2.3. The Mean and Variance of a Distribution

1. *Definition.* Let X be a random variable. The variance of X is

$$\sigma^2 = Var(X) = E((X - E(X))^2),$$

or, equivalently,

$$\sigma^2 = Var(X) = E(X^2) - (E(X))^2.$$

The standard deviation of X is

$$\sigma = \sqrt{Var(X)}.$$

2. *Definition.* Let X be a random variable, and let r be a positive integer. Then

$$E(X^r)$$

is called the r-th moment of X about the origin.

3. In particular, $E(X)$ is the first moment of X about the origin.

4. *Definition.* Let X be a random variable.

   Let r be a positive integer and b a real number. Then

$$E((X-b)^r)$$

   is called the r-th moment of X about b.

5. Note that

$$Var(X) = E((X-E(X))^2).$$

   So, the variance of X is the second moment of X about $E(X)$.

6. *Example.* Let the experiment be the toss of one die. Let X be the outcome of the die. Then X is a random variable. Find $Var(X)$.

— *Solution.* The pmf of X is given in the following table:

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A calculation from the last section shows that $E(X) = 3.5$. Also, we have

$$E(X) = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} = \frac{91}{6}.$$

By definition,

$$Var(X) = E(X^2) - (E(X))^2 = \frac{91}{6} - 3.5^2 = \frac{35}{12}.$$