Theorem 6.5

If G is a *Hamiltonian graph*, then for every *non-empty proper set* S of vertices of G,

$$k(G - S) \leq |S|$$

Proof:

Let S be a non-empty proper subset of $V(G)$. Suppose that $k(G - S) = k$ and that $G_1, G_2, \ldots, G_k$ are components of $G - S$. Since G is Hamiltonian, G contains a Hamiltonian cycle C. Whenever C encounters a vertex of G_i for the last time ($1 \leq i \leq k$), the next vertex of C must belong to S. This implies that S must contain at least k vertices, that is, $k = k(G - S) \leq |S|$.