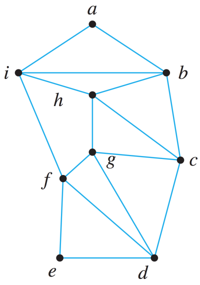
Sections 1.1 and 1.2

We covered some basic graph concepts on our first worksheet. As a review:

Write down *two true statements* you can make about the graph below using vocabulary words from Monday's worksheet.

- Example: The order of G is 9.



G

Vertex h has degree 4

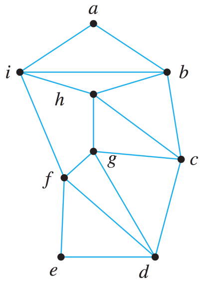
G has a Hamilton cycle

G is connected

Making things a bit more technical

Definition.

A graph G is an ordered pair of sets $G = (V, E)$, where V is a set (called the *vertex set*) and E is a set containing subsets of size 2 taken from V.



G

$V = \{a, b, c, d, e, f, g, h, i\}$

$E = \{ \underbrace{\{a,b\}, \{b,h\}, \{b,c\},}_{\text{so clunky to write!}} \ldots \}$

shortcut:

$E = \{ab, bh, bc, \ldots\}$

Important: No order is implied here.

$ab = ba$

Making things a bit more technical

Definition.

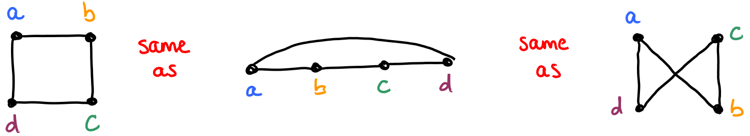
A graph G is an ordered pair of sets $G = (V, E)$, where V is a set (called the *vertex set*) and E is a set containing subsets of size 2 taken from V.

- Edge connecting vertex u and v corresponds to $\{u, v\} \in E$; often write uv instead of $\{u, v\}$
- For a graph G, we often use $V(G)$ and $E(G)$ to represent the vertices in G and the edges in G
- Note: This definition does not allow loops (i.e. vertices adjacent to themselves) or multiple edges (the same edge appearing more than once). When we wish to allow such nonsense, we will explicitly say so. (will discuss this later

* If you took MATH 2345, this is a change from Epp's textbook. No consensus on what a graph is!

Something else important

- Technically speaking, a graph is TWO SETS.
- The pictures we generally associate with graphs are merely ways to represent the graph in an easy-to-understand package.
- How we draw the graphs *doesn't matter* - it's still the same graph, no matter how the vertices and edges are positioned.



All three: $V(G) = \{a, b, c, d\}$

$E(G) = \{ab, bc, cd, da\}$

Applications

- Vertices = Facebook users, edge = Facebook friends
- Vertices = cities, edge = direct Delta flight between cities
- Vertices = computers, edge = direct network connection between them
- Vertices = cars manufactured by Ford, edge = they contain a common component

Brainstorm at least three others:

electrical hookups in a building

predator/prey relationships

family trees

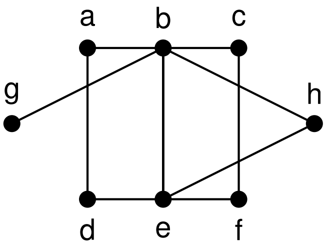
transit networks

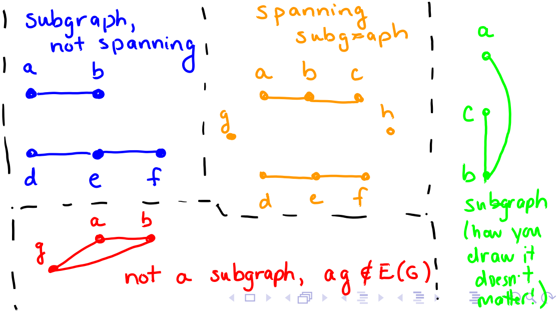
power grids

geopolitical maps

Words

- If $e = uv$ is an edge, then e is *incident* on u and v ↙ or "to"
- Edges that share a common vertex are *adjacent edges*
- A graph H is a *subgraph* of G if H is a graph, and both $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.
- If H is a subgraph of G, then G is a *supergraph* of H.
- If $H \neq G$, then H is a *proper subgraph*. → G is a subgraph of itself, but not a proper subgraph
- If $V(H) = V(G)$, then H is a *spanning subgraph*.

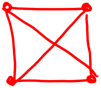




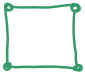
Some special graphs

- The graph with one vertex is the *trivial graph*.
- A graph with no edges is an *empty graph*. (any number of vertices)
- A graph on n vertices with all possible edges is a *complete graph*, denoted K_n.
- A path on n vertices is denoted P_n.
- A cycle on n vertices is denoted C_n.

K_4



C_4



P_4

Worksheet break...