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Model Questions

Q.N.1)

A pivot position in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A.

Solution:
Given,

$$A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

changing matrix-A into echelon form,

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$\sim \begin{bmatrix} 3 & -9 & 12 & 9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

$R_3 \rightarrow R_3 - \frac{3}{2} R_2$

$$\sim \begin{bmatrix} \boxed{3} & -9 & 12 & 9 & 6 & 15 \\ 0 & \boxed{2} & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & \boxed{1} & 4 \end{bmatrix}$$

The highlighted elements are the pivot element of the pivot column. Hence, we obtained the echelon form of the matrix-A.

Now,

For reduced echelon form,

$R_1 \to R_1 - 6R_3$, $R_2 \to R_2 - 2R_3$.

$$\sim \begin{bmatrix} 3 & -9 & 12 & -9 & 0 & -9 \\ 0 & 2 & -4 & 4 & 0 & -14 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_2 \to R_2/2$.

$$\sim \begin{bmatrix} 3 & -9 & 12 & -9 & 0 & -9 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_1 \to R_1 + 9R_2$

$$\sim \begin{bmatrix} 3 & 0 & -6 & 9 & 0 & -72 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_1 \to R_1/3$

$$\sim \begin{bmatrix} 1 & 0 & -6 & 9 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

This is the reduced echelon form of the matrix - A .

Q.N.2)

Solution:

A transformation from R^n to R^m is a rule that assigns to each vector x in R^n a vector $T(x)$ in R^m.

A transformation (or mapping) T is linear if:

i) $T(u+v) = T(u) + T(v)$ for all u, v in the domain of T.

ii) $T(cu) = c\,T(u)$ for all scalers c and all u in the doma of T.

Example:
A linear transformation of $T: R^2 \to R^2$ is defined by $T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$. Then the images under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ is given by

$$T(u) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Also, the images under T of $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is given by

$$T(v) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

If T is a linear transformation then $T(0) = 0$ and $T(cu + dv) = c\,T(u) + d\,T(v)$.
for all vector u and v and all scalers c and d.

Here,

$$T(x, y) = \begin{bmatrix} x \\ 2y \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}_{\text{standard matrix } A} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 2y \end{bmatrix}$$

let $u = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then,

$$T(u) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Also,

let $v = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$

$T(u+v) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0+2 \\ 0+4 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$

then, $T(u+v) = T(u) + T(v)$.

Here, it follows all conditions of linear transformation so $T(x,y) = (x, 2y)$ is a linear transformation.

Q.N.3.)

When x and $T(x)$ are written in column vectors, we can determine standard matrix of T by inspection, visualizing the row vector computation of each entry in Ax.

$$T(x) = \begin{bmatrix} 3x_1 + x_2 \\ 5x_1 + 7x_2 \\ x_1 + 3x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{——— (i)}$$

So, T is a linear transformation with its standard matrix A shown in (i). The columns of A are linearly independent because they are not multiples of each other. As we know that T is one-one if and only if the columns of A are linearly independent. So, By theorem, T is one-one.

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To decide if T is onto R^3, examine the span of the columns of A. Since, A is 3X2, the columns of A span R^3 if and only if A has 3 pivot positions, by theorem. This is impossible. Since, A has only 2 columns. So, the columns of A does not span R^3, and the associated linear transformation is not onto R^3.

Q.N.3)

solution:

let,

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}_{4\times5}$$

In order to find LU factorization of A, we must follow series of steps:

step 1:

Since, the dimension of A is 4X5, the dimension of L will be 4X4 and the first column of L will be the same as A but divided by the first pivot entry i.e.

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & * & 1 & 0 \\ -3 & * & * & 1 \end{bmatrix}_{4\times4}$$

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Step 2:

We determine the value of * and generate U, using row-reduction method i.e.

$$\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ -2 & -5 & -4 & 1 & 8 \\ 6 & 0 & 7 & -3 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$; $R_3 \to R_3 - R_1$; $R_4 \to R_4 + 3R_1$

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & -9 & -3 & -4 & 10 \\ 0 & 12 & 4 & 12 & -5 \end{bmatrix}$$

