UNIT-5

Random Numbers

The numbers generated by a process, whose outcome is unpredictable, and which cannot be sub sequentially reliably reproduced are called random numbers. Random numbers are basic ingredient in the simulation of almost all discrete systems. Computer languages have a subroutine, object, or function that will generate a random number.

⊛. Properties of random numbers: [Imp]

Random numbers have two important properties:

i) **Uniformity**: (i.e, they are equally probable every where).

ii) **Independence**: (i.e, the current value of random variable has no relation with the previous values).

⇒ Each random number R_i is an independent sample drawn from a continious uniform distribution between zero and one.

↳ pdf is given by; $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

↳ expectation is given by: $E(R) = \int_0^1 x\,dx = \frac{1}{2}$

↳ variance is given by: $V(R) = \int_0^1 x^2\,dx - [E(R)]^2 = \frac{1}{12}$

⇒ Some consequences of the uniformity and independence properties are:

↳ If the interval (0,1) is divided into n sub-intervals of equal length, the expected number of observations in each interval is N/n, where N is the total number of observations. Note that N has to be sufficiently large to show this trend.

↳ The probability of observing a value in a particular interval is independent of the previous values drawn.

[Imp], asked in micro-syllabus/model set — Types of random numbers

Differences between true and pseudo random numbers:

Pseudo-random	True-random
i) It is an algorithm of mathematical formula, later translated into relatively bits of programming code.	i) It is an extract randomness from physical phenomena and introduce it into a computer.
ii) It has fast responses in generating numbers.	ii) It has slow responses in generating numbers.
iii) In this, sequence of numbers can be reproduced.	iii) In this, sequence of numbers cannot be reproduced.
iv) In this, sequence of numbers is repeated.	iv) In this, sequence of numbers will or will not get repeated.
v) It is deterministic	v) It is non-deterministic.

Methods of generation of Random Numbers:

1) Linear Congruential Method: [Imp]

The linear congruential method produces a sequence of integers, X1, X2, ... between zero and m-1 according to the following recursive relationship:

$$\boxed{X_{i+1} = (aX_i + c) \bmod m, \quad i = 0,1,2,\ldots} \quad \text{--- (i)}$$

The initial value X_0 is called the seed, a is called the constant multiplier, c is the increment, and m is the modulus.

Case 1: If $c \neq 0$ in eqn (i), the form is called mixed congruential method.

Case 2: When $c = 0$, the form is known as the multiplicative congruential method. The selection of the values for a, c, m, and X_0 drastically affects the statistical properties and the cycle length.

Example1: Use the linear congruential method to generate a sequence of random numbers with $X_0=27$, $a=17$, $c=43$, and $m=100$.

Solution:

theory for understanding only

Here, the integer values generated will all be between 0 and 99 because of the value of the modulus. These random integers should appear to be uniformly distributed between the integers 0 to 99. Random numbers between 0 and 1 can be generated by $R_i = X_i/m$, $i=1,2,\ldots$

The sequence of X_i and subsequent R_i values is computed as follows:

formula $X_{i+1} = (aX_i + c) \bmod m$ where $i=0,1,2,\ldots$

$X_0 = 27$

$X_1 = (17\times 27 + 43) \bmod 100$
$= 502 \bmod 100$
$= 2$

& $R_1 = \frac{2}{100} = 0.02$ — $R_i = \frac{X_i}{m}$

$X_2 = (17\times 2 + 43) \bmod 100 = 77 \bmod 100 = 77$

$R_2 = \frac{77}{100} = 0.77$

$X_3 = (17\times 77 + 43) \bmod 100 = 1352 \bmod 100 = 52$

$R_3 = \frac{52}{100} = 0.52$

we use left side value of a if given like this — we keep right side value for actual solving

Example2: Let $a = 7^5 = 16{,}807$, $m = 2^{31}-1 = 2{,}147{,}483{,}647$ (a prime number), and $c=0$. These choices satisfy the conditions that ensure a period of $P=m-1$. Further, specify a speed $X_0 = 123{,}457$. Calculate/Generate sequence of random numbers.

