

Example 3: Solve

$$\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3$$

Solution This is a Bernoulli equation with $n = 3$, $P(x) = -5$ and $Q(x) = -\frac{5}{2}$. To transform $\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3$ into a linear equation, we first divide by y^3 to obtain

$$y^{-3}\frac{dy}{dx} - 5y^{-2} = -\frac{5}{2}x$$

Next we make the substitution $v = y^{-2}$. Since $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$, the transformed equation is

$$\begin{aligned} -\frac{1}{2}\frac{dv}{dx} - 5v &= -\frac{5}{2}x \\ \frac{dv}{dx} + 10v &= 5x \end{aligned}$$

$\frac{dv}{dx} + 10v = 5x$ is linear, so we can solve it for v . It turns out

$$v = \frac{x}{2} - \frac{1}{20 + Ce^{-1x}}$$

Not included in the last equation is the solution $y \equiv 0$ that was lost in the process of $\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3$ by y^3