Linear Combinations of Column Vectors

The matrix product Ax is a linear combination of the column vectors $a_1, a_2, \ldots, a_n$ that form the coefficient matrix A.

$$x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

Furthermore, the system

$$Ax = b$$

is consistent if and only if b can be expressed as such a linear combination, where the coefficients of the linear combination are a solution of the system.