

# CHAPTER 13

## NAVIGATIONAL ASTRONOMY

### PRELIMINARY CONSIDERATIONS

#### 1300. Definitions

The science of Astronomy studies the positions and motions of celestial bodies and seeks to understand and ex-

plain their physical properties. Navigational astronomy deals with their coordinates, time, and motions. The symbols commonly recognized in navigational astronomy are given in Table 1300.

#### Celestial Bodies

○ Sun	○  Lower limb
○ Moon	○  Center
☿ Mercury	○  Upper limb
♀ Venus	● New moon
⊕ Earth	● Crescent moon
♂ Mars	● First quarter
♃ Jupiter	○ Gibbous moon
♄ Saturn	○ Full moon
♅ Uranus	○ Gibbous moon
♆ Neptune	● Last quarter
♇ Pluto	● Crescent moon
★ Star	
★-P Star-planet altitude correction (altitude)	

#### Miscellaneous Symbols

yrs Years	* Interpolation impractical
m Months	° Degrees
d Days	' Minutes of arc
h Hours	" Seconds of arc
m Minutes of time	♂ Conjunction
s Seconds of time	♂ Opposition
■ Remains below horizon	□ Quadrature
□ Remains above horizon	☿ Ascending node
//// Twilight all night	☽ Descending node
☽ Aries (vernal equinox)	

Table 1300. Astronomical symbols.

#### 1301. The Celestial Sphere

Looking at the sky on a dark night, imagine that celestial bodies are located on the inner surface of a vast, Earth-centered sphere (see Figure 1301). This model is

useful since we are only interested in the relative positions and motions of celestial bodies on this imaginary surface. Understanding the concept of the celestial sphere is most important when discussing sight reduction in Chapter 19.

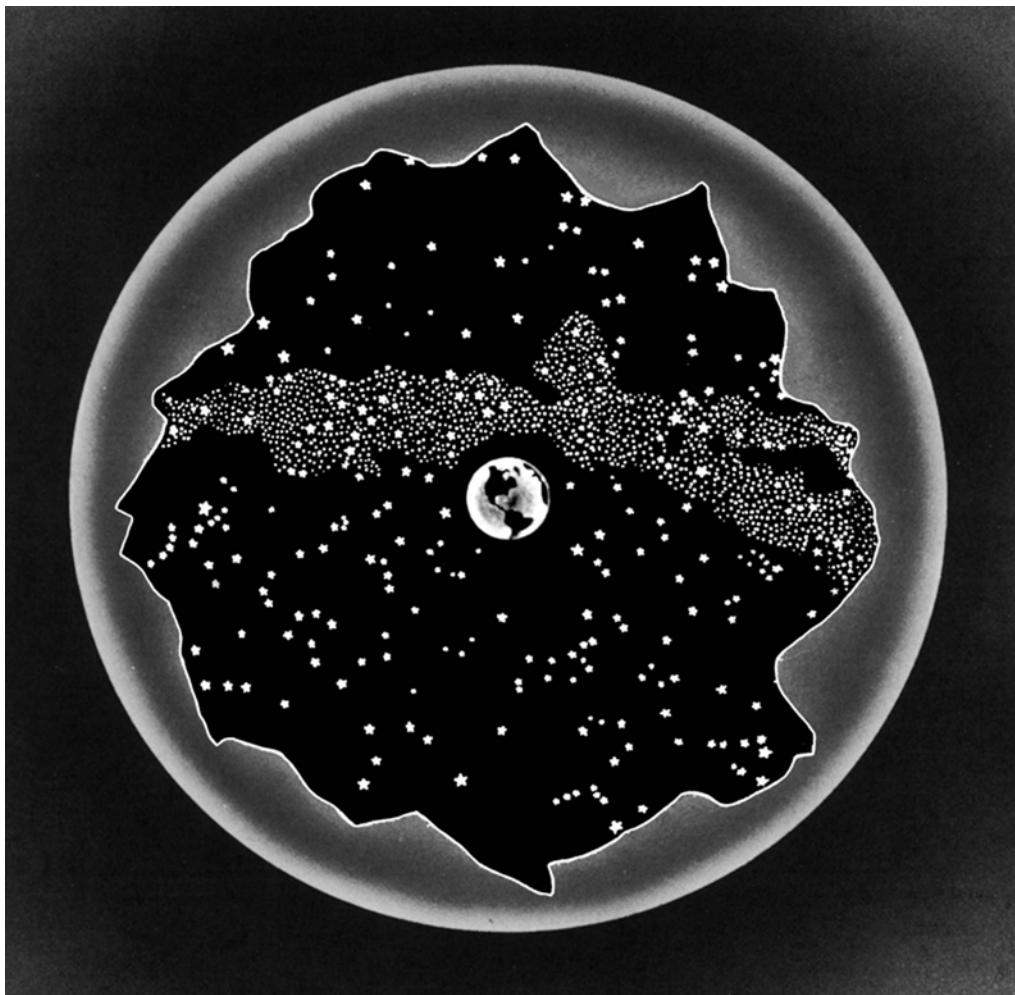


Figure 1301. The celestial sphere.

### 1302. Relative and Apparent Motion

Celestial bodies are in constant motion. There is no fixed position in space from which one can observe absolute motion. Since all motion is relative, the position of the observer must be noted when discussing planetary motion. From the Earth we see apparent motions of celestial bodies on the celestial sphere. In considering how planets follow their orbits around the Sun, we assume a hypothetical observer at some distant point in space. When discussing the rising or setting of a body on a local horizon, we must locate the observer at a particular point on the Earth because the setting Sun for one observer may be the rising Sun for another.

Apparent motion on the celestial sphere results from the motions in space of both the celestial body and the Earth. Without special instruments, motions toward and away from the Earth cannot be discerned.

### 1303. Astronomical Distances

We can consider the celestial sphere as having an infinite radius because distances between celestial bodies are so vast. For an example in scale, if the Earth were represented by a ball one inch in diameter, the Moon would be a ball one-fourth inch in diameter at a distance of 30 inches, the Sun would be a ball nine feet in diameter at a distance of nearly a fifth of a mile, and Pluto would be a ball half an inch in diameter at a distance of about seven miles. The nearest star would be one-fifth of the actual distance to the Moon.

