Example 2: Show that for *any* choice of the constants c_1 and c_2, the function

$$\phi(x) = c_1 e^{-x} + c_2 e^{2x}$$

is an explicit solution to the linear equation

$$y'' - y' - 2y = 0$$

Solution

We compute $\phi(x) = -c_1 e^{-x} + 2c_2 e^{2x}$ and $\phi''(x) = c_1 e^{-x} + 4c_2 e^{2x}$. Substitution of ϕ, ϕ' and ϕ'' for y, y' and y'' in the equation yields

$$\left(c_1 e^{-x} + 4c_2 e^{2x}\right) - \left(c_1 e^{-x} + 2c_2 e^{2x}\right) - 2\left(c_1 e^{-x} + c_2 e^{2x}\right) = \left(c_1 + c_1 - 2c_1\right)e^{-x} + \left(4c_2 - 2c_2 - 2c_2\right)e^{2x} = 0$$

Since equality holds for all x in $(-\infty, \infty)$, then $\phi(x) = c_1 e^{-x} + c_2 e^{2x}$ is an explicit solution to the equation on the interval $(-\infty, \infty)$ for any choice of the constants c_1 and c_2