Finding Similar Sets

Applications , Shingling , MinHashing

Locality-Sensitive Hashing

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Thanks for source slides and material to:
J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets
http://www.mmds.org

Outline

- Motivation for Similar Items
- Three steps to find them
  1. Shingling // documents -> sets
  2. Min-hashing // Sets -> signatures
  3. Locality-sensitive-hashing // signatures -> similarity

A Common Metaphor

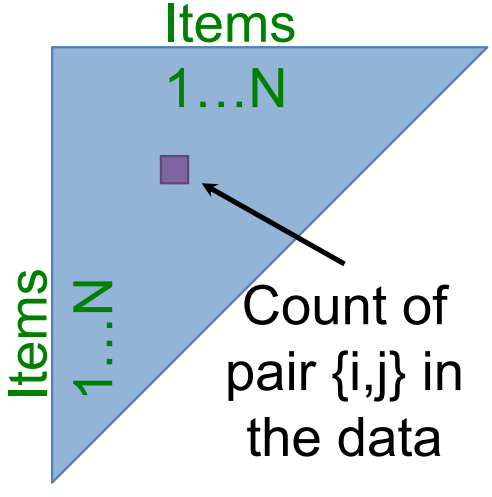
- **Many problems can be expressed as finding "similar" sets:**
  - **Find near-neighbors in high-dimensional space**
- **Examples:**
  - **Pages with similar words**
    - For duplicate/plagiarism detection, classification by topic
  - **Customers who purchased similar products**
    - Products with similar customer sets
  - **Images with similar features**
    - Users who visited similar websites

Problem to be Solved

- **Given**
  - Large set of high-dimensional data points
  - A distance function
    - That quantifies the distance between points
- **Find**
  - **All pairs of points that are within some distance threshold**
- Naive solution would take $O(N^2)$ for N points
- We'll look at O(N) solutions

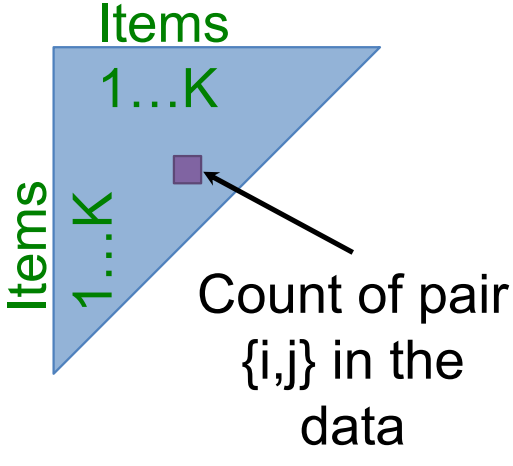
Relation to Previous Lectures

- **Ch. 6:** Finding frequent pairs



Naïve solution:
Single pass but requires space quadratic in the number of items $O(N^2)$

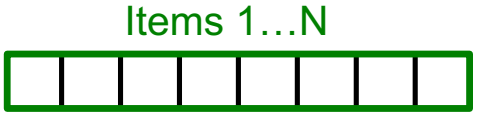
N … number of distinct items
K … number of items with support ≥ *s*



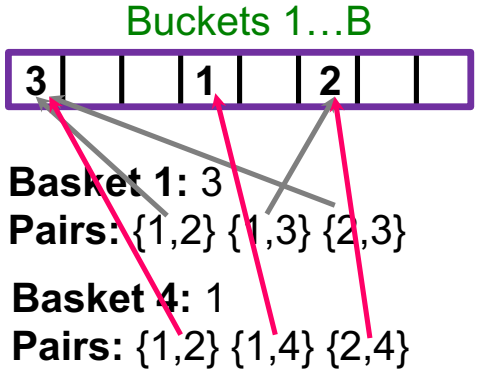
A-Priori:
First pass: Find frequent singletons
For a pair to be **a frequent pair candidate**, its singletons have to be frequent!
Second pass:
Count only candidate pairs!

Relation to Previous Lecture

- **Ch. 6:** Finding frequent pairs
- **Further improvement: PCY**
  - **Pass 1:**
    - Count exact frequency of each item:
    - Take pairs of items {i,j}, hash them into *B* buckets and count of the number of pairs that hashed to each bucket:



  - **Pass 2:**
    - For a pair {i,j} to be a **candidate for a frequent pair,** its singletons {i}, {j} have to be frequent and the pair has to hash to a frequent bucket!



Relation to Previous Lecture

- **Last time:** Finding frequent pairs
- **Fu**
  - ?
  - ?

Ch. 6: A-Priori

Main idea: Candidates

Instead of keeping a count of each pair, only keep a count of candidate pairs!

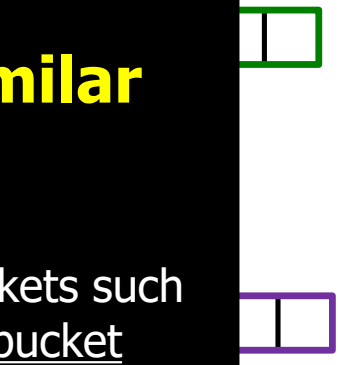
Today's lecture: Find pairs of similar docs

Main idea: Candidates

-- Pass 1: Take documents and hash them to buckets such that documents that are similar hash to the same bucket

-- Pass 2: Only compare documents that are **candidates** (i.e., they hashed to a same bucket)

Benefits: Instead of $O(N^2)$ comparisons, we need O(N) comparisons to find similar documents



Finding Similar Items
Problem Formulation

- Item represented as a set of objects (or components)
  - "baskets"=?
- Problem becomes: find similar sets
  - "finding similar items" = "finding items having similar objects"
- Challenges:
  - Large sets
  - Large number of items/sets

Distance Measures

Goal: Find near-neighbors in high-dimensional space

- We formally define "near neighbors" as points that are a "small distance" apart

- For each application, we first need to define what "**distance**" means
- **Today: Jaccard distance/similarity**

Smaller better bigger is better

Task: Finding Similar Documents

- **Goal: Given a large number (in the millions or billions) of documents, find "near duplicate" pairs**
- **Applications:**
  - Mirror websites, or approximate mirrors
    - Don't want to show both in search results
  - Similar news articles at many news sites
    - Cluster articles by "same story"
- **Problems:**
  - Many small pieces of one document can appear out of order in another
  - Too many documents to compare all pairs
  - Documents are so large or so many that they cannot fit in main memory

3 Essential Steps for Finding Similar Docs

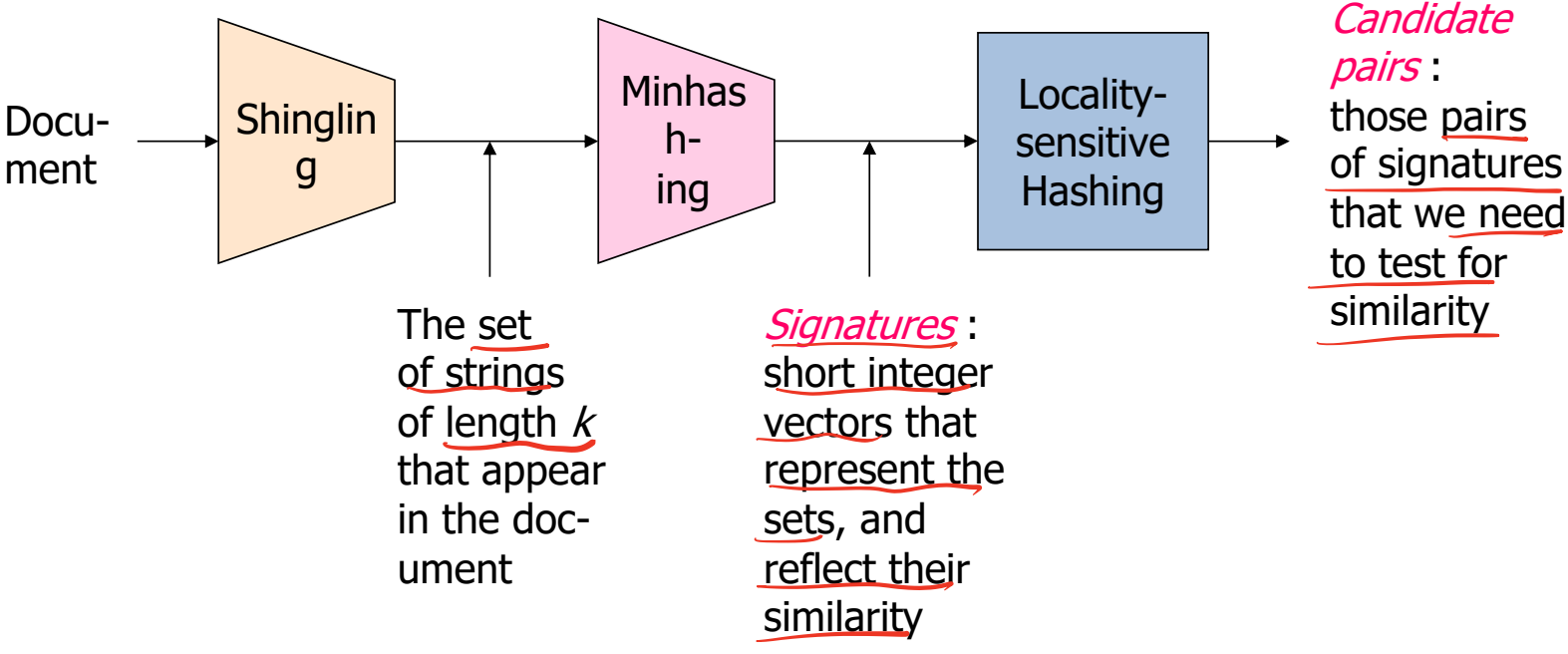
1. ***Shingling:*** Convert documents to sets

