

HDCG Study Plan — Full User Stories

One story card per chapter (visual style matches attached examples).

HDCG-01 — Finite Point Configurations

HDCG-01

Finite Point Configurations

Epic / Feature Combinatorial & Discrete Geometry

Business Value Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Finite Point Configurations).

Priority / Estimate Priority: Must

SP: 3

Persona research student of discrete geometry

Dependencies —

Assumptions / Risks time to internalize proofs vs. breadth

As a research student of discrete geometry, I want to master Finite Point Configurations so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.

Non-Functional Reliability Reproducibility

Acceptance Criteria (BDD)

Scenario Happy path

Given the chapter, examples, and any tooling are available

When I complete the *Hands-on Objective* and validations for this chapter

Then the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks

Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • **Definition of Done:** All ACs pass; Tests green; Security/ai1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Recreate classic extremal/incidence examples; compute bounds on small instances.
- Generate synthetic point sets (random, grids, clustered) and measure incidence properties.
- Write a short note contrasting combinatorial vs. metric phenomena observed.

HDCG-02 — Packing and Covering

HDCG-02	Packing and Covering
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Packing and Covering).
Priority / Estimate	Priority: Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Packing and Covering so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Derive density bounds for circle/sphere packing in small domains; validate by experiment.
- Implement a greedy packing heuristic; compare to known optimal layouts for toy sizes.
- Compute simple covering numbers for intervals/disks; visualize uncovered mass.

HDCG-03 — Tilings

HDCG-03	Tilings
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Tilings).
Priority / Estimate	Priority: Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Tilings so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Classify tilings (periodic vs. aperiodic) for given prototiles; prove a simple property.
- Implement a substitution tiling generator; render several levels of refinement.
- Measure tile frequency and boundary growth; summarize findings.

HDCG-04 — Helly-type Theorems & Transversals

HDCG-04	Helly-type Theorems & Transversals
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Helly-type Theorems & Transversals).
Priority / Estimate	Priority: Must SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Helly-type Theorems & Transversals so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- State Helly/Carathéodory/Tverberg precisely; prove a concrete low-dimensional instance.
- Model a convex-feasibility LP; empirically test Helly-style certificates of feasibility.
- Create counterexamples to naive generalizations; document assumptions.

HDCG-05 — Pseudoline Arrangements

HDCG-05	Pseudoline Arrangements
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Pseudoline Arrangements).
Priority / Estimate	Priority: Must
	SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Pseudoline Arrangements so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Draw small arrangements; compute cells/levels/zones and verify counts.
- Implement arrangement construction for segments/lines and enumerate faces.
- Explore zone theorem numerically by measuring average zone complexity.

HDCG-06 — Oriented Matroids

HDCG-06	Oriented Matroids
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Oriented Matroids).
Priority / Estimate	Priority: Must SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Oriented Matroids so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Work with sign vectors and chirotopes on small point sets; verify axioms.
- Relate realizable vs. non-realizable examples; find and reproduce a literature example.
- Map an arrangement to an oriented matroid; note dualities.

HDCG-07 — Lattice Points & Lattice Polytopes

HDCG-07	Lattice Points & Lattice Polytopes	
Epic / Feature	Combinatorial & Discrete Geometry	
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Lattice Points & Lattice Polytopes).	
Priority / Estimate	Priority: Must	SP: 3
Persona	research student of discrete geometry	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a research student of discrete geometry, I want to master Lattice Points & Lattice Polytopes so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compute Ehrhart polynomials for small lattice polytopes; verify reciprocity numerically.
- Implement lattice-point counting for boxes/simplices; validate against closed forms.
- Investigate how dilation changes counts and coefficients.

HDCG-08 — Low-Distortion Embeddings

HDCG-08	Low-Distortion Embeddings
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Low-Distortion Embeddings).
Priority / Estimate Priority:	Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Low-Distortion Embeddings so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement a Johnson–Lindenstrauss (JL) embedding; measure distortion for varying k.
- Compare PCA vs. random projections on a real dataset; report reconstruction error.
- Document trade-offs (run time, memory, accuracy) across embedding choices.

HDCG-09 — Polygonal Linkages

HDCG-09	Polygonal Linkages
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Polygonal Linkages).
Priority / Estimate	Priority: Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Polygonal Linkages so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Simulate a simple linkage; visualize configuration space qualitatively.
- Test feasibility (realisability) for a small linkage with constraints.
- Identify singular configurations and discuss rigidity vs. flexibility.

HDCG-10 — Geometric Graph Theory

HDCG-10	Geometric Graph Theory
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Geometric Graph Theory).
Priority / Estimate	Priority: Must
	SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Geometric Graph Theory so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compute visibility graphs and planarity tests on random point sets.
- Measure crossing numbers experimentally for small n; compare to bounds.
- Explore thickness and minors on selected graphs.

HDCG-11 — Euclidean Ramsey Theory

HDCG-11	Euclidean Ramsey Theory
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Euclidean Ramsey Theory).
Priority / Estimate	Priority: Must SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Euclidean Ramsey Theory so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.	

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement colorings for points/edges; search for monochromatic structures.
- Reproduce a small Euclidean Ramsey statement; provide a constructive or probabilistic proof sketch.
- Summarize growth rates and open directions.

