Theorem: Cancellation Properties

If C is an invertible matrix, then the properties below are true:

(1) If $AC = BC$, then $A = B$ Right cancellation property

(2) If $CA = CB$, then $A = B$ Left cancellation property

Proof

To prove Property 1, use the fact that C is invertible and write

$$\begin{aligned} AC &= BC \\ (AC)C^{-1} &= (BC)C^{-1} \\ A(CC^{-1}) &= B(CC^{-1}) \\ AI &= BI \\ A &= B \end{aligned}$$

The second property can be proven in a similar way.