Group member names:

Review of Proof Writing

Goals for today's activity:

1. Review how to prove and disprove universal versus existential statements
2. Review how to write a direct proof

Recall that a *universal* statement

$$\forall x \in D, P(x)$$

is TRUE if $P(x)$ is true for EVERY substitution from D, and it is FALSE if $P(x)$ is false for AT LEAST ONE substitution from D.

Recall that an *existential* statement

$$\exists x \in D \text{ s.t. } P(x)$$

is TRUE if $P(x)$ is true for AT LEAST ONE substitution from D, and it is FALSE if $P(x)$ is false for EVERY substitution from D.

Question 1. Circle ALL of the statements below that could be PROVEN using an example. Don't worry about whether they are true or not (some are and some aren't) - don't even worry about what they mean! Just focus on what *kind* of statement they are. And remember to be on the lookout for hidden universal statements.

1. Every graph can be colored with $\Delta(G) + 1$ colors. universal
2. There exists a connected 10-vertex graph with 9 edges. existential
3. Let G be a graph. If G has a vertex of degree t, then G is not $(t+1)$-connected. universal
4. There is a 5-regular graph that is not Hamiltonian. existential
5. If a graph G has a Hamilton cycle, then G has and Euler circuit. universal

Question 2. Circle ALL of the statements below that could be DISproven using an example. Don't worry about whether they are true or not (some are and some aren't) - don't even worry about what they mean! Just focus on what *kind* of statement they are. And remember to be on the lookout for hidden universal statements.

1. If $G$is a graph and it has an Euler circuit, then G has a Hamilton cycle. universal
2. 3-edge-connected graphs may not be 3-connected. existential
3. There is a 5-regular graph that is 2-colorable. existential
4. If G is a bipartite graph, then G does not contain an odd cycle. universal

What about when an example won't work? The most straightforward way to prove a universal statement (or disprove an existential statement) is using a *direct proof.* Hopefully you either remembered how to do this, or you watched the pencasts before you came as instructed.

Question 3. For each statement, (1) rewrite it formally, and (2) write what the first and last sentence would be of a direct proof of the statement. Again, you don't need to even know what the words mean!

1. Every graph can be colored with $\Delta(G) + 1$ colors.

$\forall$ graphs G, G can be colored with $\Delta(G)+1$ colors.

Proof: Let G be an arbitrary graph.

⋮

Therefore G can be colored with $\Delta(G)+1$ colors.

2. Let G be a graph. If G has a vertex of degree t, then G is not $(t+1)$-connected.

$\forall$ graphs G, if G has a vertex of degree t, then G is not (t+1)-connected.

Proof: Let G be an arbitrary graph such that G has a vertex of degree t.

⋮

Therefore G is not (t+1)-connected.

3. If G is a bipartite graph, then G does not contain an odd cycle.

$\forall$ graphs G, if G is bipartite, then G does not contain an odd cycle.

or: $\forall$ bipartite graphs G, G does not contain an odd cycle.

Proof: Let G be an arbitrary graph such that G is bipartite.

⋮

Therefore G does not contain an odd cycle.

Question 4. A statement like, "There exists a connected n-vertex graph with $n - 1$ edges," sits somewhere in the middle. Notice that this is a hidden universal statement: "For all positive integers n, there exists an n-vertex graph with $n - 1$ edges." However, we can prove it is true by choosing an arbitrary positive integer n, and then describing what an example would look like for that particular n. (Remember, when I say an *arbitrary* n, I don't mean choose $n = 13$. I mean choose $n = n$.) Go ahead and prove this statement!

$\forall n \in \mathbb{N}$, $\exists$ n-vertex graph with n-1 edges.

Proof: Let n be an arbitrary positive integer. Consider P_n. P_n has n vertices and n-1 edges by definition. Therefore there exists an n-vertex graph with n-1 edges.

* Notice: P_n isn't (technically) an example, it's infinitely many examples.