

Optimal Energy Allocation for Wireless Communications with Energy Harvesting Constraints

Chin Keong Ho, *IEEE Member*, and Rui Zhang, *IEEE Member*

Abstract

We consider the use of energy harvesters, in place of conventional batteries with fixed energy storage, for point-to-point wireless communications. In addition to the challenge of transmitting in a channel with time selective fading, energy harvesters provide a perpetual but unreliable energy source. In this paper, we consider the problem of energy allocation over a finite horizon, taking into account channel conditions and energy sources that are time varying, so as to maximize the throughput. Two types of side information (SI) on the channel conditions and harvested energy are assumed to be available: causal SI (of the past and present slots) and full SI (of the past, present and future slots). We obtain structural results for the optimal energy allocation, via the use of dynamic programming and convex optimization techniques. In particular, if unlimited energy can be stored in the battery with harvested energy and full SI is available, we prove the optimality of a water-filling energy allocation solution where the so-called water levels follow a staircase function.

Index terms: Energy harvesting, wireless communications, optimal policy, dynamic programming, convex optimization

This paper was presented in part at the IEEE International Symposium on Information Theory, Austin, TX, June 2010.

C. K. Ho is with the Institute for Infocomm Research, A*STAR, 1 Fusionopolis Way, #21-01 Connexis, Singapore 138632 (e-mail: hock@i2r.a-star.edu.sg).

R. Zhang is with the Institute for Infocomm Research, A*STAR, 1 Fusionopolis Way, #21-01 Connexis, Singapore 138632 (e-mail: rzhang@i2r.a-star.edu.sg). He is also with the Department of ECE, National University of Singapore (e-mail: elezhang@nus.edu.sg).

I. INTRODUCTION

In conventional wireless communication systems, the communication devices have access to a fixed power supply, or are powered by replaceable or rechargeable batteries for user mobility. In these cases, the transmissions are limited by power constraints for safety reasons, or by the sum energy constraint so as to prolong operating time for battery-powered devices. In other communication systems, however, a fixed power supply is not readily available, and even replacing the batteries periodically may not be a viable option if the replacement is considered to be too inconvenient (when thousands of sensor nodes are scattered throughout the building), too dangerous (the devices may be located in toxic environments) or even impossible (when the devices are embedded in building structures or inside human bodies). In such situations, the use of energy harvesting for wireless communications appears appealing or even essential. Examples of energy that can be harvested include solar energy, piezoelectric energy and thermal energy, etc.

For transmitters that are powered by energy harvesters, the energy that can potentially be harvested is unlimited. Typically, energy is replenished by the energy harvester, while expended for communications or other processing; any unused energy is then stored in an energy storage, such as a rechargeable battery. However, unlike conventional communication devices that are subject only to a power constraint or a sum energy constraint, transmitters with energy harvesting capabilities are, in addition, subject to other *energy harvesting constraints*. Specifically, in every time slot, each transmitter is constrained to use at most the amount of stored energy currently available, although more energy may become available in the future slots. Thus, a causality constraint is imposed on the use of the harvested energy.

Several contributions in the literature have considered using energy harvester as an energy source, in particular based on the technique of dynamic programming [1]. In [2], the problem of maximizing a reward that is linear with the energy used is studied. In [3], the discounted throughput is maximized over an infinite horizon, where queuing for data is also considered. In

[4], adaptive duty cycling is employed for throughput maximization and implemented in practical systems.

In this work, we consider the problem of maximizing the throughput via energy allocation over a finite horizon of $K < \infty$ time slots. The channel signal-to-noise ratios (SNRs) and the amount of energy harvested change over different slots. Our aim is to study the structure of the maximum throughput and the corresponding optimal energy allocation solution, such as concavity and monotonicity. These results may be useful for developing heuristic solutions, since the optimal solutions are often complex to obtain in practice. We consider two types of side information (SI) available to the transmitter:

- *causal SI*, consisting of past and present channel conditions, in terms of SNR, and the amount of energy harvested in the past slot, or
- *full SI*, consisting of past, present and future channel conditions and amount of energy harvested.

The case of full SI may be justified if the environment is highly predictable, e.g., the energy is harvested from the vibration of motors that are turned on only during fixed operating hours and line-of-sight is available for communications. Given causal SI, and assuming the variations in the channel conditions and energy harvested is modeled by a first-order Markov model, we obtain the optimal energy allocation solution by dynamic programming and obtain structural results to characterize the optimal solution. Given full SI, we obtain a closed-form solution for $K = 2$ slots. We also obtain the structure of this optimal solution for arbitrary K with unlimited energy storage. The optimal solution then has a water-filling interpretation, as in [5]. However, instead of a single water level, there are multiple so-called water levels that are non-decreasing over time, i.e., the water levels follow a staircase-like function. Finally, we propose a heuristic scheme that uses only causal SI. Compared to a naive scheme, the proposed scheme performs relatively close to the optimal throughput obtained with full SI in the numerical studies.

This paper is organized as follows. Section II gives the system model. Then, Section III

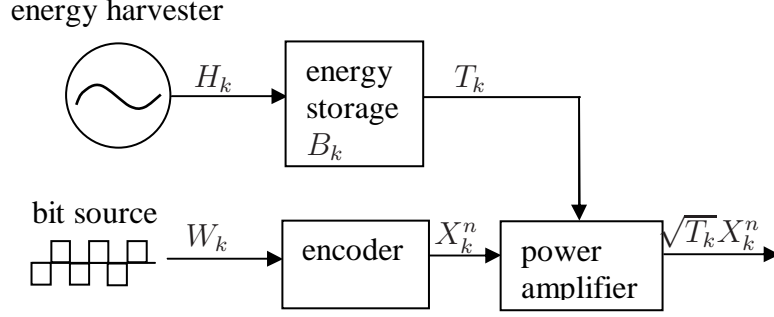


Fig. 1. Block diagram of a transmitter powered by an energy harvester. Energy is replenished by an energy harvester but is drawn for transmission.

considers optimal schemes with availability of causal SI of the channel conditions and harvested energy. Section IV considers optimal schemes with availability of full SI with a constraint on the maximum amount of energy that can be stored on the battery, while Section V considers the specific case where this constraint is removed. Section VI shows numerical results for the various schemes. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

For simplicity, each packet transmission is performed in one time slot. Each time slot allows n symbols to be transmitted, where n is assumed to be sufficiently large for reliable decoding. We index time by the slot index $k \in \mathcal{K} \triangleq \{1, \dots, K\}$. We assume that there is always back-logged data available for transmission. In slot k , a message $W_k \in \{1, \dots, 2^{nR_k}\}$ is sent, where the rate $R_k \geq 0$ in bits per symbol can be selected.

We consider a point-to-point, flat-fading, single-antenna communication system. As shown in Fig.1, the transmitter consists of an energy harvester with energy as input, an energy storage, an encoder, and a power amplifier. Energy is measured on a per symbol (or channel use) basis, hence we use the terms energy and power interchangeably.

Consider slot $k \in \mathcal{K}$. At time instant k^- , which denotes the time instant just before slot k , the battery has $B_k \geq 0$ amount of stored energy per symbol available. For transmission, the

message W_k is first encoded as data symbols $X_k^n \triangleq [X_{1k}, \dots, X_{nk}]$ of length n , where we normalize $\sum_{i=1}^n X_{ik}/n = 1$. Then the transmitter transmits packet k in slot k as $\sqrt{T_k}X_k^n$, where $0 \leq T_k \leq B_k$ is the energy per symbol used by the power amplifier. Except for transmission, we assume the other circuits in the transmitter consume negligible energy.

