A **Cartesian plane** or **two-dimensional Cartesian coordinate system** is a pictorial representation of $R \times R$ obtained by setting up a one-to-one correspondence between ordered pairs of real numbers and points in a **Eculidean plane**. To obtain it, two perpendicular lines, called the **horizontal** and **vertical axes**, are drawn in the plane. Their points of intersection is called the **origin**, and a unit of distance is chosen for each axis. An ordered pair (x, y) of real numbers corresponds to the point P that lies $|x|$ units to the right or left of the vertical axis and $|y|$ units above or below the horizontal axis.

A **real-valued function of a real variable** is a function from one set of real numbers to another. If f is such a function, then for each real number of x in the domain of f, there is a unique corresponding real number $f(x)$. Thus it is possible to define the **graph of** f:

Definition

Let f be a real-valued function of a real number variable. The **graph of** f is the set of all points (x, y) in the Caretensian coordinate plane with the property that x is in the domain of f and $y = f(x)$.

The definition of a graph means that for all x in the domain of f:

$$y = f(x) \quad \Leftrightarrow \quad \text{the point } (x, y) \text{ lies on the graph of } f$$

Note that if $f(x)$ can be written as an algebriaic expression in x, the graph of the function f is the same as the graph of the equation $y = f(x)$ where x is restricted to lie in the domain of f.

Power Functions

A function that sends a real number x to a particular power, x^a, is called a **power function**.

Definition

Let a be any non-negative real number. Define p_a, the **power function with exponent** a, as follows:

$$p_a(x) = x^a \qquad \text{for each non-negative real number } x.$$

The Flooring Function For each real number x, there exists a unique integer n such that $n \leq x < n+1$. The floor of a number is the integer immediately to its left on the number line. More formally, the floor function F is defined by the rule

$$\begin{aligned} F(x) &= \lfloor x \rfloor \\ &= \text{the greatest integer that is less than or equal to } x \\ &= \text{the unique integer } n \text{ such that } n \leq x < n+1 \end{aligned}$$

Graph of a Multiple of a Function

A multiple of a function is obtained by multiplying every value of the function by a fixed number.

Definition

Let f be a real-vaued function of a real variable that let M be any real number. The function Mf, called the **multiple of** f **by** M or M **times** f, is the real-valued function with the same domain as f that is defined by the rule

$$(Mf)(x) = M \cdot (f(x)) \qquad \text{for all } x \in \text{domain of } f$$

If f is a function and M is a real number, the height of the graph of Mf at any real number x is M times the quantity $f(x)$.

Increasing and Decreasing Functions

The **absolute value function** is defined as follows:

$$A(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

for all real numbers x.

When $x \geq 0$, the graph A is the same as the graph of $y = x$, the straight line with slop 1 that passes through the origin $(0, 0)$. For $x < 0$, the graph A is the same as the graph of $y = -x$, which is the straight line with slope -1 that passes through $(0, 0)$.

Note that as you trace from left to right along the graph to the left of the origin, the height of the graph continually decreases. For this reason, the absolute value function is said to be decreasing on the set of real numbers less than 0. As you trace from left to right along the graph to the right of the origin, the height of the graph continually increases. Consequently, the absolute value function is said to be increasing on the set of real number greater than 0.

Since the height of the graph of a function f at a point x is $f(x)$, these geomtric concepts translate to the following analytic definition.

Definition

Let f be a real-valued function defined on a set of real numbers, and suppose the domain of f contains a set S. We say that f is **increasing on the set** S if, and only if,

$$\text{for all real numbers } x_1 \text{ and } x_2 \text{ in } S, \text{ if } x_1 < x_2 \text{ then } f(x_1) < f(x_2)$$

We say that f is **decreasing on the set** S if, and only if,

$$\text{for all real numbers } x_1 \text{ and } x_2 \text{ in } S, \text{ if } x_1 < x_2 \text{ then } f(x_1) > f(x_2)$$

We say that f is an **increasing (**or **decreasing) function** if, and only if, f is increasing (or decreasing on its entire domain.