Recommendation Systems

Collaborative Filtering

Hybrid Systems

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Thanks for source slides and material to:
J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets
http://www.mmds.org

Three Approaches to Recommendation Systems

- **1) Content-based**
  - Use characteristics of an item
  - Recommend items that have similar content to items user liked in the past
  - Or items that match predefined attributes of the user
- **2) Collaborative filtering**
  - Build a model from a user's past behavior (items previously purchased or rated) and similar decisions made by other users (“neighbors”)
  - Use the model to predict items that the user may like
  - Collaborative: suggestions made to a user utilize information across the entire user base (‘neighbors”)
  - **User-Neighbors vs. Item-Neighbors**
- **3) Hybrid approaches**

1. Review

Collaborative Filtering: Overview

- CF works by **collecting user feedback**: e.g., **ratings for items**
  - Exploit similarities in rating behavior among users in determining recommendations
- **Two classes of CF algorithms:**

1. **Neighborhood-based or Memory-based approaches**
   - User-based CF (User's neighbors)
   - Item-based CF (Item's neighbors)
2. **Model-based approaches**
   - Estimate parameters of statistical models for user ratings
   - Latent factor and matrix factorization models

Neighbor-Based Memory-Based Algorithms

- Key Ideas of **Collaborative Filtering** (CF)
  - Find neighbors by similarity on "averaged rates"
  - Listen to neighbors with weights (similarity)
  - Make recommendations

User-based CF: ("User" Neighbors)

$$w_{u,v} = \frac{\sum_{i \in I} (r_{u,i} - \bar{r}_u)(r_{v,i} - \bar{r}_v)}{\sqrt{\sum_{i \in I} (r_{u,i} - \bar{r}_u)^2} \sqrt{\sum_{i \in I} (r_{v,i} - \bar{r}_v)^2}} \qquad P_{a,i} = \bar{r}_a + \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_u) \cdot w_{a,u}}{\sum_{u \in U} | w_{a,u} |}$$

Item-Based CF: ("Item" Neighbors)

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}} \qquad P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

Making User-based CF Predictions with Pearson: Weighted Sum of Neighbors Ratings

- **Find neighbors *U* of an active user *a* based co-rated items *I***

$$w_{u,v} = \frac{\sum_{i \in I} \left(r_{u,i} - \overline{r}_u\right)\left(r_{v,i} - \overline{r}_v\right)}{\sqrt{\sum_{i \in I} \left(r_{u,i} - \overline{r}_u\right)^2} \sqrt{\sum_{i \in I} \left(r_{v,i} - \overline{r}_v\right)^2}}$$

- **Making predictions**
  - **Summarize the neighbor *U's* weighted ratings for item *i***

$$P_{a,i} = \overline{r}_a + \frac{\sum_{u \in U} \left(r_{u,i} - \overline{r}_u\right) \cdot w_{a,u}}{\sum_{u \in U} \left| w_{a,u} \right|}$$

Making User-based CF Predictions with Pearson: Weighted Sum of Neighbors Ratings

- **Find neighbors *U* of an active user *a* based co-rated items *I***

  $w_{u,v}$ = Cosine Similarity of the following two vectors:

  User u's averaged rate for the co-rated items $(r_{u,i} - \bar{r}_u)$

  User v's averaged rate for the co-rated items $(r_{v,i} - \bar{r}_v)$

- **Making predictions**
  - **Summarize the neighbors $U=\{u_1, \ldots, u_n\}$'s weighted ratings for item *i***

$$P_{a,i} = \bar{r}_a + (r_{u_1,i} - \bar{r}_{u_1}) \left[\frac{w_{a,u_1}}{\sum_{u \in U} |w_{a,u}|}\right] + \cdots + (r_{u_n,i} - \bar{r}_{u_n}) \left[\frac{w_{a,u_n}}{\sum_{u \in U} |w_{a,u}|}\right]$$

You own averaged opinion

Neighbor 1's opinion

How much you trust Neighbor-1

Neighbor *n* opinion

How much you trust Neighbor n

Continued Example: User-Based CF Prediction with Pearson Correlation Coefficient

	I_1	I_2	I_3	I_4	"average other than I2"
U_1	4	?	5	5	(4+5+5)/3
U_2	4	2	1		(4+1)/2
U_3	3		2	4	
U_4	4	4			(4)/1
U_5	2	1	3	5	(2+3+5)/3

- **Want to predict rating for user U1 on item I2:** users U2, U4 and U5 rated I2
- **Similarity of U1 to these users:** $w_{1,5} = 0.756$, $w_{1,4} = 0$, $w_{1,2} = -1$

$$P_{1,2} = \bar{r}_1 + \frac{\sum_u (r_{u,2} - \bar{r}_u) \cdot w_{1,u}}{\sum_u |w_{1,u}|}$$

14/3 = 4.67

5/2=2.5

$$= \bar{r}_1 + \frac{(r_{2,2} - \bar{r}_2) w_{1,2} + (r_{4,2} - \bar{r}_4) w_{1,4} + (r_{5,2} - \bar{r}_5) w_{1,5}}{|w_{1,2}| + |w_{1,4}| + |w_{1,5}|}$$

4/1=4

$$= 4.67 + \frac{(2 - 2.5)(-1) + (4 - 4)0 + (1 - 3.33)0.756}{1 + 0 + 0.756}$$

10/3=3.33

$$= 3.95.$$

3. Item-based Collaborative Filtering

- Neighborhood-based CF algorithms do not scale well when applied to millions of users & items
  - Due to computational complexity of search for similar users (possible solutions?)
- **Item-to-item collaborative filtering**
  - Rather than matching similar users
  - **Match user's rated items to similar items**
- In practice, often leads to faster online systems and better recommendations
- **Similarities between pairs of items i and j are computed off-line**
- Predict rating of user **a** on item ***i*** with a simple weighted average

Pearson Correlation between items i, j

For the item-based algorithm, denote the set of users $u \in U$ who rated both items i and j, then the *Pearson Correlation* will be

the cosine similarity of the rate of i, j by u, averaged by the co-users

$$w_{i,j} = \frac{\sum_{u \in U} \left(r_{u,i} - \overline{r}_i\right)\left(r_{u,j} - \overline{r}_j\right)}{\sqrt{\sum_{u \in U} \left(r_{u,i} - \overline{r}_i\right)^2} \sqrt{\sum_{u \in U} \left(r_{u,j} - \overline{r}_j\right)^2}}, \qquad (2)$$

where $r_{u,i}$ is the rating of user u on item i, $\overline{r}_i$ is the average rating of the ith item by those users, see Figure 2 [40].

- **Note: Sum over set of co-users *U* who rated both items *i, j***
- $r_{u,i}$ is rating of user *u* on item *i*
- $\overline{r_i}$ is average rating of i^{th} item by those co-users

