

Theorem: Absolute Convergence

If the series $\sum |a_n|$ converges, then the series $\sum a_n$ also converges.

Proof:

Because $0 \leq a_n + |a_n| \leq 2|a_n|$ for all n , the series

$$\sum_{n=1}^{\infty} (a_n + |a_n|)$$

converges by the comparison with the convergent series

$$\sum_{n=1}^{\infty} 2|a_n|$$

Furthermore, because $a_n = (a_n + |a_n|) - |a_n|$, you can write

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (a_n + |a_n|) - \sum_{n=1}^{\infty} |a_n|$$

where both series on the right converge. So, it follows that $\sum a_n$ converges.