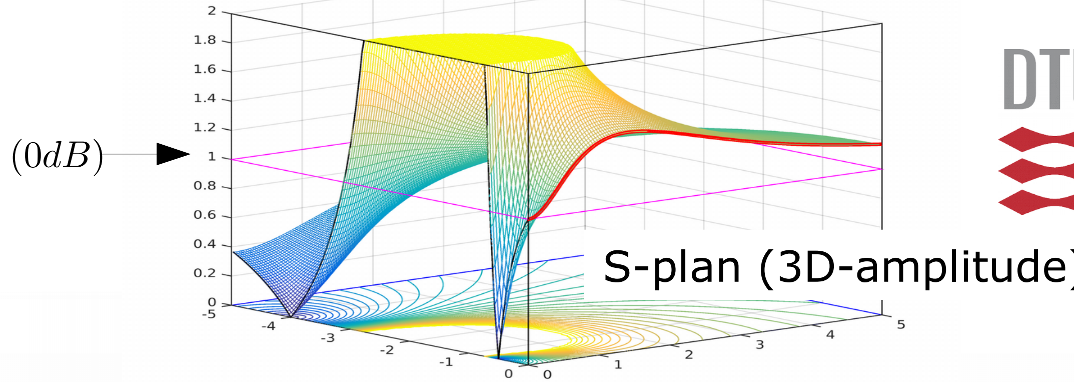
$$s = \sigma + j\omega$$

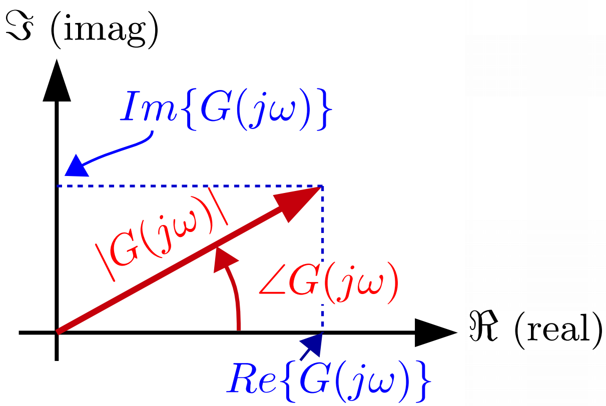
$$G(s) = \frac{(s+0.5)(s+4)}{(s+1)(s+2)}$$

$$|G(j\omega)| = \left| \frac{(j\omega+0.5)(j\omega+4)}{(j\omega+1)(j\omega+2)} \right|$$



Bodeplot





Amplitude (Magnitude)

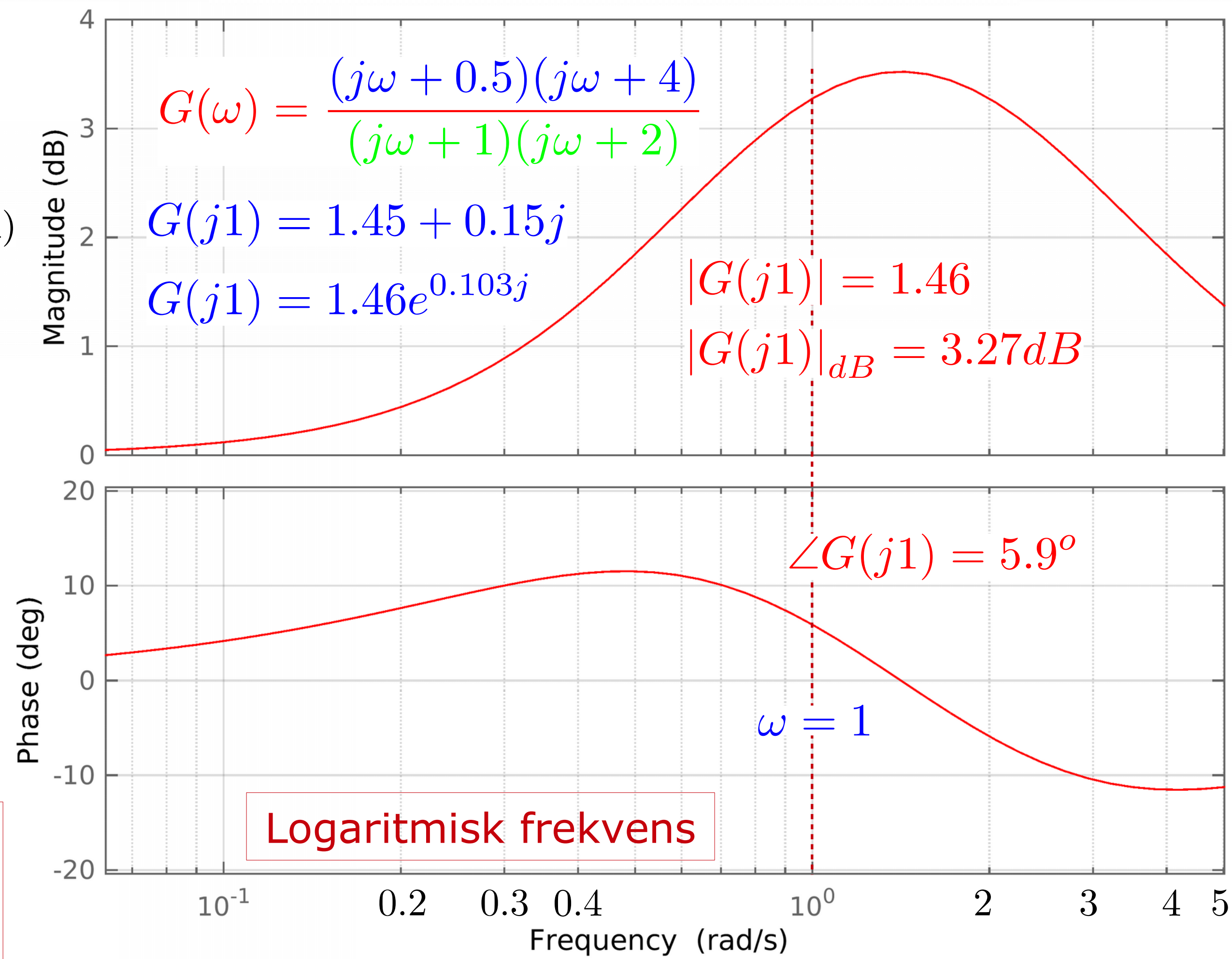
$$|G(j\omega)|_{dB} =$$

$$\boxed{20 \log_{10}(|G(j\omega)|)}$$

Fase (Phase)

$$\angle G(j\omega) =$$

$$\boxed{\tan^{-1}\left(\frac{Im\{G(j\omega)\}}{Re\{G(j\omega)\}}\right)}$$



Bodeplot - pol

$$G(s) = \frac{1}{s+\omega_0}$$

$$G(s) = \frac{1}{s+1}$$

$$G(j\omega) = \frac{1}{j\omega+1}$$

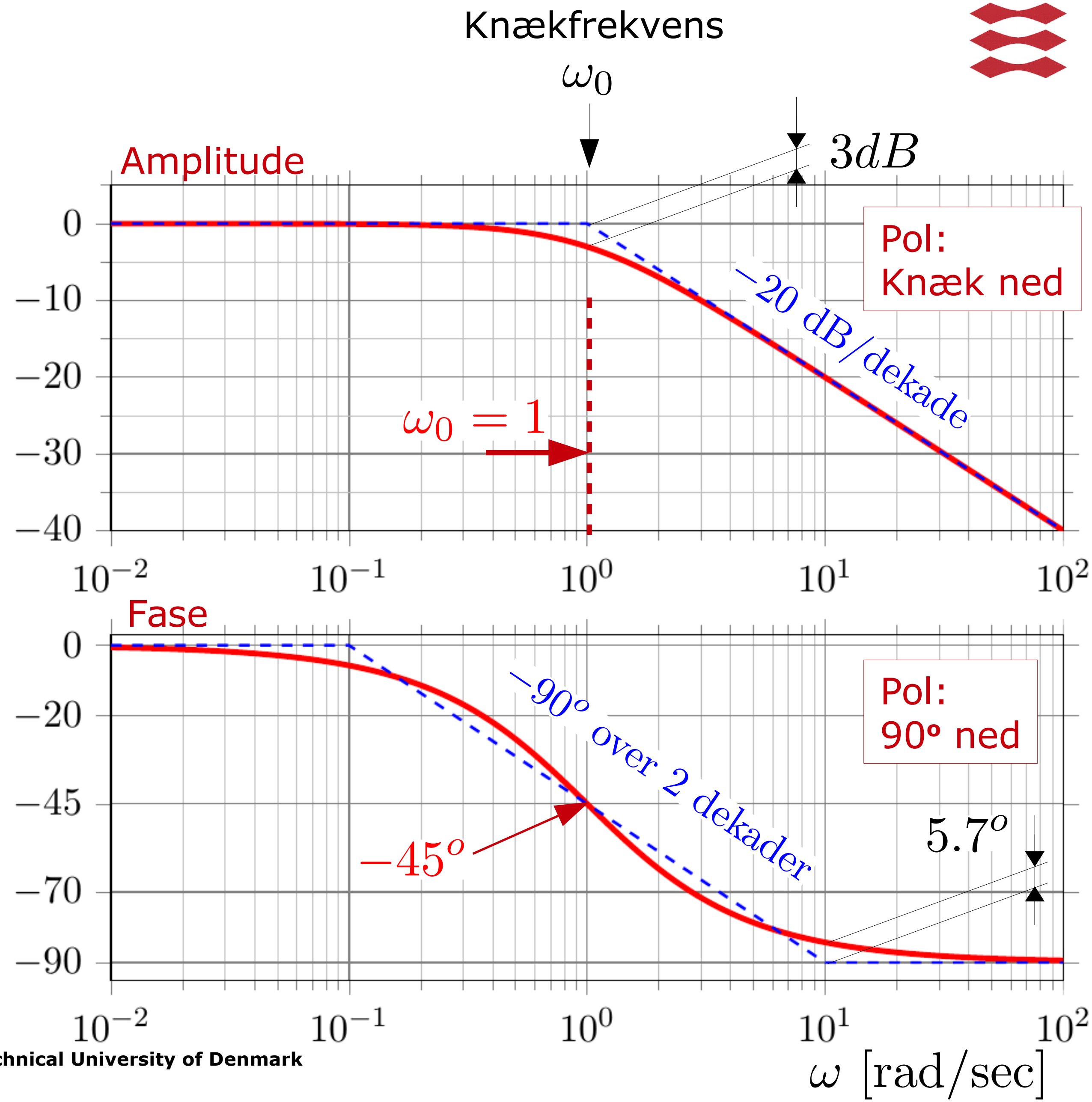
$$G(j1) = \frac{1}{j+1}$$

$$G(j1) = \frac{1}{2}(1-j)$$

$$G(j1) = \frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}$$

$$G(j1) = 0.707\angle -45^o$$

$$|G(j1)|_{dB} = -3dB$$



Bodeplot - nulpunkt

$$G(s) = s + \omega_0$$

$$G(s) = s + 0.2$$

$$G(j\omega) = j\omega + 0.2$$

$$G(0.2j) = 0.2j + 0.2$$

$$G(0.2j) = 0.283e^{\frac{\pi}{4}}$$

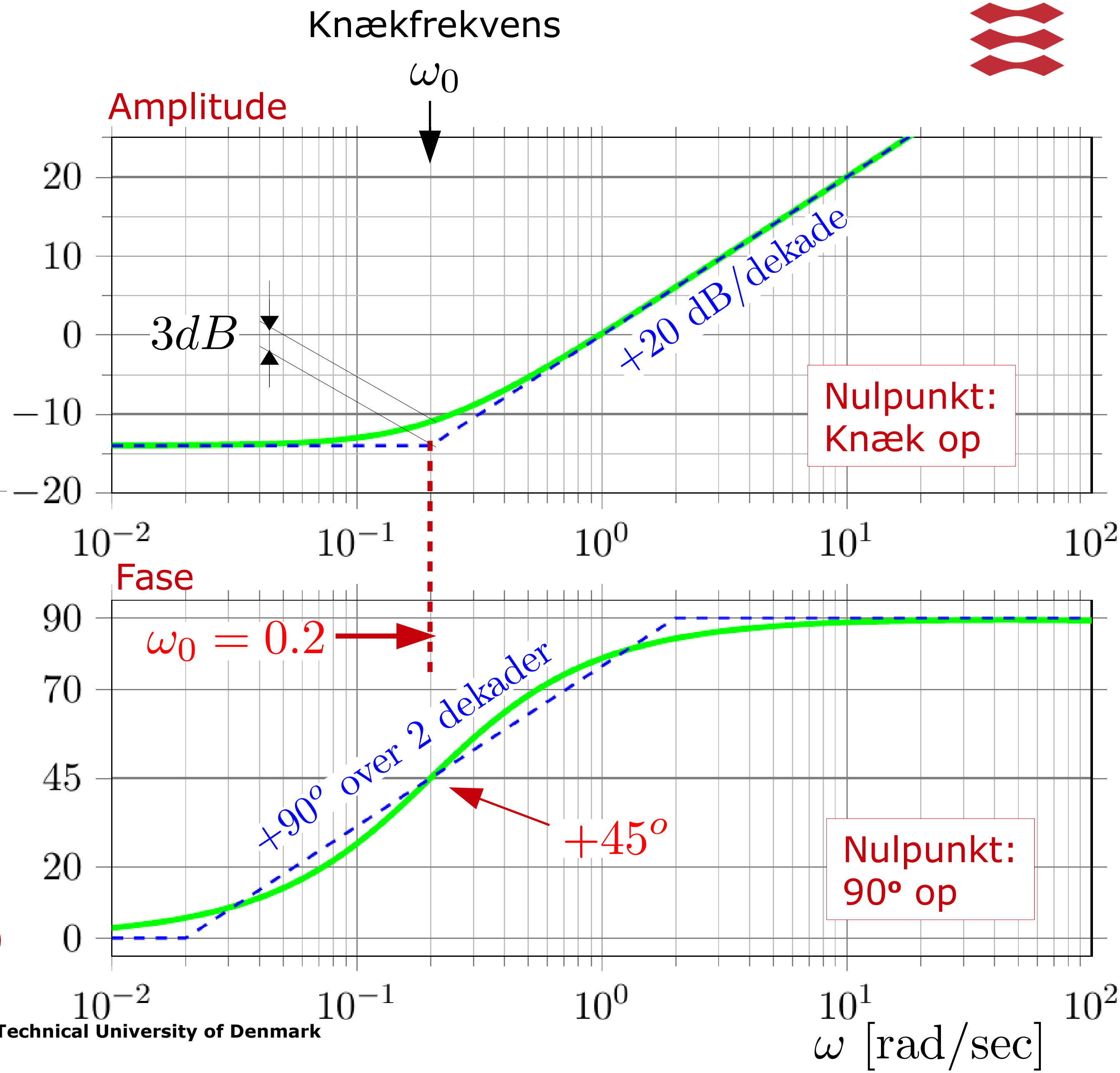
$$G(0.2j) = 0.283\angle 45^o$$

$$|G(0.2j)|_{dB} = -10.97dB$$

MATLAB

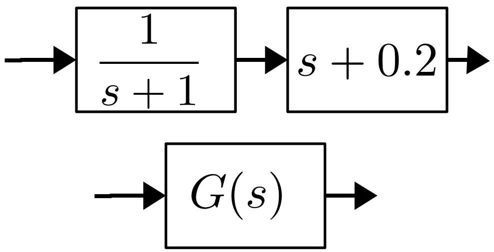
A = abs(0.2*j + 0.2)

F = angle(0.2*j + 0.2)



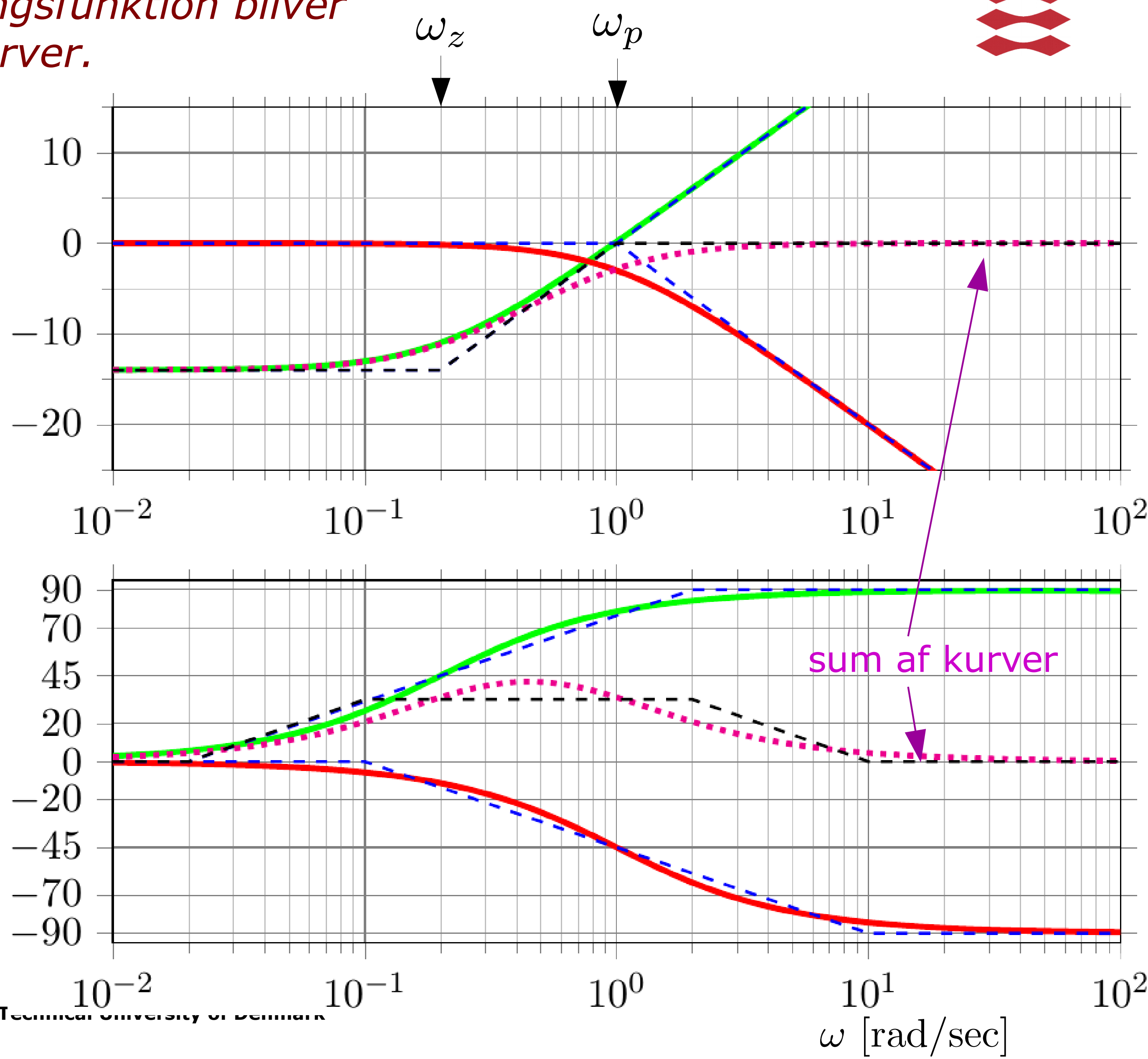
Bodeplot - pole and zero

Produkt af overføringsfunktion bliver til summation af kurver.



$$G(s) = \frac{s + \omega_z}{s + \omega_p}$$

$$G(s) = \frac{s + 0.2}{s + 1}$$



MATLAB

```
figure(1)
Gp = tf([1],[1 1])
Gz = tf([1 0.2],[1])
G = Gp * Gz
bode(Gp, Gz, G)
```

