Example 3: Suppose $v(t)$ satisfies the initial value problem

$$\frac{dv}{dt} = -3 - 2v^2, \quad v(0) = 2$$

By experimenting with Euler's method, determine to within one decimal place (± 0.1) the value of $v(0.2)$ and the time it will take $v(t)$ to reach zero.

Solution

Determining rigorous estimates of the accuracy of the answers obtained by Euler's method can be quite a challenging problem. The common practice is to repeatedly approximate $v(0.2)$ and the zero crossing, using smaller and smaller values of h, until the digits of the computed vaues stabilize at the required accuracy level. For this example, Euler's algorithm yields the following values:

$h = 0.1$	$v(0.2) \approx 0.4380$	$v(0.3) \approx 0.0996$	$v(0.4) \approx -0.2024$
$h = 0.05$	$v(0.2) \approx 0.6036$	$v(0.35) \approx 0.0935$	$v(0.4) \approx -0.0574$
$h = 0.025$	$v(0.2) \approx 0.6659$	$v(0.375) \approx 0.0750$	$v(0.4) \approx -0.0003$
$h = 0.0125$	$v(0.2) \approx 0.6938$		
$h = 0.00625$	$v(0.2) \approx 0.7071$		

Ackknowledging the remote possibility that finer values of h might revel aberrations, we state with reasonable confidence that $v(0.2) = 0.7 \pm 0.1$. The Intermediate Value Theorem would imply that $v(t_0) = 0$ at some time t_0 satisfying $0.375 < t_0 < 0.4$, if the computations were perfect; they clearly provide evidence that $t_0 = 0.4 \pm 0.1$