40 ← 47-5 → 50

CSc.163-2075

Tribhuvan University

Institute of Science and Technology

2075

Bachelor Level / First Year /Second Semester/Science
Computer Science and Information Technology (MTH. 163)
(Mathematics II)
(NEW COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Group A

Attempt any three questions: (3×10=30)

1. When a system of linear equation is consistent and inconsistent? Give an example for each. Test the consistency and solve: x + y + z = 4, x + 2y + 2z = 2, 2x + 2y + z = 5. (2+1+7)

2. What is the condition of a matrix to have an inverse? Find the inverse of the matrix $A = \begin{pmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{pmatrix}$ if exists. (2+8)

3. Define linearly independent set of vectors with an example. Show that the vectors (1, -4, 3), (0, 3, 1) and (3, -5, 4) are linearly independent. Do they form a basis? Justify. (2+5+3)

4. Find the least-square solution of Ax = b for $A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 5 \\ 7 \\ 3 \end{pmatrix}$. (10)

$(A^TA)^{-1}A^Tb$

Group B

Attempt any ten questions: (10×5=50)

5. Change into reduce echelon form of the matrix $\begin{pmatrix} 0 & 3 & -6 \\ 3 & -7 & 8 \\ 3 & -9 & 12 \end{pmatrix}$. (5)

6. Define linear transformation with an example. Is a transformation T: $\mathbb{R}^2 \to \mathbb{R}^3$ defined by T(x, y) = (3x + y, 5x + 7y, x + 3y) linear? Justify. (2+3)

7. Let $A = \begin{pmatrix} -1 & -2 \\ 5 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 2 \\ k & -1 \end{pmatrix}$. What value (s) of k if any will make AB = BA? (5)

8. Define determinant. Evaluate without expanding $\begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$. (1+4)

9. Define subspace of a vector space. Let $H = \left\{ \begin{pmatrix} s \\ t \\ 0 \end{pmatrix} : s, t \in \mathbb{R} \right\}$. Show that H is a subspace of $\mathbb{R}^3$. (1+4)

10. Find the dimension of the null space and column space of $A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$. (5)

11. Find the eigenvalues of the matrix $\begin{pmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix}$. (5)

12. Find LU factorization of the matrix $\begin{pmatrix} 2 & 5 \\ 6 & -7 \end{pmatrix}$. (5)

13. Define group. Show that the set of all integers $\mathbb{Z}$ forms group under addition operation. (1+4)

14. Define ring with an example. Compute the product in the given ring (-3, 5) (2, -4) in $\mathbb{Z}_4 \times \mathbb{Z}_{11}$. (2.5+2.5)

15. State and prove the Pythagorean theorem of two vectors and verify this for u = (1, -1) and v = (1, 1). (3+2)