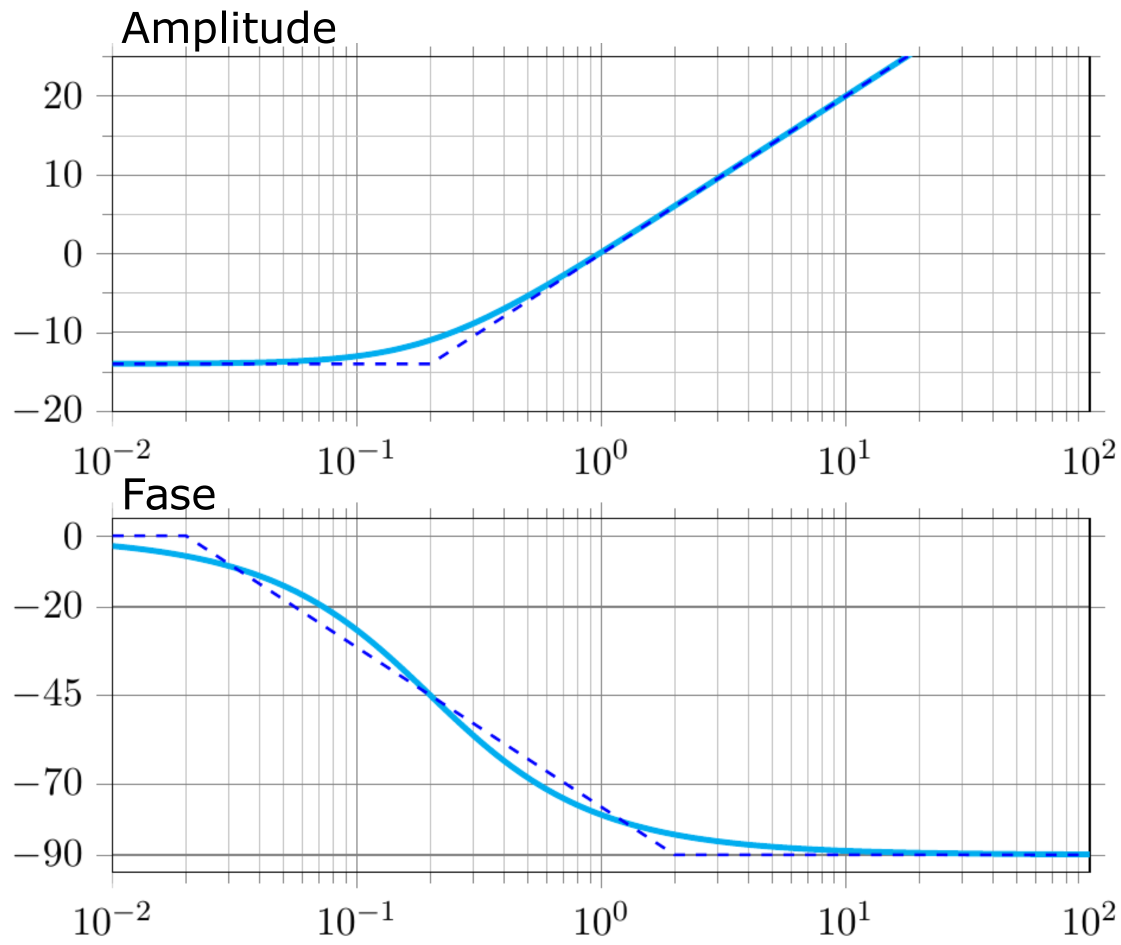
Bodeplot
- nulpunkt i højre halvplan

$$G(s) = -s + \omega_0$$

$$G(s) = -s + 0.2$$

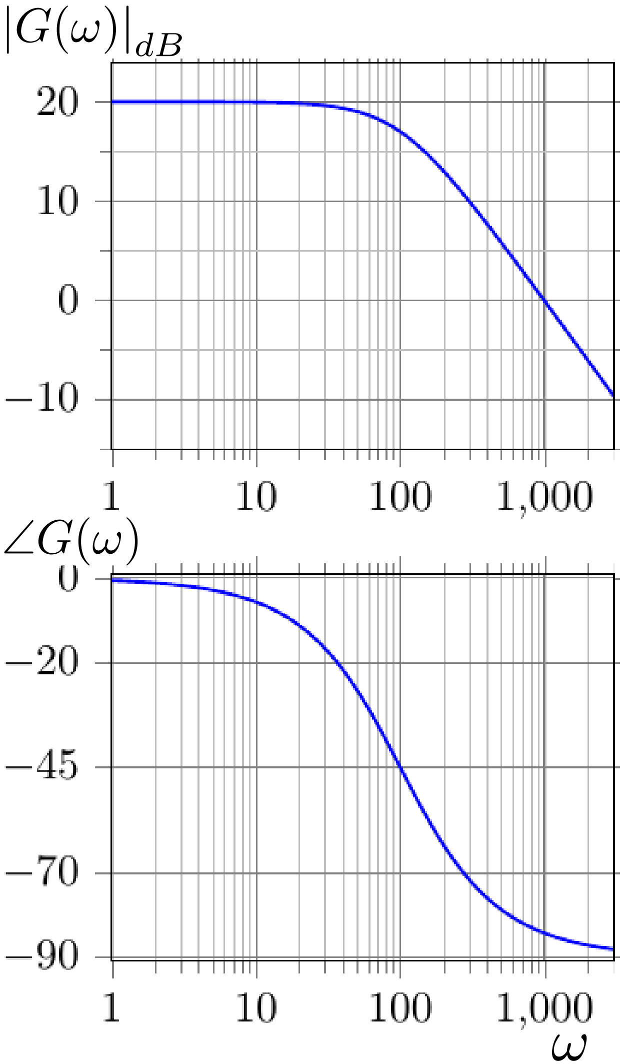
$$G(j\omega) = -j\omega + 0.2$$

Nulpunkt i højre halvplan:
Amplitude uændret, men
Fase går 90º **NED**!

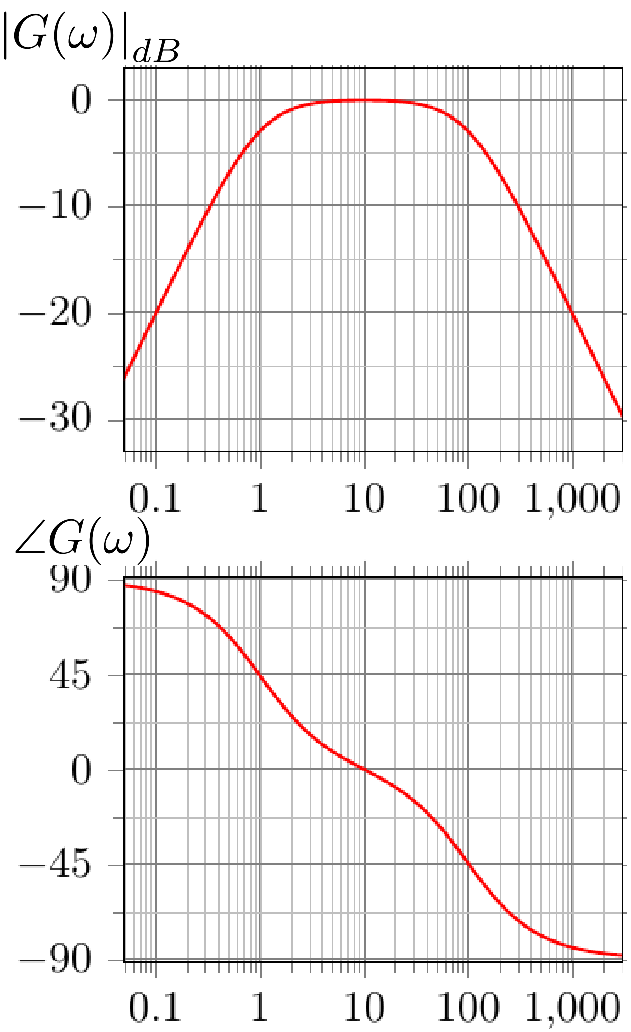


Pol i højre halvplan?

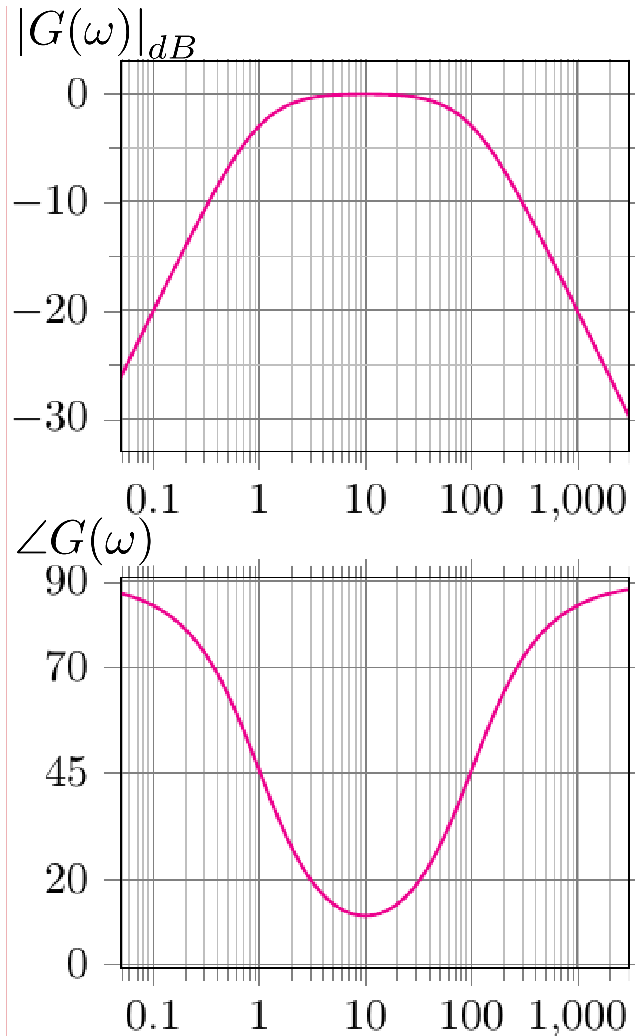
Kontrolspørgsmål – find overføringsfunktion



$G(s) = ?$

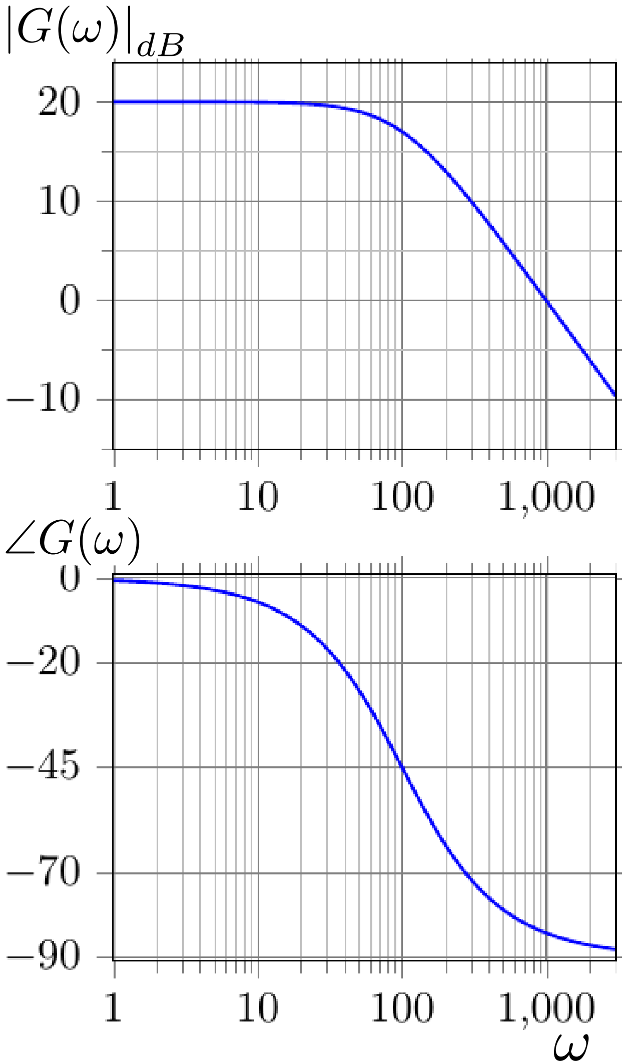


$G(s) = ?$



$G(s) = ?$

Kontrolspørgsmål – find overføringsfunktion



Knæk ned 20dB/dekade og
fase ned 90 grader,
passer med en pol.

Knækfrekvens ved 3dB ændring = 100 rad/sek

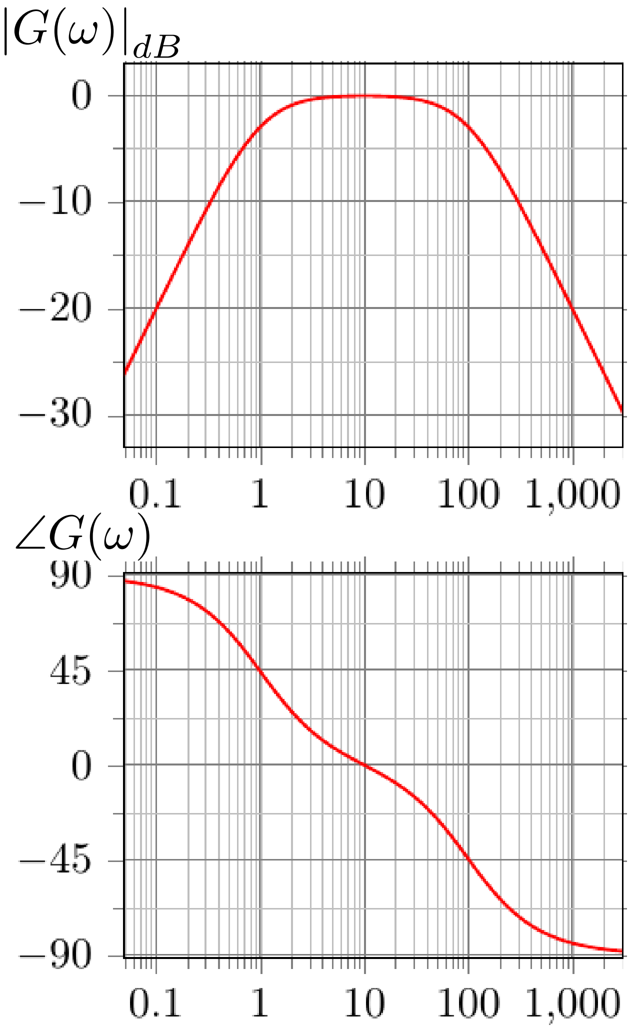
$$G(s) = \frac{k}{s+100}$$

Ved $s = j\omega$ og $\lim_{\omega \to 0} G(\omega) = 20 \text{ dB}$

$$\lim_{s \to 0} G(s) = \frac{k}{100} = 10^{\frac{20}{20}}$$

$$G(s) = \frac{1000}{s+100}$$

Kontrolspørgsmål – find overføringsfunktion



Knæk ned 20 dB/dek og
fase ned med 90 grader ved både
1 og 100 rad/sek
→ 2 poler ved s=-1 og s=-100

Starter med +20 dB/dek,
må starte med et knæk op,
passer med startvinkel på +90 grader
→ nulpunkt i s = 0

$$G(s) = \frac{k(s+0)}{(s+1)(s+100)}$$

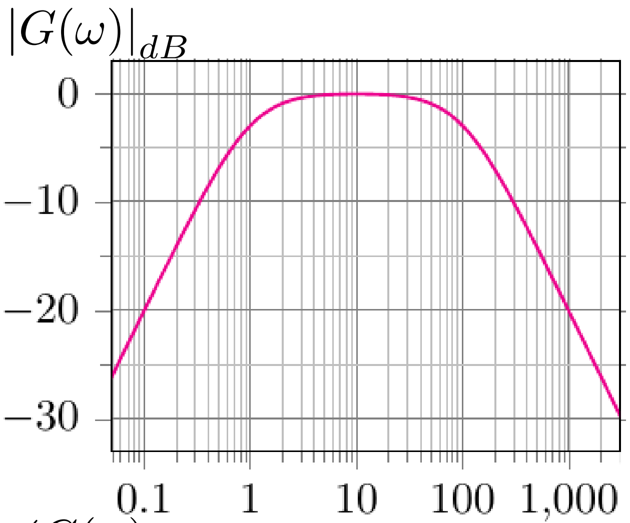
Ved $\omega = 10$ er $|G(\omega)| = 0dB = 1$

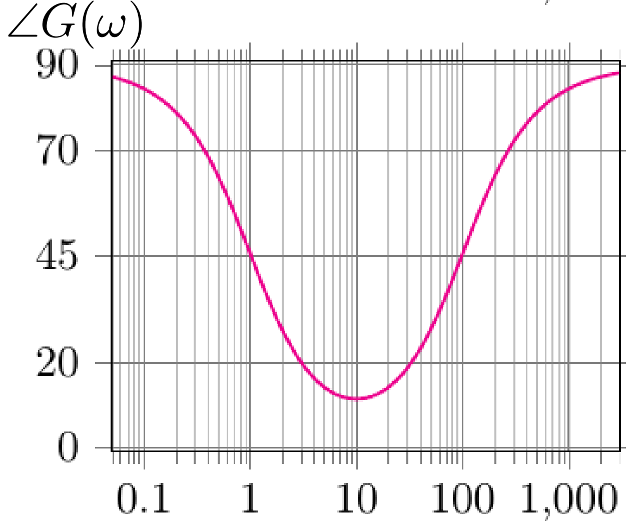
$$\left| \frac{ks}{(s+1)(s+100)} \right|_{s=10j} = 1$$

$$\left| \frac{k(10j)}{(10j)(100)} \right| = 1 \Rightarrow k = 100$$

$$G(s) = \frac{100s}{(s+1)(s+100)}$$

Kontrolspørgsmål – find overføringsfunktion





Set på amplituden
ligner kurven den forregående.
Fasen passer også, på nær den sidste pol.

Den sidste pol må være i højre
halvplan!

$$G(s) = \frac{100s}{(s+1)(s-100)}$$

$$G(s) = \frac{100s}{(s+1)(-s+100)}$$

Hvilken
er rigtig?