1) *Mutual Information:* We assume the channel is quasi-static for every slot $k \in \mathcal{K}$ with SNR γ_k . The maximum reliable transmission rate in slot k is then given by the mutual information $I(\gamma_k, T_k) \geq 0$ in bits per symbol [6]. In general, we assume that $I(\gamma, T)$ is concave in T given γ , and is increasing in T for all γ . For example, we may employ Gaussian signalling for transmission over a complex Gaussian channel [6], which gives

$$I(\gamma, T) = \log_2(1 + T\gamma). \quad (1)$$

2) *Battery Dynamics:* In general, let us denote a vector of length k as $Y^k = [Y_1, \dots, Y_k]$, e.g., the battery energy from slot 1 to slot k is given by $B^k = [B_1, \dots, B_k]$. While transmitting packet k , the energy harvester collects an average energy of $H_k \geq 0$ per symbol, which is then stored in the battery. At time instant $(k+1)^-$, the energy stored is updated in general as

$$B_{k+1} = f(B^k, T^k, H^k), k \in \mathcal{K},$$

where the function f depends on the battery dynamics, such as the storage efficiency and memory effects. Intuitively, we expect B_{k+1} to increase (or remains the same) if B_k or H_k increases, or if T_k decreases. As a good approximation in practice, we assume the stored energy increases and decreases linearly provided the maximum stored energy in the battery B_{\max} is not exceeded, i.e.,

$$B_{k+1} = \min\{B_k - T_k + H_k, B_{\max}\}, k \in \mathcal{K}. \quad (2)$$

We assume the initial stored energy B_1 is known, where $0 \leq B_1 \leq B_{\max}$. Thus, $\{B_k\}$ follows a deterministic first-order Markov model that depends only on the immediate past random variables.

3) *Channel and Harvest Dynamics:* To model the unpredictable nature of energy harvesting and the wireless channel over time, we model¹ H^{K-1} and γ^K jointly as a random process described by their joint distribution. The exact distribution depends on the energy harvester used and the wireless channel environment.

In most typical operating scenarios, both the wireless channels and the harvested energy vary slowly over time. To account for these variations, the SNR γ_k is assumed to be constant in each slot and follow a first-order stationary Markov model over time k , see e.g. similar assumptions in [7]. Also, the harvested energy H_k is modeled as first-order stationary Markov model over time k , where the accuracy of this mode is justified by empirical studies when solar energy is harvested [8]. Given $H_0 = \hat{H}_0$ and $\gamma_1 = \hat{\gamma}_0$, the joint pdf of H^{K-1} and γ^K thus becomes

$$p_{H^{K-1}, \gamma^K}(H^{K-1}, \gamma^K | \hat{H}_0, \hat{\gamma}_1) = \prod_{k=2}^K p_{H_{k-1}}(H_{k-1} | H_{k-2}) p_{\gamma_k}(\gamma_k | \gamma_{k-1}) \cdot \delta(\gamma_1 - \hat{\gamma}_1) \quad (3)$$

where $p_{H_k}(\cdot | \cdot)$ and $p_{\gamma_k}(\cdot | \cdot)$ are independent of k , and $\delta(\cdot)$ is the Dirac delta function. In (3), we have also assumed that the harvested energy and the SNR are independent, which is reasonable in most practical scenarios. In this paper, we assume that the joint distribution (3) is known, which may be obtained via long-term measurements in practice.

4) *Overall Dynamics:* Let us denote the *state* $\mathbf{s}_k = (\gamma_k, H_{k-1}, B_k)$, $k \in \mathcal{K}$, or simply \mathbf{s} if the index k is arbitrary. Let the accumulated states be $\mathbf{s}^k \triangleq (\mathbf{s}_1, \dots, \mathbf{s}_k)$, $k \in \mathcal{K}$.

We assume the initial state $\mathbf{s}_1 \triangleq (\gamma_1, H_0, B_1)$ to be always known at the transmitter, which may be obtained causally prior to any transmission. From (2) and (3), given $\mathbf{s}_1 = \hat{\mathbf{s}}_1$, the states thus follow a first-order Markov model:

$$p_{\mathbf{s}^K}(\mathbf{s}^K | \hat{\mathbf{s}}_1) = \prod_{k=2}^K p_{\mathbf{s}_k}(\mathbf{s}_k | \mathbf{s}_{k-1}) \cdot \delta(\mathbf{s}_1 - \hat{\mathbf{s}}_1). \quad (4)$$

In particular, (4) includes the special cases where the states are independent, i.e., $p_{\mathbf{s}_k}(\mathbf{s}_k | \mathbf{s}_{k-1}) = p_{\mathbf{s}_k}(\mathbf{s}_k)$, or where the states are deterministic rather than random, i.e., $p_{\mathbf{s}_k}(\mathbf{s}_k | \mathbf{s}_{k-1}) = \delta(\mathbf{s}_k - \hat{\mathbf{s}}_k)$.

¹The harvested energy in slot K , namely H_K , cannot be used for transmission in slots 1 to slot K and so does not affect the throughput.

III. CAUSAL SIDE INFORMATION

A. Problem Statement

The transmitter is given knowledge² of s_k before packet k is transmitted, where $k \in \mathcal{K}$. In practice, for instance, the receiver feeds back γ_k shortly before transmission, while the transmitter infers H_{k-1} and B_k from its energy storage device. We say that causal SI is available as future states are not *a priori* known. Thus, this allows us to model and treat the unpredictable nature of the wireless channel and harvesting environment.

The causal SI is used to decide the amount of energy T_k for transmitting packet k . We want to maximize the throughput, i.e., the expected mutual information summed over a finite horizon of K time slots, by choosing a deterministic power allocation policy $\pi = \{T_k(s_k), \forall s_k, k = 1, \dots, K\}$. The policy can be optimized offline and implemented in real time via a lookup table that is stored at the transmitter.

A policy is feasible if the energy harvesting constraints $0 \leq T_k(s_k) \leq B_k$ is satisfied for all possible s^k and all $k \in \mathcal{K}$; we denote the space of all feasible policies as Π . Mathematically, given s_1 , the maximum throughput is

$$\mathcal{T}^* = \max_{\pi \in \Pi} \mathcal{T}(\pi), \quad (5)$$

where

$$\mathcal{T}(\pi) = \sum_{k=1}^K \mathbb{E} [I(\gamma_k, T_k(s_k)) | s_1, \pi]. \quad (6)$$

In (6), the k th summation term represents the throughput of packet k (after expectation); its expectation is performed over all (relevant) random variables given initial state s_1 and policy π .

For example, if $K = 2$ and a given policy, (6) simplifies as

$$\mathcal{T} = I(\gamma_1, T_1(s_1)) + \mathbb{E}_{s_2} [I(\gamma_2, T_2(s_2)) | s_1] \quad (7)$$

²It can be shown that having knowledge of previous states s^{k-1} does not improve throughput, due to the Markovian property of the states in (4).

subject to $0 \leq T_1 \leq B_1$ for the first term and $0 \leq T_2 \leq B_2 = \min\{B_1 - T_1 + H_1, B_{\max}\}$ for the second term. Clearly, the transmission energy T_1 in the first slot affects the stored energy B_2 available in the second slot, which in turn affects the energy T_2 to be allocated.

In general the optimization of $\{T_k\}$ cannot be performed independently due to the energy harvesting constraints, as shown also in the above example. Instead, for the above example, we can first optimize T_2 given all possible T_1 (and hence all possible B_2), then optimize for T_1 with T_2 replaced by the optimized value (as a function of T_1). This approach, as will be suggested by dynamic programming in the general case, will be shown to be optimal.

B. Optimal Solution

The optimization problem (5) is solved by dynamic programming in Lemma 1.

Lemma 1: Given initial state $\mathbf{s}_1 = (\gamma_1, H_0, B_1)$, the maximum throughput \mathcal{T}^* is given by $J_1(\mathbf{s}_1)$, which can be computed recursively based on Bellman's equations, starting from $J_K(\mathbf{s}_K)$, $J_{K-1}(\mathbf{s}_{K-1})$, and so on until $J_1(\mathbf{s}_1)$:

$$J_K(\gamma, H, B) = \max_{0 \leq T \leq B} I(\gamma, T) = I(\gamma, B), \quad (8a)$$

$$J_k(\gamma, H, B) = \max_{0 \leq T \leq B} I(\gamma, T) + \bar{J}_{k+1}(\gamma, H, B - T) \quad (8b)$$

for $k = 1, \dots, K - 1$, where

$$\bar{J}_{k+1}(\gamma, H, x) = \mathbb{E}_{\tilde{H}, \tilde{\gamma}} \left[J_{k+1}(\tilde{\gamma}, \tilde{H}, \min\{B_{\max}, x + \tilde{H}\}) | \gamma, H \right]. \quad (9)$$

In (9), \tilde{H} denotes the harvested energy in the present slot given the harvested energy H in the past slot, and $\tilde{\gamma}$ denotes the SNR in the next slot given the SNR γ in the present slot. An optimal policy is $\pi^* = \{T_k^*(\mathbf{s}_k), \forall \mathbf{s}_k, k = 1, \dots, K\}$, where $T_k^*(\mathbf{s}_k)$ is the optimal T that solves (8).

