

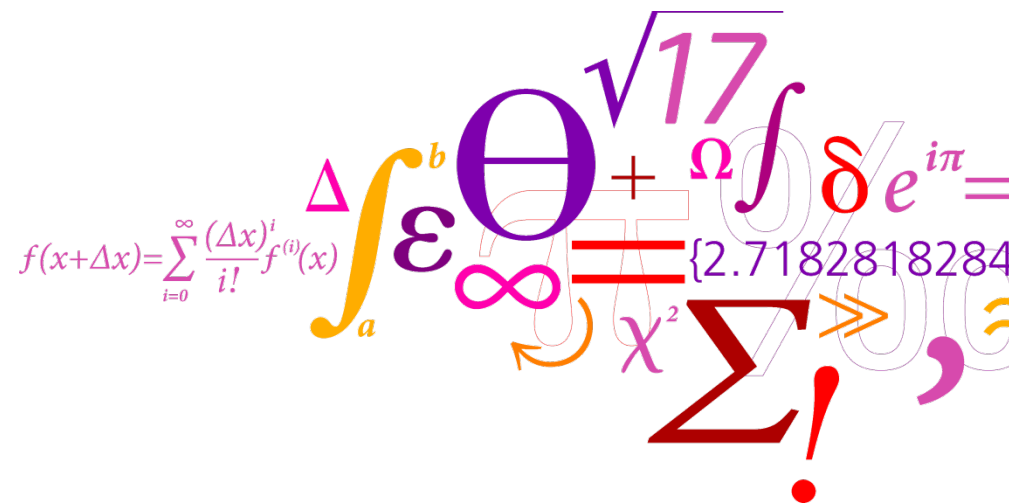
# Reguleringsteknik 1

J. Christian Andersen

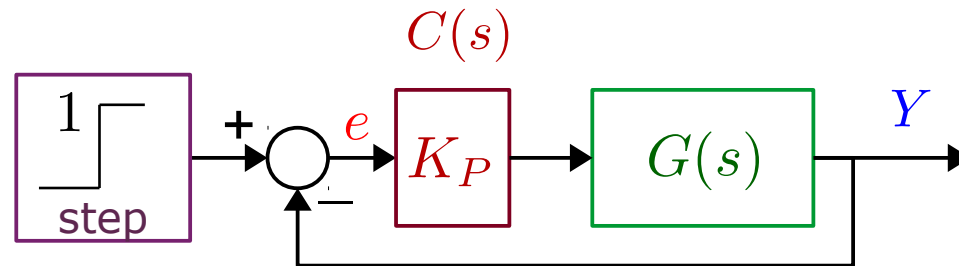
Kursusuge 6

## Plan

- Stabilitetsmargin
  - Begreber (igen)
  - Nyquist plot
- PID design ud fra frekvensanalyse
  - P- regulator
  - PI regulator
  - P-Lead regulator



# Regulator design



Håndtuning gæt:  $K_P = 1$

$$G(s) = \frac{15000}{s^3 + 44s^2 + 784s + 4800}$$

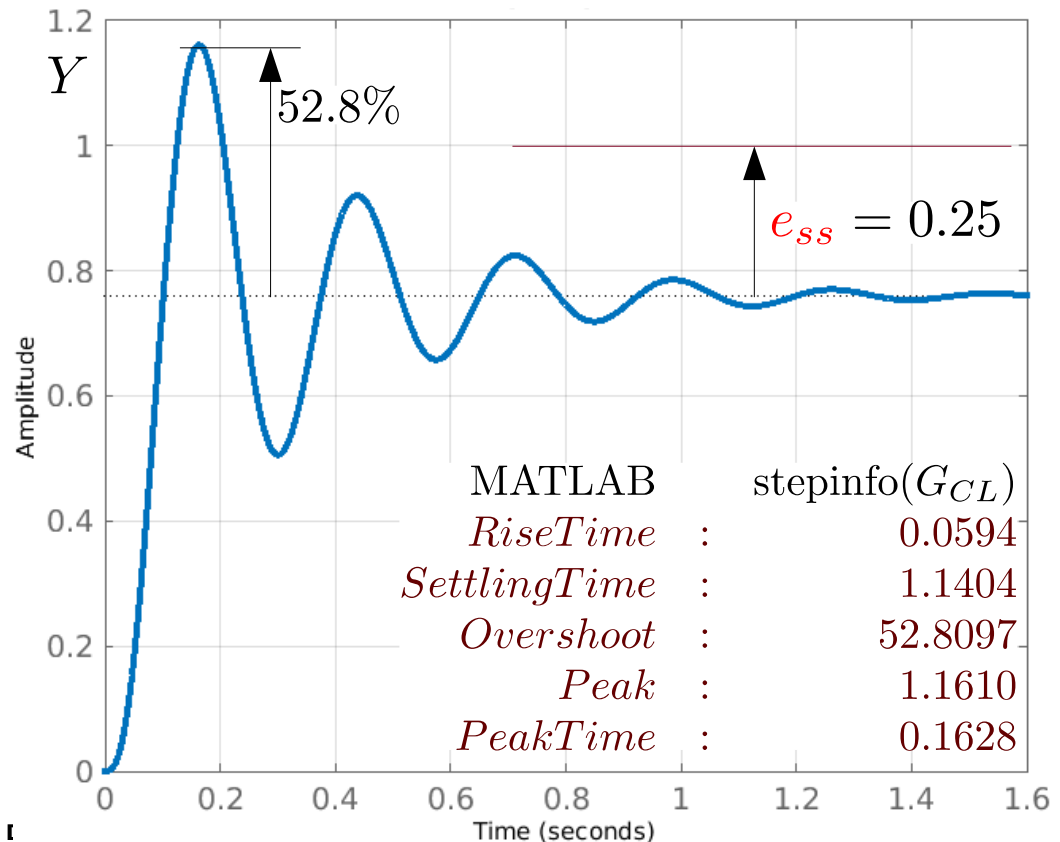
$$G_{\dot{a}} = 1 \cdot G$$

$$e_{ss,r} = \lim_{s \rightarrow 0} \left( \frac{1}{1 + G_{\dot{a}}} \right)$$

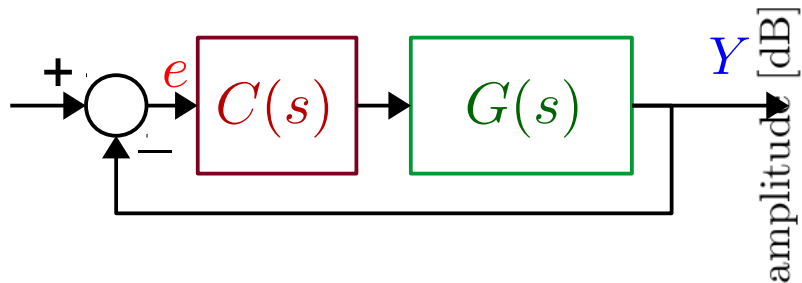
$$e_{ss,r} = \frac{1}{1 + \frac{15000}{4800}}$$

$$e_{ss,r} = 0.25$$

Godt nok?