$R_3 \to R_3 + 3R_2$; $R_4 \to R_4 - 4R_2$

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 4 & 7 \end{bmatrix}$$

$R_4 \to R_4 - 2R_3$

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\therefore U = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

step 3: '*' are given by dividing the highlighted column by pivot element,

$$\begin{bmatrix} 2 \\ -4 \\ 2 \\ -6 \end{bmatrix}_{\div 2}, \quad \begin{bmatrix} 3 \\ -9 \\ 12 \end{bmatrix}_{\div 3}, \quad \begin{bmatrix} 2 \\ 4 \end{bmatrix}_{\div 2}, \quad \begin{bmatrix} 5 \end{bmatrix}_{\div 5}$$

i.e. $\begin{bmatrix} 1 \\ -2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix}$

Then,

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -3 & 4 & 2 & 1 \end{bmatrix}$$

step 4: checking:

$$LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -3 & 4 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & 3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix} = A.$$

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Q.N.4) solution:

Given,

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

for least square solution;
we use,

$$A^T A \hat{x} = A^T b.$$

$$\text{i.e } \hat{x} = (A^T A)^{-1} A^T b \quad \text{—— (i)}$$

Then,

$$A^T = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

Now,

$$AA^T = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\therefore AA^T = \begin{bmatrix} 16 & 0 & 4 \\ 0 & 4 & 2 \\ 4 & 2 & 2 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 17 & 1 \\ 1 & 5 \end{bmatrix}$$

Now,

$$A^T b = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

Then,
using (i),

$$\hat{x} = (A^T A)^{-1} A^T b.$$

Now,
for $(ATA)^{-1}$,
$|A^T A| = 84$.

And,
$$(A^T A)^{-1} = \frac{1}{|A^T A|} \text{Adj of } A^T A$$
$$= \frac{1}{84}\begin{bmatrix} 5 & -1 \\ -1 & 17 \end{bmatrix}$$

Now,

$$\hat{x} = \frac{1}{84}\begin{bmatrix} 5 & -1 \\ -1 & 17 \end{bmatrix} \times \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Hence, the least square solution of the given system is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

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Q.N. 5)

Solution:

Given,

$$u = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

Then,

$$u + v = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$\therefore u + v = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$$

$$u - 2v = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$\therefore u - 2v = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\therefore 2u + v = \begin{bmatrix} 2 \\ -6 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \\ 7 \end{bmatrix}$$

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Q.N. 6) → Solution:

Given,

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$T: R \to R^2$ be defined by $T(x) = Ax$.

Then,

$T(u) = ?$, $T(v) = ?$

Also,

$$u = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \quad \& \quad v = \begin{bmatrix} a \\ b \end{bmatrix}$$

Then,

$$T(u) = Au = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$\therefore T(u) = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$T(v) = Av = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

$$\therefore T(v) = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

Q.N.7). Solution:

Given,

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -5 \\ 3 & K \end{bmatrix}$$

Now,

$$AB = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -5 \\ 3 & K \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4 + 5\times3 \\ -3\times-5 + 1\times K \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 23 \\ +15+K \end{bmatrix}$$

Again,

$$BA = \begin{bmatrix} 4 & -5 \\ 3 & K \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times2 - 5\times-3 \\ 3\times5 + K\times1 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 23 \\ 15+K \end{bmatrix}$$

According to the question:

For, $AB = BA$, K can by any finite number.

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Q.N.8) solution:

$$A = \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$$

If we can change the given matrix A into triangular form, then the determinant is given by the product of the diagonal elements.

Now

Using row-reduction method,

$R_2 \to R_2 - 3/2\, R_1$; $R_3 \to R_3 + 3/2\, R_1$; $R_4 \to R_4 - 1/2\, R_1$

$$\sim \begin{bmatrix} 2 & -8 & 6 & 8 \\ 0 & 3 & -4 & -2 \\ 0 & -12 & 10 & 10 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$$\sim \begin{bmatrix} 2 & -8 & -6 & 8 \\ 0 & 3 & 4 & -2 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 - R_3/2$

$$\sim \begin{bmatrix} 2 & -8 & 6 & 8 \\ 0 & 3 & -4 & -2 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence,

$|A| =$ determinant of $A = 2 \times 3 \times -6 \times 1$

$= -36$

Q.N.18. Solution:

Given,

$$H = \left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix}, s,t \in R \right\}$$

$$H = \left\{ \begin{bmatrix} s-2t \\ s+t \\ 0s+t \end{bmatrix}, s,t \in R \right\}$$

$$= \left\{ s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, s,t \in R \right\}$$

Here,

basis of the subspace H are $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right\}$

And the dimension is 2.