Proof: The proof follows by applying Bellman's equation [1] and using (2) and (3). ■

In (8a), the optimal maximization is trivial: the interpretation is that we use all available energy for transmission in slot K . We can interpret the maximization in (8b) as a tradeoff between the present and future rewards. This is because the mutual information $I(\cdot, \cdot)$ represents

the present reward, while \bar{J}_{k+1} , commonly known as the value function, is the expected future mutual information accumulated from slot $k + 1$ until slot K .

Next, we obtain structural properties of the maximum throughput \mathcal{T}^* in (5) and the corresponding optimal policy π^* in Theorems 1, 2. The proofs are given in the Appendix.

Theorem 1: Suppose $I(\gamma, T)$ is concave in T given γ . Given γ and H , then

- 1) $J_k(\gamma, H, B)$ in (8) is concave in B for $k \in \mathcal{K}$;
- 2) $\bar{J}_k(\gamma, H, x)$ in (9) is concave in x for $k \in \mathcal{K}$.

Thus, $\mathcal{T}^* = J_1(s_1)$ is concave in B_1 .

Theorem 2: Suppose $I(\gamma, T)$ is concave in T given γ . Given γ and H , then the optimal power allocation $T_k^*(\gamma, H, B)$ that solves (8) is non-decreasing in B , where $k \in \mathcal{K}$.

The structural properties in Theorems 1 and 2 simplify the numerical computation of the optimal power allocation solution in Lemma 1, as shown in the next section.

C. Numerical Computations

From (8a), we get the optimal solution for slot K as $T_K^*(s_K) = B_K$. Now, consider the problem of finding the optimal $T_k^*(s_k)$ to obtain $J_k(s_k)$, $k \in \{1, \dots, K - 1\}$. Let us fix the SNR and harvested energy as γ, H , respectively, and drop these arguments when possible to simplify notations. Consider the *unconstrained maximization* over all $T \geq 0$, i.e., not subject to any energy harvesting constraint:

$$T_k^\dagger = \arg \max_{T \geq 0} g(T) \quad (10)$$

where we denote $g(T) = I(\gamma, T) + \bar{J}_{k+1}(B - T)$. Since $I(\gamma, T)$ is concave, and $\bar{J}_{k+1}(B - T)$ is concave due to Theorem 1, the objective function $g(T)$ is concave. Thus, the maximization over all T gives a unique solution T_k^\dagger , easily solved using numerical techniques such as a bisection search [9]. Also, Theorem 2 helps to reduce the search space by restricting the search to be in one direction for different B . Alternatively, if $g(T)$ is differentiable and available in closed-form,

T_k^\dagger is given by solving $g'(T) = 0$. Finally, we get the optimal solution for (8b) by restricting the maximization in (10) to be over $0 \leq T \leq B$ to give

$$T_k^* = \begin{cases} 0, & T_k^\dagger \leq 0; \\ B, & T_k^\dagger \geq B; \\ T_k^\dagger, & 0 < T_k^\dagger < B. \end{cases} \quad (11)$$

This is because if $T_k^\dagger \leq 0$, the (concave) objective function $g(T)$ must be decreasing for $T \geq 0$; if $T_k^\dagger \geq B$, the objective function must be increasing for $T \leq B$.

D. I.I.D. SNR and Harvested Energy

We consider the *i.i.d.* SI scenario where both γ_k and H_k are i.i.d. over k for analytical tractability. Even with i.i.d. SI, the optimization problem in Lemma 1 is not decoupled as it still depends on the past harvested energy H_{k-1} . Intuitively, this is because the present transmission energy T_k (whose maximum allowable depends on H_{k-1}) will still affect the future storage energy B_{k+1}, B_{k+2}, \dots .

If we assume a *Rayleigh fading channel* with expected SNR given by $\bar{\gamma}$, i.e., the statistic of the SNR is $p_\gamma(\gamma) = 1/\bar{\gamma} \exp(\gamma/\bar{\gamma}), \gamma \geq 0$, the expected mutual information evaluates as

$$\bar{I}(T) \triangleq \mathbb{E}_\gamma[I(\gamma, T)] = \exp\left(\frac{1}{\bar{\gamma}T}\right) \text{E}_1\left(\frac{1}{\bar{\gamma}T}\right) \quad (12)$$

where the exponential integral is defined as $\text{E}_1(x) = \int_x^\infty \exp(-t)/t \, dt$. Instead, if we assume an *AWGN channel* where the channel is time-invariant with $\gamma_k = \bar{\gamma}$ for all k , then the expected mutual information is simply

$$\bar{I}(T) = I(\bar{\gamma}, T) = \log(1 + \bar{\gamma}T). \quad (13)$$

In both Rayleigh fading and AWGN channels, by inspection $J_k(\gamma, H, B)$ in Lemma 1 is also independent of γ for all k , but still dependent on H . Hence, the optimization problem for each $J_k(\gamma, H, B)$ still has to be solved recursively, rather than as decoupled optimization problems.

IV. FULL SIDE INFORMATION: ARBITRARY B_{\max}

The initial battery energy B_1 is always known by the transmitter. We say that full SI is available if the transmitter also has priori knowledge of the harvest power H^{K-1} and SNR γ^K before any transmission begins. This corresponds to the ideal case of a predictable environment where the harvest power and channel SNR are both known in advance, and also gives an upper bound to the maximum throughput \mathcal{T}^* for any distribution (3).

In this section, we consider the general case where B_{\max} may be finite. Corollary 1, as a consequence of Lemma 1, gives the optimal throughput \mathcal{T}^* for the same problem (5) but with full SI available.

Corollary 1: Given full SI $\{H^{K-1}, \gamma^K\}$, the maximum throughput is given by

$$J_1(B_1) = \max_{\pi \in \Pi} \sum_{k=1}^K I(\gamma_k, T_k), \quad (14)$$

which can be computed recursively based on Bellman's equations:

$$J_K(B) = \max_{0 \leq T \leq B} I(\gamma_K, T) = I(\gamma_K, B), \quad (15a)$$

$$J_k(B) = \max_{0 \leq T \leq B} I(\gamma_k, T) + J_{k+1}(\min\{B_{\max}, B - T + H_k\}) \quad (15b)$$

for $k = 1, \dots, K - 1$.

Proof: All side information are *a priori* known and hence the SI is deterministic rather than random. Corollary 1 thus follows immediately from Lemma 1, by replacing the pdfs in (4) by Dirac delta functions accordingly. ■

In general, power may be allocated via these modes:

- *greedy* (G): use all stored energy whenever available;
- *conservative* (C): save as much stored energy as possible (without wasting any harvested energy) to the last slot;
- *balanced* (B): stored energy is traded among slots accordingly to channel conditions.

For the last slot, or if $K = 1$ where there is only one slot, from (15a) it is optimal to allocate all power for transmission. For the case $K = 2$, Corollary 2 obtains the optimal power allocation

for the first slot. The proof is given in the Appendix.

Corollary 2: Consider $K = 2$ slots. Suppose the mutual information function is given by (1). Given full SI $\{B_1, H_1, \gamma_1, \gamma_2\}$, the optimal transmission energy for slot 1 is given by (corresponding to the G, B, C modes, respectively)

$$T_1^* = \begin{cases} B_1, & a < 0 \text{ or } B_1 < b; \\ \tilde{T}, & a \geq 0 \text{ and } |b| \leq B_1 \leq c; \\ B_1 - a, & a \geq 0 \text{ and } (B_1 > c \text{ or } B_1 < -b); \end{cases}$$

where

$$\tilde{T} = B_1/2 + (1/\gamma_2 - 1/\gamma_1 + H_1)/2 \quad (16)$$

and we denote $a = B_{\max} - H_1$, $b = H_1 + 1/\gamma_2 - 1/\gamma_1$ and $c = 2B_{\max} - H_1 + 1/\gamma_2 - 1/\gamma_1$.

In Corollary 2, the power allocation is interpreted to be in G, B, or C mode. For example, consider $b > 0$. Then all modes can be active: power allocation is greedy if the energy to be harvested is large or the stored energy is small ($a < 0$ or $B_1 < b$); power allocation is conservative if the energy to be harvested is small *and* the stored energy is large ($a \geq 0$ and $B_1 > c$); otherwise, the allocation depends on the SI.

