Set4

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x) = \dfrac{3x + |x|}{x}$ [5]

Evaluate domain and sketch the graph.

(b) If $f(x) = \begin{cases} \sqrt{x-4}, & if \ \ x > 4 \\ 8 - 2x & if \ \ x < 4 \end{cases}$, determine whether limit exits or not. [5]

2. (a) Sketch the curve : $f(x) = \dfrac{\cos x}{2 + \sin x}$. [5]

(b) Estimate the area between the curve $y = \dfrac{1}{x}$ and the lines x = 0 and x = 1,

using Trapezoidal rule. [5]

3. (a) The region enclosed by the curves y = x and $y = x^2$ and is rotated about the x-axis.
Find the volume of the resulting solid. [4]

(b) Define orthogonal trajectories. Find the orthogonal trajectories of the family of curves
$x = ky^2$, where k is an arbitrary constant. [6]

4. (a) Find parametric equations and symmetric equations of the line that passes through the points A(2, 4, -3) and B(3, -1, 1). At what point does this line intersect the -plane? [6]

(b) Find the direction angles of the vector < 1, 2, 3> . [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $y = x^5 + x$ (b) $y = 1 - x^4$ (c) $y = 2x - x^2$

6. Define continuity of a function at a point x = a. Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuous on the interval [-1, 1].

7. Evaluate the following, using L Hospital's Rule $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$.

8. A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

9. Evaluate: $\int_0^1 \ln x \, dx.$

10. Find the arc length function for the curve $y = x^2 - \dfrac{1}{8} \ln x$ taking the point P(1, 1) as the starting point.

11. What is the solution of non-homogeneous second-order linear differential equation.
Solve: $y'' - 4y = xe^x + \cos 2x.$

12. Find the Maclaurin's series for the function f(x) = x.cosx.

13. Find the sum of the series $\sum_{n=1}^{\infty} \left(\dfrac{3}{n(n+1)} + \dfrac{1}{2^n} \right)$.

14. Find the angle between the planes x + y + z = 1 and x - 3y + 2z = 1.Find symmetric equations for the line of intersection L of these two planes.

15. Find the second partial derivatives of $f(x, y) = x^3 + x^2y^3 - 2y^2$.

Set 4

Group: A

(Q.N.1.a)

= solution,

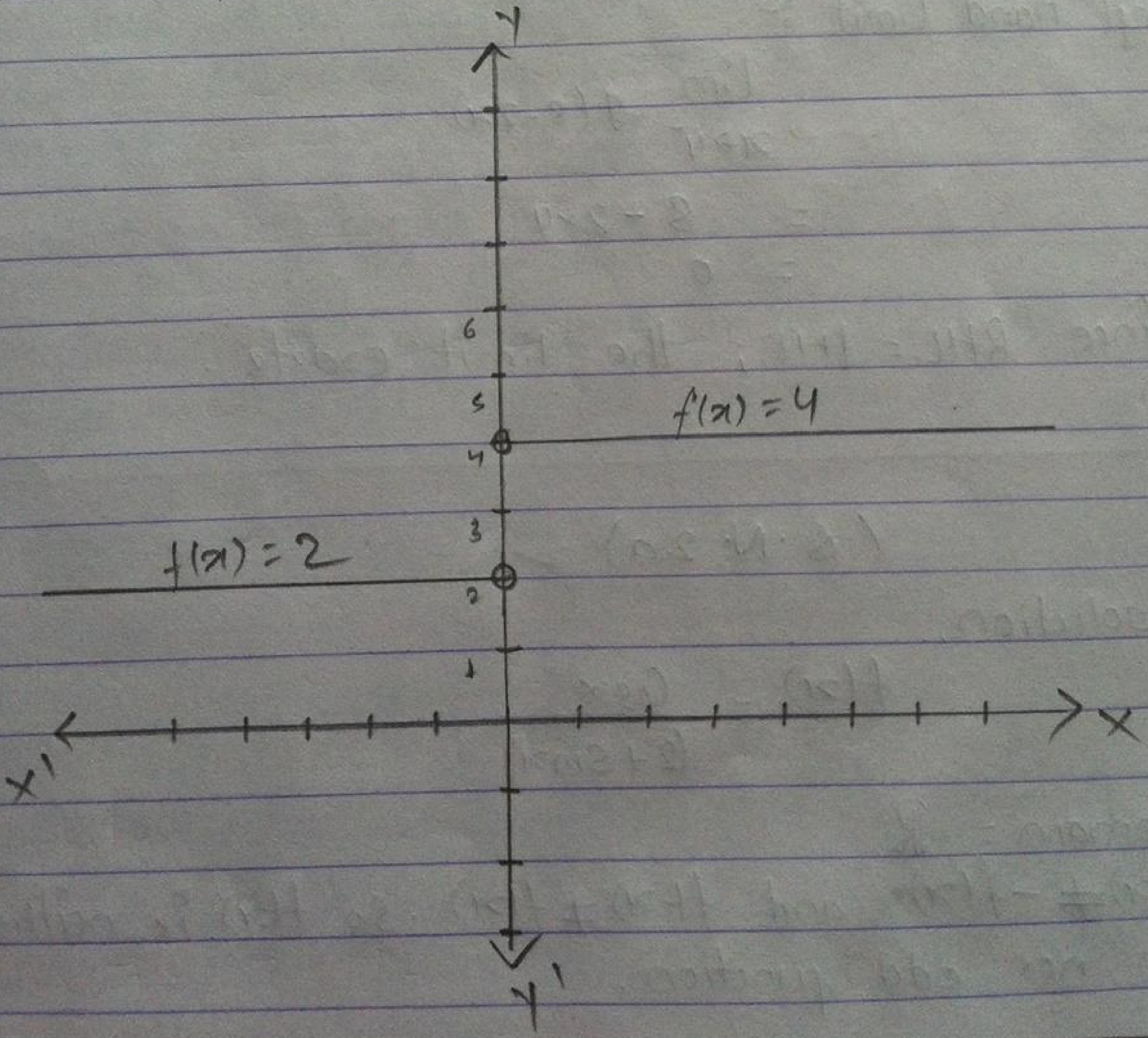
$$f(x) = \frac{3x + |x|}{x}$$

we know,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\text{so, } f(x) = \begin{cases} \frac{3x+x}{x} & \text{if } x > 0 \\ \frac{3x-x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} 4 & \text{if } x > 0 \\ 2 & \text{if } x < 0 \end{cases}$$