Pearson Correlation between items i, j

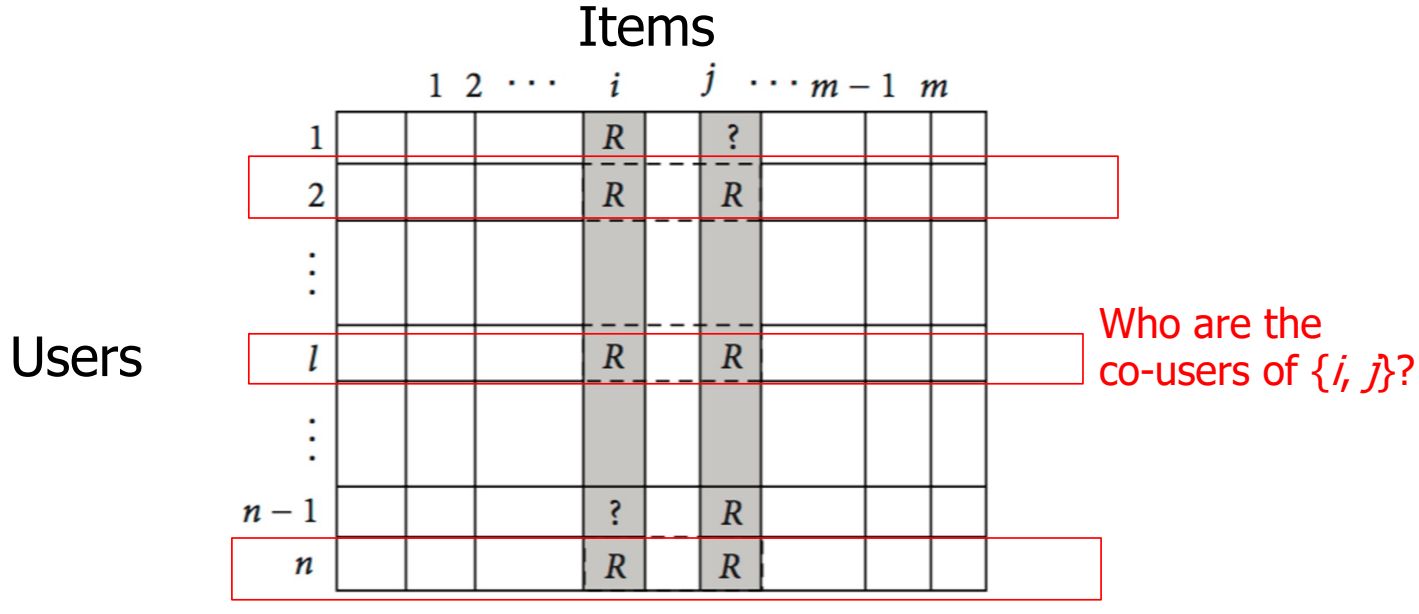


FIGURE 2: item-based similarity ($w_{i,j}$) calculation based on the co-rated items i and j from users 2, l and n.

Source: Su, X., & Khoshgoftaar, T. M. (2009). A survey of collaborative filtering techniques. *Advances in artificial intelligence*, *2009*, 4.

Make Item-Based Predictions Using a Simple Weighted Average

- Predict rating for user *u* on item *i*
- $w_{i,n}$ is weight between items *i and n*
- $r_{u,n}$ is rating for user *u* on item *n*
- Summation over **neighborhood set *N* of items** rated by *u* that are most similar to *i*

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

where the summations are over all other rated items $n \in N$ for user u, $w_{i,n}$ is the weight between items i and n, $r_{u,n}$ is the rating for user u on item n.

$$\boldsymbol{P_{u,i} = \left(r_{u,n_1}\right)\left[\frac{w_{i,n_1}}{\sum_{n \in N} |w_{i,n}|}\right] + \cdots + \left(r_{u,n_N}\right)\left[\frac{w_{i,n_N}}{\sum_{n \in N} |w_{i,n}|}\right]}$$

Make Item-Based Predictions Using a Simple Weighted Average

- Predict rating for user *u* on item *i*
- $w_{i,n}$ is weight between items *i and n*
- $r_{u,n}$ is rating for user *u* on item *n*
- Summation over **neighborhood set *N* of items** rated by *u* that are most similar to *i*

$$P_{u,i} = \left(r_{u,n_1}\right)\left[\frac{w_{i,n_1}}{\sum_{n \in N} |w_{i,n}|}\right] + \cdots + \left(r_{u,n_N}\right)\left[\frac{w_{i,n_N}}{\sum_{n \in N} |w_{i,n}|}\right]$$

