Example 3: Solve the nonlinear equation

$$\frac{dy}{dx} = \frac{6x^5 - 2x + 1}{\cos(y) + e^y}$$

Solution

Separating variables and integrating, we have

$$\begin{aligned} \big(\cos(y) + e^y\big)dy &= \big(6x^5 - 2x + 1\big)dx \\ \int \big(\cos(y) + e^y\big)dy &= \int \big(6x^5 - 2x + 1\big)dx \\ \sin(y) + e^y &= x^6 - x^2 + x + C \end{aligned}$$

At this point, we reach an impasse. We would like to solve for y explicitly, but we cannot. That is often the case in solving nonlinear first-order equations. Consequently, when we say "solve the equation," we must on occasion be content if only an implicit form of the solution has been found.