Study Plan — Discrete and Computational Geometry

User Story Template & Examples

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How to Write Effective User Stories (Quick Guide)

User Story format *As a **[persona]** on the DCG study track, I want to **[goal]** so that **[outcome/business value]**.*

Good stories are **Independent, Negotiable, Valuable, Estimable, Small, Testable** (INVEST). Keep one verifiable outcome per story.

Acceptance Criteria Use **BDD**: **Scenario, Given, When, Then**. Criteria must be objective and testable.

Non-Functional Capture qualities that matter to the learning artifact: Rigor Reproducibility Clarity Performance

Definition of Ready Persona clear; dependencies listed; estimate set; acceptance criteria drafted; risks understood.

Definition of Done All ACs pass; tests (or proofs) recorded; code and notes committed; artifacts published.

Story Card Definition

Epic / Feature The broader learning objective this story supports (e.g., *Convex Geometry Foundations*).

Business Value Why this matters (e.g., enables later algorithms, reduces confusion, increases proof fluency).

Priority / Estimate Priority tag (*Must/Should/Could*) and a small story point estimate (e.g., *SP: 3*).

Persona Who is executing (e.g., *self-learner, teaching assistant, study group member*).

Dependencies Prerequisites (e.g., vector geometry, induction, installed toolchain).

Assumptions / Risks Time-box, scope creep, fragile datasets, numerical robustness.

Story Write the *As a / I want / so that* sentence.

Non-Functional Quality tags (Rigor Reproducibility Clarity Accessibility Privacy).

Acceptance Criteria Capture BDD criteria using Scenario/Given/When/Then.

Tasks Actionable steps with checkboxes. Keep 3–7 items.

Definition of Ready: Persona clear; AC drafted; dependencies known; estimate set.

Definition of Done: All ACs pass; tests green; quality checks; docs updated; artifacts published.

Templates

Blank Story Card Template

DCG-? — *Card Title Here*

Epic / Feature *e.g., Chapter Foundations*
Business Value *e.g., establish core concepts to build advanced topics*
Priority / Estimate **Priority:** *Must* SP: 3
Persona *self-learner / study group*
Dependencies *list any math or tooling prerequisites*
Assumptions / Risks *note uncertainties, time-box, dataset fragility*
Story *As a **[persona]**, I want to **[goal]** so that **[value]**.*
Non-Functional Performance Security Reliability Accessibility Privacy i18n
Acceptance Criteria (BDD)
Scenario Happy path
Given *the chapter text, problems, and tools are available*
When *the hands-on objectives for this card are executed*
Then *the stated outcomes are demonstrated and recorded in your repo/notebook*

Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • **Definition of Done:** All ACs pass; Tests green; Quality checks; Docs updated; Published.

Tasks
☐ *Concrete task 1*
☐ *Concrete task 2*
☐ *Concrete task 3*
☐ *Concrete task 4*

Example Cards for This Study Plan

DCG-1 — Polygons — Predicates & Basics

Epic / Feature Chapter 1: Polygons
Business Value Build the robust geometric predicates used by nearly all later chapters.
Priority / Estimate **Priority:** Must SP: 3
Persona developer on a new geometry repo
Dependencies Git toolchain; `numpy`/`matplotlib` or C++ with CGAL; unit-test framework.
Assumptions / Risks Robustness of floating-point predicates; handling degenerate cases (collinearity, duplicate points).
Story As a learner building a geometry toolkit, I want to implement orientation, segment intersection, and point-in-polygon so that downstream algorithms are correct and testable.
Non-Functional Rigor Reproducibility Clarity Performance
Acceptance Criteria (BDD)
Scenario Happy path
Given target repository and test scaffold are available
When all predicate implementations and unit tests are completed
Then tests pass on random and adversarial inputs; README includes usage and complexity notes

Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • **Definition of Done:** All ACs pass; tests green; docs updated; artifacts published.

Tasks

☐ Initialize repo with `geometry` module and a `hello_world` test.
☐ Implement `orient(p,q,r)`, `intersects(a,b,c,d)`, and winding vs. ray-crossing `point_in_polygon`.
☐ Add unit tests including degenerate and adversarial cases; measure runtime on 10^5 ops.
☐ Write a short note: interior-angle sum and a property of monotone polygons.

DCG-2 — Convex Hulls — Implementation & Analysis

Epic / Feature Chapter 2: Convex Hulls
Business Value Foundational for Delaunay/Voronoi duality and many optimization problems.
Priority / Estimate **Priority:** Must SP: 3
Persona developer optimizing geometric kernels
Dependencies DCG-1 completed; plotting utility for hull edges.
Assumptions / Risks Handling duplicates/collinearity; performance on large n.
Story As a learner, I want to implement Graham scan and Andrew's monotone chain so that I can benchmark correctness and $n \log n$ scaling.
Non-Functional Performance Rigor Reproducibility
Acceptance Criteria (BDD)
Scenario Happy path
Given point sets loaded from fixtures
When hulls computed by two algorithms with identical results
Then benchmarks and correctness proofs are recorded; README explains edge cases
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • **Definition of Done:** All ACs pass; tests green; docs updated; artifacts published.

Tasks

☐ Implement Graham scan and Monotone Chain; add Quickhull via library for comparison.
☐ Create adversarial datasets (points on circle, grid with noise, many collinear).
☐ Benchmark for $n = 10^3, 10^4, 10^5$; chart time vs. n.
☐ Write a brief correctness sketch and discuss the lower bound idea.