Neighbor item n_1's rate

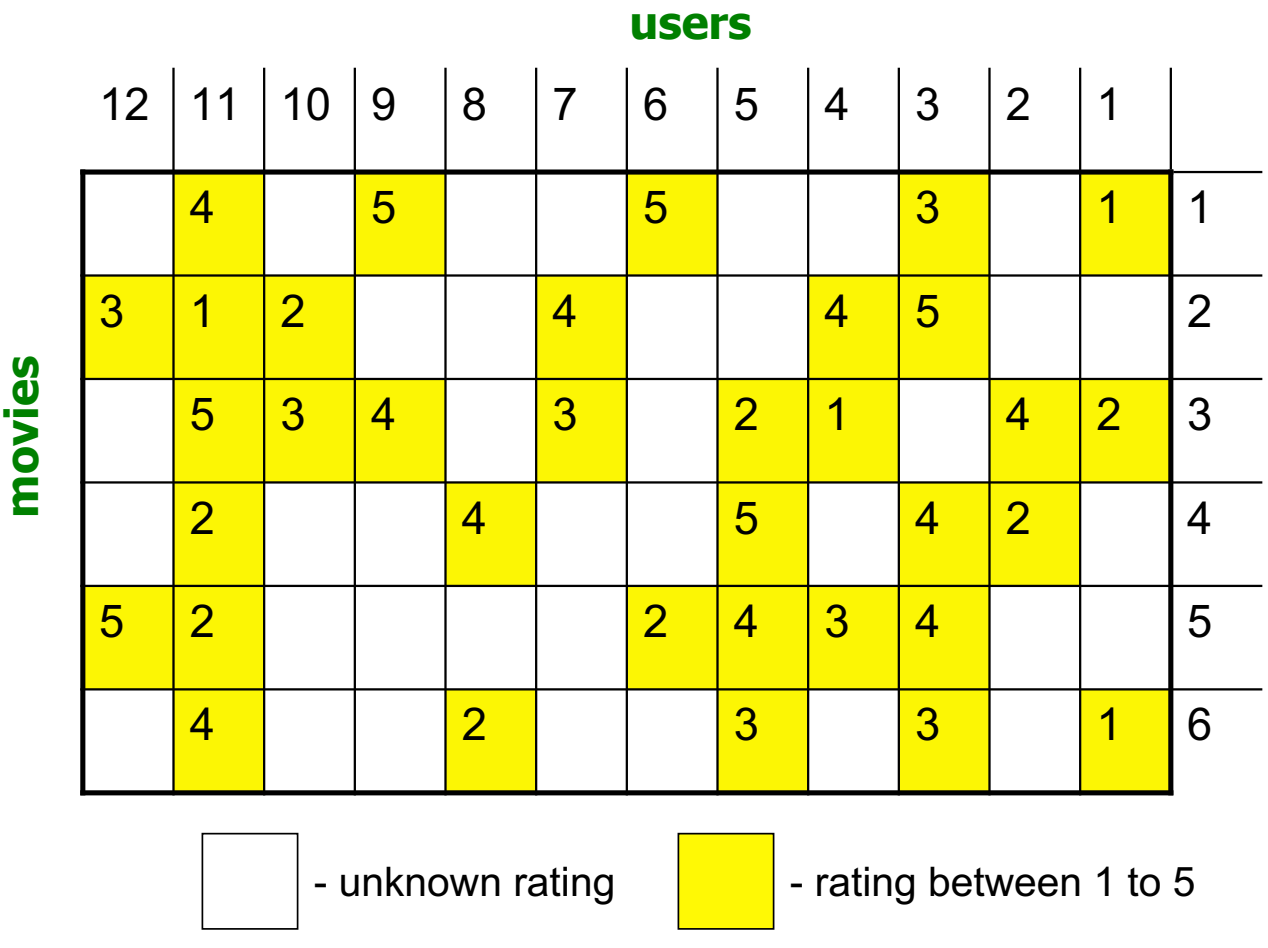
How much you trust Neighbor n_1

Neighbor n_N opinion

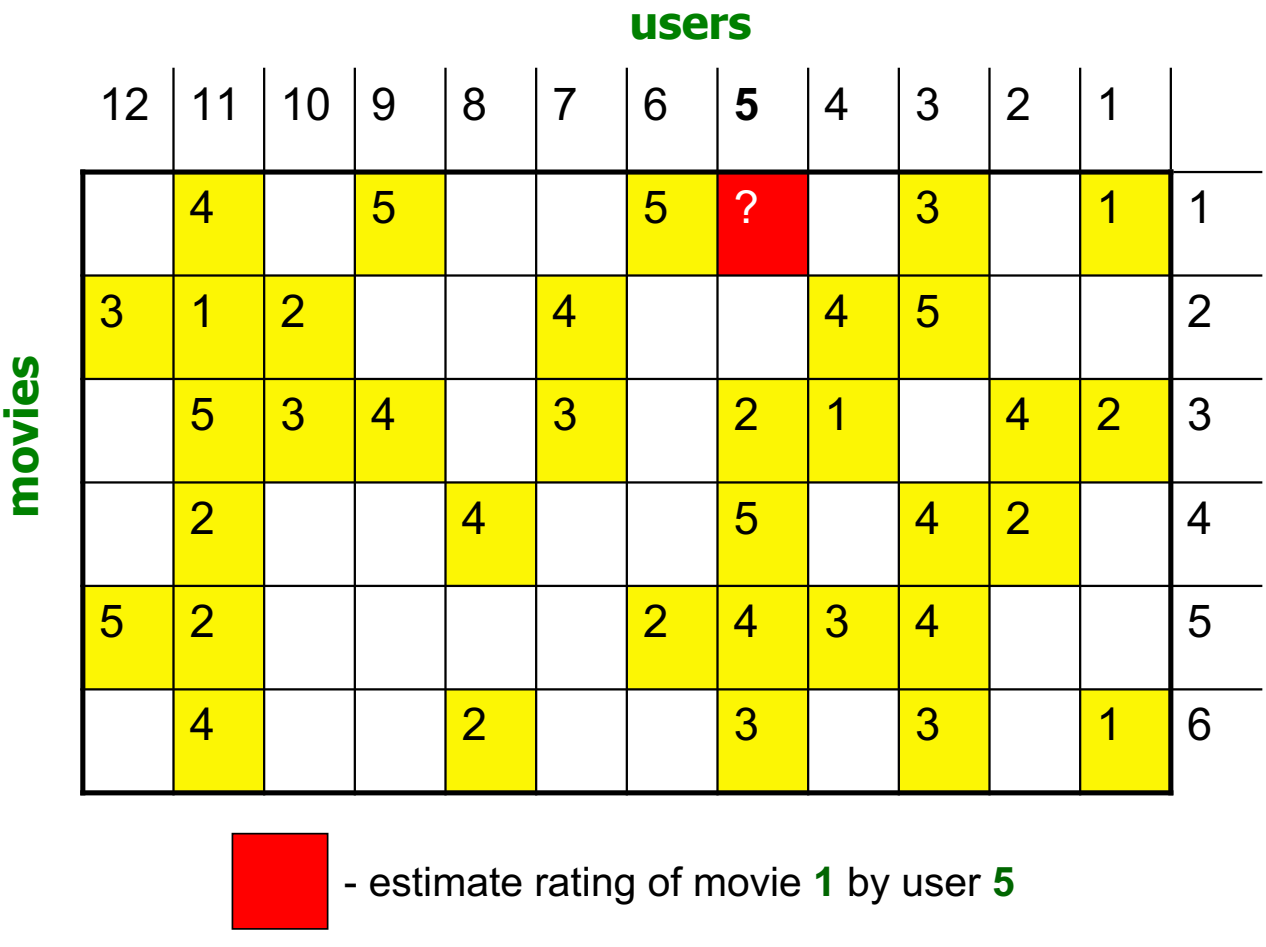
How much you trust Neighbor n_N

The neighbors of item *i are* {n_1, …, n_N}

Item-Item CF (|N|=2)



Item-Item CF (|N|=2)



Item-Item CF (|N|=2)

First: what is similarity between items?

users

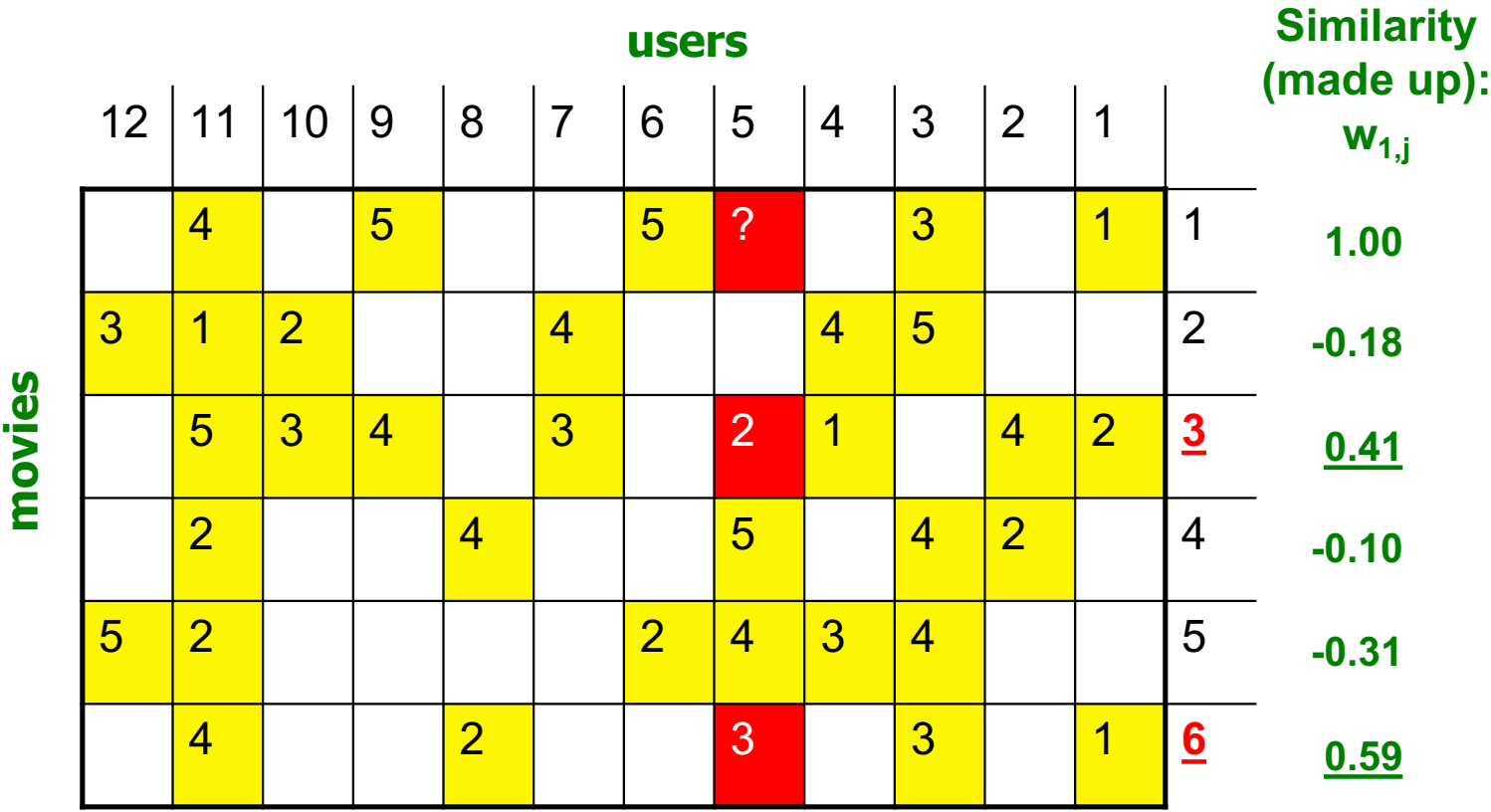
movies

12	11	10	9	8	7	6	**5**	4	3	2	1		Similarity (made up for example): $w_{1,j}$
	4		5			5	?		3		1	1	1.00
3	1	2			4			4	5			2	-0.18
	5	3	4		3		2	1		4	2	**3**	0.41
	2			4			5		4	2		4	-0.10
5	2					2	4	3	4			5	-0.31
	4			2			3		3		1	**6**	0.59

