Prove: For every positive integer n, $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$.

Proof (by induction): Let $P(n)$ be "$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$".

Basis step: We wish to show that $P(1)$ is true, that is, $\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1+1}$.

The left side simplifies to $\frac{1}{1\cdot 2} = \frac{1}{2}$ and so does the right, so $P(1)$ is true.

Inductive step: Suppose $P(k)$ is true for some arbitrary integer $k \geq 1$, that is, $\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}$.

We wish to show that $P(k+1)$ is true, that is,

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k+1}{(k+1)+1} = \frac{k+1}{k+2}$$

Separating off the last term,

$$\begin{aligned}
\sum_{i=1}^{k+1} \frac{1}{i(i+1)} &= \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+1+1)} \\
&= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} && \text{(by the inductive hypothesis)} \\
&= \frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} && \text{(common denominator)} \\
&= \frac{k^2+2k+1}{(k+1)(k+2)} && \text{(combining and expanding)} \\
&= \frac{(k+1)(k+1)}{(k+1)(k+2)} && \text{(factoring)} \\
&= \frac{k+1}{k+2} && \text{(simplifying)}
\end{aligned}$$

Hence $P(k+1)$ holds.

Therefore $P(n)$ is true for all $n \geq 1$ by PMI.