Direct proofs of universal statements

$\forall\, x \in A,\ P(x)$ or $\underbrace{\forall\, x \in A,\ \overbrace{P(x) \rightarrow Q(x)}^{\text{conditional}}}$ (universal conditional statement)

A infinite set

Idea: Choose arbitrary member of A, use only that it is a member of A

Prove: The sum of (any) two even numbers is even

Def. An even number is one that can be written as $2k$ for some integer k

Rewrite: $\underbrace{\forall\, x, y \text{ in the set of even numbers}}_{\text{"}\forall x \in A\text{"},\ \forall x,y \in A}$, $\underbrace{x+y \text{ is even}}_{P(x),\ P(x,y)}$.

Proof: Let x and y be arbitrary even numbers.
Since x is even, $x = 2k$ for some integer k. Similarly, $y = 2m$ for some integer m. Now $x+y = 2k + 2m$ by substitution. Factoring, $x+y = 2(k+m)$. Since k and m are integers, $k+m$ is an integer

Therefore $x+y$ is even by definition. — $2(\text{int})$

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" $\forall x \in A$ " " $P(x)$ "

$\forall x, y \in A$ $P(x, y)$

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