Neighbor selection: Identify movies **most similar to movie 1** and **rated by user 5**

Neighborhood size is 2: pick movies 3 and 6

Item-Item CF (|N|=2)



Similarity weights:

$w_{1,3}$=0.41, $w_{1,6}$=0.59

Item-Item CF (|N|=2)

users

movies

12	11	10	9	8	7	6	5	4	3	2	1		Similarity: $w_{1,j}$
	4		5			5	2.6		3		1	1	1.00
3	1	2			4			4	5			2	-0.18
	5	3	4		3		2	1		4	2	3	0.41
	2			4			5		4	2		4	-0.10
5	2					2	4	3	4			5	-0.31
	4			2			3		3		1	6	0.59

Predict by taking weighted average:

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} |w_{i,n}|}$$

$P_{5,1}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

Example: Item-to-Item Collaborative Filtering with Pearson Similarity

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}}$$

- What does set U represent?
  - *Set of users U who rated both items i, j*
- What are the members of the set U for $w_{1,2}$?
  - *U1 and U4 rated items I1 and I2*
- When calculating average ratings for item i, which ratings do we use? All ratings or just for co-rated items?
  - *We will show examples of* co-rated items
  - *We can use all ratings as well: based on all ratings:* *avg(r_1) =* (2+3+5)*/3, avg(r_2) =* (1+4+3)*/3*

Example: Item-to-Item Collaborative Filtering with Pearson Similarity

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$w_{i,j} = \frac{\sum_{u \in U} \left(r_{u,i} - \overline{r}_i\right)\left(r_{u,j} - \overline{r}_j\right)}{\sqrt{\sum_{u \in U} \left(r_{u,i} - \overline{r}_i\right)^2} \sqrt{\sum_{u \in U} \left(r_{u,j} - \overline{r}_j\right)^2}}$$

- *Based on all ratings: average ratings for items I1, I2: $\overline{r_1}$ = 10/3, $\overline{r_2}$ = 8/3*
- Pearson correlation for $w_{1,2.}$ *Similarity between items 1 and 2*
- *Set U includes U1 and U4*
- $w_{1,2}$ = (($r_{U1,I1}$ – 10/3)($r_{U1,I2}$ – 8/3) + ($r_{U4,I1}$ – 10/3)($r_{U4,I2}$ – 8/3)) /
  (sqrt(($r_{U1,I1}$ – 10/3) ^ 2 + ($r_{U4,I1}$ – 10/3)^2) * sqrt(($r_{U1,I2}$ – 8/3)^2) + ($r_{U4,I2}$ – 8/3)^2))
- $w_{1,2}$ = [(2-10/3)(1-8/3) + (5-10/3)(3-8/3)] /
  [sqrt((2-10/3)^2 + (5-10/3)^2) * sqrt((1-8/3)^2 + (3-8/3)^2)]
  = 2.778/3.628 = 0.765

Example: Item-to-Item Collaborative Filtering with Pearson Similarity

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}}$$

co-rated items only

- *Based on co-ratings: average ratings for items I1, I2: $\bar{r}_1$ = 7/2, $\bar{r}_2$ = 4/2*
- Pearson correlation for $w_{1,2}$. *Similarity between items 1 and 2*
- *Set U includes U1 and U4*
- $w_{1,2}$ = ((r$_{U1,I1}$ – 7/2)(r$_{U1,I2}$ – 4/2) + (r$_{U4,I1}$ – 7/2)(r$_{U4,I2}$ – 4/2)) /
  (sqrt((r$_{U1,I1}$ – 7/2) ^ 2 + (r$_{U4,I1}$ – 7/2)^2) * sqrt((r$_{U1,I2}$ – 4/2)^2) + (r$_{U4,I2}$ – 4/2)^2))
- $w_{1,2}$ = [(2-7/2)(1-4/2) + (5-7/2)(3-4/2)] /
  [sqrt((2-7/2)^2 + (5-7/2)^2) * sqrt((1-4/2)^2 + (3-4/2)^2)]
  =3/3=1

Item-Based Prediction

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

- If we have the following item similarities:

  $w_{2,1} = 0.5$, $w_{2,3} = 0.2$, $w_{2,4} = 0.3$

- Which items are in the neighborhood N for item 2 if |N| = 2?
  - Items I1 and I4
- **Predict the rating of user U2 on I2:** user = 2, item = 2, |N| = items 1,4

  $P_{2,2} = [(r_{2,1} * w_{2,1}) + (r_{2,4} * w_{2,4})] / [0.5 + 0.3]$

  $= [3*0.5 + 2*0.3] / 0.8 = 2.625$

Extensions to Memory-Based Algorithms

- Default Voting (fill in the blanks)
- Inverse User Frequency (weed out useless items)
- Case Amplifications
- Imputation-Boosted

Extensions to Memory-Based Algorithms: Default Voting

- In many collaborative filters, **pairwise similarity is computed only from the ratings in the intersection of the items both users have rated ("co-rated items")**
  - **Not reliable when there are too few votes** to generate similarity values (U is small)
  - Focusing on co-rated items ("intersection set similarity") also **neglects global rating behavior reflected in a user's entire rating history**
- Assuming some default voting values for the missing ratings: **can improve CF prediction performance**

Extensions to Memory-Based Algorithms: Default Voting (cont.)

Approaches to default voting values:

- Herlocker et al. accounts for small intersection sets (small number of co-rated items) by **reducing the weight of users that have fewer than 50 items in common**

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

- Chee et al. **use average of the clique (small group of co-rated items) as a default voting** to extend a user's rating history
- Breese et al. **use a neutral or somewhat negative preference for the unobserved ratings** and then computes similarity between users on the resulting ratings data.

Extensions to Memory-Based Algorithms: Inverse User Frequency

- **Universally liked items are not as useful in capturing similarity as less common items**
- **Inverse frequency**
  - $f_j = log\ (n/n_j)$
  - **n_j is number of users who have rated item *j***
  - ***n* is total number of users**
- If everyone has rated item *j*, then f_j is zero
  - (Note: looks a lot like Inverse Document Frequency (IDF))
- **Approach: transform the ratings**
  - For vector similarity-based CF: **new rating = original rating multiplied by f_j**

$$P_{a,i} = \bar{r}_a + \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_u) \cdot w_{a,u}}{\sum_{u \in U} |w_{a,u}|}$$

  - **For very popular items, ratings $r_{u,i}$ will be greatly reduced**
  - **Less popular items will have greater effect on prediction**

Extensions to Memory-Based Algorithms: Case Amplification

- **Transform applied to weights used in CF prediction**
- Emphasizes high weights and punishes low weights

$$w'_{i,j} = w_{i,j} \cdot |w_{i,j}|^{\rho-1}, \quad (8)$$

where ρ is the *case amplification* power, $\rho \geq 1$, and a typical choice of ρ is 2.5 [65].

- **Reduces noise in the data**
- **Favors high weights**

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} |w_{i,n}|}$$

- Small values raised to a power become negligible
- Example:
  - For $w_{i,j} = 0.9$, weight it remains high ($0.9^{2.5} \approx 0.8$)
  - For $w_{i,j} = 0.1$, weight becomes negligible ($0.1^{2.5} \approx 0.003$)

Extensions to Memory-Based Algorithms: Imputation-Boosted CF

- **When the rating data for CF tasks are extremely sparse: hard to produce accurate predictions using the Pearson correlation-based CF**
- Su et al. uses imputation-boosted collaborative filtering (IBCF)
- **First uses an imputation technique to fill in missing data**
- **Then use traditional Pearson correlation-based CF algorithm** on this completed data to predict a user rating for a specified item
  - **Example imputation techniques:** mean imputation, linear regression imputation, predictive mean matching imputation, Bayesian multiple imputation, and machine learning classifiers (including naıve Bayes, SVM, neural network, decision tree, lazy Bayesian rules)

Extensions to Memory-Based Algorithms: Imputation-Boosted CF (Cont'd)

Simple mean imputation

The data on the left below has one missing observation on variable 2, unit 10.

