Theorem: Convergence of p-Series

The p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$

converges for $p > 1$ and diverges for $0 < p \leq 1$.

Proof:

The proof follows from the Integral Test and from Theorem 8.5, which states that

$$\int_1^{\infty} \frac{1}{x^p} \, dx$$

converges for $p > 1$ and diverges for $0 < p \leq 1$.