浙江大学 2023 - 2024 学年春夏季学期

《离散数学》课程期中考试试卷

开课学院：计算机学院，考试形式：✓闭、开卷

考试时间：2024 年 5 月 14 日，所需时间：90 分钟

考生姓名：________________ 学号：________________ 课程编号：______________

1. (10 points) Determine whether the following statements are true (√) or false (×).

( ) (1) $2^{2022} \equiv 1 \pmod 7$.

( ) (2)Let m be a positive integer and let a, b, and c be integers. If $ac \equiv bc \pmod m$ then $a \equiv b \pmod m$.

( ) (3)The set Z, with the usual ≤ ordering, is well-ordered.

( ) (4)Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Then there are either three mutual friends or three mutual enemies in the group.

( ) (5)7 is an inverse of 5 modulo 17.

2. (30 points) Fill in the blanks.

(1) What form does a particular solution of the linear nonhomogeneous recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + F(n)$ have when $F(n) = (n^2 + 1)2^n$?

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(2) Find the number of permutations of the letters in the word "multiplication ".

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(3) Suppose $|A| = 6$ and $|B| = 5$. Find the number of one-to-one functions $f: A \to B$.

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(4) A computer randomly prints three-digit codes, with no repeated digits in any code (for example, 387, 072, 760). What is the minimum number of codes that must be printed in order to guarantee that at least six of the codes are identical?

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(5) How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$

Where $x_i, i = 1,2,3,4,5,$ is a nonnegative integer such that $x_i \geq 2 \, for \quad i = 1,2,3,4,5.$

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(6) Find the coefficient of x^{12} in the power series of the function $x^3/(1+4x)^2$.

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(7) An office manager has four employees and nine reports to be done. In how many ways can the reports be assigned to the employees so that each employee has at least one report to do.

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(8) Use Fermat's little theorem to find 9^{45} mod 23. ____________________

(9) Express gcd(450, 120) as a linear combination of 120 and 450

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(10) Suppose $|A| = 4$. Among all binary relations on A there are ____________________

anti-symmetric relations.

3. (8 points) Let S be the set of positive integers defined by:

Basis step: $4 \in S$.

Recursive step: If $n \in S$, then $5n + 2 \in S$ and $n^2 \in S$.

(1) Show that if $n \in S$, then $n \equiv 4 \pmod 6$.

(2) Show that there exists an integer $m \equiv 4 \pmod 6$ that does not belong to S.

4. (10 points) Use generating function to solve the recurrence relation

$$a_n = 7a_{n-1} - 10a_{n-2} \text{ with initial conditions } a_0 = 1 \text{ and } a_1 = 1.$$

5. (10 Points) There are r red balls, r blue balls and r white balls, how many ways are there for selecting 12 balls from these three piles of ball, but the red balls must be selected in even numbers?

6.(12 points) A country uses only coins of \$1 and \$2. Let a_n denote the number of ways of paying n dollars in the country by using only coins, where the order in which the coins are paid matters.(Assume that the coins with the same value are indistinguishable)
1)Find a recurrence relation for a_n and give the necessary initial condition(s).
2)Find an explicit formula for a_n by solving the recurrence relation in part (1).