Solution:

Given, $a=75$, $m=(231-1)$, $c=0$ & $X_0=123{,}457$.

i.e. 2,147,483,647 given

The first few numbers generated are as follows:

$X_1 = (75)(123{,}457) \bmod (231-1) = 2{,}074{,}941{,}799 \bmod (231-1)$

or, $X_1 = 2{,}074{,}941{,}799$

$R_1 = \frac{2{,}074{,}941{,}799}{231-1} = 0.966$

actual value is 2,147,483,647

$X_2 = (75)(2{,}074{,}941{,}799) \bmod (231-1) = 559{,}872{,}160.$

$R_2 = \frac{X_2}{231-1} = 0.2607$

$X_3 = (75)\ (559,872,160) \bmod (231-1) = 1,645,535,613$

$$R_3 = \frac{X_3}{231-1} = 0.7662$$

2) Mid-Square Method:

Mid square method is also used to generate pseudo random numbers. For this we follow following steps:

→ Select a seed number (X) with n digits ($X_1\ X_2 \ldots X_n$). X is the initial random number. Note: n is even.

→ Square X to obtain number (Y) with m-digits.

$$Y_1\ Y_2 \ldots Y_m = \{X_1X_2 \ldots X_n\}^2$$

→ Add zeros to the left of Y to form Z number with (2n) digits,

$$Z_1Z_2 \ldots Z_{2n} = 00\ldots0\,Y_1Y_2\ldots Y_m$$

→ Extract the middle n digits of Z, which is represented by number R,

$$R_1R_2 \ldots R_n = Z_{(n/2)}Z_{(n/2)+1} \ldots Z_{(n/2)}$$

Example: Calculate the first five random numbers using mid-square method for seed = 1920.

Solution:

1920, $(1920)^2 = 3686400$, 03686400, then $R_1 = 6864$

6864, $(6864)^2 = 47114496$, 47114496, then $R_2 = 1144$

1144, $(1144)^2 = 1308736$, 01308736, then $R_3 = 3087$

3087, $(3087)^2 = 9529569$, 09529569, then $R_4 = 5295$

5295, $(5295)^2 = 28037025$, 28037025, then $R_5 = 0370$.

Tests for Randomness:

a) Uniformity testing:

1) Kolmogorov-Smirnov (K-S) test: [Imp]

theory can be escaped algorithm and numerical, mostly solving numerical is only imp. ✓

→ This test compares the continuous cdf, $F(x)$, of the uniform distribution to the empirical cdf, $S_N(x)$, of the sample of N observations. By definition:

$$F(x) = x, \quad 0 \leq x \leq 1.$$

→ If the sample from the random-number generator is $R_1, R_2, \dots, R_N$, then the empirical cdf, $S_N(X)$, is defined by:

$$S_N(x) = (\text{number of } R_1, R_2, \dots, R_n \text{ which are } <= x)/N.$$

→ As N becomes larger, $S_N(x)$ should become a better approximation to $F(x)$, provided that the null hypothesis is true.

→ $D = \max|F(x) - S_N(x)|$. When the sampling distribution D is known, it is tabulated as a function of N in Table.

Algorithm for K-S Test:

Step1: Rank the data from smallest to largest. Let $R_{(i)}$ denote the ith smallest observation, so that $R_{(1)} <= R_{(2)} <= \dots <= R_{(N)}$.

Step2: Compute

$$D^+ = \max\{i/N - R_i\}$$

$$D^- = \max\{R_i - (i-1)/N\}$$

Step3: Compute $D = \max\{D^+, D^-\}$

Step4: Determine the critical value, D_α, from Table.

Step5: If the sample statistic D is greater than the critical value D_α, the null hypothesis that the data are a sample from a uniform distribution is rejected.

