Theorem: Determinant of an Invertible Matrix

A square matrix A is invertible (nonsingular) if and only if $det(A) \neq 0$.

Proof

To prove the theorem in one direction, assume A is invertible. Then $AA^{-1} = I$, and by Theorem Determinant of a Matrix Product, you can write $|A||A^{-1}| = |I|$. Now, $|I| = 1$, so you know that neither determinant on the left is zero. Specifically, $|A| \neq 0$.

To prove the theorem in the other direction, assume the determinant of A is nonzero. Then using Gauss-Jordan, find a matrix B, in reduced row-echelon form, that is row-equivalent to A. The matrix B must be the identity matrix I or it must have at least one row that consists entirely of zeros, because B is in redued row-echelon form. But if B has a row of all zeros, then by Theorem Conditions That Yield a Zero Determinant, you know that $|B| = 0$, which would imply that $|A| = 0$. You assumed that $|A|$ is nonzero, so you can conclude that $B = I$. The matrix A is, therefore, row-equivalent to the identity matrix, and by Theorem Equivalent Conditions, you know that A is invertible.