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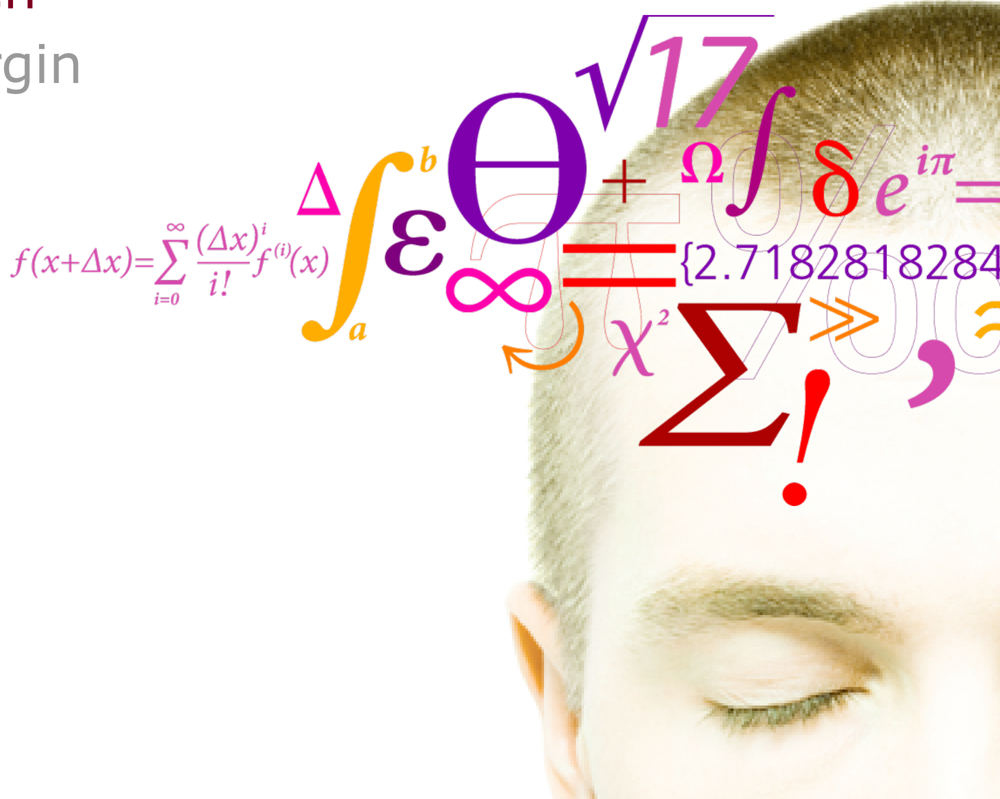


Komplekse poler

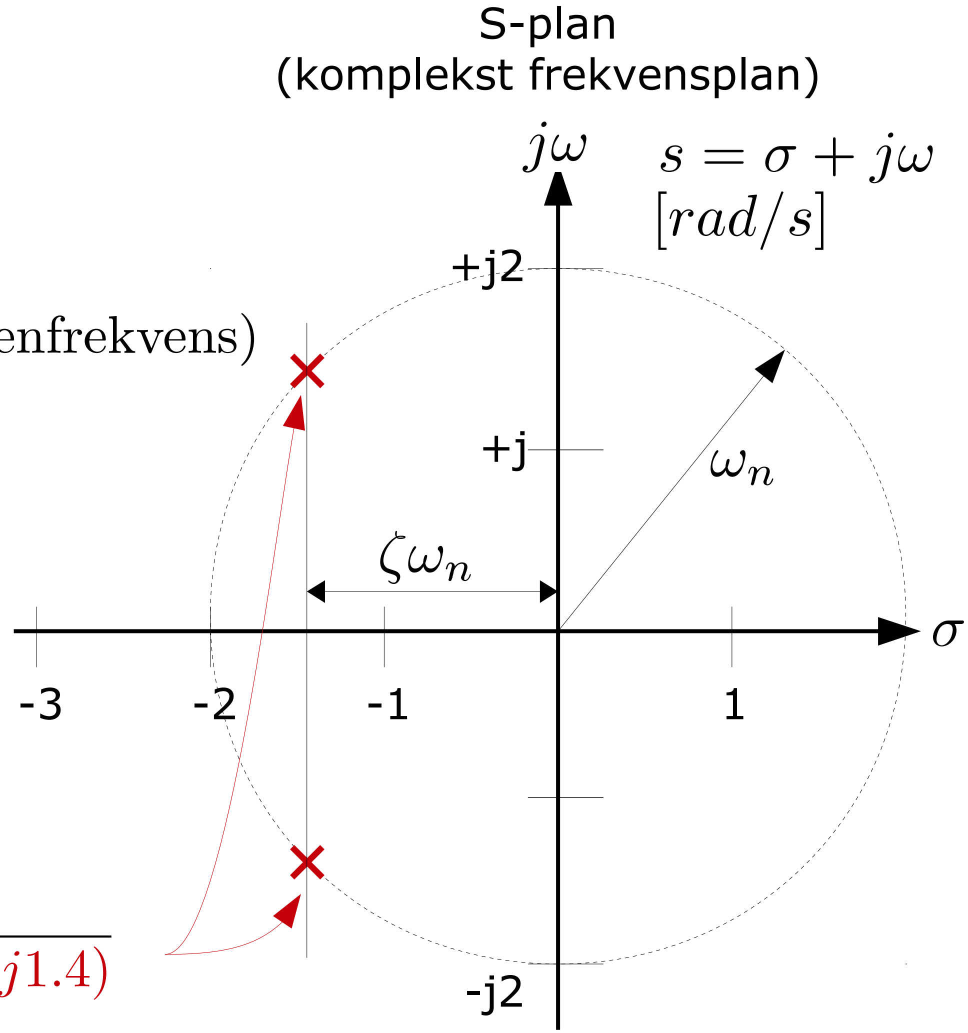
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\begin{aligned}\zeta &= \text{dæmpningsfaktor} \\ \omega_n &= \text{egenfrekvens (udæmpet egenfrekvens)}\end{aligned}$$

$$\zeta = 0.707$$
$$\omega_n = 2\ [rad/s]$$

$$G(s) = \frac{4}{(s^2 + 2 \cdot 0.707 \cdot 2s + 4)}$$

$$G(s) = \frac{4}{(s + 1.4 + j1.4)(s + 1.4 - j1.4)}$$



S-plan plot

$$G = \frac{s+0.5}{(s+1)^2}$$

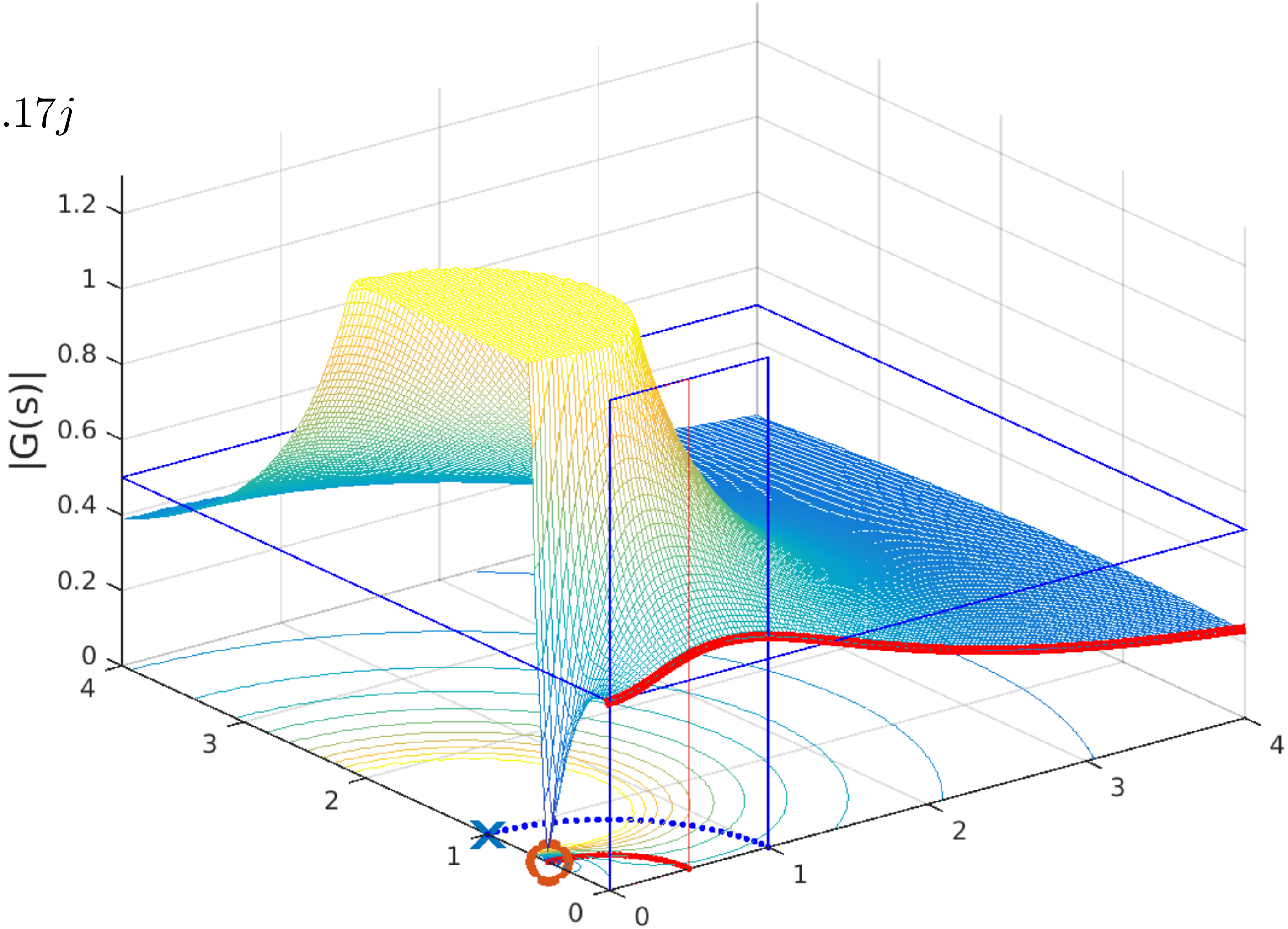
$$G = \frac{s+0.5}{s^2+2s+1}$$

$$\omega_n = 1$$

$$P = -0.98 \pm 0.17j$$

$$Z = -0.5j$$

$$\zeta = 1$$



S-plan plot 2

$$G = \frac{s + 0.5}{(s + 0.98 + 0.17j)(s + 0.98 - 0.17j)}$$

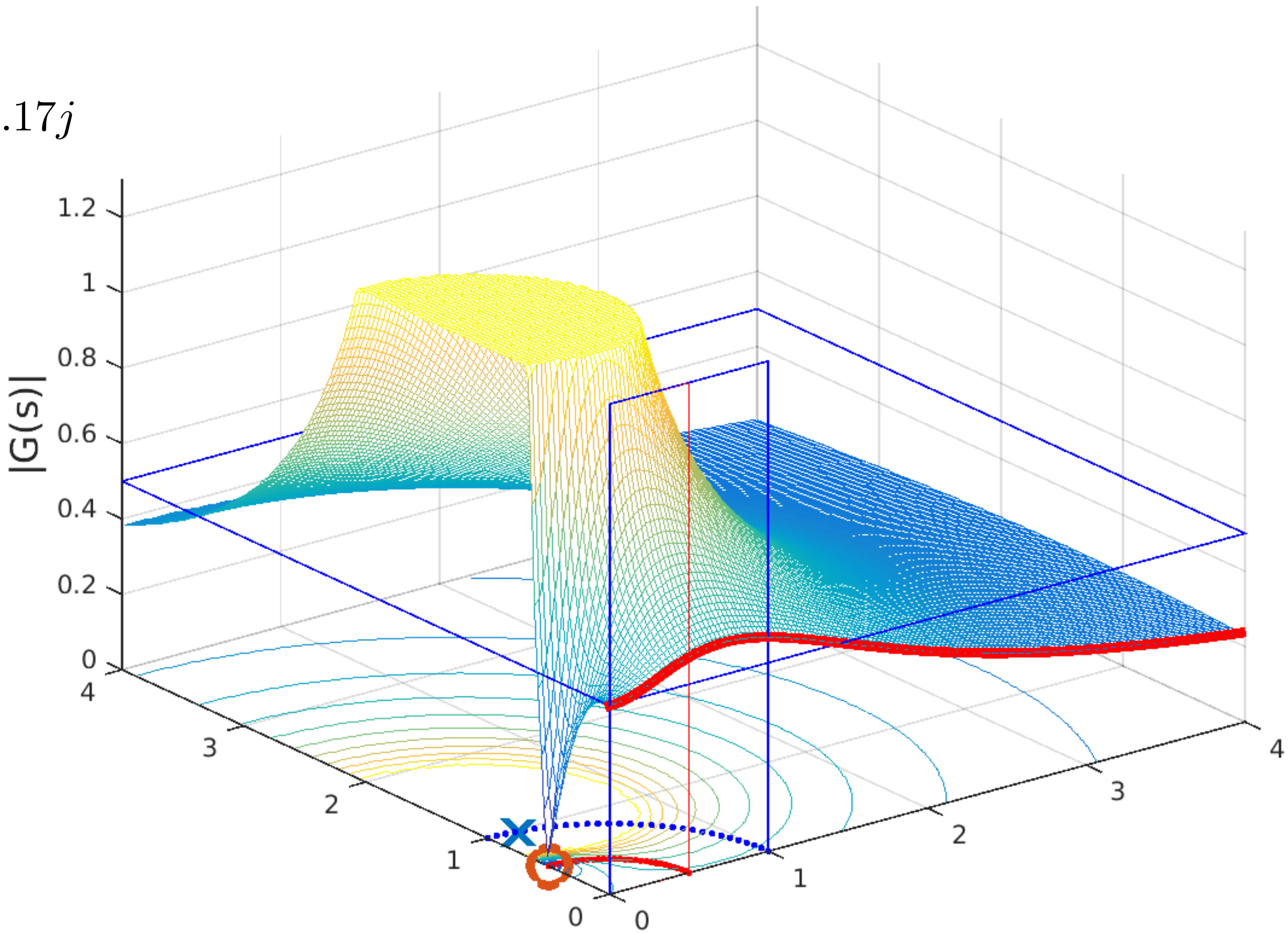
$$G = \frac{s + 0.5}{s^2 + 2 \cdot 0.98s + 1}$$

$$\omega_n = 1$$

$$P = -0.98 \pm 0.17j$$

$$Z = -0.5j$$

$$\zeta = 0.98$$



S-plan plot 3

$$G = \frac{s+0.5}{(s+0.94+0.34j)(s+0.94-0.34j)}$$

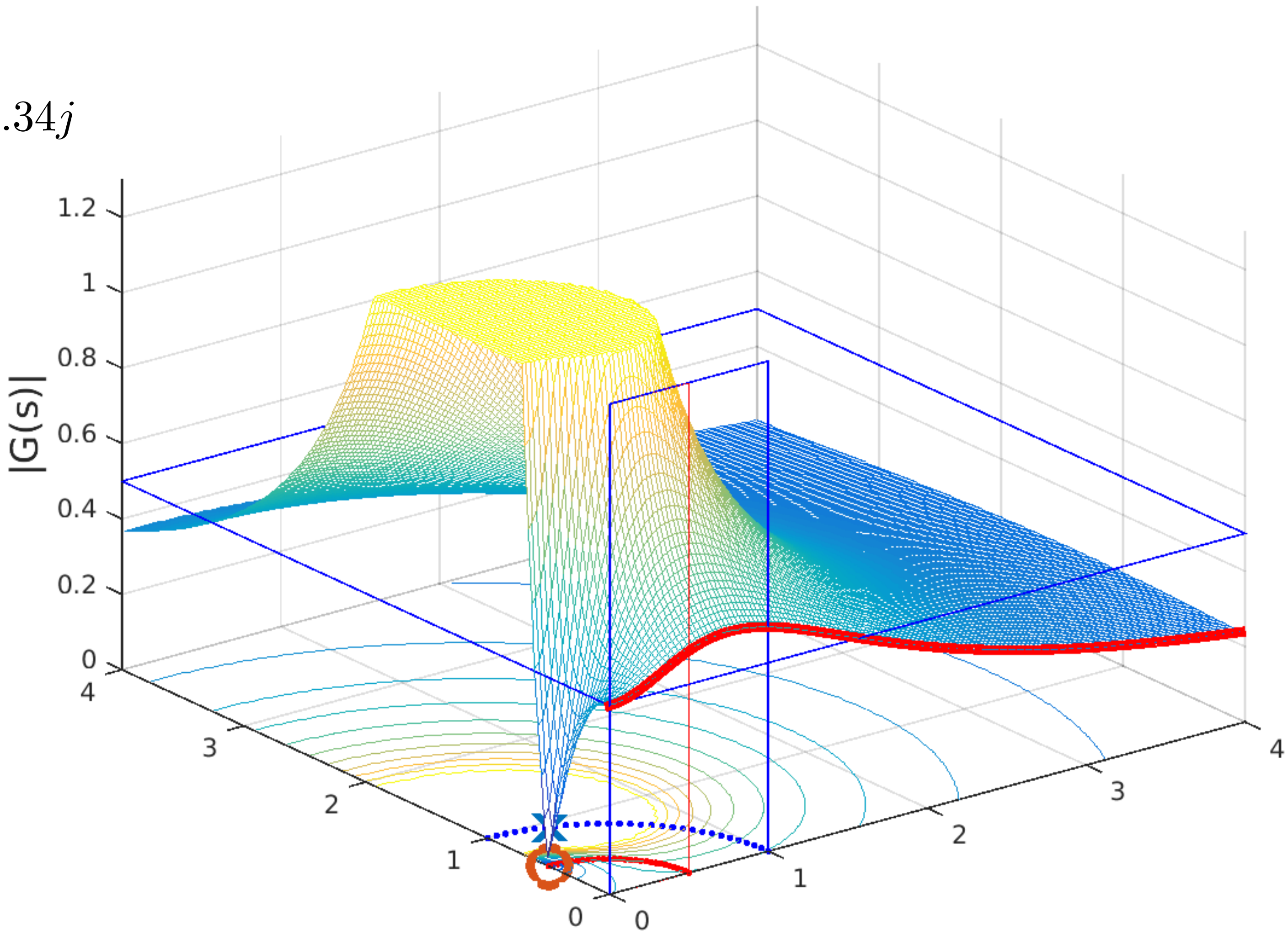
$$G = \frac{s+0.5}{s^2+2\cdot 0.94 s+1}$$

$\omega_n = 1$

$P = -0.94 \pm 0.34j$

$Z = -0.5j$

$\zeta = 0.94$



S-plan plot 4

$$G = \frac{s+0.5}{(s+0.87+0.5j)(s+0.87-0.5)}$$

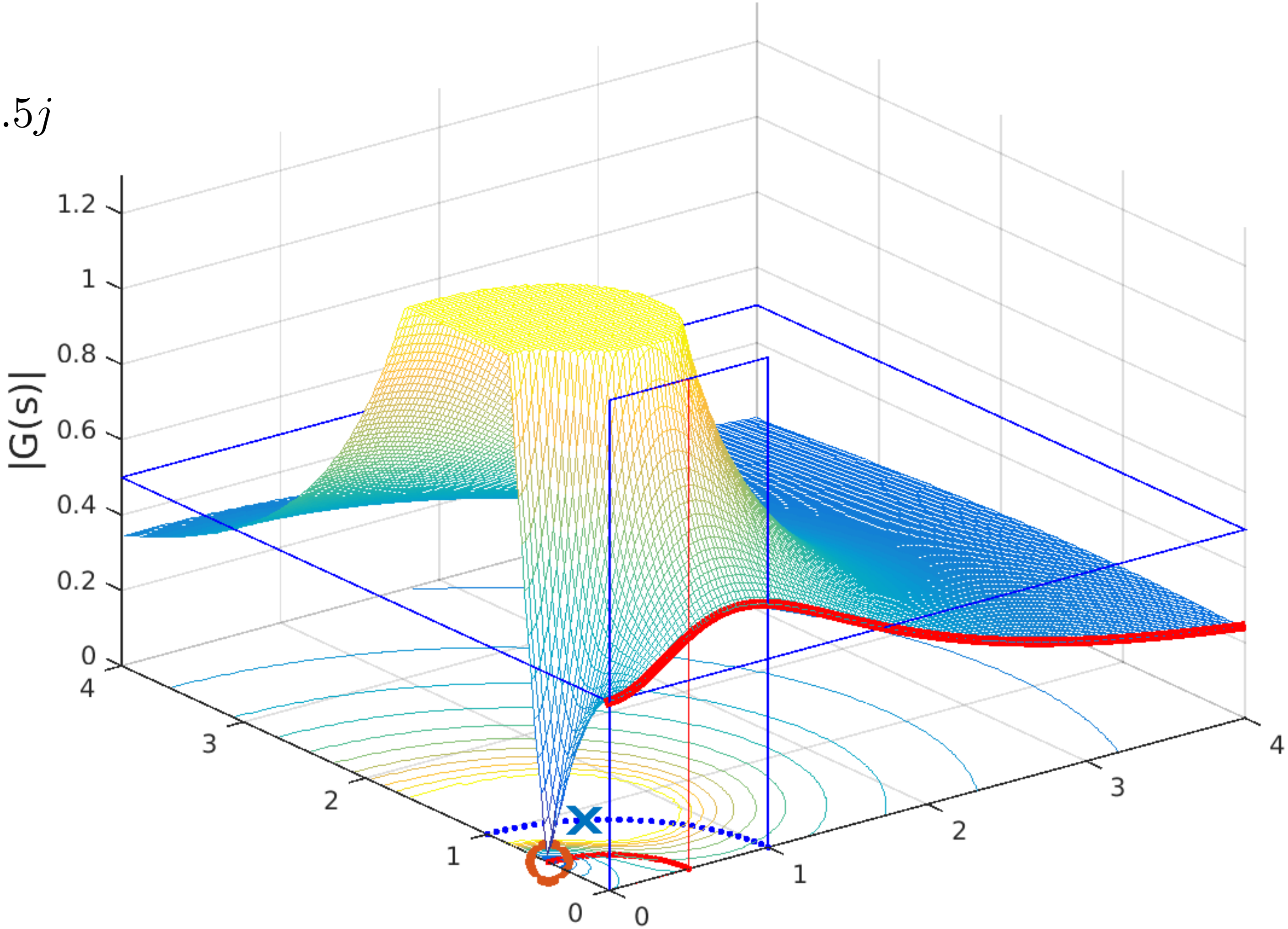
$$G = \frac{s+0.5}{s^2+2\cdot 0.87s+1}$$

