Section 1.1 Background

$y = y(x)$

x ← independent variable

y ← dependent variable

$$\frac{dy}{dx}, \quad \frac{d^2y}{dx^2}, \quad \ldots$$

$z = z(x, y)$

x, y ← independent variables

z ← dependent variable

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}, \quad \frac{\partial^2 z}{\partial x^2}, \quad \frac{\partial^2 z}{\partial x \partial y}, \quad \frac{\partial^2 z}{\partial y^2}, \quad \ldots$$

Definition

An equation involving the derivatives of a dependent variable w.r.t. one or more independent variables is called a <u>differential equation</u> (d.e.).

If only one independent variable occurs it is an <u>ordinary differential equation</u> (o.d.e.), otherwise it is a <u>partial differential equation</u> (p.d.e.).

Definition

The <u>order of a d.e.</u> is the order of the highest derivative it contains.

Definition

A 1^{ST} order o.d.e. is said to be <u>linear</u> if it can be written in the form

$$a_0(x)\frac{dy}{dx} + a_1(x)y = b(x), \quad a_0(x) \neq 0$$

A 2^{ND} order o.d.e. is said to be <u>linear</u> if it can be written in the form

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = b(x), \quad a_0(x) \neq 0$$

<u>3^{RD} order linear</u>

$$a_0(x)\frac{d^3y}{dx^3} + a_1(x)\frac{d^2y}{dx^2} + a_2(x)\frac{dy}{dx} + a_3(x)y = b(x), \quad a_0(x) \neq 0$$

<u>n^{TH} order linear</u>

$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + a_2(x)\frac{d^{n-2}y}{dx^{n-2}} + \cdots$$

$$\cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = b(x), \quad a_0(x) \neq 0$$

Example 1

Describe the following equations:

ⓐ $$\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + \ln x \cdot \frac{dy}{dx} + e^x y = 2^x$$

ode order = 3 linear

ⓑ $$\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right)^4 + 6y = \sin x$$

ode order = 2 non-linear

(c) $$\frac{d^2y}{dr^2} + r^4 \cos y = 4e^{-r}$$

ode order = 2 non-linear

(d) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

pde order = 2 linear

Example 2

Write a differential equation that fits the physical description: The velocity at time t of a particle moving along a straight line is proportional to the fourth power of it's position x.

Solution velocity $= \frac{dx}{dt} \propto x^4$

$$\therefore \frac{dx}{dt} = kx^4 \quad , \; k = \text{constant}$$

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