Trigonometry

Basic Identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \tan(x) = \frac{1}{\tan(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)} \qquad \csc(x) = \frac{1}{\sin(x)}$$

Pythagorean Identities

$$\cos^2(x) + sin^2(x) = 1 \qquad \sec^2(x) - tan^2(x) = 1$$

$$\csc^2(x) - \cot^2(x) = 1$$

Double-Angle Identities

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \cos(2x) = 1 - 2\sin^2(x)$$

$$\cos(2x) = 2\cos^2(x) - 1 \qquad \cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan(x)}$$

Sum/Difference Identities

$$\sin(s+t) = \sin(s)\cos(t) + \cos(s)\sin(t)$$

$$\sin(s-t) = \sin(s)\cos(t) - \cos(s)\sin(t)$$

$$\cos(s+t) = \cos(s)\cos(t) - \sin(s)\sin(t)$$

$$\cos(s-t) = \cos(s)\cos(t) + \sin(s)\sin(t)$$

$$\tan(s+t) = \frac{\tan(s) + \tan(t)}{1 + \tan(s)\tan(t)}$$

$$\tan(s-t) = \frac{\tan(s) - \tan(t)}{1 + \tan(s)\tan(t)}$$

Product-To-Sum Identities

$$\cos(s)\cos(t) = \frac{\cos(s-t)+\cos(s+t)}{2}$$

$$\sin(s)\sin(t) = \frac{\cos(s-t)-\cos(s+t)}{2}$$

$$\sin(s)\cos(t) = \frac{\sin(s+t)+\sin(s-t)}{2}$$

$$\cos(s)\sin(t) = \frac{\sin(s+t)-\sin(s-t)}{2}$$

Triple-Angle Identities

$$\sin(3x) = -\sin^3(x)+\cos^2(x)\sin(x)$$

$$\sin(3x) = -4\sin^3(x)$$

$$\cos(3x) = \cos^3 -3\sin^2(x)\cos(x)$$

$$\cos(3x) = 4\cos^3(x)-3\cos(x)$$

$$\tan(3x) = \frac{3\tan(x)-\tan^3(x)}{1-3\tan^2(x)}$$

$$\cot(3x) = \frac{3\cot(x)-\cot^3(x)}{1-3\cot^2(x)}$$

Function Ranges

Domain	**Range**
$y = \sin(x)$	$-1 \leq y \leq 1$
$y = \cos(x)$	$-1 \leq y \leq 1$
$y = \tan(x)$	$-\infty < y < \infty$
$y = \cot(x)$	$-\infty < y < \infty$
$y = \csc(x)$	$-\infty < y \leq 1 \cup 1 \leq y < \infty$
$y = \sec(x)$	$-\infty < y \leq y \cup 1 \leq y < \infty$
$y = \arcsin(x)$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos(x)$	$0 \leq y \leq \pi$
$y = \arctan(x)$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \cot^{-1}(x)$	$0 < x < \pi$
$y = \csc^{-1}(x)$	$0 \leq y \leq \frac{\pi}{2} \cup \pi \leq y < \frac{3\pi}{2}$
$y = \sec^{-1} x$	$-\pi < y \leq \frac{\pi}{2} \cup 0 < y < \frac{\pi}{2}$

Function Values

	$\sin(x)$	$\cos(x)$	$\tan(x)$	$\cot(x)$
0	0	1	0	Undefined
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	Undefined	0
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
π	0	-1	0	Undefined
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1
$\frac{4\pi}{3}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{3\pi}{2}$	-1	0	Undefined	0
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
2π	0	1	0	Undefined