Because of the size of celestial distances, it is inconvenient to measure them in common units such as the mile or kilometer. The mean distance to our nearest neighbor, the Moon, is 238,855 miles. For convenience this distance is sometimes expressed in units of the equatorial radius of the Earth: 60.27 Earth radii.

Distances between the planets are usually expressed in terms of the **astronomical unit (au)**, which closely corresponds to the average distance between the Earth and the

Sun. This is approximately 92,960,000 miles. Thus the mean distance of the Earth from the Sun is 1 au. The mean distance of the dwarf planet Pluto is about 39.5 au. Expressed in astronomical units, the mean distance from the Earth to the Moon is 0.00257 au.

Distances to the stars require another leap in units. A commonly-used unit is the **light-year**, the distance light travels in one year. Since the speed of light is about  $1.86 \times 10^5$  miles per second and there are about  $3.16 \times 10^7$  seconds per year, the length of one light-year is about  $5.88 \times 10^{12}$  miles. The nearest stars, Alpha Centauri and its neighbor Proxima, are 4.3 light-years away. Relatively few stars are less than 100 light-years away. The nearest galaxy of comparable size to our own Milky Way is the Andromeda Galaxy, at a distance of about 2.5 million light years. The most distant galaxies observed by astronomers are 13 billion light years away, just at the edge of the visible universe.

#### 1304. Magnitude

The relative brightness of celestial bodies is indicated by a scale of stellar **magnitudes**. Initially, astronomers divided the stars into 6 groups according to brightness. The 20 brightest were classified as of the first magnitude, and the dimmest were of the sixth magnitude. In modern times, when it became desirable to define more precisely the limits of magnitude, a first magnitude star was considered 100 times brighter than one of the sixth magnitude. Since the fifth root of 100 is 2.512, this number is considered the **magnitude ratio**. A first magnitude star is 2.512 times as

bright as a second magnitude star, which is 2.512 times as bright as a third magnitude star. A second magnitude is  $2.512 \times 2.512 = 6.310$  times as bright as a fourth magnitude star. A first magnitude star is  $2.512^{20}$  times as bright as a star of the 21st magnitude, the dimmest that can be seen through a 200-inch telescope. It is important to note the higher the magnitude, the dimmer the object.

Stars vary in color; i.e., some are more red than others. Therefore, the brightness of a star is a function of what "detector" is being used. For example, stars that are more red than others appear brighter using a detector that is most sensitive in red wavelengths. Thus, it is common when defining magnitudes to include an idea of the detector. For navigation, most magnitudes are described as "visual", or how the object would look to the unaided eye, but sometimes you will see other magnitude bands. If no band is given assume that the magnitude is visual.

Brightness is normally tabulated to the nearest 0.1 magnitude, about the smallest change that can be detected by the unaided eye of a trained observer. All stars of magnitude 1.50 or brighter are popularly called "first magnitude" stars. Those between 1.51 and 2.50 are called "second magnitude" stars, those between 2.51 and 3.50 are called "third magnitude" stars, etc. Sirius, the brightest star, has a magnitude of -1.6. The only other star with a negative magnitude is Canopus, -0.9. At greatest brilliance Venus has a magnitude of about -4.4. Mars, Jupiter, and Saturn are sometimes of negative magnitude. The full Moon has a magnitude of about -12.7, but varies somewhat. The magnitude of the Sun is about -26.7.

### THE UNIVERSE

#### 1305. The Solar System

The Sun, the most conspicuous celestial object in the sky, is the central body of the solar system. Associated with it are eight planets, five dwarf planets like Pluto, and thousands of asteroids, comets, and meteors. All planets other than Mercury and Venus have moons.

#### 1306. Motions of Bodies of the Solar System

Astronomers distinguish between two principal motions of celestial bodies. **Rotation** is a spinning motion about an axis within the body, whereas **revolution** is the motion of a body in its orbit around another body. The body around which a celestial object revolves is known as that body's **primary**. For the moons (satellites), the primary is a planet. For the planets, the primary is the Sun. The entire solar system is held together by the gravitational force of the Sun. The whole system revolves around the center of the Milky Way galaxy and the Milky Way is in motion relative to its neighboring galaxies.

The hierarchies of motions in the universe are caused

by the force of gravity. As a result of gravity, bodies attract each other in proportion to their masses and to the inverse square of the distances between them. This force causes the planets to go around the sun in nearly circular, elliptical orbits.

The laws governing the motions of planets in their orbits were discovered by Johannes Kepler, and are now known as **Kepler's laws**:

1. The orbits of the planets are ellipses, with the sun at the common focus.
2. The straight line joining the sun and a planet (the **radius vector**) sweeps over equal areas in equal intervals of time.
3. The squares of the sidereal periods of any two planets are proportional to the cubes of their mean distances from the sun.

In 1687 Isaac Newton stated three "laws of motion," which he believed were applicable to the planets. **Newton's laws of motion** are:

1. Every body continues in a state of rest or of uniform motion in a straight line unless acted upon by an external force.
2. When a body is acted upon by an external force, its acceleration is directly proportional to that force, and inversely proportional to the mass of the body, and acceleration take place in the direction in which the force acts.
3. To every action there is an equal and opposite reaction.

Newton also stated a single **universal law of gravitation**, which he believed applied to all bodies, although it was based upon observations with the solar system only:

Every particle of matter attracts every other particle with a force that varies directly as the product of their masses and inversely as the square of the distance between them.

From these fundamental laws of motion and gravitation, Newton derived Kepler's empirical laws. He proved rigorously that the gravitational interaction between any two bodies results in an orbital motion of each body about the barycenter of the two masses that form a conic section, that is a circle, ellipse, parabola, or hyperbola.

Circular and parabolic orbits are unlikely to occur in nature because of the precise speeds required. Hyperbolic orbits are open, that is one body, due to its speed, recedes into space. Therefore, a planet's orbit must be elliptical as found by Kepler.

Both the sun and each body revolve about their common center of mass. Because of the preponderance of the mass of the sun over that of the individual planets, the common center of the sun and each planet except Jupiter lies with the sun. The common center of the combined mass of the solar system moves in and out of the sun.

The various laws governing the orbits of planets apply equally well to the orbit of any body with respect to its primary.