HDCG-12 — Discrete Aspects of Stochastic Geometry

HDCG-12	Discrete Aspects of Stochastic Geometry	
Epic / Feature	Combinatorial & Discrete Geometry	
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Discrete Aspects of Stochastic Geometry).	
Priority / Estimate	Priority: Must	SP: 3
Persona	research student of discrete geometry	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth; variance in experiments; need for many trials	
<i>As a research student of discrete geometry, I want to master Discrete Aspects of Stochastic Geometry so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed/flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Simulate Poisson point processes; estimate mean area/length of derived structures.
- Compute mean widths/coverage via Monte Carlo; include confidence intervals.
- Compare empirical findings to theoretical expectations where available.

HDCG-13 — Geometric Discrepancy & Uniform Distribution

HDCG-13	Geometric Discrepancy & Uniform Distribution	
Epic / Feature	Combinatorial & Discrete Geometry	
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Geometric Discrepancy & Uniform Distribution).	
Priority / Estimate	Priority: Must	SP: 3
Persona	research student of discrete geometry	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Geometric Discrepancy & Uniform Distribution so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility	
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compute discrepancy of random vs. low-discrepancy sequences (Halton/Sobol) on test sets.
- Integrate a smooth test function using QMC vs. MC; compare convergence rates.
- Relate discrepancy to VC-dimension or range spaces from the chapter.

HDCG-14 — Polyominoes

HDCG-14	Polyominoes
Epic / Feature	Combinatorial & Discrete Geometry
Business Value	Establish theoretical tools and combinatorial principles that underpin later algorithms and applications. (Polyominoes).
Priority / Estimate	Priority: Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Polyominoes so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Use exact cover (DLX) to tile small regions; verify parity and area constraints.
- Classify tilings for a given board; produce enumerations for small sizes.
- Document invariants that quickly rule out impossible tilings.

HDCG-15 — Convex Polytopes: Basics

HDCG-15	Convex Polytopes: Basics
Epic / Feature	Polytopes & Polyhedra
Business Value	Build fluency with polytopal structures used across optimization, geometry processing, and combinatorics. (Convex Polytopes: Basics).
Priority / Estimate	Priority: Must
	SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Convex Polytopes: Basics so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Convert small polytopes between H- and V-representations; check duals.
- Illustrate faces/facets/normal fans with a plotting tool; verify Euler.
- Summarize duality relationships with examples (simplex, cube, cross-polytope).

HDCG-16 — Subdivisions & Triangulations of Polytopes

HDCG-16	Subdivisions & Triangulations of Polytopes				
Epic / Feature	Polytopes & Polyhedra				
Business Value	Build fluency with polytopal structures used across optimization, geometry processing, and combinatorics. (Subdivisions & Triangulations of Polytopes).				
Priority / Estimate	Priority: Must				SP: 3
Persona	research student of discrete geometry				
Dependencies	prior chapters as referenced, convex hulls, orientation/circumcircle predicates				
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Subdivisions & Triangulations of Polytopes so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>				
Non-Functional	Reliability	Reproducibility	Performance	Scalability	
Acceptance Criteria (BDD)					
Scenario	Happy path				
Given	the chapter, examples, and any tooling are available				
When	I complete the <i>Hands-on Objective</i> and validations for this chapter				
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks				
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed/flagged.					

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Construct regular triangulations; implement flips and track secondary polytopes qualitatively.
- Evaluate simplex quality metrics (aspect ratio, minimum angle).
- Show effect of lifting on triangulation regularity.

HDCG-17 — Face Numbers: f-, h-, and g-vectors

HDCG-17	Face Numbers: f-, h-, and g-vectors	
Epic / Feature	Polytopes & Polyhedra	
Business Value	Build fluency with polytopal structures used across optimization, geometry processing, and combinatorics. (Face Numbers: f-, h-, and g-vectors).	
Priority / Estimate	Priority: Must	SP: 3
Persona	research student of discrete geometry	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a research student of discrete geometry, I want to master Face Numbers: f-, h-, and g-vectors so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compute f- and h-vectors for classic polytopes; check Dehn–Sommerville relations.
- Explore g-theorem examples; illustrate inequalities with plots.
- Prepare a cheat sheet summarizing identities and constraints.

HDCG-18 — Symmetry of Polytopes

HDCG-18	Symmetry of Polytopes
Epic / Feature	Polytopes & Polyhedra
Business Value	Build fluency with polytopal structures used across optimization, geometry processing, and combinatorics. (Symmetry of Polytopes).
Priority / Estimate	Priority: Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Symmetry of Polytopes so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Identify automorphism groups for basic polytopes; compute group orders.
- Relate symmetry to orbits of faces; visualize symmetric embeddings.
- Discuss symmetry exploitation in algorithms (state-space reduction).

HDCG-19 — Polytope Skeletons & Paths

HDCG-19	Polytope Skeletons & Paths
Epic / Feature	Polytopes & Polyhedra
Business Value	Build fluency with polytopal structures used across optimization, geometry processing, and combinatorics. (Polytope Skeletons & Paths).
Priority / Estimate	Priority: Must SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Polytope Skeletons & Paths so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Experiment with graph diameters; simulate pivot paths on polytope graphs.
- Implement simple pivot rules; measure paths vs. diameter bounds.
- Relate results to Hirsch-type questions qualitatively.

HDCG-20 — Polyhedral Maps

HDCG-20	Polyhedral Maps
Epic / Feature	Polytopes & Polyhedra
Business Value	Build fluency with polytopal structures used across optimization, geometry processing, and combinatorics. (Polyhedral Maps).
Priority / Estimate	Priority: Must
SP:	3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Polyhedral Maps so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Embed simple polyhedral maps on surfaces; verify Euler characteristic.
- Count faces/edges/vertices; confirm orientability effects.
- Construct a small map with specified degree sequence.

