试卷参考答案

一、填空题: (每题 4 分, 共 64 分)

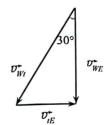
1.
$$v = 5 + \int_0^3 (3+2t) dt = 5 + (3t+t^2) \Big|_0^3 = 23 \text{ (m/s)}$$

2.
$$v = \frac{ds}{dt} = 10 - t$$
, $a_t = \frac{dv}{dt} = -1 \text{ m/s}^2$, $t = 5 \text{s}$, $v = 5 \text{ m/s}$, $a_n = \frac{v^2}{R} = 0.5 \text{ m/s}^2$

$$a = \sqrt{a_i^2 + a_n^2} = \frac{\sqrt{5}}{2} \text{ m/s}^2 = 1.1 \text{ m/s}^2$$

3. 设
$$W$$
雨滴, t 列车, E 地面,则有 $\vec{v}_{WE} = \vec{v}_{Wt} + \vec{v}_{tE}$, $v_{tE} = 10$ m/s

$$v_{WE} = v_{tE} \text{ctg} 30^{\circ} = 10\sqrt{3} \text{ m/s} = 17.3 \text{ m/s}; \quad v_{Wt} = \frac{v_{tE}}{\sin 30^{\circ}} = 20 \text{ m/s}$$



4.
$$ma\cos\theta = mg\sin\theta$$
 $a = g\tan\theta$

5.
$$F\Delta t_1 = (m_1 + m_2)v_A$$
, $v_A = \frac{F\Delta t_1}{m_1 + m_2}$, $F\Delta t_2 = m_2(v_B - v_A)$, $v_B = \frac{F\Delta t_1}{m_1 + m_2} + \frac{F\Delta t_2}{m_2}$

6.
$$W = \int F dx = \int_0^4 (6x^2 + 10) dx = 168 (J)$$

7.
$$F_{x} = -\frac{\partial E_{p}}{\partial x} = -2ax - by; \quad F_{y} = -\frac{\partial E_{p}}{\partial y} = -bx; \quad F_{z} = -\frac{\partial E_{p}}{\partial z} = -(-3cz^{2}) = 3cz^{2};$$
$$\vec{F} = -(2ax + by)\vec{i} - bx\vec{j} + 3cz^{2}\vec{k}$$

8.
$$mv_{\text{max}}3R = mv_{\text{min}}6R$$
, $\frac{1}{2}mv_{\text{max}}^2 + (-G\frac{Mm}{3R}) = \frac{1}{2}mv_{\text{min}}^2 + (-G\frac{Mm}{6R})$
 $v_{\text{max}} = \frac{2}{3}\sqrt{G\frac{M}{R}} = \frac{2}{3}\sqrt{gR}$, $v_{\text{min}} = \frac{3R}{6R}v_{\text{max}} = \frac{1}{3}\sqrt{gR}$

9.
$$x_C = \frac{\int x dm}{\int dm} = \frac{\int_0^l x k x dx}{\int_0^l k x dx} = \frac{k l^3 / 3}{k l^2 / 2} = \frac{2}{3} l$$
, $I_0 = \int x^2 dm = \int_0^l x^2 \lambda x dx = \frac{1}{4} k l^4$

10.
$$M\Delta t = J\omega - J\omega_0$$
, $M = \frac{J\omega - J\omega_0}{\Delta t} = \frac{0 - 2\pi nJ}{\Delta t} = -50\pi \text{ N} \cdot \text{m} = -157 \text{ N} \cdot \text{m}$

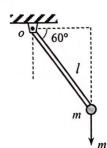
11. 变大

$$12. \int_0^{\theta} \left(-\frac{2}{3} m \mu g R\right) d\theta = 0 - \frac{1}{2} J \omega_0^2, \quad \frac{2}{3} m \mu g R \theta = \frac{1}{2} \cdot \frac{1}{2} m R^2 \omega_0^2, \quad \theta = \frac{3R \omega_0^2}{8 \mu g},$$

$$n = \frac{\theta}{2\pi} = \frac{3R\omega_0^2}{16\pi\mu g}$$

13.
$$M_1 = mgl$$
, $\alpha_1 = \frac{M_1}{I} = \frac{mgl}{ml^2} = \frac{g}{l}$,

$$M_2 = mgl\cos 60^\circ = \frac{mgl}{2}$$
, $\alpha_2 = \frac{M_2}{J} = \frac{g}{2l}$



