

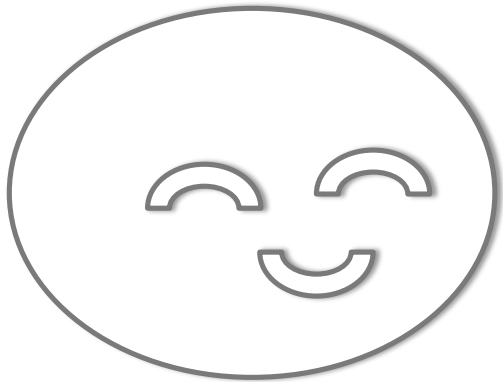
CSCI 561 - Foundation for Artificial Intelligence

Week 7 Discussion **Propositional Logic**

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Logic Concepts

- *Entailment* \models
- *Inference* \vdash



Your head

	4	Stench		Breeze	PIT
3		Stench	Breeze Stench Gold	PIT	Breeze
2		Stench		Breeze	
1		START	Breeze	PIT	Breeze
		1	2	3	4

A possible “real” world

Entailment

(between your head and the universe)

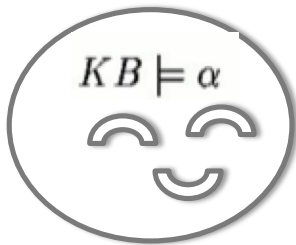
$$KB \models \alpha$$

Knowledge base KB entails sentence α (in your head)
if and only if

α is true in all worlds where KB is true (in the universe)

E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”

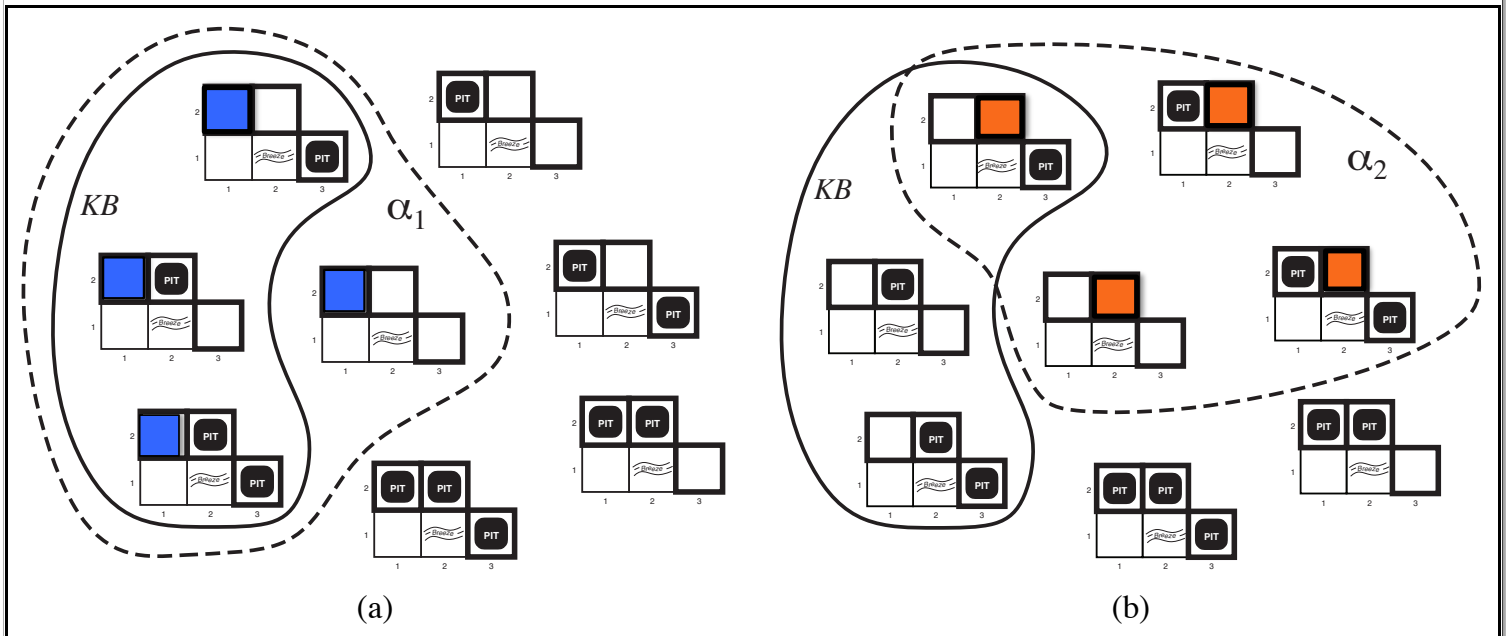
Entailment means it is impossible for this case to occur:
premises are true and the consequence is false.