so,
The domain is a set of all real numbers except zero. Now, Graph:-



(Q.N.1.b)

= solution,

here,

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \end{cases}$$

Now,

Right Hand limit :-

$$\lim_{x \to 4^+} \sqrt{(x-4)}$$

$$= \sqrt{4-4}$$

$$= 0$$

And,

left Hand Limit :-

$$\lim_{x \to 4^-} \cancel{f}(8-2x)$$

$$= 8 - 2 \times 4$$

$$= 0$$

Since RHL = LHL ; the limit exists.

(Q.N. 2a)

= solution,

$$f(x) = \frac{\cos x}{(2 + \sin x)}$$

(i) Domain = $\mathbb{R}$

(ii) $f(-x) \neq -f(x)$ and $f(-x) \neq f(x)$. So $f(x)$ is neither even, nor odd function.

(iii) When $x=0$, $f(x)=(\frac{1}{2})$ $\therefore$ $y=(\frac{1}{2})$ is the y-intercept.
When $f(x)=0$; $\cos x=0$; i.e. $x=(2n+1)\frac{\pi}{2}$ is the x-intercept, where 'n' is integer.

(iv) $f(x+2\pi)=f(x)$ for all x. So $f(x)$ has a period of 2π.

(v) Asymptotes: None

(vi)
$$f'(x)=\frac{(2+\sin x)(-\sin x)-(\cos x)(\cos x)}{(2+\sin x)^2}$$
$$=\frac{-2\sin x-\sin^2 x-\cos^2 x}{(2+\sin x)^2}$$
$$=\frac{-(1+2\sin x)}{(2+\sin x)^2}$$

Thus, $f'(x)>0$ when $(2\sin x+1)<0 \implies \sin x<-\frac{1}{2} \implies \frac{7\pi}{6}<x<\frac{11\pi}{6}$. So, f is increasing on $(\frac{7\pi}{6}, \frac{11\pi}{6})$ and decreasing on $(0, \frac{7\pi}{6})$ and $(\frac{11\pi}{6}, 2\pi)$

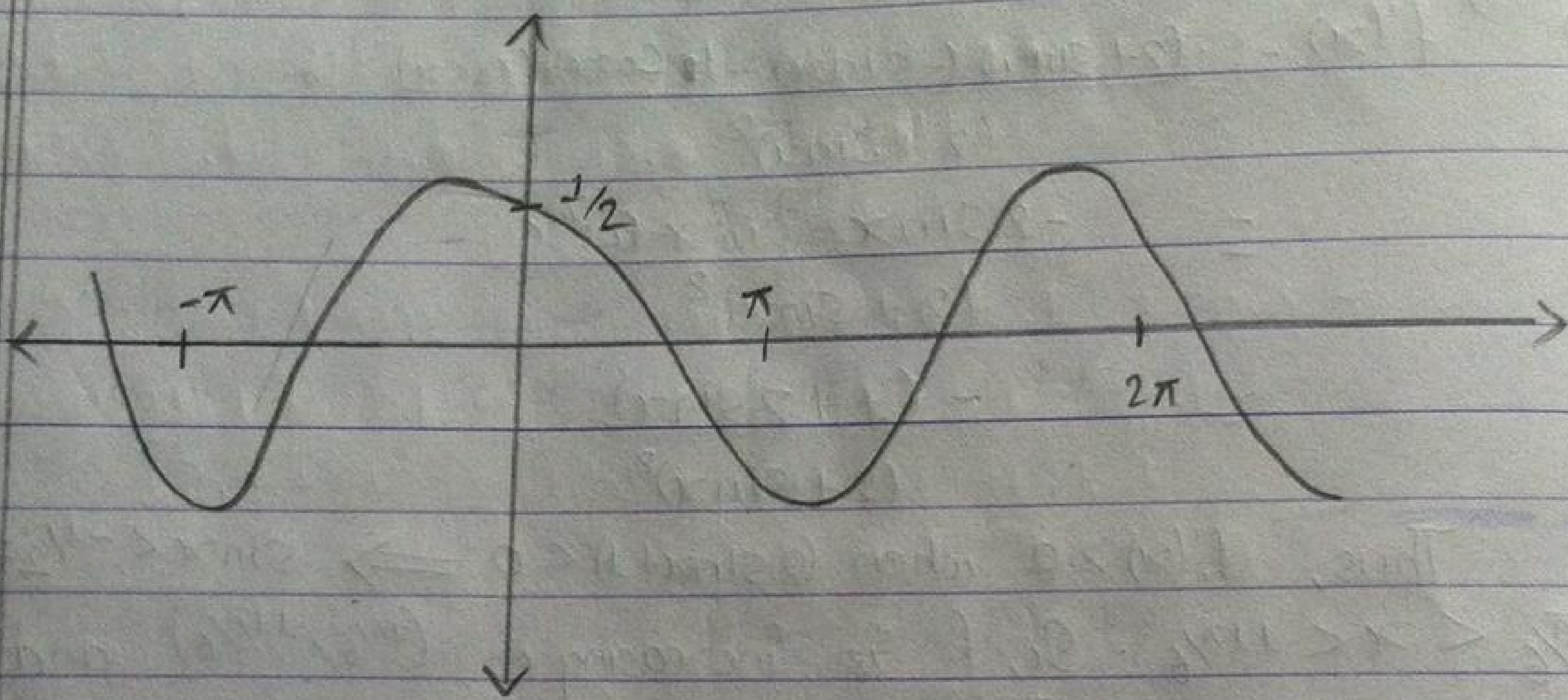
(vii) From part (vi) and the First Derivative test, we see that the local minimum value is $f(\frac{7\pi}{6})=-\frac{1}{\sqrt{3}}$ and the local maximum value is $f(\frac{11\pi}{6})=\frac{1}{\sqrt{3}}$

