Principle of Strong Mathematical Induction

Let $P(n)$ be a property that is defined for integers n, and let a and b be fixed integers with $a \leq b$. Suppose the following two statements are true:

1. $P(a), P(a+1), \ldots,$ and $P(b)$ are all true. (basis step)
2. For any integer $k \geq b$, if $P(i)$ is true for all integers i from a through k, then $P(k+1)$ is true. **(inductive step)**

Then the statement

$$\text{for all integers } n \geq a, P(n)$$

is true. (The supposition that $P(i)$ is true for all integrs i through k is called the **inductive hypothesis**. Another way to state the inductive hypothesis is to say that $P(a), P(a+1), \ldots, P(k)$ are all true.)