$\omega_n = 1$

$P = -0.87 \pm 0.5j$

$Z = -0.5j$

$\zeta = 0.87$



S-plan plot 5

$$G = \frac{s + 0.5}{(s + 0.77 + 0.64j)(s + 0.77 - 0.64)}$$

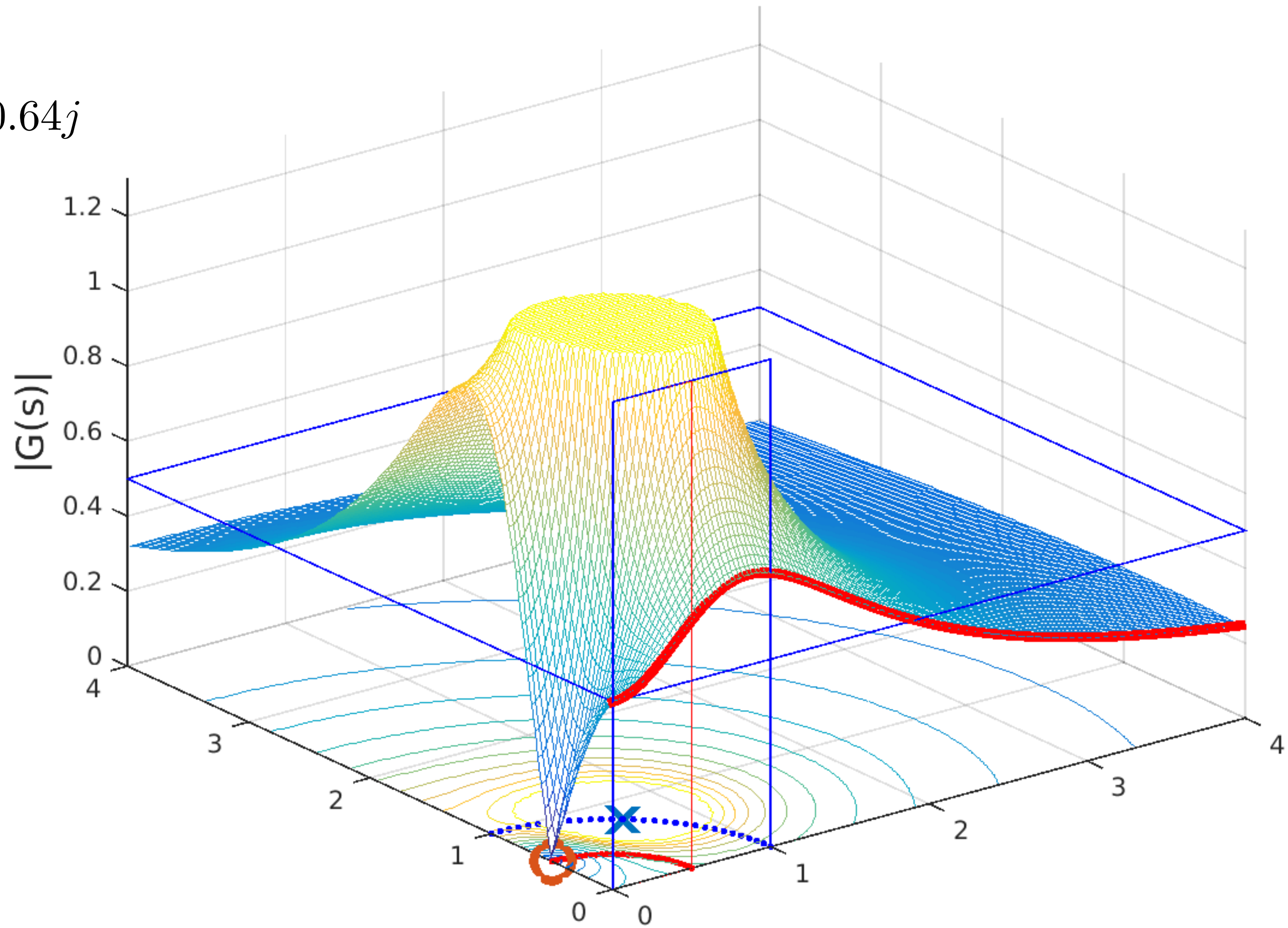
$$G = \frac{s + 0.5}{s^2 + 2 \cdot 0.77s + 1}$$

$$\omega_n = 1$$

$$P = -0.77 \pm 0.64j$$

$$Z = -0.5j$$

$$\zeta = 0.77$$



S-plan plot 6

$$G = \frac{s + 0.5}{(s + 0.64 + 0.77j)(s + 0.64 - 0.77)}$$

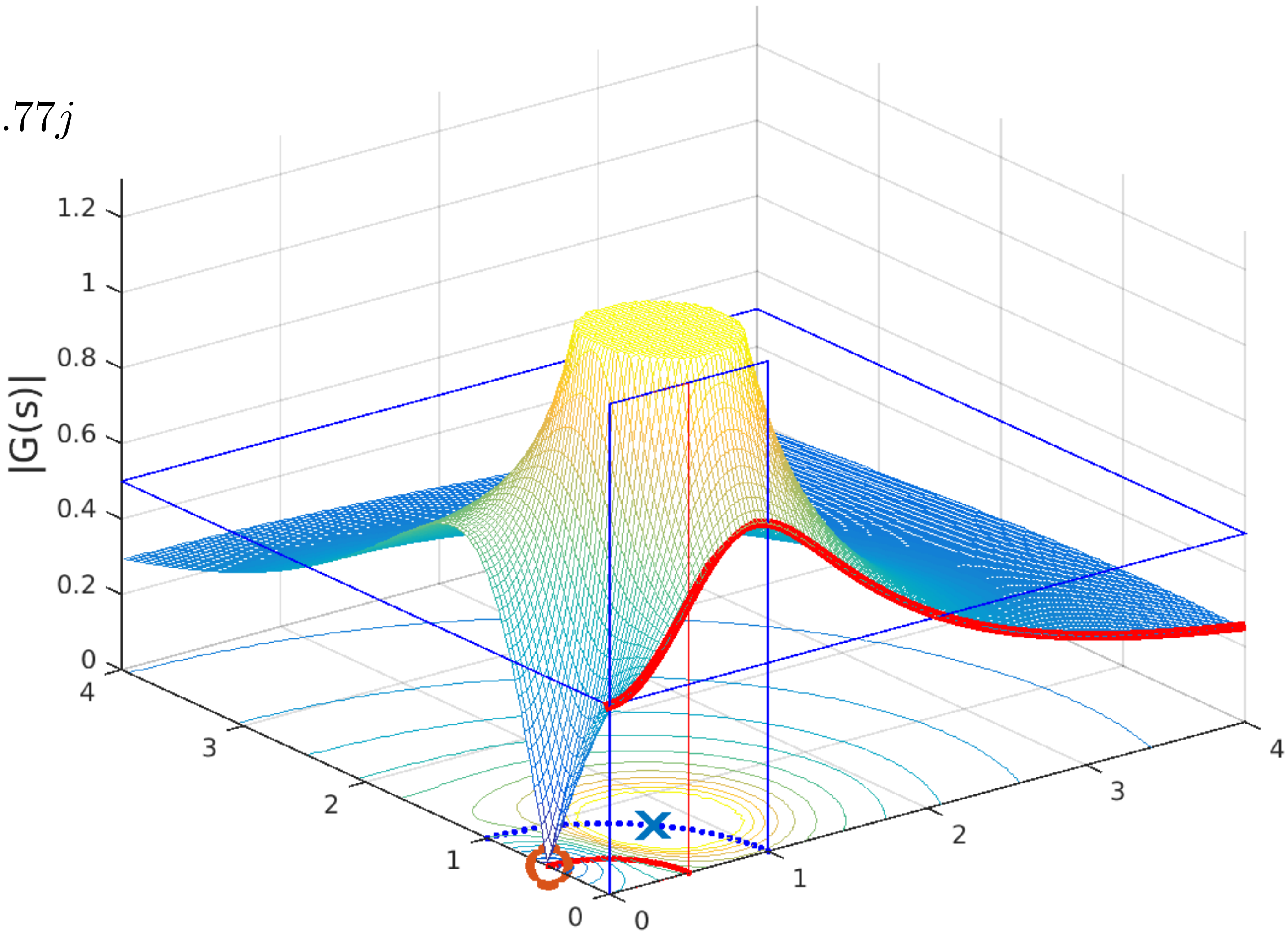
$$G = \frac{s + 0.5}{s^2 + 2 \cdot 0.64s + 1}$$

$\omega_n = 1$

$P = -0.64 \pm 0.77j$

$Z = -0.5j$

$\zeta = 0.64$



S-plan plot 7

$$G = \frac{s + 0.5}{(s + 0.5 + 0.87j)(s + 0.5 - 0.87j)}$$

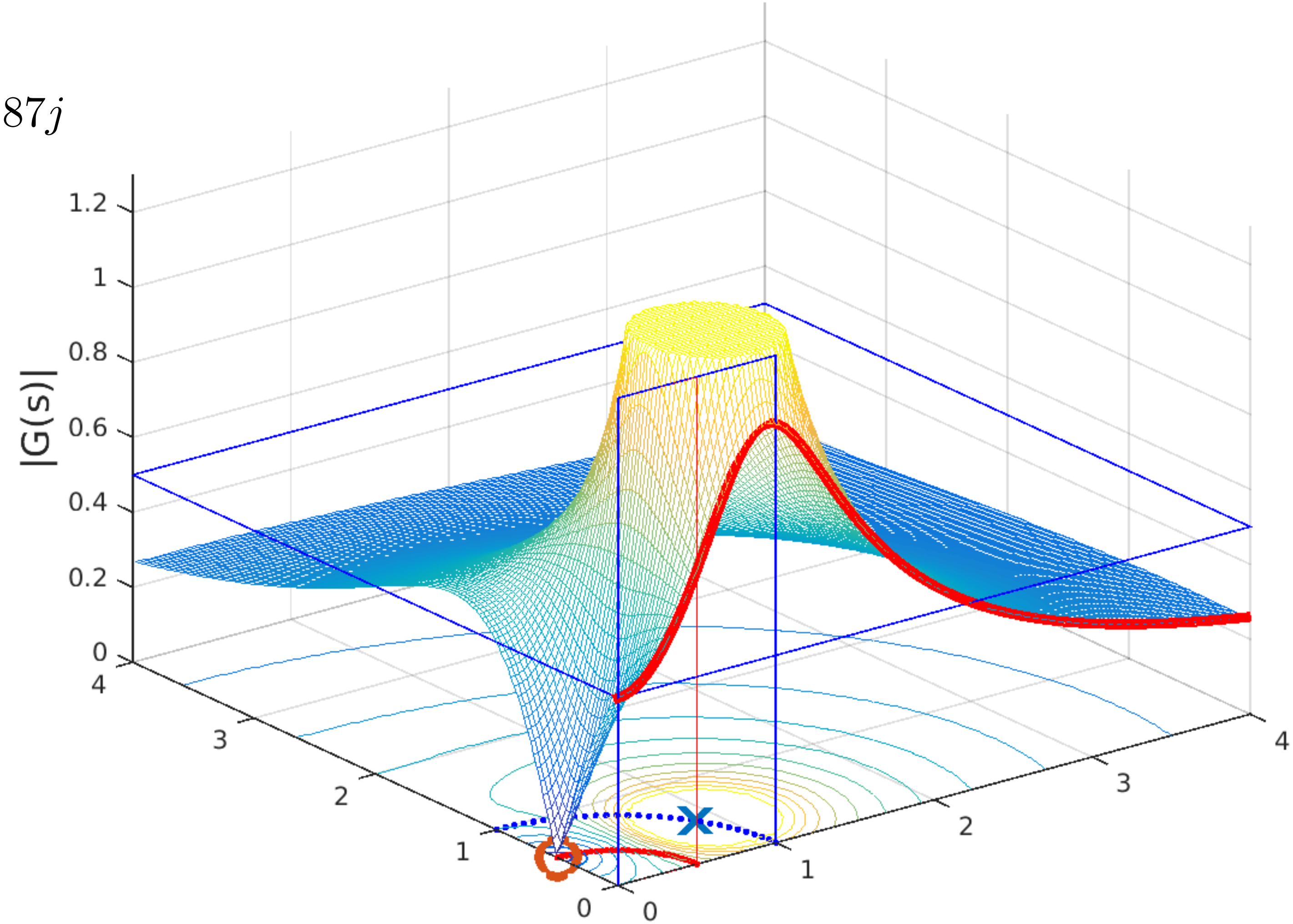
$$G = \frac{s + 0.5}{s^2 + 2 \cdot 0.5s + 1}$$

$\omega_n = 1$

$P = -0.5 \pm 0.87j$

$Z = -0.5j$

$\zeta = 0.5$



S-plan plot 8

$$G = \frac{s+0.5}{(s+0.34+0.94j)(s+0.34-0.94j)}$$

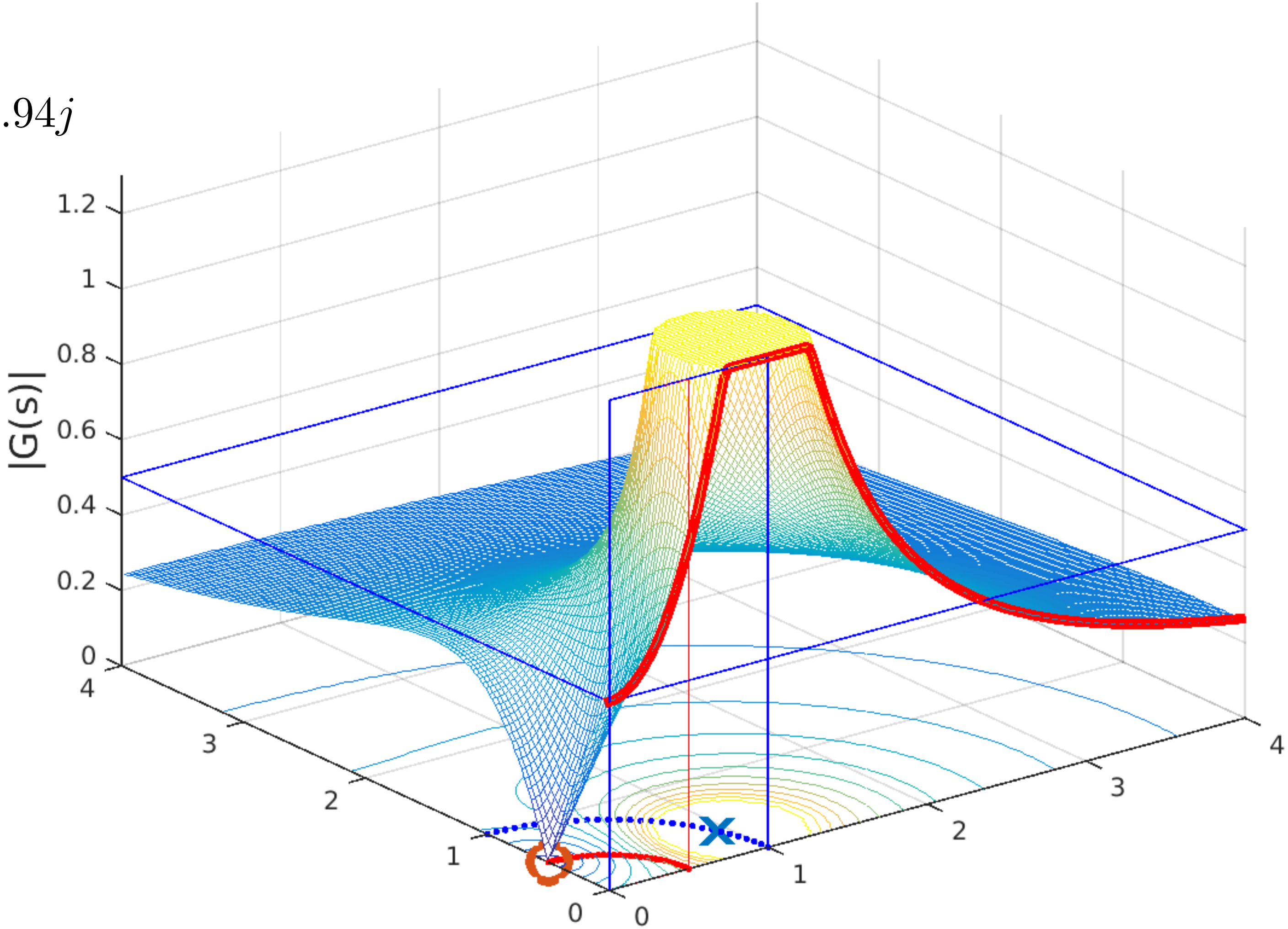
$$G = \frac{s+0.5}{s^2+2\cdot 0.34s+1}$$

$\omega_n = 1$

$P = -0.34 \pm 0.94j$

$Z = -0.5j$

$\zeta = 0.34$



S-plan plot 9

$$G = \frac{s + 0.5}{(s + 0.17 + 0.98j)(s + 0.17 - 0.98j)}$$

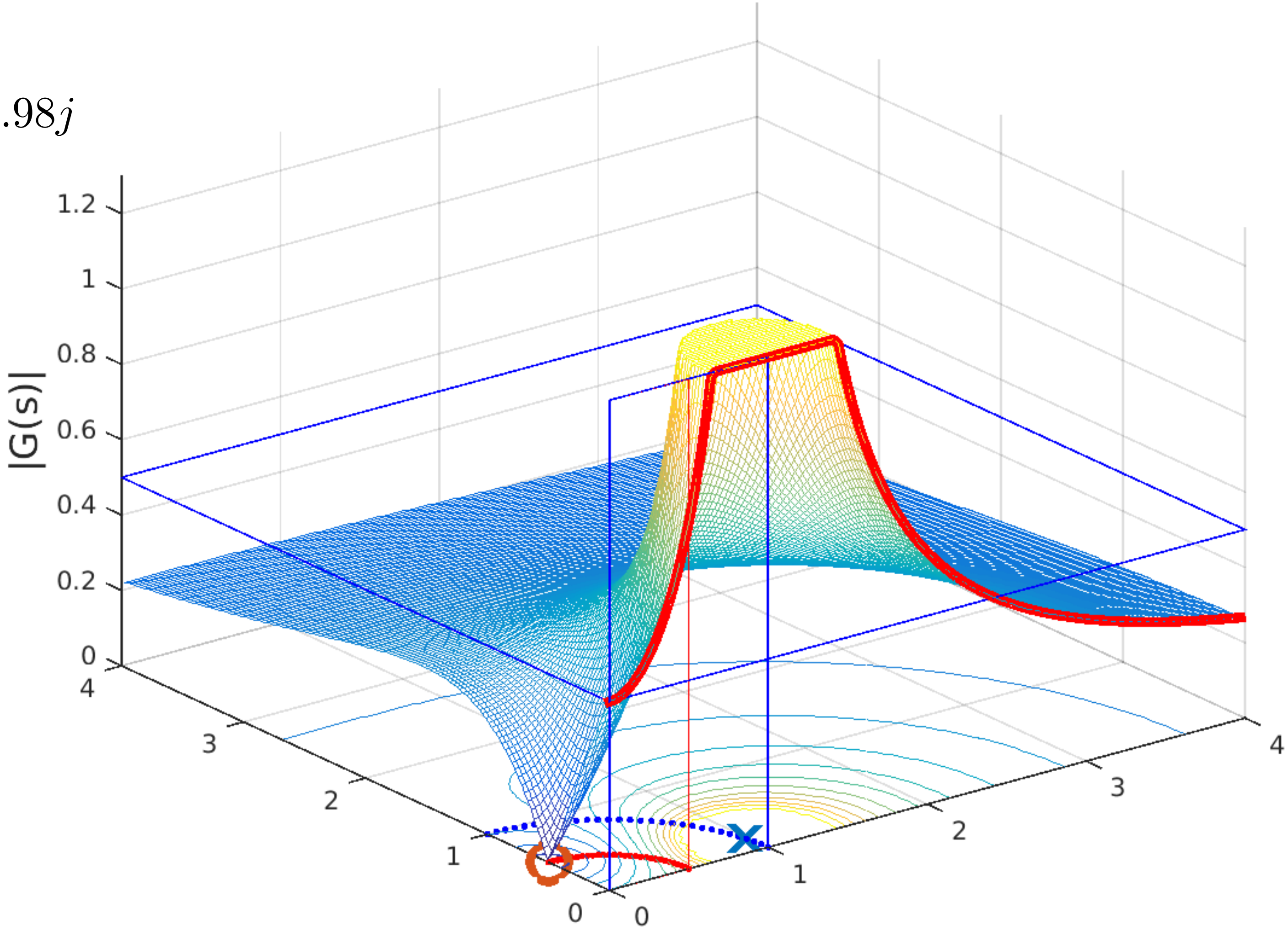
$$G = \frac{s + 0.5}{s^2 + 2 \cdot 0.17s + 1}$$

