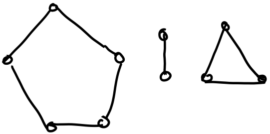
Section 1.3
Common Classes of Graphs

Already met:

- complete graphs: K_n has n vertices, every pair connected by an edge
- P_n is a path with n vertices
- C_n is a cycle with n vertices

Union of graphs: Make each graph in the union a component.

Example: $C_5 \cup P_2 \cup K_3$



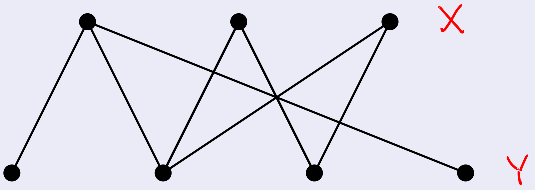
It's a single graph because I say so! It just has 3 components.

Bipartite graphs

Definition

A graph is a *bipartite graph* if there exists a partition of $V(G)$ into sets X and Y such that for all $uv \in E(G)$, either $u \in X$ and $v \in Y$, or $u \in Y$ and $v \in X$.

Two quantifiers in this definition!

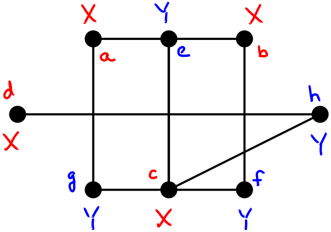


X and Y are the *partite sets*, and X, Y is the *bipartition*.

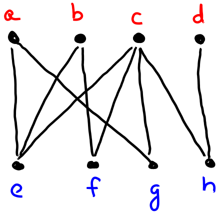
- Notice all edges for *between* X and Y, none are contained inside those sets.
- You will prove on your homework that if every component of G is bipartite, then G is bipartite.

More about bipartite graphs

Being bipartite is an isomorphic invariant of G. This means being bipartite is not a property that depends on how you draw it!

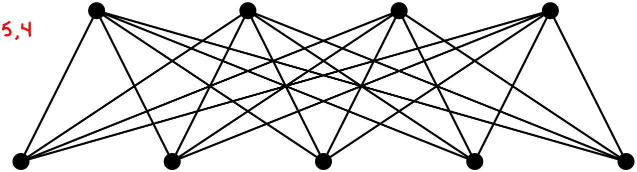


These labels illustrate the bipartition. We can also redraw it.



The complete bipartite graph $K_{m,n}$ is a bipartite graph with $|X| = m$, $|Y| = m$, and $xy \in E(G)$ for all $x \in X$ and $y \in Y$.

$K_{4,5}$ or $K_{5,4}$



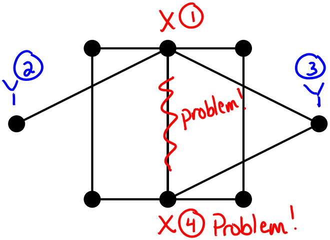
How do you show a graph is NOT bipartite?

Definition

A graph is a *bipartite graph* if there exists a partition of $V(G)$ into sets X and Y such that for all $uv \in E(G)$, either $u \in X$ and $v \in Y$, or $u \in Y$ and $v \in X$.

What does it mean for a graph NOT to be bipartite?

G is not bipartite if for every partition of $V(G)$ into sets X and Y, there exists $uv \in E(G)$ such that $u \notin X$ or $v \notin Y$ and $u \notin Y$ or $v \notin X$.



In other words: No matter how you partition the vertices, there will always be an edge with both endpoints in the same set.

How to check: Just start trying from a single vertex, branching out.