Step6: If $D <= D_\alpha$, conclude that no difference has been detected between the true distribution of $\{R_1 R_2, \dots, R_n\}$ and the uniform distribution. Hence the null hypothesis is accepted.

Example: Suppose that the five numbers 0.44, 0.81, 0.14, 0.05, 0.93 were generated, and it is desired to perform a test for uniformity using the Kolmogorov-Smirnov test with a level of significance of 0.05.

Solution:

We can also solve same method as we did in statistics by setting up hypothesis it is better

Ranking numbers from smallest to largest we get,

0.05, 0.14, 0.44, 0.81, 0.93.

The calculations can be faciliated by use of Table below:

R_i	0.05	0.14	0.44	0.81	0.93
i/N	0.2	0.4	0.6	0.8	1.0
$i/N - R_i$	0.15	0.26	0.16	—	0.07
$R_i - (i-1)/N$	0.05	—	0.04	0.21	0.13

$N=5$
$i=1,2,3,4,5$

if values go beyond given values we write, — since here we got 0.95 and −0.06 values (two) which are outside 0.05 to 0.93 so rejected

Now,

$$D^+ = \max\{i/N - R_i\}$$
$$= \max\{0.15, 0.26, 0.16, 0.07\}$$
$$= 0.26$$

$$D^- = \max\{R_i - (i-1)/N\}$$
$$= \max\{0.05, 0.04, 0.21, 0.13\}$$
$$= 0.21$$

$$\therefore D = \max\{D^+, D^-\} = \max\{0.26, 0.21\} = 0.26.$$

table A.8 given in book in appendix

The critical value of D, obtained from table for $\alpha = 0.05$ and $N=5$ is 0.565.

OR $B=5$

if $D <= D_\alpha$ accepted otherwise rejected.

Since the computed value, 0.26 is less than the tabulated value, 0.565, the hypothesis that the distribution of the generated numbers is the uniform distribution is not rejected.

2) Chi-Square Test:

The chi-square test uses the sample statistic

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where, O_i is the observed number in the ith class, E_i is the expected number in the ith class, and n is the number of classes. For the uniform distribution, E_i the expected number in each class is given by;

$E_i = N/n$. for equally spaced classes where N is the total number of observations.

It can be shown that the sampling distribution of χ^2 is approximately the chi-square distribution with $n-1$ degrees of freedom.

Example: Use the chi-square test with $\alpha = 0.05$ to test whether the data shown below are uniformly distributed.

0.34	0.83	0.96	0.47	0.79	0.99	0.37	0.72	0.06	0.18
0.90	0.76	0.99	0.30	0.71	0.17	0.51	0.43	0.39	0.26
0.25	0.79	0.77	0.17	0.23	0.99	0.54	0.56	0.84	0.97
0.89	0.64	0.67	0.82	0.19	0.46	0.01	0.97	0.24	0.88
0.87	0.70	0.56	0.56	0.82	0.05	0.81	0.30	0.40	0.64
0.44	0.81	0.41	0.05	0.93	0.66	0.28	0.94	0.64	0.47
0.12	0.94	0.52	0.45	0.65	0.10	0.69	0.96	0.40	0.60
0.21	0.74	0.73	0.31	0.37	0.42	0.34	0.58	0.19	0.11
0.46	0.22	0.99	0.78	0.39	0.18	0.75	0.73	0.79	0.29
0.67	0.74	0.02	0.05	0.42	0.49	0.49	0.05	0.62	0.78

Solution: The table for chi square statistics is:

i.e, 0.0 to 0.1 values in number

similarly 0.11 to 0.2 no. of values

0.91 to 1

Class interval (i)	O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
1	8	10	-2	4	0.4
2	8	10	-2	4	0.4
3	10	10	0	0	0.0
4	9	10	-1	1	0.1
5	12	10	2	4	0.4
6	8	10	-2	4	0.4
7	10	10	0	0	0.0
8	14	10	14	16	0.16
9	10	10	0	0	0.0
10	11	10	1	1	0.1