(viii) Again,
$$f''(x)=-\frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$$

Since $(2+\sin x)^3>0$ and $(1-\sin x)\geq 0$; for all x; we know, $f''(x)>0$ when $\cos x<0$, that is $\frac{\pi}{2}<x<\frac{3\pi}{2}$. So, $f(x)$ is concave upward in the interval $(\frac{\pi}{2}, \frac{3\pi}{2})$ and concave downward on

$(0, \pi/2)$ and $(3\pi/2, 2\pi)$. The inflection points within 0 and 2π (repeating periodically) are $(\pi/2, 0)$ and $(3\pi/2, 0)$.

The graph is :-



(Q.N. 2.b)

= solution,

$$y = f(x) = \left(\frac{1}{x}\right)$$

end points : $a = 1$, $b = 2$

There are 10 possible trapezium pieces between 0 and 1. So; $n = 10$

Now,

$$\Delta x = \frac{(b-a)}{n} = \frac{(2-1)}{10} = 0.1$$

$\therefore$ $x_0 = 1$, $x_1 = 1.1$, $x_2 = 1.2$, $x_3 = 1.3$, $x_4 = 1.4$, $x_5 = 1.5$

$x_6 = 1.6,\ x_7 = 1.7,\ x_8 = 1.8,\ x_9 = 1.9,\ x_{10} = 2$

so,

The Trapezoidal Rule gives:-

$$\int_1^2 \frac{1}{x}\cdot dx \approx T_{10} = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{0.1}{2}\left[f(1.0) + 2f(1.1) + 2f(1.2) + 2f(1.3) + 2f(1.4) + 2f(1.5) + 2f(1.6) + 2f(1.7) + 2f(1.8) + 2f(1.9) + f(2)\right]$$

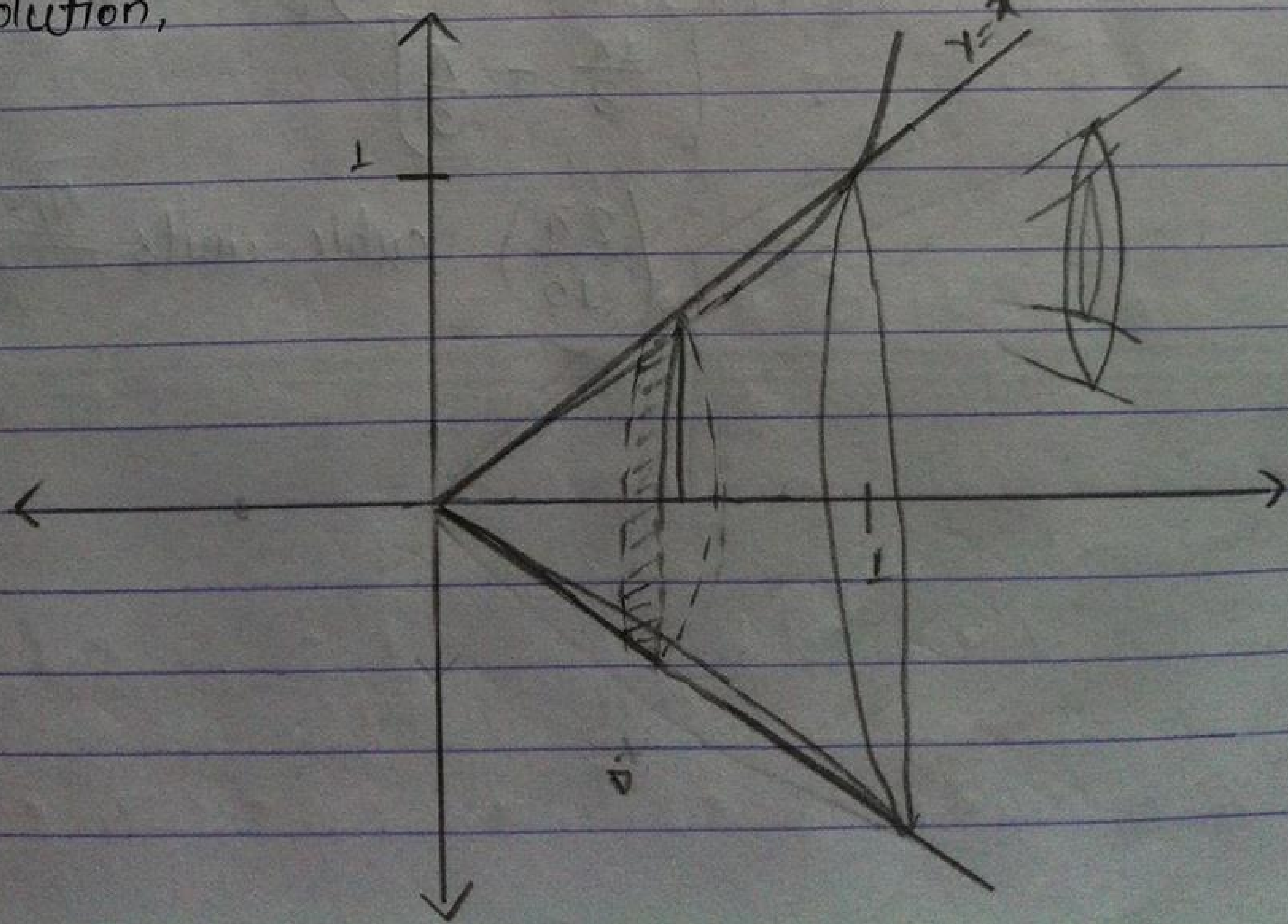
$$= 0.05 \times [1 + 1.818181818 + 1.666666667 + 1.538461538 + 1.428571429 + 1.333333333 + 1.25 + 1.176470588 + 1.111111111 + 1.052631579 + 0.5]$$

