Theorem: 1.1-5

If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof

The event $A \cup B$ can be represented as a union of mutually exclusive events, namey

$$A \cup B = A \cup (A' \cap B)$$

Hence, by property (c) of **probability**

$$P(A \cup B) = P(A) + P(A' \cap B)$$

However,

$$B = (A \cap B) \cup (A' \cap B)$$

which is a union of mutually exclusive events. Thus

$$P(B) = P(A \cap B) + P(A' \cap B)$$

and

$$P(A' \cap B) = P(B) - P(A \cap B)$$

If the right side of this equation is substituted into $P(A \cup B) = P(A) + P(A' \cap B)$, we obtain

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

which is the desired result.