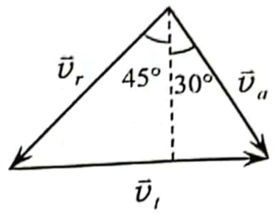
试卷参考答案

一、填空题：（每题 4 分，共 64 分）



1． $x=4t^2$ ，$y=2t+3$， $x=(y-3)^2$

2． $v=\dfrac{\mathrm{d}S}{\mathrm{d}t}=b-ct$， $a_t=\dfrac{\mathrm{d}v}{\mathrm{d}t}=-c$， $a_n=\dfrac{v^2}{R}=\dfrac{(b-ct)^2}{R}$

3． 由正弦定理

$$\frac{v_a}{\sin(90°-45°)}=\frac{v_t}{\sin(30°+45°)},\qquad \frac{v_a}{\cos45°}=\frac{v_t}{\sin30°\cos45°+\cos30°\sin45°}$$

$$v_a=\frac{v_t}{\sin30°+\sin45°\dfrac{\cos30°}{\cos45°}}=25.6\ \text{m/s}\quad 或\quad v_a=\frac{\cos45°}{\sin75°}v_t$$

4． $T=ma'$ ，$Mg+ma-T=ma'$， $a=g/2$， $T=3mg/4$

5． $a_1=\dfrac{F}{m}$， $s_1=\dfrac{1}{2}a_1t^2==\dfrac{1}{2}\dfrac{F}{m}t^2$， $I=mv_2-0$， $v_2=\dfrac{I}{m}$， $s_2=v_2t=\dfrac{I}{m}t$， $s_1=s_2$

$t=2I/F$.

6． $F=P=P_0-ky=mg-0.2gy=107.8-1.96y(\text{SI})$，

$W=\int\mathrm{d}W=\int_0^H F\,\mathrm{d}y=\int_0^{10}(107.8-1.96y)\mathrm{d}y=980\ \text{J}$

7． $F(r)=-\dfrac{\mathrm{d}U}{\mathrm{d}r}=-\dfrac{r_0}{r^2}U_0\mathrm{e}^{-r/r_0}-\dfrac{1}{r}U_0\mathrm{e}^{-r/r_0}=-\dfrac{r_0+r}{r^2}U_0\mathrm{e}^{-r/r_0}$

8． $\Delta E_k=-\Delta E_p=G\dfrac{mM}{R+h_2}-G\dfrac{mM}{R+h_1}=\dfrac{GmM(h_1-h_2)}{(R+h_1)(R+h_2)}$

9． $hmv_0=lmv$ 即 $v/v_0=h/l$ ，则动能之比为 $E_K/E_{K0}=h^2/l^2$

10． $y_c=\dfrac{3mR-mR}{8m}=\dfrac{R}{4}$， $I_o=8mR^2$ ， $I_c=I_o-8my_c^2=8mR^2-8m(\dfrac{R}{4})^2=7.5mR^2$

11． $M=\int x\mu g\mathrm{d}m=\int_0^l\dfrac{m}{l}\mu gx\mathrm{d}x=\dfrac{1}{2}\mu mgl$

12． $M_f=J\alpha$， $-k\omega=J\dfrac{\mathrm{d}\omega}{\mathrm{d}t}$， $\int_{\omega_0}^{\omega_0/4}\dfrac{\mathrm{d}\omega}{\omega}=-\dfrac{k}{J}\Delta t$， $\Delta t=\dfrac{2J}{k}\ln2$

13． $M=\dfrac{1}{2}mgl$， $\dfrac{1}{2}mgl=3m\dfrac{l^2}{4}\cdot\alpha$， $\alpha=\dfrac{2g}{3l}$

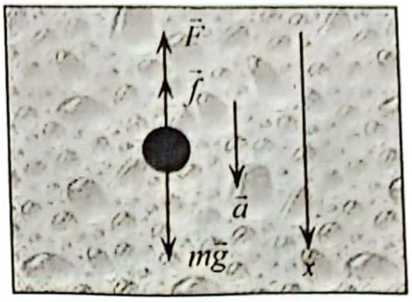
14． $\dfrac{60x_1+Mx_2}{60+M}=\dfrac{60x_{10}+Mx_{20}}{60+M}$， $60\cdot x_1+Mx_2=60\cdot x_{10}+Mx_{20}$， $x_1-x_{10}=3$ m， $x_{20}-x_2=1$ m，