In each planet's orbit, the point nearest the Sun is called the **perihelion**. The point farthest from the Sun is called the **aphelion**. The line joining perihelion and aphelion is called the **line of apsides**. In the orbit of the Moon, the point nearest the Earth is called the **perigee**, and that point farthest from the Earth is called the **apogee**. Figure 1306 shows the orbit of the Earth (with exaggerated eccentricity), and the orbit of the Moon around the Earth.

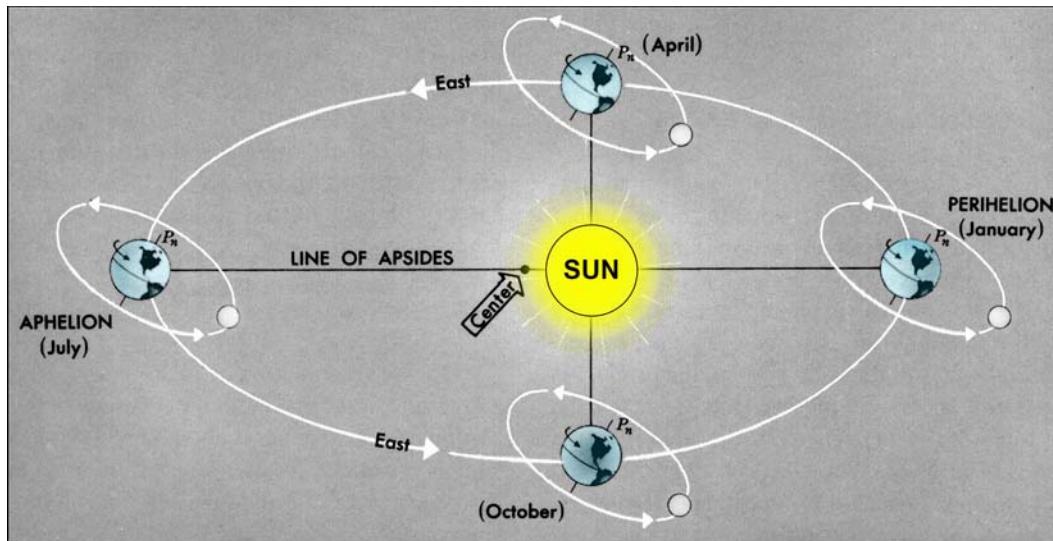


Figure 1306. Orbits of the Earth and Moon.

### 1307. The Sun

The Sun dominates our solar system. Its mass is nearly a thousand times that of all other bodies of the solar system combined. Its diameter is about 865,000 miles. Since it is a star, it generates its own energy through a thermonuclear reaction, thereby providing heat and light for the entire solar system.

The distance from the Earth to the Sun varies from 91,300,000 at perihelion to 94,500,000 miles at aphelion. When the Earth is at perihelion, which always occurs early in January, the Sun appears largest, 32.6' of arc in diameter. Six months later at aphelion, the Sun's apparent diameter is a minimum of 31.5'. Reductions of celestial navigation

sights taken of the Sun's limb take this change of apparent size into account.

Observations of the Sun's surface (called the **photosphere**) reveal small dark areas called **sunspots**. These are areas of intense magnetic fields in which relatively cool gas (at 7000°F.) appears dark in contrast to the surrounding hotter gas (10,000°F.). Sunspots vary in size from perhaps 50,000 miles in diameter to the smallest spots that can be detected (a few hundred miles in diameter). They generally appear in groups. See Figure 1307.

Surrounding the photosphere is an outer **corona** of very hot but tenuous gas. This can only be seen during an eclipse of the Sun, when the Moon blocks the light of the photosphere.

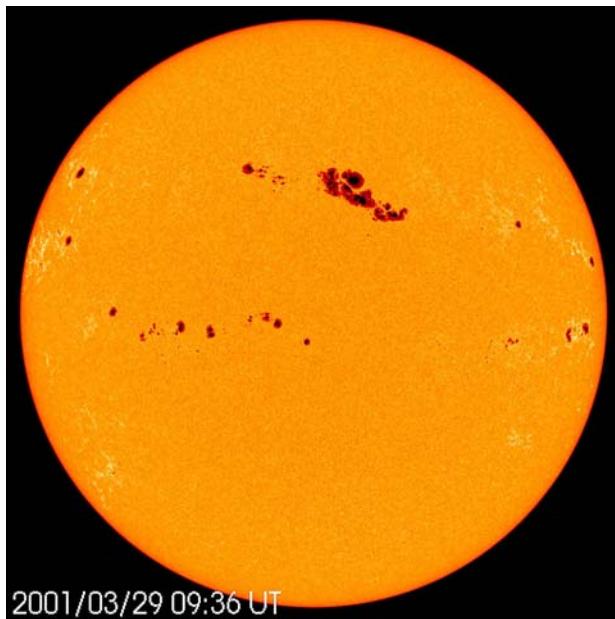


Figure 1307. The huge sunspot group observed on March 30, 2001 spanned an area 13 times the entire surface of the Earth. Courtesy of SOHO, a project of international cooperation between ESA and NASA.

The Sun is continuously emitting charged particles, which form the **solar wind**. As the solar wind sweeps past the Earth, these particles interact with the Earth's magnetic field. If the solar wind is particularly strong, the interaction can produce magnetic storms which adversely affect radio signals on the Earth and can interfere with satellite communications. At such times the **auroras** are particularly brilliant and widespread.

The Sun is moving approximately in the direction of Vega at about 12 miles per second, or about two-thirds as fast as the Earth moves in its orbit around the Sun.

### 1308. The Planets

The principal bodies orbiting the Sun are called **planets**. Eight planets are known; in order of their distance from the Sun, they are: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. Pluto, formerly considered a planet, is now classified as a dwarf planet. All of the planets revolve around the Sun in the same direction in nearly circular orbits. All of the planets are spherical or nearly so, all have regular rotation rates, and all shine by reflected sunlight. All except Mercury have substantial atmospheres. Only four of the planets are commonly used for celestial navigation: Venus, Mars, Jupiter, and Saturn.

The orbits of the planets lie in nearly the same plane as the Earth's orbit. Therefore, as seen from the Earth, the planets are confined to a strip of the celestial sphere near the **ecliptic**, which is the intersection of the mean plane of the Earth's orbit around the Sun with the celestial sphere. Ex-

cept for Uranus and Neptune, the planets are bright enough to be easily seen by the unaided eye, although the brightness of each at any given time depends on its distance from the Earth and the fraction of the sunlit part observed.