Remark 1: From Corollary 2, $T_1^*(B_1)$ is a piece-wise linear function of B_1 . We also see that $T_1^*(B_1)$ is increasing in B_1 , as predicted in Theorem 2.

Although we can derive a closed-form result for larger K , the expression becomes unwieldy and less intuitive. However, if $B_{\max} \rightarrow \infty$, for any K we can obtain a closed-form result that is a variation of the water-filling power allocation policy [5], to be presented next.

V. FULL SIDE INFORMATION: INFINITE B_{\max}

The previous section considers the general case of arbitrary B_{\max} . To develop more insights, in this section we consider that the mutual information function is given by (1). Moreover, we consider the special case where $B_{\max} \rightarrow \infty$. Then from (2), the battery stored at slot $k + 1$,

where $k \in \mathcal{K}$, is given by

$$B_{k+1} = B_1 - \sum_{i=1}^k T_i + \sum_{i=1}^k H_i. \quad (17)$$

A non-negative power allocation is feasible if and only if $B_{k+1} \geq 0, k \in \mathcal{K}$. The throughput maximization problem solved in Corollary 1 can then be formulated as follows:

$$\mathcal{T}^* = \max_{\{T_k \geq 0, k \in \mathcal{K}\}} \sum_{k=1}^K I(\gamma_k, T_k) \quad (18a)$$

$$\text{subject to } \sum_{i=1}^k T_i - B_1 - \sum_{i=1}^{k-1} H_i \leq 0, \quad k \in \mathcal{K}. \quad (18b)$$

A. Water-Filling Algorithm

Before we consider the general case where the constraint (18b) is imposed for all $k \in \mathcal{K}$, we impose the constraint (18b) only for the last slot, i.e., only for $k = K$. This then corresponds to the conventional problem of maximizing the sum throughput with a *sum* energy constraint of $P_{\max} = B_1 + \sum_{i=1}^{K-1} H_i$:

$$\mathcal{T}_{\text{WF}}^*(\gamma^K, P_{\max}) = \max_{\{T_k \geq 0, k \in \mathcal{K}\}} \sum_{k=1}^K I(\gamma_k, T_k) \quad (19a)$$

$$\text{subject to } \sum_{i=1}^K T_i \leq P_{\max}. \quad (19b)$$

Since less constraints are imposed, the maximum throughput in (19) is no smaller than that of (18). It is well known that the optimal solution for (19) is given by (see e.g. [6], [9])

$$T_{\text{WF},k}^* = \left[\nu - \frac{1}{\gamma_k} \right]^+ \quad (20)$$

where $[x]^+ \triangleq \max(0, x)$. This optimal solution is implemented by the *water-filling algorithm*, where the *water-level* (WL) $\nu \geq 0$ is chosen such that (19b) holds with equality by using the optimal power allocation in (20). For completeness, an implementation of the *water-filling algorithm*, which gives the maximum \mathcal{T} to within a tolerance of ϵ , is given in Algorithm 1.

```

input : slot size  $K$ , SNRs  $\{\gamma_k\}$ , power constraint  $P_{\max}$ , tolerance  $\epsilon$  (close to zero)
output: optimal power allocation  $\{T_{\text{WF},k}^*\}$ , optimal WL  $\lambda^*$ 

// initialization
 $P := 0$ ,  $\lambda^{\text{lo}} := 0$ ,  $\lambda^{\text{hi}} := \infty$  (a large number) ;
 $T_k^\theta := [\lambda^\theta - 1/\gamma_k]^+$ ,  $\theta \in \{\text{lo}, \text{hi}\}$ ,  $k \in \mathcal{K}$  ;

// loop until sum power  $P$  less than  $P_{\max}$  to within tolerance  $\epsilon$ 
while  $|P_{\max} - P| > \epsilon$  or  $P > P_{\max}$  do
    // improve  $P$  to be closer to  $P_{\max}$ 
     $\lambda := (\lambda^{\text{lo}} + \lambda^{\text{hi}})/2$  ;
     $T_k := [\lambda - 1/\gamma_k]^+$ ,  $k \in \mathcal{K}$  ;
     $P := \sum_{k=1}^K T_k$  ;
    // update  $\lambda_{\text{lo}}$  or  $\lambda_{\text{hi}}$ 
    if  $P > P_{\max}$  then
        |  $\lambda^{\text{hi}} := \lambda$  ;
    else
        |  $\lambda^{\text{lo}} := \lambda$  ;
    end
end
 $T_{\text{WF},k}^* := T_k$  ;
 $\lambda^* := \lambda$  ;

```

Algorithm 1: Conventional water-filling algorithm. This implementation achieves optimality to a tolerance of ϵ .

B. Staircase Water-Filling Algorithm

We now proceed to solve our original problem (18) with additional energy harvesting constraints in (18b). It turns that the water-filling algorithm is no longer optimal. Instead it is necessary to use a generalized type of water-filling where the water level is a staircase-like function.

1) *Structural Properties:* The optimization problem in (18) is convex and so can be solved by the dual problem [9]. The Lagrangian associated to the primal problem (18) is

$$\mathcal{L}(\lambda^K, T^K) = \mathcal{T}(T^K) - \sum_{k=1}^K \lambda_k \cdot \left(\sum_{i=1}^k T_i - B_1 - \sum_{i=1}^{k-1} H_i \right)$$

where $T_k \geq 0$ is the power allocation for the k th slot and $\lambda_k \geq 0$ is the Lagrangian multiplier for the k th constraint in (18b), $k \in \mathcal{K}$. Then the necessary and sufficient conditions for λ^K and T^K to be both primal and dual optimal are given by the Karush-Kuhn-Tucker (KKT) optimality

conditions:

$$\sum_{i=1}^k T_i - B_1 - \sum_{i=1}^{k-1} H_i \leq 0, \quad (21a)$$

$$T_k \geq 0, \quad (21b)$$

$$\lambda_k \geq 0, \quad (21c)$$

$$\lambda_k \left(\sum_{i=1}^k T_i - B_1 - \sum_{i=1}^{k-1} H_i \right) = 0, \quad (21d)$$

$$\left. \frac{\partial \mathcal{L}(\lambda^K, T^K)}{\partial T_k} \right|_{\lambda_k = \lambda_k, T_k = T_k, k \in \mathcal{K}} = 0, \quad (21e)$$

for $k \in \mathcal{K}$. From (21b), and imposing the constraints (21c) and (21e) via similar arguments to obtain (20) in [6], [9], we obtain the optimal power allocation as

$$T_k^* = \left[\nu_k - \frac{1}{\gamma_k} \right]^+ \quad (22)$$

for $k \in \mathcal{K}$, where $\nu_k \triangleq \left(\ln 2 \sum_{i=k}^K \lambda_i \right)^{-1} \geq 0$.

Analogous to the problem in (19) with only power constraint (19b), we say ν_k is the WL for slot k . Also, we say slot $t \in \mathcal{K}$ is a *transition slot* (TS) if the water level changes *after* slot t , i.e., $\nu_t \neq \nu_{t+1}$. We define the last slot $k = K$ also as a TS (say by defining ν_{K+1} to be infinity); hence there is at least one TS. We collect all TSs as the set $\mathcal{S} = \{t_1, t_2, \dots, t_{|\mathcal{S}|}\}$, where $t_i < t_j$ for $i < j$ and so $t_{|\mathcal{S}|} = K$. Fig. 2 gives an example of the optimal power allocation. In general the optimal WLs $\{\nu_k\}$ depend on the slot indices and there can be multiple optimal TSs, while in the conventional water-filling algorithm, the optimal WL is the same for all slot indices and thus there is no TS.

From the result in (22), we obtain the following structural properties for the optimal power allocation.

Theorem 3: The optimal power allocation in (22) satisfy these properties:

- P1: The WL is non-decreasing over slots, i.e., $\nu_1 \leq \dots \leq \nu_K$. We say that the optimal power allocation performs *staircase water-filling* over slots, since the WL is a staircase-like function (see e.g. Fig. 2).

