

Properties of Transposes

If A and B are matrices (with sizes such that the matrix operations are defined) and c is a scalar, then the properties below are true.

1. $(A^T)^T = A$ Transpose of a transpose
2. $(A + B)^T = A^T + B^T$ Transpose of a sum
3. $(cA)^T = c(A^T)$ Transpose of a scalar multiple
4. $(AB)^T = B^T A^T$ Transpose of a product

Proof

The transpose operation interchanges rows and columns. To prove Property 1, let A be an $m \times n$ matrix. Observe that A^T has size $n \times m$ and $(A^T)^T$ has size $m \times n$, the same as A . To show that $(A^T)^T = A$, you must show that the ij^{th} entries are the same. Let a_{ij} be the ij^{th} entry of A . Then a_{ij} is the j^{th} entry of A^T , and the ij^{th} entry of $(A^T)^T$. This proves Property 1.