Chapter 1st

(1) A function is define by f(x)= $\begin{cases} 1-x, if\ x \le -1 \\ x^2, if\ x > -1 \end{cases}$ Evaluate f(-2),f(-1),f(0) and sketch the graph.

(2) As dry air moves upward , it expands and cools. if the ground temperature is 20^0c and the temperature at a height of 1 km is 10^0c, express the temperature T as a function of the height h, assuming the a liner model is appropriate.(i) Draw the graph of the function (ii) What does the slope represent?(iii) What is the temperature at a height of 2.5 km?

(3) .Define Linear mathematical model.At the surface of the ocean,the water pressure is same as the air pressure above the water 15lb/m^2 . Here water pressure increased by 4.34lb/m^2 for every 10ft of decent then at what dept is pressure 100lb/m^2.

(4) Sketch the graphs of the function y= 1-sin2x

(5) if f(x)=$\sqrt{x}$ and g(x)= $\sqrt{2-x}$, find $f_o g$, $g_o f$ and their domain.

(6)If F(x)=$\cos^2$(x+9), find functions f, g, h Such that F=$f_o g_o h$.

(7) Sketch the graphs of the function y= $3-2^x$ and determine its domain and range.

(8) Sketch the graphs of the function y= ½ $e^{-x}-1$ and determine its domain and range.

(9) Define domain and range.Find domain and range of y=$(x^2-5x+6)^{1/2}$.

(10) Find the appropriate transformation used thus obtained new function of

y=1-2$\sqrt{2-x}$ and graph the function.

Chapter 2nd

(11) Prove that the limit $\lim_{x\to 0}\frac{|x|}{x}$ does not exist

(12)Define vertical asymptote.Find the vertical asymptotes of f(x)=tanx.

(13) Find $\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}$

(14) Show that $\lim_{x\to 0}|x|$=0

(15) A function is define by f(x)= $\begin{cases} 8-2x, if\ x<4 \\ \sqrt{x-4}, if\ x>4 \end{cases}$,Determine whether $\lim_{x\to 4} f(x)$ exists.

(16) Find a number ∂ such that if $|x-1| < \partial$ then $|(x^3-5x+6)-2| < 0.2$

(17) Define precise definition of infinite limit.Given that $\lim_{x\to\frac{\pi}{2}} tan^2x = \infty$ find the value of ∂ that corresponds to M=10000.

(18) Prove that $\lim_{t\to 3} 4x - 5$ =7.

(19)Define left hand limit. Prove that $\lim_{t\to 0^+} \sqrt{t} = 0$

(20) Prove that $\lim_{x\to 3} x^2 = 9$.

(21) If $\lim_{x\to a} f(x)$=L , $\lim_{x\to a} g(x)$=M then prove that $\lim_{x\to a}[f(x) + g(x)]$=L+M

(22) Define infinite limit. Prove that $\lim_{x\to 0} \frac{1}{x^2} = \infty$.

(23)Define continuity of a function at x=a. Show that the function f(x)=$1 - \sqrt{1 - x^2}$ is continuous on [-1,1].

(24)If f and g are continuous at a and c is a constant , then show that the f+g , f-g, cf, fg, f/g where g(a)≠0 are continuous at a.

(25)Prove that any polynomial is continuous every where.also prove any rational function is continuous *wherever it is defined.*

(26) Where is the function f(x)=$\frac{lnx+tan^{-1}x}{x^2-1}$ is continuous?

(27) Evaluate $\lim_{x\to\pi} \frac{sinx}{2+cosx}$

(28) Evaluate $\lim_{x\to 1} arcsin\frac{1-\sqrt{x}}{1-x}$.

(29) Where are the following functions continuous? (i)h(x)=sinx^2 (ii) f(x)= ln(1+cosx)

(30)State the intermediate value theorem. Show that there is a root of the equation $4x^3$-$6x^2$+3x-2=0 between 1 and 2.

(31) Evaluate $\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$ and indicate which properties of limits are used at each stage.

(32) Find the vertical and horizontal asymptotes of the graph of the function $f(x) = \frac{\sqrt{2x^2+1}}{3x-5}$.

(33) Use a graph to find N such that if x>N then $\left|\frac{3x^2-x-2}{5x^2+4x+1} - 0.6\right| < 0.1$

(34) Define limit at infinity. Prove that $\lim_{x\to\infty} \frac{1}{x} = 0$.

(35)Define slant asymptote. Find slant asymptote of the curve f(x)= $\frac{3x^3-x-2}{5x^2+4x+1}$

(36) Define slant asymptote. Find slant asymptote of the curve f(x)=$\frac{x^2-3x/2}{2x+1}$

Chapter 3rd

(37)Find an equation of the tangent line to the parabola y=x^2 at the point (1,1).

(38) Find an equation of the tangent line to the hyperbola y=3/x at the point (3,1).

(39) Show that the sum of the x and y intercepts of any tangent line to the curve $\sqrt{x}$ +$\sqrt{y}$ = $\sqrt{c}$ is equal to c.