HDCG-21 — Topological Methods in Discrete Geometry

HDCG-21	Topological Methods in Discrete Geometry	
Epic / Feature	Combinatorial & Computational Topology	
Business Value	Leverage topological perspectives to reason about existence, structure, and invariants in geometric problems. (Topological Methods in Discrete Geometry).	
Priority / Estimate	Priority: Must	SP: 3
Persona	research student of discrete geometry	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a research student of discrete geometry, I want to master Topological Methods in Discrete Geometry so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility	
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Use ham-sandwich or Borsuk–Ulam style arguments to prove a discrete claim.
- Illustrate topological proof vs. combinatorial alternative on the same problem.
- Document assumptions and generalization limits.

HDCG-22 — Random Simplicial Complexes

HDCG-22	Random Simplicial Complexes	
Epic / Feature	Combinatorial & Computational Topology	
Business Value	Leverage topological perspectives to reason about existence, structure, and invariants in geometric problems. (Random Simplicial Complexes).	
Priority / Estimate	Priority: Must	SP: 3
Persona	research student of discrete geometry	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth; variance in experiments; need for many trials	
<i>As a research student of discrete geometry, I want to master Random Simplicial Complexes so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/ally checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Generate Linial–Meshulam or Vietoris–Rips models; sweep probability parameters.
- Measure thresholds for connectedness or vanishing homology.
- Plot Betti numbers across regimes; discuss finite-size effects.

HDCG-23 — Graphs on Surfaces (Embeddings & Genus)

HDCG-23

Graphs on Surfaces (Embeddings & Genus)

Epic / Feature Combinatorial & Computational Topology

Business Value Leverage topological perspectives to reason about existence, structure, and invariants in geometric problems. (Graphs on Surfaces (Embeddings & Genus)).

Priority / Estimate Priority: Must

SP: 3

Persona research student of discrete geometry

Dependencies prior chapters as referenced

Assumptions / Risks time to internalize proofs vs. breadth; numerical robustness and degeneracies

As a research student of discrete geometry, I want to master Graphs on Surfaces (Embeddings & Genus) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.

Non-Functional Reliability Reproducibility Robustness Accuracy

Acceptance Criteria (BDD)

Scenario Happy path

Given the chapter, examples, and any tooling are available

When I complete the *Hands-on Objective* and validations for this chapter

Then the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks

Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set.

• **Definition of Done:** All ACs pass; Tests green; Security/ally checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement planarity testing; compute genus for small graphs.
- Route edges on a surface embedding; visualize crossings by handles.
- Compare embeddings with/without constraints.

HDCG-24 — Persistent Homology (Barcodes)

HDCG-24	Persistent Homology (Barcodes)
Epic / Feature	Combinatorial & Computational Topology
Business Value	Leverage topological perspectives to reason about existence, structure, and invariants in geometric problems. (Persistent Homology (Barcodes)).
Priority / Estimate	Priority: Must
	SP: 3
Persona	research student of discrete geometry
Dependencies	prior chapters as referenced, simplicial complexes, metrics, stability notion
Assumptions / Risks	time to internalize proofs vs. breadth
<i>As a research student of discrete geometry, I want to master Persistent Homology (Barcodes) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.	

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Build filtrations (Rips/Čech); compute 0–2D barcodes on a toy dataset.
- Study stability by adding noise and comparing bottleneck distance.
- Interpret features (lifetimes) with domain context.

HDCG-25 — High-dimensional Topological Data Analysis

HDCG-25	High-dimensional Topological Data Analysis	
Epic / Feature	Combinatorial & Computational Topology	
Business Value	Leverage topological perspectives to reason about existence, structure, and invariants in geometric problems. (High-dimensional Topological Data Analysis).	
Priority / Estimate	Priority: Must	SP: 3
Persona	data scientist working with geometric methods	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth; curse of dimensionality; memory pressure	
<i>As a data scientist working with geometric methods, I want to master High-dimensional Topological Data Analysis so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed/flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Construct sparse filtrations; apply landmarks/witness complexes.
- Use dimensionality reduction to visualize summaries; validate with metrics.
- Evaluate scalability (time/memory) vs. sample size and dimension.

HDCG-26 — Convex Hulls

HDCG-26	Convex Hulls
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Convex Hulls).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Convex Hulls so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement 2D (Andrew) and 3D (incremental) hulls; include degeneracy handling.
- Add exact/filtered predicates; benchmark accuracy and speed.
- Validate by area/volume/facet orientation; export meshes.

HDCG-27 — Voronoi Diagrams & Delaunay Triangulations

HDCG-27	Voronoi Diagrams & Delaunay Triangulations			
Epic / Feature	Algorithms of Fundamental Geometric Objects			
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Voronoi Diagrams & Delaunay Triangulations).			
Priority / Estimate	Priority: Must			SP: 3
Persona	applied geometry engineer			
Dependencies	prior chapters as referenced, convex hulls, orientation/circumcircle predicates			
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Voronoi Diagrams & Delaunay Triangulations so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>			
Non-Functional	Reliability	Reproducibility	Performance	Scalability
Acceptance Criteria (BDD)				
Scenario	Happy path			
Given	the chapter, examples, and any tooling are available			
When	I complete the <i>Hands-on Objective</i> and validations for this chapter			
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks			
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed/flagged.				