DCG-3 — Triangulations — Polygon & Delaunay

Epic / Feature Chapter 3: Triangulations
Business Value Enables meshing, interpolation, and Voronoi duality.
Priority / Estimate **Priority:** Must SP: 5
Persona student implementing mesh primitives
Dependencies DCG-1, DCG-2; simple polygon datasets.
Assumptions / Risks Numerical stability for empty-circle tests; handling holes.
Story As a learner, I want to triangulate polygons (ear clipping) and generate Delaunay triangulations so that I can reason about flips and mesh quality.
Non-Functional Rigor Reproducibility Clarity
Acceptance Criteria (BDD)
Scenario Happy path
Given sample polygons and point sets prepared
When triangulations rendered; flips demonstrated on convex polygons
Then visualizations saved; notes explain duality to Voronoi

Tasks

☐ Implement ear clipping for simple polygons; visualize diagonals.
☐ Implement or use a Delaunay routine; verify empty-circle property on random sets.
☐ Add flip operation and show that flips connect all triangulations of a convex n-gon.

DCG-4 — Voronoi Diagrams — Duality & Sweep

Epic / Feature Chapter 4: Voronoi Diagrams
Business Value Backbone for nearest-neighbor search and spatial partitioning.
Priority / Estimate **Priority:** Must SP: 5
Persona applied geometry learner
Dependencies Delaunay from DCG-3; plotting.
Assumptions / Risks Degeneracies (co-circular points); boundary treatment.
Story As a learner, I want to construct Voronoi diagrams and relate them to Delaunay triangulations so that I can reason about spatial proximity structures.
Non-Functional Performance Clarity Reproducibility
Acceptance Criteria (BDD)
Scenario Happy path
Given clean input point sets
When Voronoi computed and clipped to a bounding box
Then Delaunay/Voronoi plotted side-by-side with correctness notes

Tasks
☐ Generate Voronoi with a library or implement a simplified Fortune sweep.
☐ Clip unbounded cells; compare site degrees and Euler relations.
☐ Demonstrate weighted Voronoi variants (additive multiplicative) on toy data.

DCG-5 — Shape Recovery — α-Shapes & Crust

Epic / Feature Chapter 5: Shape Recovery
Business Value Reconstruction from samples; link to computational topology.
Priority / Estimate **Priority:** Should SP: 5
Persona research-minded learner
Dependencies Delaunay; filtration plotting.
Assumptions / Risks Sampling density assumptions; noise sensitivity.
Story As a learner, I want to explore α-complexes and Crust so that I can reconstruct curves from point samples and analyze stability.
Non-Functional Rigor Reproducibility Clarity
Acceptance Criteria (BDD)
Scenario Happy path
Given sampled curves and noise knobs ready
When α-sweep performed with snapshots
Then report compares precision/recall across α; failure cases documented

Tasks
☐ Build α-complex via Delaunay filtration; export frames across α.
☐ Implement NN-Crust; compare reconstructed boundary to ground truth.
☐ Analyze sensitivity to noise and sparsity with plots.

DCG-6 — Polygonal Chains — Motion & Shortest Paths

Epic / Feature Chapter 6: Polygonal Chains
Business Value Introduces linkages and motion planning; core to robotics/pathfinding.
Priority / Estimate **Priority:** Should SP: 5
Persona algorithms enthusiast
Dependencies Predicate library; polygon datasets.
Assumptions / Risks Collision checks; numerical stability.
Story As a learner, I want to simulate chain straightening and compute shortest paths in simple polygons so that I understand configuration spaces and the funnel algorithm.
Non-Functional Clarity Performance Reproducibility
Acceptance Criteria (BDD)
Scenario Happy path
Given chain editor and polygon inputs available
When straightening attempts and funnel paths computed
Then animations exported; complexity of funnel algorithm summarized

Tasks
☐ Build a simple linkage simulator with revolute joints.
☐ Implement the funnel algorithm; compare against visibility graph A*.
☐ Document examples of locked vs. unlockable chains.

DCG-7 — Polyhedra — Euler, Unfoldings, Rigidity

Epic / Feature Chapter 7: Polyhedra
Business Value Connects combinatorics and geometry in 3D; foundations for mesh processing.
Priority / Estimate **Priority:** Could SP: 5
Persona 3D geometry explorer
Dependencies OBJ/PLY loader; plotting.
Assumptions / Risks Handling non-manifold models; precision of angle sums.
Story As a learner, I want to verify Euler's formula and produce convex polyhedra unfoldings so that I can reason about curvature and rigidity.
Non-Functional Rigor Clarity Reproducibility
Acceptance Criteria (BDD)
Scenario Happy path
Given clean convex models loaded
When V,E,F and angle deficits computed
Then unfoldings exported as SVG; notes on Cauchy's rigidity compiled

Tasks
☐ Compute V, E, F; verify Euler for multiple models.
☐ Implement a star or source unfolding; export a gallery of nets.
☐ Summarize scissors congruence/Dehn invariants at a high level.

DCG-8 — Computational Complexity in Geometry

Epic / Feature Chapter 8: Complexity
Business Value Positions geometric problems within P/NP/#P; guides algorithm choices.
Priority / Estimate **Priority:** Should SP: 3
Persona theory-minded learner
Dependencies Survey papers/books at hand.

Assumptions / Risks Scope creep; formal reductions time-consuming.

Story As a learner, I want to survey complexities and implement a small ε-approximation so that I can justify algorithmic trade-offs.

Non-Functional Rigor Clarity Reproducibility

Acceptance Criteria (BDD)

Scenario Happy path
Given target problem chosen and datasets prepared
When exact and approximate solutions implemented
Then report compares solution quality and runtime; reduction outline included

Tasks

☐ Build a table of complexity classes for canonical DCG problems.
☐ Implement a simple ε-approximation (e.g., 2D k-center via grid/coreset).
☐ Write a two-page survey with references.