Theorem 6.8

Let u and v be *non-adjacent vertices* in a graph G of ordern n such that $deg(u) + deg(v) \geq n$. Then $G + uv$ is *Hamiltonian if and only if* G is *Hamiltonian*

Symbolically:

Let u and v be *non-adjacent vertices* in a graph G of ordern n such that $deg(u) + deg(v) \geq n$. Then $G + uv$ is *Hamiltonian* $\Leftrightarrow$ G is *Hamiltonian*

Proof

If G is a Hamiltonian graph, then certainly $G + uv$ is Hamiltonian for any non-adjacent vertices u and v of G. Thus we need only verify the converse.

Let $G+uv$ be a Hamiltonian graph for two non-adjacent vertices u and v of a graph G and assume, to the contrary, that G is not Hamiltonian. This implies that every Hamiltonian u—v path. Since $deg_G u + deg_G v \geq n$, the proof of Theorem 6.6 tells us that G contains a Hamiltonian cycle. This is a contradiction.