We replace this with the arithmetic average of the observed data *for that variable*. This value is shown in red in the table below.

Unit	Variables 1	2
1	3.4	5.67
2	3.9	4.81
3	2.6	4.93
4	1.9	6.21
5	2.2	6.83
6	3.3	5.61
7	1.7	5.45
8	2.4	4.94
9	2.8	5.73
10	3.6	5.58

- This approach is clearly inappropriate for categorical variables.
- It does not lead to proper estimates of measures of association or regression coefficients. Rather, associations tend to be diluted.
- In addition, variances will be wrongly estimated (typically under estimated) if the imputed values are treated as real. Thus inferences will be wrong too.

Imputation Example

Source:
http://missingdata.lshtm.ac.uk/index.php?option=com_content&view=article&id=68:simple-mean-imputation&catid=39:simple-ad-hoc-methods-for-coping-with-missing-data&Itemid=96

二、MODEL-BASED CF
Collaborative Filtering Overview

CF works by **collecting user feedback**: **ratings for items**

- Exploit similarities in rating behavior among users in determining recommendations

Two classes of CF algorithms:

1. **Neighborhood-based or Memory-based approaches**
   - User-based CF
   - Item-based CF
2. **Model-based approaches**
   - Estimate parameters for statistical models for user ratings
   - Latent factor and matrix factorization models

Model-based Collaborative Filtering

- Provide recommendations by estimating parameters of statistical models for user ratings
- Design and development of models can allow system to learn to recognize complex patterns
  - Based on training set – supervised learning
- Then make intelligent predictions for CF tasks based on the **learned models**
- Example models:
  - Bayesian models
  - **Clustering models**
  - Dependency networks
  - Classification algorithms (if users ratings are in categories)
  - Regression models and SVD (singular value decomposition) methods for numerical ratings

unsupervised

2. # Clustering CF Algorithms

- **Cluster = collection of data objects that are:**
  - Similar to one another within the same cluster
  - Dissimilar to objects in other clusters
- **Measurements of similarity between objects include**
  - Pearson correlation
  - Cosine similarity
    - (it's important to study your data! E.g., http://grouplens.org/blog/similarity-functions-for-user-user-collaborative-filtering/)
  - Minkowski distance
    - Two objects $X = (x_1, x_2, \ldots, x_n)$, $Y = (y_1, y_2, \ldots, y_n)$
    - Where q is a positive integer
    - If q=2: Euclidean distance

$$d(X,Y) = \sqrt[q]{\sum_{i=1}^{n} |x_i - y_i|^q}$$

Clustering Algorithms

- Common clustering method
  - K-Means
  - Hierarchical Clustering

Clustering Algorithms (Cont'd)

- **Key operation: Repeatedly combine two nearest clusters**
- **(1) How to represent a cluster of many points?**
  - **Key problem:** As you merge clusters, how do you represent the "location" of each cluster, to tell which pair of clusters is closest?
  - **Euclidean case:** each cluster has a *centroid* = average of its (data)points
- **(2) How to determine "nearness" of clusters?**
  - Measure cluster distances by distances of centroids

K–Means Algorithm(s)

- Assumes Euclidean space/distance
- Start by picking ***k***, the number of clusters
- Initialize clusters by picking one point per cluster
  - **Example:** Pick one point at random, then ***k*-1** other points, each as far away as possible from the previous points

Populating Clusters

1) For each point, place it in the cluster whose current centroid it is nearest

2) After all points are assigned, update the locations of centroids of the ***k*** clusters

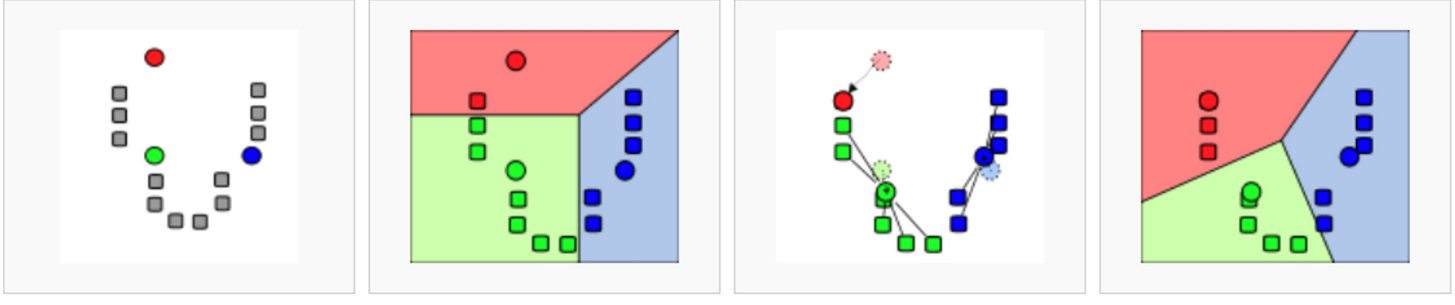
3) Reassign all points to their closest centroid

Sometimes moves points between clusters

- **Repeat 2 and 3 until convergence**
  - **Convergence:** Points don't move between clusters and centroids stabilize

Example: Standard K-Means

Demonstration of the standard algorithm



1. *k* initial "means" (in this case *k*=3) are randomly generated within the data domain (shown in color).

2. *k* clusters are created by associating every observation with the nearest mean. The partitions here represent the Voronoi diagram generated by the means.

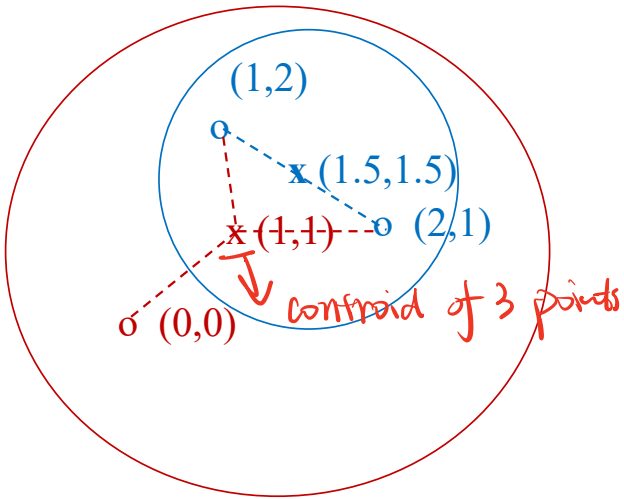
3. The centroid of each of the *k* clusters becomes the new mean.

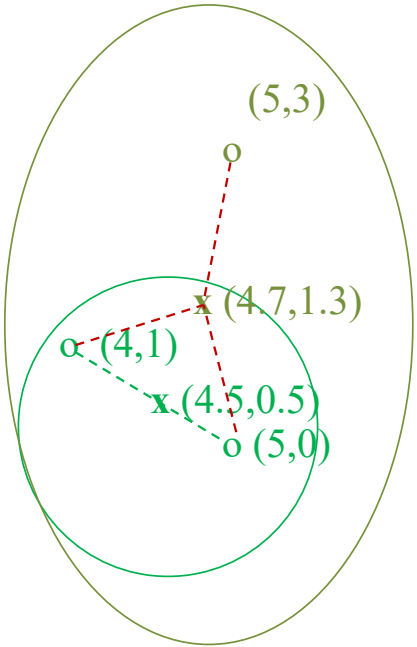
4. Steps 2 and 3 are repeated until convergence has been reached.

