

Theorem 5.5

Let G be a non-trivial connected graph and let $u \in V(G)$. If v is a vertex that is farthest from u in G , then v is not a cut-vertex of G .

Proof:

Assume, to the contrary, that v is a cut-vertex of G . Let w be a vertex belonging to a component of $G - v$ that does not contain u . Since every u — w path contains v , it follows that $d(u, w) > d(u, v)$, which is a contradiction.

Corollary 5.6

Every non-trivial connected graph contains at least two vertices that are not cut-vertices.

Proof:

Let u and v be vertices of a non-trivial connected graph G such that $d(u, v) = \text{diam}(G)$. Since each of u and v is farthest from the other, it follows by Theorem 5.5 that both u and v are not cut-vertices of G .