Mercury and Venus, the two planets with orbits closer to the Sun than that of the Earth, are called **inferior planets**, and the others, with orbits farther from the Sun are called **superior planets**. The four planets nearest the Sun (Mercury through Mars) are called the inner planets, and the others (Jupiter through Neptune) are referred to as the outer planets. The outer planets are sometimes also called gas giants because they are so much larger than the others and have deep, dense atmospheres.

Planets can sometimes be identified in the sky by their appearance, because unlike the stars—they do not twinkle. The stars are so distant that they are point sources of light. Therefore the stream of light from a star is easily disrupted by turbulence in the Earth's atmosphere, causing **scintillation** (the twinkling effect). The naked-eye planets, however, are close enough to present perceptible disks. The broader stream of light from a planet is not so easily disrupted.

The orbits of many thousands of minor planets, also called asteroids, lie chiefly between the orbits of Mars and Jupiter. These are all too faint to be seen without a telescope.

### 1309. The Earth

In common with other planets, the Earth **rotates** on its axis and **revolves** in its orbit around the Sun. These motions are the principal source of the daily apparent motions of other celestial bodies. The Earth's rotation also causes a deflection of water and air currents to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. Because of the Earth's rotation, high tides on the open sea lag behind the meridian transit of the Moon.

For most navigational purposes, the Earth can be considered a sphere. However, like the other planets, the Earth is approximately an **oblate spheroid**, or **ellipsoid of revolution**, flattened at the poles and bulged at the equator. See Figure 1309. Therefore, the polar diameter is less than the equatorial diameter, and the meridians are slightly elliptical, rather than circular. The dimensions of the Earth are recomputed from time to time, as additional and more precise measurements become available. Since the Earth is not exactly an ellipsoid, results differ slightly when equally precise and extensive measurements are made on different parts of the surface.

### 1310. Inferior Planets (Mercury and Venus)

The orbits of Mercury and Venus are closer to the Sun than the Earth's orbit, thus they always appear in the neighborhood of the Sun. Over a period of weeks or months, they appear to oscillate back and forth from one side of the Sun to the other.

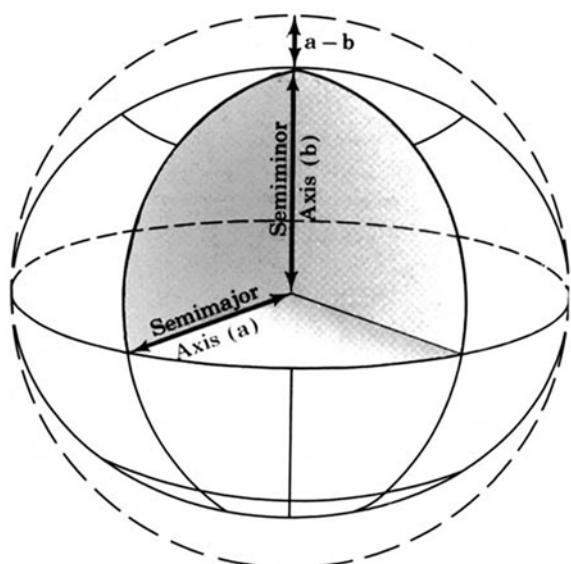


Figure 1309. Oblate spheroid or ellipsoid of revolution.

They are seen either in the eastern sky before sunrise or in the western sky after sunset. For brief periods they disappear into the Sun's glare. At this time they are between the Earth and Sun (known as **inferior conjunction**) or on the opposite side of the Sun from the Earth (**superior conjunction**). On rare occasions at inferior conjunction, the planet will cross the face of the Sun as seen from the Earth. This is known as a **transit of the Sun**.

When Mercury or Venus appears most distant from the Sun in the evening sky, it is at greatest eastern elongation. (Although the planet is in the western sky, it is at its easternmost point from the Sun.) From night to night the planet will appear to approach the Sun until it disappears into the glare of twilight. At this time it is moving between the Earth and Sun to inferior conjunction. A few days later, the planet will appear in the morning sky at dawn. It will gradually appear to move away from the Sun to its greatest western elongation, then move back toward the Sun. After disappearing in the morning twilight, it will move behind the Sun to superior conjunction. After this it will reappear in the evening sky, heading toward eastern elongation, beginning the cycle again. See Figure 1310.

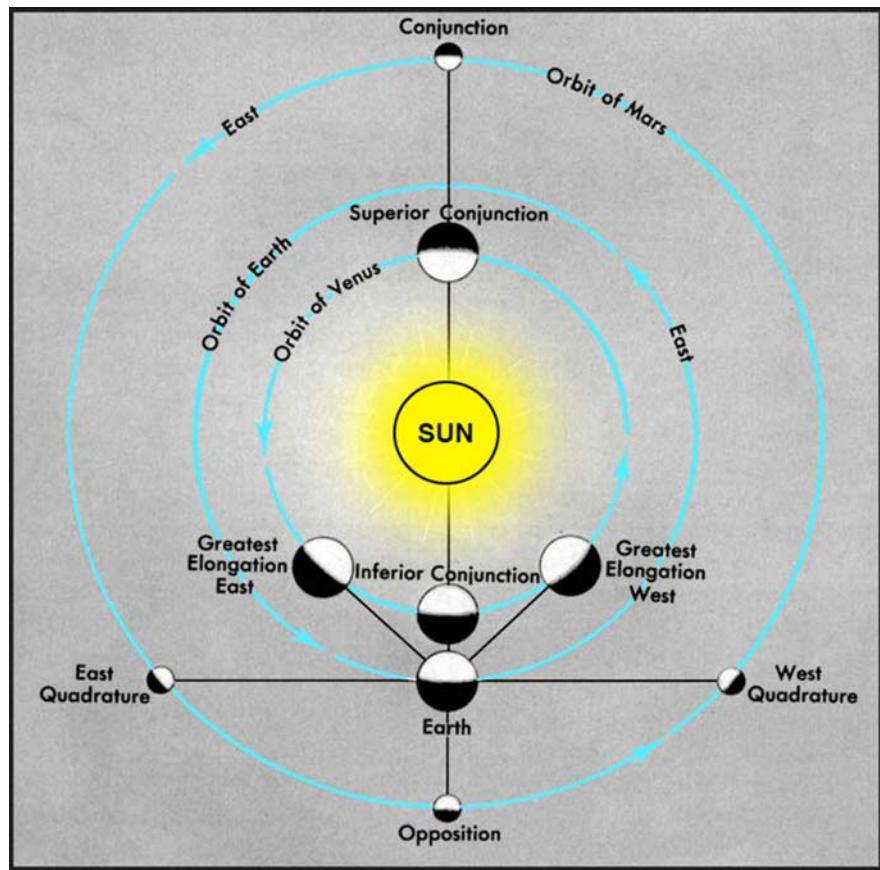


Figure 1310. Planetary configurations.