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Build Delaunay via edge flips/incremental insertion; output Voronoi by duality.
- Implement point location; compare query latency to kd-tree baseline.
- Stress-test co-circular/duplicate inputs; enable exact predicates.

HDCG-28 — Arrangements of Curves and Surfaces

HDCG-28	Arrangements of Curves and Surfaces			
Epic / Feature	Algorithms of Fundamental Geometric Objects			
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Arrangements of Curves and Surfaces).			
Priority / Estimate	Priority: Must			SP: 3
Persona	applied geometry engineer			
Dependencies	prior chapters as referenced			
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies			
<i>As a applied geometry engineer, I want to master Arrangements of Curves and Surfaces so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>				
Non-Functional	Reliability	Reproducibility	Robustness	Accuracy
Acceptance Criteria (BDD)				
Scenario	Happy path			
Given	the chapter, examples, and any tooling are available			
When	I complete the <i>Hands-on Objective</i> and validations for this chapter			
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks			
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/ally checks; Docs updated; Deployed flagged.				

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Construct arrangements of segments/lines; enumerate cells and graph structure.
- Measure zone/level complexity empirically; compare to theory.
- Demonstrate applications (motion planning cells / point location).

HDCG-29 — Triangulations & Mesh Generation

HDCG-29	Triangulations & Mesh Generation	
Epic / Feature	Algorithms of Fundamental Geometric Objects	
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Triangulations & Mesh Generation).	
Priority / Estimate	Priority: Must	SP: 3
Persona	applied geometry engineer	
Dependencies	prior chapters as referenced, convex hulls, orientation/circumcircle predicates	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a applied geometry engineer, I want to master Triangulations & Mesh Generation so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>		
Non-Functional	Reliability Reproducibility Performance Scalability	
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement Delaunay refinement; track minimum angle and element size.
- Perform boundary recovery; assess quality metrics before/after refinement.
- Export mesh and run a simple PDE/graphics demo.

HDCG-30 — Polygons (Geometry & Algorithms)

HDCG-30

Polygons (Geometry & Algorithms)

Epic / Feature Algorithms of Fundamental Geometric Objects

Business Value Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Polygons (Geometry & Algorithms)).

Priority / Estimate Priority: Must **SP:** 3

Persona applied geometry engineer

Dependencies prior chapters as referenced

Assumptions / Risks time to internalize proofs vs. breadth

As a applied geometry engineer, I want to master Polygons (Geometry & Algorithms) so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.

Non-Functional Reliability Reproducibility Performance Scalability

Acceptance Criteria (BDD)

Scenario Happy path

Given the chapter, examples, and any tooling are available

When I complete the *Hands-on Objective* and validations for this chapter

Then the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks

Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • **Definition of Done:** All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement point-in-polygon (ray casting / winding number); triangulate simple polygons.
- Handle degeneracies (collinearity, repeated vertices); compute area/centroid.
- Demonstrate art-gallery style visibility on floorplan polygons.

HDCG-31 — Shortest Paths & Networks

HDCG-31	Shortest Paths & Networks
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Shortest Paths & Networks).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Shortest Paths & Networks so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Build a visibility graph and run Dijkstra for polygonal domains.
- Implement continuous Dijkstra or funnel algorithm on triangulations.
- Compare path lengths and runtimes across methods.

HDCG-32 — Proximity Algorithms

HDCG-32	Proximity Algorithms
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Proximity Algorithms).
Priority / Estimate Priority:	Must
SP:	3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Proximity Algorithms so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Construct MST/ β -skeletons and k-NN graphs; profile runtimes.
- Analyse stability under noise; compare exact vs. approximate structures.
- Summarize use-cases (clustering, skeletonization).

HDCG-33 — Visibility & Art-Gallery Problems

HDCG-33	Visibility & Art-Gallery Problems
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Visibility & Art-Gallery Problems).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Visibility & Art-Gallery Problems so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compute visibility polygons; handle holes and reflex vertices.
- Formulate simple art-gallery coverage and test heuristics.
- Visualize guard placements and uncovered regions.

HDCG-34 — Geometric Reconstruction Problems

HDCG-34	Geometric Reconstruction Problems
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Geometric Reconstruction Problems).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Geometric Reconstruction Problems so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Recover shapes from projections or shadows; study minimal measurement sets.
- Quantify reconstruction error (Hausdorff/symmetric difference) on synthetic data.
- Discuss identifiability conditions and failure modes.

HDCG-35 — Curve & Surface Reconstruction

HDCG-35	Curve & Surface Reconstruction
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Curve & Surface Reconstruction).
Priority / Estimate	Priority: Must
SP:	3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies
<i>As a applied geometry engineer, I want to master Curve & Surface Reconstruction so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility Robustness Accuracy
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.	

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement a simple crust/ball-pivoting/Poisson pipeline on noisy samples.
- Tune parameters and measure Hausdorff distance to ground truth.
- Report topology errors and smoothing trade-offs.

HDCG-36 — Computational Convexity

HDCG-36	Computational Convexity
Epic / Feature	Algorithms of Fundamental Geometric Objects
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Computational Convexity).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Computational Convexity so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Solve membership/separation via cutting-plane or ellipsoid on toy instances.
- Compare oracle-based methods vs. explicit H/V representations.
- Document complexity and numerical behavior.

