## 浙江大学 2023 - 2024 学年春夏季学期 《离散数学》课程期中考试试卷

开课学院: <u>计算机学院</u> ,考试形式: \闭、开卷
考试时间:年 月14 日,所需时间:90分钟
考生姓名:学号:课程编号:
1. (10 points) Determine whether the following statements are true ( $$ ) or false (×).
<ul> <li>( ) (1) 2<sup>2022</sup>≡1 (mod 7).</li> <li>( ) (2)Let m be a positive integer and let a, b, and c be integers. If ac ≡ bc (mod m) then a ≡ b (mod m).</li> <li>( ) (3)The set Z, with the usual ≤ ordering, is well-ordered.</li> <li>( ) (4)Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Then there are either three mutual friends or three mutual enemies in the group.</li> <li>( ) (5)7 is an inverse of 5 modulo 17.</li> </ul>
2. (30 points) Fill in the blanks.
(1) What form does a particular solution of the linear nonhomogeneous recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + F(n)$ have when $F(n) = (n^2 + 1)2^n$ ?
(2) Find the number of permutations of the letters in the word "multiplication".
Suppose $ A  = 6$ and $ B  = 5$ . Find the number of one-to-one functions $f: A \to B$ .
(4) A computer randomly prints three-digit codes, with no repeated digits in any code (for example, 387, 072, 760). What is the minimum number of codes that must be printed in

order to guarantee that at least six of the codes are identical?

(5)	How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x$	$x_4 + x_5 = 21$
Who	ere $x_i$ , $i = 1,2,3,4,5$ , is a nonnegative integer such that $x_i \ge 2 f e^{-ix}$	or $i = 1, 2, 3, 4, 5$ .

(6)	Find the coefficient of $x^{12}$ in the power series of the function $x^3/(1+4x)^2$ .
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- (7) An office manager has four employees and nine reports to be done. In how many ways can the reports be assigned to the employees so that each employee has at least one report to do.
- (8) Use Fermat's little theorem to find 9<sup>45</sup> mod 23.
- (9) Express gcd(450, 120) as a linear combination of 120 and 450
- (10) Suppose |A| = 4. Among all binary relations on A there are \_\_\_\_\_\_ anti-symmetric relations.
- 3. (8 points) Let S be the set of positive integers defined by:

Basis step:  $4 \in S$ .

Recursive step: If  $n \in S$ , then  $5n + 2 \in S$  and  $n^2 \in S$ .

- (1) Show that if  $n \in S$ , then  $n \equiv 4 \pmod{6}$ .
- (2) Show that there exists an integer  $m \equiv 4 \pmod{6}$  that does not belong to S.

4. (10 points) Use generating function to solve the recurrence relation  $a_n = 7a_{n-1} - 10a_{n-2}$  with initial conditions  $a_0 = 1$  and  $a_1 = 1$ .

5. (10 Points) There are r red balls, r blue balls and r white balls, how many ways are there for selecting 12 balls from these three piles of ball, but the red balls must be selected in even numbers?

2)Find an explicit for	nula for $a_n$ by so	olving the re	ecurrence rela	ation in part (1	).
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