Trigonometric Identities and Formulas

Many of the following trigonometric and inverse trigonometric identities and formulas are used in calculus, such as for the integration technique of trigonometric substitution. Note that these identities, being equalities, may be applied in both directions. It is assumed the student understands the *Unit Circle* definitions for $\sin x$ and $\cos x$, and knows how to use the *Special Reference Angle* values of the trigonometric functions (also included in this topic document) for any special angle in the *Unit Circle*.

In most of these identities/formulas, the angle unit (radians, degrees, etc.) is arbitrary. However, for those containing $\pi, \pi/2$, etc., in arithmetic combination with the angle x, the angle unit must be radians; these identities may be used with degree values by replacing the fixed radian units of $\pi, \pi/2$, etc., by the equivalent angle in degrees.

A number of these identities also contain expressions of the form $a^2 - b^2$ (this includes expressions where a or b are 1, which can be thought of as 1^2.) Sometimes it is useful to apply the following equation *(Difference of Two Squares)* in either direction:

$$a^2 - b^2 = (a+b)(a-b)$$

Special Reference Angle Values

Reference Angle	Sine	Cosine	Tangent	Cosecant	Secant	Cotangent
0	0	1	0	undefined	1	undefined
$\boldsymbol{\frac{\pi}{12}}$ **(15°)**	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$	$2+\sqrt{3}$
$\boldsymbol{\frac{\pi}{6}}$ **(30°)**	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\boldsymbol{\frac{\pi}{4}}$ **(45°)**	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\boldsymbol{\frac{\pi}{3}}$ **(60°)**	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\boldsymbol{\frac{5\pi}{12}}$ **(75°)**	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$	$2-\sqrt{3}$
$\boldsymbol{\frac{\pi}{2}}$ **(90°)**	1	0	undefined	1	undefined	0

(Note: The special angles of $\pi/12$ and $5\pi/12$ are rarely seen; they fill in the “gaps” in the *Unit Circle*.)

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad (\textit{alt}: \ \cos^2 x = 1 - \sin^2 x \ ; \ \sin^2 x = 1 - \cos^2 x \ )$$

$$1 + \tan^2 x = \sec^2 x \quad (\textit{alt}: \ \tan^2 x = \sec^2 x - 1 \ ; \ \sec^2 x - \tan^2 x = 1 \ )$$

$$1 + \cot^2 x = \csc^2 x \quad (\textit{alt}: \ \cot^2 x = \csc^2 x - 1 \ ; \ \csc^2 x - \cot^2 x = 1 \ )$$

Relation of Sine and Cosine Functions

$$\sin x = \pm\sqrt{1 - \cos^2 x}$$

$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

(The $\pm$ above depends upon the quadrant of x.)

Cofunction and Related Unit Circle Identities

$$\sin\left(\frac{\pi}{2} \pm x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} \pm x\right) = \mp \sin x \qquad \tan\left(\frac{\pi}{2} \pm x\right) = \mp \cot x$$

$$\sin(\pi \pm x) = \mp \sin x \qquad \cos(\pi \pm x) = -\cos x \qquad \tan(\pi \pm x) = \pm \tan x$$

$$\sin\left(\frac{3\pi}{2} \pm x\right) = -\cos x \qquad \cos\left(\frac{3\pi}{2} \pm x\right) = \pm \sin x \qquad \tan\left(\frac{3\pi}{2} \pm x\right) = \mp \cot x$$

$$\sin(2\pi \pm x) = \pm \sin x \qquad \cos(2\pi \pm x) = \cos x \qquad \tan(2\pi \pm x) = \pm \tan x$$

(Reciprocate the sin/cos/tan identities to obtain those for csc/sec/cot, respectively.)

Even/Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Angle Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \ \cos v \ \pm \ \cos u \ \sin v$$

$$\cos(u \pm v) = \cos u \ \cos v \ \mp \ \sin u \ \sin v$$

$$\tan(u \pm v) = \frac{\tan u \ \pm \ \tan v}{1 \ \mp \ \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{1}{1 - \tan x} - \frac{1}{1 + \tan x}$$

