Theorem: The Inverse of a Product

If A and B are invertible matrices of order n, then AB is invertible and

$$(AB)^{-1} = B^{-1}A^{-1}$$

Proof

To show that $B^{-1}A^{-1}$ is the inverse of AB, you need only show that it conforms to the definition of an inverse matrix. That is,

$$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = A(I)A^{-1} = (AI)A^{-1} = AA^{-1} = I$$

In a similar way, $B^{-1}A^{-1})(AB) = I$. So, AB is invertible and its inverse is $B^{-1}A^{-1}$.