$N = 100$

$\Sigma (O_i - E_i)^2 / E_i = 3.4$

Now,

$$\chi_0^2 = \sum_{i=0}^{n} \frac{(O_i - E_i)^2}{E_i} = 3.4$$

i.e, critical value < tabulated value so accepted i.e, not rejected

The value of χ_0^2 is $3.4 < \chi^2_{0.05,9} = 16.9$. Hence, the data given are uniformly distributed.

b) Independent Testing:

1) Gap test: Gap test counts the number of digits that appear between repetitions of particular digit and then uses the Kolmogorov–Smirnov test to compare with expected size of gaps. The following example illustrates the length of gaps associated with the digit 3:

4, 1, <u>3</u>, 5, 1, 7, 2, 8, 0, 7, 9, 1, <u>3</u>, 5, 2, 7, 9, 4, 1, 6, <u>3</u>, <u>3</u>, 9, 6, <u>3</u>, 4, 8, 2, <u>3</u>, 1, 9, 4, 4, 6, 8, 4, 1, <u>3</u>.

There are 7 three's, so only 6 gaps can occur. The first gap is of length 9, second gap of length 7, third gap of length zero and so on.

Example: Based on the frequency with which gap occur, analyze the 110 digits below to test whether they are independent. Use $\alpha = 0.05$.

4, 1, 3, 5, 1, 7, 2, 8, 2, 0, 7, 9, 1, 3, 5, 2, 7, 9, 4, 1, 6, 3, 3, 9, 6,
3, 4, 8, 2, 3, 1, 9, 4, 4, 6, 8, 4, 1, 3, 8, 9, 5, 5, 7, 3, 9, 5, 9,
8, 5, 3, 2, 2, 3, 7, 4, 7, 0, 3, 6, 3, 5, 9, 9, 5, 5, 5, 0, 4, 6, 8, 0,
4, 7, 0, 3, 3, 0, 9, 5, 7, 9, 5, 1, 6, 6, 3, 8, 8, 8, 9, 2, 9, 1, 8, 5, 4, 4,
5, 0, 2, 3, 9, 7, 1, 2, 0, 3, 6, 3.

Solution: The number of gaps is given by the number of data values minus the number of distinct digits, 0 to 9 total 10.

i.e, $N = 110 - 10 = 100$

The numbers of gaps associated with the various digits are as follows:

OR look in theory of gap test topic to know how to find gap length

We are first finding for zero. So, look at data and find first 0. Now, count after first zero as 1, 2, 3, ... until next zero gets repeated. (For e.g. we get 2nd zero after 47 digits). Similarly for all

Digit	Length of Gap	No. of Gaps
0	47, 9, 3, 2, 2, 21, 6	7
1	2, 7, 6, 10, 6, 45, 9, 10	8
2	1, 6, 12, 22, 0, 38, 8, 4	8
3	10, 7, 0, 2, 3, 8, 5, 5, 2, 4, 1, 14, 0, 9, 14, 5, 1	17
4	17, 7, 5, 0, 2, 18, 12, 3, 23, 0	10
5	10, 26, 0, 3, 2, 11, 2, 0, 0, 12, 2, 12, 2	13
6	3, 9, 24, 9, 14, 0, 22	7
7	4, 5, 26, 10, 1, 16, 6, 22	8
8	19, 7, 3, 8, 21, 16, 0, 0, 4	9
9	5, 5, 7, 8, 4, 1, 14, 0, 14, 0, 14, 2, 8, 1, 9	13

Note: Frequency is given by no. of length gap in a given interval. For e.g. for interval 0–3; 0, 1, 2, and 3 are repeated above 35 times. So, frequency of interval 0–3 is 35. Similarly others.

x is upper limit of gap length

The calculation for gap test is shown in following table: $1-0.9^{x+1}$ i.e. $1-0.9^{3+1}$