2. ***Min-Hashing:*** Convert large sets to short signatures, while preserving similarity

3. ***Locality-Sensitive Hashing:*** Focus on pairs of signatures likely to be from similar documents

   - **Candidate pairs!**

The Big Picture



Documents as High-Dimensional Data

- **Step 1: *Shingling:* Convert documents to sets**

- **Simple approaches:**
  - Document = set of words appearing in document
  - Document = set of "important" words
  - Don't work well for this application. Why?

- **Need to account for ordering of words!**
- A different way: **Shingles!**

Define: Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters, for examples

- **Example: k=2**; document $\mathbf{D_1}$ = abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca} // shingles as a "set"
  - **Option:** Shingles as a "bag" (multiset), count ab twice: $\mathbf{S'(D_1)}$ = {ab, bc, ca, ab}

Working Assumption

- **Documents that have lots of shingles in common have similar text, even if the text appears in different order**

- **Caveat:** You must pick shingle size ***k*** large enough, or most documents will have most shingles
  - ***k*** = 5 is OK for short documents
  - ***k*** = 10 is better for long documents

- May want to compress long shingles

(3)

Compressing Shingles

to bunch of integers

- To **compress long shingles**, we can **hash** them to numbers
  - Each number may be represented as (say) 4 bytes
- **Represent a document by the set of hash values of its *k*-shingles**
  - **Idea:** Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared
- **Example: k=2**; document $\mathbf{D_1}$= abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  Hash the singles: $\mathbf{h(D_1)}$ = {1, 5, 7}

Why is compression needed?

- How many k-shingles?
  - Rule of thumb: imagine 20 characters in alphabet
  - Estimate of number of k-shingles is 20^k
  - 4-shingles: 20^4 or 160,000 or $2^{17.3}$
  - 9-shingles: 20^9 or 512,000,000,000 or 2^{39}
- How many buckets?
  - Assume we use 4 bytes to represent a bucket
  - Assume buckets are numbered in range 0 to 2^{32}– 1
  - This is much smaller than possible number of 9-shingles
    - (but bigger then the # of 4-singles)
  - Compression
    - Represent each shingle with 4 bytes, not 9 bytes

better

2 chars
{x, y}
2-shingles
xx
xy
yx
yy

Why hash 9-shingles to 4 bytes rather than use 4-shingles?

- With 4-shingles, $2^{17.3}$ possible singles
  - Most sequences of four bytes are unlikely or impossible to find in typical documents
  - Effective number of different shingles is much less than the number of buckets $2^{32} - 1$
    - Not efficient use of memory
- With 9-shingles, 2^{39} possible shingles
  - Many more than 2^{32} buckets
  - After hashing, may get any sequence of 4 bytes
    - Effectient use of memory

Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**
- Equivalently, each document is a vector of 0s,1s in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse
- **A natural similarity measure is the Jaccard similarity**

Jaccard Similarity of Sets

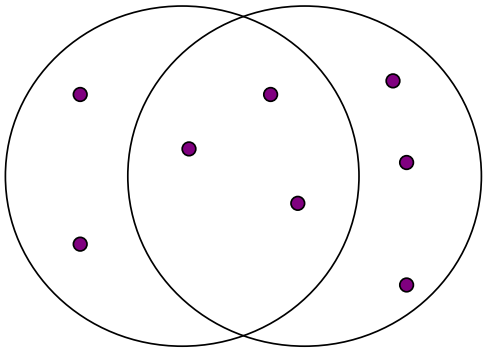
- The *Jaccard similarity* of two sets is the size of their intersection divided by the size of their union

$$Sim\ (C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$

intersection / union

[0, 1]

Example: Jaccard Similarity



3 in intersection
8 in union

Jaccard similarity = 3/8

- **Jaccard distance** = 1 – Jaccard Similarity
  or 5/8 in this example

Motivation for Minhash/LSH

Use k-shingles to create Signatures: short integer vectors that represent sets and reflect their similarity

- **Suppose we need to find near-duplicate documents among million documents**
- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of docs**
  - **≈ 5*10^11** comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **5 days**
- For million, it takes more than a year…

From Sets to Boolean Matrices

- Rows = elements of the universal set
- Columns = sets
- 1 in row *e* and column *S* if and only if element *e* is a member of set *S*
- Column similarity is the Jaccard similarity of the sets of their rows with 1: intersection/union of sets
- Typical matrix is sparse (many 0 values)
  - May not really represent the data by a boolean matrix
  - Sparse matrices are usually better represented by the list of non-zero values (e.g., triples)
  - But the matrix picture is conceptually useful

Example 3.6

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

- Universal set: {a, b, c, d, e}
- Matrix represents sets chosen from universal set
- S1 = {a, d}, S2 = {c}, S3 = {b, d, e} and S4 = {a, c, d}
- Example: rows are products and columns are customers, represented by set of items they bought
- Jacquard similarity of S1, S4: intersection/union = 2/3

Example: Jaccard Similarity of Columns

C_1	C_2		
0	1		*
1	0		*
1	1	*	*
0	0		
1	1	*	*
0	1		*

Sim (C_1, C_2) = ⅖ = 0.4

When Is Similarity Interesting?

1. When the sets are so large or so many that they cannot fit in main memory
2. Or, when there are so many sets that comparing all pairs of sets takes too much time
3. Or both

Outline: Finding Similar Columns

1. Compute signatures of columns = small summaries of columns
2. Examine pairs of signatures to find similar signatures
   - Essential: "similarities of signatures" and "similarities of columns" are related
3. Optional: check that columns with similar signatures are really similar

Warnings

1. Comparing all pairs of signatures may take too much time, even if not too much space
   - A job for "Locality-Sensitive Hashing" (later, step#3)
2. These methods can produce false negatives, and even false positives (if the optional check is not made)

(3)

Signatures

- Key idea: "hash" each column *C* to a small *signature* *Sig*(C), such that:
  1. *Sig* (C) is small enough that we can fit a signature in main memory for each column
  2. *Sim* (C_1, C_2) is the same as the "similarity" of *Sig* (C_1) and *Sig* (C_2)

Sim(C1,C2) same as Sim(Sig(C1), Sig(C2)).

eg:

Four Types of Rows

- Given columns C_1 and C_2, there are 4 types of rows and may be classified as:

	C_1	C_2	
a	1	1	☐ type "a"
b	1	0	☐ type "b"
c	0	1	☐ type "c"
d	0	0	☐ type "d"

- Also, *a* = "# rows of type *a*" , etc.
- Note *Sim* (C_1, C_2) = *a* /(*a* +*b* +*c*)
  - Jaccard similarity: intersection/union
  - "*a*" is intersection, "*a*+*b*+*c*" is union

Minhashing

- To ***minhash*** a set represented by a column of the matrix, **pick a random permutation of the rows**
- **Define "hash" function *h* (*C*) = the "index" number of the first row (in the permuted order) in which column *C* has 1**
- Use several (e.g., 100) independent hash functions to create a signature

