**Theorem: Cramer's Rule**

If a system of $n$ linear equations in $n$ variables has a coefficient matrix $A$ with a nonzero determinant $|A|$, then the solution of the system is

$$x_1 = \frac{det(A_1)}{det(A)}, x_2 = \frac{det(A_2)}{det(A)}, \cdots, x_n = \frac{det(A_n)}{det(A)}$$

where the $i^{th}$ column of $A_i$ is the column of constants in the system of equations.

**Proof**

Let the system be represented by $AX = B$. The determinant of $A$ is nonzero, so you can write

$$X = A^{-1}B = \frac{1}{|A|}adj(A)B = \begin{bmatrix} x_1 \; x_2 \cdots \; x_n \end{bmatrix}^T$$

If the entries of $B$ are $b_1, b_2, \ldots, b_n$, then $x_1 = \frac{1}{|A|}(b_1C_{1i} + b_2C_{2i} + \cdots + b_nC_{ni})$, but the sum (in parentheses) is precisely the cofactor expansion of $A_i$, which means that $x_i = |A_i|/|A|$, and the proof is complete.