$$= 0.6937714028$$

Ans

(Q.N.3.a)

= solution,



Here, the curves $y = x$ and $y = x^2$ intersect at the origin and at $(1, 1)$. A typical shell can be constructed, whose cross-sectional area is given by :-

$A(x)$ = Area of bigger circle − Area of inner smaller circle

$$= \pi(x)^2 - \pi(x^2)^2$$

$$= (\pi x^2 - \pi x^4)$$

Then,

Volume of solid $= \int_0^1 A(x) \cdot dx$

$$= \int_0^1 (\pi x^2 - \pi x^4) \cdot dx$$

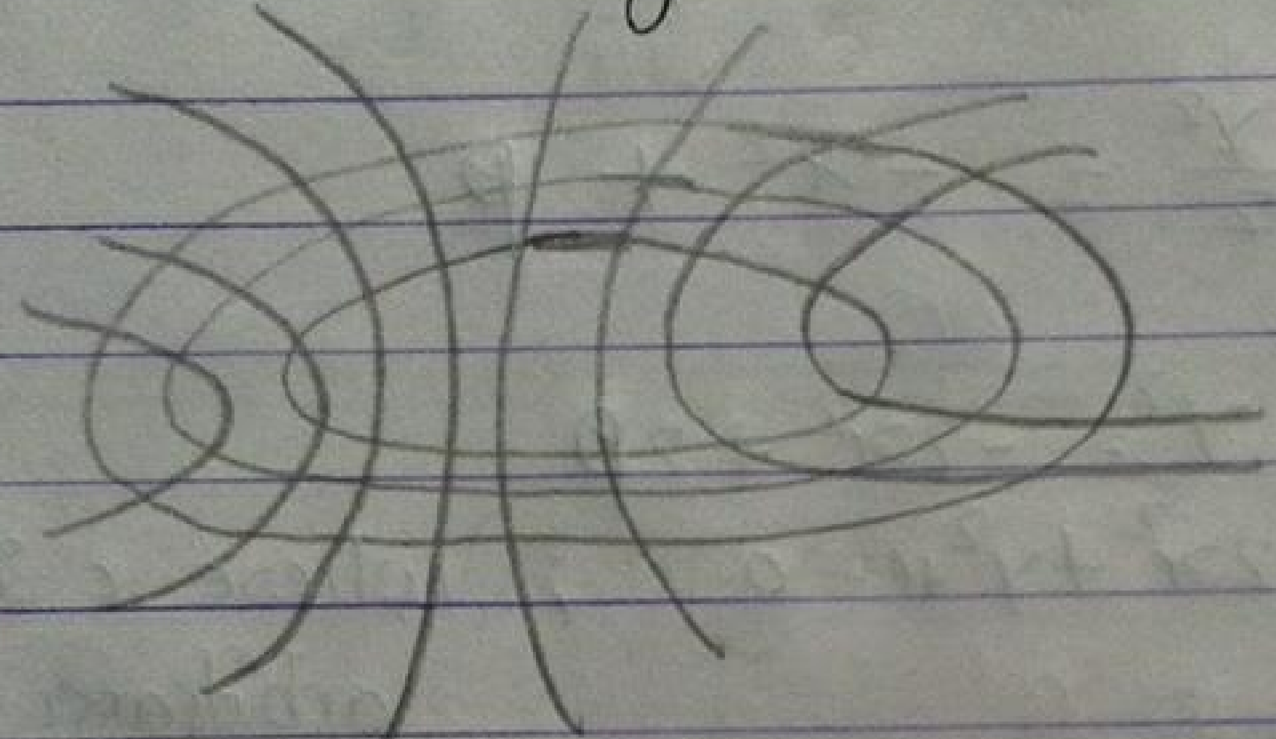
$$= \left[\frac{\pi x^3}{3} - \frac{\pi x^5}{5}\right]_0^1$$

$$= \left[\frac{1\pi}{3} - \frac{\pi}{5}\right]$$

$$= \left(\frac{2\pi}{15}\right) \text{ cubic units}$$ Ans

(Q·N·3.a)

If two families of curves are such that each member of a family cuts each member of the other family at right angles, then the members of one family are said to be the orthogonal trajectories of the other family. eg: The orthogonal trajectories of rectangular hyperbolas is $xy = c^2$.



