

Written Homework.

1. Analyze the big-oh run time of each method in terms of the variable n:

```
void A(int n)
{
    if (n < 10)
        time++;
    else
    {
        for (int i = 0; i < n; i++)
            time++;
        A(n - 2);
    }
}
```

```
void B(int n)
{
    time = 0;
    for (i = 0; i < n; i++)
        for (j = 0; j < i * i; j++)
            for (k = 0; k < j*j; k++)
                time++;
}
```

```
void C(int n)
{
    if (n < 100)
        time++;
    else
    {
        for (int i = 0; i < n; i++)
            time++;
        C(n/2);
    }
}
```

```
void D(int n)
{
    if (n < 8)
        time++;
    else
    {
        time++;
        D(n/2);
    }
}
```

```

void E(int n)
{
    if (n < 15)
        time++;
    else
    {
        time++;
        E(n/2);
        E(n/2);
    }
}

```

```

void F(int n)
{
    if (n < 20)
        time++;
    else
    {
        F(n/2);
        for (int i = 0; i < n; i++)
            time++;
        F(n/2);
    }
}

```

2. Let F_n denote the n th Fibonacci number. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Show that $A^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$. (Hint: use induction).
3. Use the equality proven in question 2 to design an algorithm to compute the n th Fibonacci number in $O(\log n)$ time.
4. Show that $\log(n!) = \Theta(n \log n)$.
5. Suppose Dr. WhyLie comes up to you and claims that he has invented a super-fast new data structure that supports two operations (insert and extract-min), and that these operations are guaranteed to run in the following run times: (variable n denotes the current number of items currently contained inside the data structure when the operation is performed)
 - a. insert a new item into the data structure in $O(\sqrt{\log n})$ time
 - b. extract (remove and return) the smallest item from the data structure in $O(\sqrt{\log n})$ time.

Dr. WhyLie says this data structure will work this fast for any collection of comparable items, as it only uses comparisons to determine which items to remove. Explain why Dr. WhyLie must be lying.