3.2. An example

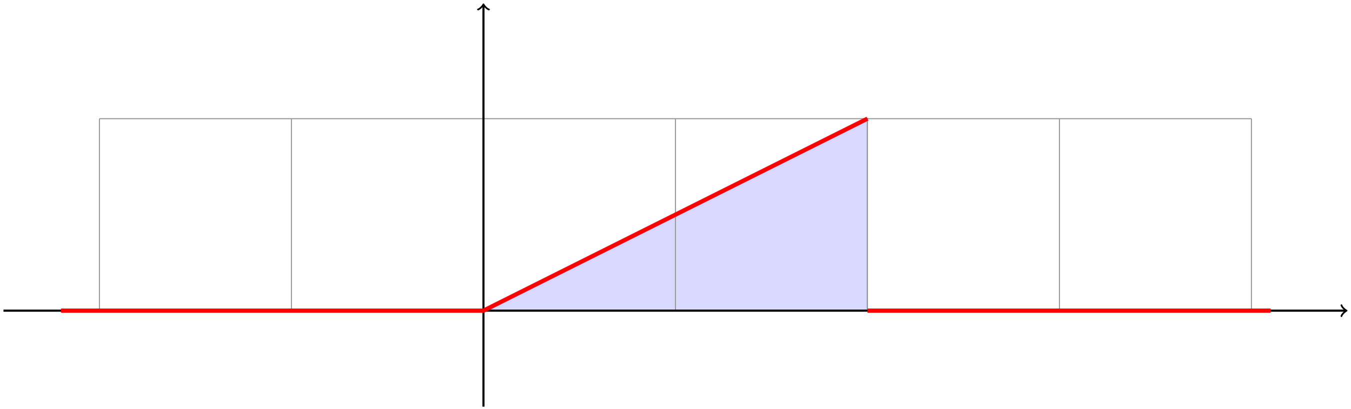
1. Define the function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Then, $f(x)$ qualifies as a pdf, because $f(x) \geq 0$, and

$$\int_{-\infty}^{\infty} f(x)dx = \int_0^2 f(x)dx = 1.$$

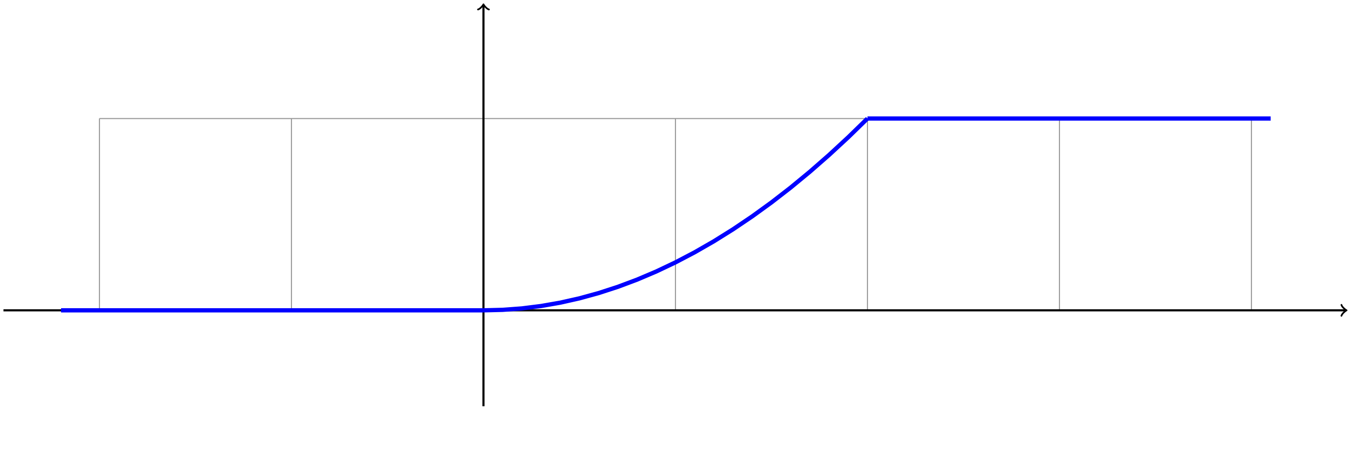
The graph of $f(x)$ is plotted (in red) on the next page.



