Topic 0: cid, vector, contd, operations, productoftwovectors

- Lecture Vector Analysis Outline Scalars Vectors Negativeofavector FourVectorOperations AdditionofTwoVectors MultiplicationbyaScalar Dot(orScalar)ProductofTwoVectors Cross(orVector)ProductofTwoVectors VectorAlgebra ComponentForm AdditionofTwoVectors MultiplicationbyaScalar Outline (contd)
- DotProductofTwoVectors CrossProductofTwoVectors Tripleproducts Scalartripleproduct Vectortripleproduct Scalars Scalarshavemagnitudeonly
- Examples masstemperaturechargeelectricpotentialworkenergy Vectors Vectorshavebothmagnitudeanddirection(mnorth)andobeythe rulesofvectoralgebra
- Examples displacementvelocityforce momentumtorqueelectricfieldmagneticfieldetcIndiagrams vectorisdenotedbyarrow thelengthofthearrowisproportionalto themagnitudeofthevectorandthearrowheadindicatesitsdirection
- Vectors Negativeofavector Minus(cid)A((cid)A)isavectorwiththesamemagnitudeas(cid)Abutof Four Vector Operations AdditionofTwoVectors Placethetailof(cid)Battheheadof(cid)Athesum(cid)A(cid)Bisthevector fromthetailof(cid)Atotheheadof(cid)B Four Vector Operations AdditionofTwoVectors(contd)
- TriangleLawofVectorAddition Iftwosidesofatriangletakeninthesameorderrepresentthe twovectorsinmagnitudeanddirectionthenthethirdsideinthe oppositeorderrepresentstheresultantoftwovectors
- Four Vector Operations AdditionofTwoVectors(contd)
- ParallelogramLawofVectorAdditionIftwovectorsare representedinmagnitudeanddirectionbythetwosidesofa parallelogramdrawnfromapointthentheirresultantisgivenin magnitudeanddirectionbythediagonaloftheparallelogram passingthroughthatpoint
- R (cid) (cid)(cid)P(cid)Q (cid) (cid) (cid) PQPQcos (cid) (cid) Qsin tan PQcos Additioniscommutative(cid)A(cid)B(cid)B(cid)A (cid) (cid) (cid) (cid) Additionisassociative (cid)A(cid)B (cid)C(cid)A (cid)B(cid)C Four Vector Operations MultiplicationbyaScalar Multiplicationofavectorbyapositivescalaramultiplesthe magnitudebutleavesthedirectionunchanged
- (Ifaisnegative thedirectionisreversed) (cid) (cid) Scalarmultiplicationisdistributive a (cid)A(cid)B a(cid)Aa(cid)B Four Vector Operations Dot(orScalar)ProductofTwoVectors Thedotproductoftwovectorsisdefinedby (cid)A(cid)BABcos () andisascalar