HDCG-37 — Algorithmic Real Algebraic Geometry

HDCG-37	Algorithmic Real Algebraic Geometry			
Epic / Feature	Algorithms of Fundamental Geometric Objects			
Business Value	Master core geometric algorithms that power search, reconstruction, meshing, and planning pipelines. (Algorithmic Real Algebraic Geometry).			
Priority / Estimate	Priority: Must	SP: 3		
Persona	applied geometry engineer			
Dependencies	prior chapters as referenced			
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies			
<i>As a applied geometry engineer, I want to master Algorithmic Real Algebraic Geometry so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>				
Non-Functional	Reliability	Reproducibility	Robustness	Accuracy
Acceptance Criteria (BDD)				
Scenario	Happy path			
Given	the chapter, examples, and any tooling are available			
When	I complete the <i>Hands-on Objective</i> and validations for this chapter			
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks			
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.	• Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.	

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Experiment with solving small semi-algebraic systems; visualize solution sets.
- Use cylindrical algebraic decomposition (CAD) conceptually or via a CAS for toy inputs.
- Discuss complexity blowups and practical workarounds.

HDCG-38 — Point Location

HDCG-38	Point Location
Epic / Feature	Geometric Data Structures & Searching
Business Value	Develop data structures for fast queries, intersections, and proximity at scale. (Point Location).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Point Location so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Build a randomized trapezoidal map; benchmark query vs. build time.
- Test degeneracies and dynamic updates (insertions).
- Compare to persistent search structures where applicable.

HDCG-39 — Collision Detection & Proximity Queries

HDCG-39	Collision Detection & Proximity Queries						
Epic / Feature	Geometric Data Structures & Searching						
Business Value	Develop data structures for fast queries, intersections, and proximity at scale. (Collision Detection & Proximity Queries).						
Priority / Estimate	Priority: Must						
Persona	applied geometry engineer						
Dependencies	prior chapters as referenced						
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies						
<i>As a applied geometry engineer, I want to master Collision Detection & Proximity Queries so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>							
Non-Functional	Reliability	Reproducibility	Robustness	Accuracy			
Acceptance Criteria (BDD)							
Scenario	Happy path						
Given	the chapter, examples, and any tooling are available						
When	I complete the <i>Hands-on Objective</i> and validations for this chapter						
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks						
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/ally checks; Docs updated; Deployed flagged.							

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement BVH (AABB/OBB) construction and traversal; add narrow-phase tests.
- Evaluate continuous collision detection for moving segments/triangles.
- Profiling: queries/sec vs. object count; document worst-case scenes.

HDCG-40 — Range Searching

HDCG-40	Range Searching
Epic / Feature	Geometric Data Structures & Searching
Business Value	Develop data structures for fast queries, intersections, and proximity at scale. (Range Searching).
Priority / Estimate	Priority: Must
SP:	3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Range Searching so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement kd/interval/segment trees; test orthogonal range counting/reporting.
- Measure query/update trade-offs; visualize pruning behavior.
- Scale experiments to large n and report memory footprints.

HDCG-41 — Ray Shooting & Lines in Space

HDCG-41	Ray Shooting & Lines in Space
Epic / Feature	Geometric Data Structures & Searching
Business Value	Develop data structures for fast queries, intersections, and proximity at scale. (Ray Shooting & Lines in Space).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Ray Shooting & Lines in Space so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement ray-scene intersection with uniform grid and BVH; compare.
- Validate against analytic scenes; collect miss/hit statistics.
- Profile coherent vs. incoherent rays.

HDCG-42 — Geometric Intersection

HDCG-42	Geometric Intersection
Epic / Feature	Geometric Data Structures & Searching
Business Value	Develop data structures for fast queries, intersections, and proximity at scale. (Geometric Intersection).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies
<i>As a applied geometry engineer, I want to master Geometric Intersection so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.	

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Write exact segment/triangle intersection predicates; fuzz edge cases.
- Build sweep-line for segment intersection; report complexity and statistics.
- Summarize robustness fixes (epsilon vs. exact arithmetic).

HDCG-43 — Nearest Neighbors in High Dimension

HDCG-43	Nearest Neighbors in High Dimension
Epic / Feature	Geometric Data Structures & Searching
Business Value	Develop data structures for fast queries, intersections, and proximity at scale. (Nearest Neighbors in High Dimension).
Priority / Estimate	Priority: Must
SP:	3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth; curse of dimensionality; memory pressure
<i>As a applied geometry engineer, I want to master Nearest Neighbors in High Dimension so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/ally checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compare kd-tree, LSH, and HNSW on real vector data; tune parameters.
- Measure recall/latency trade-offs; draw accuracy-speed curves.
- Discuss curse-of-dimensionality and mitigation strategies.

HDCG-44 — Randomization & Derandomization in Geometry

HDCG-44	Randomization & Derandomization in Geometry	
Epic / Feature	Computational Techniques	
Business Value	Adopt practical computation techniques for speed, robustness, generalization, and summaries. (Randomization & Derandomization in Geometry).	
Priority / Estimate	Priority: Must	SP: 3
Persona	applied geometry engineer	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth; variance in experiments; need for many trials	
<i>As a applied geometry engineer, I want to master Randomization & Derandomization in Geometry so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/ally checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement random sampling/ ε -nets for a range space; validate bounds empirically.
- Replace with a deterministic construction (derandomization) on small cases.
- Compare quality and runtime for both approaches.

HDCG-45 — Robust Geometric Computation

HDCG-45	Robust Geometric Computation
Epic / Feature	Computational Techniques
Business Value	Adopt practical computation techniques for speed, robustness, generalization, and summaries. (Robust Geometric Computation).
Priority / Estimate Priority:	Must
SP:	3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies
<i>As a applied geometry engineer, I want to master Robust Geometric Computation so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility Robustness Accuracy
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Integrate exact/filtered predicates into one pipeline; catalog failures with floats.
- Re-run earlier algorithms (hull, Delaunay, intersections) under robustness modes.
- Summarize cost of robustness vs. correctness benefits.

