Induction Pitfalls

A few common mistakes to watch out for:

(a) **Skipping the basis step**
Sure, it's usually easy, but it's *absolutely necessary*. If you don't actually knock a domino down, then none of the dominoes fall.

(b) **Not proving anything in the inductive step**
Sometimes people are tempted to prove the basis step, then claim that the Principle of Mathematical Induction tells us that $P(n)$ being true implies that $P(n+1)$ is true, so therefore the statement is true for all positive integers. Here's the thing: The Principle of Mathematical Induction is pretty powerful, but it's not THAT powerful. It says that if (and this is a BIG IF) **you**, the mathematician, can **prove** that $P(n)$ implies $P(n+1)$, **then** (and ONLY then, assuming you have also done the basis step) can you conclude that $P(n)$ holds for all positive integers.

(c) **Not using the inductive hypothesis when proving the inductive step**
If you never say, "By the inductive hypothesis..." you have done something wrong. The inductive hypothesis is what makes induction work. You have to use the fact that $P(n)$ is true (by assumption) to show that $P(n+1)$ is true.

(d) **In the inductive hypothesis, assuming $P(n)$ is true for *every* $n \geq 1$.**
That's assuming what you need to prove!

(e) **STARTING from the "n" version of the object, and BUILDING the $n+1$ version from it.**
You need to START with the $n+1$ object, and find the n object inside it - then build back up.