Tribhuvan University

Institute of Science and Technology

2066

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Bachelor Level/ Second Year/ Third Semester/ Science — Full Marks: 60

Computer Science and Information Technology (CSc. 204) — Pass Marks: 24

(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

1. Define the fixed-point iteration method. Given the function $f(x) = x^2 - 2x - 3 = 0$, rearrange the function in such a way that the iteration method converses to its roots. **(2+3+3)**
2. What do you mean by interpolation problem? Define divided difference table and construct the table from the following data set. **(2+2+4)**

X_i	3.2	2.7	1.0	4.8	5.6
F_i	22.0	17.8	14.2	38.3	51.7

OR

Find the least squares line that fits the following data.

X	1	2	3	4	5	6
Y	5.04	8.12	10.64	13.18	16.20	20.04

What do you mean by linear least square approximation?

3. Derive the composite formula for the trapezoidal rule with its geometrical figure. Evaluate $I = \int_0^1 e^{-x^2}\, dx$ using this rule with n=5, upto 6 decimal places. **(4+4)**
4. Solve the following system of algebraic linear equations using Jacobi or Gauss-Seidel iterative **(8)**

$$6x_1 - 2x_2 + x_3 = 11$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$X_1 + 2x_2 - 5x_3 = -1$$

5. Write an algorithm and computer program to fit a curve $y = ax^2 + bx + c$ for given sets of (x_i, y_i, g. 0 = 1, …, x) values by least square method. **(4+8)**
6. Derive a difference equation to represent a Poison's equation. Solve the Poison's equation $\nabla^2 f = 2x^2y^2$ over the domain $0 \le x \le 3, 0 \le y \le 3$ with $f = 0$ on the boundary and h = 1. **(3+5)**

7. Define ordinary differential equation of the first order. What do you mean by initial value problem? Find by Taylor's series method, the values of y at x = 0.1 and x = 0.2 to find places of decimal form

$\frac{dy}{dx} = x^2y - 1$, when y(0) = 1 **(2+6)**

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Attempt all questions:

1. Discuss methods of Half Interval and Newton's for solving the nonlinear equation f(x) = 0. Illustrate the methods by figure and compare them stating their advantages and disadvantages. **(8)**
2. Derive the equation for Lagrange's interpolating polynomial and find the value of f(x) at x = 1 for the following: **(4+4)**

X	-1	-2	2	4
F(x)	-1	-9	11	69

3. Write Newton-cotes integration formulas in basic form for x = 1, 2, 3 and give their composite rules. Evaluate $I = \int_{.2}^{1.5} e^{-x^2}\, dx$ using the Gaussian integration three point formula. **(4+4)**
4. Solve the following algebraic system of linear equations by Gauss-Jordan algorithm. **(8)**

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 2 & 3 & 2 \\ 4 & -3 & 0 & 1 \\ 6 & 1 & -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -7 \\ 6 \end{bmatrix}$$

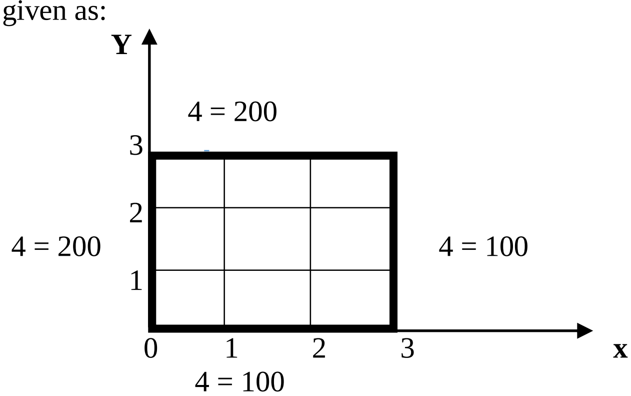
5. Write an algorithm and program to solve system of linear equations using Gauss-Seidel iterative method. **(4+8)**
6. Explain the Picard's proves of successive approximation. Obtain a solution upto the fifth approximation of the equation

$\frac{dy}{dx} = y + x$ such that y = 1 when x = 0

using Picard's process of successive approximations . **(2+6)**

7. Define a difference equation to represent a Laplace's equation. Solve the following Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within $0 \le x \le 3, 0 \le y \le 3$