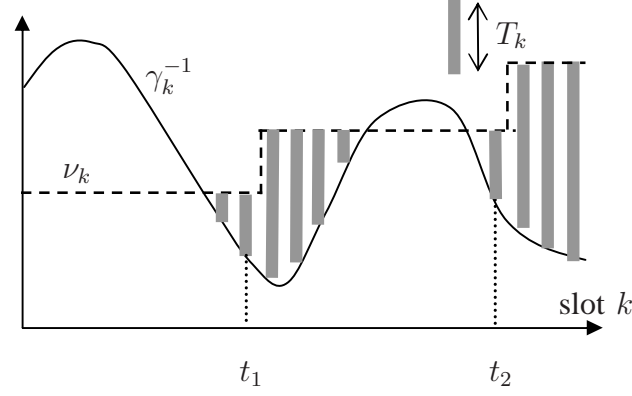


Fig. 2. Structure of optimal power allocation T_k^* with full SI and infinite B_{\max} . We assume two optimal TSs and hence three distinct water levels for ν_k . Here, the SNRs $\{\gamma_k\}$ are arbitrary.

P2: If slot t is a TS, then the battery storage is empty, i.e., (21a) holds with equality if $k \in \mathcal{S}$.

Proof: Since $\lambda_k \geq 0$, it follows that $\nu_k \geq 0$ and also that ν_k is non-decreasing with k . This proves property P1.

Suppose slot t is an TS, i.e., $t \in \mathcal{S}$ and so $\nu_t \neq \nu_{t+1}$. By definition $\nu_t = 1/\ln 2 \sum_{i=t}^K \lambda_i$, thus $\lambda_t \neq 0$. From (21c), we get $\lambda_t > 0$. It then follows from the complementary slackness condition (21d) (with k replaced by t) that (21a) holds with equality for $k = t$. This proves property P2. ■

From Theorem 3, we have the following further structural properties in Corollary 3 and Corollary 4.

Corollary 3: If the SNR is non-decreasing over slots, then the optimal power allocation is non-decreasing over slots.

Proof: This follows immediately from (22) and property P1, which implies that $T_l \leq T_k$ if $\gamma_l \leq \gamma_k$ for $l < k$. ■

An example that illustrates Corollary 3 is given in Fig. 3, where we see that the inverse of the SNR is non-increasing. It is easy to see that the converse of Corollary 3 is not true in general. That is, if the SNR is non-increasing, then the optimal power allocation may not be non-increasing over slots (in particular for the slot immediately after the TS). In conventional

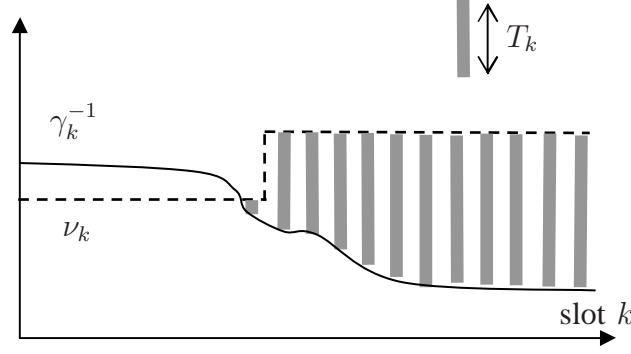


Fig. 3. Structure of optimal power allocation T_k with full SI and infinite B_{\max} , with increasing SNR γ_k , i.e, decreasing γ_k^{-1} , over slot k . In this case, T_k^* must increase over slot k .

water-filling, however, both Corollary 3 and its converse hold, i.e., the optimal power allocation is non-decreasing over slots *if and only if* the SNR is non-decreasing over slots.

We give an intuitive understanding of Corollary 3, and why the converse does not hold. First, let us consider the AWGN channel where the SNR is constant over slots. If all the harvested energy is already available in the first slot, i.e., there is only a single sum-power constraint, a uniform power allocation is optimal for an AWGN channel. However, in an energy harvesting system, maintaining a uniform power allocation may not be always possible due to the causal arrival of the harvested energy. Due to this non-uniform availability of harvested energy over slots, more energy only becomes available for transmission in the latter slots. Intuitively, we also expect more energy to be allocated for transmission in the latter slots such that the energy harvesting constraints in (18b) are satisfied. This strategy is optimal from Corollary 3, which applies since the SNR is constant and hence also non-decreasing. Next, consider the case where the SNR is non-decreasing over slots. From the water-filling algorithm under a single sum-power constraint, to achieve the maximum throughput it is optimal to allocate more power to the latter slots that have higher SNRs. This is consistent with the earlier observation that more power should be allocated to the latter slots such that the constraints in (18b) are satisfied. Hence, it is also optimal to allocate more power to the latter slots. In general if the SNR is arbitrary,

however, the high-SNR slots may not correspond to the latter slots; hence intuitively the converse of Corollary 3 may not hold in general.

2) *Efficient Implementation:* Based on the structural properties, we now develop an efficient algorithm to implement the staircase water-filling. Some definitions are in order. For convenience, let $t_0 = 0$. We refer to the i th slot interval as the slots in the set $\tilde{\mathcal{S}}_i \triangleq \{t_{i-1} + 1, \dots, t_i\}$, where $i = 1, \dots, |\mathcal{S}|$. Thus, $\bigcup_i \tilde{\mathcal{S}}_i = \mathcal{K}$ and $\tilde{\mathcal{S}}_i \cap \tilde{\mathcal{S}}_j = \emptyset$ for $i \neq j$. The optimal set of TSs corresponding to an optimal power allocation is denoted as $\mathcal{S}^* = \{t_1^*, t_2^*, \dots, t_{|\mathcal{S}^*|}^*\}$

Corollary 4: The optimal power allocation performs staircase water-filling as follows: for every i th slot interval, where $i = 1, \dots, |\mathcal{S}^*|$, conventional water-filling is performed subject to the sum power constraint of $P(i) \triangleq \sum_{k \in \tilde{\mathcal{S}}_i} H_{k-1}$, where we denote $H_0 = B_1$ for notational simplicity.

Proof: From property P2, all the harvested energy available in the i th slot interval, namely $P(i)$, is used during the i th slot interval. This follows by induction for $i = 1, \dots, |\mathcal{S}|$. Moreover, the optimal power allocation in (22) is equivalent to conventional water-filling. To maximize throughput, the optimal power allocation must then be to use conventional water-filling with sum power constraint of $P(i)$ for every i th slot interval. ■

From Corollary 4, the original optimization problem (18) can be reduced to searching for the optimal \mathcal{S}^* that has a size from 1 to at most K :

$$\begin{aligned} \mathcal{T}^* = \max_{1 \leq |\mathcal{S}| \leq K} \max_{\mathcal{S}} & \mathcal{T}_{\text{WF}}^*(\gamma_1^{t_1^*}, P(1)) + \mathcal{T}_{\text{WF}}^*(\gamma_{t_1^*+1}^{t_2^*}, P(2)) \\ & + \dots + \mathcal{T}_{\text{WF}}^*(\gamma_{t_{|\mathcal{S}^*|}-1}^{t_{|\mathcal{S}^*|}^*}, P(|\mathcal{S}|)) \end{aligned} \quad (23)$$

subject to the power allocation $P(1), \dots, P(|\mathcal{S}|)$ satisfying the constraints in (18b). A brute force search based on (23) is of a high computational complexity, since the size $|\mathcal{S}^*|$ of the optimal set of TSs is not known in advance. Nevertheless, it turns out that it is optimal to simply employ a forward-search procedure, starting with the search of the optimal t_1^* , then of the optimal t_2^* , and so on until the last optimal TS $t_{|\mathcal{S}^*|}^*$ equals K , at which point the optimal size $|\mathcal{S}^*|$ is also obtained.

The first optimal TS t_1^* can be found in Lemma 2; by induction, the search of the subsequent optimal TSs will follow similarly. Lemma 2 requires the following *feasible-search procedure* for a given optimization problem (18):

- 1) Initialize \mathcal{S}_1 as an empty set.
- 2) For $t_1 = 1, \dots, K$, obtain the optimal power allocation from slot 1 to slot t_1 by using a water-filling algorithm (such as Algorithm 1) assuming that all harvested energy is available, i.e., the sum power constraint is $P_{\max} = B_1 + \sum_{i=1}^{t_1-1} H_i$.
- 3) Admit t_1 in the set \mathcal{S}_1 if the corresponding optimal power allocation satisfies the constraint (18b) for $k = 1, \dots, t_1$.

