

Theorem 5.3

Let v be a cut-vertex in a connected graph G and let u and w be vertices in distinct components of $G - v$. Then v lies on every u — w path in G .

Corollary 5.4

A vertex v of a connected graph G is a cut-vertex of G *if and only if* there exists u and w distinct from v such that v lies on every u — w path of G .

Proof:

Suppose that v is a cut-vertex of G . Then $G - v$ is disconnected. Let u and w be vertices in different components of $G - v$. It then follows by Theorem 5.3 that every u — w path in G contains v .

On the other hand, if G contains two vertices u and w such that every u — w path in G contains v , then there is no u — w path in $G - v$. Thus u and w are not connected in $G - v$ and so $G - v$ is disconnected. Therefore, v is a cut-vertex of G .