= solution,

here,

$$x = ky^2 \text{ ——— } (1)$$

or, $ky^2 = x$

Differentiating both sides w.r.t. x; we get--

$$2ky \cdot \frac{dy}{dx} = 1$$

or, $\frac{dy}{dx} = \frac{1}{2ky}$

or, $\frac{dy}{dx} = \frac{y}{2x}$ $\left[\because k = \frac{x}{y^2}\right]$

∴ The slope of orthogonal trajectories is :-

$$\left(\frac{dy}{dx}\right) = -\frac{2x}{y}$$

or, $y \times dy = (-2x) \cdot dx$

Integrating both sides,

$$\int y \cdot dy = (-2) \int x \cdot dx$$

or, $\frac{y^2}{2} = -x^2 + k$

or, $y^2 = -2x^2 + 2k$

or, $2x^2 + y^2 + c = 0$; where c is an arbitrary constant.

∴ $2x^2 + y^2 + c = 0$ are the orthogonal trajectories of the parabolas $x = ky^2$.

(Q. N. 4. a)

= solution,

$A(2, 4, -3)$ and $B(3, -1, 1)$ be the points on any line.

Then,

$$\overrightarrow{AB} = (3-2, -1-4, 1+3) = (\vec{i} - 5\vec{j} + 4\vec{j})$$

Here,

Either A or B can be used to find the parametic equation. let us use $A(2, 4, -3)$. Then;

$x = (2+t),\ y = (4-5t),\ z = (-3+4t)$

These are the parametric equations.

Now,

solving for 't';

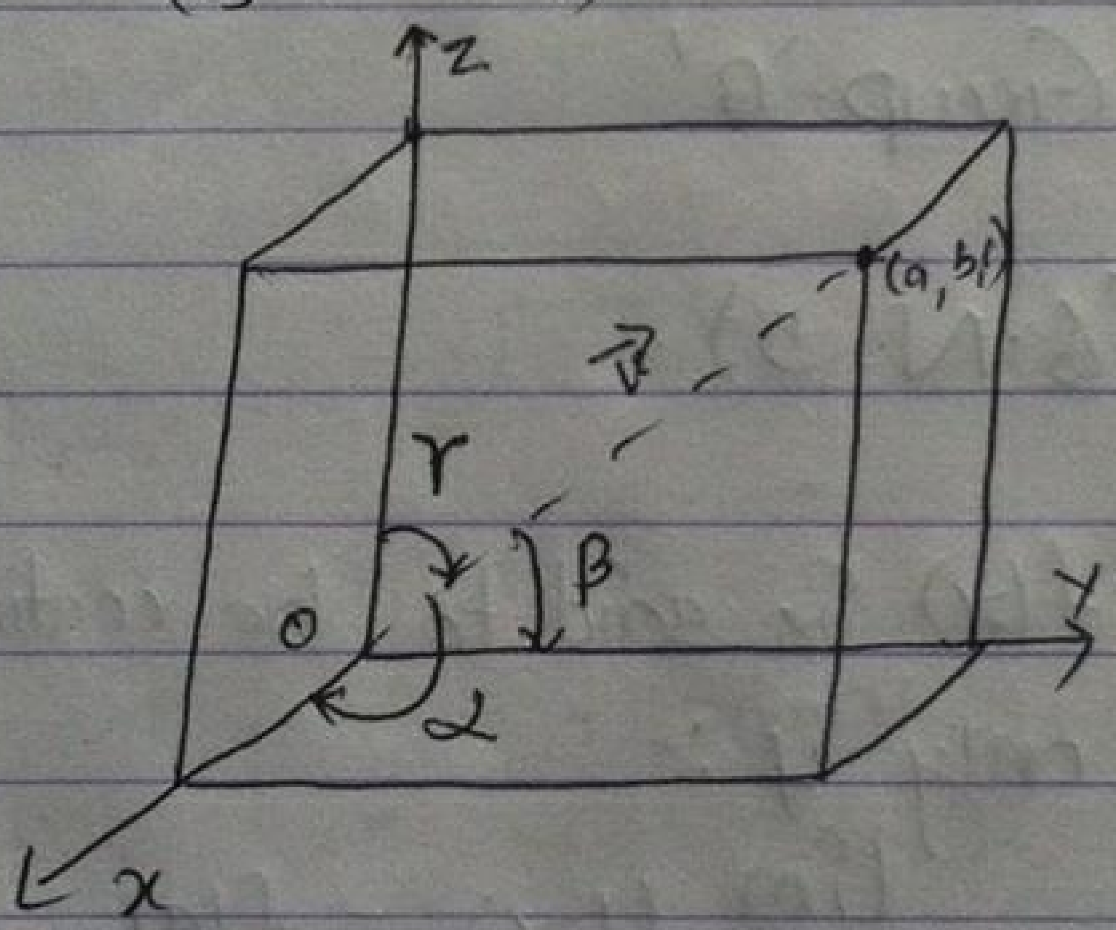
$$t = \frac{(x-2)}{1} = \frac{-y+4}{+5} = \frac{(z+3)}{4}$$

$$\Rightarrow \frac{(x-2)}{1} = \frac{(4-y)}{5} = \frac{(z+3)}{4}$$; which are the symmetric eqns.

Ans

(Q.N. 4.b)

= solution,



given vector $\vec{v} = (1, 2, 3)$

α be the angle formed between $\vec{v}$ and x-axis.