$M=180$ kg

15． $l=l_0\sqrt{1-\dfrac{v^2}{c^2}}$ $\qquad v=c\sqrt{1-\dfrac{l^2}{l_0^2}}$ $\qquad p=mv=m_0c\dfrac{l_0}{l}\sqrt{1-\dfrac{l^2}{l_0^2}}=m_0c\sqrt{\dfrac{l_0^2}{l^2}-1}$

16． $\Delta x'=\dfrac{\Delta x-u\Delta t}{\sqrt{1-u^2/c^2}}=\dfrac{1}{\sqrt{1-u^2/c^2}}(\text{m})$

二、计算题：（共 4 题，共 36 分）



1．解：小球受力如图，根据牛顿第二定律

$$mg - kv - F = ma = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

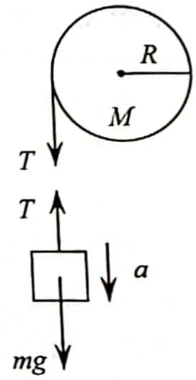
$$\frac{\mathrm{d}v}{(mg - kv - F)/m} = \mathrm{d}t$$

初始条件： $t = 0$, $v = 0$.

$$\int_0^v \frac{\mathrm{d}v}{(mg - kv - F)/m} = \int_0^t \mathrm{d}t \qquad \therefore\ v = (mg - F)(1 - \mathrm{e}^{-kt/m})/k$$

2．解：(1) ∵ $mg - T = ma$　　$TR = I\alpha$　　$a = R\alpha$　　$I = \frac{1}{2}MR^2$

$$\therefore\ \alpha = mgR/(mR^2 + J) = \frac{mgR}{mR^2 + \frac{1}{2}MR^2} = \frac{2mg}{(2m+M)R} = 81.7\ \mathrm{rad/s^2}$$ 垂直纸面向外．



(2) ∵ $\omega^2 = \omega_0^2 - 2\alpha\theta$

当 $\omega = 0$ 时 $\theta = \frac{\omega_0^2}{2\alpha} = \frac{(2m+M)R\omega_0^2}{4mg} = 0.612\ \mathrm{rad}$

物体上升的高度 $h = R\theta = \frac{(2m+M)R^2\omega_0^2}{4mg} = 6.12\times10^{-2}\ \mathrm{m}$

3．解：（1） $x_c = \frac{mL/2 + mL}{2m} = \frac{3}{4}L$

（2） $I = mL^2 + \frac{1}{3}mL^2 = \frac{4}{3}mL^2$，　$mv_0L = \frac{4}{3}mL^2\omega$，　$\omega = \frac{3v_0}{4L}$

$$E_k = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{4}{3}mL^2\times\left(\frac{3v_0}{4L}\right)^2 = \frac{3}{8}mv_0^2$$

（3） $N - 2mg = 2ma_{cn} = 2m\omega^2 x_c$，　$N = 2mg + \frac{27mv_0^2}{32L}$，方向向上

4．解：据相对论动能公式　$E_K = mc^2 - m_0c^2$　　$m = \frac{m_0}{\sqrt{1-(v/c)^2}}$

得 $E_K = m_0c^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right)$　即 $\frac{1}{\sqrt{1-(v/c)^2}} - 1 = \frac{E_K}{m_0c^2} = 1.419$

$$v = c\sqrt{\frac{E_k^2 + 2E_0E_K}{(E_k + E_0)^2}} = c\sqrt{\frac{E_k^2 + 2m_0c^2E_K}{(E_k + m_0c^2)^2}} = 0.91c$$

平均寿命为　$\tau = \frac{\tau_0}{\sqrt{1-(v/c)^2}}\ \mathrm{s}$　　$\tau = 5.31\times10^{-8}$