

Theorem: Determinant of an Inverse Matrix

If A is an $n \times n$ invertible matrix, then

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Proof

The matrix A is invertible, so $AA^{-1} = I$, and using Theorem Determinant of a Matrix Product, $|A||A^{-1}| = |I| = 1$. By Theorem Determinant of an Invertible Matrix, you know that $|A| \neq 0$, so you can divide each side by $|A|$ to obtain

$$|A^{-1}| = \frac{1}{|A|}$$