**Mercury** is never seen more than about  $28^\circ$  from the Sun. For this reason it is not commonly used for navigation. Near greatest elongation it appears near the western horizon after sunset or the eastern horizon before sunrise. At these

times it resembles a first magnitude star and is sometimes reported as a new or strange object in the sky. The interval during which it appears as a morning or evening star can vary from about 30 to 50 days. Around inferior conjunction,

Mercury is difficult to observe for about 5 days; near superior conjunction, it is as long as 35 days. Observed with a telescope, Mercury is seen to go through phases similar to those of the Moon.

**Venus** can reach a distance of  $47^{\circ}$  from the Sun, allowing it to dominate the morning or evening sky. At maximum brilliance, about five weeks before and after inferior conjunction, it has a magnitude of about  $-4.4$  and is brighter than any other object in the sky except the Sun and Moon. At these times it can be seen during the day and is sometimes observed for a celestial line of position. It appears as a morning or evening “star” for approximately 263 days in succession. Near inferior conjunction Venus disappears for 8 days; around superior conjunction it disappears for 50 days. Through strong binoculars or a telescope, Venus can be seen to go through a full set of phases. This actually has the effect of offsetting Venus’ center of light from its center of mass. Reductions of celestial navigation sights taken of Venus take this offset into account.

### 1311. Superior Planets (Mars, Jupiter, Saturn, Uranus, and Neptune)

All other planets besides Mercury and Venus have orbits further from the Sun than Earth’s orbit; these are called superior planets. While Mercury and Venus never appear too far from the Sun, the superior planets are not confined to the proximity of the Sun as seen from the Earth. They can pass behind the Sun (**conjunction**), but they cannot pass between the Sun and the Earth. We see them move away from the Sun until they are opposite the Sun in the sky (**opposition**). When a superior planet is near conjunction, it rises and sets approximately with the Sun and is thus lost in the Sun’s glare. Gradually it becomes visible in the early morning sky before sunrise. From day to day, it rises and sets earlier, becoming increasingly visible through the late night hours until dawn. At opposition, it will rise about when the Sun sets, be visible throughout the night, and set about when the Sun rises.

Observed against the background stars, the planets normally move eastward in what is called **direct motion**. Approaching opposition, however, a planet will slow down, pause (at a stationary point), and begin moving westward (**retrograde motion**), until it reaches the next stationary point and resumes its direct motion. This is not because the planet is moving strangely in space. This relative, observed motion results because the faster moving Earth is “catching up” with and “passing” by the slower moving superior planet.

The superior planets are brightest and closest to the Earth at opposition, when they are visible throughout the night. The interval between oppositions is known as the **synodic period**. This period is longest for the closest planet, Mars, and becomes increasingly shorter for the outer planets.

Unlike Mercury and Venus, the superior planets do not go through a full cycle of phases. They are always full or highly gibbous. With the exception of Mars, the offset between a superior planet’s center of light from its center of mass (due to phase) does not need to be accounted for in traditional celestial navigation. Reductions of celestial navigation sights of Mars often take this offset into account.

**Mars** can usually be identified by its orange color. It can become as bright as magnitude  $-2.8$  but is more often between  $-1.0$  and  $-2.0$  at opposition. Oppositions occur at intervals of about 780 days. The planet is visible for about 330 days on either side of opposition. Near conjunction it is lost from view for about 120 days. Its two satellites can only be seen in a large telescope.

**Jupiter**, largest of the known planets, normally outshines Mars, regularly reaching magnitude  $-2.0$  or brighter at opposition. Oppositions occur at intervals of about 400 days, with the planet being visible for about 180 days before and after opposition. The planet disappears for about 32 days at conjunction. Four satellites (of a total 67 currently known) are bright enough to be seen with binoculars. Their motions around Jupiter can be observed over the course of several hours.

**Saturn**, the outermost of the navigational planets, comes to opposition at intervals of about 380 days. It is visible for about 175 days before and after opposition, and disappears for about 25 days near conjunction. At opposition it becomes as bright as magnitude  $+0.8$  to  $-0.2$ . Through good, high powered binoculars, Saturn appears as elongated because of its system of rings. A telescope is needed to examine the rings in any detail. Saturn is now known to have at least 62 satellites, none of which are visible to the unaided eye.

**Uranus**, **Neptune** and the dwarf planet, **Pluto**, are too faint to be used for navigation; Uranus, at about magnitude 5.5, is faintly visible to the unaided eye.

### 1312. The Moon

The **Moon** is the only satellite of direct navigational interest. It revolves around the Earth once in about 27.3 days, as measured with respect to the stars. This is called the **sideral month**. Because the Moon rotates on its axis with the same period with which it revolves around the Earth, the same side of the Moon is always turned toward the Earth. The cycle of phases depends on the Moon’s revolution with respect to the Sun. This **synodic month** is approximately 29.53 days, but can vary from this average by up to a quarter of a day during any given month.

