Table of Integrals Involving the Form $a + bu$

$$\int \frac{u\ du}{a+bu} = \frac{1}{b^2}(a + bu - a\ln|a+bu|) + C$$

$$\int \frac{u^2 du}{a+bu} = \frac{1}{2b^3}[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|] + C$$

$$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int \frac{du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$$

$$\int u\sqrt{a+bu}\ \ du = \frac{2}{15b^2}(3bu - 2a)(a+bu)^{3/2} + C$$

$$\int \frac{u\ du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a+bu} + \ C$$

$$\int \frac{u^2\ du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a+bu} + \ C$$

$$\int \frac{du}{u\sqrt{a+bu}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ \ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + \ C\ , & if\ \ a > 0 \\ \dfrac{2}{\sqrt{-a}}\ \tan^{-1}\sqrt{\dfrac{a+bu}{-a}} + \ C\ , & if\ \ a < 0 \end{cases}$$

$$\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$$

$$\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$$

$$\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$$

$$\int \frac{u^n du}{\sqrt{a+bu}} = \frac{2}{b(2n+1)}\left[u^n\sqrt{a+bu} - na\int \frac{u^{n-1}du}{\sqrt{a+bu}}\right]$$

$$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{1}{a(n-1)}\left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{b(2n-3)}{2}\int \frac{du}{u^{n-1}\sqrt{a+bu}}\right]$$