Source: https://en.wikipedia.org/wiki/K-means_clustering

(2)

Example: Hierarchical clustering

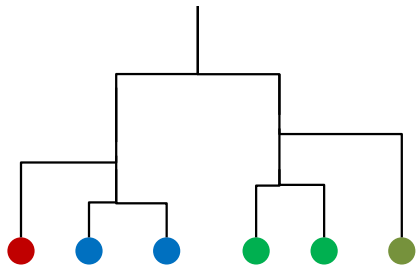




Data:

o … data point

x … centroid



Dendrogram

Clustering CF Algorithms

- **Clustering is an intermediate step**
- Resulting clusters used for further analysis or processing
  - For classification and other tasks
  - Example: **partition data into clusters; then use memory-based CF algorithm like Pearson correlation to make predictions within each cluster**
- Clustering algorithms have **better scalability than typical CF methods** because they **make predictions on smaller clusters rather than whole customer base**
- **Complex and expensive clustering computation run offline**
- **Recommendation quality is generally low**
- Optimal clustering over large data sets is impractical
  - Most applications use greedy cluster generation techniques

3. Regression-based CF Algorithms

- Numerical ratings are common in real recommender systems
- **Regression methods: good at making predictions for numerical values**
- **Uses an approximation of the ratings to make predictions based on a regression model**

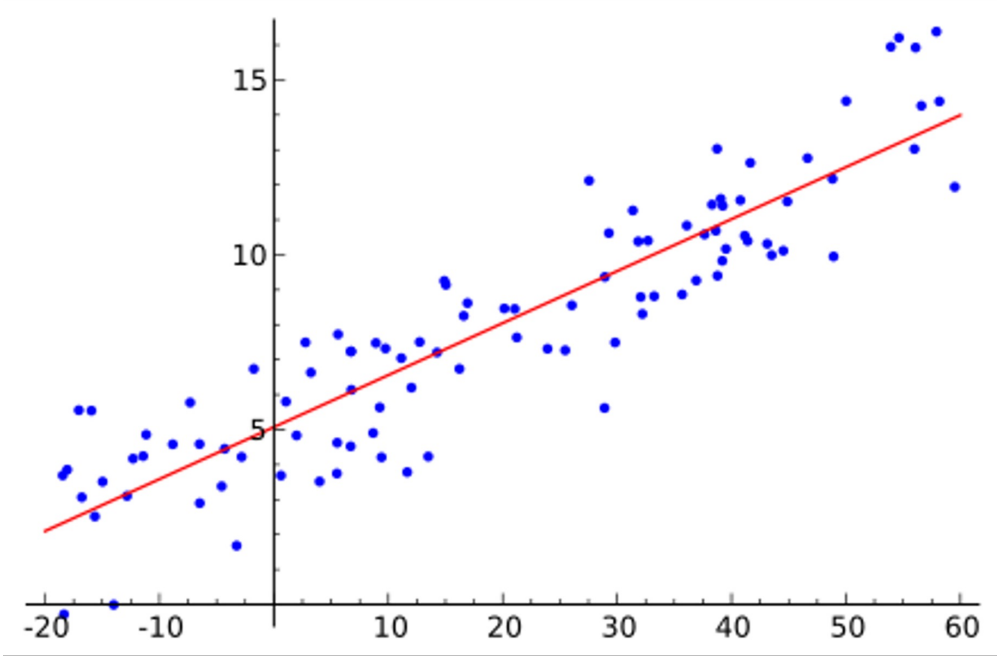
Linear Regression

- Data are modeled using linear predictor functions, and unknown model parameters are estimated from the data
- Such models are called *linear models*
- If the goal is prediction, forecasting, or reduction, linear regression can be used to **fit a predictive model to an observed data set of *y* and *X* values**
- After developing such a model, if an additional value of *X* is then given without its accompanying value of *y*
  - the fitted model can be used to **make a prediction of the value of *y***

Regression method

- **Let X = (X1, X2, …, XN) be a random variable representing user's preference on different items**
- Linear regression method:
  - **Y = MX + N**
  - Where **M is an n x k matrix**
  - **N = (N_1, …., N_n) is a random variable representing noise** in user choices
  - **Y is an n x m matrix where Y_{ij} is rating of user i on item** j (typically very sparse)
  - **X is a k x m matrix with each column as estimate of the value of the random variable X** (user's rating in k-dimensional rating space for one user)

Example: Linear Regression



Source: https://en.wikipedia.org/wiki/Linear_regression

4. Latent Factor Models

AIM3 Scalable Data Analysis and Data Mining

hidden

not obvious

11 – Latent factor models for Collaborative Filtering

Sebastian Schelter, Christoph Boden, Volker Markl

Fachgebiet Datenbanksysteme und Informationsmanagement
Technische Universität Berlin

http://www.dima.tu-berlin.de/

Latent Structure

- Given a matrix that "encodes" data ...
- Potential problems
  - too large
  - too complicated
  - missing entries
  - noisy entries
  - lack of structure
  - ...

shingles ↓ document

$$\mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1m} \\ \dots & \dots & \dots & \dots & \dots \\ a_{i1} & \dots & a_{ij} & \dots & a_{im} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{nj} & \dots & a_{nm} \end{pmatrix}$$

- Is there a **simpler** way to **explain** entries?
- There might be a **latent structure** underlying the data.
- How can we "find" or "reveal" this structure?

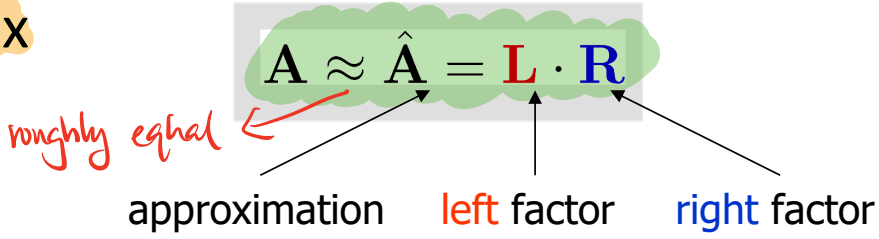
Latent Factor Models

Idea

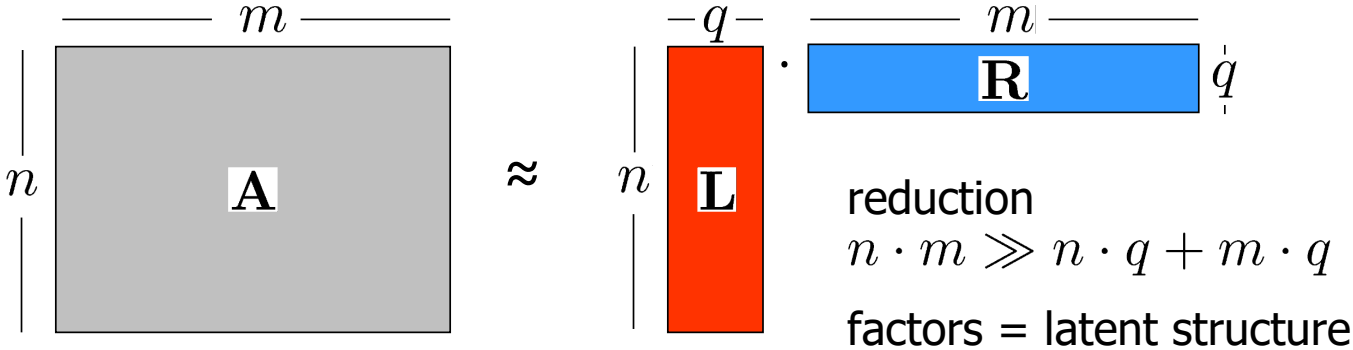
- ratings are deeply influenced by a set of **factors** that are very **specific to the domain** (e.g. amount of action in movies, complexity of characters)
- these factors are in general **not obvious**, we might be able to think of some of them but it's hard to estimate their impact on the ratings
- the goal is to infer those so called **latent factors** from the rating data by using mathematical techniques

Matrix Decomposition

- Common approach: approximately factorize matrix



- Factors are typically constrained to be "**thin**"



reduction

$$n \cdot m \gg n \cdot q + m \cdot q$$

factors = latent structure

Latent Factor Models

- **Approach**
  - users and items are characterized by **latent factors**, each user and item is mapped onto a **latent feature space**

$$u_i, m_j \in R^f$$

  - each rating is approximated by the dot product of the **user feature vector** and the **item feature vector**

