

Theorem 8.13

The Petersen graph is not 1-factorable.

Proof:

Assume, to the contrary, that the Petersen graph PG is 1-factorable. Thus PG can be factored into three 1-factors F_1, F_2, F_3 . Hence the spanning subgraph H of PG with $E(H) = E(F_1) \cup E(F_2)$ is 2-regular and so H is either a single cycle or a union of two or more cycles. Since PG is not Hamiltonian, H cannot be a single cycle and is therefore the union of two or more cycles. On the other hand, since the length of a smallest cycle in PG is 5, it follows that $H = 2C_5$. This is impossible, however, since $2C_5$ does not contain a 1-factor.