Minhashing Example (3.7)

boolean matrix

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

shuffle

Permute

Element	S_1	S_2	S_3	S_4
b	0	0	1	0
e	0	0	1	0
a	1	0	0	1
d	1	0	1	1
c	0	1	0	1

smallest index with one after shuffling

- To minhash a set represented by a column of the characteristic matrix, pick a permutation of the rows
- The minhash value of any column is the "index" number of the first row, in the permuted order, in which that column has a 1
- For set S1, first 1 appears in row a, so:

h(S1) = a

h(S2) = c

h(S3) = b

h(S4) = a

Minhashing Example

7 rows → 3 rows

# of shuffling

Input matrix

1	4	3	1	0	1	0
3	2	4	1	0	0	1
7	1	7	0	1	0	1
6	3	6	0	1	0	1
2	6	1	0	1	0	1
5	7	2	1	0	1	0
4	5	5	1	0	1	0

smallest index with one

min{3,4,2,5}

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Surprising Property: Connection between Minhashing and Jaccard Similarity

- The probability that minhash function for a random permutation of rows produces same value for two sets equals Jaccard similarity of those sets
  - **"The probability that $h(C_1)=h(C_2)$" is the same as "*Sim* (C_1, C_2)"**
- Recall four types of rows:

	C_1	C_2
a	1	1
b	1	0
c	0	1
d	0	0

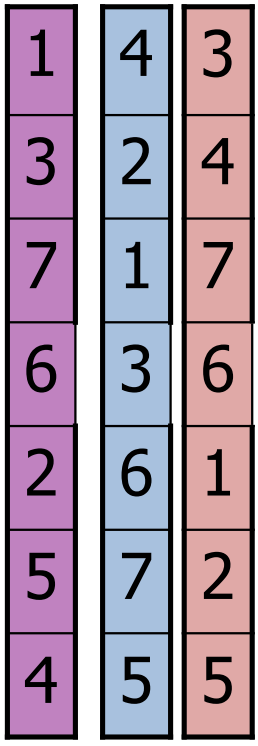
- **Sim(C_1, C_2) for both Jacquard and Minhash are $a/(a+b+c)$!**
  - Why? Look down the permuted columns C_1 and C_2 until we see a 1
  - If it's a type-*a* row, then $h(C_1) = h(C_2)$. If a type-*b* or type-*c* row, then not. (Don't count the *type-d* rows)

Similarity for Signatures

- Sets represented by characteristic matrix M
- To represent sets (columns): pick at random some number n of permutations of the rows of M
  - 100 permutations or several hundred // n << I
- Call minhash functions determined by these permutations

  $h_1, h_2, \ldots, h_n$
- From column representing set S, **construct minhash signature for S**:
  - vector $[h_1(S), h_2(S), \ldots, h_n(S)]$, usually represented as column
- Construct a ***signature matrix***: i^{th} column of M replaced by minhash signature for i^{th} column
- **The *similarity of signatures* is the fraction of the hash functions in which they agree**

Min Hashing – Example



Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	**¾**	**¾**	0	0
Sig/Sig	**2/3**	**3/3**	0	0

Note: these two measures are correlated!

Minhash Signatures

- Given a matrix of *R* rows
- Pick (say) *n*=100 random permutations of the rows
- Think of *Sig* (C) as a column vector
- Let *Sig* (C)[i] =

  according to the *i* th permutation, the number of the first row that has a 1 in column *C*

# of permutations

Note: the size of signature |Sig(C)|=n is much smaller than the size of column (R rows)

Implementation – (1)

- Not feasible to permute a large characteristic matrix explicitly
  - Suppose 1 billion rows
  - Hard to pick a random permutation from 1…billion
  - Representing a random permutation requires 1 billion entries
  - Accessing rows in permuted order leads to thrashing

- **Can simulate the effect of a random permutation by a random hash function**
  - Maps row numbers to as many buckets as there are rows
  - May have collisions on buckets
  - Not important as long as number of buckets is large

Implementation – (2)

- A good approximation to permuting rows: pick around 100 hash functions
- For each:
  - column c (set representing a document)
  - hash function h_i
- Keep a "slot" in signature matrix $M(i,c)$
- **Intent: $M(i,c)$ will become the smallest value of $h_i(r)$ for which column c has 1 in row r**
  - $h_i(r)$ gives order of rows for i^{th} permutation

Implementation – (3)

for each row *r*

  for each column *c*

    if c has 1 in row *r*

      for each hash function h_i **do**

        if $h_i(r)$ is a smaller value than $M(i, c)$ **then**

          $M(i, c) := h_i(r)$;

Computing Minhash Signatures: Example 3.8

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

Two hash functions give permutations of rows:
h1 = x+1 mod 5, h2 = 3x +1 mod 5

	S_1	S_2	S_3	S_4
h_1	∞	∞	∞	∞
h_2	∞	∞	∞	∞

Initial signature matrix

	S_1	S_2	S_3	S_4
h_1	1	∞	∞	1
h_2	1	∞	∞	1

For row 0: Replace existing signature values with lower hash values for S1 and S4, since both

Computing Minhash Signatures: Example 3.8 (part 2)

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

	S_1	S_2	S_3	S_4
h_1	1	∞	2	1
h_2	1	∞	4	1

For row 1: replace h1 and h2 values for S3, since row has a 1 and values are lower

	S_1	S_2	S_3	S_4
h_1	1	3	2	1
h_2	1	2	4	1

For row 2: replace values for S2 since set has a 1 value. Do not replace values for S4, because existing values are lower

Computing Minhash Signatures: Example 3.8 (part 3)

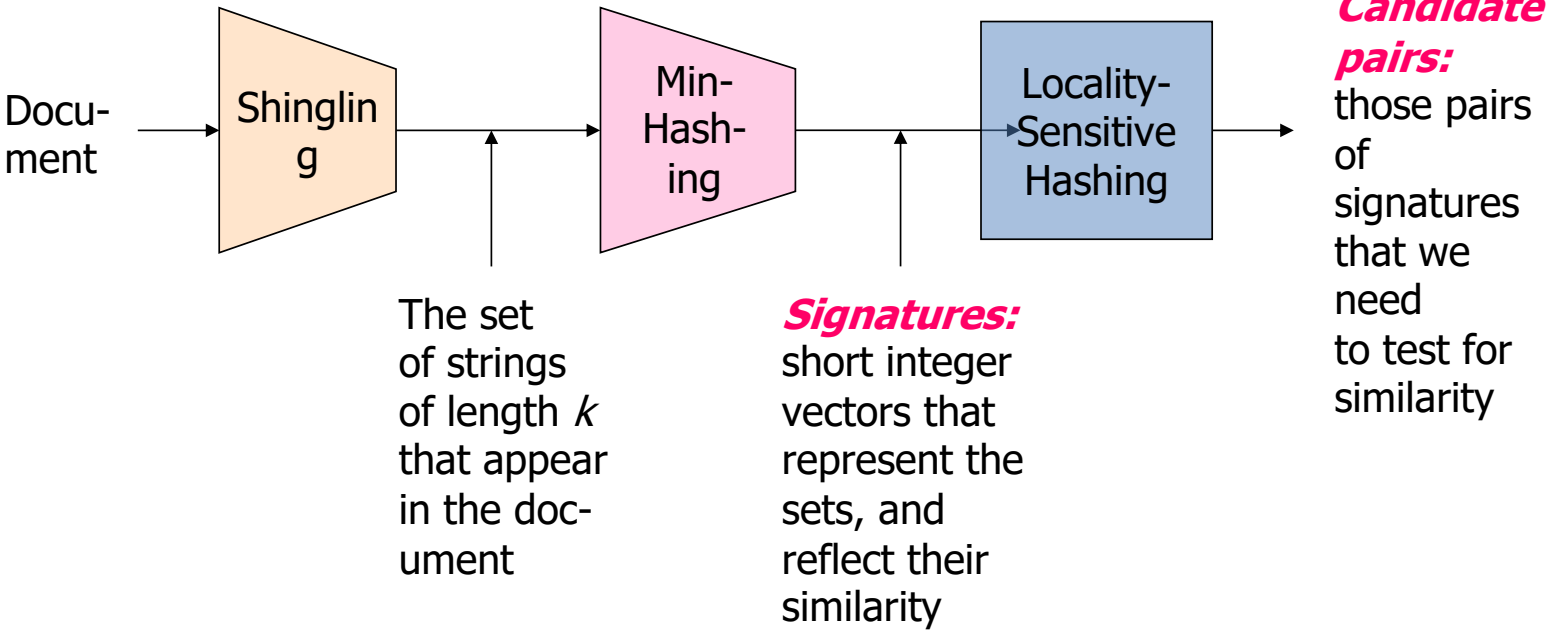
Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

	S_1	S_2	S_3	S_4
h_1	1	3	2	1
h_2	0	2	0	0

For row 3: don't replace h1 values--all are below 4; replace h2 values with 0 for S1, S3, S4

	S_1	S_2	S_3	S_4
h_1	1	3	0	1
h_2	0	2	0	0

For row 4: replace h1 value for S3, don't replace h2 value since current value is lower
Note: result is same as permutations to find first 1



Locality Sensitive Hashing

3.

