

Theorem: Absolute Value Theorem

For the sequence (a_n) , if

$$\lim_{n \rightarrow \infty} |a_n| = 0 \text{ then } \lim_{n \rightarrow \infty} a_n = 0$$

Proof:

Consider the two sequences $(|a_n|)$ and $-(|a_n|)$. Because both of these sequences converge to 0 and

$$-|a_n| \leq a_n \leq |a_n|$$

you can use the Squeeze Theorem to conclude that (a_n) converges to 0.