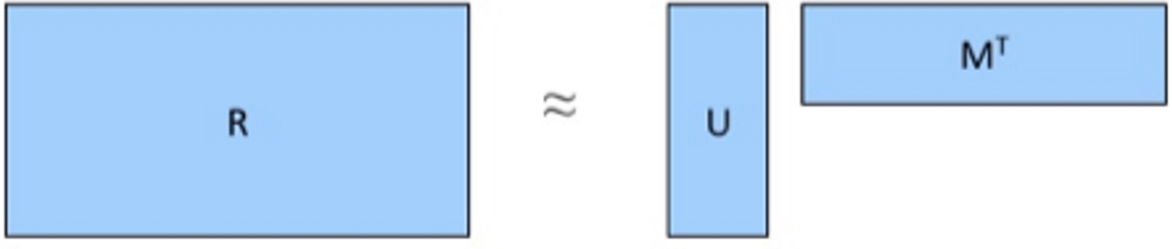
$$r_{ij} \approx m_j^T u_i$$

  - **prediction of unknown ratings** also uses this dot product
  - **squared error** as a measure of loss

$$\left(r_{ij} - m_j^T u_i\right)^2$$

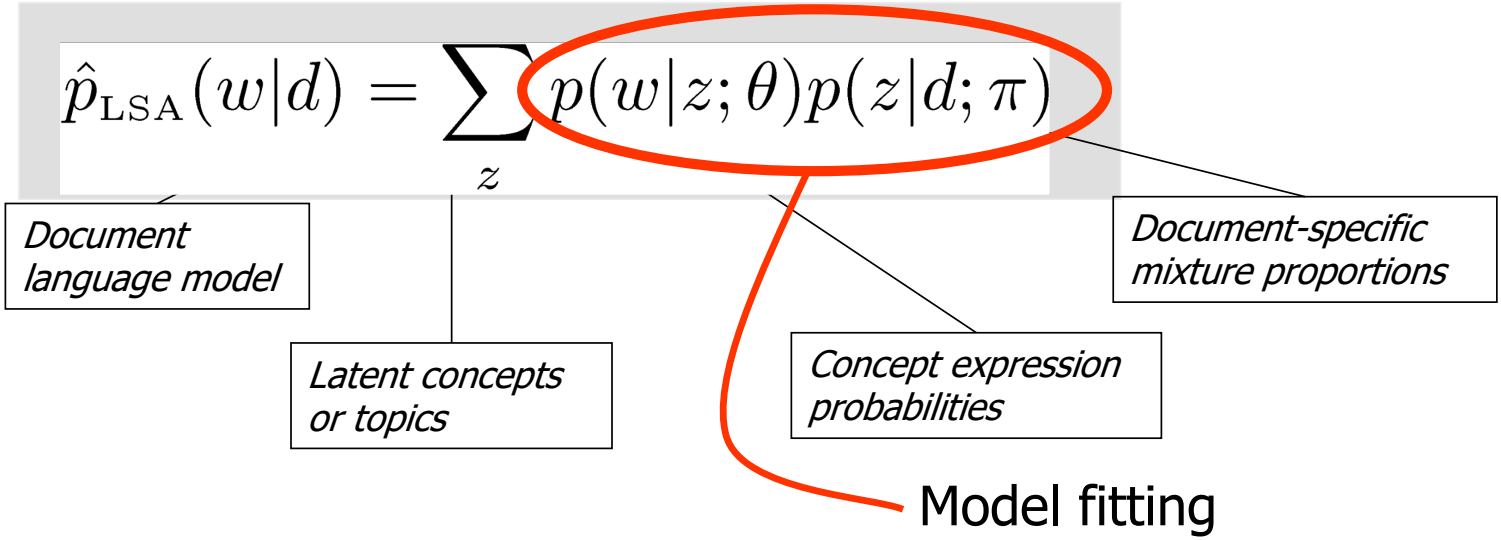
Latent Factor Models

- **Approach**
  - **decomposition of the rating matrix** into the product of a user feature and an item feature matrix
  - row in U: vector of a user's affinity to the features
  - row in M: vector of an item's relation to the features
  - closely related to **Singular Value Decomposition** which produces an optimal low-rank optimization of a matrix



pLSA – Latent Variable Model

- Structural modeling assumption (mixture model)

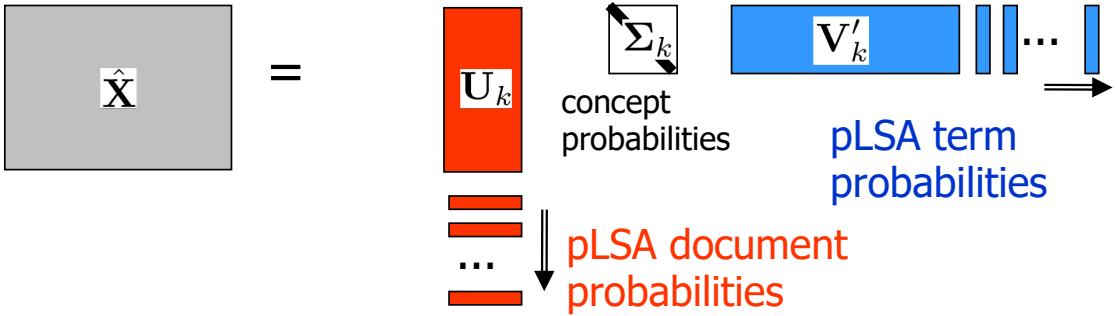


- [Hofmann, Proceedings ACM SIGIR, 1999]

pLSA: Matrix Decomposition

- Mixture model can be written as a **matrix factorization**
- Equivalent symmetric (joint) model

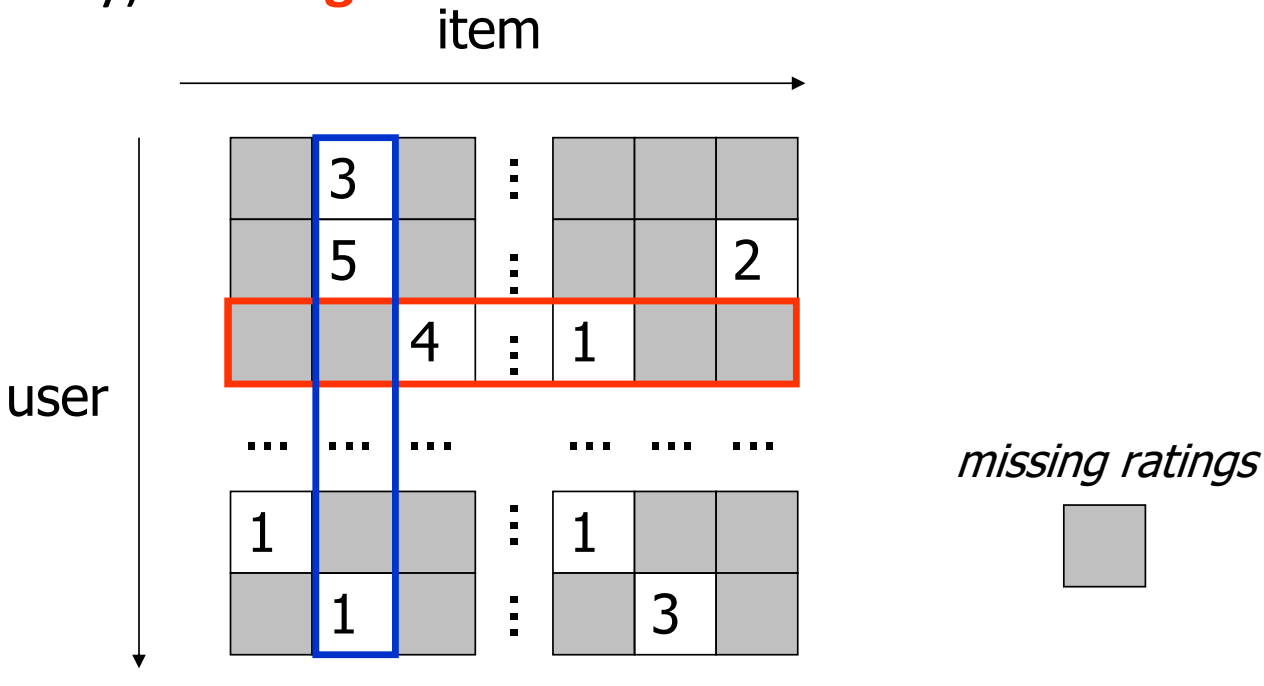
$$\hat{p}_{\mathrm{LSA}}(d,w) = \sum_z p(d|z)\, p(z)\, p(w|z)$$



