DATE

Section 2.6 Substitutions and Transformations

Bernoulli Equations

An equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$

is called a Bernoulli differential equation.

Solution If $n=0$ or 1 then the Bernoulli Equation is a linear equation.

If $n \neq 0$ or 1 then the transformation $v=y^{1-n}$ reduces the Bernoulli Eqn to a linear equation in v.

Example Solve $\frac{dy}{dx} + y = xy^3$.

Solution Let $v = y^{1-3} = y^{-2}$

$$\text{Then } \frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$$

Multiply through the given equation by y^{-3}:

$$y^{-3}\frac{dy}{dx} + y^{-2} = x$$

Rewrite in terms of v : $-\frac{1}{2}\frac{dv}{dx} + v = x$

$$\therefore \frac{dv}{dx} - 2v = -2x \quad \leftarrow 1^{st} \text{ Order Linear}$$

$$\mu = e^{\int p(x)\,dx} = e^{\int -2\,dx} = e^{-2x}$$

$$\therefore \underbrace{e^{-2x}\frac{dv}{dx} - 2e^{-2x}v} = -2x e^{-2x}$$

$$\frac{d}{dx}\left(e^{-2x}v\right) = -2xe^{-2x}$$

$$\therefore e^{-2x}v = \int -2x\, e^{-2x}\,dx$$

$$= \int u\,dv, \quad u = -2x, \quad dv = e^{-2x}\,dx$$

$$du = -2\,dx, \quad v = -\tfrac{1}{2}e^{-2x}$$

$$= uv - \int v\,du$$

$$= (-2x)\left(-\tfrac{1}{2}e^{-2x}\right) - \int \left(-\tfrac{1}{2}\right)e^{-2x}(-2)\,dx$$

$$= xe^{-2x} - \int e^{-2x}\,dx$$

$$= xe^{-2x} + \tfrac{1}{2}e^{-2x} + C$$

$$\therefore v = x + \tfrac{1}{2} + Ce^{2x}$$

$$\therefore \frac{1}{y^2} = x + \tfrac{1}{2} + Ce^{2x}$$

<u>HW</u> Page 74, #'s 21, 23, 25, 27