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Ch-2 : Polynomials

Polynomial: An algebric expression whole number as the exponents of the variable are called Polynomials in one variable. It is denoted by p or x.
Example: $3y^2 + 5y, x^3 - x^2 + 4x + 7$.

Degree of Polynomial: If ~~p of x~~ P(x) is a polynomial in x then the highest power of x in ~~p of x~~ P(x) is called the degree of Polynomial
Example: $7u^6 + u^2 - 9$ is a polynomial of degree 6

Types of polynomials:
Standard form
$ax + b$ - Linear polynomial
$ax^2 + bx + c$ - Quadratic polynomial
$ax^3 + bx^2 + cx + d$ - Cubic polynomial

Note: If ~~t of x~~ p(x) is a polynomial in x, and if k is any real number, then the value ~~of~~ obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k)

Example: $p(x) = x^2 - 3x - 4$

$$p(-1) = (-1)^2 - 3(-1) + 4$$
$$= 1 + 3 + 4$$
$$= 8$$

$$p(4) = x^2 - 3x + 4$$
$$= (4)^2 - 3(4) + 4$$
$$= 16 - 12 + 4$$
$$= 8$$

Relationship between ~~zeros~~ and coefficient of polynomial.

1. zeroes of linear polynomial is $-\dfrac{b \text{ (constant)}}{a \text{ (coefficient of } x)}$

2. zeroes of quadratic polynomial : $\alpha + \beta = \frac{-b}{a}$
   $\alpha\beta = \frac{c}{a}$
   let zeroes of the polynomial be α and β.

3. zeroes of cubic polynomial: be α, β and γ
   $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
   $\alpha\beta\gamma = \frac{d}{a}$

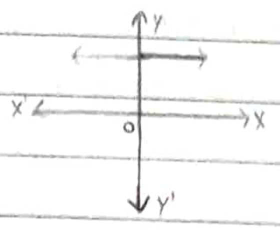
Geometrical meaning of the zeroes of a polynomial

1. The graph of a linear polynomial is a straight line which intersects the x-axis exactly at one point.

2. The graph of quadratic polynomial represents parabola. It intersects maximum 2 points.

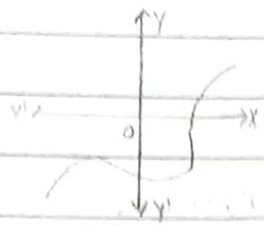
Exercise 2.1

(i)



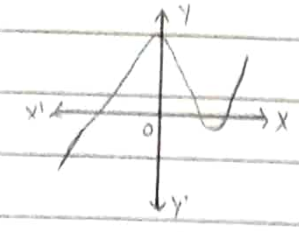
Sol:- This graph shows p(x) has no zero.

(ii)



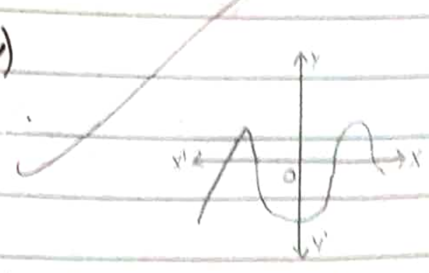
Sol:- This graph shows p(x) has one zero.

(iii)



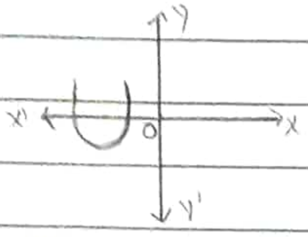
Sol:- This graph shows p(x) has three zeroes

(iv)



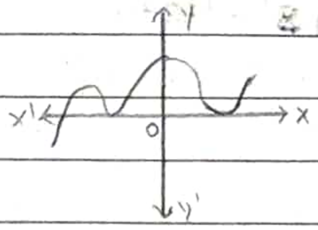
Sol:- This graph shows p(x) has four zeroes

(iv)



Sol: This graph shows p(x) has two zeroes.

(vi)



Sol:- This graph shows p(x) has three zeroes

Note: If α and β are the zeroes of the quadratic polynomial
$\therefore (x-\alpha), (x-\beta)$ are the factors of polynomial

$$\therefore ax^2+bx+c = k(x-\alpha)(x-\beta)$$
$$= k(x^2-(\alpha+\beta)x+\alpha \times \beta)$$

where $\alpha+\beta$ and $\alpha\beta$ are the sum is product if zeroes of a polynomial.

Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients

(i) x^2-2x-8

Sol:- $x^2-2x-8 = x^2-4x+2x-8$
$= x(x-4)+2(x-4)$
$= (x+2)(x-4)$

zeroes of polynomial are:- $x-4=0$ $\quad x+2=0$
$x=4$ $\quad x=-2$

<u>Verification</u>:

$\alpha = 4$ and $\beta = -2$

$\alpha+\beta = 4-2 = -2$

$\frac{-b}{a} = \frac{-(-2)}{1} = 2$

$\therefore \alpha\beta = \frac{-b}{a}$

$\alpha\beta = 4\times-2 = -2$

$\frac{c}{a} = \frac{-8}{1} = -8$

$\therefore \alpha\beta = \frac{c}{a}$

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(ii) $4s^2 - 4s + 1$

Sol: $4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1$

$= 2s(2s-1) - 1(2s-1)$

$= (2s-1)(2s-1)$

Zeroes of polynomial are:

$2s - 1 = 0$	$2s - 1 = 0$
$2s = 1$	$2s = 1$
$s = \frac{1}{2}$	$s = \frac{1}{2}$

Verification:

$\alpha = \frac{1}{2}, \beta = \frac{1}{2}$

$\alpha + \beta = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

$\frac{-b}{a} = \frac{-(-4)}{4} = 1$

$\therefore \alpha + \beta = \frac{-1}{2}$

$\alpha\beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{c}{a} = \frac{1}{4}$

$\therefore \alpha\beta = \frac{c}{a}$

(iii) $6x^2 - 3 - 7x$

Sol: $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= 6x^2 - 9 + 2x - 3$

$= 3x(2x-3) + 1(2x-3)$

$= (3x+1)(2x-3)$

Zeroes of polynomial are:

$2x - 3 = 0$ | $3x + 1 = 0$

$2x = 3$ | $3x = -1$

$x = \frac{3}{2}$ | $x = \frac{-1}{3}$

$\alpha = \frac{3}{2}$ | $\beta = \frac{-1}{3}$

Verification:

$\alpha + \beta = \frac{-7}{3} + \frac{3}{2} = \frac{-2}{6} + \frac{9}{6} = \frac{7}{6}$

$\frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$

$\therefore \alpha + \beta = \frac{1}{2}$

$\alpha\beta = \frac{3}{2} \times \frac{-1}{3} = \frac{-1}{2}$

$\frac{c}{a} = \frac{-3}{6} = \frac{1}{2}$

$\therefore \alpha\beta = \frac{c}{a}$

(iv) $4u^2 + 8u$

Sol: $4u^2 + 8u = 4u(u + 2)$

Zeroes of polynomial are:

$4u = 0$ | $u + 2 = 0$

$u = \frac{0}{4} = 0$ | $u = -2$

$\alpha = 0$ | $\beta = -2$

Verification:

$\alpha + \beta = 0 - 2 = -2$

$\frac{-b}{a} = \frac{-8}{4} = -2$

$\therefore \alpha + b = \frac{-b}{a}$

$\alpha\beta = 0 \times -2 = 0$

$\frac{c}{a} = \frac{0}{4} = 0$

$\therefore \alpha\beta = \frac{c}{a}$

(v) $t^2 - 15$

Sol:- $\cancel{t^2 = (\sqrt{15})^2}$ $t^2 - 15 = t^2 - (\sqrt{15})^2$

$a^2 - b^2 = (a+b)(a-b)$

$(t + \sqrt{15})(t - \sqrt{15})$

zeroes of the polynomial are:

$t + \sqrt{5} = 0$	$t - \sqrt{15} = 0$
$t = -\sqrt{15}$	$t = \sqrt{15}$

$\alpha = -\sqrt{15}$ $\qquad \beta = \sqrt{15}$

$\cancel{\alpha +}$

Verification:

$\alpha + \beta = -\sqrt{15} + \sqrt{15}$
$= 0$

$\frac{-b}{a} = \frac{0}{1} = 0$

$\therefore \alpha + \beta = \frac{-b}{a}$

$\alpha\beta = -\sqrt{15} \times \sqrt{15}$

$\cancel{\sqrt{15}} = 15$

$\frac{c}{a} = \frac{-15}{1} = -15$

$\therefore \alpha\beta = \frac{c}{a}$

(vi) $3x^2 - x - 4$

Sol:- $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x-4)+1(3x-4)$
$= (3x-4)(x+1)$

zeroes of polynomial

$3x-4=0 \mid x=\frac{4}{3}$
$x+1=0 \mid x=-1$
$\alpha=\frac{4}{3} \quad \beta=1$

Verification:

$\alpha+\beta=\frac{4}{3}+(-1)$
$=\frac{1}{3}=\frac{-(-1)}{3}$
$\therefore \alpha+\beta=\frac{-b}{a}$

