Theorem: Properties of Matrixs Multiplication

If A, B and C are matrices (with sizes such that matrix produts are defined), and c is scalar, then the properties below are true.

1. $A(BC) = (AB)C$ Associative Property of Multiplication
2. $A(B + C) = AB + AC$ Distributive Property
3. $(A + B)C = AB + AC$ Distributive Property
4. $c(AB) = (cA)B = A(cB)$

Proof

To prove property 2, show that corresponding entries of matrices $A(B + C)$ and $AB + AC$ are equal. Assume A has size $m \times n$, B has size $n \times p$, and C has size $n \times p$. Using the definition of matrix multiplication, the entry in the i^{th} row and j^{th} column of $A(B + C)$ is $a_{i1}(b_{1j} + c_{1j}) + a_{i2}(b_{2j} + c_{2j}) + \cdots + a_{in}(b_{nj} + c_{nj})$. Moreover, the entry in the i^{th} row and j^{th} column of $AB + AC$ is

$$(a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}) + (a_{i1}c_{1j} + a_{i2}c_{2j} + \cdots + a_{in}c_{nj})$$

By distributing and regrouping, you can see that these two ij^{th} entries are equal. So,

$$A(B + C) = AB + AC$$