Gap Length	Frequency (f)	Relative frequency (f/N)	Cumulative frequency S(x) (adding each freq)	Theoretical Frequency F(x)	$D=\lvert F(x)-S(x)\rvert$
0–3	35	0.35	0.35	0.3439	0.0061
4–7	22	0.22	0.57	0.5695	0.0005
8–11	17	0.17	0.74	0.7176	0.0224
12–15	9	0.09	0.83	0.8147	0.0153
16–19	5	0.05	0.88	0.8784	0.0016
20–23	6	0.06	0.94	0.9202	0.0198
24–27	3	0.03	0.97	0.9497	0.0223
28–31	0	0	0.97	0.9657	0.0043
32–35	0	0	0.97	0.9775	0.0075
36–39	2	0.02	0.99	0.9852	0.0043
40–43	0	0	0.99	0.9903	0.0003
44–47	1	0.01	1.00	0.9936	0.0064

The critical value of D is given by $D_{0.05} = 1.36/\sqrt{100} = 0.136$. ($\sqrt{N}$)

Since $D = \max|F(x) - S(x)| = 0.0224$ is less than $D_{0.05}$, so we do not reject the hypothesis of independence on the basis of this test.

2) Auto Correlation test:

Auto correlation test is a statistical test that determines whether a random number generator is producing independent random number in a sequence. The test for the auto correlation is concerned with the dependence between numbers in sequence. The variables involved in this test are:

$m \rightarrow$ It is the lag, the space between the number being tested.
$i \rightarrow$ It is the index or number from which we start.
$N \rightarrow$ It is the number of random numbers generated.
$M \rightarrow$ It is the largest integer such that $i+(M+1)m <= N$.

Now, the autocorrelation between $R_i, R_{i+m}, R_{i+2m}, \ldots, R_{i+(M+1)m}$ is computed as;

$$\rho_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} R_{i+(k+1)m}\right] - 0.25$$

Now the test statistics is;

$$Z_0 = \frac{\rho_{im}}{\sigma_{im}}$$

where, $\sigma_{im} = \frac{\sqrt{13M+7}}{12(M+1)}$

After computing Z_0, accept the null hypothesis if independence of $-Z_{\alpha/2} \le Z_0 \le Z_{\alpha/2}$. where, α is level of significance.

Example:

0.19, 0.16, 0.82, 0.63, 0.04, 0.16, 0.3, 0.22, 0.88, 0.48, 0.23, 0.56, 0.44, 0.05, 0.81, 0.38, 0.59, 0.37, 0.75, 0.43, 0.92, 0.45, 0.57, 0.99, 0.2, 0.14, 0.64, 0.5, 0.73, 0.15, 0.02, 0.49, 0.86, 0.24, 0.9, 0.74, 0.41, 0.09, 0.86, 0.42, 0.16, 0.23, 0.77, 0.08, 0.69, 0.46, 0.39, 0.18, 0.21, 0.98.

Take $\alpha = 0.05$, Test numbers at position 2nd, 7th, 12th are auto-correlated.

Solution:

Here, $i=2$ and $m=5$

Now, we have,

$$i+(M+1)m \le N$$
$$\Rightarrow 2+(M+1)5 \le 50$$
$$\Rightarrow 2+5M+5 \le 50$$
$$\Rightarrow 5M \le 43$$
$$\Rightarrow M \le 8.6 \quad \text{or, } M=8$$

Now,

$$\rho_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} \cdot R_{i+(k+1)m}\right] - 0.25$$

or, $$\rho_{25} = \frac{1}{8+1}\left[\sum_{k=0}^{8} R_{2+k\times 5} \cdot R_{2+(k+1)5}\right] - 0.25$$

$$= \frac{1}{9}\left[R_2 R_7 + R_7 \cdot R_{12} + R_{12} R_{17} + R_{17} R_{22} + R_{22} R_{27} + R_{27} R_{32} + R_{32} R_{37} + R_{37} \cdot R_{42} + R_{42} \cdot R_{47}\right] - 0.25$$

