Theorem 5.15

For every two integers r and n with $2 \leq r < n$,

$$\kappa(H_{r,n}) = r$$

If r is even or if r is odd and n is even, then $H_{r,n}$ is an r–regular graph of order n and so has size $m = \frac{rn}{2}$. Thus $\left\lfloor \frac{2m}{n} \right\rfloor = r$. On the other hand, if r and n are both odd, then $H_{r,n}$ contains $n-1$ vertices of degree r and one vertex of degree $r+1$ and so $m = \frac{(rn+1)}{2}$. In this case as well, $\left\lfloor \frac{2m}{n} \right\rfloor = r$ and so by Theorem 5.13 $\kappa(H_{r,n}) \leq r$. In fact, $\kappa(H_{r,n}) \leq \delta(H_{r,n}) = 1$.