

Example 8: For the initial value problem

$$3 \frac{dy}{dx} = x^2 - xy^3, \quad y(1) = 6$$

does the Existence and Uniqueness of Solution Theorem imply the existence of a unique solution?

Solution

Dividing by 3 to conform to the statement of the theorem, we identify $f(x, y)$ as $\frac{(x^2 - xy^3)}{3}$ and $\frac{\partial f}{\partial y}$ as $-xy^2$. Both of these functions are continuous in any rectangle containing the point $(1, 6)$, so the hypotheses of the Existence and Uniqueness of Solution Theorem are satisfied. It then follows from the theorem that the initial value problem $3 \frac{dy}{dx} = x^2 - xy^3, \quad y(1) = 6$ has a unique solution in an interval about $x = 1$ of the form $(1 - \delta, 1 + \delta)$, where δ is some positive number.