$$= \frac{1}{9}\left[0.16\times 0.3 + 0.3\times 0.56 + 0.56\times 0.59 + 0.59\times 0.45 + 0.45\times 0.64 + 0.64\times 0.49 + 0.49\times 0.41 + 0.41\times 0.23 + 0.23\times 0.39\right] - 0.25$$

$$= 0.172$$

Again, $$\sigma_{\rho_{im}} = \sigma_{25} = \frac{\sqrt{13M+7}}{12(M+1)}$$

$$= \frac{\sqrt{13\times 8+7}}{12(8+1)}$$

$$= 0.0975$$

and $$Z_0 = \frac{\rho_{25}}{\sigma_{25}} = \frac{0.172}{0.0975} = 1.764$$

The tabulated value of $Z_{0.05/2} = 1.96$

Since $-Z_{\alpha/2} \le Z_0 \le Z_{\alpha/2}$. Therefore numbers are independent.

3) Poker Test: [Imp],

mainly focus on numerical. Numericals are more imp

The poker test for independence is based on the frequency with which certain digits are repeated in a series of numbers. The poker test uses the chi-square statistics to accept or reject the hypothesis. The following example shows an unsual amount of repetition:

0.255, 0.577, 0.331, 0.414, 0.828, 0.909, 0.303, 0.001, ...

In three digit numbers there are only three possibilities as follows:

i) The individual numbers can all be different.

ii) The individual numbers can all be same.

iii) There can be one pair of like digits.

The probability associated with each of these possibilities is given by the following:

i) P(three different digits) = P(second different from the first) × P(third different from the first and second)

$= (0.9)(0.8) = 0.72$

ii) P(three like digits) = P(second digit same as the first) × P(third digit same as the first)

$= (0.1)(0.1) = 0.01.$

iii) P(exactly one pair) $= 1 - 0.72 - 0.01$

$= 0.27$

<u>Again For Four digit:</u> there are five possibilities:

i) P(four different digits) = C(4,4) × P(second different from first) × P(third different from first and second) × P(fourth different from 1st, 2nd and 3rd).

$= 1 \times 0.9 \times 0.8 \times 0.7$

$= 0.504$

ii) P(one pair) = C(4,2) × P(second similar as first) × P(third different from first) × P(fourth different from first and third)

$= 6 \times 0.1 \times 0.9 \times 0.8$

$= 0.432$

iii) P(three digits of a kind) = C(4,3) × P(second same as first) × P(third same as first) × P(fourth not same as first)

$= 4 \times 0.1 \times 0.1 \times 0.9$

$= 0.036$

iv) P(four digits of a kind) = C(4,4) × P(second same as first) × P(third same as first) × P(fourth same as first)

$= 1 \times 0.1 \times 0.1 \times 0.1$

$= 0.001$

v) P(two pair) = 1 − P(four different digits) − P(three digits of a kind) − P(four digits of a kind) − P(one pair).

$= 1 - 0.504 - 0.036 - 0.001 - 0.432$

$= 0.027$

Example: (Three digit):

A sequence of 1000 three-digit numbers has been generated and an analysis indicates that 680 have three different digits, 289 contain exactly one pair of like digits and 31 contain three like digits. Based on poker test, are the numbers independent? let $\alpha = 0.05$.

Solution:

Defining Hypothesis:

$H_0: R_i \sim$ independent

$H_1: R_i \nsim$ independent.

Calculation:

Combination	Observed frequency (O_i)	Expected Frequency (E_i)	$\frac{(O_i - E_i)^2}{E_i}$
Three different digit	680	$0.72 \times 1000 = 720$	2.22
One pair	289	$0.27 \times 1000 = 270$	1.33
Three like digit	31	$0.01 \times 1000 = 10$	44.1
			$\Sigma = 47.65$

Here, $\chi^2 = 47.65$

And, Tabulated $\chi^2_{\alpha, n-1} = \chi^2_{0.05, 3} = 5.99$

Since, calculated $\chi^2 >$ tabulated χ^2. So, H_0 is rejected and we conclude numbers are dependent.