Chebyshev Multiple Angle Recursion Formulas $(n \geq 2)$

$$\sin nx = 2 \cos x \sin[(n-1)x] - \sin[(n-2)x]$$

$$\cos nx = 2 \cos x \cos[(n-1)x] - \cos[(n-2)x]$$

$$\tan nx = \frac{\tan[(n-1)x] + \tan x}{1 - \tan[(n-1)x] \tan x}$$

Half-Angle Formulas

$$\sin\left(\tfrac{1}{2}\, x\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\tfrac{1}{2}\, x\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\tfrac{1}{2}\, x\right) = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x} = \pm\frac{\sqrt{1 + \tan^2 x} \ - \ 1}{\tan x}$$

(The $\pm$ above depends upon the quadrant of $x/2$.)

$$\tan\left(\frac{u+v}{2}\right) = \frac{\sin u + \sin v}{\cos u + \cos v} = \frac{\cos u - \cos v}{\sin u - \sin v}$$

$$\cos x = \frac{1 \ - \ \tan^2(\tfrac{1}{2}\, x)}{1 \ + \ \tan^2(\tfrac{1}{2}\, x)} \qquad \textit{(used in the Weierstrass substitution technique)}$$

$$\sin x = \frac{2 \tan(\tfrac{1}{2}\, x)}{1 \ + \ \tan^2(\tfrac{1}{2}\, x)} \qquad \textit{(used in the Weierstrass substitution technique)}$$

Power-Reducing Formulas for Powers of 2, 3, and 4

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \sin^3 x = \frac{3\sin x - \sin 3x}{4} \qquad \sin^4 x = \frac{3 - 4\cos 2x + \cos 4x}{8}$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \cos^3 x = \frac{3\cos x + \cos 3x}{4} \qquad \cos^4 x = \frac{3 + 4\cos 2x + \cos 4x}{8}$$

$$\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x} \qquad \tan^3 x = \frac{3\sin x - \sin 3x}{3\cos x + \cos 3x} \qquad \tan^4 x = \frac{3 - 4\cos 2x + \cos 4x}{3 + 4\cos 2x + \cos 4x}$$

$$\boldsymbol{\sin^2 x \cdot \cos^2 x = \frac{1-\cos 4x}{8}} \qquad \boldsymbol{\sin^3 x \cdot \cos^3 x = \frac{3\sin 2x - \sin 6x}{32}}$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}\,[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}\,[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}\,[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}\,[\sin(u+v) - \sin(u-v)]$$

$$\tan u \tan v = \frac{\cos(u-v) - \cos(u+v)}{\cos(u-v) + \cos(u+v)}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Sum of the Sine and Cosine of the Same Angle Formula

General: $a\sin x + b\cos x = R\cos(x-\alpha)$ *where* $R = \sqrt{a^2+b^2}$ and

$\tan\alpha = \dfrac{b}{a}$; α is in the quadrant where $\cos\alpha = \dfrac{a}{R}$ and $\sin\alpha = \dfrac{b}{R}$

$$\sin x + \cos x = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

$$\sin x - \cos x = \sqrt{2}\cos\left(x - \frac{3\pi}{4}\right) = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

$$\cos x - \sin x = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) = \sqrt{2}\sin\left(x + \frac{3\pi}{4}\right)$$

Miscellaneous and Curious Identities and Formulas

If $u + v + w = \pi \quad (or\ 180°)$, then

$$\tan u + \tan v + \tan w = \tan u \tan v \tan w$$

$$\sin 2u + \sin 2v + \sin 2w = 4 \sin u \sin v \sin w$$

$$\cot u \cot v + \cot u \cot w + \cot v \cot w = 1$$

$$\tan x + \sec x = \tan\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

$$\cos\frac{x}{2} \cdot \cos\frac{x}{4} \cdot \cos\frac{x}{8} \cdots = \prod_{n=1}^{\infty} \cos\left(\frac{x}{2^n}\right) = \frac{\sin x}{x} \quad (Vi\grave{e}te' Infinite\ Product)$$

$$|\sec x + \tan x| = \sqrt{\frac{1 + \sin x}{1 - \sin x}} \quad \text{and} \quad |\csc x + \cot x| = \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

Identities and Formulas For Specific Angles

$$\cos\frac{\pi}{5} = \cos 36° = \frac{\sqrt{5} + 1}{4} \quad (also\ the\ value\ for\ \sin 54°)$$

$$\sin\frac{\pi}{10} = \sin 18° = \frac{\sqrt{5} - 1}{4} \quad (also\ the\ value\ for\ \cos 72°)$$

$$\tan\frac{\pi}{8} = \tan 22.5° = \sqrt{2} - 1$$

$$\tan\frac{3\pi}{8} = \tan 67.5° = \sqrt{2} + 1$$

(Note that for many other terminal angles there are exact expressions, but they are more complicated than the ones above, containing nested square roots.)

