Tribhuvan University

Institute of Science and Technology

2065

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Bachelor Level/First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology — Pass Marks: 32

(MTH.155 – Linear Algebra) — Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. Illustrate by an example that a system of linear equations has either equations has either exactly one solution or infinitely many solutions.

2. When is a linear transformation invertible?

3. Solve the system
$$3x_1 + 4x_2 = 3, \qquad 5x_1 + 6x_2 = 7$$
by using the inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

4. State the numerical importance of determinant calculation by row operation.

5. State Cramer's rule for an invertible n x n matrix A and vector $b \in R^n$ to solve the system $Ax = b$. Is this method efficient from computational point of view?

6. Determine if $\{v_1, v_2 v_3\}$ is basis for R^3, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

7. Determine if $W = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is a Nul(A) for $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

8. Show that 7 is an eigen value of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.

9. If $S = \{u_1, \cdots\cdots\cdots, u_p\}$ is an orthogonal set of nonzero vectors in R^2, show S is linearly independent and hence is a basis for the subspace spanned by S.

10. Let $W = span\{x_1,\ x_2\}$ where $x_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$. Their construct orthogonal basis for W.

Group B **(5 x 4 = 20)**

11. Determine if the given set is linearly dependent:

a) $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$

12. Find the 3 x 3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^0, and finally a translation that adds (-0.5, 2) to each point of a figure.

OR

Describe the Leontief Input-Output model for certain economy and derive formula for $(I\text{-}C)^{-1}$, where symbols have their usual meanings.

13. Find the coordinate vector $[X]_B$ of a x relative to the given basis $B = \{b_1,\ b_2\}$, where
$$b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix},\quad b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix},\qquad x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

14. Let $A = \begin{bmatrix} 4 & -8 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1,\ b_2\}$. Find the B-matrix for the transformation $x \to Ax$ with $P = \{b_1,\ b_2\}$.

15. Let u and v be non-zero vectors in R^3 and the angle between them be ϕ. Then prove that
$$u.v = \|u\|\|v\| \cos \emptyset,$$
where the symbols have their usual meanings.

Group C **(5 x 8 = 40)**

16. Let $T\colon R^n \to R^m$ be a linear transformation. Then T is one-to-one if and only if the equation T(x) = 0 has only the trivial solution, prove the statement.

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and define $T\colon R^2 \to R^3$ by $T(x) = Ax$. Then

a) Find T(u)
b) Find an $x \in R^2$ whose image under T is b.
c) Is there more than one x whose image under T is b?
d) Determine if c is the range of T.

17. Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & -2 & 7 & -1 \end{array}\right] \quad and \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right].$$

18. What do you mean by change of basis in R^n? Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and consider the bases for R^2 given by $B = \{b_1,\ b_2\}$ and $C = \{c_1,\ c_2\}$.
    a) Find the change of coordinate matrix from C to B.
    b) Find the change of coordinate matrix from B to C.

OR

Define vector spaces, subspaces, basis of vector space with suitable examples. What do you mean by linearly independent set and linearly dependent set of vectors?

19. Diagonalize the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$, if possible.

20. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3). What do you mean by least squares lines?

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Attempt all questions:

Group A **(10 x 2 = 20)**

1. When is system of linear equation consistent or inconsistent?

2. Write numerical importance of partitioning matrices.

3. How do you distinguish singular and non-singular matrices?

4. If A and B are n x n matrices, then verify with an example that det(AB) = det(A)det(B).

5. Calculate the area of the parallelogram determined by the columns of
$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}.$$

6. Determine if $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is Nul(A), where, $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

7. Determine if $\{v_1, v_2, v_3\}$ is a basis for λ^3, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

8. Find the characteristic polynomial for the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.

9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.