Example: (Four digits):

Q. A sequence of 1000 four digit numbers have been generated and an analysis indicates the following combinations and frequencies.

Combination	Observed frequency O_i
Four different digits	560
One pair	394
Two pair	32
Three digits of kind	13
Four digits of kind	1

Test the independence.

Solution:

Define Hypothesis:

$H_0: R_i \sim$ independent

$H_1: R_i \nsim$ independent.

Calculation:

Combination.	Observed Frequency (O_i)	Expected Frequency (E_i)	$\frac{(O_i-E_i)^2}{E_i}$
Four different digits	560	0.504×1000=504	6.22
One pair	394	0.432×1000=432	3.434
Two pairs	32	0.027×1000=27	0.926
Three digits of kind	13	0.036×1000=36	14.694
Four digits of kind	1	0.001×1000=1	0
			Σ = 25.165

Here, $\chi^2 = 25.165$

And Tabulated $\chi^2_{\alpha, n-1} = \chi^2_{0.05, 4} = 9.49$

Since, Calculated value is greater than tabulated value. So, H_0 is rejected. Hence numbers are dependent.

⊛. Random Variate Generation:

1> Inverse Transform Method:

The inverse transform method can be used to sample from the exponential, uniform, triangular distribution etc. by inversing the cdf of those probability distributions. The inverse transform technique can be utilized for any distribution when the cdf, $F(x)$, is of a form that its inverse, F^{-1} can be computed caisly.

Uniform Distribution:

The pdf for random variable X in uniform distribution is given by;

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$$

and the cdf is given by;

$$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b. \end{cases}$$

Now, set $F(X) = \frac{X-a}{b-a} = R$.

$X = a + (b-a)R$ is the random variate generator for uniform distribution.

2) Acceptance / Rejection Method:

It is useful particularly when inverse cdf does not exist in closed form. Suppose that an analyst needed to think up a method for generating random variates, X uniformly distributed between 1/4 and 1. One way to proceed would be to follow these steps:

Step1: Generate a random number R.

Step2: (a) If $R \geq 1/4$, accept $X=R$, then go to step 3.

(b) If $R < 1/4$, reject R, and return to step 1.

Step3: If another uniform random variate on [1/4, 1] is needed, repeat the procedure beginning at step1. If not, stop.

The efficiency of an acceptance-rejection technique depend heavily on being able to minimize the number of rejections.

Additional: [Imp]

Q. Write a computer program in C that will generate four digit random numbers using the multiplicative congruential method. Allow the user to input values of X_0, a, c and m.

Solution:

```
#include<stdio.h>
#include<conio.h>
```

~~//Function to generate random numbers~~
~~void linearCongruentialMethod(int X0, int m, int a, int c~~

```
int main(){
    int X0, X1; /* X0=seed, X1=next random no. that
                   we will generate */
    int a,c,m; /*a=constant multiplier, c=increment,
                 m=modulus */
    int i,n; /*i for loop control, n for no. of random numbers*/
    n=4;  /* Given in question */
    int array[20]; /* to store random numbers generated */
    printf ("Enter the seed value X0: ");
    scanf ("%d", &X0);
    printf ("\n");

    printf("Enter the constant multiplier a: ");
    scanf ("%d", &a);
    printf ("\n");

    printf ("Enter the increment c: ");
    scanf ( "%d", &c);
    printf ("\n");

    printf ("Enter the modulus m: ");
    scanf ("%d", &m);
    printf ("\n");
```

```
for (i=0; i<n; i++)   /*loop to generate random numbers */
{
    X1 = (a* X0+c) % m;
    array[i] = X1;
    X0 = X1;
}
printf ("The generated random numbers are : ");
for (i=0; i<n; i++)
{
    printf ("%d", array[i]);
    printf ("\t");
}
getch ();
return (0);
}
```