Step 3: ***Locality-Sensitive Hashing:***

Focus on pairs of signatures likely to be from similar documents

Motivation for Locality Sensitive Hashing

- *Used k-shingles to create sets that summarize documents*
- *Used Minhashing to generate signatures that represent sets of shingles, reflect their similarity*
- **Suppose we need to find near-duplicate documents among a million 10^6 documents**
- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of signatures**
  - 10^6 choose 2
  - Recall: for large n, $\binom{n}{2}$ is approximately $n^2/2$
  - **≈ 5*10^{11}** comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **6 days**

Locality Sensitive Hashing Overview

- **Hash items several times**
  - In a way that **similar items are more likely to be hashed to the same bucket than dissimilar items**
- **Candidate Pair:** Any pair that hashes to the same bucket for **any** of the hashings
- **Check only the candidate pairs for similarity**
- **False positives:** dissimilar pairs that hash to the same bucket
- **False negatives:** truly similar pairs do not hash to the same bucket for at least one of the hash functions

2	1	4	1
1	2	1	2
2	1	2	1

LSH: First Cut

- **Goal: Find documents with Jaccard similarity at least *s*** for some similarity threshold ***s*** (e.g. ***s***=0.8)

- **LSH – General idea:** Use a function ***f(x,y)*** that tells whether ***x*** and ***y*** are a ***candidate pair:*** a pair of elements whose similarity must be evaluated

- **For Min-Hash matrix:**
  - Hash columns of signature matrix ***M*** to many buckets
  - Each pair of documents that hashes into the same bucket is a **candidate pair**

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- **Pick a similarity threshold *s* (0 < s < 1)**

- Columns ***x*** and ***y*** of ***M*** are a **candidate pair** if their signatures agree on at least fraction ***s*** of their rows:
  M* (*i*, *x*) = *M* (*i*, *y*)** for at least frac. ***s values of ***i***
  - We expect documents ***x*** and ***y*** to have the same (Jaccard) similarity as their signatures

LSH for Min-Hash

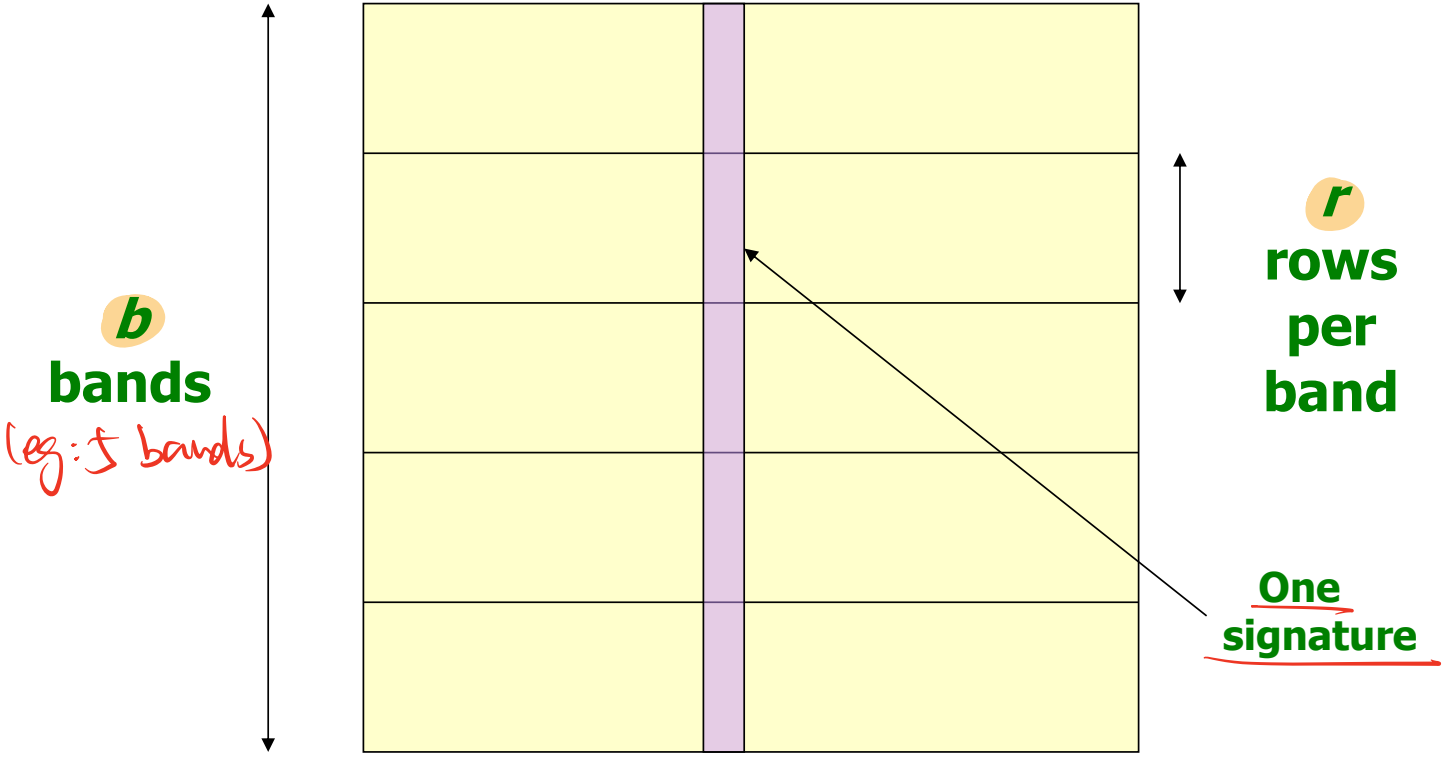
2	1	4	1
1	2	1	2
2	1	2	1

- **Big idea: Hash columns of signature matrix *M* several times**
- Arrange that (only) **similar columns** are likely to **hash to the same bucket**, with high probability
- **Candidate pairs are those that hash to the same bucket**

1 signature → b bands, r rows/band

2	1	4	1
1	2	1	2
2	1	2	1

Partition *M* into *b* Bands

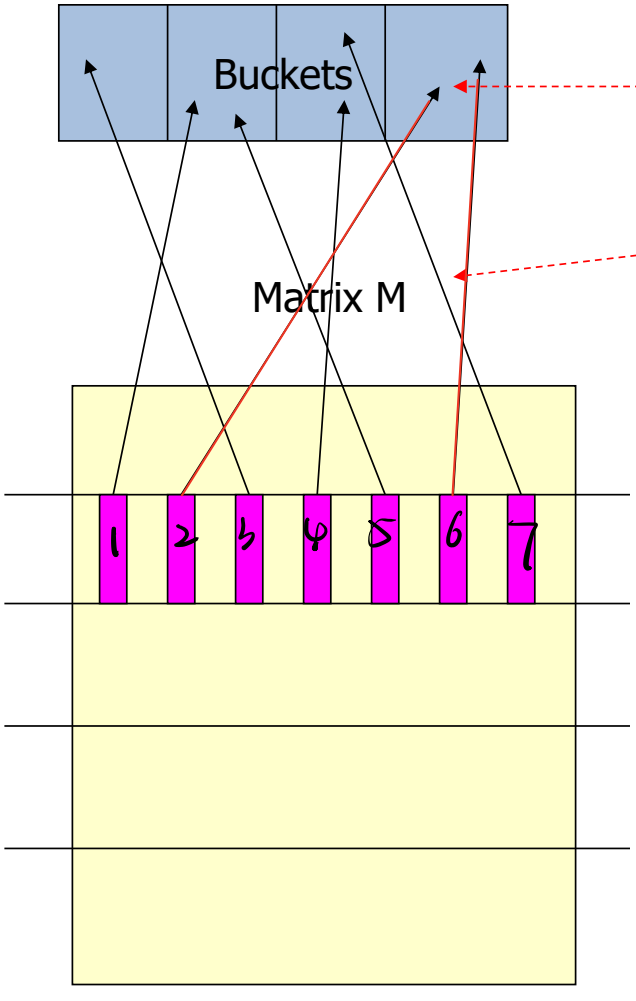


Signature matrix *M*

3) Step:

Partition Signatures M into Bands

- Divide matrix ***M*** into ***b*** bands of ***r*** rows
  - Signatures are still too big, so we check band by band
  - So, check "similar signatures" becomes check "similar bands"
- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible
  - Use a separate bucket array for each band so columns with the same vector in different bands don't hash to same bucket
- ***Candidate*** **column pairs** are those that **hash to the same bucket for ≥ 1 band**
- Tune ***b*** and ***r*** to catch most similar pairs, but few non-similar pairs



(1,5)(2,6)(4,7)

Columns 2 and 6 are probably identical.

