Basic "U-Form" Integration Formulas

The following table lists the "basic" indefinite integrals students should memorize or at least be aware of. They are sorted into "families" as noted by the colored boxes. This table differs from the *Table of Fundamental Indefinite Integrals* in that the variable of integration is "u" instead of "x" (which really is irrelevant!), and the integrands are not generalized in any way except for the *Real* number exponent n for the ubiquitous *Power Rule* and a for the exponential a^u and logarithmic $\log_a u$ formulas. Most formalized tables of integrals include various generalized constants so a specific indefinite integral of interest may be directly integrated just by "plugging into" the formula with a minimum of algebraic manipulation and/or transformation.

It is assumed that students are familiar with the techniques of integration so any specific indefinite integral may be integrated if it is transformable into one of the following simple forms using algebra, "u" substitution, and other techniques. Since these are indefinite integrals (that is, antiderivatives of the integrand), the integrated results must include the arbitrary constant of integration C.

(Several of these indefinite integrals have alternative/equivalent forms which are highlighted in blue font. Some of the alternatives are inverse hyperbolic functions, a family of functions related to the natural logarithm which are either ignored or only cursorily studied in Calculus, but which are important, along with regular hyperbolic functions which are related to the exponential function, in the solutions of various differential equations encountered in mathematics, science, engineering, etc.)

$\int du = u + C$	$\int \sin u\,du = -\cos u + C$
$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1) \quad$ (*The Power Rule*)	$\int \cos u\,du = \sin u + C$
$\int u^{-1}\,du = \int \frac{du}{u} = \ln\lvert u\rvert + C$	$\int \tan u\,du = \ln\lvert\sec u\rvert + C = -\ln\lvert\cos u\rvert + C$
$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C = \ln\left\lvert\frac{\sin u}{1+\cos u}\right\rvert + C$
$\int \frac{du}{\sqrt{u^2-1}} = \ln\left\lvert u + \sqrt{u^2-1}\right\rvert + C = \cosh^{-1} u + C$ *for* $u \geq 1$	$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C = \ln\left\lvert\frac{1+\sin u}{\cos u}\right\rvert + C$
$\int \frac{du}{\sqrt{1+u^2}} = \ln\left(u + \sqrt{u^2+1}\right) + C = \sinh^{-1} u + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \frac{du}{1+u^2} = \tan^{-1} u + C$	$\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$
$\int \frac{du}{1-u^2} = \frac{1}{2}\ln\left\lvert\frac{1+u}{1-u}\right\rvert + C = \tanh^{-1} u + C$ *for* $\lvert u\rvert < 1$	$\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$
$\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}\lvert\, u \,\rvert + C = \cos^{-1}\left\lvert\frac{1}{u}\right\rvert + C$	$\int \tan^{-1} u\,du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$
$\int \frac{du}{u\sqrt{1-u^2}} = \ln\left\lvert\frac{u}{1+\sqrt{1-u^2}}\right\rvert + C = -\cosh^{-1}\left\lvert\frac{1}{u}\right\rvert + C$	$\int \csc^{-1} u\,du = u\csc^{-1} u + \ln\left\lvert u + \sqrt{u^2-1}\right\rvert + C$
$\int \frac{du}{u\sqrt{1+u^2}} = \ln\left\lvert\frac{u}{1+\sqrt{1+u^2}}\right\rvert + C = -\sinh^{-1}\left\lvert\frac{1}{u}\right\rvert + C$	$\int \sec^{-1} u\,du = u\sec^{-1} u - \ln\left\lvert u + \sqrt{u^2-1}\right\rvert + C$
$\int e^u\,du = e^u + C$	$\int \cot^{-1} u\,du = u\cot^{-1} u + \frac{1}{2}\ln(1+u^2) + C$
$\int \ln u\,du = u(\ln u - 1) + C = u\ln\frac{u}{e} + C$	$\int \sec u \tan u\,du = \sec u + C$
$\int a^u\,du = \frac{a^u}{\ln a} + C \quad (a > 0, a \neq 1)$	$\int \csc u \cot u\,du = -\csc u + C$
$\int \log_a u\,du = u\log_a \frac{u}{e} + C \quad (a > 0, a \neq 1)$ (*several forms*)	$\int \sec^2 u\,du = \tan u + C$
	$\int \csc^2 u\,du = -\cot u + C$