Example 5: Verify that for every constant C the relation $4x^2 - y^2 = C$ is an implicit solution to

$$y\frac{dy}{dx} - 4x = 0$$

Solution

When we implicitly differentiate the equation $4x^2 - y^2 = C$ with respect to x, we find

$$8x - 2y\frac{dy}{dx} = 0$$

which is equivalent to $y\frac{dy}{dx} - 4x = 0$.If we sketched the implict solution for $C = 0, \pm 1, \pm 4$. The curves are hyperbolas with common asymptotes $y = \pm 2x$. Notice that the implicit solution curves (with C arbitrary) fill the entire plane and are nonintersecting for $C \neq 0$. For $C = 0$, the implicit solution gives rise to the two explicit solutions $y = 2x$ and $y = -2x$, both of which pass through the origin.