Columns 6 and 7 are surely different.

r rows

b bands

Example of Bands

Assume the following case:

- Suppose 100,000 columns of ***M***
  - Correspond to signatures for 100,000 documents
- Signatures of 100 integers (rows)
  - Correspond to 100 hash functions used in minhashing
- 4 bytes per integer
- Therefore, signatures take 40Mb (4 x 100 x 100,000)
- Choose ***b*** = 20 bands of ***r*** = 5 rows of integers/band

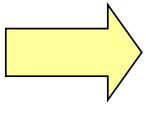
- **Goal:** Find pairs of documents that are at least ***s*** = *0.8* or 80% similar

Recall: Minhashing Example

Input matrix

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0



Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

(4) 证明:

Analysis of Banding Technique

- Use b bands of r rows each
- **Pair of documents have Jaccard similarity *t***
  - Probability that minhash signatures for the documents agree in any one particular row of the signature matrix is *t*
- Columns C_1 and C_2 in signature matrix have similarity *t*
- Pick any band (*r* rows)
  - **Prob. that all rows in band are equal = t^r**
  - **Prob. that not all *r* rows are equal (some row in band is unequal) = $1 - t^r$**
- **Prob. that no band has rows that are all equal = $(1 - t^r)^b$**
- **Prob. that at least 1 band has rows that are all equal (which is the probability of being a candidate pair) = $1 - (1 - t^r)^b$**

P(one row is equal) t

2	1	4	1
1	2	1	2
2	1	2	1

C_1, C_2 are 80% Similar

- **Find pairs of ≥ *s*=0.8 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) ≥ **s**, we want C_1, C_2 to be a **candidate pair**
  - We want them to hash to at **least 1 common bucket** (at least one band is identical)
- **Probability C_1, C_2 identical in one particular band:**

  $t^r = (0.8)^5 = 0.328$
- Probability C_1, C_2 are ***not*** similar in all of the 20 bands:

  $(1 - t^r)^b = (1\text{-}0.328)^{20} = 0.00035$
  - i.e., about .035% of the 80%-similar column pairs are **false negatives** (truly similar pairs that we miss)
  - **We would find 99.965% pairs of truly similar documents**

2	1	4	1
1	2	1	2
2	1	2	1

C_1, C_2 are 30% Similar

- **Find pairs of ≥ *s*=0.3 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < **s** we want C_1, C_2 to hash to **NO common buckets** (all bands should be different)
  - **Should NOT be a candidate pair!**
- **Probability C_1, C_2 identical in one particular band:**

  $t^r = (0.3)^5 = 0.00243$
- Will identify C1, C2 as candidate pair if they are **identical in at least one band**
- Probability C_1, C_2 identical in at least 1 of 20 bands:

  $1 - (1 - t^r)^b = 1 - (1 - 0.00243)^{20} = 0.0474$
  - Approximately 4.74% pairs of docs with similarity 0.3% end up becoming **candidate pairs**
    - They are **false positives** (dissimilar documents that must be examined as candidate pairs but will have similarity below

Other False Positives and False Negatives

For b=20, r=5, we may compute ALL the false positives and false negatives as follows:

similar 看作 dissimilar

For false negatives: (using the procedure on slide#15 to compute "?" below):

- How many true 100%-similar pairs we may miss: 0.0% (obvious)
- How many true 90%-similar pairs we may miss: ?.?%
- How many true 80%-similar pairs we may miss: 0.035% (see slide#15)
- How many true 70%-similar pairs we may miss: ?.?%
- How many true 60%-similar pairs we may miss: ?.?%
- How many true 50%-similar pairs we may miss: ?.?%

dissimilar 看作 similar

For false positives: (using the procedure on slide#16 to compute "?" below):

- How many 40%-similar pairs we may mistakenly taken as positive: ?.?%
- How many 30%-similar pairs we may mistakenly taken: 4.74% (see slide#16)
- How many 20%-similar pairs we may mistakenly taken: ?.?%
- How many 10%-similar pairs we may mistakenly taken: ?.?%
- How many 0%-similar pairs we may mistakenly taken: 0.0% (obvious)

LSH Involves a Tradeoff

- **Pick:**
  - The number of Min-Hashes (rows of ***M***)
  - The number of bands ***b***, and
  - The number of rows ***r*** per band

  to balance false positives/negatives

- **Example:** If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Example of Tradeoffs

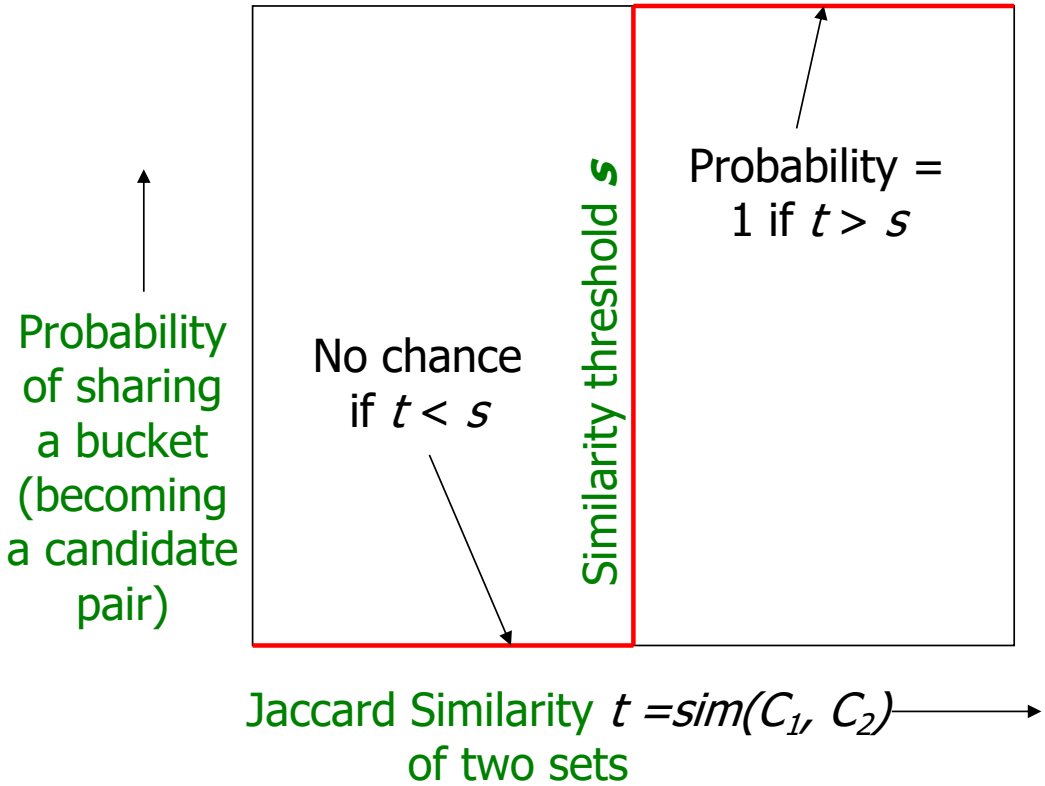
b=20, r=5

- Previous example: 20 bands of 5 rows each
  - Probability of false negatives when C1, C2 are 80% similar: 0.00035
  - Probability of false positives when C1, C2 are 30% similar: 0.0474

b=5, r=15

- What if we use 15 rows of 5 bands each (smaller signature matrix)?
  - **Probability of false negatives higher** when C1, C2 are 80% similar:
    - $(1 - t^r)^b = (1 - 0.8^{15})^5 = (1 - 0.035)^5 = 0.837$
  - **Probability of false positives lower** when C1, C2 are 30% similar:
    - $1 - (1 - t^r)^b = 1 - (1 - 0.3^{15})^5 = 7.174^{-8}$