(40)Suppose that a ball is dropped from the upper observation deck of the CN tower,450m above the ground.(i)What is the velocity of the ball after 5 seconds?(ii)How fast is the ball traveling when it hits the ground?

(41)A manufacturer produces bolts of a fabric with a fixed width. The cost of producing x meters of this fabric is C=f(x) dollars.(i)What is the meaning of the derivative f'(x)?What are its units?(ii)What does it mean to say that f'(1000)=9?(iii) Which do you think is greater, f'(50) or f'(500)?What about f'(5000)?

(42) Let D(t) be the Canadian gross public debt at time t.The table in the margin gives approximate *values of this function* by providing midyear estimates, in billions of dollars, from 1994 to 2002.Interpret *and estimate the value of* D'(1998)

t	1994	1996	1998	2000	2002
D(t)	414	469.5	467.3	456.4	442.3

(43) Find the derivative of the function f(x)=x^2- 8x+9 at the number a.

(44) Find the derivative of the function f(x)=$\frac{4}{\sqrt{1-x}}$ at the number a i.e. f'(a).

(45)When does a function differentiable on interval ? Where is the function f(x) =$|x|$ is differentiable?

(46) Prove that every differentiable function is continuous but converse is not true.

(47) Find $\lim_{x\to 1}\frac{lnx}{x-1}$

(48)Calculate $\lim_{x\to\infty}\frac{e^x}{x^2}$

(49) Calculate $\lim_{x\to\infty}\frac{lnx}{\sqrt[3]{x}}$

(50) Calculate $\lim_{x\to 0}\frac{tanx-x}{x^3}$

(51) Calculate $\lim_{x\to\pi^-}\frac{sinx}{1-cosx}$

(52) Evaluate $\lim_{x\to 0^+} x\ln x$

(53) Evaluate $\lim_{x\to(\frac{\pi}{2})^-}(\sec x - \tan x)$

(54) Evaluate $\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$

(55) Evaluate $\lim_{x\to 0^+} x^x$

(56) State and prove Rolle's theorem.

(57) Prove that the equation $x^3 + x - 1 = 0$ has exactly one real root.

(58) State and prove the mean value theorem.

(59) Verify mean value theorem for f(x)= x/x+2 on [1,4].

(60)Verify Rolle's theorem for f(x)=cos2x on $[\frac{\pi}{8} . \frac{7\pi}{8}]$

(*)Verify Rolle's theorem for $f(x)=x^3-x^2-6x+2$ on [0,3].

(61) If f(0)=-3 and f '(x)≤5 for all value of x. How large can f(2) possibly be?

(62) Does there exist a function f such that f(0)=-1,f(2)=4 and f '(x)≤2 for all x?

(63) If f(1)=10 and f '(x)≥ 2 for $1\le x\le 4$, how small can f(4) possibly be?

(52) Evaluate $\lim_{x \to 0^+} x \ln x$

(53) Evaluate $\lim_{x \to (\frac{\pi}{2})^-} (secx - tanx)$

(54) Evaluate $\lim_{x \to 0^+} (1 + sin4x)^{cotx}$

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(63) If f(1)=10 and f '(x)≥ 2 for $1 \leq x \leq 4$, how small can f(4) possibly be?

(64)Prove that if f '(x)=0 for all x in an interval (a, b), then f is constant on (a, b).

(65) Prove the identity $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(66) Prove the identity arcsin $\frac{x-1}{x+1}$ =2 arctan$\sqrt{x} - \frac{\pi}{2}$

(67) Let f(x) =tanx.Show that f(0)=f(π) but there is no number c in (0, π) such that f '(c) = 0.Why does this not contradict Rolle's theorem?

(68) Let f(x)=2-|$2x - 1$|. Show that there is no value of c such that f(3)- f(0)=f '(c)(3-0).Why does this not contradict the mean value theorem?

Chapter 4th

(69) Sketch the graph of f(x)=$\frac{2x^2}{x^2-1}$

(70) Sketch the graph of f(x)=$\frac{x^2}{\sqrt{x+1}}$

(71) Sketch the graph of f(x)= xe^x.

(72) Sketch the graph of f(x)= $\frac{\cos x}{2+\sin x}$

(73) Sketch the graph of f(x)=ln(4-x^2)

(74) Sketch the graph of f(x)= $\frac{x^3}{x^2+1}$

(75) Sketch the graph of f(x)= $\frac{x^3}{x-2}$

(76) Sketch the graph of f(x)= $\frac{x^2-4}{x^2-2x}$

(77) A farmer has 1200m of fencing and wants to fence off a rectangular field that borders a straight river .He needs to fence along the river .What are the dimensions of the field that has the largest area? (ans 300,600)

(78)A cylindrical can is to be made to hold 1L of oil.Find the dimensions that will minimize the cost of the metal to manufacture the can. (r=$\sqrt[3]{500/\pi}$,h=2r)

(*) Find the point on the parabola y^2=2x that is closest to the point (1,4)   ans (2,2)

