I'm considering how to construct the test shock wave - perhaps I can use the analytical shock definition on the RT element with the edge nodes, etc, then attenuate the modal coefficients of a Vandermonde matrix for the RT element and re-project the nodal values using the RT matrix for interpolation to all RT nodes?

Steps for Constructing an RT-Compatible Shock Profile

Step 1: Define the Analytical Shock Profile

We start with a standard smooth shock transition, modeled as a hyperbolic tangent function:

$$\rho(x) = \rho_1 + \frac{\rho_2 - \rho_1}{2}\left(1 + \tanh\left(\frac{x - x_s}{\epsilon}\right)\right)$$

where:

- x_s is the shock position,
- ϵ controls the smoothness of the transition.

For a shock positioned diagonally across the RT element, define:

$$\rho(r, s) = \rho_1 + \frac{\rho_2 - \rho_1}{2}\left(1 + \tanh\left(\frac{r + s}{\epsilon}\right)\right)$$

which places the shock along the diagonal $r + s = 0$ of the reference triangle.

Step 2: Evaluate at RT Nodes

Using the RT element node set, evaluate the shock profile at each node:

$$\mathbf{u}_{\text{nodes}} = [u(\mathbf{x}_1), u(\mathbf{x}_2), \ldots, u(\mathbf{x}_N)]$$

where $u(r, s)$ represents the shock profile evaluated at each RT node.

Step 3: Compute Modal Coefficients (Vandermonde Projection)

Project the nodal values into the RT modal basis using the inverse Vandermonde matrix:

$$\mathbf{c} = V^{-1}\mathbf{u}_{\text{nodes}}$$

where:

- V is the Vandermonde matrix for the RT basis.
- $\mathbf{c}$ contains the modal coefficients.

Step 4: Apply Modal Filtering (Persson Dissipation)

Apply an artificial dissipation filter to damp high-order modal coefficients:

$$c_k \leftarrow \left(1 - \nu\left(\frac{k}{N}\right)^{2p}\right) c_k$$

where:

- k indexes the modal "degree," not the node number.
- N is the max polynomial degree in the basis.
- ν is a user-defined dissipation strength (tunable),
- p controls sharpness (typically 1 to 4 — sharper if you only want to hit the highest modes).

For RT, you may want to split this by **type**:

- Edge modes may need lighter filtering (they interact with neighbors directly).
- Interior modes could get heavier filtering.

This is flexible, but the basic form holds.

Step 5: Reconstruct Nodal Values

After filtering, reconstruct the nodal values by applying the Vandermonde matrix:

$$\mathbf{u}_{\text{nodes}}^{\text{filtered}} = V\mathbf{c}_{\text{filtered}}$$

Step 6: (Optional) Interpolate to Quadrature Points

If you want to check how this looks at quadrature points (for flux projection, edge flux reconstruction, etc.), you can do:

$$\mathbf{u}_{\text{quad}} = V_{\text{quad}}\mathbf{c}_{\text{filtered}}$$

where V_{quad} is the Vandermonde matrix evaluated at quadrature points.

Summary Table

Step	Description
1	Define shock profile in (r, s)
2	Evaluate at RT nodes
3	Project to RT modal coefficients (Vandermonde inverse)
4	Apply Persson modal filter to coefficients
5	Reconstruct filtered nodal values
6	(Optional) Interpolate to quadrature points