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Logical equivalence

In logic, statements p and q are **logically equivalent** if they have the same logical content. That is, if they have the same truth value in every model (Mendelson 1979:56). The logical equivalence of p and q is sometimes expressed as $p \equiv q$, $\mathsf{E}pq$, or $p \iff q$. However, these symbols are also used for material equivalence. Proper interpretation depends on the context. Logical equivalence is different from material equivalence, although the two concepts are closely related.

Contents

Logical equivalences

Equivalence	***Name***
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Logical equivalences involving conditional statements:

1. $p \implies q \equiv \neg p \vee q$
2. $p \implies q \equiv \neg q \implies \neg p$
3. $p \vee q \equiv \neg p \implies q$
4. $p \wedge q \equiv \neg(p \implies \neg q)$

5. $\neg(p \implies q) \equiv p \land \neg q$
6. $(p \implies q) \land (p \implies r) \equiv p \implies (q \land r)$
7. $(p \implies q) \lor (p \implies r) \equiv p \implies (q \lor r)$
8. $(p \implies r) \land (q \implies r) \equiv (p \lor q) \implies r$
9. $(p \implies r) \lor (q \implies r) \equiv (p \land q) \implies r$

Logical equivalences involving biconditionals:

1. $p \iff q \equiv (p \implies q) \land (q \implies p)$
2. $p \iff q \equiv \neg p \iff \neg q$
3. $p \iff q \equiv (p \land q) \lor (\neg p \land \neg q)$
4. $\neg(p \iff q) \equiv p \iff \neg q$

Example

The following statements are logically equivalent:

1. If Lisa is in France, then she is in Europe. (In symbols, $f \implies e$.)
2. If Lisa is not in Europe, then she is not in France (In symbols, $\neg e \implies \neg f$.)

Syntactically, (1) and (2) are derivable from each other via the rules of contraposition and double negation. Semantically, (1) and (2) are true in exactly the same models (interpretations, valuations); namely, those in which either *Lisa is in France* is false or *Lisa is in Europe* is true.

(Note that in this example classical logic is assumed. Some non-classical logics do not deem (1) and (2) logically equivalent.)

Relation to material equivalence

Logical equivalence is different from material equivalence. Formulas p and q are logically equivalent if and only if the statement of their material equivalence ($p \iff q$) is a tautology (Copi et at. 2014:348).

The material equivalence of p and q (often written $p \iff q$) is itself another statement in the same object language as p and q. This statement expresses the idea "'p if and only if q'". In particular, the truth value of $p \iff q$ can change from one model to another.

The claim that two formulas are logically equivalent is a statement in the metalanguage, expressing a relationship between two statements p and q. The statements are logically equivalent if, in every model, they have the same truth value.

See also

- Entailment
- Equisatisfiability
- If and only if
- Logical biconditional
- Logical equality

References

- Irving M. Copi, Carl Cohen, and Kenneth McMahon. Introduction to Logic, 14th edition, Pearson New International Edition, 2014.
- Elliot Mendelson, *Introduction to Mathematical Logic*, second edition, 1979.