For the rectangular plate given as:



(3 + 5)

OR

Derive a difference equation to represent a Poison's equation. Solve the Poison's equation
$\nabla^2 f = 2x^2y^2$
Over the domain $0 \le x \le 3, 0 \le y \le 3$ with $f = 0$ on the boundary and h = 1. **(3+5)**

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(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
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Attempt all questions:

1. Define the types of errors in numerical calculations. Derive the formula for secant method and illustrate the method by figure. **(4+4)**
2. Define the linear least squares approximations. Give the data set (x_i, y_i) as (20.5, 765), (32.7, 826), (51.0, 873), (73.2, 942), (95.7, 1032) find the linear least square to fit given data. **(2+6)**
3. Evaluate $I = \int_0^1 e^{-x^2}\ dx$ using trapezoidal rule with n=10. Also evaluate the same integral using Grossion 3 point formula and compare the result. **(4+4)**
4. Solve the following system of linear equations using Gauss-elimination method (use partial pivoting if necessary);

$$\begin{aligned} 2x_2 + x_4 &= 0 \\ 2x_1 + 2x_2 + 3x_3 + 2x_4 &= -2 \\ 4x_1 - 3x_2 + x_4 &= -7 \\ 6x_1 + x_2 - 6x_3 - 5x_4 &= 6 \end{aligned}$$

(8)

OR

What do you mean by eigen -value eigen- vector problems? Find the largest eigen value correct to two significant digits and corresponding eigen vectors of the following matrix using power method.

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(2+6)

5. Write an algorithm and program to solve system of linear equations using Gauss- Jordan method. **(4+8)**
6. Apply Runge-Kutta method of second order and fourth order to find an approximate value of y when x = 0.2 given that

$$\frac{\partial y}{\partial x} = x + y \text{ and } y(0) = 1.$$

(8)

7. How can you solve Laplace's equation? Explain. The steady-state two dimensional heat flow in a metal plate is defined by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.$

A steel plate of size 30 x 30cm is given. Two adjacent sides are placed at 100℃ and other side held at 0℃ . Find the temperature at interior points, assuming the grid size of 10 x 10cm.

(3+5)

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(Numerical Method) — Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
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Attempt all questions:

1. Derive the formula to solve nonlinear equation using secant method. Using your formula estimate a real root of following nonlinear equation using secant method correct up to two decimal places $x^2 + \ln x = 3$. **(3+5)**
2. Estimate $f(3)$ from the following data using Cubic Spline interpolation.

x	1	2.5	4	5.7
f(x)	-2.0	4.2	14.4	31.2

OR

Find the best fitting quadratic polynomial from following data using least square approximation.

x	-2	-1.2	0	1	1.2	2.5	3	4.5	6.3
f(x)	10.39	2.96	-2.0	-2.63	-2.46	0.83	3.1	12.8	30.4

3. a) For the function $f(x) = e^x \sqrt{sinx + lnx}$ estimate $f'(6.3)$ and $f''(6.3)$ [take h = 0.01] **(4)**

   b) Evaluate $\int_1^2 (lnx + x^2 \, sinx)dx$ using Gaussian integration 3 point formula. **(4)**

4. Solve the following set of equation using Gauss elimination or Gauss Jordan method

$$3x_1 + 5x_2 - 3x_3 + x_4 = 16$$
$$2x_1 + x_2 + x_3 + 4x_4 = 9$$
$$3x_1 - 4x_2 - x_4 = 1$$
$$2x_1 + x_2 - 3x_3 + 9x_4 = 5$$

(8)

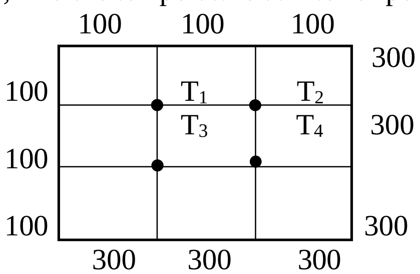
5. How can you solve higher order differential equation? Explain. Solve the following differential within $0 \le x \le 1$ using Heun's method.

