Determinant of a Triangular Matrix

If A is a triangular matrix of order n, then its determinant is the product of the entries on the main diagonal. That is

$$det(A) = |A| = a_{11}a_{22}a_{33}\cdots a_{nn}$$

Proof(By induction)

The proof of the case in which A is lower triangular is similar. If A has order 1, then $A = \begin{bmatrix} a_{11} \end{bmatrix}$ and the determinant is $|A| = a_{11}$. Assuming the theorem is true for any upper triangular matrix of order $k-1$, consider an upper triangular matrix A of order k. Expanding in the k^{th} row, you obtain

$$|A| = 0C_{k1} + 0C_{k2} + \cdots + 0C_{k(k-1)} + a_{kk}C_{kk} = a_{kk}C_{kk}$$

Now, note that $C_{kk} = (-1)^{2k}M_{kk} = M_{kk}$ is the determinant of the upper triangular matrix found by deleting the k^{th} row and k^{th} column of A. This matrix is of order $k-1$, so apply the induction assumption to write

$$|A| = a_{kk}M_{kk} = a_{kk}(a_{11}a_{22}a_{33}\cdots a_{k-1,k-1} = a_{11}a_{22}a_{33}\cdots a_{kk}$$