Probability

Probability is a real-values set functon P that assigns, to each event A in the sample space S, a number $P(A)$, called the probability off the event A, such that the following properties are satisfied:

(a) $P(A) \geq 0$

(b) $P(S) = 1$

(c) if $A_1, A_2, A_3, \ldots$ are events and $A_i \cap A_j = \emptyset$, $i \neq j$, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$$

for each positive integer k, and

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

for an infinite, but countable, number of events.

Permutation

Each of the $n!$ arrangements (in a row) of n different objects is called a permutation of the n objects.

$$n(n-1)\cdots(2)(1) = n!$$

Permutation of n objects taken r at a time

Each of the ${}_nP_r$ arrangmenets is called a permutation of n objects taken r at a time.

$${}_nP_r = \frac{n(n-1)\cdots(n-r+1)(n-r)\cdots(3)(2)(1)}{(n-r)\cdots(3)(2)(1)} = \frac{n!}{(n-r)!}$$

Ordered Sample of Size r

If r objects are selected from a set of n objects, and if the order of selection is noted, then the selected set of r objects is called an ordered sample of size r

---

Sampling with Replacement

Sampling with replacement occurs when an object is selected and then replaced before the next object is selected.

Sampling without Replacement

Sampling without replacement occurs when an object is not replaced after it has been selected

$$n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

Combination of n objects taken r at a time

Each of the ${}_nC_r$ unordered subsets is called a combination of n objects taken r at a time, where

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Distinguishable Permutation

Each of the ${}_nC_r$ permutations of n objects, r of one type and $n-r$ of another type, is called a distinguishable permutation.

Conditional Probability

The conditional probability of an event A, given that event B has occurred, is defined by

$$\frac{P(A|B) = P(A \cap B)}{P(B)}$$

provided that $P(B) > 0$

Multiplication Rule

The probability that two events A and B, both occur is given by the multiplication rule

$$P(A \cap B) = P(A)P(B|A)$$

provided $P(A) > 0$ or by

$$P(A \cap B) = P(B)P(A|B)$$

provided $P(B) > 0$

Independent Events

Events A and B are indepednent if and only if $P(A \cap B) = P(A)P(B)$. Otherwise, A and B are called depdendent events.

Mutually Independent

Events A, B, and C are mutually independent if and only if the following two conditions hold:

(a) A, b, and C are pairwise independent; that is

$$P(A \cap B) = P(A)P(B_, \quad P(A \cap C) = P(A)P(C)$$

and

$$P(B \cap C) = P(B)P(C)$$

(b) $P(A \cap B \cap C) = P(A)P(B)P(C)$
