Theorem 8.14

For each positive integer km the complete graph K_{2k} is 1-factorable.

Proof:

Since the result is true for $k = 1$ and $k = 2$, we assume that $k \geq 3$. Let $G = K_{2k}$, where $V(G) = \{v_0, v_1, v_2 \ldots v_{2k-1}\}$. Let $v_0, v_1, v_2 \ldots v_{2k-1}$ be the vertices of a regular $(2k-1)$-gon and place v_0 in the center of the $(2k-1)$-gon. Draw each edge of G as a straight line segment. Let F_1 be the 1-factor of G consisting of the edge v_0v_1 and all edges of G perpendicular to v_0v_1, namely $v_2v_{2k-1}, v_3v_{2k-2}, \ldots, v_kv_{k+1}$. In general, for $1 \leq i \leq 2k-1$ let F_i be the 1-factor of G consisting of the edge v_0v_i and all edges of G perpendicular to v_0v_i. Then G has a factorization into the 1-factors $F_1, F_2, \ldots, F_{2k-1}$.