Theorem 5.10

For every positive integer n, $\lambda(K_n) = n - 1$.

Proof:

By definition, $\lambda(K_1) = 0$. Let $G = K_n$ for $n \geq 2$. Since every vertex of G has degree $n-1$, if we remove the $n-1$ edges incident with a vertex, then a disconnected graph results. Thus $\lambda(G) \leq n-1$. Now let X be a minimum edge-cut of G. So $|X| = \lambda(G)$. Then $G - X$ has exactly two components of G_1 and G_2, where G_1 has order k, and G_2 has order $n - k$. Since (1) X consists of all edges joining G_1 and G_2 and (2) G is complete, it follows that $|X| = k(n-k)$. Because $k \geq 1$ and $n-k \geq 1$, we have $(k-1)(n-k-1) \geq 0$ and so

$$(k-1)(n-k-1) = k(n-k) - n + 1 \geq 0$$

Hence $\lambda(G) = |X| = k(n-k) \geq n-1$. Therefore, $\lambda(K_n) = n-1$.