Theorem 8.6

Every r–regular bipartite graph ($r \geq 1$) has a perfect matching.

Proof:

Let G be an r–regular bipartite graph with partite sets U and W. Necessarily $|U| = |W|$. Let X be a non-empty subset of U. Suppose that $|X| = k \geq 1$. Since every vertex of X has degree r in G, there are kr edges of G incident with vertices of X. Furthermore, since each vertex of W is incident with at most r of these kr edges, every vertex in $N(X)$ is incident with at most r edges and so $|N(X)| \geq k = |X|$. By theorem 8.3, G has a perfect matching.