Theorem 8.11 (Petersen's Theorem)

Every 3-regular bridgeless graph contains a 1-factor.

Proof:

Let G be a 3-regular bridgeless graph and let S be a subset of $V(G)$ of cardinality $k \geq 1$. We show that the number $k_o(G-S)$ of odd components of $G-S$ is at most $|S|$. Since this is certainly the case if $G-S$ has no odd components, we may assume that $G-S$ has $\ell \geq 1$ odd components $G_1, G_2, \ldots G_\ell$. Let $X_i (1 \leq i \leq \ell)$ denote the set of edges joining the vertices of S and the vertices of G_i. Since every vertex of each graph G_i has degree 3 in G and the sum of the degrees of the vertices in the graph G_i is even, $|X_i|$ is odd. Because G is bridgeless, $|X_i| \neq 1$ for each $i(1 \leq i \leq \ell$ and so $|X_i| \geq 3$. Therefore, there are at least 3ℓ edges joining the vertices of S has degree 3 in G, at most $3k$ edges join the vertices of S and the vertices of $G-S$. Therefore,

$$3k_o(G-S) = 3\ell \leq 3k = 3|S|$$

and so $k_o(G-S) \leq |S|$. By Theorem 8.10, G has a 1-factor.