Ch-1: Real Numbers

Exercise 1.2

1. Express each number as a product of its prime factors:

(i) 140

Sol:-

2	140
2	70
5	35
7	7
	1

$140 = 2 \times 2 \times 5 \times 7$
$= 2^2 \times 5 \times 7$

(ii) 156

Sol:-

2	156
2	78
3	39
13	13
	1

$156 = 2 \times 2 \times 3 \times 13$
$= 2^2 \times 3 \times 13$

(iii) 3825

Sol:-

3	3825
3	1275
5	425
5	85
17	17
	1

$3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$

(iv) 5005

Sol:-

5	5005
7	1001
11	143
13	13
	1

$5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Sol:-

17	7429
19	437
23	23
	1

$7429 = 17 \times 19 \times 23$

The Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of times and this factorisation is unique, apart from the order in which the prime factor occurs.

HCF: Highest Common Factor
It is a product of the smallest power of each common prime factors in the numbers.

LCM: ~~Least~~ Lowest Common Factor
It is a product of the greatest power of each prime factors involved in the numbers.

Relation between HCF, LCM and the numbers
$HCF \times LCM = a \times b$ where a and b are two numbers

2. Find the LCM and HCF of the following pairs of integers and verify that LCM X HCF = product of the two numbers.

(i) 26 and 91

Sol:-

2	26
13	13
	1

$26 = 2 \times 13$

7	91
13	13
	1

$91 = 7 \times 13$

Prime factor of 26 and 91	Greatest power
2	1
7	1
13	1

$\therefore LCM = 2^1 \times 7^1 \times 13^1$
$= 182$

Common factor of 26 and 91	Smallest power
13	1

$\therefore HCF = 13$

Verification:

$HCF \times LCM = 182 \times 13 = 2366$
$a \times b = 26 \times 91 = 2366$

(ii) 510 and 92

Sol:-

$$\begin{array}{r|l} 2 & 510 \\ \hline 3 & 255 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$510 = 2 \times 3 \times 5 \times 17$

$$\begin{array}{r|l} 2 & 92 \\ \hline 2 & 46 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$92 = 2 \times 2 \times 23$

Prime factor of 510 and 92	Greatest power
2	2
3	1
5	1
17	1
23	1

$\therefore LCM = 2^2 \times 3^1 \times 5^1 \times 17^1 \times 23^1$
$= 23460$

Common factor of 510 and 92	Smallest power
2	1

$\therefore HCF = 2$

Verification:

$HCF \times LCM = 2 \times 23460 = 46920$
$a \times b = 510 \times 92 = 46920$

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(iii) 336 and 54

Sol:

2	336
2	168
2	84
2	42
3	21
7	7
	1

$336 = 2^4 \times 3 \times 7$

2	54
3	27
3	9
3	3
	1

$54 = 2 \times 3^3$

Prime factor of 336 and 54	Greatest power
2	4
3	3
7	1

$\therefore LCM = 2^4 \times 3^3 \times 7$

$= 3024$

Common factor of 336 and 54	Smallest power
2	1
3	1

$\therefore HCF = 6$

Verification:

$HCF \times LCM = 6 \times 3024 = 18144$

$a \times b = 336 \times 54 = 18144$

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3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Sol:

2	12
2	6
3	3
	1

$12 = 2^2 \times 3$

3	15
5	5
	1

$15 = 3 \times 5$

3	21
7	7
	1

$21 = 3 \times 7$

$\therefore HCF = 3$

Prime factor of 12, 15 and 21	Greatest power
2	2
3	1
5	1
7	1

$\therefore LCM = 2^2 \times 3^1 \times 5^1 \times 7^1$
$= 420$

(ii) 17, 23 and 29

Sol:- $17 \mid 17$, 1 ; $17 = 17 \times 1$; $23 \mid 23$, 1 ; $23 = 23 \times 1$; $29 \mid 29$, 1 ; $29 = 29 \times 1$