10. Let $\{u_1, \cdots\cdots\cdots, u_p\}$ be an orthogonal basis for a subspace W of R^n. Then prove that for each $y \epsilon W$, the weights in $y = c_1u_1 + \cdots\cdots\cdots + c_pu_p$ are given by
$$c_j = \frac{y.u_j}{u_j.u_j} \qquad (j = 1, \cdots\cdots\cdots, p)$$

<u>Group B</u> **(5 x 4 = 20)**

11. Prove that any set $\{v_1, \cdots\cdots\cdots, v_p\}$ in R^n is linearly dependent if $p > n$.

12. Consider the Leontief input – output model equation $x = cx + d$, where the consumption matrix is

$$C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}.$$

Suppose the final demand is 50 units of manufacturing, 30 units of agriculture, 20 units for services. Find the production level x that will satisfy the demand.

13. What do you mean by basis of a vector space? Find the basis for the row space of

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

OR

State and prove the unique representation theorem for coordinate systems.

14. What do you mean by eigen values, eigen vectors and characteristic polynomial of a matrix? Explain with suitable examples.

15. Define the Gram-Schmidt process. Let W=span$\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Then construct an orthogonal basis $\{v_1, v_2\}$ for w.

<u>Group C</u> **(5 x 8 = 40)**

16. Given the matrix

$$\begin{bmatrix} 0 & 3 & -6 & 6 & -5 \\ 3 & -7 & 8 & -5 & 9 \\ 3 & -9 & 12 & -9 & 15 \end{bmatrix},$$

discuss the for word phase and backward phase of the row reduction algorithm.

17. Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists, by using elementary row reduce the augmented matrix.

18. What do you mean by change of basis in R^n? Let $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and consider the bases for R^2 given by B={b_1, b_2} and C={c_1, c_2}. Find the change of coordinates matrix from B to C.

19. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, if possible

OR

Find the eigen value of $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$, and find a basis for each eigen space.

20. Find a least-square solution for Ax = b with $A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$. What do you mean by least squares problems?

OR

Define a least-squares solution of Ax = b, prove that the set of least squares solutions of Ax = b coincides with the non-empty set of solutions of the normal equations $A^TAx = A^Tb$.

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Attempt all questions:

Group A (10x2=20)

1. Illustrate by an example that a system of linear equations has either no solution or exactly one solution.
2. Define singular and nonsingular matrices.
3. Using the Invertible matrix Theorem or otherwise, show that
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$
is invertible.
4. What is numerical drawback of the direct calculation of the determinants?
5. Verify with an example that det (AB) = det (A) det (B) for any n x n matrices A and B.
6. Find a matrix A such that $w = col(A)$.
$$w = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in R \right\}.$$
7. Define subspace of a vector with an example.
8. Are the vectors;
$u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ eigen vectors of $\begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$?
9. Find the distance between vectors $u = (7, 1)$ and $v = (3, 2)$. Define the distance between two vectors in R^n.
10. Let w = span $\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.
Then construct orthogonal basis for w.

Group B (5x4=20)

11. If a set $s = \{v_1, v_2, \ldots \ldots, v_p\}$ in R^n contains the zero vector, then prove that the set is linearly dependent. Determine if the set
$$\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$
is linearly dependent.
12. Given the Leontief input-output model x = Cx + d, where the symbols have their usual meanings, consider any economy whose consumption matrix is given by
$$C \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$$

Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level x that will satisfy this demand.

13. Define rank of a matrix and state Rank Theorem. If A is a 7 x 9 matrix with a two-dimensional null space, find the rank of A.

14. Determine the eigen values and eigen vectors of $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ in complex numbers.

OR

Let $A = \begin{bmatrix} 4 & -9 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1, b_2\}$.

Find the B-matrix for the transformation $x \to x$ with $P = [b_1, b_2]$.

15. Let u and v be nonzero vectors in R^2 and the angle between them be θ then prove that $u.v = \|u\| \, \|v\| \cos\theta$,
where the symbols have their usual meanings.

16. Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set.
$3x_1 + 5x_2 - 4x_3 = 0,\ -3x_1 - 2x_2 + 4x_3 = 0,\ 6x_1 + x_2 - 8x_3 = 0.$

17. An n x n matrix A is invertible if and only if A is row equivalent to I_n , and in this case, any sequence of elementary row operations that reduces A to I_n also transform $I_{n\,x\,m}$ into A^{-1}.
Use this statement to find the inverse of the matrix $= \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if exists.