When the Moon is in conjunction with the Sun (new Moon), it rises and sets with the Sun and is lost in the Sun’s glare. The Moon is always moving eastward at about  $12.2^{\circ}$  per day, so that sometime after conjunction (as little as 16 hours, or as long as two days), the thin lunar crescent can be observed after sunset, low in the west. For the next couple of weeks, the Moon will **wax**, becoming more fully illum-

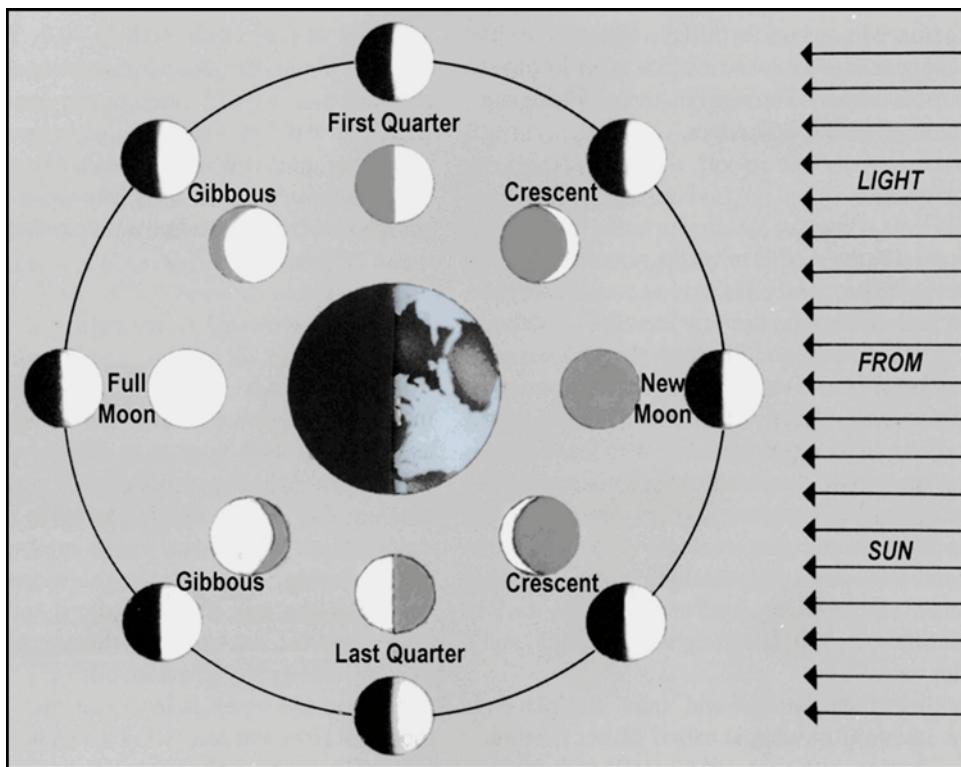


Figure 1312a. Phases of the Moon. The inner figures of the Moon represent its appearance from the Earth.

nated. From day to day, the Moon will rise (and set) later, becoming increasingly visible in the evening sky, until (about 7 days after new Moon) it reaches first quarter, when the Moon rises about noon and sets about midnight. Over the next week the Moon will rise later and later in the afternoon until full Moon, when it rises about sunset and dominates the sky throughout the night. During the next couple of weeks the Moon will **wane**, rising later and later at night. By last quarter (a week after full Moon), the Moon rises about midnight and sets at noon. As it approaches new Moon, the Moon becomes an increasingly thin crescent, and is seen only in the early morning sky. Sometime before conjunction (16 hours to 2 days before conjunction) the thin crescent will disappear in the glare of morning twilight.

At full Moon, the Sun and Moon are on opposite sides of the ecliptic. Therefore, in the winter the full Moon rises early, crosses the celestial meridian high in the sky, and sets late; as the Sun does in the summer. In the summer the full Moon rises in the southeastern part of the sky (Northern Hemisphere), remains relatively low in the sky, and sets along the southwestern horizon after a short time above the horizon.

At the time of the autumnal equinox, the part of the ecliptic opposite the Sun is most nearly parallel to the horizon. Since the eastward motion of the Moon is approximately along the ecliptic, the delay in the time of rising of the full Moon from night to night is less than at other times of the year. The full Moon nearest the autumnal equinox is called the **Harvest Moon**; the full Moon a

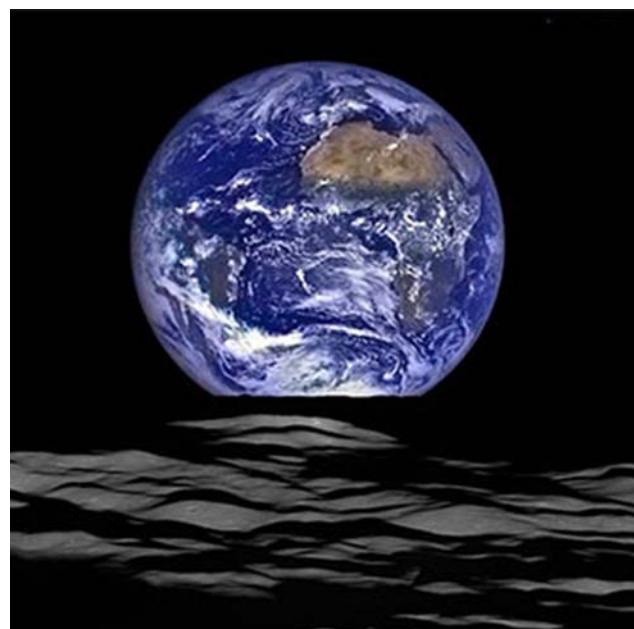


Figure 1312b. Earthrise from the surface of the Moon.  
Image courtesy of NASA.

month later is called the **Hunter's Moon**. See Figure 1312a for an image of the Phases of the Moon.

See Figure 1312b for a depiction of Earthrise from the surface of the moon.

### 1313. Comets and Meteors

Although **comets** are noted as great spectacles of nature, very few are visible without a telescope. Those that become widely visible do so because they develop long, glowing tails. Comets consist of a solid, irregularly shaped nucleus, a few kilometers across, composed of rock and ice. As the nucleus approaches the Sun in its orbit, the ice evaporates and forms an atmosphere around the nucleus, called the coma, and the tail. The tail, which may eventually extend tens of millions of kilometers or more, consists of both gas and dust; the dust reflects sunlight while the gases fluoresce. The tail is driven away from the direction of the Sun by radiation pressure and solar wind. The tail is so thin that stars can easily be seen through it.

Compared to the well-ordered orbits of the planets, comets are erratic and inconsistent. Some travel east to west and some west to east, in highly eccentric orbits inclined at any angle to the ecliptic. Periods of revolution range from about 3 years to thousands of years. Some comets may speed away from the solar system after gaining velocity as they pass by Jupiter or Saturn.

