Theorem: Systems of Equations with Unique Solutions

If A is an invertible matrix, then the system of linear equations $Ax = b$ has a unique solution $x = A^{-1}b$.

Proof

The matrix A is nonsingular, so the steps shown are valid.

$$\begin{aligned} Ax &= b \\ A^{-1}Ax &= A^{-1}b \\ Ix &= A^{-1}b \\ x &= A^{-1}b \end{aligned}$$

This solution is unique because if x_1 and x_2 were two solutions, then you could apply the cancellation property to the equation $Ax_1 = b = Ax_2$ to conclude that $x_1 = x_2$