18. What do you mean by basis change? Consider two bases $B = \{b_1, b_2\}$ and $c = \{c_1, c_2\}$ for a vector space V, such that $b_1 = 4c_1 + c_2$ and $b_2 = 6c_1 + c_2$. Suppose $[x]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ i.e., $x = 3b_1 + b_2$. Find $[x]_C$.

OR

Define basis of a subspace of a vector space.

Let $v_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$, where $v_3 = 5v_1 + 3v_2$, and let $H = \text{span}\,\{v_1, v_2, v_3\}$.

Show that $\text{span}\,\{v_1, v_2, v_3\} = \text{span}\,\{v_1, v_2\}$ and find a basis for the subspace H.

19. Diagonalize the matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible.

20. What do you mean by least-squares lines? Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

OR

Find the least-squares solution of Ax = b for

$= \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}.$

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Prepared By ASCOL CSIT 2070

Year 2068

Attempt all question:

Group A (10 × 2 = 20)

1. Write down the conditions for consistent of non-homogenous system of linear equations.
2. What is meant by independent of vectors?
3. What is normal form of a matrix?
4. Define nonsingular linear transformation with suitable example.
5. Consider the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$ as a linear mapping. Write the corresponding co-ordinate equations.
6. State the numerical importance of determinant calculation by row operation.
7. Show that $\{(1,1), (-1,0)\}$ form a bias for R^2.
8. Let $T: R^2 \to R^2$ be a linear transformation defined by
   $$T(x, y) = (x + y, y).$$ Find Ker T.
9. If λ is an eigen values of matrix A, find the eigen values of A^{-1}.
10. Let $u = (1,2,-1,3)$ and $v = (3,0,2,-2)$. Compute the inner product $(u, u + v)$.

Group B (5 × 4 = 20)

11. Determine whether the following vectors in R^3 are linearly dependent:
    a. $(1,0,1), (1,1,0), (-1,0,-1)$,
    b. $(2,1,1), (3,-2,2), (-1,2,-1)$.
12. Investigate and interpret geometrically the transformation of the unit square whose vertices are $O(0,0,1), A(1,0,1), B(0,1,1)$ *and* $C(1,1,1)$ effected by the 3×3 matrix:

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

OR

Is the set of vectors $\{(1,0,1),(0,1,0),(-1,0,1)\}$ orthrogonal? Obtain the corresponding orthonormal set in R^3.

13. In the vector space R^2, express the given vector (1,2) as a linear combination of the vectors $(1,-1)$ *and* $(0,1)$
14. Find the matrix representation of the linear transformation $T: R^2 \to R^2$ defined by $T(x,y) = (x, x+2y)$ relative to the basis (1,0) and (1,1)
15. Let u and v be nonzero vector in R^n and the angle between them be θ. Then prove that

$$u.v = ||u||\,||v||\,cos\theta$$

Where the symbol have their usual meanings.

Group C **(5×8=40)**

16. Test for consistency and solve:

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

17. Let U and V be vector spaces over a field and assume that dim U=dim V. If $T: U \to V$ is a linear transformation, then prove that the following are equivalent;
    i. T is invertable
    ii. T is one-one and onto, and
    iii. T is non-singular

OR

Verify that the set of matrices of the form $\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$ is a subspace of the vector space of 3×3 matrices.

18. Verify Cayley-Hamilton Theorem for matrix:

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}$$

19. Diagonalize the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

OR

Compute the multiplication of partitioned matrices for

$$A = \begin{bmatrix} 1 & 2 & 4 & 6 & 7 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 3 & 6 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 4 & 5 \\ 2 & 2 & 0 \\ 0 & 7 & 6 \end{bmatrix}$$

20. Find the equation $y = \beta_0 + \beta_1 x$ for the least squares line that best fits the data points (2, 0), (3, 4), (4, 10), (5, 16).

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(Linear Algebra)

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Attempt all questions:

Group A (10x2=20)

1. What do you mean by linearly independent set and linearly dependent set of vectors?

2. Verify that $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigen ~~values~~ vector of $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$.