$\omega_n = 1$

$P = -0.17 \pm 0.98j$

$Z = -0.5j$

$\zeta = 0.17$



S-plan plot 10

$$G = \frac{s+0.5}{(s+j)(s-j)}$$

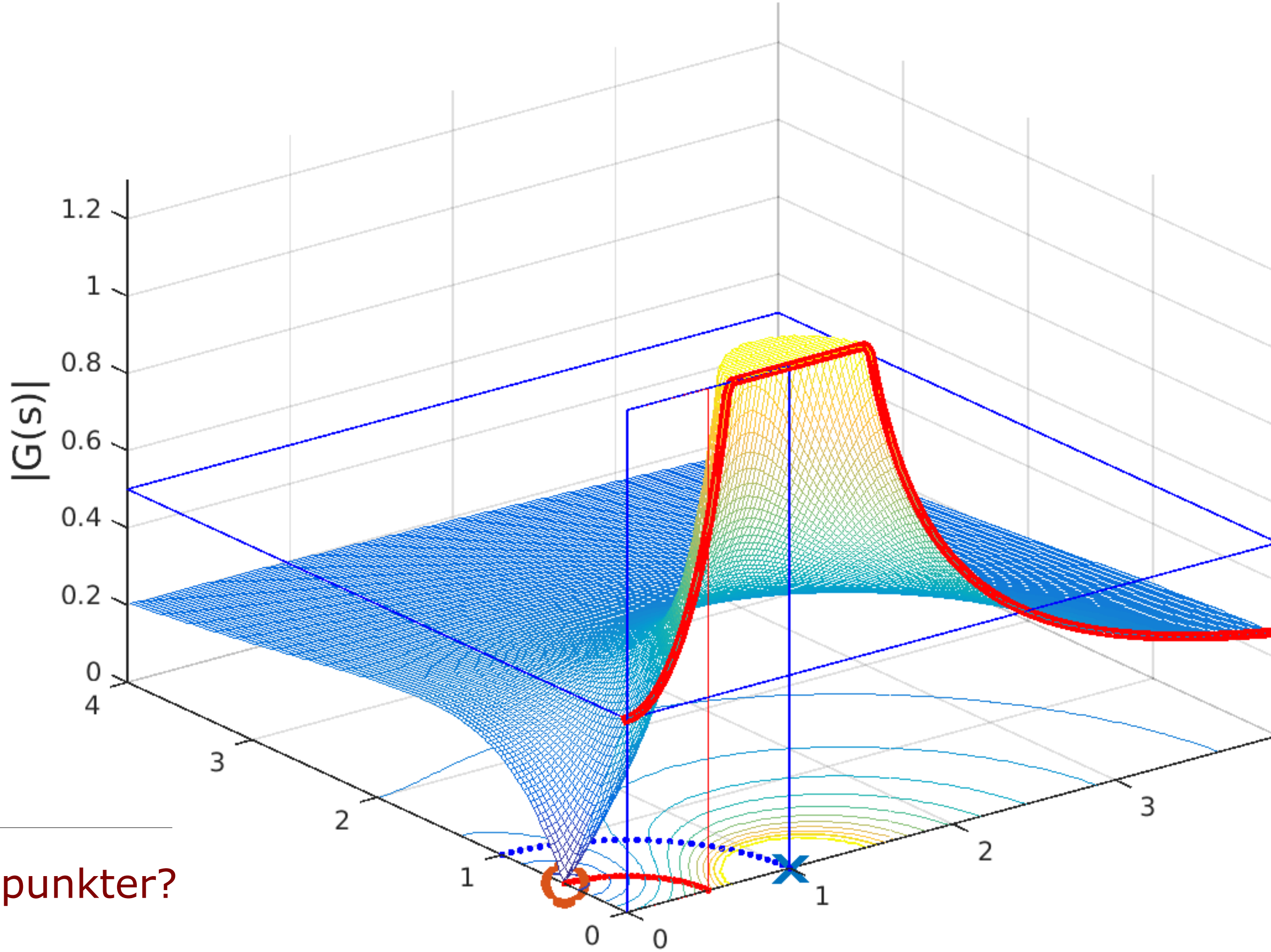
$$G = \frac{s+0.5}{s^2+1}$$

$\omega_n = 1$

$P = \pm j$

$Z = -0.5j$

$\zeta = 0$



Komplekse nulpunkter?

Bode plot
– 2. orden

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s = j\omega$$

$$\omega \to 0 \Rightarrow G(\omega) \to 1\angle 0$$

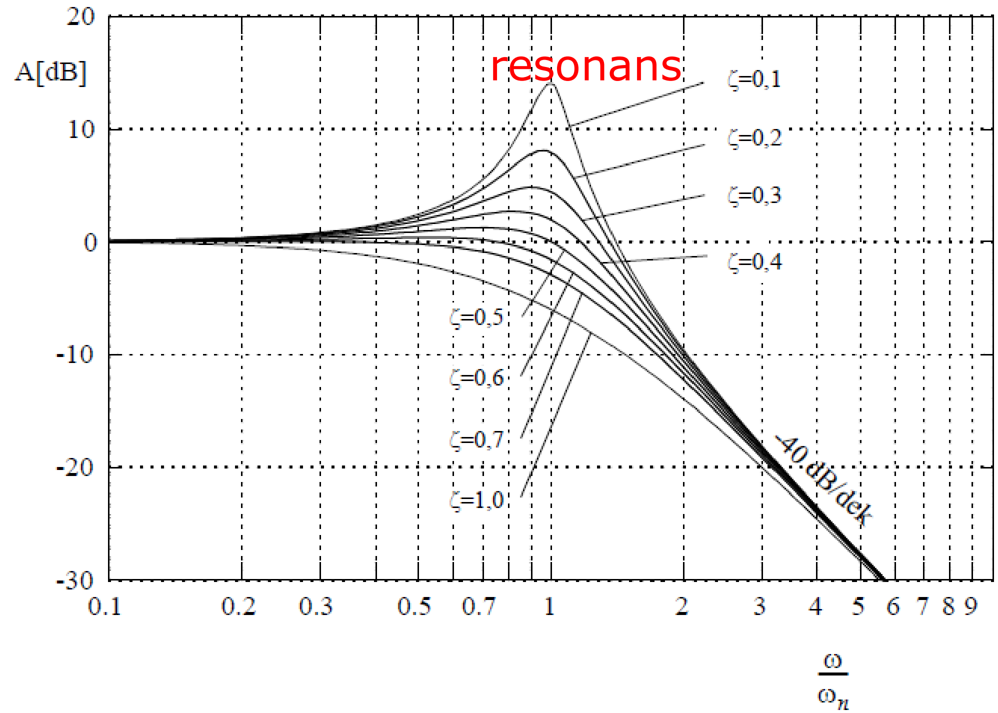
$$\omega \to \infty \Rightarrow G(\omega) \to 0\angle -180^o$$

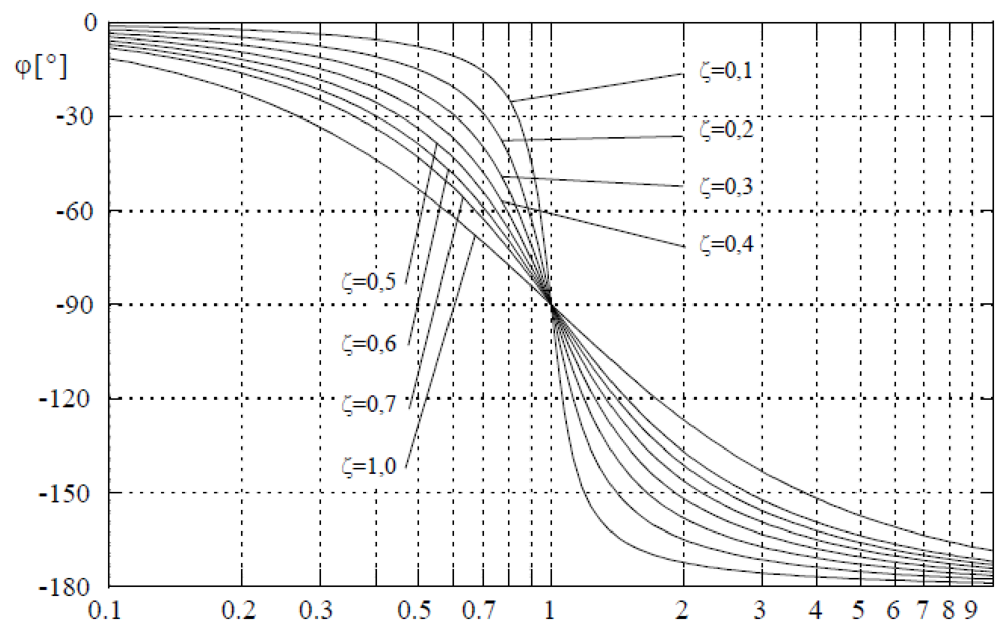
Ved egenfrekvensen ω_n

$$G(\omega_n) = \frac{\omega_n^2}{2\zeta\omega_n\omega_n j} = \frac{1}{2\zeta}\angle -90^o$$

Spørgsmål:

$$\zeta = 0.05 \Rightarrow \ |G(\omega_n)|_{dB} \ ?$$



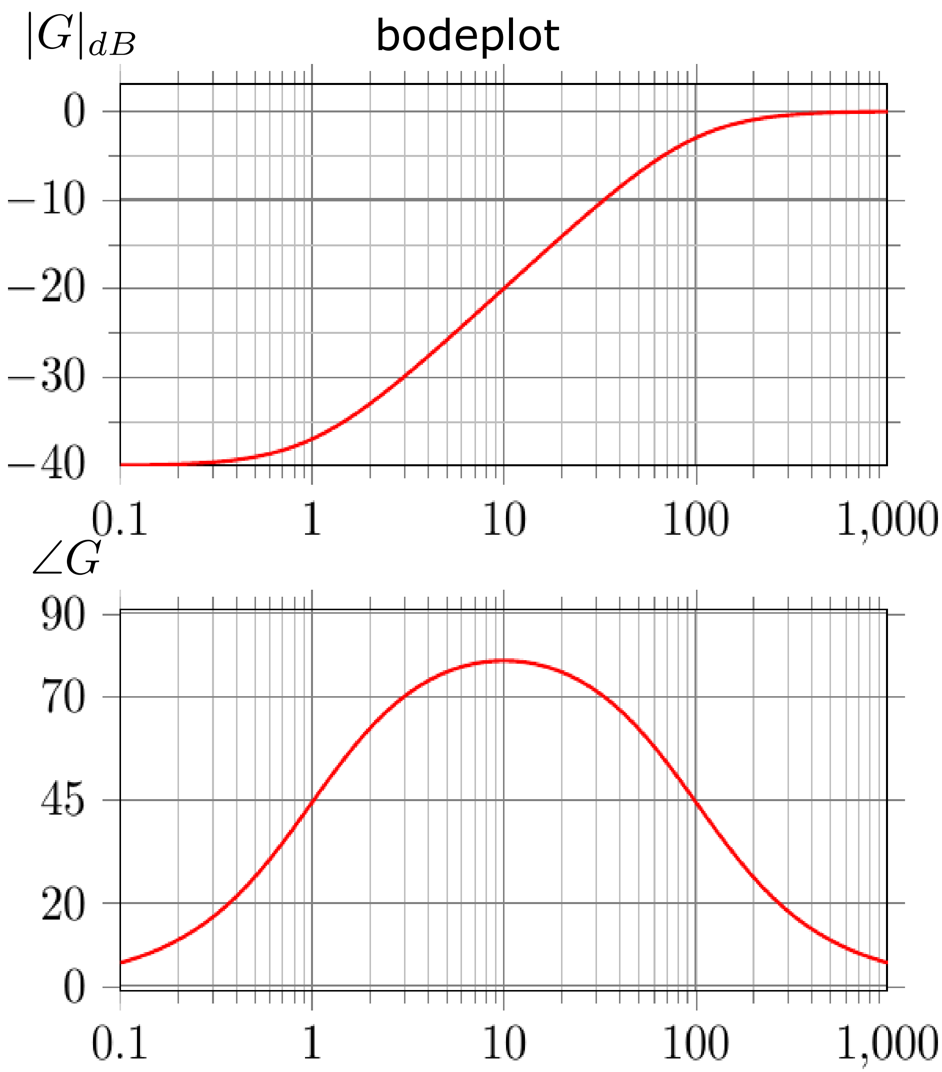


Bode:

- Minimum fase:
  Alle poler og nulpunkter i venstre halvplan
- Ikke minimum fase:
  Mindst en pol eller nulpunkt i højre halvplan

Minimum fase system:

- *Hvis amplitudeforløbet er kendt kan faseforløbet udledes direkte*

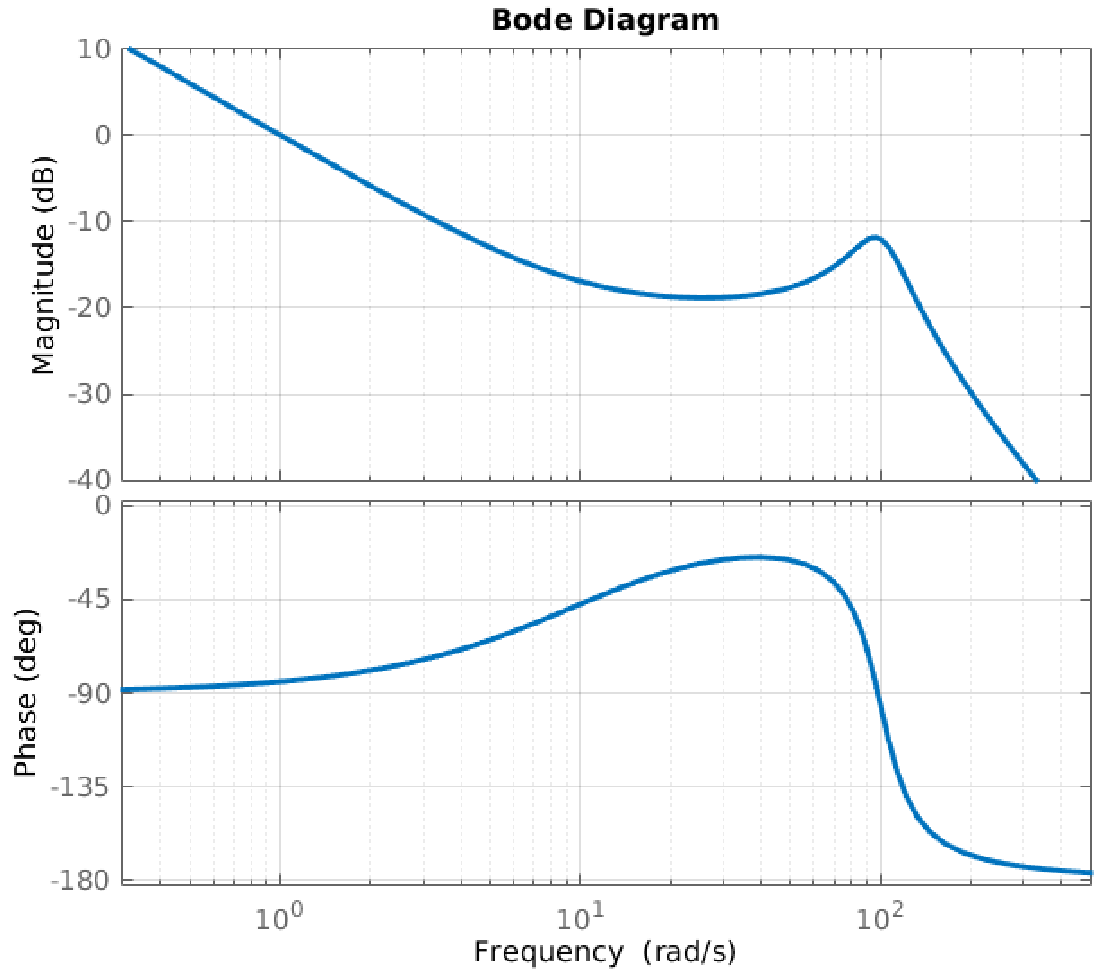


Kontrolspørgsmål

For et system giver en måling det viste bodeplot,

Hvad er systemets overføringsfunktion (ca.)?

$$G(s) = \frac{\cdots}{\cdots}$$

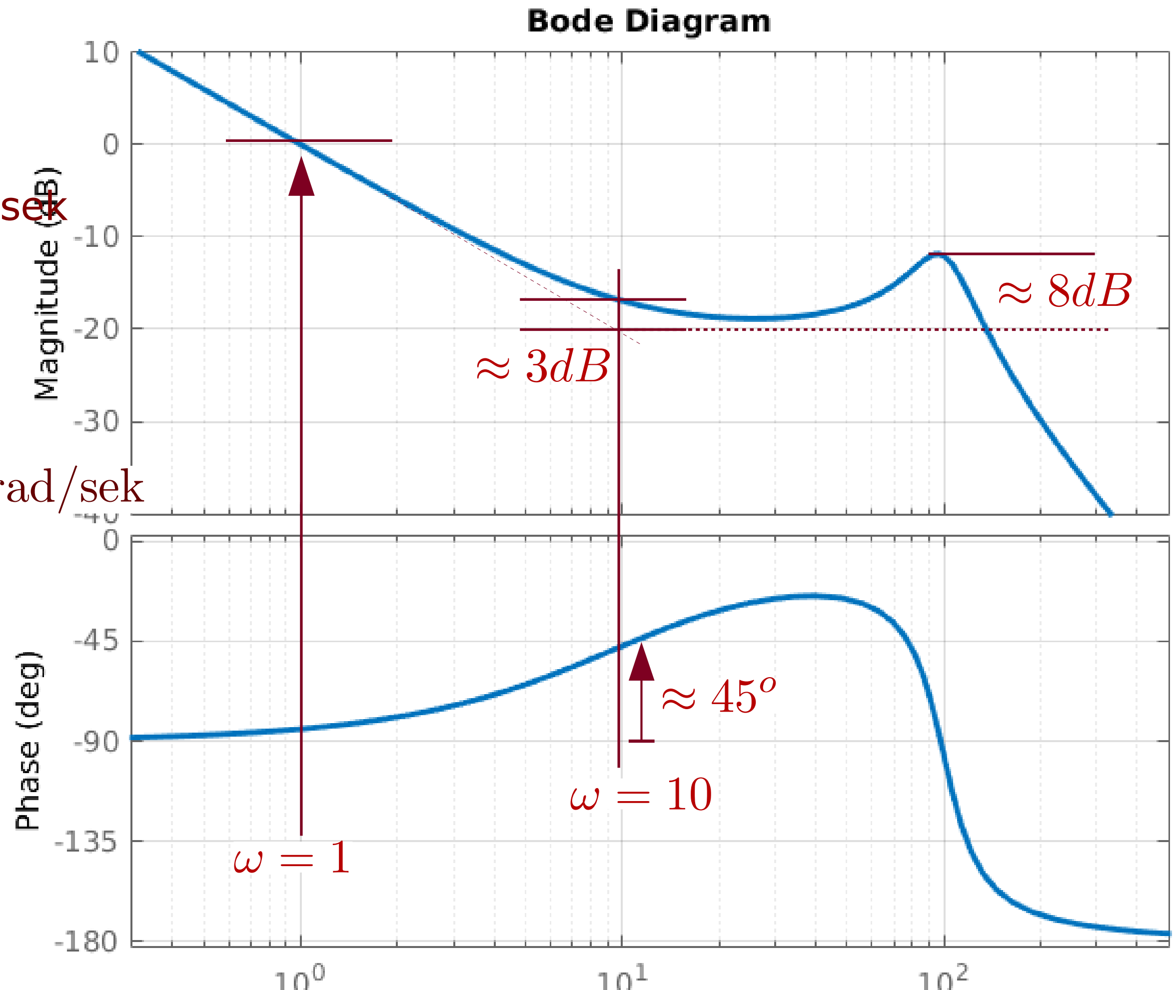


Kontrolspørgsmål

2. orden led: $s^2 + 2\zeta\omega_n s + \omega_n^2$

Overføringsfunktion (ca.)?