HDCG-46 — Parallel Algorithms in Geometry

HDCG-46	Parallel Algorithms in Geometry
Epic / Feature	Computational Techniques
Business Value	Adopt practical computation techniques for speed, robustness, generalization, and summaries. (Parallel Algorithms in Geometry).
Priority / Estimate Priority:	Must
SP:	3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced, task parallelism basics, threading model
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Parallel Algorithms in Geometry so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Parallelize a hull or Delaunay implementation; measure speedup and scalability.
- Identify contention hotspots; propose work partitioning.
- Test on varying cores; produce a scalability plot.

HDCG-47 — ε -nets & ε -approximations

HDCG-47	ε -nets & ε -approximations
Epic / Feature	Computational Techniques
Business Value	Adopt practical computation techniques for speed, robustness, generalization, and summaries. (ε -nets & ε -approximations).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master ε-nets & ε-approximations so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Construct ε -nets/approximations for basic range spaces; verify hitting/approximation properties.
- Relate to VC-dimension; compute sample sizes for target ε, δ .
- Apply to a small learning-like problem (set cover/active sampling).

HDCG-48 — Coresets & Sketches

HDCG-48	Coresets & Sketches
Epic / Feature	Computational Techniques
Business Value	Adopt practical computation techniques for speed, robustness, generalization, and summaries. (Coresets & Sketches).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master Coresets & Sketches so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Performance Scalability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Build k-means/median coresets; evaluate clustering error vs. exact.
- Profile construction time vs. coreset size; plot trade-offs.
- Demonstrate downstream speedups with negligible loss.

HDCG-49 — Linear Programming (Low-dimensional & Randomized)

HDCG-49	Linear Programming (Low-dimensional & Randomized)				
Epic / Feature	Applications of Discrete & Computational Geometry				
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Linear Programming (Low-dimensional & Randomized)).				
Priority / Estimate	Priority: Must				SP: 3
Persona	practitioner building geometry-driven applications				
Dependencies	prior chapters as referenced				
Assumptions / Risks	time to internalize proofs vs. breadth; variance in experiments; need for many trials				
<i>As a practitioner building geometry-driven applications, I want to master Linear Programming (Low-dimensional & Randomized) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>					
Non-Functional	Reliability	Reproducibility	Performance	Scalability	
Acceptance Criteria (BDD)					
Scenario	Happy path				
Given	the chapter, examples, and any tooling are available				
When	I complete the <i>Hands-on Objective</i> and validations for this chapter				
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks				
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.					

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement randomized incremental LP in low dimension; visualize feasible region.
- Benchmark vs. simplex on small test sets; record degeneracy behavior.
- Apply to a geometric optimization mini-problem (smallest enclosing ball).

HDCG-50 — Algorithmic Motion Planning

HDCG-50	Algorithmic Motion Planning	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Algorithmic Motion Planning).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Algorithmic Motion Planning so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement PRM/RRT on a 2D environment; measure coverage and path quality.
- Add collision checks via BVH; compare planners across seeds.
- Export paths and visualize milestones/edges.

HDCG-51 — Robotics: Configuration Spaces

HDCG-51	Robotics: Configuration Spaces	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Robotics: Configuration Spaces).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Robotics: Configuration Spaces so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Model a planar arm's configuration space with obstacles; compute free space components.
- Plan a collision-free path; validate in a simple simulator.
- Discuss DOF scaling and sampling strategies.

HDCG-52 — Computer Graphics: Geometric Pipelines

HDCG-52	Computer Graphics: Geometric Pipelines		
Epic / Feature	Applications of Discrete & Computational Geometry		
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Computer Graphics: Geometric Pipelines).		
Priority / Estimate	Priority: Must		SP: 3
Persona	practitioner building geometry-driven applications		
Dependencies	prior chapters as referenced		
Assumptions / Risks	time to internalize proofs vs. breadth		
<i>As a practitioner building geometry-driven applications, I want to master Computer Graphics: Geometric Pipelines so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>			
Non-Functional	Reliability	Reproducibility	Usability
Acceptance Criteria (BDD)			
Scenario	Happy path		
Given	the chapter, examples, and any tooling are available		
When	I complete the <i>Hands-on Objective</i> and validations for this chapter		
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks		
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.			

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement geometric clipping and rasterization of a simple scene.
- Run mesh simplification; report quality vs. decimation.
- Profile the pipeline stages you implemented.

HDCG-53 — Modeling Motion (Rigid & Affine)

HDCG-53	Modeling Motion (Rigid & Affine)	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Modeling Motion (Rigid & Affine)).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Modeling Motion (Rigid & Affine) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Derive rigid/affine transforms; implement screw interpolation demo.
- Track errors under concatenation; verify invariants.
- Compare different interpolation schemes for stability.

HDCG-54 — Pattern Recognition (Geometric View)

HDCG-54	Pattern Recognition (Geometric View)	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Pattern Recognition (Geometric View)).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Pattern Recognition (Geometric View) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement geometric classifiers (nearest-center/Voronoi) on a toy dataset.
- Compare to an SVM baseline; report decision boundary shapes.
- Analyze robustness to outliers using geometric medians.