3. What do you mean by consistent equations? Give suitable examples.

4. What do you mean by change of basis in $\mathbb{R}^n$?

5. Find the dimension of the vector space spanned by (1, 1, 0) and (o, 1, 0).

6. Solve the system $3x_1 + 4x_2 = 3$, $5x_1 + 6x_2 = 7$ by using the inverse of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

7. When is a linear transformation invertable?

8. Find the rank of AB where

$$A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \text{ and } B = [\ 1\ 4\ 5\ ]$$

9. Define Kernel and image of linear transformation.

10. What is meant by Discrete dynamical system? Give suitable example.

Group B (5x4=20)

11. Let $T : R^3 \to R^3$ be the linear transformation defined by $T(x, y, z) = (x, y, x - 2y)$. Find a basis and dimension of (a) Ker T (b) Im T.

12. Show that the following sets of vectors are linearly independent: (1,1,2), (3,1,2), (0,1,4).

13. Find the matrix representation of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + 2y)$ relative to the standard basis.

14. Is the set of vectors $\{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$ orthogonal? Obtain the corresponding orthonormal set in $\mathbb{R}^3$.

15. Let the four vertices O(0, 0), A(1, 0), B(0, 1) and C(1, 1) of a unit square be represented by 2×4 matrix : $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. Inverstigate and interpret geometrically the effect of pre-multiplication f this matrix by the 2×2 matrix: $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.

OR

State and prove orthogonality property for any two non-zero vectors in $\mathbf{R}^n$.

Group C

(5x8=40)

16. Find a matrix A whose inverse in

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$

17. Test the consistency and slove:

$x + y + z = 4$

$x + 2y + 2z = 2$

$2x + 2y + z = 5$

OR

Verify Cayley Hamilton theorem for a matrix $A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}$.

18. The set of matrices of the form

$$\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

is a subspace of the vector space of 3×3 matrices. Verify it.

19. Let V and W be vector spaces over a field F of real numbers. Let dim V = n and dim W = m. Let $\{e_1, e_2, \ldots, e_n\}$ be a basis of V and $\{f_1, f_2, \ldots, f_m\}$ be a basis of W. Then, prove that each linear transformation $T: V \to W$ can be represented by an $m \times n$ matrix A with elements from F such that

$$Y = AX$$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$

are column matrices of coordinates of $v \in V$ relative to its basis and coordinates of $w \in W$ relative to its basis, respectively.

OR

Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc:cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hdashline 0 & 4 & -2 & 7 & -1 \end{array}\right] \quad \text{and} \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hdashline -1 & 3 \\ 5 & 2 \end{array}\right].$$

20. Find the equation $y = a_0 + a_1 x$ for the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

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Attempt all questions:

Group A (10×2=20)

1. Why the system $x_1 - 3x_2 = 4$; $-3x_1 + 9x_2 = 8$ is inconsistent? Give the graphical representation?

2. Define linear combination of vectors. If v_1, v_2, v_3 are vectors, write the linear combination of $3v_1 - 5v_2 + 7v_3$ as a matrix times a vector.

3. Is $\begin{pmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 2 & 3 & 4 \end{pmatrix}$ invertible matrix?

4. Define invertible linear transformation.

5. Let S be the parallelogram determined by the rectors $b_1 = (1, 3)$ and $b_2 = (5, 1)$ and let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$. Compute the area of the image S under the mapping $x \to Ax$.

6. Define vector space.

7. Show that the entries in the vector $x = (1, 6)$ are the co-ordinates of x relative to the standard basis (e_1, e_2).

8. Is $\lambda = -2$ an eigen value of $\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$?

9. Find the inner product of (1, 2, 3) and (2, 3, 4).

10. Compute the norm between the vectors $4 = (7, 1)$ and $v = (3, 2)$.

Group B
(5×4=20)

11. A linear transformation T: $\mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}.$$

Find the image under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

12. If $A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ and $x = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ compute $(Ax)^T$, $x^T A^T$ and xx^T. Can you compute $x^T A^T$?

13. If $b_1 = (2, 1)$, $b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $B = \{b_1, b_2\}$, find the co-ordinate vector $[x]_B$ of x relative to B.