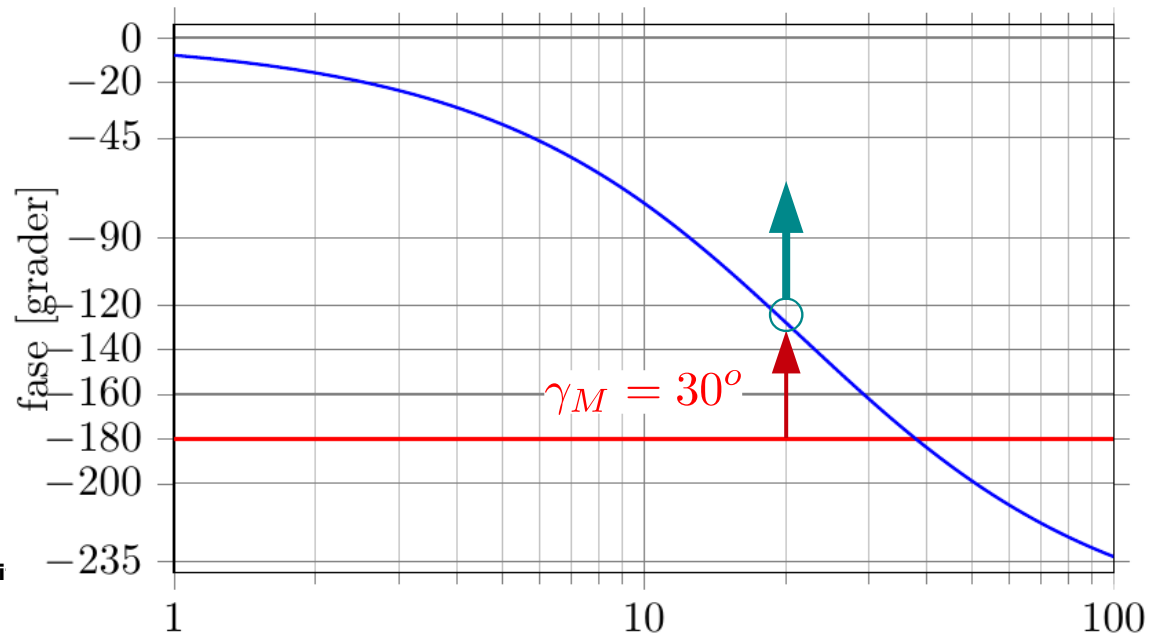
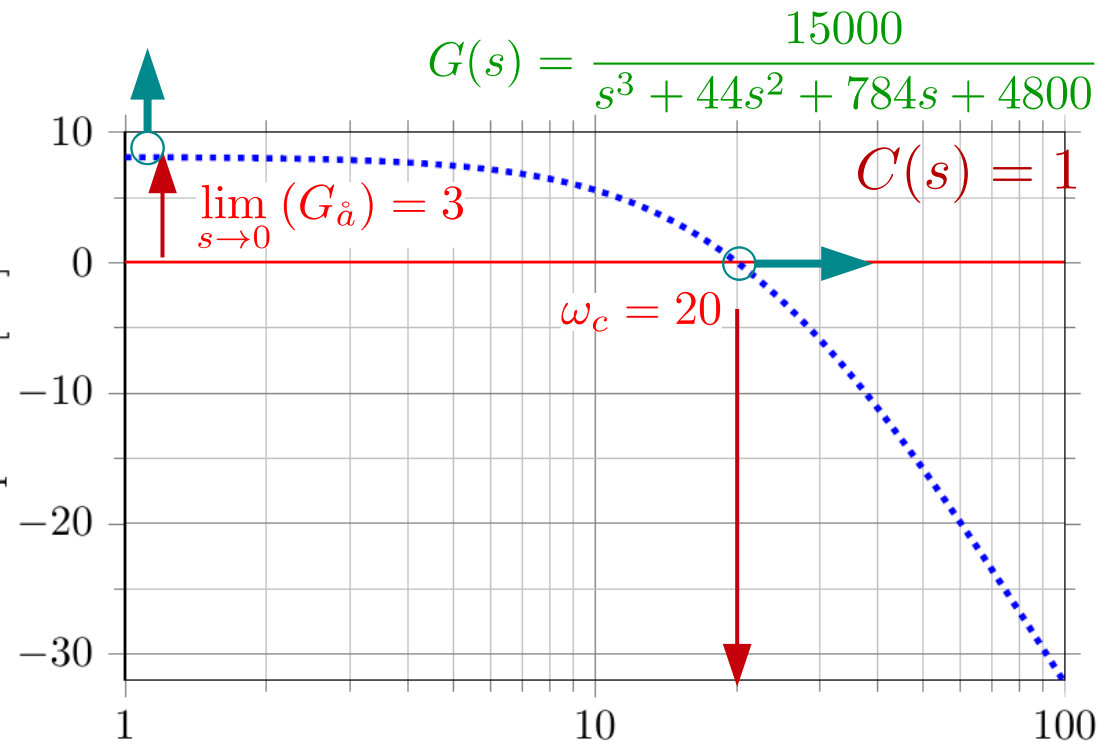
# Målsætning



Målsætning (som oftest):

- Lille statisk fejl:  $\lim_{s \rightarrow 0} (G_{\dot{a}}) \rightarrow \infty$
- Hurtig stigetid (risetime)  
→ høj krydsfrekvens
- Stabilt med lille oversving (overshoot)  
→ stor fasemargin  
→ stor gainmargin

