Theorem: Covering a Board with Trominoes

For any integer $n \geq 1$, if one square is removed from a $2^n \times 2^n$ checkerboard, the remaining squares can be completely covered by L-shaped trominoes.

Proof (by mathematical induction)

Let the property $P(n)$ be the sentence

If any square is removed from a $2^n \times 2^n$ checkerboard,
then the remaining squares can be completely covered.
by L-shaped trominoes.

Show that $P(1)$ is true;

A $2^1 \times 2^1$ chekerboard just consists of any four squares. If one square is removed, the remaining squares form an L, which can be covered by a single L-shaped tromino. Hence $P(1)$ is true.

Show that for all integers $k \geq 1$, if $P(k)$is true then $P(k+1)$ is also true:

[Suppose that $P(k)$ is true for a particular but arbitrarily chosen integer $k \geq 3$. That is:] Let k be any integer such that $k \geq 1$, and suppose that

If any square is removed from a $2^n \times 2^n$ checkerboard,
then the remaining squares can be completely covered. $\longleftarrow P(k)$
by L-shaped trominoes.

$P(k)$ is the inductive hypothesis.

[We must show that $P(k+1)$ is true, That is:] We must show that

If any square is removed from a $2^{k+1} \times 2^{k+1}$ checkerboard,
then the remaining squares can be completely covered. $\longleftarrow P(k+1)$
by L-shaped trominoes.

Consider a $2^{k+1} \times 2^{k+1}$ checkerboard with one square removed. Divide it into four equal quadrants. Each will consist of a $2^{k+1} \times 2^{k+1}$ checkerboard. In one of the quadrants, one square will have been removed, and so, by inductive hypothesis, all the remaining squares in this quadrant can be completely covered by L-shaped trominoes. The other three quadrants meet at the center of the checkerboard, and the center of the checkerboard serves as a corner of a square from each of those quadrants. An L-shaped tromino can, therefore, be placed on those three central squares. By inductive hypoyhesis, the remaining squares in each of the three quadrants can be ompletely covered by L-shaped trominoes. Thus every square in the $2^{k+1} \times 2^{k+1}$ checkerboard except the one that was removed can be completely covered by L-shaped trominoes.