14. Find the eigen values of $A = \begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$.

15. Show that $\{v_1, v_2, v_3\}$ is an orthogonal set, where $v_1 = (3, 1, 1)$, $v_2 = (-1, 2, 1)$, $v_3 = (-1/2, -2, 7/2)$.

Group C
(5×8=40)

16. Let $a_1 = (1, 2, -5)$, $a_2 = (2, 5, 6)$ and $b = (7, 4, -3)$. Determine whether b can be generated as a linear combination of a_1 and a_2. That is, determine whether x_1 and x_2 exist such that

$$x_1 a_1 + x_2 a_2 = b$$

has solution, find it.

OR

Determine if the following system is consistent

$$x_2 - 4x_3 = 8$$

$$2x_1 - 3x_2 + 2x_3 = 1$$

$$5x_1 - 8x_2 + 7x_3 = 1.$$

17. Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc|cc} 1 & -3 & 2 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & -2 & 7 & -1 \end{array}\right] \text{ and } B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right].$$

18. Let $b_1 = (1, 0, 3)$, $b_2 = (2, 1, 8)$, $b_3 = (1, -1, 2)$ and $x = (3, -5, 4)$. Does $B = \{b_1, b_2, b_3\}$ form a basis? Find $[x]_B$, for x.

19. Diagonalize the matrix , if possible

$$A = \begin{bmatrix} -1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

20. When two vectors u and v are orthogonal? If u and v are vectors, prove that $[\text{dist}\,(u, -v)]^2 = [\text{dist}\,(u, v)]^2$ iff $u \,.\, v = 0$.

OR

Find a least square solution of $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}.$$

Tribhuvan University

Institute of Science and Technology

2071

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Bachelor Level / First Year /Second Semester/Science — Full Marks: 80

Computer Science and Information Technology — Pass Marks: 32

(MTH. 155 – Linear Algebra) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A (10×2=20)

1. What is a system of linear equations? When the system is consistent and inconsistent?

2. Define linearly dependent and independent vectors. If (1, 2) and (3, 6) are vectors then the vectors are linearly dependent or independent?

3. Define invertible matrix transformation.

4. Let S be the parallelogram determined by the vectors $b_1 = (1, 3)$ and $b_2 = (5, 1)$ and let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$. Compute the area of the image S under the mapping $x \to Ax$.

5. Show that the matrices $A = \begin{pmatrix} 5 & 1 \\ 3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}$ do not commute.

6. Define vector space.

7. Determine if w = (1, 3, -4) is in Nul A, where $\begin{pmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{pmatrix}$.

8. Is u = (3, -2) is an eigen value of $\begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$?

9. Find the inner product of (2, -5, -1) and (3, 2, -3).

10. Find the norm between the vectors u = (1, 2, 3, 4) and v = (0, 1, 2, 3).

Group B (5×4=20)

11. Let $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, u = (1, 0, -3) and v = (5, -1, 4). If $T: R^3 \to R^3$ defined by T (x) = Ax, find T (u) and T (v).

12. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, show that det (A + B) = det A + det B iff a + d = 0.

13. If v_1 and v_2 are the vectors of a vector space V and H = span $\{v_1, v_2\}$, then show that H is a subspace of V.

14. Find the eigen values of $A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$.

15. Show that (v_1, v_2, v_3) is an orthogonal basis of R^3, where

$$v_1 = \left(3/\sqrt{11}, 1/\sqrt{11}, 1/\sqrt{11}\right),\ v_2 = \left(-1/\sqrt{6}, 2/\sqrt{6}, 1/\sqrt{6}\right), v_3 = \left(-1/\sqrt{66}, -4/\sqrt{66}, 7/\sqrt{66}\right).$$

OR

Find an orthogonal projection of y onto u, where y = (7, 6), u = (4, 2).

Group C

(5×8=40)

16. Determine if the following system is inconsistent.

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

OR

Let $a_1 = (1, -2, -5)$, $a_2 = (2, 5, 6)$ and b = (7, 4, -3), are the vectors. Determine whether b can be generated as a linear combination of a_1 and a_2. That is determine whether x_1 and x_2 exist such that $x_1 a_1 + x_2 a_2 = b$ has solution, find it.

