## Section 3.5 Electrical Circuits

**Notation** $Q = Q(t)$ = instantaneous charge (Coulombs)

$I = I(t) = \frac{dQ}{dt}$ = current (Amps)

$R$ = resistance (Ohms)

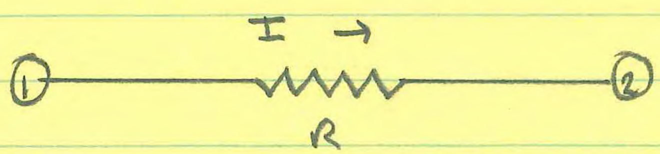
$L$ = inductance (Henries)

$C$ = capacitance (Farads)

$E$ or $V$ = EMF or Potential (Volts)

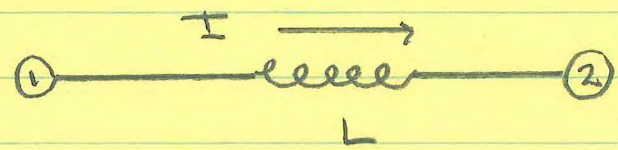
**Laws**

**Ohms Law**



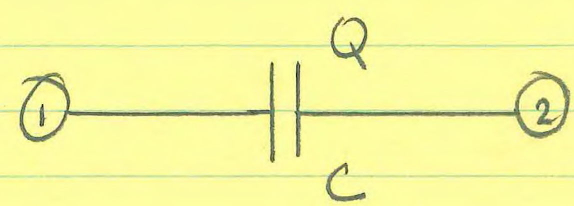

$$V_1 - V_2 = V_R = RI$$

**Law of Inductance**



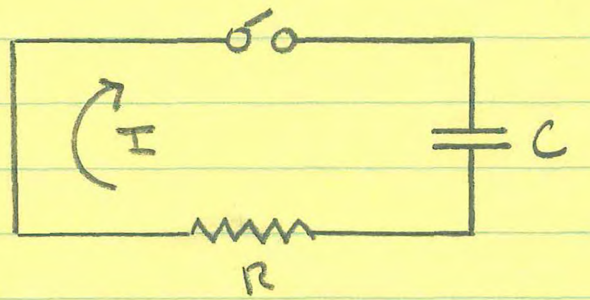

$$V_1 - V_2 = V_L = L\frac{dI}{dt}$$

**Law of Capacitance**




$$V_1 - V_2 = V_C = \frac{Q}{C}$$

Example 1




At time $t=0$ the key in the circuit opposite is closed. If the initial charge on the capacitor is $Q_0$ Coulombs, describe the state of the circuit at time $t>0$.

Solution Let $Q = Q(t)$ = charge on the capacitor at time $t$.

$$V_R + V_C = 0$$

$$RI + \frac{Q}{C} = 0$$

$$\therefore R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

$$\frac{dQ}{dt} + \frac{1}{RC}Q = 0 \quad \leftarrow 1^{st} \text{ Order Linear}$$

$$\mu(t) = e^{\int \frac{1}{RC}dt} = e^{t/RC}$$

Mult. above eqn by $\mu$: $\underbrace{e^{t/RC}\frac{dQ}{dt} + \frac{1}{RC}e^{t/RC}Q}_{} = 0$

$$\frac{d}{dt}\left(e^{t/RC}Q\right) = 0$$

Int $\quad e^{t/RC}Q = A$

$$Q = Ae^{-t/RC}$$

$\underline{t=0,\ Q=Q_0}$ : $\quad Q_0 = A e^0 \Rightarrow A = Q_0$

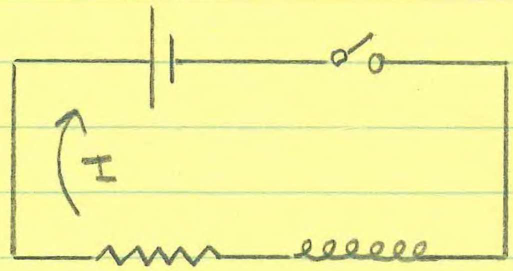
$$\therefore \quad Q = Q_0 e^{-t/RC}$$

Further, $\quad I = \dfrac{dQ}{dt} = -\dfrac{Q_0}{RC} e^{-t/RC}$

<u>Example 2</u>

At time $t=0$ the key in the circuit opposite is closed. If the initial current is zero, find the current at time $t>0$.

$E = 40$ Volts




$R = 10\,\Omega$ $\qquad L = 0.2\,H$

<u>Solution</u>

$$V_R + V_L = E$$

$$RI + L\frac{dI}{dt} = E$$

$$10I + \frac{1}{5}\frac{dI}{dt} = 40$$

$$\frac{dI}{dt} + 50I = 200, \quad I(0) = 0$$

$$\mu(t) = e^{\int 50\,dt} = e^{50t}$$

Mult by $\mu$ : $\quad e^{50t}\dfrac{dI}{dt} + 50e^{50t}I = 200e^{50t}$

$$\therefore \frac{d}{dt}\left[ e^{50t} I \right] = 200 e^{50t}$$

<u>Int</u> $$e^{50t} I = \int 200 e^{50t}\, dt + A$$

$$= 4e^{50t} + A$$

$$\therefore I = 4 + Ae^{-50t}$$

<u>$t=0, I=0$</u> $$0 = 4 + A(1) \Rightarrow A = -4$$

$$\therefore I = 4 - 4e^{-50t}$$

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