Of the known comets in our solar system, Halley's comet is the most famous because it returns about every 75 years. Its appearance in 1910 was spectacular but its 1986 apparition was hardly noticed, especially in the northern hemisphere. It will return in 2061. Comet Hale-Bopp, easily visible from the northern hemisphere in the spring of 1997, is said to have been seen by more people than any other comet in history. Other recent bright comets include Comet Ikeya-Seki (1965), Comet West (1976), and Comet McNaught (2007), the last of which was most spectacular from the southern hemisphere.

The short-period comets long ago lost the gasses needed to form a tail. Long period comets, such as comet Hyakutake, are more likely to develop tails. See Figure 1313. The visibility of a comet depends very much on how close it approaches the Earth. Hyakutake's passage on March 25, 1996 was one of the closest cometary approaches of the previous 200 years.

The visibility of a comet depends very much on how close it approaches the Earth. In 1910, Halley's comet spread across the sky. Yet when it returned in 1986, the Earth was not well situated to get a good view, and it was barely visible to the unaided eye.

**Meteors**, popularly called **shooting stars**, are rocks or rock particles from space that fall toward the Earth and are heated to incandescence by air friction in the Earth's upper atmosphere. They are visible as streaks of light in the night sky that generally last no longer than a few seconds. The particles involved, called meteoroids, range in size from dust grains to boulders, with the former much more frequent than the latter. A particularly bright meteor is called a **fireball**. One that explodes is called a **bolide**. The rare meteoroid that survives its trip through the atmosphere and lands as a solid particle is called a **meteorite**.



Figure 1313. Comet Hyakutake made its closest approach to the Earth on March 25, 1996. Image courtesy of NASA.

Millions of meteors large enough to be seen enter the Earth's atmosphere each hour, and many times this number undoubtedly enter, but are too small to be seen. The cosmic dust they create constantly rains down on the Earth, tons per day. Meteors are seen more frequently in the pre-dawn hours than at other times of the night because the pre-dawn sky is on the leading side of the Earth as it moves along its orbit, where more meteoroid particles collect.

**Meteor showers** occur at certain times of the year when the Earth passes through **meteor swarms** (streams of meteoroid particles), the scattered remains of comets that have broken apart. At these times the number of meteors observed is many times the usual number.

A faint glow sometimes observed extending upward approximately along the ecliptic before sunrise and after sunset has been attributed to the reflection of sunlight from quantities of this material. This glow is called **zodiacal light**. A faint glow at that point of the ecliptic 180° from the Sun is called the **gegenschein** or **counterglow**.

### 1314. Stars

**Stars** are distant Suns, in many ways resembling our own. Like the Sun, stars are massive balls of gas that create their own energy through thermonuclear reactions.

Although stars differ in size and temperature, these differences are apparent only through analysis by astronomers. Some differences in color are noticeable to the unaided eye. While most stars appear white, some (those of lower temperature) have a reddish hue. Orion, blue Rigel and red Betelgeuse, located on opposite sides of the belt, constitute a noticeable contrast.

The stars are not distributed uniformly around the sky. Stars appearing in the same area of the sky can bring to mind patterns. Ancient peoples supplied star patterns with names and myths; today we call them **constellations**. Today professional astronomers recognize 88 "modern"

constellations, used to identify areas of the sky.

Under ideal viewing conditions, the dimmest star that can be seen with the unaided eye is of the sixth magnitude. In the entire sky there are about 6,000 stars of this magnitude or brighter. Half of these are below the horizon at any time. Because of the greater absorption of light near the horizon, where the path of a ray travels for a greater distance through the atmosphere, not more than perhaps 2,500 stars are visible to the unaided eye at any time. However, the average navigator seldom uses more than perhaps 20 or 30 of the brighter stars.

Stars which exhibit a noticeable change of magnitude are called **variable stars**. A star which suddenly becomes several magnitudes brighter and then gradually fades is called a **nova**. A particularly bright nova is called a **supernova**. Supernovae that are visible to the unaided eye are very rare, occurring less than once per century on average.

Two stars which appear to be very close together are called a **double star system**. They may just lie in the same direction of the sky and not be physically related to each other. If they are gravitational bound to each other, they are known as a **binary star system**. The bright star Sirius is actually one component of a binary star system; the other component is too faint to be seen without a telescope. If more than two stars are included in a group, it is called a **multiple star system**.

A group of a few dozen to several hundred stars moving through space together is called an **open cluster**. The Pleiades is an example of an open cluster. There are also spherically symmetric clusters of hundreds of thousands of stars known as **globular clusters**. The globular clusters are all too distant to be seen with the naked eye.

A cloudy patch of matter in the heavens is called a **nebula**. If it is within the galaxy of which the Sun is a part, it is called a **galactic nebula**; if outside, it is called an **extragalactic nebula**.

Motion of a star through space can be classified by its vector components. That component in the line of sight is called **radial motion**, while that component across the line of sight, causing a star to change its apparent position relative to the background of more distant stars, is called **proper motion**.

## APPARENT MOTION

### 1316. Apparent Motion due to Rotation of the Earth

The **apparent motion** of the heavens arising from the Earth's rotation is much greater than other motions of celestial bodies. This motion causes celestial bodies to appear to rise along the eastern half of the horizon, climb to their maximum altitude as they cross the meridian, and set along the western horizon, at about the same point relative

### 1315. Galaxies

A **galaxy** is a vast collection of clusters of stars and clouds of gas. In many galaxies the stars tend to congregate in groups called **star clouds** arranged in long spiral arms. The spiral nature is believed due to matter density waves that propagate through the galaxy over time (Figure 1315).



Figure 1315. Spiral nebula Messier 51. Image courtesy of NASA.

The Earth is located in the Milky Way galaxy, a slowly spinning disk more than 100,000 light years in diameter. All the bright stars in the sky are in the Milky Way. However, the most dense portions of the galaxy are seen as the great, broad band that glows in the summer nighttime sky. When we look toward the constellation Sagittarius, we are looking toward the center of the Milky Way, 25,000 light years away.