HDCG-55 — Graph Drawing

HDCG-55	Graph Drawing
Epic / Feature	Applications of Discrete & Computational Geometry
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Graph Drawing).
Priority / Estimate	Priority: Must
SP:	3
Persona	practitioner building geometry-driven applications
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a practitioner building geometry-driven applications, I want to master Graph Drawing so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility Usability
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement force-directed layout with planarity constraints where possible.
- Test layered/Sugiyama layout; manage crossings.
- Quantify edge length variance and crossing counts.

HDCG-56 — Splines & Geometric Modeling

HDCG-56	Splines & Geometric Modeling
Epic / Feature	Applications of Discrete & Computational Geometry
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Splines & Geometric Modeling).
Priority / Estimate	Priority: Must
SP:	3
Persona	practitioner building geometry-driven applications
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a practitioner building geometry-driven applications, I want to master Splines & Geometric Modeling so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement B-spline/NURBS evaluation; verify C1/C2 continuity on examples.
- Fit curves to sample points; measure error sensitivity.
- Render and annotate control polygon effects.

HDCG-57 — Solid Modeling (B-Rep & CSG)

HDCG-57	Solid Modeling (B-Rep & CSG)
Epic / Feature	Applications of Discrete & Computational Geometry
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Solid Modeling (B-Rep & CSG)).
Priority / Estimate	Priority: Must SP: 3
Persona	practitioner building geometry-driven applications
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies
<i>As a practitioner building geometry-driven applications, I want to master Solid Modeling (B-Rep & CSG) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>	
Non-Functional	Reliability Reproducibility Robustness Accuracy
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Convert between CSG and B-rep for simple solids; implement Boolean ops.
- Detect and repair non-manifold issues; validate watertightness.
- Export to a CAD-friendly format and roundtrip.

HDCG-58 — Robust Statistics: Data Depth & Medians

HDCG-58	Robust Statistics: Data Depth & Medians						
Epic / Feature	Applications of Discrete & Computational Geometry						
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Robust Statistics: Data Depth & Medians).						
Priority / Estimate	Priority: Must						
Persona	data scientist working with geometric methods						
Dependencies	prior chapters as referenced						
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies						
<i>As a data scientist working with geometric methods, I want to master Robust Statistics: Data Depth & Medians so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>							
Non-Functional	Reliability	Reproducibility	Robustness	Accuracy			
Acceptance Criteria (BDD)							
Scenario	Happy path						
Given	the chapter, examples, and any tooling are available						
When	I complete the <i>Hands-on Objective</i> and validations for this chapter						
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks						
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.							

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compute Tukey depth and halfspace medians on 2D data; visualize contours.
- Compare robust vs. least-squares fits under outliers.
- Report breakdown points and runtime.

HDCG-59 — Geographic Information Systems (GIS)

HDCG-59	Geographic Information Systems (GIS)				
Epic / Feature	Applications of Discrete & Computational Geometry				
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Geographic Information Systems (GIS)).				
Priority / Estimate	Priority: Must				SP: 3
Persona	practitioner building geometry-driven applications				
Dependencies	prior chapters as referenced, map projections, spherical geometry basics				
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies				
<i>As a practitioner building geometry-driven applications, I want to master Geographic Information Systems (GIS) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>					
Non-Functional	Reliability	Reproducibility	Robustness	Accuracy	
Acceptance Criteria (BDD)					
Scenario	Happy path				
Given	the chapter, examples, and any tooling are available				
When	I complete the <i>Hands-on Objective</i> and validations for this chapter				
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks				
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.					

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Implement point-in-polygon on geodesic coordinates; handle holes and winding.
- Project between WGS84 and a planar CRS; quantify distortion.
- Run spatial joins at scale; profile performance.

HDCG-60 — Grassmann–Cayley Algebra

HDCG-60	Grassmann–Cayley Algebra	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Grassmann–Cayley Algebra).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Grassmann–Cayley Algebra so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Represent joins/meets using Grassmann–Cayley algebra; solve a projective incidence problem.
- Work a small numerical example to verify identities.
- Document where this algebra simplifies proofs.

HDCG-61 — Rigidity & Scene Analysis

HDCG-61	Rigidity & Scene Analysis
Epic / Feature	Applications of Discrete & Computational Geometry
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Rigidity & Scene Analysis).
Priority / Estimate	Priority: Must
	SP: 3
Persona	practitioner building geometry-driven applications
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a practitioner building geometry-driven applications, I want to master Rigidity & Scene Analysis so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Test bar-and-joint rigidity via rank conditions; visualize stresses.
- Generate minimally rigid (Laman) graphs; perturb to test stability.
- Relate rigidity to structure-from-motion intuition.

HDCG-62 — Rigidity of Symmetric Frameworks

HDCG-62	Rigidity of Symmetric Frameworks	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Rigidity of Symmetric Frameworks).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Rigidity of Symmetric Frameworks so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Use group actions to adapt rigidity counts; build symmetric examples.
- Check how symmetry changes generic rigidity; demonstrate a case study.
- Summarize computational implications.

HDCG-63 — Global Rigidity

HDCG-63	Global Rigidity
Epic / Feature	Applications of Discrete & Computational Geometry
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Global Rigidity).
Priority / Estimate	Priority: Must
SP:	3
Persona	practitioner building geometry-driven applications
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a practitioner building geometry-driven applications, I want to master Global Rigidity so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Differentiate local vs. global rigidity; check generic conditions on small graphs.
- Search for counterexamples; visualize multiple embeddings.
- Note algorithmic challenges and complexity hints.