$$\cos 20° \cdot \cos 40° \cdot \cos 80° = \frac{1}{8}$$

$$\sin 20° \cdot \sin 40° \cdot \sin 80° = \frac{\sqrt{3}}{8}$$

$$\tan 50° \cdot \tan 60° \cdot \tan 70° = \tan 80°$$

$$\tan 40° \cdot \tan 30° \cdot \tan 20° = \tan 10°$$

$$\cos 24° + \cos 48° + \cos 96° + \cos 168° = \frac{1}{2}$$

$$\sin^2 18° + \sin^2 30° = \sin^2 36°$$

Inverse Trigonometric Functions: Properties, Identities, Formulas, Etc.

Domains and Defined Ranges Using Interval Notation

Function	Domain	Range (radians)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *except* 0
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi]$ *except* $\frac{\pi}{2}$
$\cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$

Special Values

The table *Special Reference Angle Values* for the trigonometric functions may be "inverted" to determine the inverse trigonometric value (the angle) for a particular special "displacement" argument. The following list gives the special angle values (radians only) for each inverse trigonometric function for special values in its domain. This list does not include the rarely seen special reference angles of $\pi/12$ and $5\pi/12$.

$\sin^{-1}(-1) = -\frac{\pi}{2}$	$\cos^{-1}(-1) = \pi$	$\tan^{-1}(-\infty) = -\frac{\pi}{2}$	$\csc^{-1}(-1) = -\frac{\pi}{2}$	$\sec^{-1}(-1) = \pi$	$\cot^{-1}(-\infty) = \pi$
$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$	$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$	$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$	$\csc^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = -\frac{\pi}{3}$	$\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \frac{5\pi}{6}$	$\cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$
$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$	$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$	$\tan^{-1}(-1) = -\frac{\pi}{4}$	$\csc^{-1}(-\sqrt{2}) = -\frac{\pi}{4}$	$\sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$	$\cot^{-1}(-1) = \frac{3\pi}{4}$
$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$	$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$	$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$	$\csc^{-1}(-2) = -\frac{\pi}{6}$	$\sec^{-1}(-2) = \frac{2\pi}{3}$	$\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \frac{2\pi}{3}$
$\sin^{-1} 0 = 0$	$\cos^{-1} 0 = \frac{\pi}{2}$	$\tan^{-1} 0 = 0$	$\csc^{-1}(\pm\infty) = 0$	$\sec^{-1}(\pm\infty) = \frac{\pi}{2}$	$\cot^{-1} 0 = \frac{\pi}{2}$
$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$	$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$	$\tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$	$\csc^{-1} 2 = \frac{\pi}{6}$	$\sec^{-1} 2 = \frac{\pi}{3}$	$\cot^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{3}$
$\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$	$\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$	$\tan^{-1} 1 = \frac{\pi}{4}$	$\csc^{-1}\sqrt{2} = \frac{\pi}{4}$	$\sec^{-1}\sqrt{2} = \frac{\pi}{4}$	$\cot^{-1} 1 = \frac{\pi}{4}$
$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$	$\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$	$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$	$\csc^{-1}\frac{2\sqrt{3}}{3} = \frac{\pi}{3}$	$\sec^{-1}\frac{2\sqrt{3}}{3} = \frac{\pi}{6}$	$\cot^{-1}\sqrt{3} = \frac{\pi}{6}$
$\sin^{-1} 1 = \frac{\pi}{2}$	$\cos^{-1} 1 = 0$	$\tan^{-1}(+\infty) = \frac{\pi}{2}$	$\csc^{-1} 1 = \frac{\pi}{2}$	$\sec^{-1} 1 = 0$	$\cot^{-1}(+\infty) = 0$

Inverse-Forward Identities (e.g., $\sin^{-1}(\sin x)$)

If we define $\operatorname{trg} x$ to be any one of the six trigonometric functions, and $\operatorname{trg}^{-1} x$ to be its inverse trigonometric function, then the composite function:

$$\operatorname{trg}^{-1}(\operatorname{trg}(x)) = x \qquad \textit{only} \text{ for } x \text{ in the Range of } \operatorname{trg}^{-1} x$$

