Principle of Mathematical Induction

Let $P(n)$ be a property that is deined for integers n and let a be a fixed integer. Suppose the following two statements are true:

1. $P(a)$ is true
2. For all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true. The the statement

$$\text{for all integers } n \geq a, P(n)$$

is true.

Method of Proof by Mathematical Induction

Consider a statement of the form, "For all integers $n \geq a$, a property $P(n)$ is true." To prove such a statement, perform the following steps.

Step 1 (basis step): Show that $P(a)$ is true.

Step 2 (inductive step): Show that for all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true. To perform this step,

suppose that $P(k)$ is true, where k is any particular but arbitrarily chosen integer with $k \geq a$

[This supposition is called the **inductive hypothesis.**]

Then

show that $P(k+1)$ is true.