Background

Differential Equation: An equation involving the derivatives of a dependent variable with respect to one or more independent variables.

Ordinary Differential Equation: A differential equation involving only ordinary derivatives with respect to a single independent variable.

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + kx = 0$$

Partial Differential Equation: A differential equation involving partial derivatives with respect to more than one independent variable.

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = x - 2y$$

Order: The order of a differential equation is the order of the highest-order derivatives present in the equation.

A 1^{st} **order ordinary differential equation** is said to be linear if it can be written in the form

$$a_0(x)\frac{dy}{dx} + a_1(x)y = b(x), \qquad a_0(x) \neq 0$$

A 2^{nd} **order ordinary differential equation** is said to be linear if it can be written in the form

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = b(x), \qquad a_0(x) \neq 0$$

A 3^{rd} **order ordinary differential equation** is said to be linear if it can be written in the form

$$a_0(x)\frac{d^3y}{dx^3} + a_1(x)\frac{d^2y}{dx^2} + a_2(x)\frac{dy}{dx} + a_3(x)y = b(x), \qquad a_0(x) \neq 0$$

A n^{th} **order ordinary differential equation** is said to be linear if it can be written in the form

$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + a_2(x)\frac{d^{n-2}y}{dx^{n-2}} + \ldots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = b(x), \qquad a_0(x) \neq 0$$