Then;

$$\cos\alpha = \frac{(1,2,3)\cdot(1,0,0)}{\sqrt{1+4+9} \times 1} = \frac{1}{\sqrt{14}}$$

$\therefore \alpha = 74.499°$

If β is the angle between $\vec{v}$ and the y-axis;

$$\cos\beta = \frac{(1,2,3)\cdot(0,1,0)}{\sqrt{1+4+9}\times 1}$$

$$= \frac{2}{\sqrt{14}}$$

$$\therefore \beta = 57.688^\circ$$

If r be the angle between $\vec{v}$ and z-axis;

$$\cos r = \frac{(1,2,3)\cdot(0,0,1)}{\sqrt{14}\times 1} = \frac{3}{\sqrt{14}}$$

$$\therefore r = 36.699^\circ$$

Ans

Group-'B'

(Q.N. 5)

A function $f(x)$ is said to be continuous at a point $x = a$ if and only if :-

$$\lim_{x\to a} f(x) = f(a)$$

This definition implicitly requires following conditions:-

(i) $f(a)$ is defined. (i.e. 'a' is in the domain of f)

(ii) $\lim_{x\to a} f(x)$ exists. and,

(iii) $$\lim_{x\to a} f(x) = f(a)$$

A function is continuous on an interval if it is continuous at every point in the interval.

= solution,

If $-1 < a < 1$, then, using the limit laws, we have.

$$\lim_{x \to a} f(x) = \lim_{x \to a} (1 - \sqrt{1-x^2})$$

$$= (1 - \lim_{x \to a} \sqrt{1-x^2})$$

$$= (1 - \sqrt{1-a^2})$$

$$= f(a)$$

similarly,

$$\lim_{x \to -1^+} f(x) = 1 - \sqrt{1-1} = 1$$

$$\lim_{x \to 1^-} f(x) = 1 - \sqrt{1-1} = 1$$

equal & thus $\lim_{x \to a} f(x)$ exists.

So, $f(x)$ is continuous in the interval $[-1, 1]$.

(Q·N·7)

= solution,

$$\lim_{x \to 0} \frac{\tan x - x}{x^3} \quad \left[\because - \frac{0}{0} \text{ form} \right]$$

Using L-Hospital's Rule;

$$= \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} \quad \left[\because \frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to 0} \frac{\tan^2 x}{3x^2}$$

$$= \frac{1}{3}\left[\lim_{x\to 0} \frac{\tan x}{x}\right]^2$$

$$= 1/3 \times 1^2$$

$$= (1/3) \quad \underline{\text{Ans}}$$

(Q.N. 9)

= solution,

let: $I = \int_0^1 \ln x \cdot dx$

This is an improper integral since $\ln 0$ is not defined. Thus;

$$I = \lim_{b\to 0^+} \int_b^1 \ln x \cdot dx$$

$$= \lim_{b\to 0^+} \int_b^1 1 \cdot \ln x \cdot dx$$

$$= \lim_{b\to 0^+} \left[x \ln x - \int \frac{1}{x} \cdot x \cdot dx\right]_b^1$$

$$= \lim_{b\to 0^+} \left[x \ln x - x\right]_b^1$$

$$= -1 - \lim_{b \to 0^+} \frac{\ln b}{b^{-1}} \qquad \left[\because -\frac{\infty}{\infty} \text{ form}\right]$$

$$= -1 - \lim_{b \to 0^+} \frac{(1/b)}{-1\, b^{-2}}$$

$$= -1 + \lim_{b \to 0^+} b$$

$$= -1 + 0$$

$$= (-1) \qquad \underline{\underline{\text{Ans}}}$$

(Q.N. 11)

The solution of non-homogeneous second order linear differential equation is the sum of complementary function (CF) and particular indication (P.I) ; i.e.

$$Y = CF + PI$$

$$\text{i.e. } Y = (Y_c + Y_p)$$

= solution,

here,

$$y'' - 4y = xe^x + \cos 2x \quad \text{--- (i)}$$

This is a non-homogeneous second order linear differential equation. so,

For Complementary solution,

The auxiliary equation is :-

$$m^2 - 4 = 0$$

$$\Rightarrow m = -2, +2$$

$\therefore$ Complementary Function (CF) $= (c_1 e^{-2x} + c_2 e^{2x})$

$$\therefore y_c = c_1 e^{-2x} + c_2 e^{2x} \quad \text{--- (ii)}$$

For Particular solution, we have :-

$$G(x) = G_1(x) + G_2(x)$$

We find y_p separately for both $G_1(x)$ and $G_2(x)$.

For $G_1(x) = xe^x$;

let $y_{p1} = (Ax + B)e^x$

$\therefore y' = (Ax+B)e^x + Ae^x$

$\therefore y'' = (Ax+B)e^x + 2Ae^x$

$\therefore$ eq^n (i) can be transformed as :-

$$Axe^x + Be^x + 2Ae^x - 4Axe^x - 4Be^x = xe^x$$

or, $-3Axe^x - 3Be^x + 2Ae^x = xe^x + 0e^x$

or, $-3Axe^x + (2A - 3B)e^x = xe^x + 0e^x$

Equating corresponding coefficients; $A = \left(-\frac{1}{3}\right)$,

$2A - 3B = 0$

or, $3B = 2A$

or, $B = \frac{-2}{3} \times \frac{1}{3}$

$\therefore \quad B = \left(-\frac{2}{9}\right)$

$\therefore Y_{P1} = (Ax + B)\,e^x \quad = e^x \cdot \left(-\frac{x}{3} - \frac{2}{9}\right)$

$$= -\frac{e^x}{3}\left(x + \frac{2}{3}\right)$$

Now,

let $Y_{P2} = (A\cos 2x + B\sin 2x)$

$\therefore y' = (-2A\sin 2x + 2B\cos 2x)$

$\therefore y'' = (-4A\cos 2x - 4B\sin 2x)$

$\therefore$ eq^n ① can be modified to :-

$$-4A\cos 2x - 4B\sin 2x - 4A\cos 2x - 4B\sin 2x = \cos 2x$$

or, $\quad -8A\cos 2x - 8B\sin 2x = \cos 2x + 0\sin 2x$

Equating the corresponding coefficients;

