Study Plan — Handbook of Calculus

User Stories for Each Chapter

This deck includes one story card per chapter. Each card ends with `\clearpage` to ensure print separation.

P — Preparation for Calculus

Epic / Feature	Precalculus Fluency
Business Value	Reduce rework during calculus by restoring algebra/trig/log skills and modeling literacy.
Priority / Estimate	Priority: Must SP: 2
Persona	learner returning to math or entering Calculus I
Dependencies	None
Assumptions / Risks	Risk: hidden skill gaps; plan daily 20-minute drills and spaced repetition.

Story *As a calculus learner, I want to refresh function operations, trig, and exponential/log rules so that I can focus on calculus concepts instead of algebraic manipulation.*

Skills Emphasis Functions Graphs Trig Exp/Log

Acceptance Criteria (BDD)

Scenario	Fluency check
Given	a mixed set of 25 precalculus items (domains, inverse transformations)
When	I solve without a key and justify steps
Then	I score at least 80 percent and record topics for targ

Definition of Ready: Drill bank prepared; schedule set. Definition of Done: Diagnostic 80+; error log created; review plan scheduled.

Tasks

- ☐ Complete diagnostic (25 questions) across functions, trig, and logs.
- ☐ Build a formula sheet: trig identities, log laws, function transformations.
- ☐ Graph five function families with key features (intercepts, asymptotes).
- ☐ Mini-project: model a cooling or population scenario with exp/log.
- ☐ Reflection: list three algebra pitfalls and fixes.

Ch. 1 — Limits and Their Properties

Epic / Feature	Foundations
Business Value	Ground the derivative and integral in precise limiting behavior.
Priority / Estimate	Priority: Must SP: 3
Persona	calculus student
Dependencies	P Preparation
Assumptions / Risks	Confusing value with limit; overusing numeric tables without justification.

Story *As a learner, I want to compute and reason about limits and continuity so that I can validate derivative rules and model transitions reliably.*

Skills Emphasis Analytic Graphical Continuity IVT

Acceptance Criteria (BDD)

Scenario	Happy path
Given	piecewise and rational functions
When	I compute one-sided and two-sided limits, detect and apply continuity tests
Then	My conclusions match graphs/tables and I can ci (Limit Laws, Squeeze, IVT) correctly.

DoR: Persona clear; AC drafted; dependencies known. DoD: All ACs pass; proof sketch for Squeeze or IVT written; summary logged.

Tasks

- ☐ Build a concept map for limits (notation, properties, pitfalls).
- ☐ Evaluate 15 mixed limits (algebraic, trigonometric, infinity).
- ☐ Prove continuity of polynomials and rational functions on domains.
- ☐ Investigate a removable vs. jump discontinuity with a custom example.
- ☐ Reflection: three conditions where substitution fails.

Ch. 2 — Differentiation

Epic / Feature	Core Techniques
Business Value	Compute derivatives efficiently for modeling instantaneous change.
Priority / Estimate	Priority: Must SP: 3
Persona	learner preparing for STEM applications
Dependencies	Ch. 1
Assumptions / Risks	Over-memorizing rules without linearization intuition.

Story *As a learner, I want to differentiate using rules, implicit methods, and differentials so that I can analyze local behavior quickly and accurately.*

Skills Emphasis Rules Implicit Related Rates Linearization

	Scenario	Technique selection
	Given	compositions and products/quotients
Acceptance Criteria (BDD)	**When**	I select and apply the appropriate rules and simpl
	Then	Results match a CAS on random checks and I ca units.

DoR: Problems sourced; checklist ready. DoD: ACs pass; linearization mini-lab submitted; error analysis done.

Tasks

- ☐ Drill set (20): power, product, quotient, chain in mixed order.
- ☐ Implicit differentiation including circles, exponentials, and logs.
- ☐ Related rates with units (ladder, tank, or shadow problem).
- ☐ Linear approximation of a nontrivial value; compute relative error.
- ☐ Reflection: create a rule-selection flowchart.

