Example 1: Solve the differential equation

$$\frac{dy}{dx} = \frac{2xy^2+1}{2x^2y}$$

Solution

Some of the choices of differential forms corresponding to this equation are

$$\begin{aligned}(2xy^2+1)dx + 2x^2y\,dy &= 0\\ \frac{2xy^2+1}{2x^y}dx + dy &= 0\\ dx + \frac{2x^2y}{2xy^2+1}dy &= 0\end{aligned}$$

However, the firt form is best for our purposes because it is a total differential of the function $F(x,y) = x^2y^2+x$:

$$\begin{aligned}(2xy^2+1)dx + 2x^2y\,dy &= d\big[x^2y^2+x\big]\\ &= \frac{\partial}{\partial x}(x^2y^2+x)dx + \frac{\partial}{\partial y}(x^2y^2+x)dy\end{aligned}$$

Thus, the solutions are given implicitly by the formula $x^2y^2+x=C$