Værktøj: PI-Lead



# PID designværktøj

## I-led

$$G_I = \frac{\tau_i s + 1}{\tau_i s}$$

$$\lim_{s \rightarrow 0} (G_I) \rightarrow \infty$$

$$N_i = \frac{\omega_c}{\omega_i} \quad \tau_i = \frac{1}{\omega_i}$$

$$\tau_i = \frac{N_i}{\omega_c}$$

Gavner statisk fejl,  
Skader fasemargin

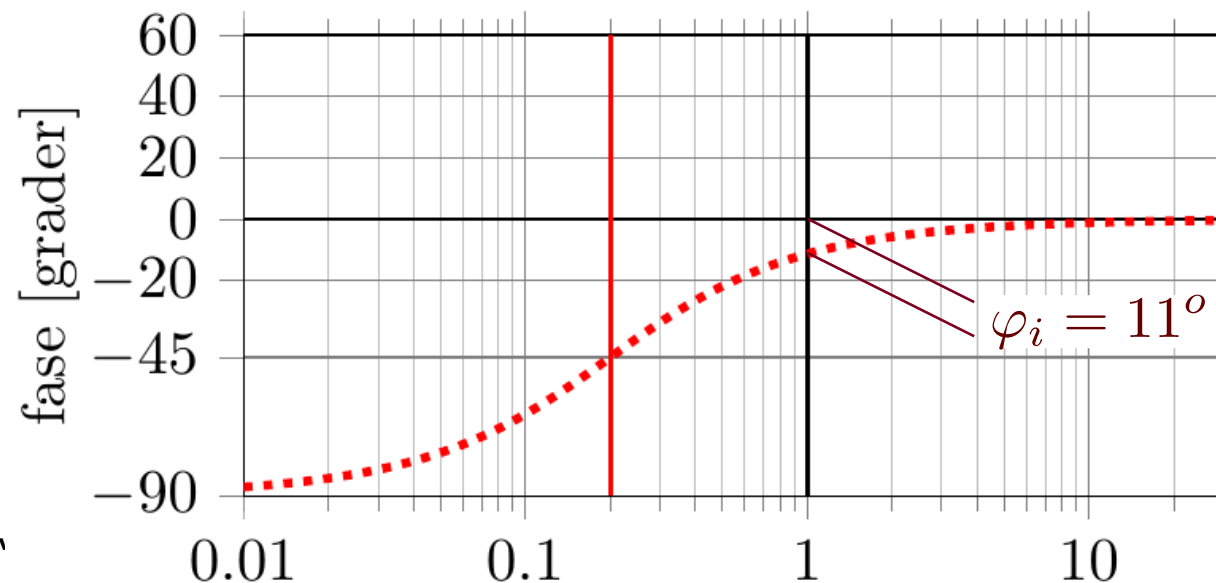
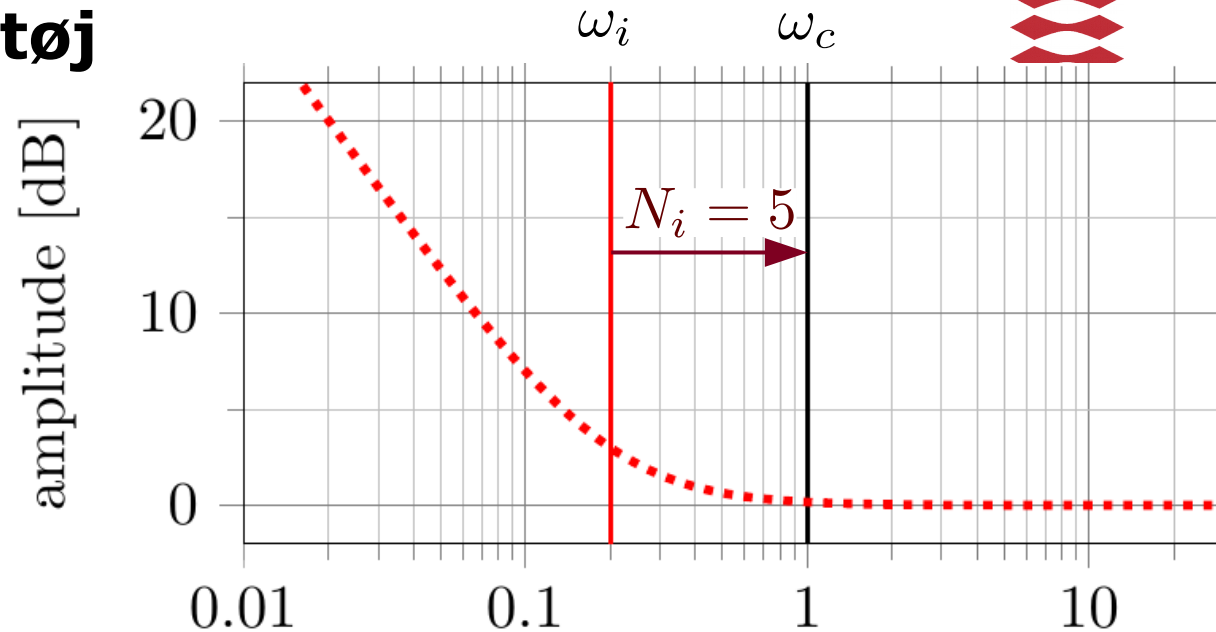
$$\varphi_i = \arctan \frac{-1}{N_i}$$

F.eks.

