Sequences

A **sequence** is a function whose domain is either all the integers between two given integers or all the integers greater than or equal to a given integer.

An **infinite sequence** is a function having for its domain the set of positive integers $\{1, 2, 3, 4, \ldots\}$

A **finite sequence** is a function having for its domain a set of positive integers $\{1, 2, 3, 4, \ldots, n\}$, for some positive integer n.

Summation Notation

If m and n are integers and $m \leq n$, the symbol $\sum_{k=m}^{n} a_k$, read the **summation from k equals m to n of a-sub-k**, is the sum of all the terms $a_m, a_{m+1}, a_{m+2}, \ldots, a_n$. We say that $a_m + a_{m+1} + a_{m+2} + \ldots + a_n$ is the **expanded form** of the sum, and we write

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + a_{m+2} + \ldots + a_n$$

We call k the **index** of the summation, m the **lower limit** of the summation, and n the **upper limit** of the summation.

Product Notation

If m and n are integers and $m \leq n$, the symbol $\prod_{k=m}^{n} a_k$, read the **product from k equals m to n of a-sub-k**, is the product of all the terms $a_m, a_{m+1}, a_{m+2}, \ldots, a_n$. We write

$$\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdot \cdots \cdot a_n$$

We call k the **index** of the product, m the **lower limit** of the product, and n the **upper limit** of the product.

Properties of Summations and Products

If $a_m + a_{m+1} + a_{m+2} + \ldots$ and $b_m + b_{m+1} + b_{m+2} + \ldots$ are sequences of real numbers and c is any real number, then the following equations hold for any integer $m \leq n$

$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$$c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c \cdot a_k \qquad \text{generalized distribution law}$$

$$\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right) = \prod_{k=m}^{n} (a_k \cdot b_k)$$