17. If the consumption matrix C is

$$C = \begin{pmatrix} 0.5 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 01 & 0.3 \end{pmatrix}$$

and the final demand is 50 units for manufacturing, 30 units for agriculture and 20 units for services, find the production level x that will satisfy this demand.

OR

Compute the multiplication of partitioned matrices for

$$A = \left(\begin{array}{ccc:cc} 1 & -3 & 2 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hdashline 0 & 4 & 2 & 7 & -1 \end{array}\right) \text{ and } B = \left(\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hdashline -1 & 3 \\ 5 & 2 \end{array}\right)$$

5 × 2

18. Let $b_1 = (1, 0, 0)$, $b_2 = (-3, 4, 0)$, $b_3 = (3, -6, 3)$ and $x = (-8, 2, 3)$ then

(a) Show that $B = \{b_1, b_2, b_3\}$ is a basis of R^3.

(b) Find the change of co-ordinates matrix from B to the standard basis.

(c) Find $[x]_B$, for the given x.

19. Diagonalize the matrix, if possible

$$\begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}.$$

20. What is a least – squares solution? Find a least – squares solution of $Ax = b$, where

$$A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}.$$

Tribhuvan University

Institute of Science and Technology

2072

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Bachelor Level / First Year /Second Semester/Science
Computer Science and Information Technology (MTH. 155)
(Linear Algebra)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A (10×2=20)

1. Define linear combination of vectors. When the vectors are linearly dependent and independent?

2. Define linear transformation between two vector spaces.

3. Show that the matrix $\begin{bmatrix} 6 & -9 \\ 4 & 6 \end{bmatrix}$ is not invertible.

4. Define invertible matrix transformation.

5. Let S be the parallelogram determined by the vectors $b_1 = (1, 3)$ and $b_2 = (5, 1)$ and let $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$. Compute the area of the image S under the mapping $x \to Ax$.

6. Define subspace of a vector space.

7. If $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and let u = (5, 3, 2), then show that u is in the Nul A.

8. Is u = (6, -5) is an eigen vector of $\begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$?

9. Find the unit vector u of v = (1, -2, 2, 0) along the direction of v.

10. Find the norm of vector v = (1, -2, 3, 0).

Group B (5×4=20)

11. Let $A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ and define $T: R^2 \to R^2$ by $T(x) = Ax$. Find the images under T of $u = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $v = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$.

12. Find the determinant of $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{bmatrix}$.

13. Show that the vectors (1, 0, 0), (1, 1, 0) and (1, 1, 1) are linearly independent.

14. Find the eigen values of $A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$.

15. If $v_1 = (3, 6, 0)$, $v_2 = (0, 0, 2)$ are the orthogonal basis then find the orthonal basis of v_1 and v_2.

OR

Find an orthogonal projection of y onto u, where y = (7, 6), u = (4, 2).

Group C (5×8=40)

16. Determine if the following system is consistent, if consistent solve the system.

$-2x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_3 = 4$

$x_1 + 3x_2 - x_3 = 2$

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ and define a transformation T: R^2 by $T(x) = Ax$, so that $T(x) = Ax = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

a) Find T(u)
b) Find x in R^2 whose image under T is b.

17. If the consumption matrix C is

$$C = \begin{pmatrix} 0.5 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{pmatrix}$$

and the final demand is 50 units for manufacturing, 30 units for agriculture and 20 units for services, find the production level x that will satisfy this demand.

18. Let $v_1 = (3, 6, 2)$, $v_2 = (-1, 0, 1)$, $x = (3, 12, 7)$ and $B = \{v_1, v_2\}$. Then B is a basis for H = span $\{v_1, v_2\}$. Determine if x is in H, and if it is, find the co-ordinate vector of x relative to B.

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19. Diagonalize the matrix , if possible

$$\begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix}.$$

20. Find the equation $y = a_0 + a_1x$ for the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

OR

When two vectors 4 and v are orthogonal? If u and v are vectors, prove that $[\text{dist}(u, -v]^2 = [\text{dist}(u, v)]^2$ iff u.v = 0.