Plot starter med -20dB/dek
og -90 grader fase → pol i s=0
Der er noget der ligner et
knæk opad ved omkring 10 rad/sek
→ reelt nulpunkt ved s=-10
Amplitudespidsen og
vinkedfald med ca.
-180 grader
→ komplekst polpar, med
resonansfrekvens på $\omega_n \approx 100 \text{ rad/sek}$
og peak er ca. 8dB over det
"flade" niveau → $\zeta \approx 0.2$



$$G(s) = \frac{k(s+10)}{s(s^2+40s+10000)}$$

Konstanten k, der er
ikke noget fladt niveau,
men så f.eks. ved

$$s = 1j \Rightarrow |G(j)| \approx 0dB$$

$$|G(s=j)| = \left|\frac{k(j+10)}{j(j^2+40j+10000)}\right| = 1 \Rightarrow \left|\frac{10k}{10000j}\right| = 1 \Rightarrow k = 1000$$

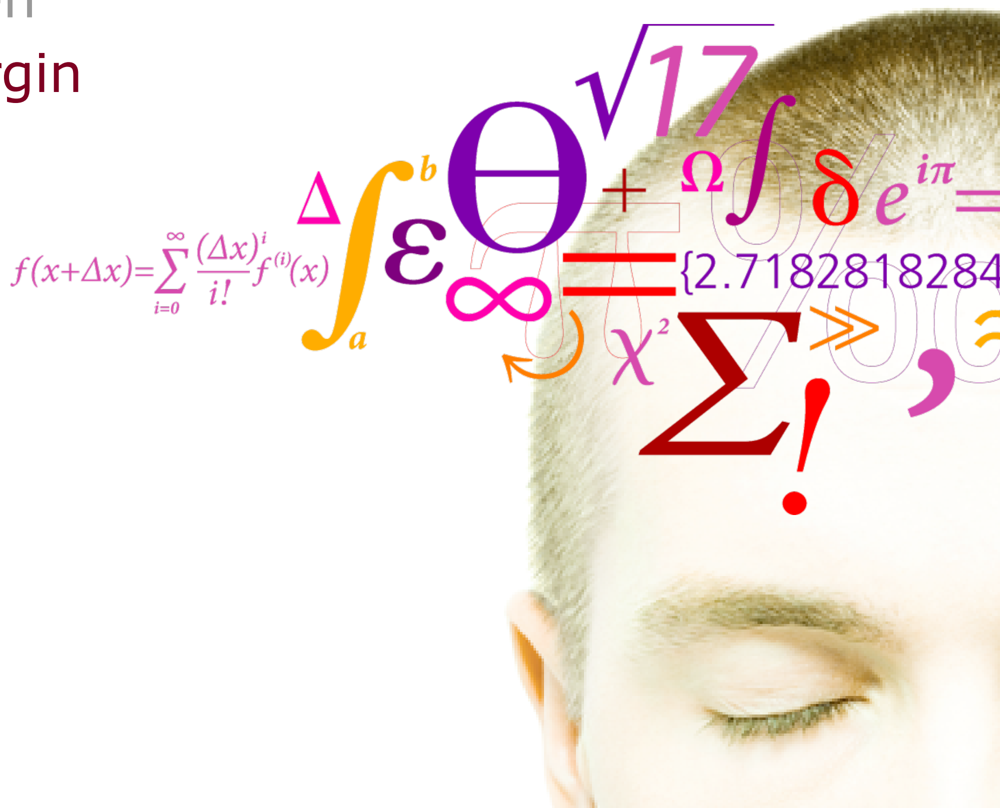
Reguleringsteknik 1

J. Christian Andersen

Kursusuge 5

Plan

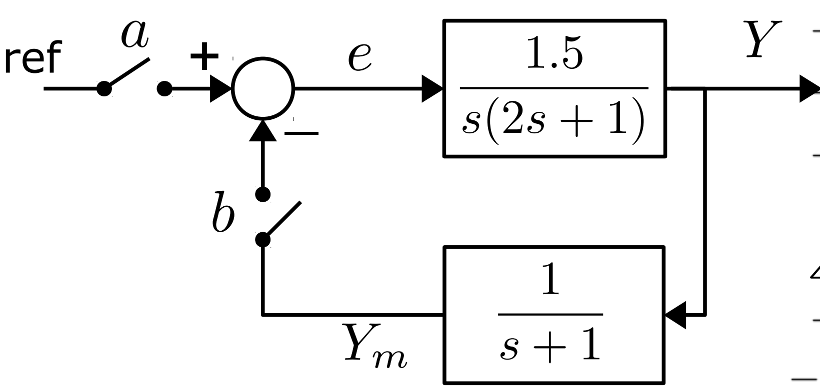
-
- Frekvensanalyse
  - Bodeplot – 1. og 2. orden
  - Stabilitet, Stabilitetsmargin
- Grupperegning
  - Frekvensanalyse
-



Stabilitet

$$G_å(s) = \frac{1.5}{s(2s+1)(s+1)}$$

$$ref = \sin(\omega_\pi t) = \sin(0.707t)$$



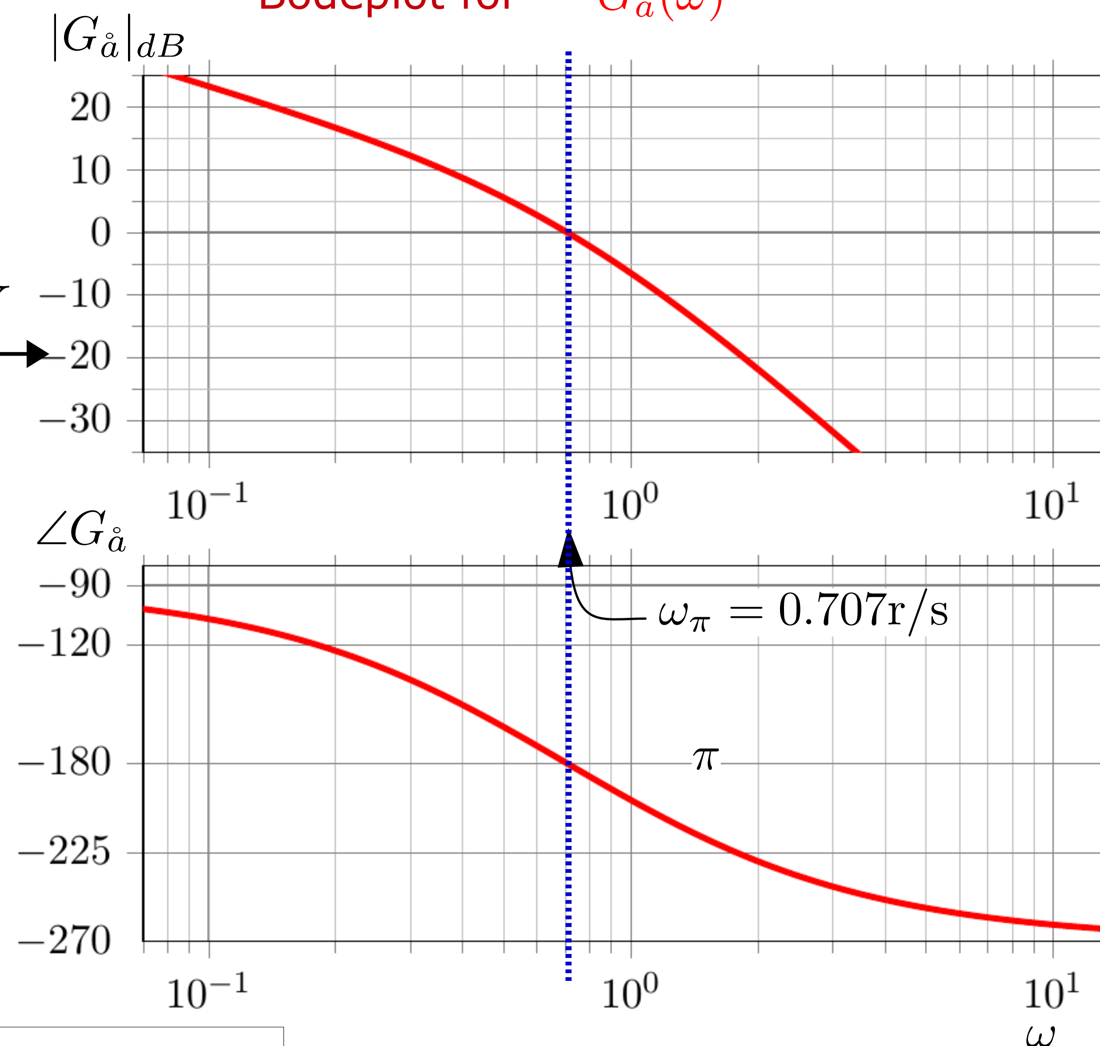
a lukkes, så er $Y_m(t)$ = -$e(t)$

Når så *b* lukkes og *a* åbnes

bliver $e(t)$ = -$Y_m(t)$ = $e(t)$

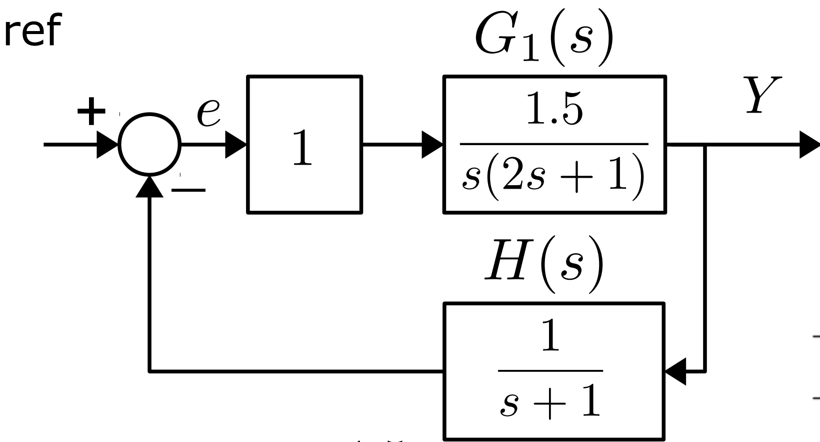
- marginalt stabilt

Bodeplot for $G_å(\omega)$



Stabilitetsmæssigt kritisk punkt $1\angle -180^o$

Stabilitet



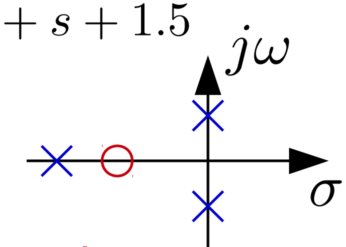
$$G_å(s) = \frac{1.5}{s(2s+1)(s+1)}$$

Lukket sløjfe

$$G_{LS}(s) = \frac{G_1}{1 + G_1 H}$$

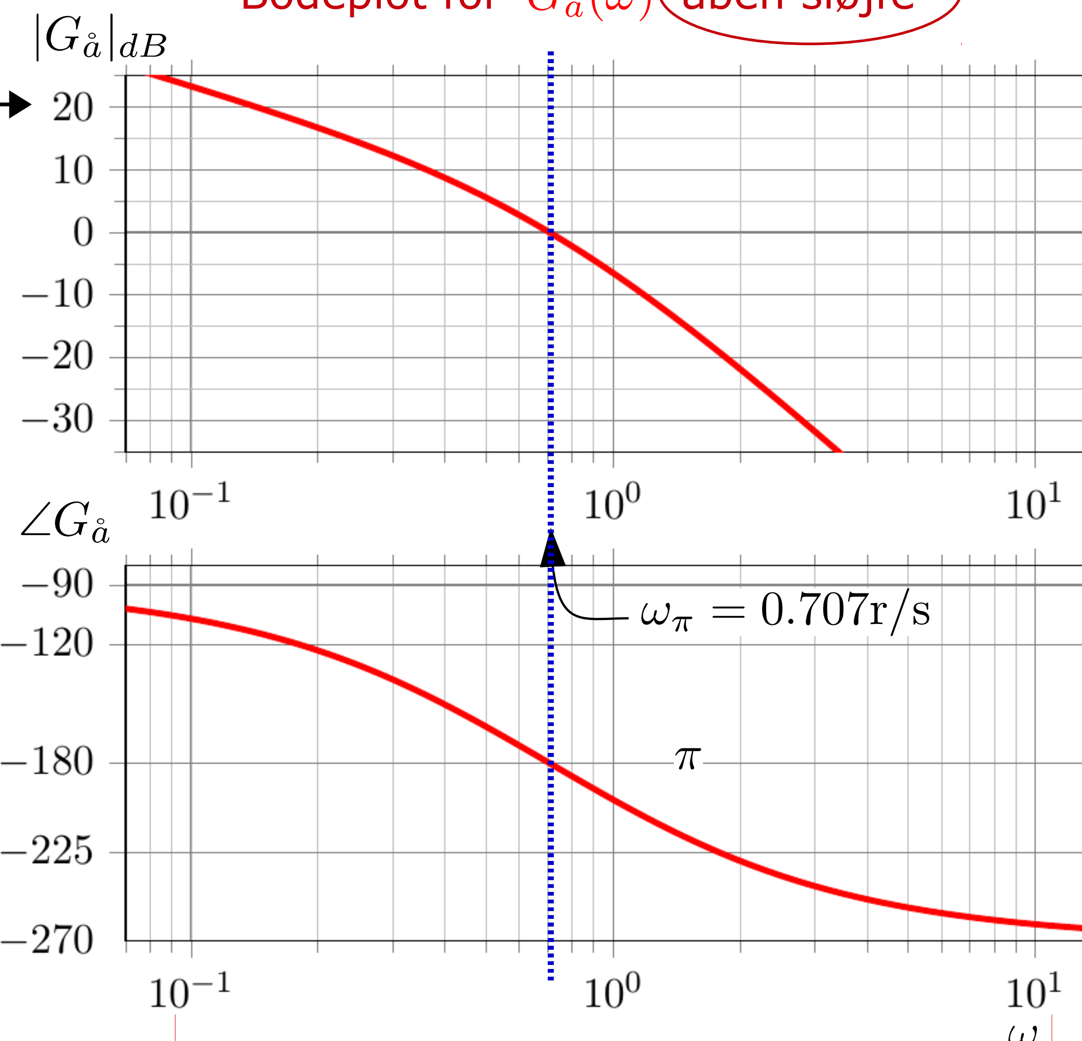
$$G_{LS}(s) = \frac{1.5(s+1)}{2s^3 + 3s^2 + s + 1.5}$$

$$\begin{aligned} p_1 &= -1.5 \\ p_2 &= +0.707j \\ p_3 &= -0.707j \end{aligned}$$



s-plan

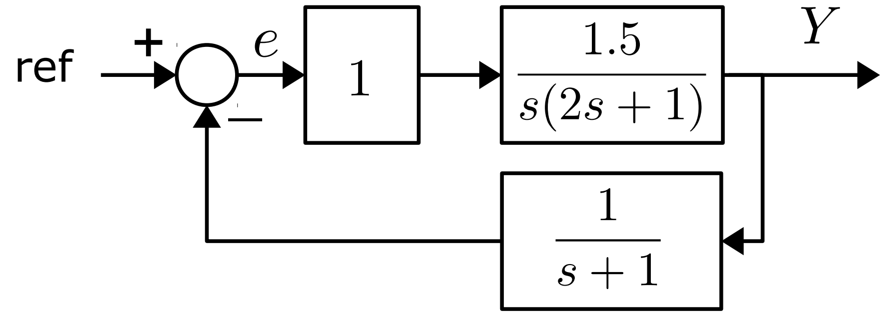
Bodeplot for $G_å(\omega)$ åben sløjfe



Stabilitetsmæssigt kritisk punkt $1\angle -180^o$

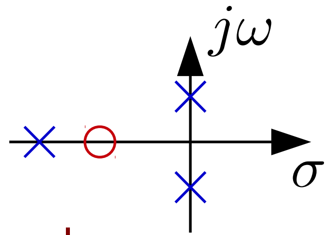
For lukket sløjfe

Lukket sløjfe step respons

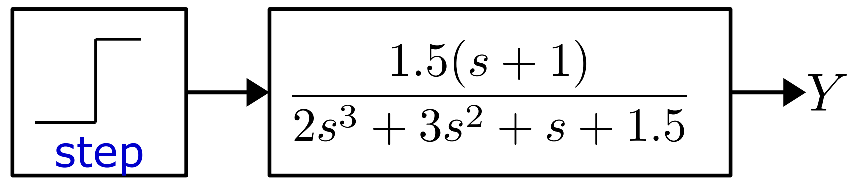


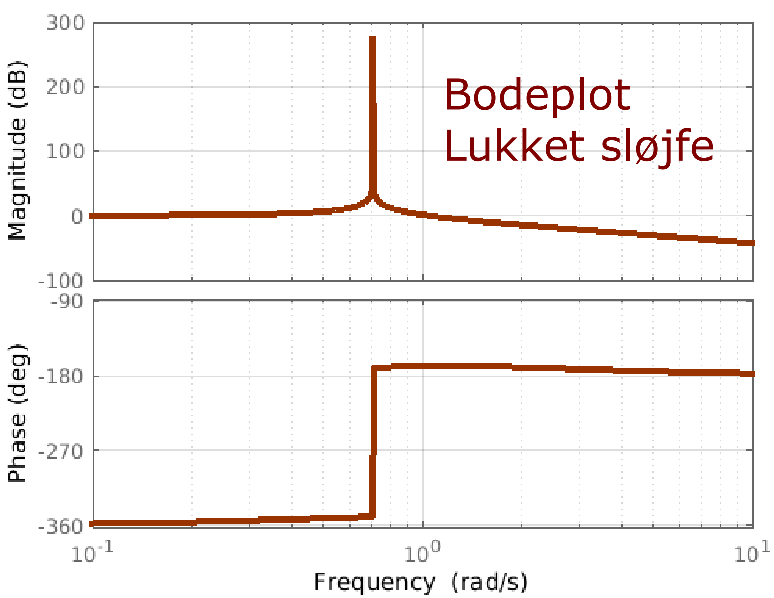
$$G_{LS}(s) = \frac{1.5(s+1)}{2s^3 + 3s^2 + s + 1.5}$$

$$\begin{aligned} p_1 &= -1.5 \\ p_2 &= +0.707j \\ p_3 &= -0.707j \end{aligned}$$



