Quizzes by HJ

Quiz 1: Numerical Computations

Perform the following computations (i) exactlly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. (iv) Compute the relative errors in parts (ii) and (iii).

a. $\frac{4}{5} \times \frac{1}{3}$

b. $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

Quiz 2: Root of a Polynomial

Use a fixed-iteration method to determine a solution with upper bound of error 10^{-2} for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $p_0 = 1$.

Quiz 3: Solve Linear Systems

Use the Gaussian Elimination Algorithm to solve the following two linear systems, if possible, and determine whether row interchanges are necessary.

(a)

$$\begin{aligned} x_1 + x_2 + x_4 &= 2 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ 4x_1 - x_2 - 2x_3 + 2x_4 &= 0 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + x_2 + x_4 &= 2 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ -x_1 + 2x_2 + 3x_3 - 4x_4 &= 4 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \end{aligned}$$

Quiz 4: Matrix Factorization

Obtain a factorization of the form $A = P'LU$ for the following matrix:

$$A = \begin{pmatrix} 1 & -2 & 3 & 0 \\ 1 & -2 & 3 & 1 \\ 1 & -2 & 2 & -2 \\ 2 & 1 & 3 & -1 \end{pmatrix}$$

Quiz 5: SOR

Find the first iteration of the SOR method with $\omega = 1.1$ for the following linear system using $x^{(0)} = 0$:

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1 \\ 3x_1 + 6x_2 + 2x_3 &= 0 \\ 3x_1 + 3x_2 + 7x_3 &= 4 \end{aligned}$$

Quiz 6: Condition Number

Compute the condition numbers of the following matrices relative to $\|\cdot\| = \infty$:

(a)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 2 \\ 1.00001 & 2 \end{pmatrix}$$

Quiz 7: Interpolation

Approximate $f(0.05)$ using the following data and the Newton forward divided-difference formula:

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

Quiz 8: LSA

In the lead example of this chapter, an experiment was describe to determine the spring constant k in Hooke' s low: $F(l) = k(l - E)$.

The function F is the force required to stretch the spring l units, where the constant $E = 5.3$ is the length of unstretched spring.

Suppose measurements are made of the length l, in inches, for applied weights $F(l)$, in pounds, as given in the following table.

$F(l)$	2	4	6
l	7.0	9.4	12.3

Find the least squares approximation for k.

Quiz 9: Least Squares Approximation

Find the linear least squares polynomial approximation on the interval $[-1, 1]$ for the following function.

$$f(x) = x^2 - 2x + 3$$

Quiz 10: Chebyshev Polynomial

Show that for each Chebyshev polynomial $T_n(x)$, we have

$$\int_{-1}^{1} \frac{(T_n(x))^2}{\sqrt{1 - x^2}} dx = \frac{\pi}{2}$$

Quiz 11: Composite Numerical Integration

The midpoint rule

$$\int_a^b f(x)dx = f\left(\frac{a+b}{2}\right)(b - a)$$

for approximating

$$\int_{-1}^{1} f(x)dx$$

gives the value 12. With $n = 2$, the composite Midpoint rule gives 5, and Composite Simpson’s rule gives 6. Use the fact that $f(-1) = f(1)$ and $f(-0.5) = f(0.5) - 1$ to determine $f(-1), f(-0.5), f(0), f(0.5)$ and $f(1)$.

Quiz 12: Gaussian Quadrature

Show that the formula

$$Q(P) = \sum_{i=1}^{n} c_i P(x_i)$$

cannot have degree of precision greater than $2n - 1$, regardless of the choice of $c_1, ..., c_n$ and $x_1, ..., x_n$.

Quiz 13: Runge-Kutta Methods

Show that the Midpoint method

$$w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}f(t_i, w_i)\right)$$

and the Modified Euler method give the same apprximations to the initial-value problem

$$y' = -y + t + 1, \quad 0 \leq t \leq 1, \quad y(0) = 1$$

for any choice of h. Why is this true?

Quiz 14: Multistep Methods

Please derive the formula of the Adams-Bashforth two-step explicit method, i.e., a formula of order 2 with the form:

$$w_{i+1} = w_i + h(b_1 f_i + b_0 f_{i-1})$$

Quiz 15: Stability

Investigate stability for the Trapezoidal method

$$w_{i+1} = w_i + \frac{h}{2}(f(t_i, w_i) + f(t_{i+1}, w_{i+1}))$$