$\alpha\beta=\frac{4}{3}(-1)=\frac{-4}{3}$

$\frac{c}{a}=\frac{4(-1)}{3}=\frac{-4}{3}$

$\therefore \alpha\beta=\frac{c}{a}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

Sol: Given $= \alpha+\beta=\frac{1}{4}$ - Sum of zeroes
$\alpha\beta=-1$ - Product of zeroes

Quadratic polynomial is $K(x^2-(\alpha+\beta)x+\alpha\beta)$

$K(x^2-\frac{1}{4}x-1)$

$K(4x^2-\frac{x}{4}-4)$

Substituting $K=4$, one of the Quadratic polynomial is

$4(\frac{4x^2-x^2-4}{4}) \Rightarrow 4x^2-x-4$

(ii) $\sqrt{2}, \frac{1}{3}$

Sol:- Given: $\alpha + \beta = \sqrt{2}$

$\alpha\beta = \frac{1}{3}$

Quadratic polynomial is $K\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$

$K\left(\frac{3x^2 - \sqrt{2}x + 1}{3}\right)$

When $K = 3$ one of the Quadratic polynomial is

$3\left(\frac{3x^2 - 3\sqrt{2}x + 1}{3}\right)$

$\Rightarrow 3x^2 - 3\sqrt{2}x + 1$

(iii) $0, \sqrt{5}$

Sol:- Given: $\alpha + \beta = 0$

$\alpha\beta = \sqrt{5}$

Quadratic polynomial is $K(x^2 - 0x + \sqrt{5})$

When $K = 1$, Quadratic polynomial is $(x^2 + \sqrt{5})$

(iv) $1, 1$

Sol:- Given: $\alpha + \beta = 1$

$\alpha\beta = 1$

Quadratic polynomial is $K(x^2 - (x + 1))$

$K\left(\frac{x^2 - x + 1}{x}\right)$

$= x^2 - x + 1$

(v) $-\frac{1}{4}, \frac{1}{4}$

Sol:- Given: $\alpha + \beta = -\frac{1}{4}$

$\alpha\beta = \frac{1}{4}$

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Quadratic polynomial is $K(x^2-(\alpha+\beta)x+(\alpha\beta)$

$K(x^2-(\frac{-1}{4})x+\frac{1}{4}$

$K\left(4x^2+\frac{1}{4}x+\frac{1}{4}\right)$

$K=4$

$\cancel{4}\left(\frac{4x^2+x+1}{\cancel{4}}\right)$

$\Rightarrow 4x^2+x+1$

(vi) 4, 1

Sol:- Given: $\alpha+\beta=4$

$\alpha\beta=1$

Quadratic polynomial is $K(x^2-(\alpha+\beta)x+\alpha\beta)$

$K(x^2-4x+1)$

$\cancel{1}\left(\frac{x^2-4x+1}{\cancel{1}}\right)$

$\Rightarrow x^2-4x+1$

Example Question

Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x)=3x^3-5x^2-11x-3$ and then verify the relationship between the zeroes and the coefficients.

Sol:- $x=3, p(3)=3(3)^3-5(3)^2-11(3)-3$

$=81-45-33-3$

$=81-81=0$

$x=-1, p(-1)=3(-1)^3-5(-1)^2-11(-1)-3$

$=-3-5+11-3$

$=11-11=0$

$x=\frac{-1}{3}, p\left(\frac{-1}{3}\right)=3\left(\frac{-1}{3}\right)^3-5\left(\frac{-1}{3}\right)^2-11\left(\frac{-1}{3}\right)-3$

$p\left(\frac{-1}{3}\right)=3\times\frac{-1}{27}-5\left(\frac{1}{9}\right)+\frac{11}{3}-3$

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$= \frac{-1}{9} - \frac{5}{9} + \frac{11}{3} - 3$

$= \frac{2}{3} + \frac{2}{3} = 0$

$\alpha + \beta + \gamma = 3 + (-1) + \left(\frac{-1}{3}\right) = 2 - \frac{1}{3} = \frac{5}{3} = -\left(\frac{-5}{3}\right) = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times (-1) + (-1) \times \left(\frac{-1}{3}\right) + \left(\frac{-1}{3}\right)$

$= -3 + \frac{1}{3} - 1 = \frac{-9 + 1 - 3}{3} = \frac{-11}{3}$

$\frac{c}{a} = \frac{-11}{3}$

Sn 29/3/23

$\alpha\beta\gamma = \frac{-d}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{11}{3}$

$\alpha\beta\gamma = 3 \times -1 \times \frac{-1}{3} = \frac{3}{3} = 1$

$-\frac{d}{a} = -\frac{(-3)}{3} = \frac{3}{3} = 1$