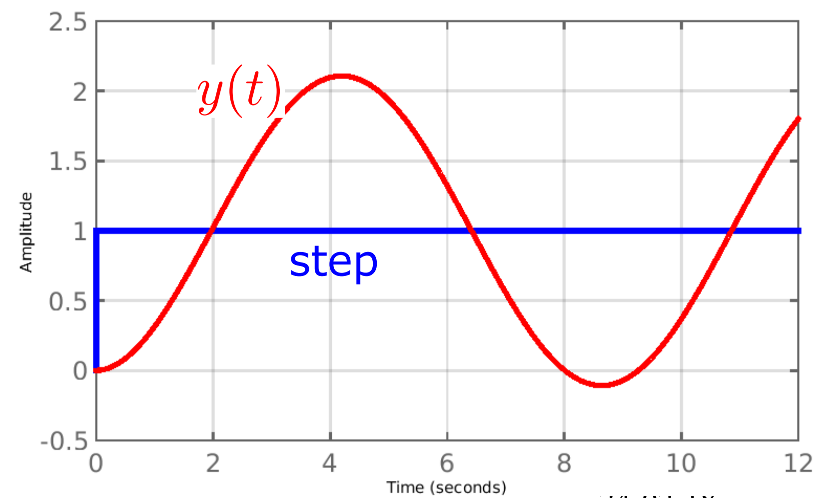
s-plan



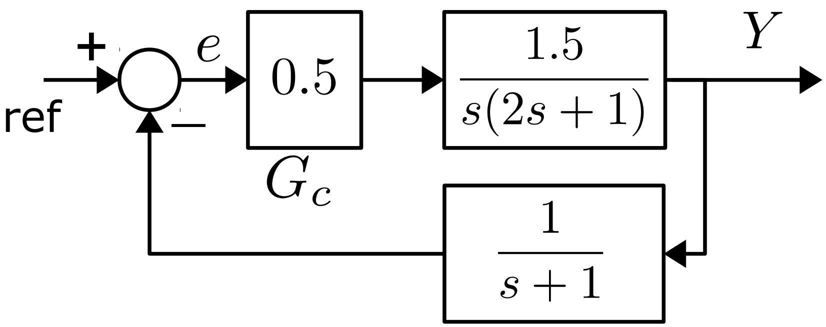


$$y(t) = \mathscr{L}^{-1}\{\tfrac{1}{s}G_{LS}(s)\}$$

$$y(t) = -1.1\cos(0.707t) + 0.2\sin(0.707t) + 1 + 0.1e^{-1.5t}$$



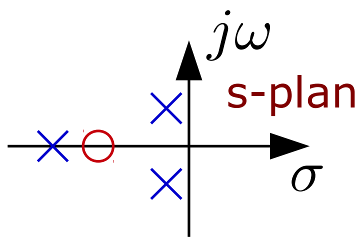
Stabilitet
Fasemargin

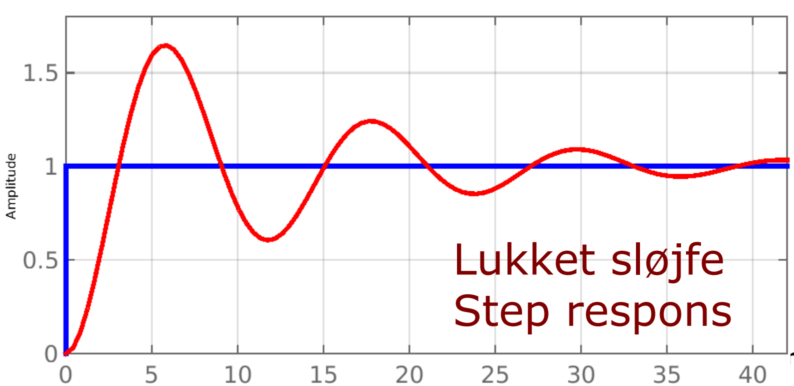


Lukket sløjfe

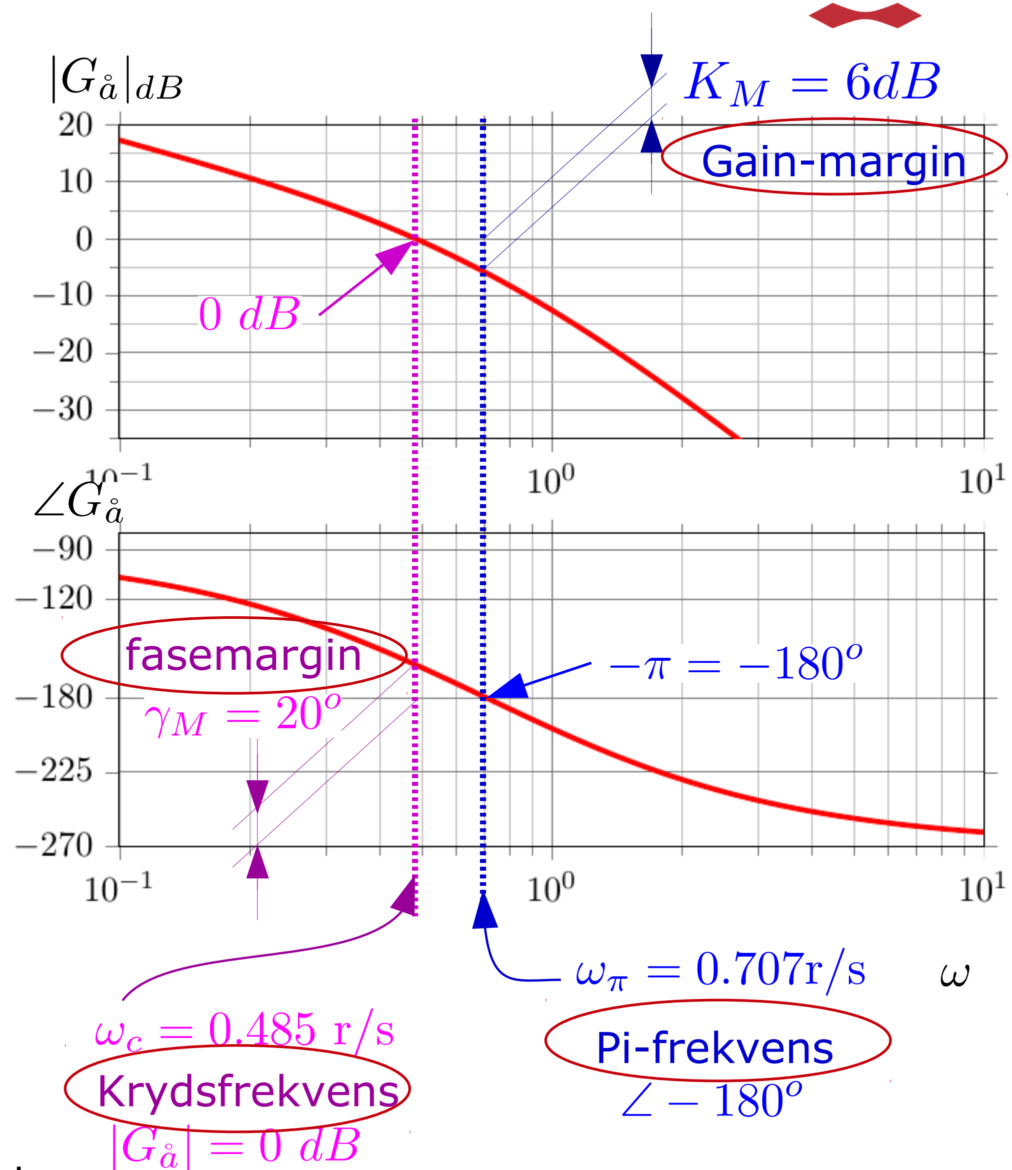
$$G_{LS}(s) = \frac{0.375(s+1)}{s^3 + 1.5s^2 + 0.5s + 0.375}$$

$$\begin{aligned} p_1 &= -1.34 \\ p_2 &= -0.08 + 0.5j \\ p_3 &= -0.08 - 0.5j \end{aligned}$$

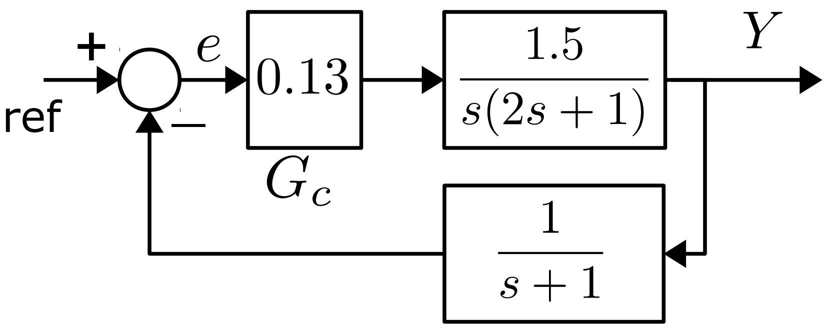




Bodeplot for $G_å(\omega)$ åben sløjfe



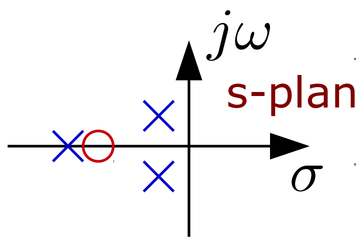
Stabilitet
Fasemargin II

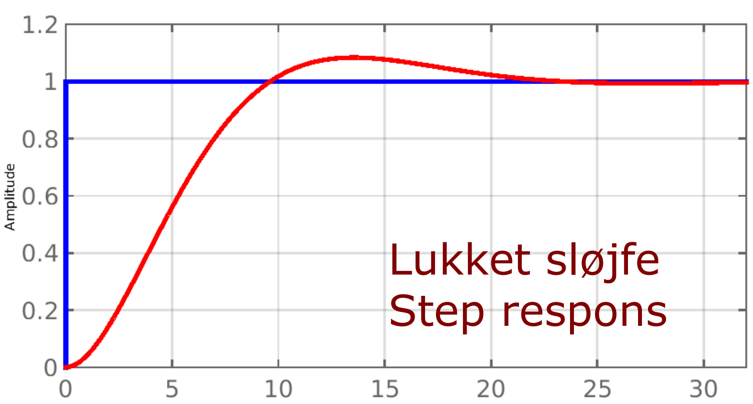


Lukket sløjfe

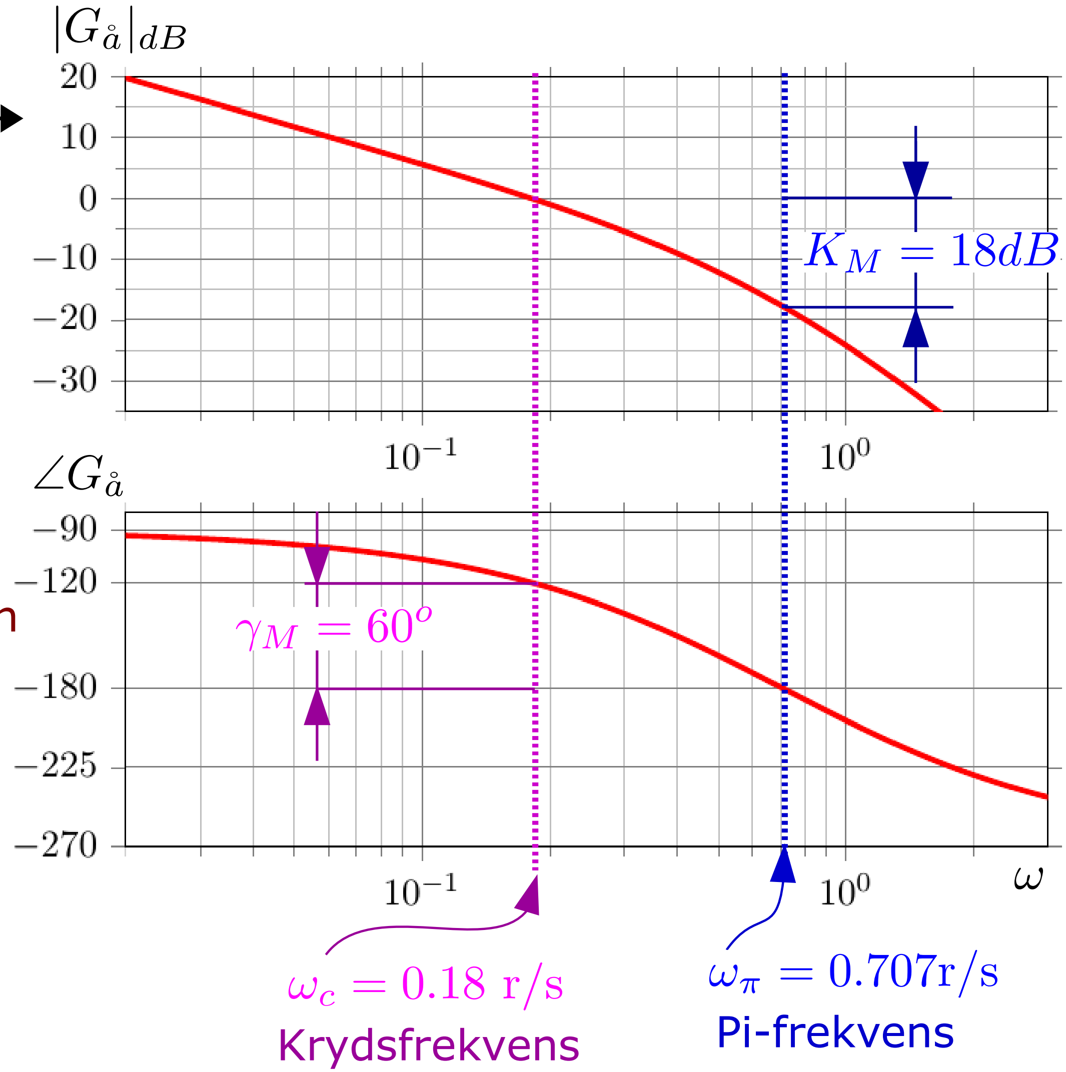
$$G_{LS}(s) = \frac{0.0975(s+1)}{s^3 + 1.5s^2 + 0.5s + 0.0975}$$

$$\begin{aligned} p_1 &= -1.14 \\ p_2 &= -0.18 + 0.23j \\ p_3 &= -0.18 - 0.23j \end{aligned}$$