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2xy = 1$$ with y(0)=1 and y'(0) = 1 (take h = 0.5)

(3+5)

6. a) How can you obtain numerical solution of a partial differential equation? Explain. **(3)**

   b) The steady-state two-dimensional heat-flow in a metal plate is defined by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$. Given the boundary conditions as shown in figure below, find the temperature at interior points T_1, T_2, T_3 and T_4. **(5)**



7. Write an algorithm and C-program code to solve non-linear equation using Newton's method. Your program should read an initial guess from keyboard and display the followings if the solution is obtained: **(5+7)**

   - Estimated root of the equation
   - Functional value at calculated root
   - Required number of iterations

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Attempt all questions:

1. What is bracketing and non-bracketing method? Explain with the help of example. Estimate a real root of following nonlinear equation using bisection method correct up to two significant figures

$$x^2 sinx + e^{-x} = 3.$$ **(3+5)**

2. Define interpolation. Find the functional value at x = 3.6 from the following data using forward difference table.

x	2	2.5	3	3.5	4	4.5
f(x)	1.43	1.03	0.76	0.6	0.48	0.39

(2+6)

3. Derive Simpson's 1/3 rule to evaluate numerical integration. Using this formula evaluate $\int_{0.2}^{1.2}(x^2 + lnx - sinx)dx$. [Take h = 0.1] **(4+4)**

4. What is pivoting? Why is it necessary? Explain. Solve the following set of equations using Gauss elimination or Gauss Seidel method.

$$x_1 + 10x_2 + x_3 = 24$$
$$10x_1 + x_2 + x_3 = 15$$
$$x_1 + x_2 + 10x_3 = 33$$

(3+5)

5. Compare Euler's method with Heun's method for solving differential equation. Obtain y(1.5) from given differential equation using Runge-Kutta 4th order method.
$\frac{dy}{dx} + 2x^2y = 1$ with y(1) = 0 (take h = 0.25) **(4+4)**

OR

Solve the following boundary value problem using shooting method.
$\frac{d^2y}{dx^2} - 2x^2y = 1$, with y(0) = 1 and y(1) = 1 [Take h = 0.5]. **(8)**

6. Solve the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 3x^2y$ over the square domain $0 \leq x \leq 1.5$ and $0 \leq y \leq 1.5$ with *f=0* on the boundary [Take h = 0.5]. **(8)**

7. Write an algorithm and C-program to approximate the functional value at any given x from given n no. of data using Lagrange's interpolation. **(5+7)**

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Attempt all questions:

1. How is the bisection method convergent to a root of an equation? Apply the bisection method to find a root of the equation
$$x\tan x - 1 = 0$$ **(3+5)**
2. Define interpolation. Find the Lagrange interpolation polynomial to fit the following data. Estimate the value

i	0	1	2	3
x_i	0	1	2	3
e^{x_i}	0	1.7183	6.3891	19.0855

of $e^{1.9}$ **(1+6+1)**

3. Derive Simpson's 1/3 rule to evaluate numerical integration. Using this formula evaluate $\int_0^2 \left(e^{x^2} - 1\right) dx$ with n = 8. **(4+4)**
4. What do you mean by ill-conditioned systems? Solve the following system using Dolittle LU decomposition method.
$$3x_1 + 2x_2 + x_3 = 24$$
$$2x_1 + 3x_2 + 2x_3 = 14$$
$$x_1 + 2x_2 + 3x_3 = 14$$
(2+6)
5. Solve the following boundary value problem using shooting method.
$\frac{d^2y}{dx^2} - 2x^2y = 1$, with y(0) = 1 and y(1) = 1 [Take h = 0.5]. **(8)**

6. Write the finite difference formula for solving Poisson's equation. Hence solve the Poisson's equation
$$\nabla^2 f = 2x^2y^2$$
over the domain $0 \le x \le 3$ and $0 \le y \le 3$ with *f = 0* on the boundary and h = 1. **(1+7)**

7. Write an algorithm and a C-program for the fixed point iteration method to find the roots of non-linear equation. **(4+8)**

OR

Write an algorithm and a C-program for the Lagrange's interpolation to approximate the functional value at any given x from given n data. **(4+8)**

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