You must check in the whole universe for this !!!

$$M(KB) \subseteq M(\alpha)$$

Entailment in Wumpus World



$$\alpha_1 = \neg P_{1,2}$$

$$KB \models^? \alpha_1$$

$$M(KB) \subseteq M(\alpha_1)$$

$$\alpha_2 = \neg P_{2,2}$$

$$KB \models^? \alpha_2$$

$$M(KB) \subseteq M(\alpha_2)$$

Inference (all in your head)

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

Soundness: i is sound if

whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if

whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Exercise 7.4

False \models True. TRUE

empty set \subseteq full set

- **False has no models and hence entails every sentence**
- **True is true in all models and hence is entailed by every sentence.**

True \models False. FALSE

- False is not true in any models

Exercise 7.4

$$A \cap B \leq A \cap B + A \text{假} B \text{假}$$

$(A \wedge B) \models (A \Leftrightarrow B)$. **TRUE**

- The left-hand side $(A \wedge B)$ has exactly one model:
 $A=\text{True}$ and $B=\text{True}$ then $(A \wedge B)=\text{True}$
- That model is one of the two models of the right-hand side $(A \Leftrightarrow B)$. Two models:
 - $A=\text{True}$ and $B=\text{True}$ then $(A \Leftrightarrow B) = \text{True}$
 - $A=\text{False}$ and $B=\text{False}$ then $(A \Leftrightarrow B) = \text{True}$

Exercise 7.4

A真B真 + A假B假 \neq AB至少一个为真

$A \Leftrightarrow B \models A \vee B$. **FALSE**

- **$(A \Leftrightarrow B)$ has two models:**
 - $A=\text{True}$ and $B=\text{True}$ then $(A \Leftrightarrow B) = \text{True}$
 - $A=\text{False}$ and $B=\text{False}$ then $(A \Leftrightarrow B) = \text{True}$
- **$(A \vee B)$ is not True in this model**
 - $A=\text{False}$ and $B=\text{False}$ then $(A \vee B) = \text{False}$

Exercise 7.4

$(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable. **TRUE**

- This sentence $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is True, when $A=\text{True}$ and $B=\text{True}$

(A真B真 + A假B假) and ($\neg A$ 和B至少一个真)

\Rightarrow A假B假 / A真B真 \checkmark

Exercise 7.4

- 1. $\text{False} \models \text{True}.$
- 2. $\text{True} \models \text{False}.$
- 3. $(A \wedge B) \models (A \Leftrightarrow B).$
- 4. $A \Leftrightarrow B \models A \vee B.$
- 5. $A \Leftrightarrow B \models \neg A \vee B.$
- 6. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C). (C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C)).$
- 7. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B).$
- 8. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E).$
- 9. $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.
- 10. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.
- 11. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

Proof methods

Proof methods divide into (roughly) two kinds:

Model checking

- truth table enumeration (sound and complete for propositional)
- heuristic search in model space (sound but incomplete)
 - e.g., the GSAT algorithm (Ex. 6.15)

Application of inference rules

Legitimate (sound) generation of new sentences from old

Proof = a sequence of inference rule applications

Can use inference rules as operators in a standard search alg.

Basic manipulation rules

$$\neg(\neg A) = A$$

Double negation

$$\neg(A \wedge B) = (\neg A) \vee (\neg B)$$

Negated “and”

$$\neg(A \vee B) = (\neg A) \wedge (\neg B)$$

Negated “or”

$$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$$

Distributivity of \wedge on \vee

$$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$$

Distributivity of \vee on \wedge

$$A \Rightarrow B = (\neg A) \vee B$$

by definition

$$\neg(A \Rightarrow B) = A \wedge (\neg B)$$

using negated or

$$A \Leftrightarrow B = (A \Rightarrow B) \wedge (B \Rightarrow A)$$

by definition

$$\neg(A \Leftrightarrow B) = (A \wedge (\neg B)) \vee (B \wedge (\neg A))$$

using negated and & or

...

Inference Rules

- ◇ **Modus Ponens or Implication-Elimination:** (From an implication and the premise of the implication, you can infer the conclusion.)

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

Modus Tollens:z $\frac{\alpha \Rightarrow \beta, \quad \neg \beta}{\neg \alpha}$

- ◇ **And-Elimination:** (From a conjunction, you can infer any of the conjuncts.)

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n}{\alpha_i}$$

- ◇ **And-Introduction:** (From a list of sentences, you can infer their conjunction.)

$$\frac{\alpha_1, \alpha_2, \dots, \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n}$$

- ◇ **Or-Introduction:** (From a sentence, you can infer its disjunction with anything else at all.)

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \dots \vee \alpha_n}$$

Inference Rules

- ◇ **Double-Negation Elimination:** (From a doubly negated sentence, you can infer a positive sentence.)

$$\frac{\neg\neg\alpha}{\alpha}$$

- ◇ **Unit Resolution:** (From a disjunction, if one of the disjuncts is false, then you can infer the other one is true.)

$$\frac{\alpha \vee \beta, \quad \neg\beta}{\alpha}$$

- ◇ **Resolution:** (This is the most difficult. Because β cannot be both true and false, one of the other disjuncts must be true in one of the premises. Or equivalently, implication is transitive.)

$$\frac{\alpha \vee \beta, \quad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

or equivalently

$$\frac{\neg\alpha \Rightarrow \beta, \quad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

What you should know

- **What is entailment and inference? How do they differ?**
- **What are examples of sound or complete inference techniques?**
- **What does satisfiable or valid mean?**
- **What is propositional logic? Basic manipulation rules? Inference rules? What are some of its limitations?**

Want More?

- **Check out some of these exercises in the book:**

7.1, 7.4-8, 10

Chap 8: 8.1-3, 8.6, 8.9-10, 8.14,17, 8.28