Example 3: Show that the relation

$$y^2 - x^3 + 8 = 0$$

implicitly defines a solution to the nonlinear equation

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

on the interval $(2, \infty)$

Solution

When we solve this equation for y, we obtain $y = \pm\sqrt{x^3 - 8}$. Let's try $\phi(x) = \sqrt{x^3 - 8}$ to see if it is an explicit solution. Since $\frac{d\phi}{dx} = \frac{3x^2}{2\sqrt{x^3-8}}$, both ϕ and $\frac{d\phi}{dx}$ are defined on $(2, \infty)$. Substituting them into $\frac{dy}{dx} = \frac{3x^2}{2y}$ yields

$$\frac{3x^2}{2\sqrt{x^3 - 8}} = \frac{3x^2}{2\left(\sqrt{x^3 - 8}\right)}$$

which is indeed valid for all x in $(2, \infty)$. You can check if $-\sqrt{x^3 - 8}$ is also an explicit solution to $\frac{dy}{dx} = \frac{3x^2}{2y}$