Some Basic Limits

Students are often required to evaluate various limits, such as for determining the convergence of sequences and series. When direct substitution of the limit value yields one of the following indeterminate forms:

Indeterminate Forms: $\frac{0}{0}$ $\quad \frac{\infty}{\infty}$ $\quad \infty \cdot 0$ $\quad \infty - \infty$ $\quad 1^\infty$ $\quad 0^0$ $\quad \infty^0$

then various techniques are available to resolve the indeterminate limit, such as *l'Hôpital's Rule*.

The following table gives the most important (or more interesting) limits, some of which are indeterminate. Many of the limits which students encounter are algebraic/compositional combinations of these basic "building block" limits.

Limits of Continuous Functions (x, a, p **are** ***Real***)	**Limits Relevant to Sequences and Series** (n **is a positive** ***Integer***; x, p **are** ***Real***)
$\lim\limits_{x\to\infty} x^p = \begin{cases} \infty & \text{for } p>0 \\ 1 & \text{for } p=0 \\ 0 & \text{for } p<0 \end{cases}$	$\lim\limits_{n\to\infty} n^p = \begin{cases} \infty & \text{for } p>0 \\ 1 & \text{for } p=0 \\ 0 & \text{for } p<0 \end{cases}$
$\lim\limits_{x\to\infty} \frac{1}{x^p} = \begin{cases} 0 & \text{for } p>0 \\ 1 & \text{for } p=0 \\ \infty & \text{for } p<0 \end{cases}$	$\lim\limits_{n\to\infty} \frac{1}{n^p} = \begin{cases} 0 & \text{for } p>0 \\ 1 & \text{for } p=0 \\ \infty & \text{for } p<0 \end{cases}$
$\lim\limits_{x\to\infty} x^{1/x} = 1 \qquad \lim\limits_{x\to 0^+} x^{1/x} = 0$	$\lim\limits_{n\to\infty} n^{1/n} = \lim\limits_{n\to\infty} \sqrt[n]{n} = 1$
$\lim\limits_{x\to\infty} (\ln x) = \infty$	$\lim\limits_{n\to\infty} \frac{\ln n}{n} = 0$
$\lim\limits_{x\to 0^+} (\ln x) = -\infty$	$\lim\limits_{n\to\infty} \frac{n}{\sqrt[n]{n!}} = e$
$\lim\limits_{x\to\infty} \left(\frac{\ln x}{x}\right) = 0$	$\lim\limits_{n\to\infty} \frac{n!}{n^n} = 0$
$\lim\limits_{x\to\infty} e^{-x} = 0$	$\lim\limits_{n\to\infty} x^{1/n} = \lim\limits_{n\to\infty} \sqrt[n]{x} = 1 \;\text{ for all } x>0$
$\lim\limits_{x\to\infty} \left(1+\frac{a}{x}\right)^x = \lim\limits_{x\to 0} (1+ax)^{\frac{1}{x}} = e^a$	$\lim\limits_{n\to\infty} x^n = 0 \quad \textit{for } \lvert x\rvert < 1$
$\lim\limits_{x\to 0} \frac{\sin(ax)}{x} = a$	$\lim\limits_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x \quad \text{for all } x$
$\lim\limits_{x\to 0} \frac{1-\cos(ax)}{x} = 0$	$\lim\limits_{n\to\infty} \frac{x^n}{n!} = 0 \quad \text{for all } x$
$\lim\limits_{x\to 0} \frac{\tan(ax)}{x} = a$	
$\lim\limits_{x\to\infty} \tan^{-1} x = \lim\limits_{x\to\infty} \sec^{-1} x = \frac{\pi}{2}$	