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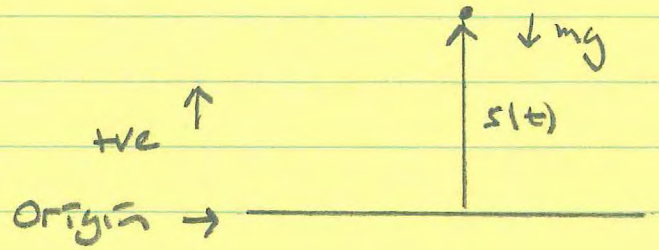
Section 3.4 Newtonian Mechanics

Example1 A body of mass m is projected vertically upwards from the ground at time $t=0$ with velocity v_0. Find (i) the velocity of the body at time $t>0$, (ii) the position of the body at time $t>0$, and (iii) the time t_f at which the body returns to the ground.

Solution



Neglect air resistance

(N2) $\Rightarrow \frac{d}{dt}(mv) = -mg$

$\therefore \frac{dv}{dt} = -g = -32$

Int $\quad v = -32t + c_0$

$t=0, v=v_0 \qquad v_0 = 0 + c_0 \Rightarrow c_0 = v_0$

$\therefore v = -32t + v_0$

$v = \frac{ds}{dt} = -32t + v_0$

Int $\quad s = -16t^2 + v_0 t + c_1$

$t=0, s=0 \qquad 0 = 0 + 0 + c_1 \Rightarrow c_1 = 0$

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$\therefore \quad s = -16t^2 + v_0 t$

Body is on the ground when $s = 0$.

$\therefore \quad -16t^2 + v_0 t = 0$

$t(-16t + v_0) = 0$

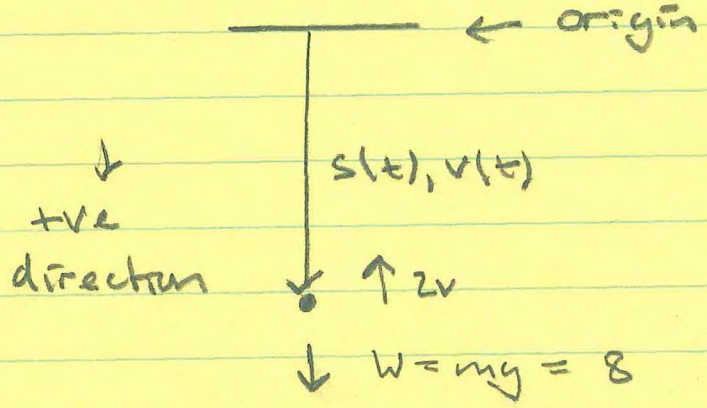
$\therefore \quad t = 0 \quad$ or $\quad t = \dfrac{v_0}{16} \leftarrow$ final state

$\Vert$

t_f

Example 2 A body weighing 8 lb falls from rest toward the earth from a great height. The body encounters air resistance numerically equal to $2v$, where v is the velocity (ft/sec). Find the velocity and distance fallen at time t seconds.

Solution



$\Rightarrow m = \dfrac{8}{32} = \dfrac{1}{4}$

(N2) $\rightarrow \quad \dfrac{d}{dt}(mv) = 8 - 2v$

$\dfrac{1}{4}\dfrac{dv}{dt} = 8 - 2v$

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$$\therefore \frac{dv}{dt} + 8v = 32$$

$$N = e^{\int 8\,dt} = e^{8t}$$

$$\therefore \underbrace{e^{8t}\frac{dv}{dt} + 8e^{8t}v}_{\frac{d}{dt}} = 32e^{8t}$$

$$\frac{d}{dt}\left(e^{8t}v\right) = 32e^{8t}$$

$$e^{8t}v = \int 32e^{8t}\,dt + C_0$$

$$= 4e^{8t} + C_0$$

$$\therefore v = 4 + C_0 e^{-8t}$$

$\underline{t=0, v=0}$ $\quad 0 = 4 + C_0(1) \Rightarrow C_0 = -4$

$$\therefore v = 4 - 4e^{-8t}$$

$$v = \frac{ds}{dt} = 4 - 4e^{-8t}$$

$$s = 4t + \frac{1}{2}e^{-8t} + C_1$$

$\underline{t=0, s=0}$ $\quad 0 = 0 + \frac{1}{2} + C_1 \Rightarrow C_1 = -\frac{1}{2}$

$$\therefore s = 4t + \frac{1}{2}\left(e^{-8t} - 1\right)$$

HW Pgs 114 – 115, #'s 1, 5, 9, 13