Name: **Jordan Suber**

Mid-Term 2

$(1a)$ The symbol $\sum a_n$ means the sequence of numbers.

$(1b)$ The expression $\sum_{n=1}^{\infty} a_n$ is a number and the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} a_j$$

$(1c)$ We say $\sum a_n$ is **convergent** if the limit exists and is finite.

(2) **nth term criterion for Divergence**

Definitions

(S_n) — n^{th} partial sum
L — The limit of a sequence (a_n). L is a real number $\{L \in \Re\}$
(a_n) — A sequence of real numbers

Proof

$$\lim_{n\to\infty} = 0$$

Assume that $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = L$. The n^{th} term in a recursively defined sequence is equal to

$$S_n = S_{n-1} + a_n$$

and

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} S_{n-1} = L$$

it follows that

$$\begin{aligned} L &= \lim_{n\to\infty} S_n \\ &= \lim_{n\to\infty} (S_{n-1} + a_n) \\ &= \lim_{n\to\infty} S_{n-1} + \lim_{n\to\infty} a_n \\ &= L + \lim_{n\to\infty} a_n \end{aligned}$$

Which implies that (a_n) converges to 0.

$(3a)$ a_n **converges**.
$(3b)$ No conclusion can be made about b_n.
$(3c)$ No conclusion can be made about a_n.
$(3d)$ b_n **diverges**.

(5a)

$$\lim_{n\to\infty}\frac{n^2+2}{n^4-4n^3}=0$$

$$\begin{aligned}
\lim_{n\to\infty}\frac{n^2/n^4+2/n^4}{n^4/n^4-4n^3/n^4} &= lim_{n\to\infty}\frac{1/n^2+2/n^4}{1-4n/n}\\
&=\frac{0+0}{1-0}\\
&=\frac{0}{1}\\
&=0
\end{aligned}$$

(5b)

$$\lim_{n\to\infty}\frac{n^2}{3^n}=0$$

$$\begin{aligned}
\lim_{n\to\infty}\frac{(n+1)^2/3^{n+1}}{n^2/3^n} &= \lim_{n\to\infty}\frac{(n+1)^2 3^n}{3^{n+1}n^2}\\
&=\lim_{n\to\infty}\frac{(n+1)^2}{3n^2}\\
&=\frac{1}{3}
\end{aligned}$$

$\lim_{n\to\infty}\frac{n^2}{3^n}$ converges to $\frac{1}{3}$.

(5c)

$$\begin{aligned}
\lim_{n\to\infty}\frac{3^n((n))^2}{(2n)!} &= \lim_{n\to\infty}\frac{3^{n+1}((n+1))^2/(2(n)!}{3^n((n))^2/(2(n+1)!}\\
&=\lim_{n\to\infty}\frac{3^{n+1}((n+1)!)^2(2n)!}{3^n((n))^2 2(n+1)!}\\
&=\lim_{n\to\infty}\frac{3(n+1)(n+1)}{2(n+1)(n+1)}\\
&=\frac{3}{2}>1
\end{aligned}$$

The limit of $\lim_{n\to\infty}\frac{3^n((n))^2}{(2n)!}$, diverges

(5d) From the conclusion of above equation we can make the conclusion that

$$\lim_{n\to\infty}\frac{4^n((n))^2}{(2n)!}$$

diverges, We use the direct comparision test.

(6)

$$\begin{aligned}
\frac{x^n}{n!} &= \frac{x^{(n+1)}/(n+1)!}{x^n/n!}\\
&=\frac{x}{n+1}\\
&=\frac{x}{\infty}\\
&=0<1
\end{aligned}$$

Interval of convergence is

$$-\infty<x<\infty$$

$(8a)$

$$\frac{x^n}{n!} = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$f(0) = 1$

$(8b)$

$$\begin{aligned} f'(x) &= \frac{d}{dx}\Big(1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\Big) \\ &= 0 + \frac{1x^0}{1!} + \frac{2x^1}{2!} + \frac{3x^2}{3!} + \cdots \\ &= 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ &= f(x) \end{aligned}$$