Mechanical Vibrations

Undamped Free System (Rudeced Form)

$$m\frac{d^y}{dt^2} + ky = 0$$

dividing by m we get

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

where $\omega = \sqrt{\frac{k}{m}}$. The auxiliary equation associated with $\frac{d^2y}{dt^2} + \omega y = 0$ is $r^2 + \omega^2 = 0$, which has complex conjugte roots $\pm\omega i$.

Undamped Free System (General Solution)

$$y(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t)$$

Undamped Free System (General Solution Convient Form)

$$y(t) = A\sin(\omega t + \phi)$$

with $A \geq 0$, by letting $C_1 = A\sin(\phi)$ and $C_2 = A\cos(\phi)$ That is

$$\begin{aligned} A\sin(\omega t + \phi) &= A\sin(\omega t)\cos(\phi) + A\cos(\omega t)\sin(\phi) \\ &= C_1 \cos(\omega t) + C_2 \sin(\omega t) \end{aligned}$$

Solving for A and ϕ in terms of C_1 and C_2, we find

$$A = \sqrt{C_1^2 + C_1^2} \quad \text{and} \quad \tan(\phi) = \frac{C_1}{C_2}$$

Simple Harmonic Motion (Undamped free system)

$$\begin{aligned} \text{angular frequency} = \omega &= \sqrt{\frac{k}{m}} \qquad \text{(rad/sec)} \\ \text{natural frequency} &= \frac{\omega}{2\pi} \qquad \text{(cycles/sec)} \\ \text{period} &= \frac{2\pi}{\omega} \qquad \text{(sec)} \end{aligned}$$

The constant A is the amplitude of the motion and ϕ is the phase angle.

The amplitude and phase angle depend on the constants C_1 and C_2, which in turn, are determined by the initial position and initial velocity of the mass. The period and frequency depend only on k and m and not on the initial conditions.