This is based on the general property that the composition of any two functions of x which are exact inverses of each other is equal to x. For example,

$$\sin^{-1}(\sin x) = x \qquad \text{for } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

In general, we do not have expressions for the other inverse-forward combinations except for the following which are derived from the trigonometric *Cofunction Identities*:

$$\left.\begin{array}{r}\sin^{-1}(\cos x)\\ \cos^{-1}(\sin x)\\ \tan^{-1}(\cot x)\\ \cot^{-1}(\tan x)\end{array}\right\} = \frac{\pi}{2} - x \qquad \text{for } \; 0 \le x \le \pi$$

$$\left.\begin{array}{r}\csc^{-1}(\sec x)\\ \sec^{-1}(\csc x)\end{array}\right\} = \frac{\pi}{2} - x \qquad \text{for } \; 0 \le x \le \frac{\pi}{2}$$

Forward-Inverse Identities (e.g., $\sin(\tan^{-1} x)$)

These are the composite functions of the form: $\operatorname{trga}(\operatorname{trgb}^{-1} x)$, where trga and trgb are any of the six trigonometric functions. Obviously, x must be in the domain of the inner inverse trigonometric function.

	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$	$\csc^{-1} x$	$\sec^{-1} x$	$\cot^{-1} x$
sin( )	x	$\sqrt{1-x^2}$	$\frac{x}{\sqrt{1+x^2}}$	$\frac{1}{x}$	$\frac{\sqrt{x^2-1}}{x}$	$\frac{1}{\sqrt{1+x^2}}$
cos( )	$\sqrt{1-x^2}$	x	$\frac{1}{\sqrt{1+x^2}}$	$\frac{\sqrt{x^2-1}}{x}$	$\frac{1}{x}$	$\frac{x}{\sqrt{1+x^2}}$
tan( )	$\frac{x}{\sqrt{1-x^2}}$	$\frac{\sqrt{1-x^2}}{x}$	x	$\frac{1}{\sqrt{x^2-1}}$	$\sqrt{x^2-1}$	$\frac{1}{x}$
csc( )	$\frac{1}{x}$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{\sqrt{1+x^2}}{x}$	x	$\frac{x}{\sqrt{x^2-1}}$	$\sqrt{1+x^2}$
sec( )	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{x}$	$\sqrt{1+x^2}$	$\frac{x}{\sqrt{x^2-1}}$	x	$\frac{\sqrt{1+x^2}}{x}$
cot( )	$\frac{\sqrt{1-x^2}}{x}$	$\frac{x}{\sqrt{1-x^2}}$	$\frac{1}{x}$	$\sqrt{x^2-1}$	$\frac{1}{\sqrt{x^2-1}}$	x

For example, from the above table:

$$\csc(\tan^{-1} x) = \frac{\sqrt{1+x^2}}{x}$$

Reciprocal Identities $(x \neq 0)$

$$\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right) \quad \rightarrow \quad \sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\sin^{-1} x = \csc^{-1}\left(\frac{1}{x}\right) \quad \rightarrow \quad \csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\text{For } x > 0\text{:} \quad \tan^{-1} x = \cot^{-1}\frac{1}{x} \quad \rightarrow \quad \cot^{-1} x = \tan^{-1}\frac{1}{x}$$

$$\text{For } x < 0\text{:} \quad \tan^{-1} x = \cot^{-1}\frac{1}{x} - \pi \quad \rightarrow \quad \cot^{-1} x = \tan^{-1}\frac{1}{x} + \pi$$

Negative Argument Identities

$$\sin^{-1}(-x) = -\sin^{-1} x \qquad (\textit{alt}\text{: } \sin^{-1} x + \sin^{-1}(-x) = 0\ )$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x \qquad (\textit{alt}\text{: } \cos^{-1} x + \cos^{-1}(-x) = \pi\ )$$

$$\tan^{-1}(-x) = -\tan^{-1} x \qquad (\textit{alt}\text{: } \tan^{-1} x + \tan^{-1}(-x) = 0\ )$$

$$\csc^{-1}(-x) = -\csc^{-1} x \qquad (\textit{alt}\text{: } \csc^{-1} x + \csc^{-1}(-x) = 0\ )$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x \qquad (\textit{alt}\text{: } \sec^{-1} x + \sec^{-1}(-x) = \pi\ )$$

$$\cot^{-1}(-x) = -\cot^{-1} x \qquad (\textit{alt}\text{: } \cot^{-1} x + \cot^{-1}(-x) = 0\ )$$

