# How Simplified Models of Different Variability Affects Performance of Ordinal Transformation\*

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Abstract—Ordinal transformation is a technique of ordinal optimization that utilizes a simplified model for performance evaluation and ranking to further reduce computational effort. This presentation-only paper will be focused on investigating how simplified models of different variability levels affect ranking. The simulation-based study investigates capacity allocation of a re-entrant line in the context of semiconductor manufacturing by using two queuing network approximation models, Jackson network approximation (JNA) and queuing network analyzer (QNA). Both are based on parametric decomposition method and JNA is a special case of QNA with a unity squared coefficient of variation because of the exponential assumptions. Mean cycle time (MCT) is the performance index. Simulation studies of a five-station re-entrant line demonstrate that QNA capture of heterogeneous variability greatly improves the MCT ranking correlation of top-10 allocations out of 415 designs by almost 8 times over JNA at the cost of less than 3% computation time increase, i.e., the value of keeping a good model of variability from simplification.

Keywords—model selection; ordinal transformation; ranking; heterogeneous variability; re-entrant line; capacity allocation

# SIMPLIFIED MODEDLS FOR ORDINAL TRANSFORMATION

Ordinal optimization (OO) focuses on ranking in performances among designs instead of their values. It exploits a goal softening strategy aiming at finding "good enough" designs with high probability and computation efficiency as opposed to an optimal design for sure. Ordinal transformation (OT) [1] is an OO technique that utilizes some simplified models to further reduce computation. OT transforms the original design space into a new onedimensional space where all designs are positioned according to their ordinal ranks using the simplified model. The original design space may be high-dimensional, have multiple local optimums spread far apart, and include a mix of integervalued and categorical variables. Although the performance values obtained by using a low-fidelity simulation model may be quite different from the true values, OT may frequently determine the relative order of performance among designs and has the potential to effectively perform optimization when simplified models are appropriate [1].

However, there are often multiple choices of simplified models that capture different levels of details or aspects of a

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system. Generally speaking, the higher fidelity level a model is, the closer the OT ranking of designs to the true one but the longer the computation time. Given a system and the corresponding design optimization problem, how to systematically select an appropriate simplified model is key to effectiveness of OT [2].

# RANKING CAPACITY ALLOCATIONS OF A RE-ENTRANT LINE

Motivated by the significance of optimizing machine capacity allocation for semiconductor manufacturing [3], this paper extracts from semiconductor manufacturing systems an exemplary queueing network with re-entrant flows (QN-R). Without going into its full complexity aspects, the QN-R system captures the complexity of optimal machine allocation for re-entrant production, where the process flow of one product may revisit one machine several times for processing. Optimal machine capacity allocation of QN-R system is the conveyor problem for investigating how simplified models of different variability affect OT performance.

We consider two frequently used queuing network approximation models of different fidelity in capturing variability, Jackson network approximation (JNA) [4] and queuing network analyzer (QNA) [5]. Mean cycle time, mean time from entry into to departure from the QN-R system is the performance index. Both JNA and QNA are parametric decomposition-based methods. In addition to the assumption of independence among service nodes, QNA uses both mean and variance parameters to model the arrival and service process of each network node. As JNA makes exponential assumptions in its approximations, JNA is thus a special case of QNA. In specific, we focus in this paper on investigating how simplified models of different variability levels affect ranking and analyzing the performance of OT using a simplified model with respect to variability levels.

# **OUEUING NETWORK APPROXIMATION MODELS**

QNA is a free software package first developed at Bell Laboratories to calculate approximate performance measures for general open queueing networks. QNA analyzes each station in a network as a single GI/G/m queue characterized by the first two moments of the inter-arrival time and service time distributions. Segal and Whitt [5] provided refinements of QNA to model a queueing network with re-entrant flows.

JNA approximates an OQN as a Jackson network, where the arrival process to a node is a Poisson process, node service time is exponentially distributed, and all are independent

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among all nodes. The squared coefficients of variance equal one (SCV=1). Network routing of each flow is independently routed from station m to station n with a probability  $q_{mn}$ . When  $q_{mn} = 1$ , the routing is deterministic.

## FIVE-STATION RE-ENTRANT LINE EXPERIMENT

Here we conduct a machine allocation experiment over a five-station re-entrant line as Fig. 1. More details of simulation setting are in Tables 1, 2, and 3.

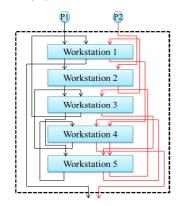


Fig. 1 Five-station re-entrant line model

There are 37 identical machines in total, and each station has at least 5 machines and at most 10 machines. Thus, we have 415 allocation designs in total. We evaluate all designs by simulation, QNA and JNA respectively, rank designs based on their approximated performance, and compare their rank correlation to simulation [6]. The rank correlation of top-K designs by QNA and JNA are given in Fig. 2.



Fig. 2 Comparison of rank correlation of top-K designs

Table. 1 Product Release Setting

Product	Inter-arrival time distribution	Mean (sec)	SCV
1	Log-normal	1	0.8
2	Log-normal	0.8	0.8

The result shows that QNA outperforms JNA. Most importantly, the difference of their rank correlations is especially large when K is small. This implies that QNA is

much likely to correctly rank true top designs. The characterization of heterogeneous SCVs is beneficial to identify top designs.

Table. 2 Processing steps and routing of each product

Product 1, P1						
Step	Station	Distribution	Mean (sec)	SCV		
1	1	Log-normal	1	0.5		
2	2	Uniform	1.66	0.8		
3	3	Erlang Order 2	1	0.5		
4	5	Log-normal	1.25	0.5		
5	4	Erlang Order 2	1.66	0.8		
6	3	Log-normal	1	0.5		
7	1	Uniform	1	0.5		
	Product 2, P2					
Step			3.5 ( )			
ыср	Station	Distribution	Mean (sec)	SCV		
1	Station 2	Distribution Uniform	Mean (sec) 1.66	SCV 0.8		
1 2						
1		Uniform		0.8		
1 2		Uniform Uniform	1.66	0.8 0.5		
1 2 3	2 1 4	Uniform Uniform Erlang Order 2	1.66 1 1.66	0.8 0.5 0.8		
1 2 3 4	2 1 4 5	Uniform Uniform Erlang Order 2 Log-normal	1.66 1 1.66	0.8 0.5 0.8 0.5		

Table. 3 Machine failure setting

Station	MTTF <sup>1</sup> (sec)	MTTR <sup>2</sup> (sec)	Distributions
1	40	10	Exponential
2	60	10	Exponential
3	40	10	Exponential
4	60	10	Exponential
5	40	10	Exponential

MTTF<sup>1</sup>: Mean Time To Failure, MTTR<sup>2</sup>: Mean Time To Repair

Table. 4 Rank correlation of top-10 designs and all designs

Rank Correlation	QNA	JNA
Top-10 designs	0.5556	0.0667
All 415 designs	0.8545	0.8245

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