- W (cid)F(cid)S Four Vector Operations Dot(orScalar)ProductofTwoVectors(contd)
- Thedotproductiscommutative(cid)A(cid)B(cid)B(cid)A (cid) (cid) Thedotproductisdistributive(cid)A (cid)B(cid)C (cid)A(cid)B(cid)A(cid)C Four Vector Operations Dot(orScalar)ProductofTwoVectors(contd)
- Foranyvector(cid)E (cid)E(cid)EE (cid) E (cid)E(cid)E Four Vector Operations Dot(orScalar)ProductofTwoVectors(contd)
- Example calculate(cid)C(cid)C Solution (cid) (cid) (cid) (cid) (cid)C(cid)C (cid)A(cid)B (cid)A(cid)B (cid)A(cid)A(cid)A(cid)B(cid)B(cid)A(cid)B(cid)B CABABcos Thisisthelawofcosines
- Four Vector Operations Cross(orVector)ProductofTwoVectors Thecrossproductoftwovectorsisdefinedby (cid)A(cid)BABsin nˆ () isavectorasanexampleoftorque(cid) (cid)r(cid)F
- Herenˆ isaunit vectorpointingperpendiculartotheplaneof(cid)Aand(cid)BThe directionofnˆ isdeterminedbyusingrighthandrule letyour fingerspointinthedirectionofthefirstvectorandcurlaround (viathesmallerangle)towardthesecondthenyourthumb Four Vector Operations Cross(orVector)ProductofTwoVectors(contd)
- Thecrossproductisnotcommutative(cid)A(cid)B(cid)(cid)B(cid)A (cid) (cid) Thecrossproductisdistributive(cid)A (cid)B(cid)C (cid)A(cid)B(cid)A(cid)C Four Vector Operations Cross(orVector)ProductofTwoVectors(contd)
- (cid) (cid) Vector Algebra Component Form Letˆiˆjandkˆ beunitvectorsparalleltoxy andzaxesrespectively Vectors(cid)Aand(cid)Bcanbeexpressedintermsofbasisvectorsˆiˆjandkˆ as (cid)AA ˆiA ˆjA kˆ and(cid)BB ˆiB ˆjB kˆ x y z x y z Vector Algebra Component Form AdditionofTwoVectors (cid)A(cid)B(A B)ˆi(A B)ˆj(A B)kˆ x x y y z z Vector Algebra Component Form MultiplicationbyaScalar a(cid)A(aA)ˆi(aA)ˆj(aA)kˆ x y z Vector Algebra Component Form DotProductofTwoVectors (cid)A(cid)B(A ˆiA ˆjA kˆ)(B ˆiB ˆjB kˆ) x y z x y z A B A B A B x x y y z z Since ˆiˆiˆjˆjkˆkˆ and ˆiˆjˆjkˆ kˆˆi (cid) Foranyvector(cid)A A A A A x y z Vector Algebra Component Form CrossProductofTwoVectors (cid)A(cid)B(A ˆiA ˆjA kˆ)(B ˆiB ˆjB kˆ) x y z x y z (A B A B)ˆi(A B A B)ˆj(A B A B)kˆ y z z y z x x z x y y x (cid) (cid) (cid) ˆi ˆj kˆ (cid) (cid) (cid) (cid) (cid) (cid)A A A (cid) x y z (cid) (cid) (cid) (cid) (cid)B x B y B z(cid) Vector Algebra Component Form CrossProductofTwoVectors(contd)
- Since ˆiˆiˆjˆjkˆkˆ ˆiˆjkˆ ˆjkˆ ˆi kˆˆiˆj ˆjˆikˆ kˆˆjˆi ˆikˆ ˆj Triple productsScalartripleproduct Thescalartripleproductofthreevectors(cid)A(cid)Band(cid)Cisdefinedas (cid)A((cid)B(cid)C) Foraparallelepipedgeneratedby(cid)A(cid)Band(cid)C Triple

productsScalartripleproduct(contd)

- (cid) (cid) (cid)A((cid)B(cid)C)(cid)(cid)B(cid)C(cid)(Acos) (cid) (cid)

Areaofthebaseofparallelepiped Altitudeoftheparallelepiped

Volumeoftheparallelepipedgeneratedby(cid)A(cid)Band(cid)C

Geometrically(cid)A((cid)B(cid)C)isthevolumeoftheparallelepiped

generatedby(cid)A(cid)Band(cid)C (cid)A((cid)B(cid)C)(cid)B((cid)C(cid)A)(cid)C((cid)A(cid)B)

Triple productsScalartripleproduct(contd)

- Incomponentform (cid) (cid) (cid)A A A (cid) (cid) x y z(cid) (cid)A((cid)B(cid)C) (cid) (cid)B B B (cid) (cid) x y z (cid) (cid) (cid) (cid) (cid)C x C y C z(cid)

Thedotandcrosscanbeinterchanged(cid)A((cid)B(cid)C)((cid)A(cid)B)(cid)C Triple productsVectortripleproduct

Thevectortripleproductofthreevectors(cid)A(cid)Band(cid)Cisdefinedas (cid)A((cid)B(cid)C)

Thevectortripleproductcanbesimplifiedbythe BACCAB rule

(cid)A((cid)B(cid)C)(cid)B((cid)A(cid)C)(cid)C((cid)A(cid)B) End of Lecture Thank you