The set \mathcal{S}_1 is non-empty; it contains at least the element $t_1 = 1$, as the constraint (18b) in Step 3 is equivalent to the sum power constraint in Step 2. Moreover, the set \mathcal{S}_1 includes all possible candidates for the optimal t_1^* ; the only candidates that are not included are those where at least one of the constraint in (18b) is not satisfied for slot $k = 1, \dots, t_1^*$.

Lemma 2: Let \mathcal{S}_1 be the feasible set of t_1 obtained by the feasible-search procedure. Then the optimal TS is given by the largest element in \mathcal{S}_1 , i.e.,

$$t_1^* = \max_{t_1 \in \mathcal{S}_1} t_1. \quad (24)$$

Proof: If $|\mathcal{S}_1| = 1$, then that only t_1 must be optimal. Henceforth, assume $|\mathcal{S}_1| \geq 2$. Consider two TSs $t', t'' \in \mathcal{S}_1$, where $t' < t''$. Denote their respective optimal WLs obtained from the water-filling algorithm as ν', ν'' . Then $\nu' \geq \nu''$. Otherwise if $\nu' < \nu''$, more power is allocated for time slot $k = 1 \dots, t'$ with water level ν'' than with water level ν' . But since the power allocation with water level ν' has already used all available power, the power allocation with water level ν'' is infeasible and thus cannot be optimal.

We now show that t' cannot be the optimal t_1^* by contradiction. Suppose $t_1^* = t'$, i.e., water-filling is used from slot 1 to slot t' with WL ν' . The WL for the power allocation must then subsequently decrease at some slot $t' < k \leq t''$, otherwise the sum power allocated from slots 1 to slot t'' will be more than the power allocated with the (constant) WL ν'' , which violates the

sum power constraint. But from property P1, the optimal WL is non-decreasing. Thus, $t_1^* \neq t'$ by contradiction. By induction, all elements in \mathcal{S}_1 , except for the largest one, is suboptimal. The only candidate left, namely the largest element, must then be optimal. ■

We now propose Algorithm 2 to solve (23), which is optimal according to Theorem 4.

```

input : slot size  $K$ ; SNRs  $\{\gamma_k\}$ ; harvested power  $\{H_k\}$  where we let  $H_0 = B_1$ ; tolerance  $\epsilon$ 
output: optimal set of TSs  $\mathcal{S}^* = \{t_i^*\}$ 

// initialization
 $t_0 = 0$ ;
for  $i = 1, 2, \dots, K$  do
    // Outer iteration: find  $t_1^*$ , then  $t_2^*$ , and so on
    for  $k = K, K-1, \dots, t_{i-1}+1$  do
        // Inner iteration: find the largest feasible  $t_i$  in (24)
        Use Algorithm 1 for slot  $t_{i-1}+1$  to slot  $k$  with inputs
        (i) SNRs  $\{\gamma_{t_{i-1}+1}, \dots, \gamma_k\}$ 
        (ii)  $P_{\max} = \sum_{i=t_{i-1}}^{k-1} H_i$ 
        (iii) tolerance  $\epsilon$ ,
        to give output  $\{T_{t_{i-1}+1}^*, \dots, T_k^*\}$ ;
        if  $\{T_1^*, \dots, T_k^*\}$  satisfy the constraints (18b) then
            |  $t_i^* := k$ ;
        end
    end
    if  $t_i^* = K$  then
        | exit;
    end
end

```

Algorithm 2: Finding optimal TSs

Theorem 4: Algorithm 2 obtains the optimal \mathcal{S}^* that solves the optimization problem in (23).

Proof: The i th outer iteration of Algorithm 2 finds the optimal t_i^* . Consider $i = 1$. From Lemma 2, we can determine the optimal t_1^* by finding the largest feasible t_1 . Without loss of optimality, we can modify the feasible-search procedure such that the search in Step 2 starts from the largest slot index to the smallest, and Step 3 terminates to give the optimal t_1^* once a feasible t_1 is found. These modifications lead to the inner iteration in Algorithm 2.

From Property P2 in Theorem 3, in the first slot interval all power available would be used. Since no power is available for subsequent slot intervals given t_1^* , the power allocation for subsequent slot intervals can be optimized independently of the actual power allocated in the first slot interval. The throughput maximization problem from slot $t_1^* + 1$ onwards can be solved similarly as before (with the i th time slot removed). Thus, we apply the inner iteration again to determine t_2^* , similarly for t_3^* and so on, as reflected in the outer iteration of Algorithm 2. The iteration ends if the optimal TS equals K , which is the largest possible value as stated in optimization problem in (23). ■

3) *Update Algorithm when New Slots Become Available:* Suppose that we have obtained the optimal \mathcal{S}^* based on Algorithm 2 for a K -slot system. Now, a new slot becomes available for our use where its SI is known. We wish to obtain the new optimal solution for this $(K + 1)$ -slot system, say $\mathcal{S}_{\text{new}}^* = \{t_{1,\text{new}}^*, t_{2,\text{new}}^*, \dots\}$. Instead of implementing Algorithm 2 afresh, we can obtain an update of $\mathcal{S}_{\text{new}}^*$ from \mathcal{S}^* as follows:

- Consider $i = 1$ in the outer iteration of Algorithm 2. We only need to execute $k = K + 1$ in the inner iteration. If the constraints (18b) are satisfied, then we have obtained $t_{1,\text{new}}^* = K + 1$ and hence $\mathcal{S}_{\text{new}}^* = \{t_{1,\text{new}}^*\}$ and Algorithm 2 terminates. Otherwise, since we already know that t_1^* is the largest element in \mathcal{S}^* , we obtain immediately $t_{1,\text{new}}^* = t_1^*$.
- The subsequent iterations are executed similarly. For the i th outer iteration, where $i = 2, 3, \dots$, we only execute $k = K + 1$ in the inner iteration. If the constraints (18b) are satisfied, then $t_{i,\text{new}}^* = K + 1$ and Algorithm 2 terminates; otherwise $t_{i,\text{new}}^* = t_i^*$.

Hence, for every outer iteration, only one inner iteration is executed until the constraints (18b) are satisfied. Since there are at most K outer iterations, we need to execute at most K inner iterations in total. In cases when the number of available slots can increase dynamically in a multi-user system, say when other users give up their slots and is assigned to our energy harvesting system, the above proposed update algorithm allows an efficient way to update \mathcal{S}^* .

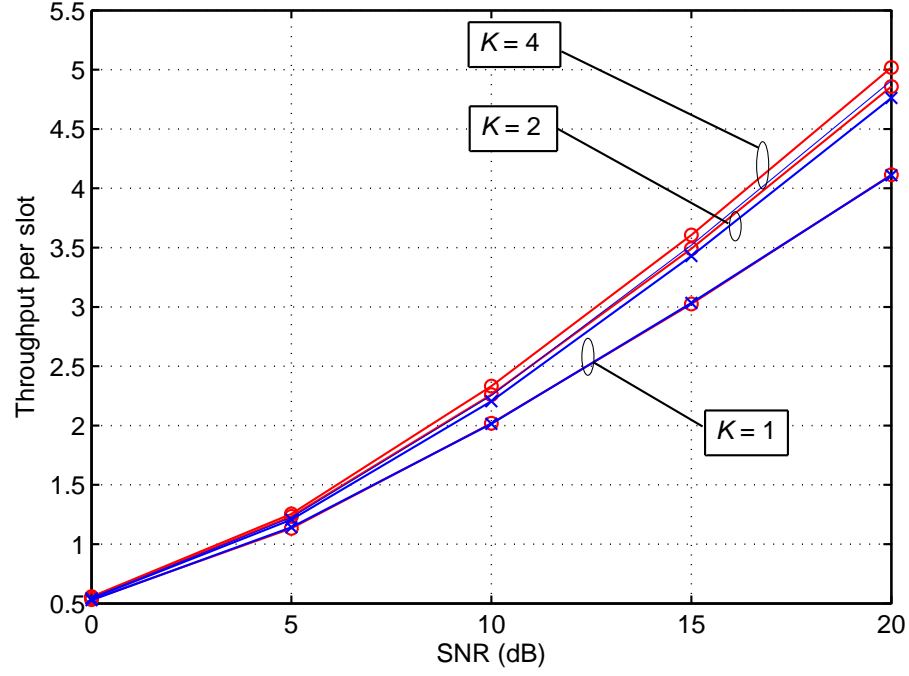


Fig. 4. AWGN channel: optimal throughput when causal SI (blue with “x” markers) or full SI (red with “o” markers) is available for $K = 1, 2, 4$.