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Q.N. 11) Solution:

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$$

for eigen value,

let λ be the eigen value. Then,

for $(A-\lambda I)$,

$$A - \lambda I = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{bmatrix}$$

Then,

$$|A - \lambda I| = (2-\lambda)(-6-\lambda) - 9 = 0$$

or, $-12 - 2\lambda + 6\lambda + \lambda^2 - 9 = 0$

or, $\lambda^2 + 4\lambda - 21 = 0$

or, $\lambda^2 + 7\lambda - 3\lambda - 21 = 0$

or $(\lambda - 3)(\lambda + 7) = 0$

Either,

$\lambda = 3$ or $\lambda = -7$

The eigen vector is given by,

When $\lambda = 3$,

$$(A - 3I) = \begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix}$$

We know, $(A-3I)x=0$, then writing in augmented form as $[A-3I : 0]$

i.e.

$$\left[\begin{array}{cc|c} -1 & 3 & 0 \\ 3 & -9 & 0 \end{array}\right]$$

$R_2 \to R_2 + 3R_1$

$$\sim \left[\begin{array}{cc|c} -1 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Here, x_2 is free variable. Then, from 1st row,

$$-x_1 + 3x_2 = 0$$
$$x_1 = 3x_2$$

Then,

eigen vector $= V_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

i.e. $V_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

When $\lambda = -7$,

$$(A+7I) = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$$

We know, $(A+7I)y=0$, then writing in augmented f
as $[A+7I : 0]$

i.e.

$$\left[\begin{array}{cc|c} 9 & 3 & 0 \\ 3 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1/3$

$$\sim \left[\begin{array}{cc:c} 9 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Here, y_2 is free variable. Then,

from 1st row,

$$9y_1 + 3y_2 = 0$$
$$y_1 = -y_2/3.$$

Then,

eigen vector $= \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -y_2/3 \\ y_2 \end{bmatrix} = y_2 \begin{bmatrix} -1/3 \\ 1 \end{bmatrix}.$

Hence, $V_2 = \begin{bmatrix} -1/3 \\ 1 \end{bmatrix}$

12)

A set of vectors $\{u_1, u_2, \cdots u_p\}$ in R^n is said to be an orthogonal set if $u_i \cdot u_j = 0$ for all $i \neq j$.

Solution:

Given,

$$u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$$

Now,

$u_1 \cdot u_2 = 3 \times -1 + 1 \times 2 + 1 \times 1$
$= 0.$

$u_1 \cdot u_3 = 3 \times -\frac{1}{2} + 1 \times -2 + 1 \times \frac{7}{2}$

$= 0$

$u_2 \cdot u_3 = -1 \times -\frac{1}{2} + 2 \times -2 + 1 \times \frac{7}{2}$

$= 0$

since, $u_1 \cdot u_2 = u_1 \cdot u_3 = u_2 \cdot u_3 = 0$. Which verifies the definition of orthogonal set, i.e. they are orthogonal set.

(Q.N.13) Solution:

Given,

$x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

Using Gram-schmidt process,

let, $v_1 = x_1$. Then, $v_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$

Also,

v_2

Also,

$$v_2 = x_2 - \frac{x_2 \cdot v_1}{v_1 \cdot v_1} \cdot v_1$$

$$= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}}{\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}} \times \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \frac{(3+12+0)}{(9+36+0)} \times \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \frac{15}{45} \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$\therefore \{v_1, v_2\}$ is orthogonal set of nonzero vectors in W.

Since, $\dim W = 2$, the set $\{v_1, v_2\}$ is a basis for W.

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Q.N.19) solution:

let $*$ be defined on Q^+ by $a * b = \frac{ab}{2}$.