- Contrast to LSA/SVD: **non-negativity** and **normalization** (intimate relation to non-negative matrix factorization)

Rating Matrix

- Rating matrix is typically a large matrix with many (mostly) **missing values**



pLSA-like Decomposition

- Generalization of pLSA (additional rating variable)

$$p_{\mathrm{LSA}}(r, y|u) = \sum_z p(r|y, z; \rho)p(y|z; \theta)p(z|u; \pi)$$

extension to predict ratings

standard pLSA model to explain sparseness pattern

Explicit decomposition of user preferences (each user can have **multiple interests**)

- Probabilistic model can be used to optimize specific **objectives**
- Data **compression** and **privacy** preservation

- Details
  - multinomial or Gaussian sampling model for rating variable
  - EM algorithm for (approximate) model fitting

T. Hofmann, *Latent Semantic Models for Collaborative Filtering*, ACM Transactions on Information Systems, 2004, Vol 22(1), pp. 89-115.

Challenges of CF

- Data Sparsity
- Cold Start Problem
- Synonyms
- Scalability
- Gray Sheep and Black Sheep
- Shilling attacks

Characteristics and Challenges of Collaborative Filtering

- **Data Sparsity**
  - Many commercial recommender systems are used with very large product sets
  - Most users do not rate most items: User-item matrix is extremely sparse
  - For CF: reduces probability of finding set of users with similar ratings
- **Approaches:**
  - **Dimensionality reduction techniques**
    - **Singular Value Decomposition (SVD):** remove unrepresentative or insignificant users or items to reduce size of user-item matrix
    - **Latent semantic Indexing:** similarity between users is determined by representation of users in reduced space
    - **Principle Component Analysis**

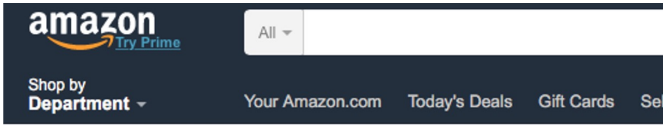
Characteristics and Challenges of Collaborative Filtering

- **Data Sparsity (cont.)**
  - **Dimensionality reduction techniques**
  - **When users or items are discarded:**
    - **Useful information for recommendations may be lost**
    - **Recommendation quality may be degraded**

Challenges (cont.)

- **Cold start problem**
  - **When a new user or item has just entered the system**
    - Hard to find similarities: not enough information to make good recommendations
  - **New item problem:** can't be recommended until some users rate it
    - Also applies to obscure items
    - Also **called "first-rater problem"**
  - **New users**: not given good recommendations because of lack of rating or purchase history
- **Approaches:**
  - **Content-based systems** do not rely on ratings from other users
  - **Hybrid CF (content-boosted CF)**: external content information can be used to produce predictions for new users or new items
  - Research on **effectively selecting items to be rated by a user to rapidly improve recommendation performance**

Amazon Vine



What is Amazon Vine?

Amazon Vine invites the most trusted reviewers on Amazon to post opinions
rank, which is a reflection of the quality and helpfulness of their reviews as j
independent opinions of the Vine Voices. The vendor cannot influence, modif
Customer review from the Amazon Vine Program.

Why do you have the Amazon Vine program?

The program was created to provide customers with more information includ

How can I join the program?

Amazon Vine is an invitation-only program. Vine Voices are selected based o
in the program. Customers who consistently write helpful reviews and develc

How are Vine Voices selected?

We want the Voice program to reflect the best of our growing body of custom
their reviews, factoring in the number of reviews they have written. More we

How To Become An Amazon Vine Reviewer & Get Free Stuff

Written by Bakari Chavanu
August 4, 2010

If you're an Amazon.com customer who regularly reads product reviews by other customers, you may have noticed that some reviewers have a little badge (one of several) under their name identifying them as a ï»¿"Vine Voice" reviewer.

These Amazon Vine reviewers were invited by Amazon to receive a monthly newsletter of pre- and recently-released books and other products that Vine members can select from and review. Selected products are sent to Amazon Vine reviewers for free. So how do you become a Vine reviewer?

I'm not a prolific reviewer on Amazon, but about six months ago I was invited to become a member of its "exclusive club of influential Amazon voices."

Getting free items such as a 1 TB Western Digital External Hard Drive, a Duracell iPhone charger, a very expensive photography bag, computer software, and numerous books in exchange for honest reviews of these products has been a pretty good deal in my view.

Challenges (cont.)

Synonyms

- **Same or very similar items that have different names or entries**
- Most recommender systems are unable to discover this latent association
- Treat these products differently
- **Synonyms decrease recommendation performance of CF systems**

- Approaches
  - **Automatic term expansion** or construction of thesaurus
    - Some added terms may have different meanings than intended
  - **SVD techniques: Latent Semantic Indexing (LSI)**: construct a semantic space where terms and documents that are closely associated are placed close to each other

Example: Latent Semantic Indexing (LSI)

Searches related to usc

usc **next coach**
university of southern california notable alumni
usc **address**
usc **clothing**
usc **tuition**
usc **ranking**
usc **jobs**
usc **athletics**

Searches related to ucla

ucla **vs stanford predictions**
ucla **football**
ucla **requirements**
ucla **tuition**
ucla **ranking**
ucla **admissions**
ucla **majors**
ucla **medical center**

Challenges (cont.)

- **Scalability**
  - **Traditional CF systems suffer scalability problems at very large scale**
  - With tens of millions of customers (M), millions of catalog items (N): and ***O(n)* algorithm is too large**
- Approaches
  - **Dimensionality reduction (SVD)** can scale and quickly produce good recommendations
    - But have to do expensive matrix factorization
  - Memory-based CF algorithms (e.g., **item-based Pearson correlation CF algorithm**) have good scalability
    - Instead of calculating similarities between all pairs of items, **calculate similarity only between pairs of co-rated items by a user**

Challenges (cont.)

- **Gray Sheep**
  - Users whose opinions **do not consistently agree or disagree with any group of people**
  - Do not benefit from collaborative filtering
- Approaches:
  - **Hybrid approach combining content-based and CF recommendations**
  - Base prediction on **weighted average of content-based prediction and CF prediction**
- **Black sheep**
  - **Idiosyncratic tastes make recommendations nearly impossible: considered an acceptable failure**

Challenges (cont.)

b. Shilling attacks

- Shill definition not true users
  - **Noun:** an accomplice of a hawker, gambler, or swindler who acts as an enthusiastic customer to entice or encourage others
  - **Verb:** act or work as a shill
- **In systems where anyone can provide ratings (Yelp, Amazon):**
  - People may give many positive ratings for their own materials
  - Negative recommendations for competitors
- CF systems want to discourage this phenomenon
- Approaches (research)
  - **Item-based less affected than user-based**
  - **Hybrid CF systems provide partial solutions**

Pros/Cons of Collaborative Filtering

+ Works for any kind of item

No feature selection needed

- Cold Start:

Need enough users in the system to find a match

- Sparsity:

The user/ratings matrix is sparse

Hard to find users that have rated the same items

- First rater:

Cannot recommend an item that has not been previously rated

New items, Esoteric items

- Popularity bias:

Cannot recommend items to someone with unique taste

Tends to recommend popular items