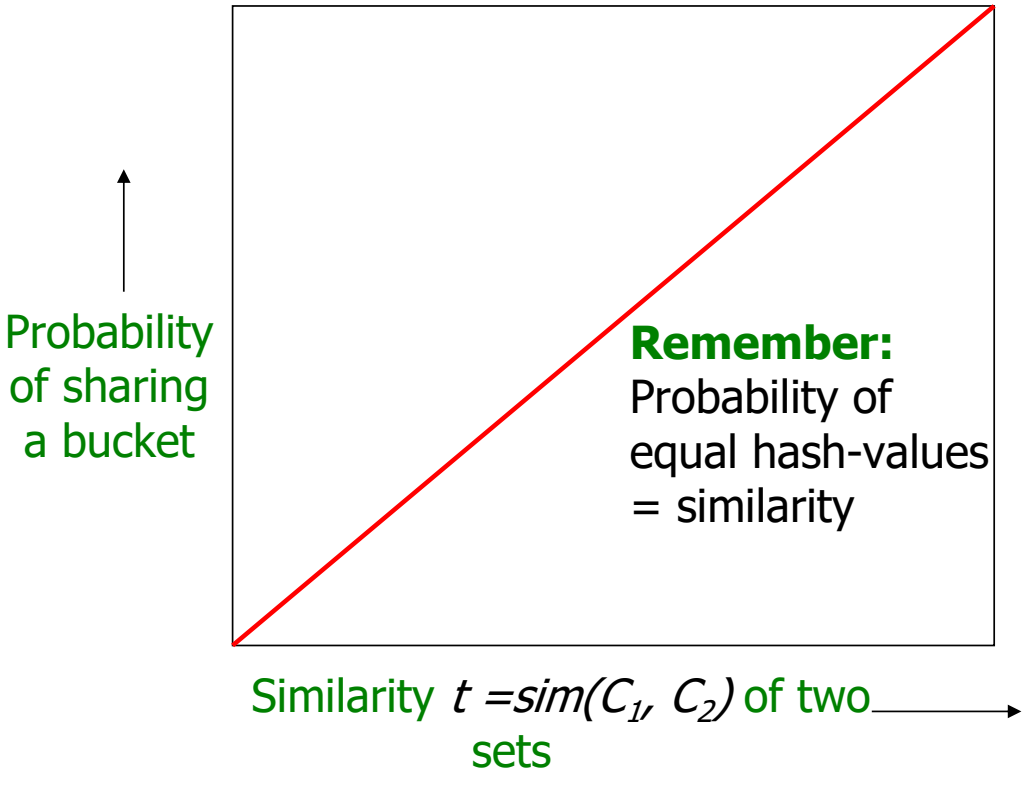
Analysis of LSH – What We Want



What 1 Band (b=1) of 1 Row (r=1) Gives You

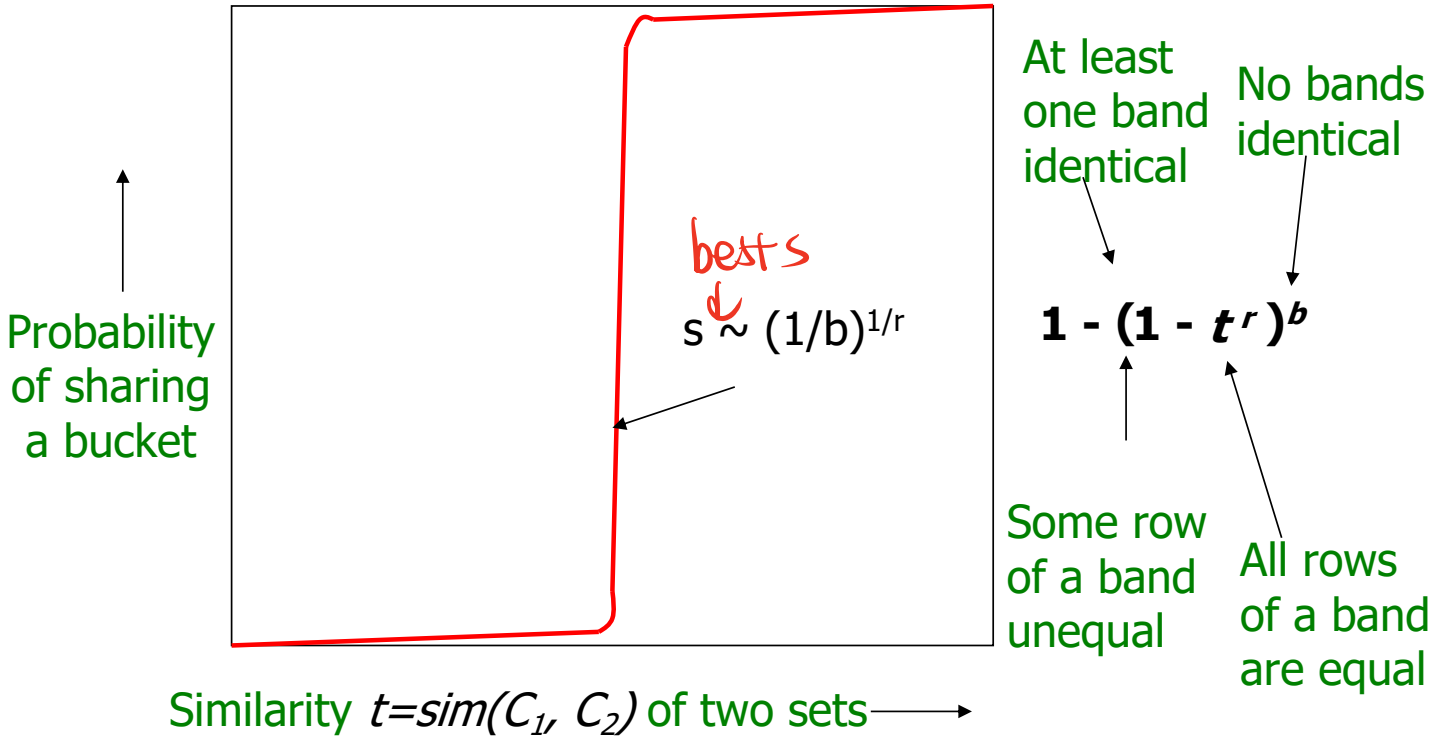
- Compare two values in similarity matrix

$$1-(1-t^{r})^{b} = 1-(1-t^{1})^{1} = t$$



What *b* Bands of *r* Rows Gives You: $1 - (1 - t^r)^b$

- Form of an S-curve, regardless of values of b and r
- Threshold *s* is where rise of curve is steepest: approximately $(1/b)^{1/r}$



r=5, b=20, for t=0.9: $1-(1-t^r)^b$=0.99999; for t=0.1: 0.0000199

eg:

b,r减, t↑ ⇒ FP↑, FN↓

Example: $b = 20$; $r = 5$

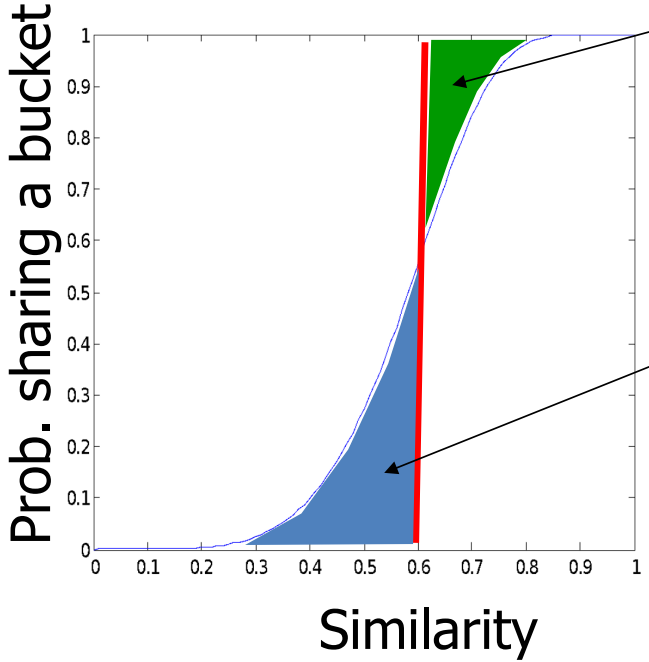
- Similarity t of two columns
- **Prob. that at least 1 band is identical (so a candidate pair):**

t	$1-(1-t^r)^b$ → FP?
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

- Not an ideal step function
- Probability rises by more than 0.6 going from similarity t = 0.4 to t = 0.6
- Slope in middle > 3.0
- 0.6/(0.6-0.4)=3.0

Picking *r* and *b*: The S-curve

- **Picking *r* and *b* to get the best S-curve**
  - 50 hash-functions (r=5, b=10)

