Elementary Row Operations and Determinants

Let A and B be square matrices.

(1) When B is obtained from A by interchanging two rows of A, $det(B) = -det(A)$.

(2) When B is obtained from A by adding a multiple of a row of A to another row of A, $det(B) = det(A)$.

(3) When B is obtained from A by multiplying a row of A by a nonzero constant c, $det(B) = c\ det(A)$.

Proof

Assume that A and B are 2×2 matrices

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix}$$

Then, you have $|A| = a_{11}a_{22} - a_{21}a_{12}$ and $|B| = a_{21}a_{12} - a_{11}a_{22}$. So $|B| = -|A|$. Using mathematical induction, assume the property is true for matrices of order $(n-1)$. Let A be an $n \times n$ matrix such that B is obtained from A by interchanging two rows of A. Then, to find $|A|$ and $|B|$, expand in a row other than the two interchanged rows. By the induction assumption, the cofactors of B will be the negatives of the cofactors of A because the corresponding $(n-1) \times (n-1)$ matrices have two rows interchanged. Finally, $|B| = -|A|$ and the proof is complete.