Ch. 3 — Applications of Differentiation

Epic / Feature	Analysis and Design
Business Value	Optimize systems and understand graph behavior for decision-making.
Priority / Estimate	Priority: Must SP: 3
Persona	applied learner
Dependencies	Ch. 2
Assumptions / Risks	Confusing local and global extrema; Newton method divergence if not guarded.

Story *As a learner, I want to analyze first and second derivatives to optimize and sketch curves so that I can solve design and root-finding problems.*

Skills Emphasis Extrema Concavity Optimization Newton

Acceptance Criteria (BDD)

Scenario	Optimization
Given	a constrained word problem with meaningful units
When	I model, differentiate, and apply derivative tests
Then	I obtain a defendable optimum and interpret sensi

DoR: Realistic contexts selected. DoD: ACs pass; sketching gallery created; Newton method guardrails documented.

Tasks

- ☐ Classification practice using first/second derivative tests (12 items).
- ☐ Model and solve two optimization problems (geometry and economics).
- ☐ Limits at infinity and asymptotes set (8 items).
- ☐ Implement Newton method on two functions; document failure cases.
- ☐ Reflection: checklists for model assumptions and units.

Ch. 4 — Integration

Epic / Feature	Accumulation
Business Value	Connect areas and accumulations to definite integrals and the FTC.
Priority / Estimate	Priority: Must SP: 3
Persona	calculus student
Dependencies	Ch. 1–3
Assumptions / Risks	Confusing antiderivative with definite integral; unit interpretation neglected.

Story *As a learner, I want to interpret the definite integral as an accumulation and compute it using substitution so that I can solve area and total-change problems.*

Skills Emphasis Riemann FTC Substitution Units

	Scenario	Area and accumulation
	Given	a rate function with units
Acceptance Criteria (BDD)	**When**	I set up and evaluate the definite integral and exp
	Then	My answer matches a numeric check and the inter correct.

DoR: Contexts chosen; notation sheet prepared. DoD: ACs pass; summary card created; error log updated.

Tasks

- ☐ Derive FTC parts I and II from area accumulation arguments (sketch).
- ☐ Evaluate 15 integrals (basic antiderivatives and substitution).
- ☐ Area between a curve and axis; signed vs total area comparison.
- ☐ Create a units table linking rate, integral, and accumulation.
- ☐ Reflection: three cases where substitution is not suitable.

Ch. 5 — Logarithmic, Exponential, and Other Transcendentals

Epic / Feature	Transcendentals
Business Value	Model growth/decay and handle inverse trig/hyperbolic functions; resolve indeterminate forms.
Priority / Estimate	Priority: Must SP: 3
Persona	STEM learner
Dependencies	Ch. 2, 4
Assumptions / Risks	Misusing L'Hopital; domain issues for logs and inverse trig.

Story *As a learner, I want to differentiate and integrate logarithmic, exponential, inverse trig, and hyperbolic functions so that I can solve real growth/decay and geometry problems.*

Skills Emphasis Exp/Log InverseTr Hyperbolic LHopital

Acceptance Criteria (BDD)

Scenario	Model fit
Given	data compatible with exponential or logarithmic b
When	I fit a model, compute derivatives/integrals, and a
Then	Predictions match baseline within tolerance and c straints are respected.

DoR: Data chosen; domain notes prepared. DoD: ACs pass; L'Hopital applicability checklist written.

Tasks

- ☐ Log differentiation exercises, including products and powers.
- ☐ Integrate forms with inverse trig; note geometry interpretations.
- ☐ Apply L'Hopital to limits with proof of conditions.
- ☐ Growth/decay mini-project with parameter interpretation.
- ☐ Reflection: domain/range pitfalls encountered.

Ch. 6 — Differential Equations (Intro)

Epic / Feature	Modeling Change
Business Value	Translate real processes into solvable first-order ODEs.
Priority / Estimate	Priority: Should SP: 3
Persona	applied learner
Dependencies	Ch. 4, 5
Assumptions / Risks	Overfitting models; ignoring equilibrium analysis.