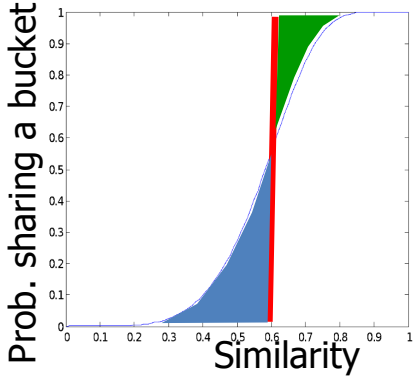


Green area:
False Negative rate
Similar documents that are not identified as candidate pairs

Blue area:
False Positive rate
Dissimilar documents that are identified as candidate pairs

Picking *b* and *r*

- **Threshold *s* defines how similar documents have to be for them to be regarded as a similar pair** (e.g., s = 0.8)
- **Length *n* for minhash signatures**
- Pick number of bands ***b*** and number of rows ***r*** such that ***br = n*** and threshold ***s*** is approximately ***$(1/b)^{1/r}$***
- **To avoid false negatives (green area):**
  - **Select *b* and *r* to produce a threshold lower than *s***
- **To avoid false positives (blue area):**
  - **Select *b* and *r* to produce a higher threshold than *s***



Example : n=100

b	r	$(1/b)^{1/r}$
50	2	0.1414
20	5	0.5493
10	10	0.7943
5	20	0.9227

Example

- $(1/b)^{1/r}$ represents the threshold of the S curve for function
- $1 - (1 - t^r)^b$, the probability of being a candidate pair
- If **s=0.6** (similarity of documents to be a candidate pair) what values should you choose for b and r to reduce the number of **false negatives**?

- **To avoid false negatives: Select *b* and *r* to produce a threshold lower than *s***
- **To avoid false positives: Select *b* and *r* to produce a higher threshold than *s***

- **Could choose (b=20, r=5) or (b=50, r=2): both give threshold lower than s**

- **Better answer probably b=20, r=5**
- **Because b=50, r=20 will have a higher rate of false positives: TRADEOFFS**

Example : n=100

b	r	$(1/b)^{1/r}$
50	2	0.1414
20	5	0.5493
10	10	0.7943
5	20	0.9227

(6)

LSH Summary

- Tune ***M, b, r*** to identify **almost all candidate pairs with similar signatures**, but **eliminate most pairs that do not have similar signatures**
- Then **check in main memory that candidate pairs really do have similar signatures**
- **Optional:** In another 2nd pass through data, check that **the remaining candidate pairs really represent similar documents**

Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity ≥ **s**

Combining the techniques (1)

1. Pick a value of *k* and **construct from each document the set of *k*-shingles**
   - Optionally **hash the k-shingles** to shorter bucket numbers
2. **Sort the document-shingle pairs** to order them by shingle
   - Which sets contain which elements (shingles)
3. Pick a length *n* for minhash signatures corresponding to *n* minhash functions and **compute the minhash signatures** for all the documents

Combining the techniques (2)

4. **Choose threshold s** that defines how similar documents have to be for them to be regarded as a "similar pair"
   - **Pick number of bands b and number of rows r** such that br = n
   - **Adjust b and r to limit false positives or negatives**

5. **Construct candidate pairs with LSH** technique

6. **Examine candidate pair signatures** and determine whether fraction of components where they agree is at least s

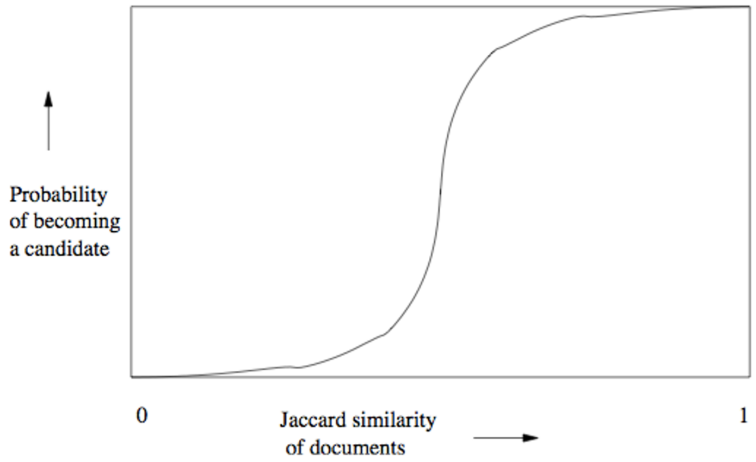
7. **Optionally,** if signatures are sufficiently similar, **compare documents** to check they are truly similar

3. CHARACTERISTICS OF LSH

1. Locality Sensitive Hashing

depends on location → S curve

- Or ***Near-neighbor search***
- Minhashing is one example of a **family of functions** (the minhash functions) **that can be combined** (by the banding technique) **to distinguish strongly between pairs at a low distance from pairs at a high distance**
- Steepness of the S-curve reflects how effectively we can **avoid false positives and false negatives** among the candidate pairs
- Section 3.6: more general theory of Locality Sensitive Functions



Families of Functions for LSH

- **Families of functions** (including minhash functions) that can serve to **produce candidate pairs efficiently**
  - Space of sets and Jaccard distance OR other space and/or distance measure
- **Three conditions for family of functions:**

1. **More likely to make close pairs be candidate pairs than distant pairs**
2. **Statistically independent**
3. **Efficient** in two ways
   1. **Be able to identify candidate pairs in time much less than time to look at all pairs**
   2. **Combinable to build functions better at avoiding false positives and negatives** (e.g., banding technique takes single minhash functions, combines them to produce S-curve shape we want)

Locality-Sensitive Functions

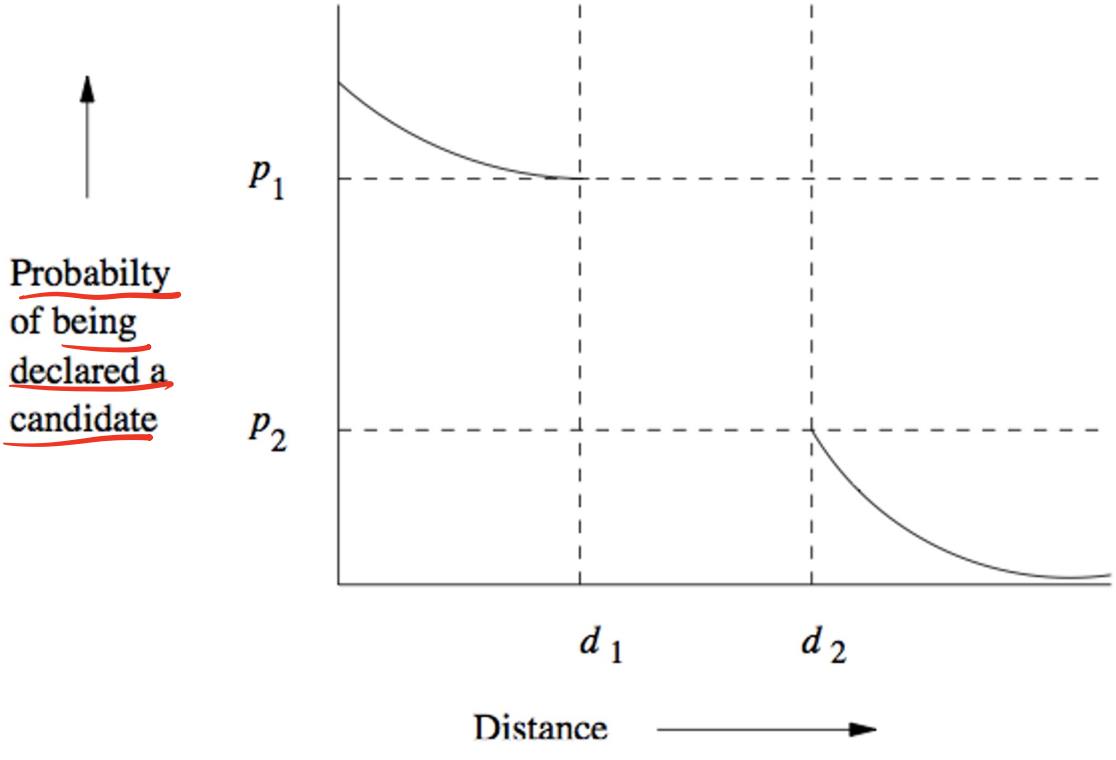


Figure 3.9: Behavior of a (d_1, d_2, p_1, p_2)-sensitive function

LS Families of Hash Functions

- Suppose we have a **space *S* of points** with a **distance measure *d***
- A **family H of hash functions** is said to be ***(d_1,d_2,p_1,p_2)-sensitive*** if for any *x* and *y* in *S*:
  1. **If d(x,y) ≤ d_1, then prob. over all *h* in H, that h(x) = h(y) is at least p_1**
  2. **If d(x,y) ≥ d_2, then prob. over all *h* in H, that h(x) = h(y) is at most p_2**
- Note: we say nothing about what happens when the distance between items is between d1 and d2
  - But can make d1 and d2 as close as we wish
  - Can drive p1 and p2 apart while keeping d1 and d2 fixed