Prime factor of 17, 23 and 29	Greatest power
17	1
23	1
29	1

$\therefore LCM = 17^1 \times 23^1 \times 29^1 = 11339$
$\therefore HCF = 1$

(iii) 8, 9 and 25

Sol:-

2	8
2	4
2	2
	1

$8 = 2 \times 2 \times 2$
$= 2^3$

3	9
3	3
	1

$9 = 3 \times 3$
$= 3^2$

5	25
5	5
	1

$25 = 5 \times 5$
$= 5^2$

$\therefore HCF = 1$.

Prime factor of 8, 9 and 25	Greatest power
2	3
3	2
5	2

$\therefore LCM = 2^3 \times 3^2 \times 5^2$
$= 1800$

4. Given that HCF (306, 657) = 9, find LCM (306, 657)

Sol:- HCF (306, 657) = 9

HCF × LCM = Product of two numbers ($a \times b$)

we get, $9 \times LCM = 306 \times 657$

$\Rightarrow LCM = \frac{306 \times 657}{9} = 306 \times 73 = 22338$

5. Check whether 6^n can end with the digit 0 for any number n

Sol:- Here, $6^n = (2 \times 3)^n = 2^n \times 3^n$

The only prime factors of 6 is 2 and 3 so by fundamental theorem of arthematic there are no other prime factor of 6^n so there is no natural number n for which 6^n ends with a digit 0

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol:- $7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1) = 13 \times 78$

$= 13 \times 13 \times 2 \times 3 = 2 \times 3 \times 13^2$

Factorisation of the number contain more than one prime

$\therefore$ number is composite

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5(1008 + 1) = 5 \times 1009$

Product of two numbers

$\therefore$ It is not a prime

$\therefore 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the

same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol:- They will meet again at the starting point in the time which will be equal to the LCM of 18 and 12.

Now

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \quad 18 = 2 \times 3^2 \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \quad 12 = 2^2 \times 3.$$

$\therefore LCM = 2^2 \times 3^2 = 36$

$\therefore$ They will meet after 36 minutes

<u>Irrational Numbers</u>: A number is called Irrational if it cannot be written in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

<u>Theorem 1.3</u>: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

<u>Theorem 1.4</u>: $\sqrt{2}$ is irrational.

<u>Proof</u>: Let us assume $\sqrt{2}$ is a rational number

$\sqrt{2} = \frac{a}{b}$, where a and b have no common factors (a and b are coprime)

square a, b $= 2b^2 = a^2 \rightarrow (1)$

Clearly 2 divides a^2 then 2 divides a

$\Rightarrow a = 2c$, c is any two integer

Substitute a in (1)

$2b^2 = (2c)^2$

$2b^2 = 4c^2$

$b^2 = 2c^2$

$\Rightarrow$ 2 divides b^2

2 divides b

$\Rightarrow$ b is a midpoint of 2

Since, a and b lies a common factors 2. It contradicts are assumptions.
$\therefore \sqrt{2}$ is irrational.

Prove that $\sqrt{3}$ is irrational
Proof: Let us assume $\sqrt{3}$ is a rational number.
$\sqrt{3} = \frac{a}{b}$, where a and b have no common factors (a and b are copri
squaring $a, b = 3b^2 = a^2 \rightarrow (1)$
Clearly 3 divides a^2 then 3 divides a
$\Rightarrow a = 3c$, c is any two integer
Substitute a in (1)
$3b^2 = (3c)^2$
$3b^2 = 9c^2$
$b^2 = 3c^2$
$\Rightarrow$ 3 divides b^2
3 divides b
$\Rightarrow$ b is a midpoint of 3
Since, a and b lies a common factors 3. It contradicts are assuptions
$\therefore \sqrt{3}$ is irrational

Date: 21/03/2023
Exercise 1.3
1. Prove that $\sqrt{5}$ is an irrational
Sol: Homework. Refer Homework book.

