Example 1: Use Euler's method with step size $h = 0.1$ to approximate the solution to the initial value problem

$$y' = x\sqrt{y}, \quad y(1) = 4$$

at the points $x = 1.1, 1.2, 1.3, 1.4$ and $1, 5$

Solution

Here $x_0 = 1, y_0 = 4, h = 0.1$, and $f(x, y) = x\sqrt{y}$. Thus, the recursive formula for y_n is

$$y_{n+1} = y_n + hf(x_n, y_n) = y_n + (0.1)x_n\sqrt{y_n}$$

Substituting $n = 0$, we get

$$\begin{aligned} x_1 &= x_0 + 0.1 = 1 + 0.1 = 1.1 \\ y_1 &= y_0 + (0.1)x_0\sqrt{y_0} = 4 + (0.1)(1)\sqrt{4} = 4.2 \end{aligned}$$

Putting $n = 1$ yields

$$\begin{aligned} x_2 &= x_1 + 0.1 = 1.1 + 0.1 = 1.2 \\ y_2 &= y_1 + (0.1)x_1\sqrt{y_1} = 4.2 + (0.1)(1.1)\sqrt{4.2} \approx 4.42543 \end{aligned}$$

As one might expect, the approximation deteriorates as x moves farther away from 1.