Differential Equations Formulas

Differential Equation in Linear Format

$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = F(x)$

General Form for an n^{th}-order equation with x independent, y dependent

$F\left(x, y, \frac{dy}{dx}, \ldots, \frac{d^n y}{dx^n}\right) = 0$

General Form for an n^{th}-order equation (Highest-order term $\frac{d^n y}{dx^n}$ Isolated)

$\frac{d^n y}{dx^n} = f\left(x, y, \frac{dy}{dx}, \ldots, \frac{d^{n-1}y}{dx^{n-1}}\right) = 0$

First-Order Differential Equation:

$y' = f(x, y), \quad y(x_0) = y_0$

Recursive Definition of Euler's Method

$x_{n+1} = x_n + h$

$y_{n+1} = y_n + hf(x_n, y_n), \quad n = 0, 1, 2, \ldots$

First-Order Differential Equation (Separable Form):

$\frac{dy}{dx} = g(x)p(y), \quad \text{where } p(y) = \frac{1}{h(y)}$

Linear First-Order Differential Equation

$a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$

Linear First-Order Differential Equation (Standard Form)

$\frac{dy}{dx} + P(x)y = Q(x) \quad \text{where } P(x) = \frac{a_0(x)}{a_1(x)} \quad \text{and} \quad Q(x) = \frac{b(x)}{a_1(x)}$

Linear First-Order Differential Equation (Standard Form) General Solution

$y(x) = \frac{1}{\mu(x)}\left[\int \mu(x)Q(x)dx + C\right]$

Total Differential of F

$dF := \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$