14.
$$x_C = \frac{m_1 x_{10} + m_2 x_{20}}{m_1 + m_2}$$
, $\Delta x_2 = x_{20} - x_2$, $\Delta x_1 = x_{10} - x_1 = l - \Delta x_2$; $\Delta x_2 = \frac{m_1}{m_1 + m_2} l = 1.2 \text{ m}$

15:
$$x = x_0 \sqrt{1 - \frac{v^2}{c^2}}$$
, $y = y_0$, $z = z_0$. $V = xyz = V_0 \sqrt{1 - \frac{v^2}{c^2}}$, $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

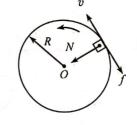
$$\rho = m/V = \frac{m_0 / \sqrt{1 - \frac{v^2}{c^2}}}{V_0 \sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{V_0 (1 - \frac{v^2}{c^2})}$$

16.
$$\Delta t' = \frac{\Delta t}{\sqrt{1 - u^2/c^2}}$$
 $u = c\sqrt{1 - (1/2)^2} = \frac{\sqrt{3}}{2}c$ $\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1 - u^2/c^2}} = -\sqrt{3}c$

二、计算题: (共4题,共36分)

1. 解: (1) 在法向
$$N = m\frac{v^2}{R}$$
, 切向摩擦力 $f = \mu N = \mu m\frac{v^2}{R}$

(2) 在切向
$$-f = ma_t$$
, $a_t = -\frac{f}{m} = -\mu \frac{v^2}{R}$



(3)
$$a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = -\mu \frac{v^2}{R}$$
, $\frac{\mathrm{d}v}{v^2} = -\frac{\mu}{R} \mathrm{d}t$, $\int_{v_0}^{v_0/3} \frac{\mathrm{d}v}{v^2} = \int_0^t -\frac{\mu}{R} \mathrm{d}t$, $t = \frac{2R}{\mu v_0}$

2.
$$M: m_B g - T_B = m_B a$$
; $T_A = m_A a$; $(T_B - T_A)R = \frac{1}{2} m_C R^2 \frac{a}{R}$;
$$a = \frac{m_B g}{m_A + m_B + m_C/2}; T_A = m_A a = \frac{m_A m_B g}{m_A + m_B + m_C/2}$$
$$T_B = m_B g - m_B a = \frac{m_A m_B + m_B m_C/2}{m_A + m_B + m_C/2} g$$

3. 解: 机械能守恒:
$$\frac{1}{2} \cdot \frac{1}{3} M l^2 \omega^2 = M g \frac{l}{2}$$
, 角动量守恒: $m v_0 \frac{l}{2} = m \frac{v_0}{2} \cdot \frac{l}{2} + \frac{1}{3} M l^2 \omega$ $v_0 = \frac{4 M l}{3 m} \omega$; $\omega = \sqrt{\frac{3 g}{l}}$; $v_0 = \frac{4 M}{3 m} \sqrt{3 g l}$

4.
$$A = m_0 c^2 \left(\frac{1}{\sqrt{1 - 0.6^2}} - 1\right) = \frac{1}{4} m_0 c^2$$
, $m_1 = \frac{m_0}{\sqrt{1 - v_1^2/c^2}} = \frac{m_0}{\sqrt{1 - 0.6^2}} = \frac{5}{4} m_0$

(2)
$$\Delta p = m_0 \left(\frac{0.8c}{\sqrt{1 - 0.8^2}} - \frac{0.6c}{\sqrt{1 - 0.6^2}} \right) = m_0 c \left(\frac{4}{3} - \frac{3}{4} \right) = \frac{7}{12} m_0 c$$

$$m_2 = \frac{m_0}{\sqrt{1 - v_2^2/c^2}} = \frac{m_0}{\sqrt{1 - 0.8^2}} = \frac{5}{3} m_0; \quad E_k = m_2 c^2 - m_0 c^2 = \frac{2}{3} m_0 c^2$$