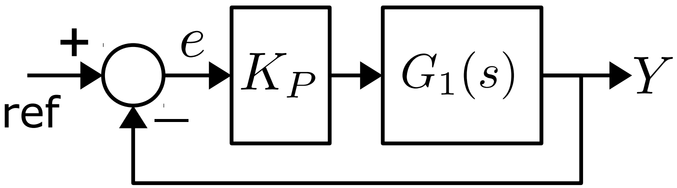


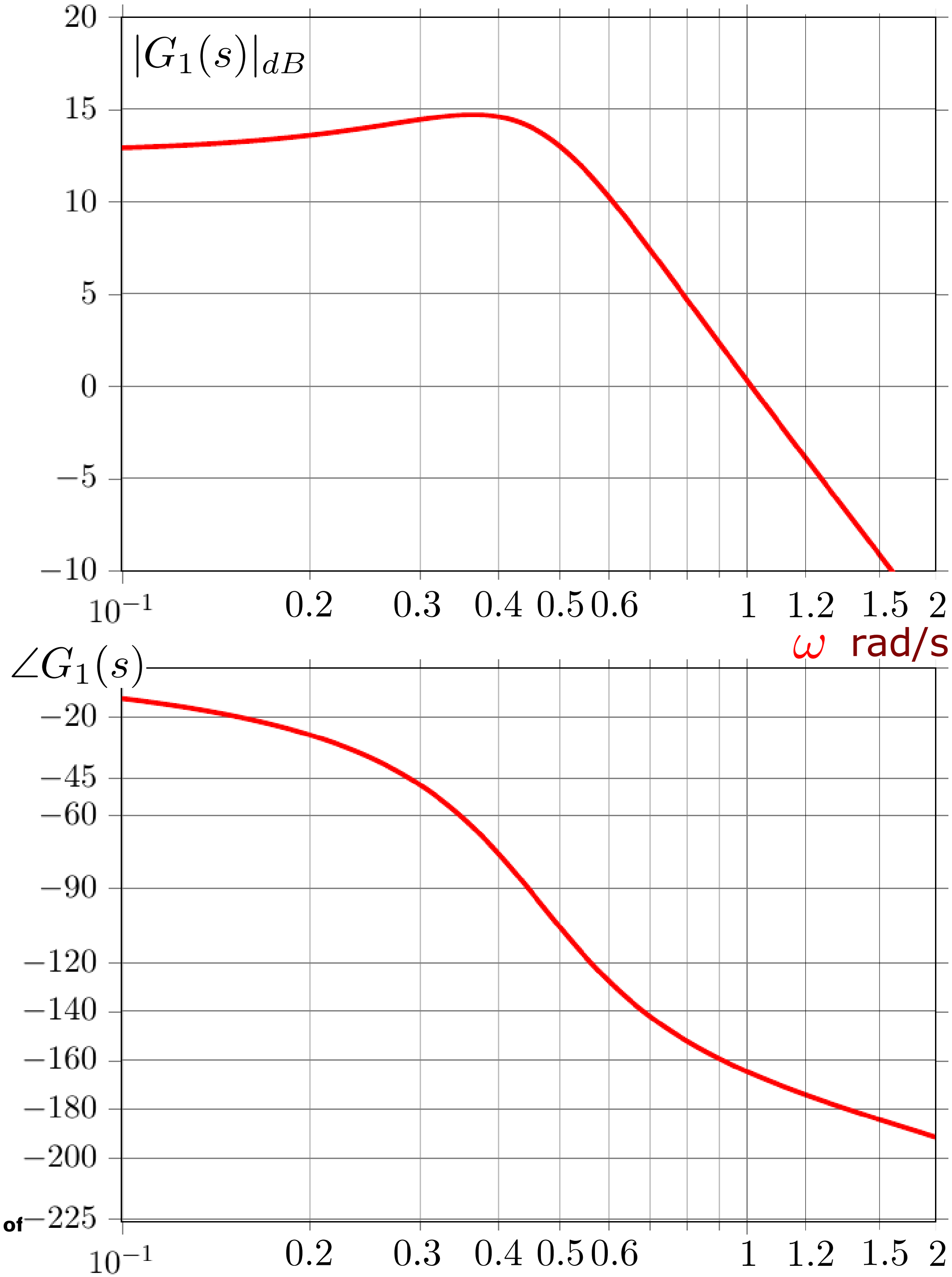


Bodeplot for $G_å(\omega)$ åben sløjfe



Kontrolspørgsmål





Opgave 1

$K_P = 1$

Fasemargin $\gamma_M = \ ?$

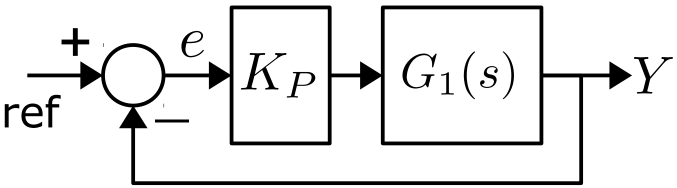
Gain-margin $K_M = \ ?$

Opgave 2

Find K_P så åben sløjfe

fasemargin $\gamma_M = 60^o$

Kontrolspørgsmål



Opgave 1

$$K_P = 1 \Rightarrow G_å = G_1(s)$$

Fasemargin $\gamma_M = \ ?$

Gain-margin $K_M = \ ?$

Opgave 2

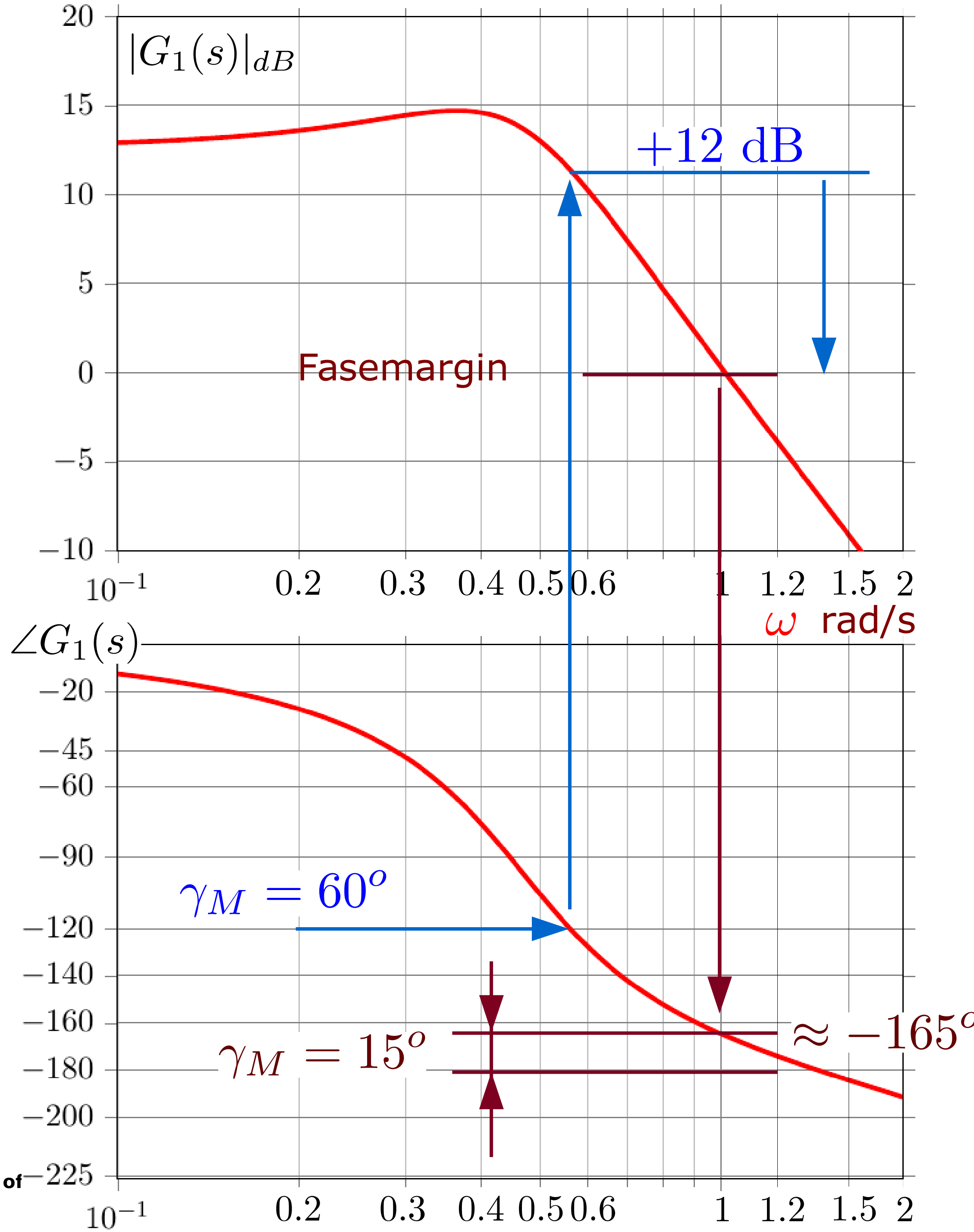
Find K_P så åben sløjfe

fasemargin $\gamma_M = 60^o$

$$\Rightarrow K_P = -12 \text{ dB}$$

$$\Rightarrow K_P = 10^{\frac{-12}{20}}$$

$$\Rightarrow K_P = 0.25$$



Tavle eksempel

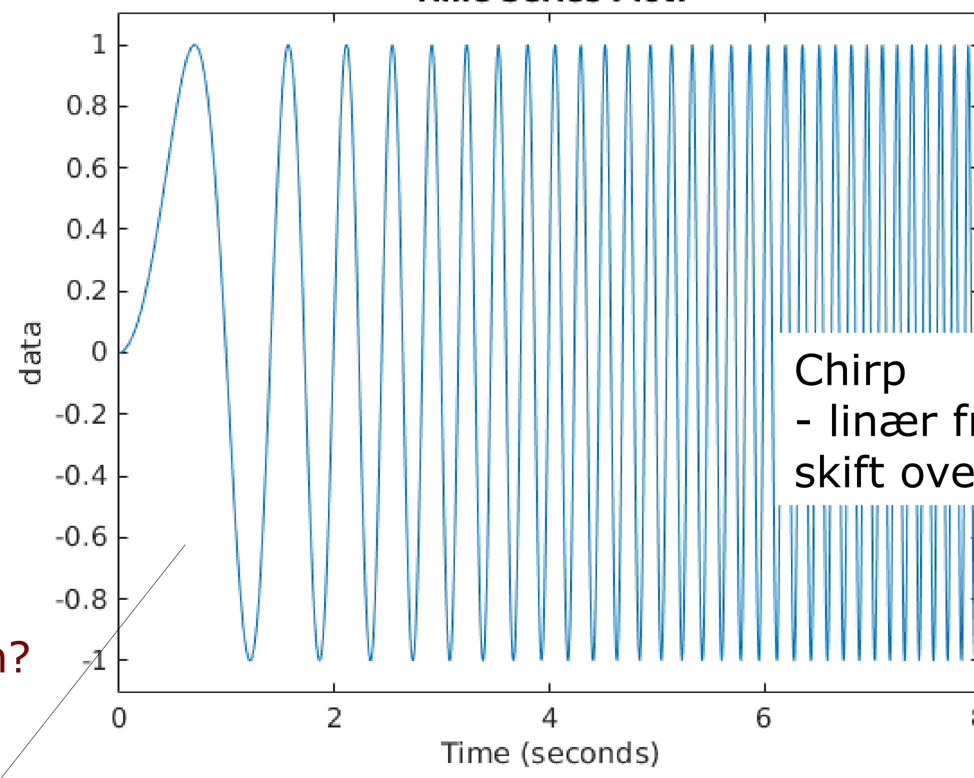
Et mekanisk system – f.eks. drejestyring af en autonom traktor – skal undersøges:
overføringsfunktionen findes af hensyn til stabilitet i en kommende regulator.

$$u(s) \longrightarrow \boxed{G(s)} \longrightarrow \theta(s)$$

drejestyring

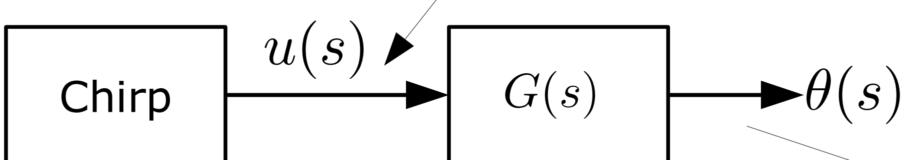
Tavle eksempel

Måleopstilling



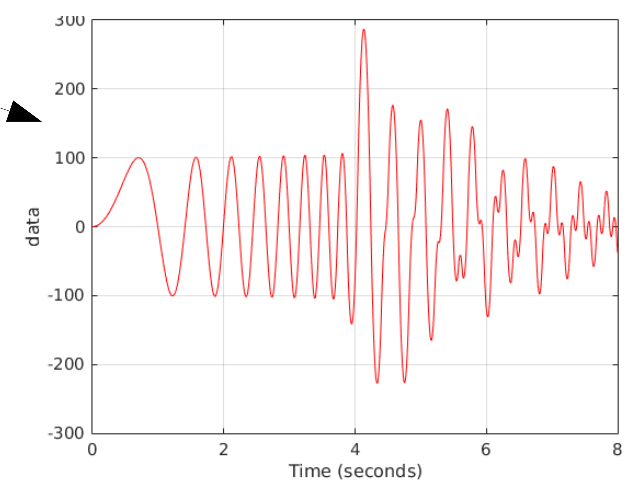
Chirp
- linær frekvens-skift over tid

Overføringsfunktion?

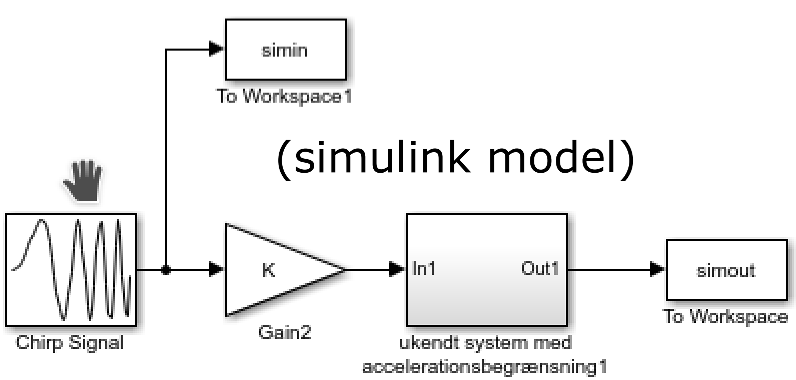


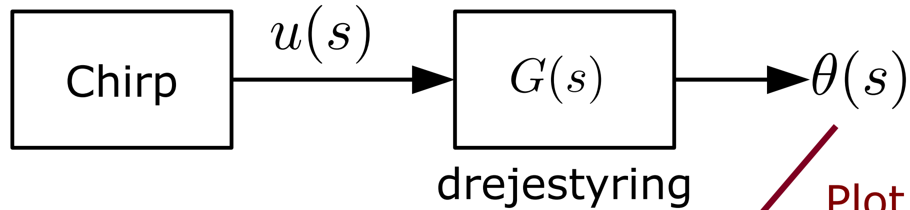
drejestyring

Måling
af output



(simulink model)



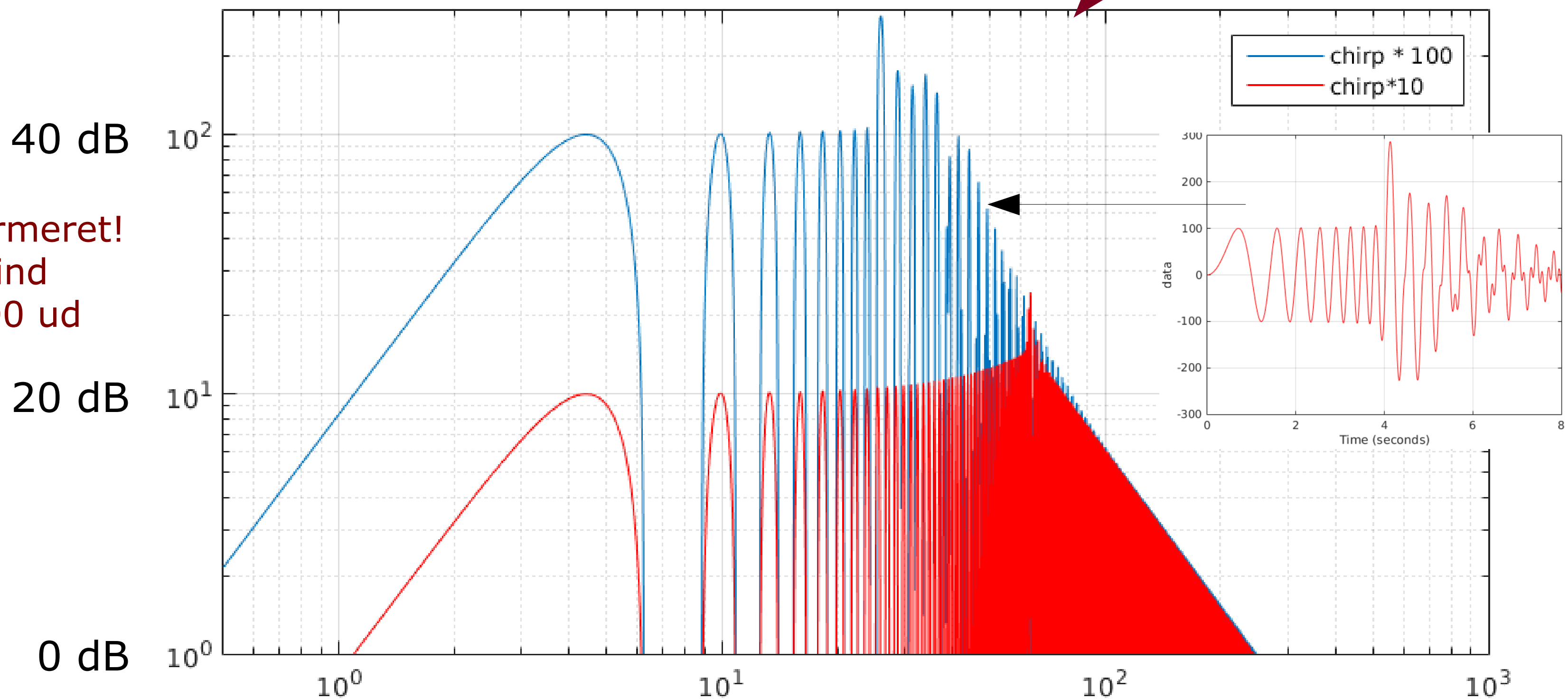


Tavle eksempel

Plot logaritmisk, som bodeplot?

Plot af output

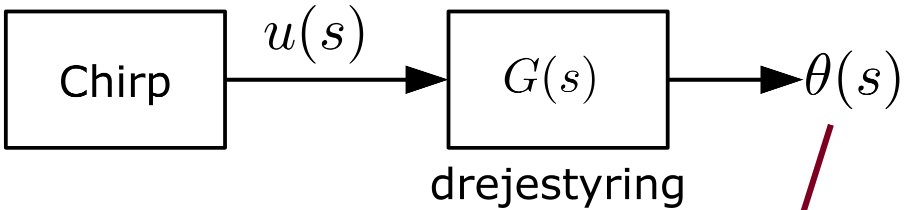
Ikke normeret!
Så 100 ind
giver 100 ud



Tid konverteret til frekvens i rad/sek

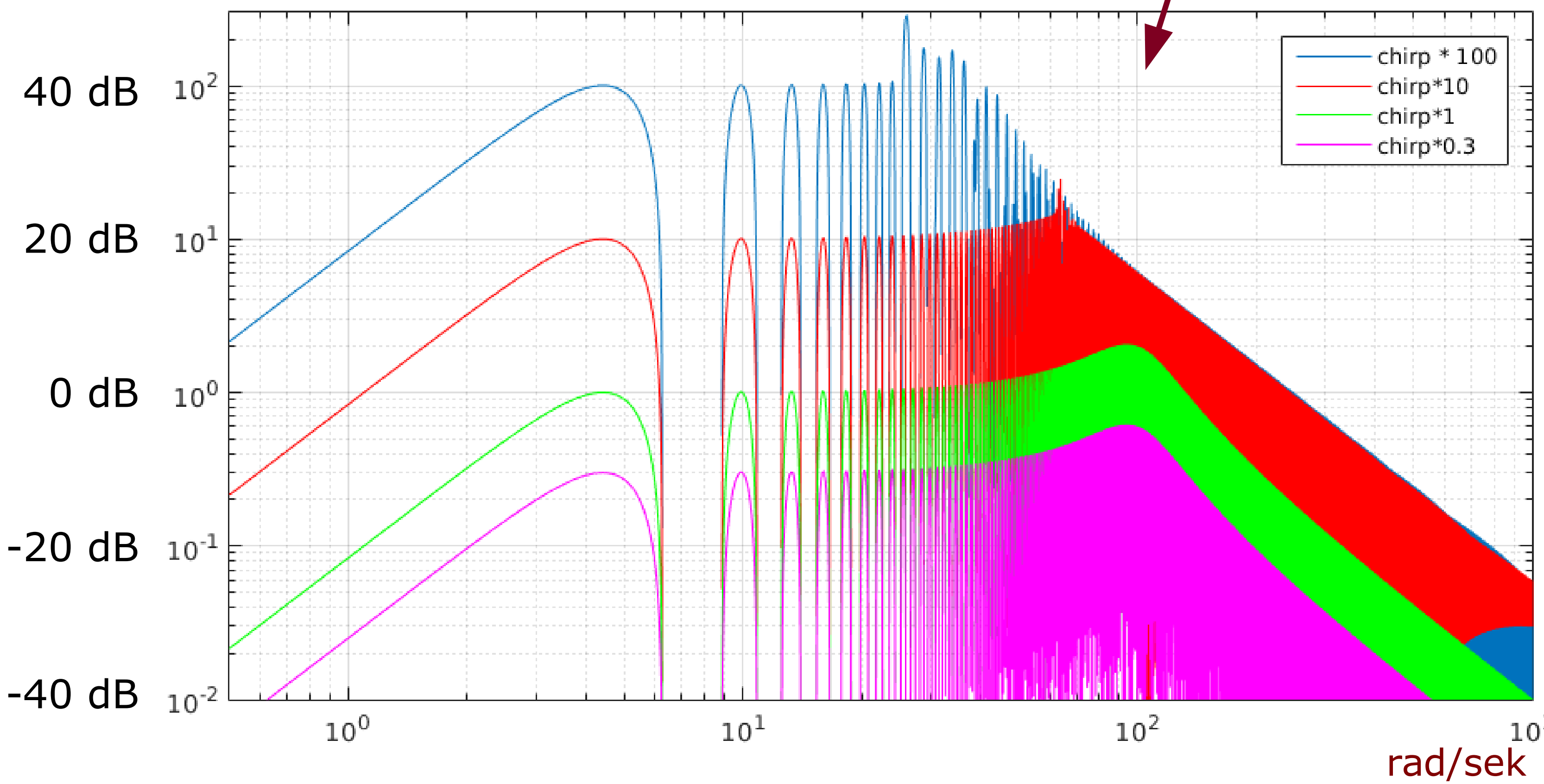
MATLAB
```
loglog(simout.time*2*pi, simout100.data);
axis([0.5,1000,1,300]);
grid on
```

Er systemet lineært?
Hvorfor er der forskel
på chirp*10 og chirp*100?

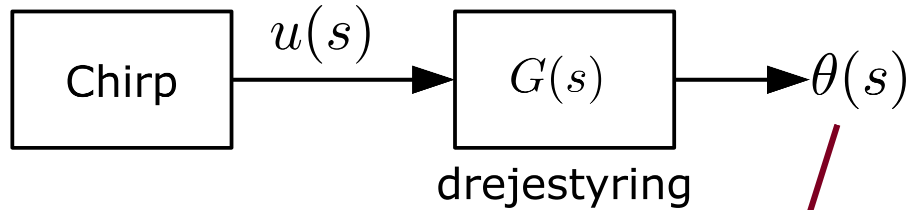


Tavle eksempel

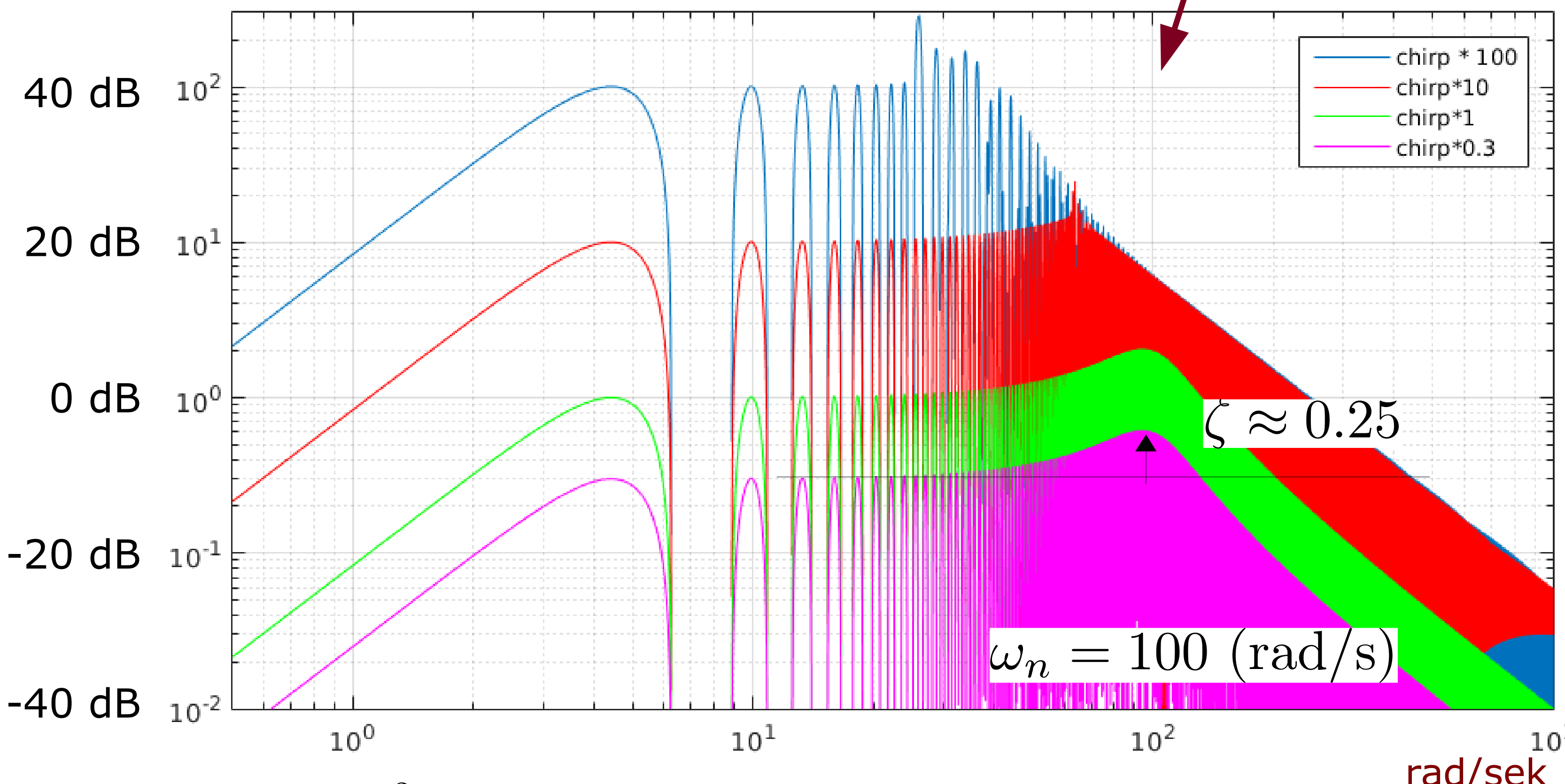
Plot output logaritmisk, som bodeplot?



Linært for amplitude under chirp*10?



Tavle eksempel



$$\frac{\theta(s)}{u(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \frac{\theta(s)}{u(s)} = \frac{10000}{s^2 + 50s + 10000}$$

Reguleringsteknik 1

J. Christian Andersen

Kursusuge 5

Plan

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  - Bodeplot – 1. og 2. orden
  - Stabilitet, Stabilitetsmargin
- Grupperegning
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