2. Prove that $3+2\sqrt{5}$ is irrational.
Sol: Let us assume that $3+2\sqrt{5}$ is rational such that $3+2\sqrt{5} = \frac{a}{b}$ where a and b are coprime and $b \neq 0$
$2\sqrt{5} = \frac{a}{b} - 3$

$2\sqrt{5} = \frac{a-3b}{b}$

$\sqrt{5} = \frac{a-3b}{2b}$

$\frac{a-3b}{2b}$ is a rational number which follows by $\sqrt{5}$ is rational

But. It constructs to the fact $\sqrt{5}$ is irrational

$\therefore 3+2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

Sol:- As $\sqrt{2}$ is irrational.

Now, let us assume that $\frac{1}{\sqrt{2}}$ is a rational number and $\frac{1}{\sqrt{2}} = \frac{a}{b}$, where a and b are coprime and $b \neq 0$.

$\Rightarrow \sqrt{2} = \frac{b}{a}$, $\because \frac{b}{a}$ are integers.

$\therefore \frac{b}{a}$ is a rational. So $\sqrt{2}$ is also rational

This contradicts the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our assumption is wrong.

Hence $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Sol:- As $\sqrt{5}$ is irrational.

Now let us assume that $7\sqrt{5}$ is a ~~ir~~rational

$\Rightarrow 7\sqrt{5} = \frac{a}{b}$, where a and b are coprime, $b \neq 0$.

Now, $7\sqrt{5} = \frac{a}{b} \Rightarrow \sqrt{5} = \frac{a}{7b}$

$\because$ a and b are integers.

$\therefore \frac{a}{7b}$ is rational $\Rightarrow \sqrt{5}$ is also rational

This contradicts the fact that $\sqrt{5}$ is irrational.

$\therefore$ Our assumption is wrong.

Hence $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Sol:- As $\sqrt{2}$ is irrational.

Now let us assume that $6+\sqrt{2}$ is a rational number and

$6+\sqrt{2} = \frac{a}{b}$ where a and b are coprime, $b \neq 0$.

$\Rightarrow \sqrt{2} = \frac{a}{b} - 6 \Rightarrow \sqrt{2} = \frac{a-6b}{b}$

Now a and b are integers.

$\Rightarrow \frac{a-6b}{b}$ is rational. So $\sqrt{2}$ is also rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our assumption is wrong.

Hence, $6 + \sqrt{2}$ is irrational.

Extra Questions

1. Prove that $2\sqrt{3} + \sqrt{5}$ is an irrational number.

Sol: Let us assume that $2\sqrt{3} + 5$ is rational such that

$2\sqrt{3} + \sqrt{5} = \frac{a}{b}$, where a and b are coprime and $b \neq 0$

$2\sqrt{3} = \frac{a}{b} - \sqrt{5}$

Square on both the sides

$(2\sqrt{3})^2 = \left(\frac{a}{b} - \sqrt{5}\right)^2$

$4(3) = \frac{a^2}{b^2} + (5)^2 - \frac{2a\sqrt{5}}{b}$

$12 = \frac{a^2}{b^2} + 5 - \frac{2a\sqrt{5}}{b}$

$\frac{2a\sqrt{5}}{b} = \frac{a^2}{b^2} + 5 - 12$

$\frac{2a\sqrt{5}}{b} = \frac{a^2}{b^2} - 7$

$\sqrt{5} = \frac{b}{2a}\left(\frac{a^2}{b^2} - 7\right) = \frac{a}{2b} - 7\frac{b}{2a}$ $\frac{7b}{2a}$

$\frac{a}{2b} - \frac{7b}{2a}$ is rational and so is $\sqrt{5}$

But it contradicts to the facts that $\sqrt{5}$ is irrational

$\therefore 2\sqrt{3} + \sqrt{5}$ is irrational.

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