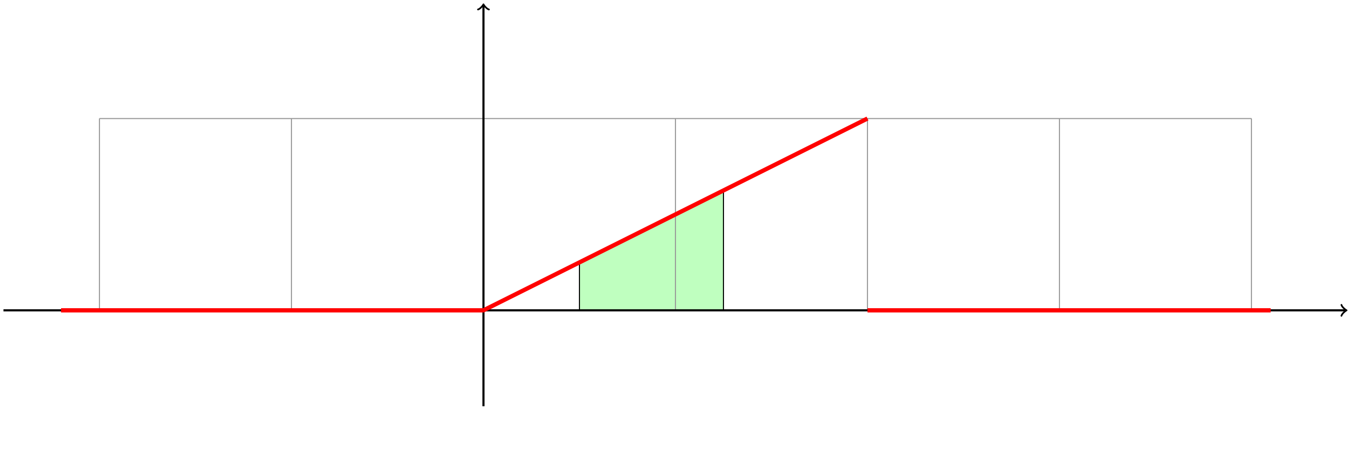
2. If a random variable X has $f(x)$ as its pdf, then X has cdf

$$F(x) = \begin{cases} 1, & x \geq 2 \\ \frac{x^2}{4}, & 0 < x < 2, \\ 0, & x \leq 0. \end{cases} \tag{2}$$

The graph of $F(x)$ is plotted (in blue) on the next page.



3. Suppose that X is a random variable, and X has pdf $f(x)$ as defined in (1). Find
$$P(0.5 < X < 1.25).$$
— *Solution.* We have
$$P(0.5 < X < 1.25) = \int_{0.5}^{1.25} f(x)dx = \int_{0.5}^{1.25} \frac{x}{2}dx = \frac{21}{64}.$$
This integral is geometrically an area — the area that is under the pdf $f(x)$, above the x-axis, and between $x = 0.5$ and $x = 1.25$. The area is plotted (in light green) on the next page.



4. If a random variable X has $f(x)$ as its pdf (that $f(x)$ defined on page 1 of this section), then

$$E(X) = \int_0^2 xf(x)dx = \int_0^2 \frac{x^2}{2}dx = \frac{4}{3},$$

$$E(X^2) = \int_0^2 x^2 f(x)dx = \int_0^2 \frac{x^3}{2}dx = 2,$$

and

$$Var(X) = E(X^2) - (E(X))^2 = 2 - (4/3)^2 = \frac{2}{9},$$

$$\sqrt{Var(X)} = \sqrt{2/9} \approx 0.47140.$$