$-8A = 1 \quad , \quad -8B\sin 2x = 0\sin 2x$

$\Rightarrow A = \left(-\frac{1}{8}\right) \quad \Rightarrow \quad B = 0$

$\therefore \quad Y_{P2} = \left(-\frac{\cos 2x}{8}\right)$

so;

Particular Indication (PI) $= Y_{P1} + Y_{P2}$

$$= -\frac{e^x}{3}\left(x + \frac{2}{3}\right) - \frac{\cos 2x}{8}$$

So, The required general solution is :-

$$y_c + y_p$$
$$= c_1 e^{-2x} + c_2 e^{2x} - \frac{e^x}{3}\left(x + 2/3\right) - \frac{\cos 2x}{8}$$

Ans

(Q.N. 15)

= solution,

$$f(x,y) = (x^3 + x^2 y^3 - 2y^2)$$

so,

$$\frac{\partial f}{\partial x} = (3x^2 + 2xy^3)$$

And;

$$\frac{\partial^2 f}{\partial x^2} = (6x + 2y^3)$$

$$\frac{\partial^2 f}{\partial x \partial y} = (6xy^2)$$

Now;

$$\frac{\partial f}{\partial y} = (3x^2y^2 - 4y)$$

And,

$$\frac{\partial^2 f}{\partial y^2} = (6x^2 y - 4), \quad \frac{\partial^2 f}{\partial y \partial x} = (6xy^2)$$

Ans

(Q.N. 12)

= solution,

$$f(x) = x\cos x \text{ ——— } ①$$

Firstly;
Consider $g'(x) = \cos x$

And;

$$g(x) = \cos x \quad , \quad g(0) = 1$$
$$g'(x) = -\sin x \quad , \quad g'(0) = 0$$
$$g''(x) = -\cos x \quad , \quad g''(0) = -1$$
$$g'''(x) = \sin x \quad , \quad g'''(0) = 0$$
$$g^{iv}(x) = \cos x \quad , \quad g^{iv}(0) = 1$$
$$g^{v}(x) = -\sin x \quad , \quad g^{v}(0) = 0$$

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as:-

$$g(0) + \frac{g'(0)}{1!}x + \frac{g''(0)}{2!}x^2 + \frac{g'''(0)}{3!}x^3 + \cdots$$

$$= 1 + 0 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \text{ for all } x$$

So;

$f(x) = x\cos x$ gives :-

$$f(x) = x\,g(x)$$

$$= x\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots\right]$$

$$= x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n)!} \quad ; \text{ for all } x$$

This is the required expression.

(Q.N. 10)

= solution,

$$Y = f(x) = \left(x^2 - \tfrac{1}{8}\ln x\right)$$

starting point = $P(1,1)$

arc length $(L) = ?$

Here,

$$f'(x) = \left(2x - \frac{1}{8x}\right)$$

$$\therefore \quad 1 + [f'(x)]^2 = 1 + \left(2x - \tfrac{1}{8x}\right)^2$$

$$= 1 + (2x)^2 - 2\cdot 2x \cdot \tfrac{1}{8x} + \left(\frac{1}{8x}\right)^2$$

$$= (2x)^2 + \tfrac{1}{2} + \left(\frac{1}{8x}\right)^2$$

$$= (2x)^2 + 2\cdot 2x \cdot \frac{1}{8x} + \left(\frac{1}{8x}\right)^2$$

$$\therefore \quad 1 + [f'(x)]^2 = \left(2x + \frac{1}{8x}\right)^2$$

$$\Longrightarrow \sqrt{1 + [f'(x)]^2} = \left(2x + \frac{1}{8x}\right)$$

Now,

The arc-length function is given by:-

$$s(x) = \int_a^x \sqrt{1+[f'(t)]^2}\,.\,dt$$

$$= \int_a^x \sqrt{(2t+\frac{1}{8t})^2}\,.\,dt$$

$$= \int_a^x \left(2t+\frac{1}{8t}\right).\,dt$$

$$= \left[t^2 + \frac{\ln t}{8}\right]_1^x$$

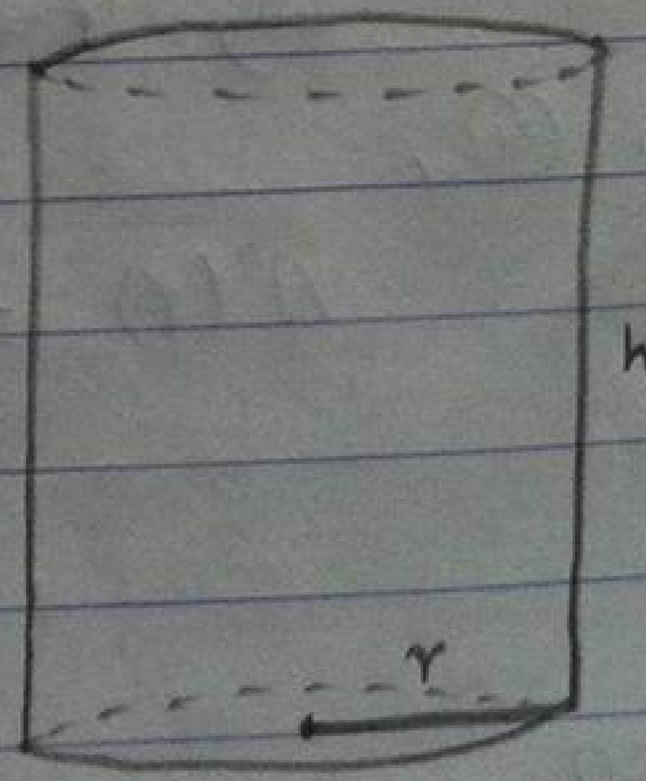
$$= \left[x^2 - 1 + \frac{\ln x}{8}\right]$$

For instance, the arc length along the curve from $(1,1)$ to $(3, f(3))$ is :-

$$s(3) = \left(3^2 - 1 + \frac{\ln 3}{8}\right) \approx 8.1373$$

Ans

(Q.N.8)



= solution,

let 'r' and 'h' be the radius and the height of the cylinder.

and,

Volume = 1 L

or, $\pi r^2 h = 1000\ cm^3$

$\Rightarrow\ h = \left(\frac{1000}{\pi r^2}\right)$

In order to minimize the manufacturing cost, we need to ~~manufa~~ minimize the total surface area of cylinder.