$$N_i = 3 \Rightarrow \varphi_i = -18.4^\circ$$

$$N_i = 5 \Rightarrow \varphi_i = -11.3^\circ$$

$$N_i = 8 \Rightarrow \varphi_i = -7.1^\circ$$



# PID designværktøjer

## Lead led

$$G_L(s) = \frac{\tau_d s + 1}{\alpha \tau_d s + 1}$$

$$\tau_d = \frac{1}{\sqrt{\alpha} \omega_c}$$

- Forbedre fasemargin med op til  $\varphi_m$
- Øger gain ved  $\omega_c$

$$\varphi_m = \arcsin \frac{1 - \alpha}{1 + \alpha}$$

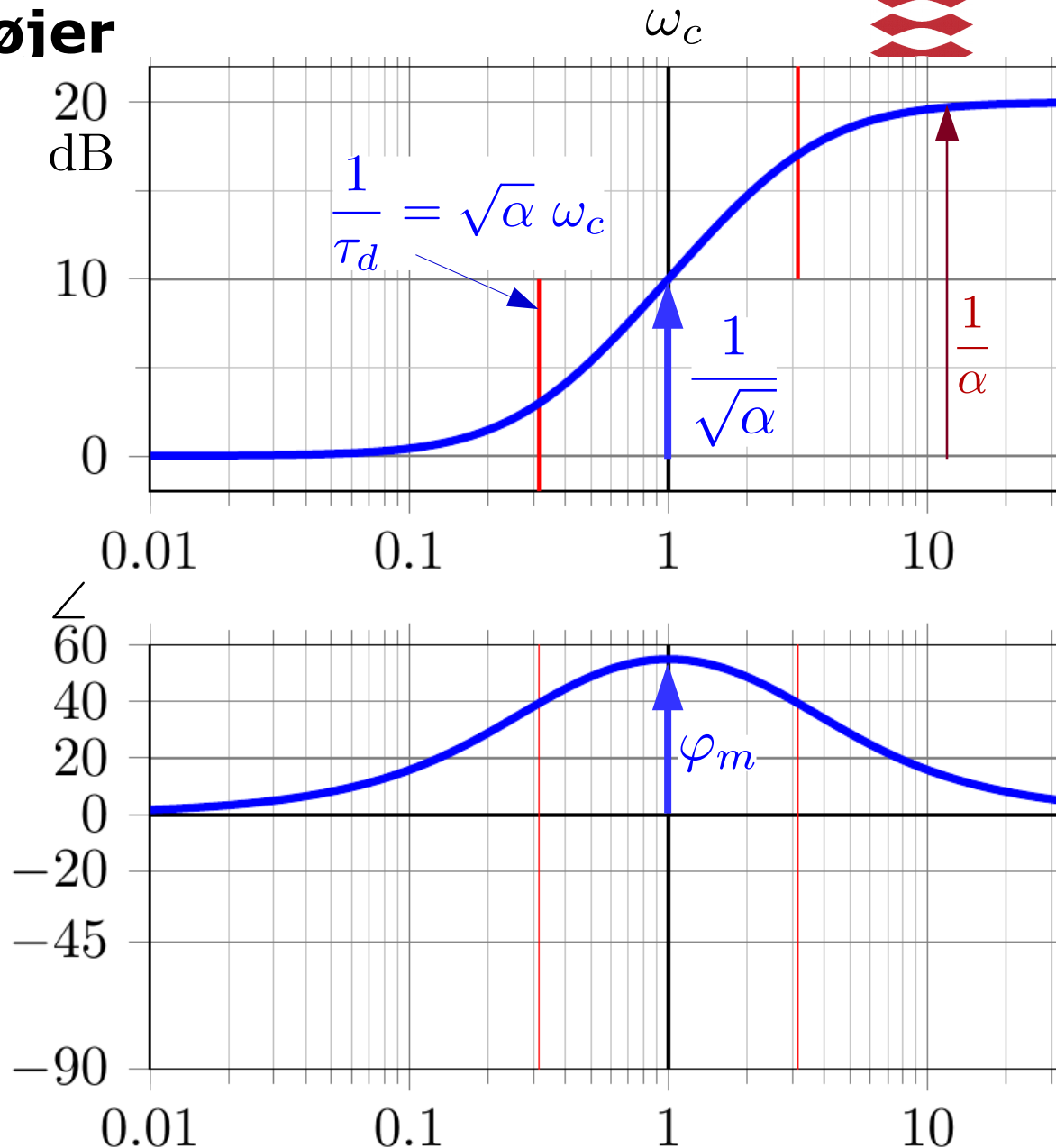
$$|G_L(j\omega_c)| = \frac{1}{\sqrt{\alpha}}$$

F.eks.

$$\alpha = 0.1 \Rightarrow \varphi_m = 54^\circ, 3.2 \text{ (10dB)}$$

$$\alpha = 0.2 \Rightarrow \varphi_m = 42^\circ, 2.2 \text{ (7dB)}$$

$$\alpha = 0.5 \Rightarrow \varphi_m = 19^\circ, 1.4 \text{ (3 dB)}$$



# PID-værktøj

## I-led + Lead led

Samlet forbedring  
af fasemargin:

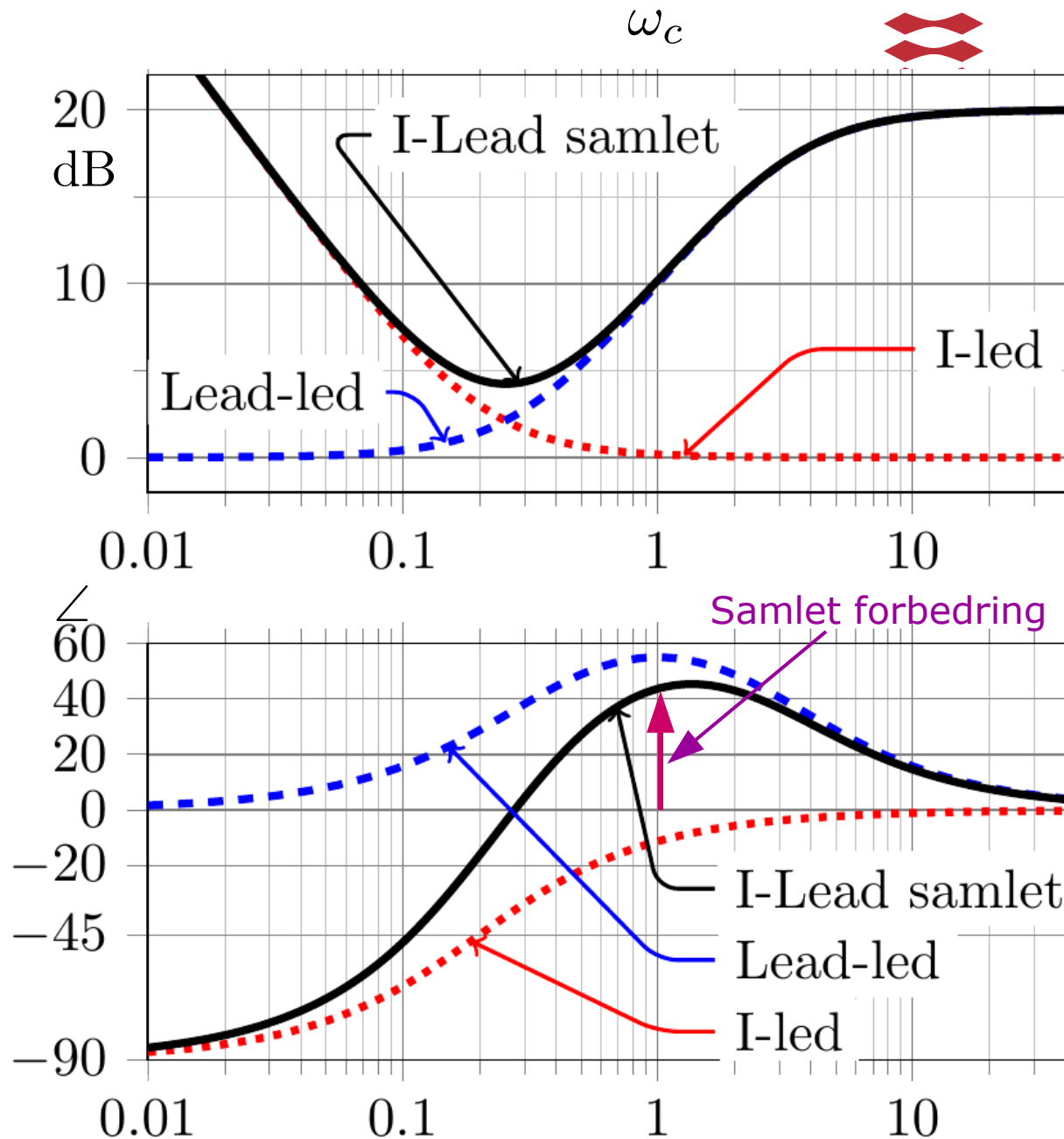
$$\varphi_{m+i} = \varphi_m + \varphi_i$$

positiv      negativ

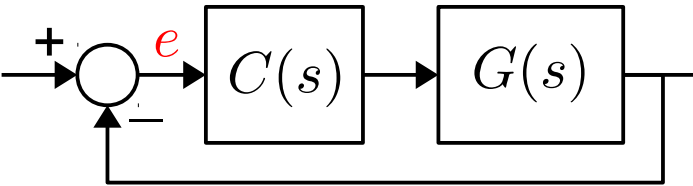
F.eks.:

$$N_i = 5, \quad \alpha = 0.1$$

$$\varphi_{m+i} = 44^\circ$$



# Metode

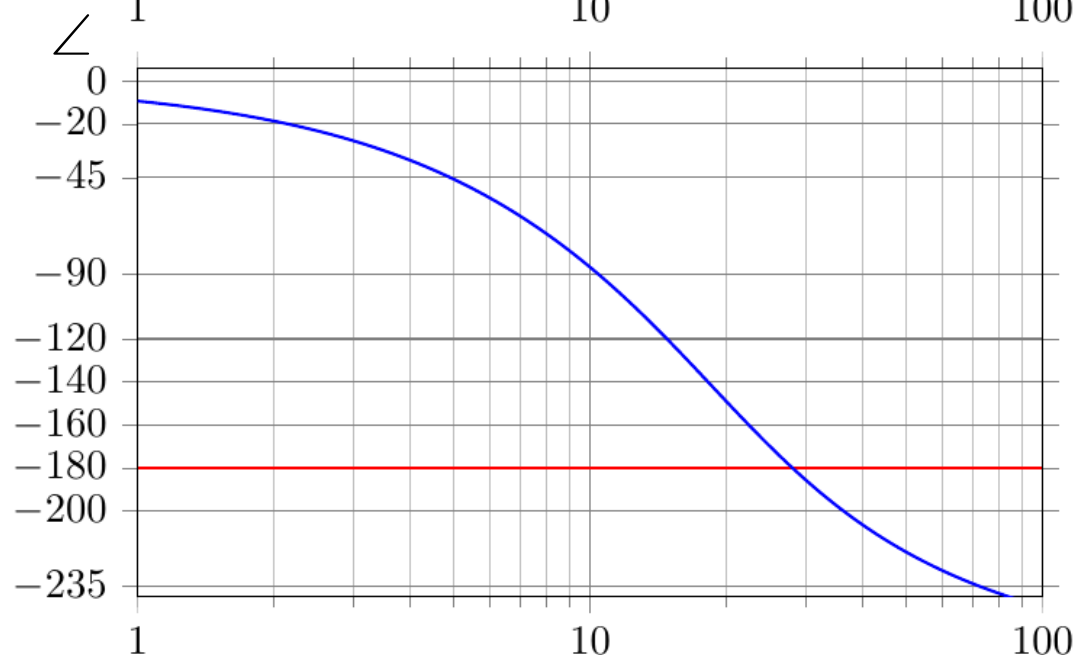
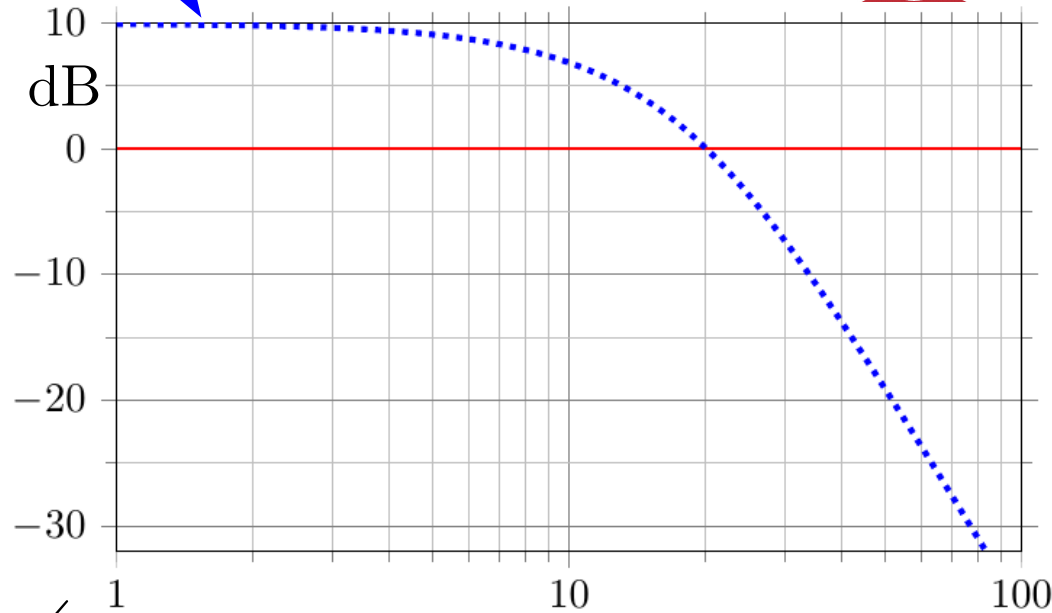