Story *As a learner, I want to read slope fields and solve separable and linear ODEs so that I can model growth/decay and approach-to-equilibrium systems.*

Skills Emphasis SlopeFields Separable Linear Logistic

Acceptance Criteria (BDD)

Scenario	Model validation
Given	data and a plausible first-order model
When	I solve the ODE and compare to data
Then	Residuals are small; equilibria and stability are in

DoR: Context picked; numeric/analytic comparison plan. DoD: ACs pass; slope-field sketch; equilibrium table submitted.

Tasks

- ☐ Sketch slope fields; identify isoclines and equilibria.
- ☐ Solve three separable ODEs and two linear ODEs.
- ☐ Fit a logistic model and interpret parameters.
- ☐ Euler method approximation for one problem; compare error.
- ☐ Reflection: modeling assumptions check.

Ch. 7 — Applications of Integration

Epic / Feature	Geometry and Physics
Business Value	Solve area, volume, work, and centroid problems via integrals.
Priority / Estimate	Priority: Must SP: 3
Persona	applied learner
Dependencies	Ch. 4
Assumptions / Risks	Setup errors with bounds or radii; units confusion.

Story *As a learner, I want to formulate geometry and physics problems as definite integrals so that I can compute areas, volumes, and work reliably.*

Skills Emphasis Areas Volumes ArcLength Work/Centroids

Acceptance Criteria (BDD)

Scenario	Volume calculation
Given	a region described by curves
When	I compute volume by disks/washers and by shells
Then	Both methods agree and units are correct.

DoR: Diagrams prepared. DoD: ACs pass; summary of method choice rules written.

Tasks

- ☐ Areas between curves (5 problems).
- ☐ Volumes by disks/washers and shells (6 problems).
- ☐ Arc length and surface area (3 problems).
- ☐ Work and centroids with densities (2 problems).
- ☐ Reflection: method selection decision tree.

Ch. 8 — Integration Techniques and Improper Integrals

Epic / Feature	Toolbox Expansion
Business Value	Handle complex integrals and convergence questions.
Priority / Estimate	Priority: Must SP: 3
Persona	calculus student
Dependencies	Ch. 4, 5
Assumptions / Risks	Choosing wrong technique; algebra slips increase.

Story *As a learner, I want a reliable process for choosing integration techniques and judging convergence so that I can solve challenging integrals.*

Skills Emphasis ByParts TrigInt PartialFrac Improper

	Scenario	Technique selection
	Given	a mixed set of 15 integrals
Acceptance Criteria (BDD)	**When**	I categorize and solve them with appropriate meth
	Then	My solutions check by differentiation and converg fied.

DoR: Mixed set prepared. DoD: ACs pass; personal technique guide created.

Tasks

- ☐ Create a technique-selection flowchart.
- ☐ Solve by parts (5), trig integrals/substitutions (5), partial fractions (5).
- ☐ Evaluate three improper integrals with tests.
- ☐ Compare exact vs numeric approximations and discuss error.
- ☐ Reflection: log frequent algebra errors.

Ch. 9 — Infinite Series

Epic / Feature	Approximation
Business Value	Decide convergence and approximate functions via power series.
Priority / Estimate	Priority: Must SP: 3
Persona	STEM learner
Dependencies	Ch. 4, 5, 8
Assumptions / Risks	Confusing necessary vs sufficient conditions; ignoring remainder bounds.

Story *As a learner, I want to test series for convergence and construct Taylor approximations so that I can approximate difficult functions and analyze error.*

Skills Emphasis ConvergenceTests PowerSeries Taylor ErrorBounds

Acceptance Criteria (BDD)	**Scenario**	Power series construction
	Given	a differentiable function near a point
	When	I build its Taylor series, determine radius/interva gence, and estimate remainder
	Then	Approximations meet a stated error tolerance on a t

DoR: Test catalogue ready. DoD: ACs pass; summary sheet of tests with conditions written.

Tasks

- ☐ Apply comparison, ratio/root, alternating, and integral tests (12 items).
- ☐ Build Taylor polynomials of degree 2, 4, 6 for target functions.
- ☐ Determine radius/interval of convergence for five power series.
- ☐ Use remainder estimates to choose degree for tolerance.
- ☐ Reflection: create a tests decision table with cues.