In order to show Q^+ forms a group, it must satisfy following properties of group:

i) Closure property:

If $a, b \in Q^+$ then $a * b \in Q^+ \ \forall\ a, b \in Q$.

for example,

$a = \frac{1}{2}$, $b = \frac{1}{3} \in Q^+$

then,

$a * b = \frac{ab}{2} = \frac{1/2 \times 1/3}{2} = \frac{1}{2} \in Q^+$

Hence, it satisfies closure property.

ii) Associative property:

If $a, b, c \in Q^+$ then $a * (b * c) = (a * b) * c$ $\forall\ a, b, c \in Q^+$.

for example,

$a = 1/2$, $b = 1/3$, $c = 1/4$

Then,

$a * (b * c) = a * \left(\frac{bc}{2}\right) = a * \frac{1}{24}$

$= \frac{1/2 \cdot 1/24}{2}$

$= \frac{1}{96} \in Q^+$

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$(a*b)*c = \frac{ab}{2} * c = \frac{1}{12} * c$

$= \frac{1/12 \cdot 1/4}{2}$

$= \frac{1}{96} \in Q^+$

$\therefore (a*(b*c) = (a*b)*c$ is also satisfied.

(iii) Existence of identity:

If $a \in Q^+$ then $a*e = a$ where e is an identity.

for example,

$a = 1/12$

Then,

$a*e = a$

$\frac{1/2 \times e}{2} = \frac{1}{2}$.

or, $e = 2 \in Q^+$

i.e 2 is the identity element for Q^+.

iv) Existance of inverse:

If $a \in Q^+$ then there exists another element $b \in Q^+$ such that $a*b = e = b*a$, then,

$a*b = \frac{ab}{2} = e$.

$$\frac{a \cdot a^{-1}}{2} = e$$

$\therefore e = \frac{1}{2} \in Q^+$

$$b * a = \frac{a^{-1} \cdot a}{2}$$
$$= \frac{1}{2} = e \in Q^+$$

since all the properties are satisfied Q^+ forms a group.

Q.N.15. let R be a non-empty set. An algebraic structure (R, +, ·) together with two binary operations addition and multiplication defined on R is called a ring if this structure satisfies following axioms.

let, $Z = \{ \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \}$

The properties to be satisfied to be a ring are :

i) closed under addition:

for all $a, b \in Z$.

$a + b \in Z$.

eg:

$1, 2 \in Z$.

$1 + 2 = 3 \in Z$.

ii) Associative under addition:-

for all $a, b, c \in R$

$a+(b+c) = (a+b)+c$

eg: $1, 2, 3 \in Z$,

$1+(2+3) = 1+5 = 6 \in Z$

$(1+2)+3 = 3+3 = 6 \in Z$

iii) Existence of additive identity:

for all $a \in Z$.

$0+a = a+0 = a$.

eg:

$3 \in Z$.

$0+3 = 3+0 = 3. \in Z$

$\therefore$ 0 is the additive identity.

(iv) Existence of additive inverse:

for all $a \in Z$.

$a+(-a) = -a+a = 0$.

eg:

$5 \in Z$

$5+(-5) = 5-5 = 0$

$-5+5 = 0$

$\therefore -5$ is the additive inverse of 5.

v) Commutative under addition:-

for all $a, b \in Z$.

$a+b = b+a$.

Eg: $2, -3 \in Z$.

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so, $2 + (-3) = 2 - 3 = -1$
$-3 + 2 = -1.$

vii) Closed under multiplication:-
for all $a, b \in Z$
$a \cdot b \in Z$

eg:
$2, -4 \in Z.$
Then,
$2 \cdot (-4) = -8 \in Z$

viii) Associative under multiplication:-
for all $a, b, c \in Z.$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

eg: $2, -3, 5 \in Z.$
Then,
$2 \cdot (-3 \cdot 5) = 2(-15) = -30$
$(2 \cdot -3) \cdot 5 = -6 \cdot 5 = -30$

viii) distributive law:-
first,
$a \cdot (b + c) = a \cdot b + a \cdot c$

for $-1, 1, 2 \in Z$
$-1(1+2) = -1 \cdot 1 - 1 \cdot 2$
or, $-3 = -3 \in Z.$

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second,
$(b+c).a = a.b + a.c$
for : $-1, 1, 2 \in Z$

$(1+2).-1 = -1.1 + 2.-1$
$-3 = -3 \in Z.$

Hence set of Z forms a ring.

Solution:
$(12)(16)$ in Z_{15}
Here
192 in Z_{15}

Now
$15 \,\rceil\, 192 \,\lfloor\, 12$
-180
$12. = R.$

$\therefore$ Required answer $= 12.$

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