Theorem: 1.4-1

If A and B are independent events, then the followin pairs of events are also independent:

(a) A and B'

(b) A' and B

(c) A' and B'

Proof

We know that conditional probability satisfies the axioms for a probability function. Hence, if $P(A) > 0$, then $P(B'|A) = 1 - P(B|A)$. Thus

$$\begin{aligned} P(A \cap B') &= P(A)P(B'|A) = P(A)\big[1 - P(B|A)\big] \\ &= P(A)\big[1 - P(B)\big] \\ &= P(A)P(B') \end{aligned}$$

because $P(B|A) = P(B)$ by hypothesis. If $P(A) = 0$, then $P(A \cap B') = 0$, so in this case we also have $P(A \cap B') = P(A)P(B')$. Consequently, A and B' are independent events.