(80) *A man launches his boat from point A on a bank of a st. river,3km wide, and wants to reach point B, 8km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point C and then run to B, or he could row directly to B,or he could row to some point D* between C and B and then run to B. If he can row 6km/h and run 8km/h . where should he land to reach B as soon as possible?   ($9/7^{1/2}$km downstream)

(81)Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.   (r^2)

(82) A store has been selling 200 Blu-ray disc players a week at 350rs each.A market survey indicates that for each 10rs rebate offered to buyers , the number of units sold will increase by 20 a week. Find the demand function and revenue function. How large a rebate should the store offer to maximize its revenue?   (125rs)

(83)Starting with x_1=2, find the third approximation x_3 to the root of the equation x^3-2x-5=0.   (2.094)

(84) Use Newton's method to find $2^{1/6}$ correct to eight decimal places.   (1.12246205)

(85) Find , correct to six decimal places , the root of the equation cosx=x.   (0.739085)

(86) Fine f if f ''(x)= 12x^2+6x-4, f(0)=4, f(1)=1.

(87) A particle moves in a st. line and has acceleration given by a(t)=6t+4. Its initial velocity is v(0)=-6cm/s and its initial displacement is s(0)=9cm.Find its position function s(t). (s(t)=t^3+2t^2-6t+9) (pg347)

(88) A ball is thrown upward with a speed of 15m/s from the edge of a cliff 140m above the ground.Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground? (ans; s(t)=-4.9t^2+15t+140,maximum height if t=1.53,it reach ground if t=7.1s)

Chapter 5th

(89)Under the parabola y=x^2 from 0 to 1,Show that the sum of the areas of the upper approximating rectangles approaches 1/3 i.e. $\lim_{n\to\infty} R_n = \frac{1}{3}$

(90)Let A be the area of the region that lies under the graph of f(x)=e^{-x} between x=0 and x=2.(i) Using right end points, find an expression for A as limit. (ii)Estimate the area by taking the sample points to be midpoints and using four subintervals and then ten subintervals. (A=$\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n} e^{\frac{-2i}{n}}$,0.8557,0.8632)

(91) Suppose the odometer on our car is broken and we want to estimate the distance driven over a 30 seconds time interval.We take speedometer reading every five seconds and record them in the following table;

Times(s)	0	5	10	15	20	25	30
Velocity(km/h)	27	34	38	46	51	50	45

Fine upper and lower estimates that the distance traveled. (ans 342m,367, we use 1km/h=1000/3600m/s)

(92) Evaluate the Riemann sum for f(x)=x^3-6x , taking the sample points to be right endpoints and a=0 ,b=3, and n=6. Also evaluate $\int_0^3(x^3-6x)dx$ (-3.9375,-6.75)

(93) Evaluate the following integrals by interpreting each in terms of areas.(i) $\int_0^1\sqrt{1-x^2}dx$ (ii) $\int_0^3(x-1)dx$ $(\pi/4, 1.5)$

(94) Use midpoint rule with n=5 to approximate $\int_1^2 1/x dx$ (0.6919)

(95) Prove that (i) $\int_a^b[f(x)+g(x)]dx = \int_a^b f(x)dx+\int_a^b g(x)dx$ (ii) $\int_a^b[cf(x)]dx=c\int_a^b f(x)dx$

(96) State and prove max-min inequality for definite integral.By using this estimate $\int_0^1 e^{-x^2}dx$ (between 0.367 and 1)

(97) state and prove the fundamental theorem of calculus, part 1.Using it find the derivative of $\int_1^{x^4} \sec t\, dt$. (4x^{3}secx^4)

(98) state and prove the fundamental theorem of calculus, part 2.Using it find $\int_1^3 e^x\ dx$.

(99)A particle is moves along a line so that its velocity at time t is v(t)=t^2-t-6(i) Find the displacement of the particle during the time period 1≤ $t \leq 4 \leq$. (ii) Find the distance traveled during this time period. (-4.5,10.17)

(100) Define improper integral of type 1.When does it convergent. Does $\int_{-\infty}^{0} xe^{x}dx$ convergent? (convergent to -1)

(101) Evaluate the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ converges or diverges. (converges to π)

(102) For what values of p is the integral $\int_{1}^{\infty} \frac{1}{x^p} dx$ convergent? (p> 1)

(103) Define improper integral of type 2.When does it convergent. Does $\int_{2}^{5} \frac{1}{\sqrt{x-2}} dx$ convergent? (convergent 2 x$3^{1/2}$)

(104) Evaluate (i) $\int_{0}^{\frac{\pi}{2}} secxdx$ (ii) $\int_{0}^{3} \frac{1}{x-1} dx$ (iii) $\int_{0}^{1} lnxdx$ $(\infty, -\infty, -1)$

(105) State comparison theorem for improper integrals. Show that $\int_{0}^{\infty} e^{-x^2} dx$ convergent .

(106) Show that $\int_{1}^{\infty} \frac{1+e^{-x}}{x}$ dx is divergent by the comparison theorem.

(107) Show that $\int_{0}^{\infty} \frac{arctanx}{2+e^x}$ dx is convergent by the comparison theorem.