Direct proofs of universal conditional statements

$\forall x \in A,\ P(x) \to Q(x)$

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

vacuously true

Goal: Focus on only those $x \in A$ that make $P(x)$ true, show that $Q(x)$ is also true in that situation

Start: Choose arbitrary member of A, satisfying $P(x)$.

Prove: The sum of any two even integers is even.
Last time: $\forall x, y$ in the set of even integers, $x+y$ is even

Rewrite: $\forall x, y \in \mathbb{Z}$, if x and y are even, then $x+y$ is even

Proof: Let x and y be arbitrary integers such that x and y are even. By definition, $x = 2k$ and $y = 2m$ for integers k and m. Now $x+y = 2k+2m$ by substitution, and $x+y = 2(k+m)$ by factoring. Since k and m are integers, $k+m$ is an integer.

Therefore $x+y$ is even by definition.

First sentence: Choose arbitrary member of given set (satisfying the hypothesis)

ALL THE WORK

Last sentence: conclusion

Direct proofs of universal conditional statements

$$\forall x \in A,\ P(x) \to Q(x)$$

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

vacuously true

Goal: Focus on only those $x \in A$ that make $P(x)$ true, show that $Q(x)$ is also true in that situation

Start: Choose arbitrary member of A, satisfying $P(x)$.

Prove: The sum of any two even integers is even.

Last time: $\forall\ x, y$ in the set of even integers, $x+y$ is even

Rewrite: $\forall\ x, y \in \mathbb{Z}$, if x and y are even, then $x+y$ is even.

Proof: Let x and y be arbitrary integers such that x and y are even. By definition, $x = 2k$ and $y = 2m$ for integers k and m. Now $x+y = 2k+2m$ by substitution, and $x+y = 2(k+m)$ by factoring. Since k and m are integers, $k+m$ is an integer.

Therefore $x+y$ is even by definition.

First sentence: Choose arbitrary member of given set (satisfying the hypothesis)

ALL THE WORK

Last sentence: conclusion