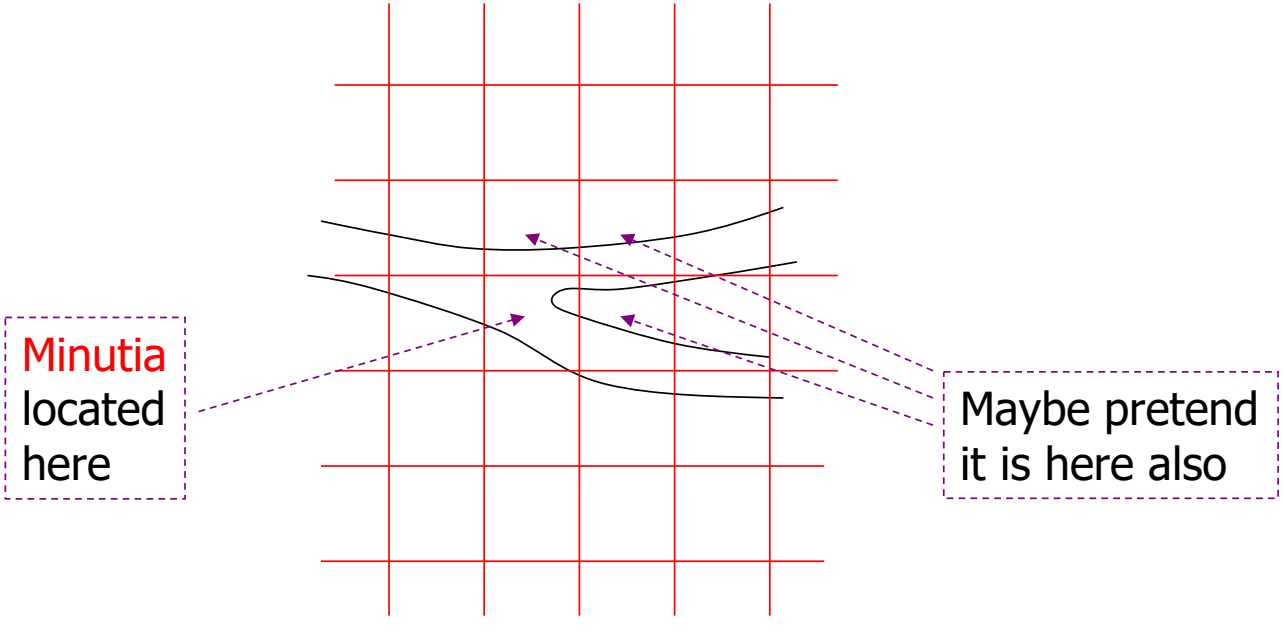
Locality Sensitive Hashing for Other Distance Measures

- We focused on **minhashing**, a locality sensitive hashing family that uses **Jaccard distance**
  - Based on sets representing documents and their Jaccard similarity
- Book covers LSH families for **other distance measures**:
  - **Euclidean distance:** based on the locations of points in a *Euclidean space* with some number of real-valued dimensions
  - **Cosine distance:** angle between vectors from the origin to the points in question
  - **Edit distance:** number of inserts and deletes to change one string into another
  - **Hamming Distance:** number of positions in which bit vectors differ

LSH for Fingerprints

- Typical representation is not an image, but **set of locations in which minutiae are located**
  - **Place where something unusual happens**: two ridges merging or a ridge ending
- Place a grid over a fingerprint
  - Normalize for size and orientation so that identical prints will overlap
- Represent fingerprint by set of grid points where minutiae are located
  - Possibly, treat minutiae near a grid boundary as if also present in adjacent grid points

Discretizing Minutiae



Place a minutia in several adjacent grid squares if it lies close to the border of the squares

Applying LSH to Fingerprints

- **Make a bit vector for each fingerprint's set of grid points with minutiae**
  - Similar to set representing a document: 1 if the shingle is in the document, 0 otherwise
- We could minhash the bit vectors to obtain signatures
  - But since there probably aren't too many grid points, we can work from the bit-vectors directly

Matching Fingerprints with LSH: Many-to-many problem

- **Many-to-many version of fingerprint matching:** take an entire database of fingerprints and identify if there are any pairs that represent the same individual
  - **Analogous to finding similar documents among millions of documents**
- **Define a locality-sensitive family of hash functions:**
  - **Each function f in the family F is defined by 3 grid squares**
  - **Function f says "yes" for two fingerprints if both have minutiae in all three grid squares, otherwise, f says "no"**
  - **"Yes" means the two fingerprints are candidate pairs**
- **Sort of "bucketization"**
  - Each set of three points creates one bucket
  - **Function f sends fingerprints to its bucket that have minutiae in all three grid points of f**
- **Compare all fingerprints in each of the buckets**

Matching Fingerprints with LSH: Many-to-One Problem

- **Many-to-one version :** A fingerprint has been found at a crime scene, and we want to **compare it with all fingerprints in a large database to see if there is a match**
- Could use many functions f from family F
- **Precompute their buckets of fingerprints to which they answer "yes" on the large database**
- **For a new fingerprint:**
  - **Determine which buckets it belongs to**
  - **Compare it with all fingerprints found in any of those buckets**

Example 3.22

- 1024 functions chosen randomly from F
  - **Each function f says "yes" for two fingerprints if both have minutiae in all three grid squares, otherwise, f says "no"**
- Suppose **typical fingerprints have minutiae in 20% of the grid points**
- Suppose **fingerprints from the same finger agree in at least 80% of their points**
- **Probability two random fingerprints each have 1 in all three points = $(0.2)^6 = .000064$**
  - 2 fingerprints, 3 points each, all independent events

First image has 1 in a point

Example: Continued

Second image of same finger also has 1

- Probability **two fingerprints from the same finger each have 1's in three given points** =

$$((0.2)(0.8))^3 = .004096$$

(Analogy: t^r)

- Prob. for **at least one of 1024 sets of three points** = $1-(1-.004096)^{1024} = .985$

(Analogy: $1 - (1 - t^r)^b$)

- But for **random fingerprints**:
- $1-(1-.000064)^{1024} = .063$

1.5% false negatives

6.3% false positives

Choosing the number of functions from F

- **Want to use many functions from F, but not too many**
- **Want a good probability of matching fingerprints from the same finger while not having too many false positives**
- Previous example: only 1.5% chance we fail to identify a print on the gun (false negative), but have to look at 6.3% of entire database (due to false positives)
- **Increasing number of functions from F increases number of false positives**
  - Only a small benefit in reducing false negatives below 1.5%
- **Can use constructions/combinations of functions**
  - Several examples in the chapter

2. Finding Same/Similar News Articles

- **Want to organize large repository of online news articles**
  - **Group together web pages derived from same basic text**
- **Scenario:** the same article, say from the Associated Press, appears on the Web site of many newspapers, but looks quite different
- **Each newspaper surrounds the text of the article with:**
  - Its own logo and text
  - Ads
  - Perhaps links to other articles
- **A newspaper may also "crop" the article (delete parts)**

Variation on Shingling

- **Looks like earlier problem:** find documents whose shingles have high Jaccard similarity
- **But: Shingling treats all parts of document equally**
- For this application, **we want to ignore certain parts of the documents** (e.g., ads, links to other articles, etc.)
- **There is a difference between text that appears in prose and text in ads or headlines/links**
  - Prose contains greater frequency of ***stop_words***
    - E.g., common words like “and” or “the”
  - Common to use list of several hundred most frequent words

New Shingling Technique

- **News articles have a lot of stop words, while ads do not**
  - "Buy Sudzo" vs. "I recommend that you buy Sudzo for your laundry."
- **Define a *shingle* to be a stop word plus the next two following words**
  - Shingles are: "I recommend that", "that you buy", "you buy Sudzo", "for your laundry", "your laundry <nextword>"
- **Then compare the similarity of the sets of shingles that represent each document**
  - Don't use minhashing or LSH in this example

Why it Works

- By requiring each shingle to have a stop word**: bias the mapping from documents to shingles** so it picked more shingles from the article than from the ads

- **Pages with the same article, but different ads, have higher Jaccard similarity** than those with the same ads, but different articles