Theorem 5.1

Let v be a vertex incident with a bridge in a connected graph G. Then v is a cut-vertex of G *if and only if* $deg(v) \geq 2$.

Proof:

Suppose that uv is a bridge of G. Then $deg(v) \geq 1$. Assume that $deg(v) = 1$. Since v is an end-vertex of G, the graph $G - v$ is connected and so v is not a cut-vertex of G.

For the converse, assume that $deg(v) \geq 2$. Then there is a vertex w different from u that is adjacent to v. Assume, to the contrary, that v is not a cut-vertex. Thus $G - v$ is connected and so there is a u—v path P in G—v. However then, P together with v and the two edges uv and vw form a cycle containing the bridge uv. This contradicts Theorem 4.1.

Corollary 5.2

Let G be a connected graph or order 3 or more. If G *contains a bridge, then* G *contains a cut-vertex.*

If v is a cut-vertex in a connected graph G, then, of course $G - v$ contains two or more components. If u and w are vertices in distinct components of $G - v$, then u and w are not connected in $G - v$. On the other hand, u and w are necessarily connected in G.