Example 1: Solve the nonlinear equation

$$\frac{dy}{dx} = \frac{x-5}{y^2}$$

Solution

Following the streamlined approach, we separate the variables and rewrite the equation in the form

$$y^2 dy = (x-5)dx$$

Integrating, we have

$$\int y^2 dy = \int (x-5)dx$$
$$\frac{y^3}{3} = \frac{x^2}{2} - 5x + C$$

and solving this last equation for y gives

$$y = \left(\frac{3x^2}{2} - 15x + 3C\right)^{\frac{1}{3}}$$

Since C is a constant of integration that can be any real number, $3C$ can also be any real number. Replacing $3C$ by the single symbol C, we have

$$y = \left(\frac{3x^2}{2} - 15x + C\right)^{\frac{1}{3}}$$