Ch. 10 — Conics, Parametric Equations, and Polar Coordinates

Epic / Feature	Beyond y=f(x)
Business Value	Model plane curves and compute areas/lengths in alternate coordinates.
Priority / Estimate	Priority: Should SP: 3
Persona	STEM learner
Dependencies	Ch. 4
Assumptions / Risks	Mismatched parameter ranges; orientation issues in polar.

Story *As a learner, I want to analyze parametric and polar curves and conics so that I can compute tangents, areas, and lengths beyond Cartesian form.*

Skills Emphasis Parametric PolarArea ArcLength Conics

Acceptance Criteria (BDD)

Scenario	Polar area
Given	a polar curve with specified theta bounds
When	I compute enclosed area and verify with a numeric
Then	Results match within tolerance and orientation is

DoR: Plotting tool ready. DoD: ACs pass; gallery of curves with annotations produced.

Tasks

- ☐ Parametric derivatives and tangents; eliminate parameter where possible.
- ☐ Polar area for petals and loops (3 problems).
- ☐ Arc length for one parametric and one polar curve.
- ☐ Conic classification and focus-directrix properties.
- ☐ Reflection: parameter range and orientation pitfalls.

Ch. 11 — Vectors and the Geometry of Space

Epic / Feature	3D Foundations
Business Value	Operate in R3 with vectors, lines, planes, and coordinate systems.
Priority / Estimate	Priority: Must SP: 3
Persona	multivariable learner
Dependencies	Ch. 1–3 (conceptual), algebra
Assumptions / Risks	Cross product direction errors; sign conventions.

Story *As a learner, I want to compute vector operations and equations of lines and planes so that I can model geometry and motion in space.*

Skills Emphasis Dot/Cross | Lines/Planes | Surfaces | Coordinates

Acceptance Criteria (BDD)

Scenario	Line-plane problems
Given	two points and a normal vector
When	I compute the line through points and plane throug
Then	Intersections and distances are computed with cor

DoR: Practice set selected. DoD: ACs pass; coordinate conversion table created.

Tasks

- ☐ Compute dot and cross products with geometric interpretation.
- ☐ Write parametric equations for lines; plane equations from points/normals.
- ☐ Classify quadric surfaces from equations.
- ☐ Convert between rectangular, cylindrical, spherical coordinates.
- ☐ Reflection: mnemonic for right-hand rule and orientation.

Ch. 12 — Vector-Valued Functions

Epic / Feature	Space Curves
Business Value	Model motion and curvature in R3.
Priority / Estimate	Priority: Should SP: 3
Persona	STEM learner
Dependencies	Ch. 11
Assumptions / Risks	Arc-length parameterization mistakes.

Story *As a learner, I want to differentiate/integrate vector functions and compute curvature so that I can analyze motion and turning behavior.*

Skills Emphasis Velocity/Accel T,N,B Curvature ArcLength

Acceptance Criteria (BDD)

Scenario	Curvature analysis
Given	a space curve r(t)
When	I compute T, N, curvature, and speed profiles
Then	Results are consistent with plots and units are co

DoR: Curve library chosen. DoD: ACs pass; annotated plot pack produced.

Tasks

- ☐ Compute derivatives/integrals of vector functions (5 problems).
- ☐ Find velocity/acceleration; tangential/normal components.
- ☐ Curvature and osculating circle at selected points.
- ☐ Arc length and reparameterization by arc length.
- ☐ Reflection: unit vectors and interpretation checklist.

Ch. 13 — Functions of Several Variables

Epic / Feature	Multivariable Foundations
Business Value	Generalize limits, derivatives, and optimization in higher dimensions.
Priority / Estimate	Priority: Must SP: 3
Persona	multivariable learner
Dependencies	Ch. 11
Assumptions / Risks	Incorrect limit conclusions from single-path checks.

Story *As a learner, I want to compute partial and directional derivatives and use gradients so that I can linearize and optimize multivariable functions.*

Skills Emphasis Limits/Cont Partials ChainRule Grad/Extrema

Acceptance Criteria (BDD)

Scenario	Critical point classification
Given	a twice-differentiable function
When	I find critical points and analyze the Hessian
Then	Local minima/maxima/saddles are correctly cla constraints are handled via Lagrange multipliers.