HDCG-64 — Crystals: Periodic & Aperiodic Structures

HDCG-64	Crystals: Periodic & Aperiodic Structures	
Epic / Feature	Applications of Discrete & Computational Geometry	
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Crystals: Periodic & Aperiodic Structures).	
Priority / Estimate	Priority: Must	SP: 3
Persona	practitioner building geometry-driven applications	
Dependencies	prior chapters as referenced	
Assumptions / Risks	time to internalize proofs vs. breadth	
<i>As a practitioner building geometry-driven applications, I want to master Crystals: Periodic & Aperiodic Structures so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>		
Non-Functional	Reliability	Reproducibility
Acceptance Criteria (BDD)		
Scenario	Happy path	
Given	the chapter, examples, and any tooling are available	
When	I complete the <i>Hands-on Objective</i> and validations for this chapter	
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks	
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.		

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Generate periodic nets; compute basic invariants and visualize unit cells.
- Contrast with an aperiodic example; discuss diffraction-like signatures.
- Explore stability under perturbations.

HDCG-65 — Structural Molecular Biology (Distance Geometry)

HDCG-65	Structural Molecular Biology (Distance Geometry)			
Epic / Feature	Applications of Discrete & Computational Geometry			
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Structural Molecular Biology (Distance Geometry)).			
Priority / Estimate	Priority: Must			SP: 3
Persona	practitioner building geometry-driven applications			
Dependencies	prior chapters as referenced			
Assumptions / Risks	time to internalize proofs vs. breadth; numerical robustness and degeneracies			
<i>As a practitioner building geometry-driven applications, I want to master Structural Molecular Biology (Distance Geometry) so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>				
Non-Functional	Reliability	Reproducibility	Robustness	Accuracy
Acceptance Criteria (BDD)				
Scenario	Happy path			
Given	the chapter, examples, and any tooling are available			
When	I complete the <i>Hands-on Objective</i> and validations for this chapter			
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks			
Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • Definition of Done: All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.				

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Solve a small distance-geometry problem for a peptide backbone; reconstruct coordinates.
- Quantify reconstruction error under noise and missing distances.
- Discuss ambiguities and constraints (chirality, bond lengths).

HDCG-66 — Geometry & Topology of Genomic Data

HDCG-66	Geometry & Topology of Genomic Data
Epic / Feature	Applications of Discrete & Computational Geometry
Business Value	Apply geometric concepts to real domains to deliver measurable outcomes and demos. (Geometry & Topology of Genomic Data).
Priority / Estimate	Priority: Must
SP:	3
Persona	data scientist working with geometric methods
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a data scientist working with geometric methods, I want to master Geometry & Topology of Genomic Data so that I can apply it to real problems and communicate theoretical insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Embed genomic relationships using geometric or topological summaries.
- Compare metrics/embeddings (edit distance, Hamming, phylogenetic).
- Interpret structures in terms of recombination/phylogeny signals.

HDCG-67 — Geometric Software (Survey)

HDCG-67

Geometric Software (Survey)

Epic / Feature Geometric Software

Business Value Select, compile, and use trusted geometry libraries to accelerate development. (Geometric Software (Survey)).

Priority / Estimate Priority: Must

SP: 3

Persona applied geometry engineer

Dependencies prior chapters as referenced

Assumptions / Risks time to internalize proofs vs. breadth

As a applied geometry engineer, I want to master Geometric Software (Survey) so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.

Non-Functional Reliability Reproducibility

Acceptance Criteria (BDD)

Scenario Happy path

Given the chapter, examples, and any tooling are available

When I complete the *Hands-on Objective* and validations for this chapter

Then the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks

Definition of Ready: Persona clear; AC drafted; Dependencies known; Estimate set. • **Definition of Done:** All ACs pass; Tests green; Security/a11y checks; Docs updated; Deployed/flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Inventory CGAL/LEDA/libigl/VTK; record kernel differences and license terms.
- Build a minimal 'hello geometry' app creating a hull and a Delaunay triangulation.
- Decide on a primary stack for subsequent labs; justify trade-offs.

HDCG-68 — LEDA & CGAL Case Studies

HDCG-68	LEDA & CGAL Case Studies
Epic / Feature	Geometric Software
Business Value	Select, compile, and use trusted geometry libraries to accelerate development. (LEDA & CGAL Case Studies).
Priority / Estimate	Priority: Must
	SP: 3
Persona	applied geometry engineer
Dependencies	prior chapters as referenced
Assumptions / Risks	time to internalize proofs vs. breadth <i>As a applied geometry engineer, I want to master LEDA & CGAL Case Studies so that I can apply it to real problems and communicate algorithmic insights clearly and reproducibly.</i>
Non-Functional	Reliability Reproducibility
Acceptance Criteria (BDD)	
Scenario	Happy path
Given	the chapter, examples, and any tooling are available
When	I complete the <i>Hands-on Objective</i> and validations for this chapter
Then	the stated outcomes are produced (proof/code/summary) and recorded in the repo with passing checks
Definition of Ready:	Persona clear; AC drafted; Dependencies known; Estimate set.
Definition of Done:	All ACs pass; Tests green; Security/all1y checks; Docs updated; Deployed flagged.

Tasks

- Extract key definitions/lemmas; compile a one-page summary with references to the chapter.
- Compile and run sample kernels; implement hull→Delaunay→point-location pipeline.
- Add robust predicates and exact constructions where available.
- Package as a reusable module with unit tests and CLI demo.