Complementary Angle Identities

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\csc^{-1} x + \sec^{-1} x = \frac{\pi}{2}$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

Miscellaneous Identities and Formulas

The table of *Forward-Inverse Identities* will generate a long list of inverse trigonometric identities which need not be tabulated here. For example,

$$\tan\left(\cos^{-1} x\right) = \frac{\sqrt{1-x^2}}{x} \quad \rightarrow \quad \cos^{-1} x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Some other interesting and occasionally useful inverse trigonometric identities include:

$$\sin^{-1} x = 2\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$$

$$\cos^{-1} x = 2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

$$\tan^{-1} x = 2\tan^{-1}\left(\frac{x}{1+\sqrt{1+x^2}}\right)$$

$$\sin^{-1} u + \sin^{-1} v =$$

$$\begin{cases} \sin^{-1}\big(u\sqrt{1-v^2} + v\sqrt{1-u^2}\big) & u^2+v^2 \leq 1 \;\text{ or }\; u^2+v^2 > 1, uv < 0 \\ \pi - \sin^{-1}\big(u\sqrt{1-v^2} + v\sqrt{1-u^2}\big) & u^2+v^2 > 1, (u,v) > 0 \\ -\pi - \sin^{-1}\big(u\sqrt{1-v^2} + v\sqrt{1-u^2}\big) & u^2+v^2 > 1, (u,v) < 0 \end{cases}$$

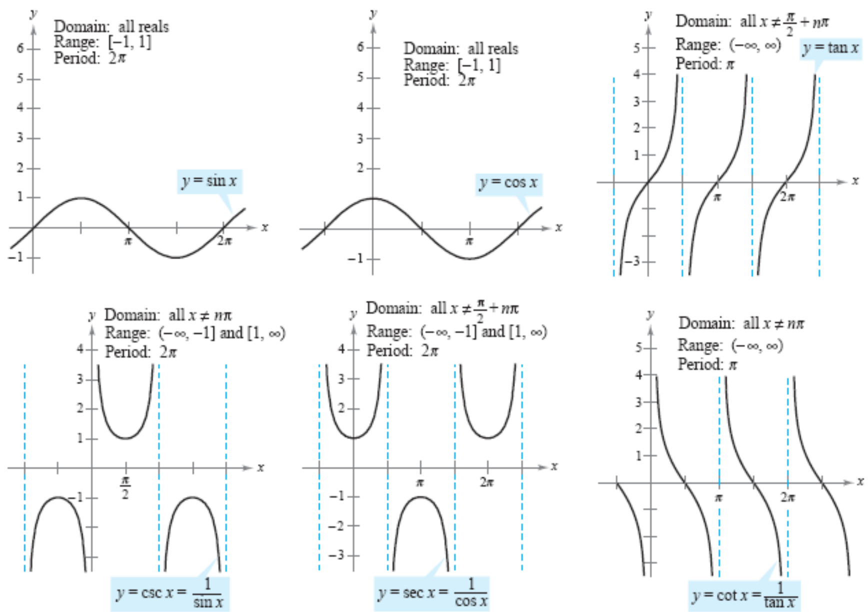
$$\cos^{-1} u + \cos^{-1} v = \begin{cases} \cos^{-1}\big(uv - \sqrt{(1-u^2)(1-v^2)}\big) & u+v \geq 0 \\ 2\pi - \cos^{-1}\big(uv - \sqrt{(1-u^2)(1-v^2)}\big) & u+v < 0 \end{cases}$$

$$\tan^{-1} u + \tan^{-1} v = \begin{cases} \tan^{-1}\left(\frac{u+v}{1-uv}\right) & uv < 1 \\ \tan^{-1}\left(\frac{u+v}{1-uv}\right) + \pi \cdot sign(u) & uv > 1 \end{cases}$$

The above identity for the sum of two inverse tangents has been used as a starting point to generate what are termed *Machin-Like Formulas* used to calculate the number pi (π) to extreme precision. (Refer to the topic document *Application of Power Series* for detailed information on the use of this identity.)

Addenda: Graphs of the Trigonometric and Inverse Trigonometric Functions

Trigonometric Functions



Inverse Trigonometric Functions