$$G_{\hat{a}}(s) = K_P \frac{\tau_i s + 1}{\tau_i s} \frac{\tau_d s + 1}{\alpha \tau_d s + 1} G(s)$$

## Metode

- 1) Vælg en  $N_i$  og  $\alpha$
- 2) Find forbedring af fasemargin  $\varphi_{m+i}$
- 3) Vælg en ønsket fasemargin  $\gamma_M$
- 4) Find ny krydsfrekvens  $\omega_c$   
ved  $\angle G = -180 + \gamma_M - \varphi_{m+i}$
- 5)  $\omega_m = \omega_c$  Beregn  $\tau_i, \tau_d$
- 6) Find ny  $K_P = \left| \frac{1}{C(j\omega)G(j\omega)} \right|_{\omega=\omega_c}$

$$G(s) = \frac{15000}{s^3 + 44s^2 + 784s + 4800}$$



# PID design

## - find ny krydsfrekvens

Valg:  $\alpha = 0.2$

$$N_i = 3$$

$$\gamma_m = 60^\circ$$

Beregn:

$$\varphi_i = \arctan\left(\frac{-1}{N_i}\right) = -18.4^\circ$$

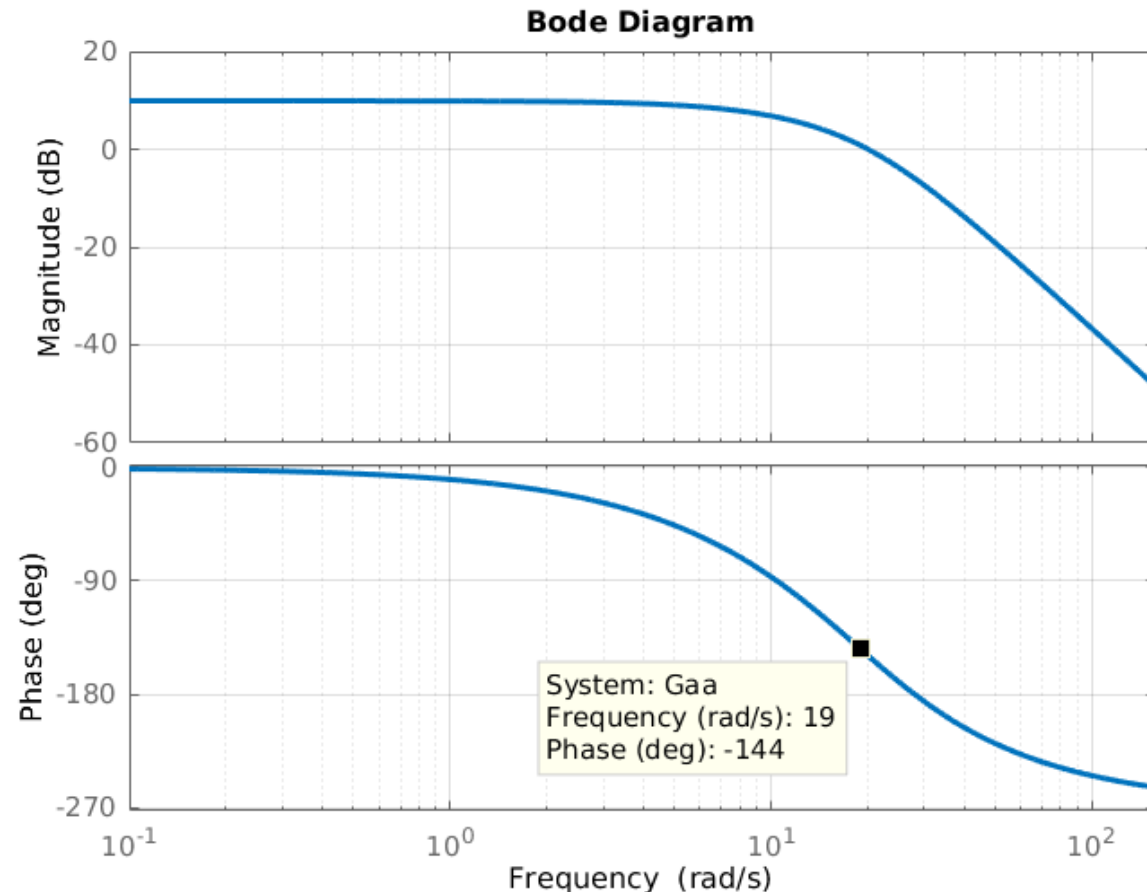
$$\varphi_m = \arcsin \frac{1 - \alpha}{1 + \alpha} = 42^\circ$$

$$-180 + \gamma_M - \varphi_m - \varphi_i = -143.6^\circ$$

$$\omega_c = \omega \mid \angle G(j\omega) = -143.6^\circ$$

Find krydsfrekvens:

$$\omega_c = 19 \text{ rad/sec}$$





# PID design

## - beregn I- og Lead-led

Kendt

$$\alpha = 0.2$$

$$N_i = 3$$

$$\gamma_m = 60^\circ$$

$$\omega_c = 19 \text{ rad/sec}$$

I-led

$$G_i = \frac{\tau_i s + 1}{\tau_i s}$$

$$\tau_i = \frac{N_i}{\omega_c}$$

$$\tau_i = \frac{3}{19} = 0.158$$

$$G_i = \frac{0.158s + 1}{0.158s}$$

Lead-led

$$G_d = \frac{\tau_d s + 1}{\alpha \tau_d s + 1}$$

$$\tau_d = \frac{1}{\omega_c \sqrt{\alpha}}$$

$$\tau_d = \frac{1}{19\sqrt{0.2}} = 0.118$$

$$G_d = \frac{0.118s + 1}{0.0236s + 1}$$

# PID design - find Kp

$$\omega_c = 19 \text{ rad/sec}$$

$$G_i = \frac{0.158s + 1}{0.158s}$$

$$G_d = \frac{0.118s + 1}{0.0236s + 1}$$

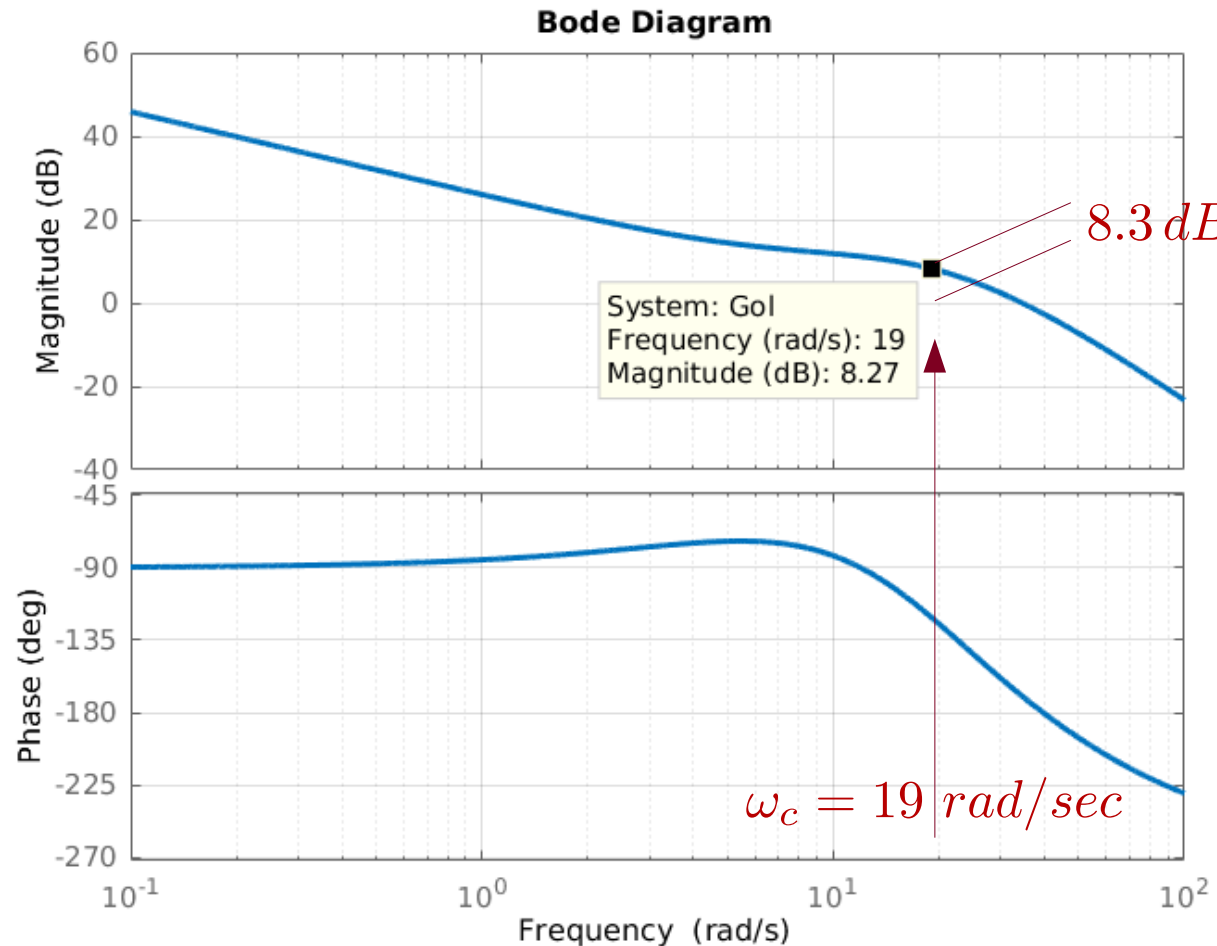
$$K_P = \left| \frac{1}{G_i G_d G} \right|_{s=j\omega_c}$$

MATLAB:

`[M,P] = bode(Gi*Gd*G, 19)`

`Kp = 1/M`

$$K_P = 0.38 \quad (-8.3 \text{ dB})$$



# PID design

## Check open loop bodeplot

$$\alpha = 0.2$$

$$N_i = 3$$

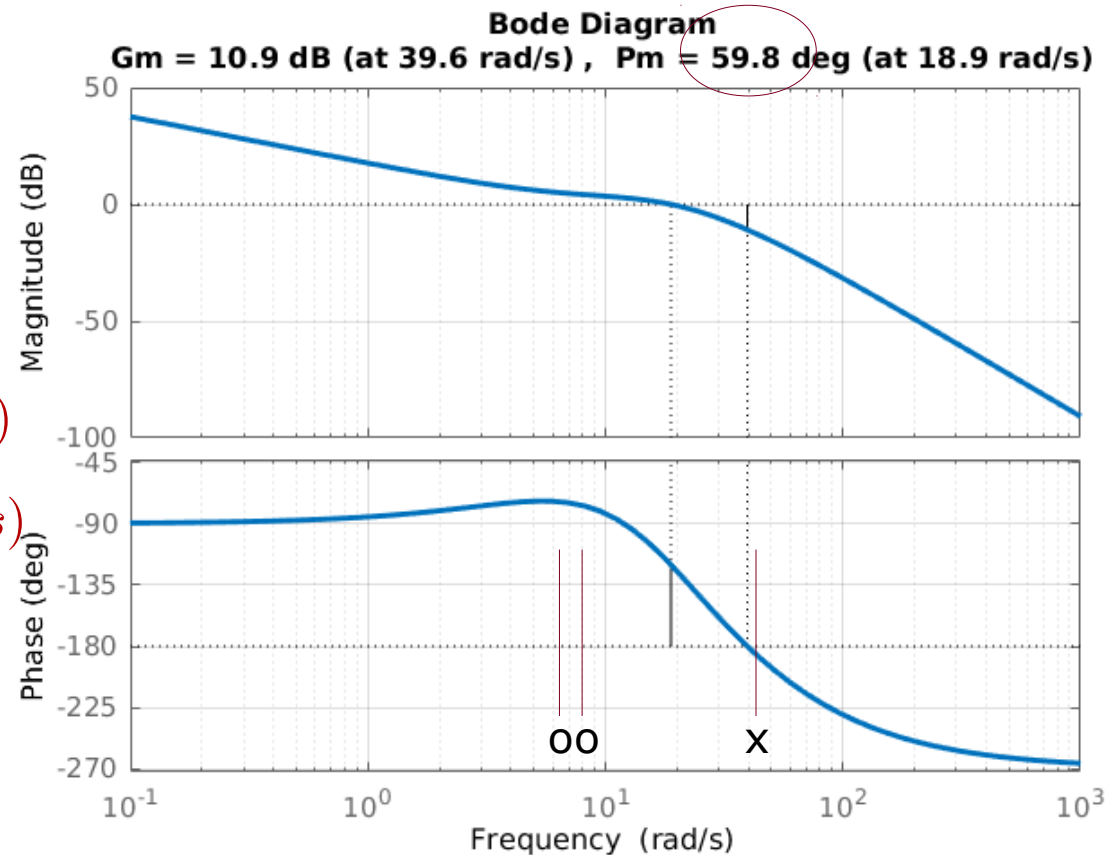
$$\gamma_m = 60^\circ$$

$$\omega_c = 19 \text{ rad/sec}$$

$$\tau_i = 0.158 \quad (\omega_i = 6.3 \text{ rad/s})$$

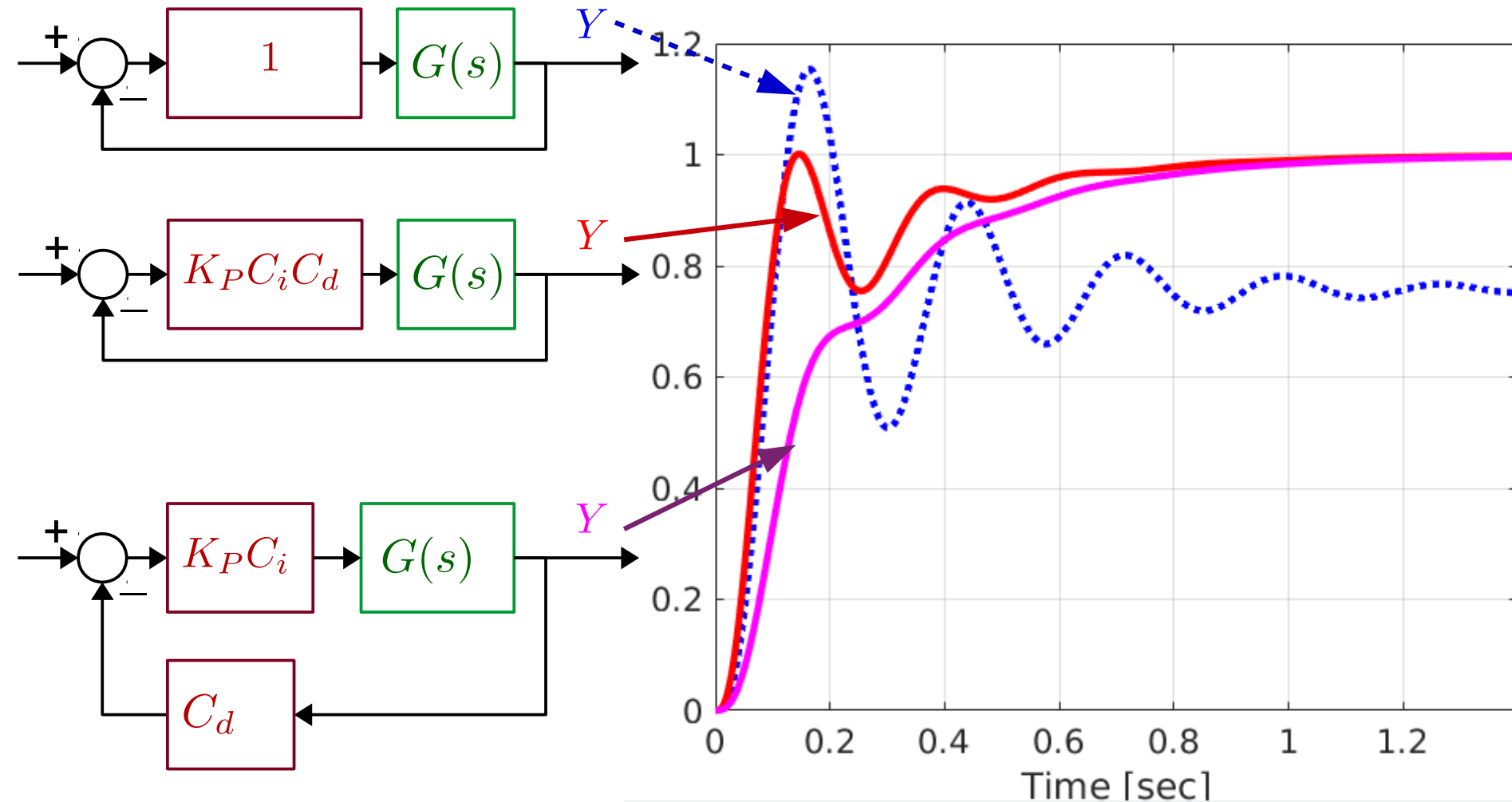
$$\tau_d = 0.118 \quad (\omega_d = 8.4 \text{ rad/s})$$

$$K_P = 0.38$$



# PID-design

## Step respons vurdering



# Grupperegning og Dagens øvelse

- Grupperegning – PI-Lead
- Øvelse 6+7+8 dampmaskineprojekt (til rapport 2)
- Næste gang: bl.a. midtvejstest og midtvejsevaluering