Theorem 8.15

Every r-regular bipartite graph, $r \geq 1$, is 1-factorable.

Proof:

Let G be an r-regular bipartite graph, where $r \geq 1$. By Theorem 8.6, G contains a perfect matching M_1. Hence $G - M_1$ is $(r-1)$-regular. If $r \geq 2$, then $G - M_1$ contains a perfect matchin M_2. Containing in this manner and applying Theorem 8.6 r times, we see that $E(G)$ can be partitioned into perfect matchings, which gives rise to a 1-factorization of G.