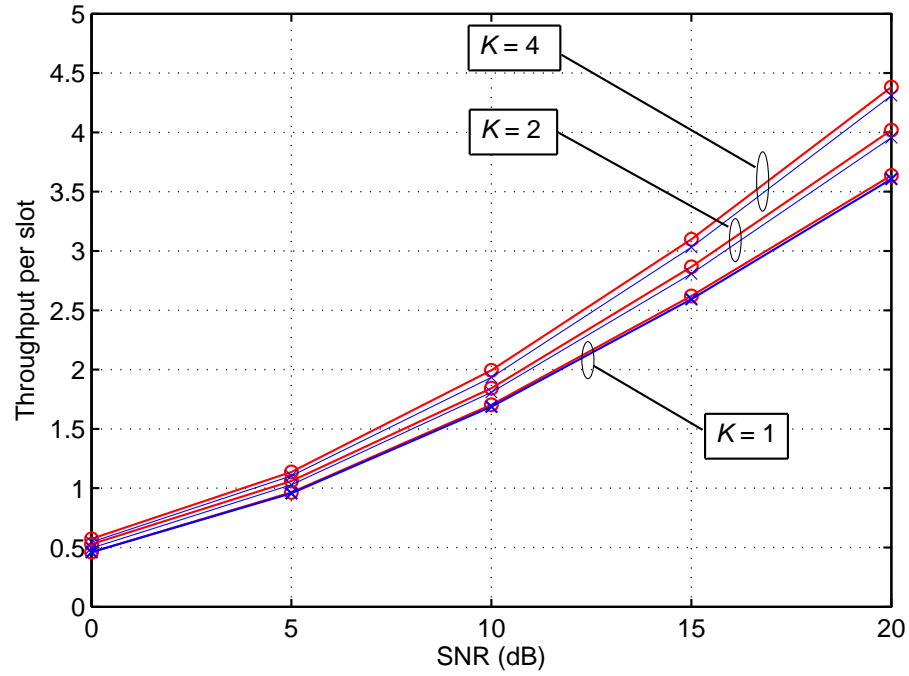


Fig. 5. Fading channel: optimal throughput when causal SI (blue with “x” markers) or full SI (red with “o” markers) is available for $K = 1, 2, 4$.

VI. NUMERICAL RESULTS

To obtain numerical results, we assume the SNR γ_k and the harvested energy H_k are i.i.d. over time slot k and the channel is either an AWGN or Rayleigh fading channel, as described in Section III-D. We assume H_k to take values in $\{0, 0.5, 1\}$ with equal probability and $B_{\max} \rightarrow \infty$. To measure the performance of various schemes, we plotted the throughput per slot, i.e., the sum throughput divided by the number of slots K , as the average SNR $\bar{\gamma}$ is increased.

If causal SI is available, the optimal policy is obtained recursively by applying Lemma 1. Specifically, we first obtain $\bar{J}_K(B)$ in (9) via the closed-form results in Section III-D; we drop the γ and H arguments due to the i.i.d. assumption. Then we obtain $J_{K-1}(\gamma, B)$ in (8b), say by an iterative bisection method. The throughput are averaged over 10^4 independent realizations of γ and H to give $\bar{J}_{K-1}(B)$. This procedure is performed for different B , discretized in step size of 0.01, and stored to be used for the next recursion. The iteration is repeated for $k = K-2, \dots, 1$. If instead full SI is available, the optimal policy is obtained by Algorithm 2; we have verified that our proposed algorithm is significantly faster but is equivalent to solving the problem via a standard optimization software. The throughput per slot is obtained from averaging the results from 10^4 independent Monte Carlo runs.

The results are shown in Fig 4 for AWGN channels and in Fig. 5 for Rayleigh fading channels. The throughput in both cases, when either full SI or causal SI is available, is the same for $K = 1$, because any SI cannot be exploited for future slots. However, in both cases the throughput per slot increases as K increases. The increment is more substantial when full SI is available, intuitively because the SI can then be much better exploited. The incremental improvement as K increases is significant when K is small, but becomes smaller when K is large. The throughput with either full SI or causal SI does not differ significantly, possibly because the SI that can be further exploited from full SI is limited in our i.i.d. scenario.

A. Heuristic Schemes with Causal SI

Next, we consider two heuristic schemes that use some causal SI so that it can be implemented in practice, namely the naive scheme and the power-halving scheme.

In the *naive scheme*, all stored energy B_k is used in every slot k , i.e., $T_k = B_k$. This is equivalent to the case of $K = 1$ in our optimization problem regardless of whether causal SI is available (see Lemma 1) or full SI is available (see Theorem 4). In both cases, it is optimal to use all stored energy. As seen earlier, the case of $K = 1$ performs significantly worse than the optimal schemes for $K > 2$ in both cases. To obtain further improvement in the per-slot throughput, we need to further exploit the causal SI available.

In the *power-halving scheme*, all stored power is used for transmission in the last slot, while for all other slots half of the stored energy is used, i.e., $T_k = w_k B_k$ where $w_k = 1$ if $k = K$ and $w_k = 1/2$ otherwise. Although this scheme is simple to implement, we note that the present throughput is traded with the future throughput by splitting the power into two halves by implicitly exploiting causal information of the harvested energy (which accumulates as the stored energy B_k). Moreover, the power-halving scheme satisfies the following characteristics and thus tends to improve the throughput as K increases:

- T_k increases with B_k , in accordance with Theorem 2 in the causal SI case.
- This scheme corresponds to (16) in Corollary 2 by setting the second term to be zero, in the case of full SI where $K = 2$. That is, only the harvested energy that has been accumulated in the battery is used as SI.
- More stored energy is deferred to be used in the latter slots, thus resembling the optimal policy with staircase WLs in the full SI case.

Fig. 6 shows the throughput per slot obtained by averaging the numerical results from 2×10^4 independent runs of Monte Carlo simulations, for both AWGN channels and Rayleigh fading channels. We fix the SNR at 20 dB. As benchmarks, we also plot the optimal throughput when full SI is available. This is because the computational complexity in solving the Bellman's

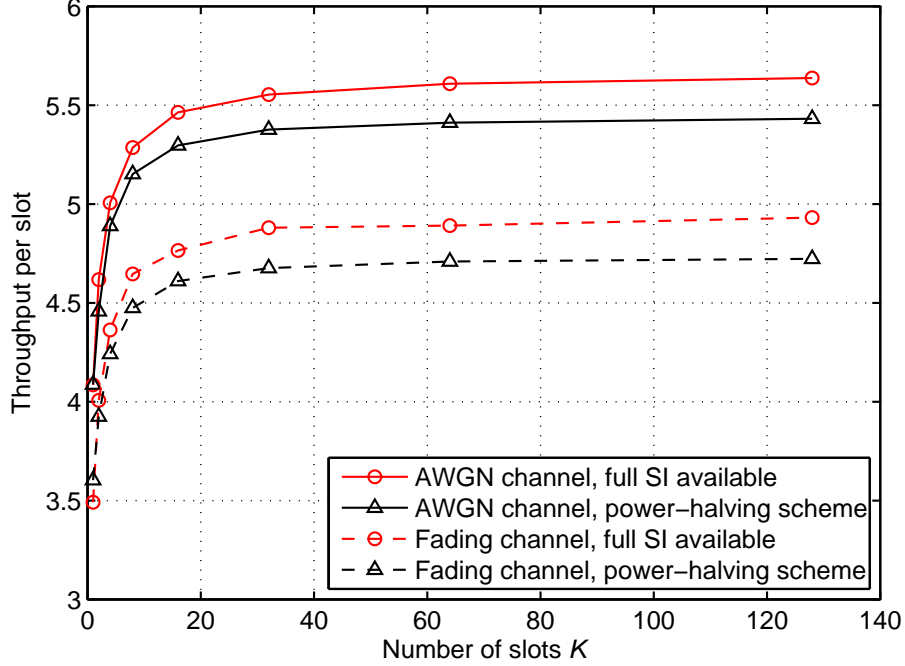


Fig. 6. Throughput based on the power-halving scheme. The optimal throughput when full SI is also plotted for comparison.

equations in Lemma 1 becomes prohibitive for large K ; moreover we observed earlier that the performance with causal SI is close to the performance with full SI. The results show that the power-halving scheme is able to improve on the per-slot throughput as K is increased, and is only within about 0.2 bits away from the case when full SI is available. Similar results are obtained at lower SNR, with an even smaller throughput degradation compared to the case when full SI is available. Further performance gain may also be obtained by optimizing this tradeoff by considering the channel conditions explicitly.