And,

Total Surface Area (TSA) $= (2\pi r h + 2\pi r^2)$

or, $A = 2\pi r^2 + 2\cancel{\pi}\cancel{r} \cdot \frac{1000}{\cancel{\pi} r^{\cancel{2}}}$

or, $A = \left(2\pi r^2 + \frac{2000}{r}\right)$

$\Rightarrow A(r) = \left(2\pi r^2 + \frac{2000}{r}\right)$ ——— (i) ; $r > 0$

Differentiating above eqn. w.r.t. r;

$A'(r) = \left(4\pi r - \frac{2000}{r^2}\right)$ ——— (ii)

For minima;

$A'(r) = 0$

$\Rightarrow 4\pi r^3 - 2000 = 0$

$\Rightarrow r = \sqrt[3]{\frac{500}{\pi}}$

From (ii),

$A''(r) = \left(4\pi - \frac{4000}{r^3}\right)$

$= 4\pi - \frac{\cancel{4000}^{8} \times \pi}{\cancel{500}} = -4\pi > 0$

$\therefore$ TSA is maximum at $r = \sqrt[3]{\frac{500}{\pi}}$.

$\therefore$ Corresponding height $= \dfrac{1000}{\pi r^2}$

$$= \frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}}$$

$$= \frac{2 \times 500}{\pi \times \pi^{-2/3} \times (500)^{2/3}}$$

$$= 2 \times \frac{500^{1/3}}{\pi^{1/3}}$$

$$= 2\sqrt[3]{\frac{500}{\pi}}$$

$$\Rightarrow \quad h = 2r$$

Thus, to minimise the cost, the radius of cylinder should be $\sqrt[3]{\frac{500}{\pi}}$ and the height should be twice this value of r.

(Q.N.13)

= solution,

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right)$$

$$= \sum_{n=1}^{\infty}\frac{3}{n(n+1)} + \sum_{n=1}^{\infty}\frac{1}{2^n}$$

$$= 3\left[\sum_{n=1}^{\infty}\frac{1}{n(n+1)}\right] + \left[\sum_{n=1}^{\infty}\frac{1}{2^n}\right]$$

Here,

let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ & $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{2^n}$

so,

$$\sum_{n=1}^{\infty} b_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$= \frac{(1/2)}{(1 - 1/2)} \qquad [\because r = (1/2) < 1]$$

$$= \frac{(1/2)}{(1/2)}$$

$$= 1$$

And;

$$\sum_{i=1}^{n} a_n = \left[\frac{1}{i} - \frac{1}{i+1}\right] = \sum_{i=1}^{n} \frac{1}{i} - \sum_{i=1}^{n} \frac{1}{(i+1)}$$

$$= \left[\frac{1}{1} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \cdots \cancel{\frac{1}{n}}\right] - \left[\cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \cdots + \cancel{\frac{1}{n}} + \frac{1}{(n+1)}\right]$$

$$= \left(1 - \frac{1}{n+1}\right)$$

so;

$$S_{a_n} = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = 1 \qquad [\because \text{Convergent series}]$$

Thus, The sum of the given series is :-

$$S_n = 3S_{a_n} + S_{b_n}$$

$$= 3 + 1$$

$$= 4$$

Ans

(Set - 4 Q.N.14)

= solution,

Consider the given equation of planes as :-

$$x + y + z = 1 \text{ ——— (i)}$$

$$x - 3y + 2z = 1 \text{ ——— (ii)}$$

The vectors normal to plane (i) and (ii) are (1, 1, 1) & (1, -3, 2) respectively. The angle between the vectors is :-

$$\cos\theta = \frac{|A_1A_2 + B_1B_2 + C_1C_2|}{|\sqrt{A_1^2 + B_1^2 + C_1^2}| \, |\sqrt{A_2^2 + (B_2)^2 + C_2^2}|}$$

$$= \frac{|1 - 3 + 2|}{|\sqrt{3}| \times |\sqrt{1+9+4}|}$$

$$\cos\theta = 0$$

$$\Rightarrow \theta = 180^\circ$$

∴ The angle between two planes is 180°. This implies that the planes are parallel.

Now, for symmetric equations,

Setting $z = 0$; the point of intersection of the line on x-y plane is :-

$$x + y = 1 \Rightarrow 3x + 3y = 3$$

$$x - 3y = 1 \Rightarrow x - 3y = 1$$

$$4x = 4$$

$$\Rightarrow x = 1, \; y = 0$$

∴ The line intersects the xy-plane at (1, 0, 0).

The cross product of (1, 1, 1) and (1, -3, 2) is :-

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= (5\vec{i} - \vec{j} + (-4)\vec{k})$$

$$= (5\vec{i} - \vec{j} - 4\vec{k})$$

So, The symmetric equations for the line of intersection are :-

$$\frac{x-1}{5} = \frac{y-0}{-1} = \frac{z-0}{-4}$$

$$\Rightarrow \frac{x-1}{5} = -y = -\frac{z}{4} \quad \underline{\underline{\text{Ans}}}$$