Theorem: Properties of Functions Defined by Power Series

If the function

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \dots$$

has a radius of convergence of $R > 0$, then, on the interval

$$(c-R, c+R)$$

f is differentiable (and therefore continuous). Moreover, the derivative and antiderivative of f are as follows.

1. $$f'(x) = \sum_{n=1}^{\infty} n a_n (x-1)^{n-1} = a_1 + 2a_2(x-c) + 3a_3(x-c)^2 + \dots$$

2. $$f(x)dx = C + \sum_{n=0}^{\infty} a_n \frac{(x-c)^{n+1}}{n+1} = C + a_0(x-c) + a_1\frac{)x-c)^2}{2} + a_2\frac{(x-c)^3}{3} + \dots$$

The *radius of convergence* of the series obtained by differentiating or integrating a power series is the same as that of the original power series. The *interval of convergence*, however, may differ as a result of the behavior at the endpoints.