VII. CONCLUSION

We considered a communication system where the energy available for transmission varies from slot to slot, depending on how much energy is harvested from the environment and expended for transmission in the previous slot. We studied the problem of maximizing the throughput via power allocation over a finite horizon of K slots, given either causal SI or full SI. We obtained

structural results for the optimal power allocation in both cases, which allows us to obtain efficient computation of the optimal throughput. Finally, we proposed a heuristic scheme where numerical results show that the throughput per slot increases as K increases and performs relatively well compared to a naive scheme.

APPENDIX A

PROOF OF THEOREM 1

With γ, H fixed, we prove by induction that $J_k(\gamma, H, B)$ and $\bar{J}_{k+1}(\gamma, H, x)$ are concave in B and x , respectively, for decreasing $k = K, \dots, 1$.

Consider $k \in \{1, \dots, K-1\}$. Suppose that $J_{k+1}(\gamma, H, B)$ is concave in B . We note that $J_{k+1}(\gamma, H, \min\{B_{\max}, x + H\})$ is concave in x , as it is the minimum of $J_{k+1}(\gamma, H, B_{\max})$ (a constant independent of x) and the concave function $J_{k+1}(\gamma, H, x + H)$. It follows that $\bar{J}_{k+1}(\gamma, H, x)$ is concave in x , since expectation preserves concavity. From (8b), J_k is a supremal convolution of two concave functions in B , namely I and \bar{J}_{k+1} (with γ, H fixed). It follows that J_k is concave in B , since the infimal convolution of convex functions is convex [10, Theorem 5.4]. To complete the proof by induction, we note that $J_K(\gamma, H, B) = I(\gamma, B)$ is concave in B by assumption on the mutual information function $I(\cdot, \cdot)$.

APPENDIX B

PROOF OF THEOREM 2

We need Lemma 3 to prove Theorem 2.

Lemma 3: Consider $T^*(B) = \arg \max F(B, T)$, where the maximization is over interval $T_l(B) \leq T \leq T_u(B)$ that depends on B . If $T_l(B), T_u(B)$ are non-decreasing in B , and if F has non-decreasing differences in (B, T) , i.e., $\forall T' \geq T, B' \geq B$,

$$F(B', T') - F(B, T') \geq F(B', T) - F(B, T), \quad (25)$$

then the maximal and minimal selections of $T^*(B)$, denoted as $\bar{T}(B), \underline{T}(B)$, are non-decreasing.

Proof: See proof in [11, Theorem 2]. ■

We now prove Theorem 2 with γ, H fixed; we drop these arguments from all functions. From (8a), the optimal transmission power is $T_K^*(B) = B$, which is increasing in B . We now apply Lemma 3 to establish that Theorem 2 hold for $k < K$. Let $F(B, T) = I(T) + \bar{J}_{k+1}(B - T)$, according to (8b). Let $T_l(B) = 0$, $T_u(B) = B$, which are non-decreasing in B . To apply Lemma 3, it is sufficient to show that each term in F has non-decreasing differences in (B, T) . Since $I(T)$ is independent of B , trivially $I(T)$ has non-decreasing differences in (B, T) . To show that $g(B - T) \triangleq \bar{J}_{k+1}(B - T)$ has non-decreasing differences in (B, T) , we note that $g(y + \delta) - g(y) \leq g(x + \delta) - g(x)$ for $x \leq y, \delta \geq 0$, since $g(x) = \bar{J}_{k+1}(x)$ is concave in x from Theorem 1. Substituting $x = B - T', y = B - T, \delta = B' - B$, we then obtain (25) with $F(B, T) = g(B - T)$. From Theorem 1, the objective function in (8) is concave, thus $T^*(B)$ is unique. From Lemma 3, $T^*(B)$ is thus non-decreasing in B , $k \in \mathcal{K}$.

APPENDIX C

PROOF OF COROLLARY 2

Since $K = 2$ and full SI is available, from (1), (15) we get

$$J_1(\gamma_1, B_1) = \max_{0 \leq T \leq B_1} g(T), \quad (26)$$

where the objective function is given by $g(T) \triangleq \log_2(1 + \gamma_1 T) + \log_2(1 + \gamma_2 \min\{B_{\max}, B_1 - T + H_1\})$.

Suppose $H_1 > B_{\max}$. Then $\min\{B_{\max}, B_1 - T + H_1\} = B_{\max}$ given $T \leq B_1$. The optimal T that solves (26) is then

$$T_1^* = B_1 \text{ if } H > B_{\max}. \quad (27)$$

Suppose $H_1 \leq B_{\max}$. Consider $0 \leq T \leq B_1 + H_1 - B_{\max}$. Then $\min\{B_{\max}, B_1 - T + H_1\} = B_{\max}$, and so the optimal T to maximize $g(T)$ subject to $0 \leq T \leq B_1 + H_1 - B_{\max}$ is given by the largest value in the variable space, namely $B_1 + H_1 - B_{\max}$. Thus, in general the optimal

solution in (26) satisfies $T_1^* \geq [B_1 + H_1 - B_{\max}]^+$, where $[x]^+ \triangleq \max(0, x)$. Without loss of generality, we can thus express T_1^* as

$$T_1^* = \arg \max_{[B_1 + H_1 - B_{\max}]^+ \leq T \leq B_1} g(T) \quad (28)$$

if $H_1 \leq B_{\max}$. Now if $T \geq [B_1 + H_1 - B_{\max}]^+$, we have $g(T) = \log_2(1 + \gamma_1 T) + \log_2(1 + \gamma_2(B_1 - T + H_1))$, which is differentiable and concave. Observe that \tilde{T} in (16) solves the equation $g'(\tilde{T}) = 0$, i.e., \tilde{T} is the optimal solution for the *unconstrained* optimization problem $\max g(T)$.

By concavity of $g(T)$, we can then obtain (28) as

$$T_1^* = \arg \max_{[B_1 + H_1 - B_{\max}]^+ \leq T \leq B_1} g(T) \\ = \begin{cases} B_1, & \tilde{T} > B_1; \\ \tilde{T}, & [B_1 + H_1 - B_{\max}]^+ \leq \tilde{T} \leq B_1; \\ B_1 + H_1 - B_{\max}, & \tilde{T} < [B_1 + H_1 - B_{\max}]^+ \end{cases}$$

if $H_1 \leq B_{\max}$. By re-writing the above conditions in terms of B_1 and combining the result with (27), we then obtain T_1^* as stated in Corollary 2.

REFERENCES

- [1] D. P. Bertsekas, *Dynamic Programming and Optimal Control Vol. 1*. Belmont, MA: Athena Scientific, 1995.
- [2] A. Fu, E. Modiano, and J. Tsitsiklis, "Optimal energy allocation and admission control for communications satellites," *IEEE/ACM Trans. Netw.*, vol. 11, no. 3, pp. 488–50, Jun. 2003.
- [3] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1326–1336, Apr. 2010.
- [4] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, "Power management in energy harvesting sensor networks," *ACM Trans. Embed. Comput. Syst.*, vol. 6, no. 4, Sep. 2007.
- [5] A. J. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.
- [6] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. John Wiley & Sons, Inc., 2006.
- [7] P. Sadeghi, R. Kennedy, P. Rapajic, and R. Shams, "Finite-state markov modeling of fading channels - a survey of principles and applications," *IEEE Signal Process. Mag.*, vol. 25, no. 5, pp. 57–80, Sep. 2008.
- [8] C. K. Ho, P. D. Khoa, and P. C. Ming, "Markovian models for harvested energy in wireless communications," in *Proc. IEEE Int. Conf. on Commun. Syst. (ICCS), Singapore, Nov. 17-20 2010*, Singapore, Nov.

- [9] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.
- [10] R. T. Rockafellar, *Convex Analysis*. Princeton University Press, 1970.
- [11] R. Amir, “Supermodularity and complementarity in economics: An elementary survey,” *Southern Economic Journal*, vol. 71, no. 3, pp. 636–660, Jan. 2005.