Theorem: Properties of Matrix Addition and Scalar Multiplication

If A, B and C are $m \times n$ matrices, and c and d are scalars, then the properties

1. $A + B = B + A$	Commutative Property of Addition
2. $A + (B + C) = (A + B) + C$	Associative Property of Addition
3. $(cd)A = c(dA)$	Associative Property of Multiplication
4. $1A = A$	Multiplicative Identity
5. $c(A + B) = cA + cB$	Distributive Property
6. $(c + d)A = cA + dA$	Distributive Property

Proof To prove the commuratuve property of matrix addition, let $A = [a_{ij}]$ and $B = [b_{ij}]$. Then, using the commutative property of *addition of real numbers*, write

$$A + B = [a_{ij} + b_{ij}] = [b_{ij} + a_{ij}] = B + A$$

Similarly, to prove Property 5, use the *distributive property for real numbers* of multiplication over addition to write

$$c(A + B) = \big[c(a_{ij} + b_{ij})\big] = \big[(ca_{ij} + cb_{ij})\big] = cA + cB$$