DoR: Examples chosen. DoD: ACs pass; linear approximation examples documented.

Tasks

- ☐ Compute multivariable limits with counterexamples.
- ☐ Partial derivatives and gradient fields for 6 functions.
- ☐ Multivariable chain rule exercises.
- ☐ Hessian-based classification; Lagrange multipliers on two problems.
- ☐ Reflection: checklist for limit proofs and pitfalls.

Ch. 14 — Multiple Integration

Epic / Feature	Volume and Mass
Business Value	Compute mass/volume/area and probabilities via double/triple integrals.
Priority / Estimate	Priority: Must SP: 3
Persona	multivariable learner
Dependencies	Ch. 13
Assumptions / Risks	Incorrect region bounds; missed Jacobian in coordinate changes.

Story *As a learner, I want to set up and evaluate double and triple integrals with coordinate changes so that I can compute geometric and physical quantities.*

Skills Emphasis Iterated Polar/Cyl/Sph Jacobian Apps

Acceptance Criteria (BDD)

Scenario	Region conversion
Given	a planar region suitable for polar coordinates
When	I convert the integral and include the Jacobian
Then	Results match the Cartesian computation and setup

DoR: Region diagrams prepared. DoD: ACs pass; method comparison table created.

Tasks

- ☐ Set up and evaluate 4 double integrals over non-rectangular regions.
- ☐ Change order of integration on two examples.
- ☐ Convert to polar, cylindrical, spherical for appropriate problems.
- ☐ Compute center of mass or moment of inertia for a lamina/solid.
- ☐ Reflection: Jacobian mnemonic and common mistakes.

Epic / Feature	Field Integrals
Business Value	Use line/surface integrals and fundamental theorems (Green, Divergence, Stokes).
Priority / Estimate	Priority: Must SP: 3
Persona	multivariable learner
Dependencies	Ch. 11, 13, 14
Assumptions / Risks	Orientation errors; confusing conservative fields conditions.

Story *As a learner, I want to compute line and surface integrals and apply Green, Divergence, and Stokes so that I can relate local derivatives to global flux/circulation.*

Skills Emphasis LineInt SurfaceInt Green Stokes/Div

Acceptance Criteria (BDD)

Scenario	Theorem verification
Given	a vector field and suitable region/surface
When	I compute both sides of the relevant theorem
Then	The equality holds and orientation is explained.

DoR: Fields and regions selected. DoD: ACs pass; conservative field tests summarized.

Tasks

- ☐ Determine whether a field is conservative; find potentials where possible.
- ☐ Evaluate line integrals directly and via Fundamental Theorem for line integrals.
- ☐ Apply Green's theorem on a planar region for circulation/flux.
- ☐ Compute surface integral and verify with Stokes or Divergence theorem.
- ☐ Reflection: orientation rules and right-hand conventions.

Ch. 16 — Additional Topics in Differential Equations

Epic / Feature	ODE Extensions
Business Value	Extend models with exact equations and second-order linear ODEs.
Priority / Estimate	Priority: Should SP: 3
Persona	applied learner
Dependencies	Ch. 6
Assumptions / Risks	Characteristic equation sign errors; initial condition handling.

Story *As a learner, I want to solve exact first-order and second-order linear ODEs so that I can model oscillations and forced systems.*

Skills Emphasis Exact 2nd Order Hom/Nonhom SeriesIntro

Acceptance Criteria (BDD)

Scenario	Oscillator model
Given	parameters for mass-spring-damper systems
When	I solve the ODE for homogeneous and forced case initial conditions
Then	Solutions match expected qualitative (over/critically/under-damped).

DoR: Parameter sets chosen. DoD: ACs pass; solution plots with phase interpretation provided.

Tasks

- ☐ Identify exact equations and integrate factors where needed.
- ☐ Solve homogeneous constant-coefficient ODEs; classify damping.
- ☐ Solve nonhomogeneous with method of undetermined coefficients.
- ☐ Sketch phase portraits for representative parameter regimes.
- ☐ Reflection: checklist for initial condition application.