Logical equivalence

In <u>logic</u>, statements p and q are **logically equivalent** if they have the same logical content. That is, if they have the same <u>truth value</u> in every <u>model</u> (Mendelson 1979:56). The logical equivalence of p and q is sometimes expressed as $p \equiv q$, Epq, or $p \iff q$. However, these symbols are also used for <u>material equivalence</u> Proper interpretation depends on the context. Logical equivalence is different from material equivalence, although the wo concepts are closely related.

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Logical equivalences

Equivalence	Name
$egin{aligned} oldsymbol{p} \wedge \mathbf{T} &\equiv oldsymbol{p} \ oldsymbol{p} ee \mathbf{F} &\equiv oldsymbol{p} \end{aligned}$	Identity laws
$egin{aligned} oldsymbol{p} \lor \mathbf{T} &\equiv \mathbf{T} \ oldsymbol{p} \land \mathbf{F} &\equiv \mathbf{F} \end{aligned}$	Domination laws
$egin{aligned} pee p&\equiv p\ p\wedge p&\equiv p \end{aligned}$	Idempotent laws
$ eg(eg p) \equiv p$	Double negation law
$egin{aligned} pee q \equiv qee p\ p\wedge q \equiv q\wedge p \end{aligned}$	Commutative laws
$(p ee q) ee r \equiv p ee (q ee r) \ (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$pee (q\wedge r)\equiv (pee q)\wedge (pee r) \ p\wedge (qee r)\equiv (p\wedge q)ee (p\wedge r)$	Distributive laws
$ egin{array}{l} \neg(p \wedge q) \equiv \neg p \vee \neg q \\ \neg(p \vee q) \equiv \neg p \wedge \neg q \end{array}$	De Morgan's laws
$egin{aligned} pee(p\wedge q)&\equiv p\ p\wedge(pee q)&\equiv p \end{aligned}$	Absorption laws
$egin{aligned} oldsymbol{p}ee eg oldsymbol{\gamma}oldsymbol{p} \equiv \mathbf{T}\ oldsymbol{p}\wedge eg oldsymbol{p} \equiv \mathbf{F} \end{aligned}$	Negation laws

Logical equivalences involving conditional statements

1.
$$p \implies q \equiv \neg p \lor q$$

2.
$$p \implies q \equiv \neg q \implies \neg p$$

3.
$$p \lor q \equiv \neg p \implies q$$

4.
$$p \land q \equiv \neg (p \implies \neg q)$$

5.
$$\neg (p \implies q) \equiv p \land \neg q$$

6. $(p \implies q) \land (p \implies r) \equiv p \implies (q \land r)$
7. $(p \implies q) \lor (p \implies r) \equiv p \implies (q \lor r)$
8. $(p \implies r) \land (q \implies r) \equiv (p \lor q) \implies r$
9. $(p \implies r) \lor (q \implies r) \equiv (p \land q) \implies r$

Logical equivalences involving biconditionals:

1.
$$p \iff q \equiv (p \implies q) \land (q \implies p)$$

2. $p \iff q \equiv \neg p \iff \neg q$
3. $p \iff q \equiv (p \land q) \lor (\neg p \land \neg q)$
4. $\neg (p \iff q) \equiv p \iff \neg q$

Example

The following statements are logically equivalent:

- 1. If Lisa is in France, then she is in Europe. (In symbols, $f \implies e$.)
- 2. If Lisa is not in Europe, then she is not in France(In symbols, $\neg e \implies \neg f$.)

Syntactically, (1) and (2) are derivable from each other via the rules of <u>contraposition</u> and <u>double negation</u> Semantically, (1) and (2) are true in exactly the same models (interpretations, valuations); namely, those in which either *Lisa* is in *France* is false or *Lisa* is in *Europe* is true.

(Note that in this example classical logic is assumed. Some non-classical logics do not deem (1) and (2) logically equivalent.)

Relation to material equivalence

Logical equivalence is different from material equivalence. Formulas p and q are logically equivalent if and only if the statement of their material equivalence $(p \iff q)$ is a tautology (Copi et at. 2014:348).

The material equivalence of p and q (often written $p \iff q$) is itself another statement in the same object language as p and q. This statement expresses the idea "p if and only if q". In particular, the truth value of $p \iff q$ can change from one model to another

The claim that two formulas are logically equivalent is a statement in the <u>metalanguage</u>, expressing a relationship between two statements p and q. The statements are logically equivalent if, in every model, they have the same truth value.

See also

- Entailment
- Equisatisfiability
- If and only if
- Logical biconditional
- Logical equality

References

- Irving M. Copi, Carl Cohen, and Kenneth McMahon<u>Introduction to Logic</u> 14th edition, Pearson New International Edition, 2014.
- Elliot Mendelson, Introduction to Mathematical Logic second edition, 1979.

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