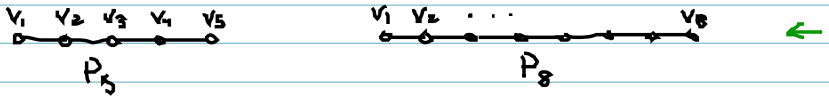
Definition of P_n Graph whose vertex set is $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set is $\{v_i v_{i+1} : 1 \le i \le n-1\}$



Proper coloring of a graph: Assignment of colors to the vertices so that adjacent vertices get different colors

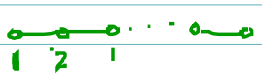
Prove: P_n can be properly colored with two colors for any integer $n \ge 1$.

Rewrite: $\forall$ integers $n \ge 1$, P_n can be properly colored with two colors.

or: $\forall$ graphs G, if $G = P_n$ for some integer $n \ge 1$, then G can be properly colored with two colors.

Proof: Let P_n be a path for some $n \ge 1$
Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(P_n) = \{v_i v_{i+1} : 1 \le i \le n-1\}$
Let v_1 be colored with color 1. Color v_2 with color 2. ~~Now v_{2i-1} can be colored with color 1 for all $1 \le i \le \lceil \frac{n}{2} \rceil$, and v_{2i} can be colored with color 2 for all $1 \le i \le \lfloor \frac{n}{2} \rfloor$.~~
Therefore P_n can be properly colored with two colors.

Now all odd-indexed vertices can be given color 1, and all even-indexed vertices can be given color 2. Adjacent vertices will then be given different colors.



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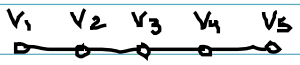

P_5

P_8 ←

Proper coloring of a graph Assignment of colors to the vertices so that adjacent vertices get different colors

Prove P_n can be properly colored with ~~two~~ colors for any integer $n \ge 1$.

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