Despite their size and luminance, almost all other galaxies are too far away to be seen with the unaided eye. An exception in the northern hemisphere is the Great Galaxy (sometimes called the Great Nebula) in Andromeda, which appears as a faint glow. In the southern hemisphere, the Large and Small Magellanic Clouds (named after Ferdinand Magellan) are the nearest known neighbors of the Milky Way. They are approximately 200,000 light years distant. The Magellanic Clouds can be seen as sizable glowing patches in the southern sky.

to due west as the rising point was to due east. This apparent motion of a body along the daily path, or **diurnal circle**, is approximately parallel to the plane of the equator. It would be exactly so if rotation of the Earth were the only motion and the axis of rotation of the Earth were stationary in space.

The apparent effect due to rotation of the Earth varies with the latitude of the observer. At the equator, where the

equatorial plane is perpendicular to the horizon (since the axis of rotation of the Earth is parallel to the plane of the horizon), bodies appear to rise and set vertically. Every celestial body is above the horizon approximately half the time. The celestial sphere as seen by an observer at the equator is called the right sphere, shown in Figure 1316a.

For an observer at one of the poles, bodies having constant declination neither rise nor set, remaining parallel to the horizon

(neglecting precession of the equinoxes and changes in refraction). They circle the sky, always at the same altitude, making one complete trip around the horizon each sidereal day (See Section 1611). At the North Pole the motion is clockwise, and at the South Pole it is counterclockwise. Approximately half the stars are always above the horizon and the other half never are. The parallel sphere at the poles is illustrated in Figure 1316b.

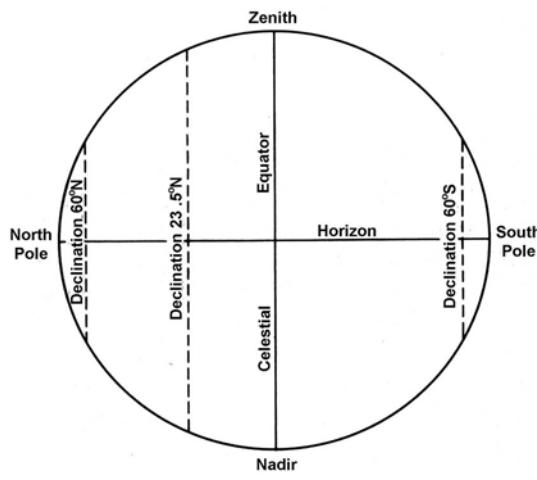


Figure 1316a. The right sphere.

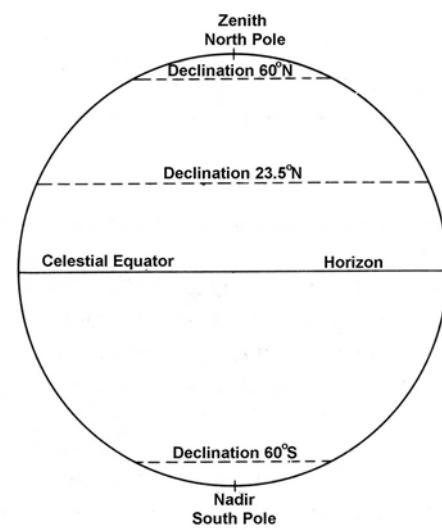


Figure 1316b. The parallel sphere.

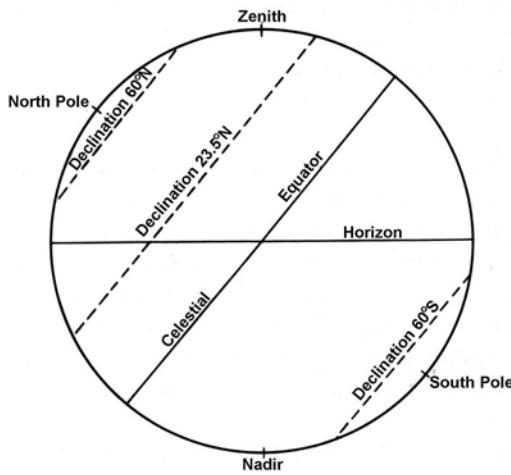


Figure 1316c. The oblique sphere at latitude 40°N.

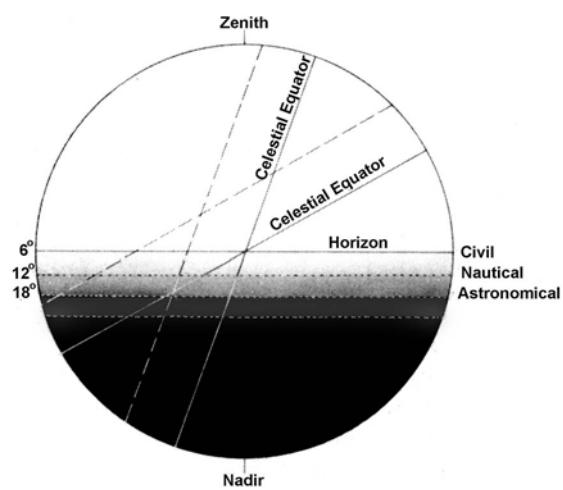


Figure 1316d. The various twilight at latitude 20°N and latitude 60°N.

Between these two extremes, the apparent motion is a combination of the two. On this oblique sphere, illustrated in Figure 1316c, circumpolar celestial bodies are those that remain above the horizon during the entire 24 hours, circling the elevated celestial pole. The portion of the sky where bodies are circumpolar extends from the elevated pole to approximately the declination equal to 90° minus the observer's latitude. For example, the stars of Ursa Major

(the Big Dipper) and Cassiopeia are circumpolar for many observers in the United States.

An area of the celestial sphere approximately equal to the circumpolar area around the depressed pole remains constantly below the horizon. For example, Crux is not visible to most observers in the United States. Other celestial bodies rise obliquely along the eastern horizon, climb to maximum altitude at the celestial meridian, and set along

Twilight	Lighter limit	Darker limit	At darker limit
Civil	sunrise/set	$-6^\circ$	Horizon clear; bright stars visible
Nautical	$-6^\circ$	$-12^\circ$	Horizon not visible
Astronomical	$-12^\circ$	$-18^\circ$	Full night

Table 1316. Limits of the three twilights.

the western horizon. The length of time above the horizon and the altitude at meridian transit vary with both the latitude of the observer and the declination of the body. Days and nights are always about the same length in the tropics. At higher latitudes the increased obliquity result in a greater change in the length of the day and longer periods of twilight. North of the Arctic Circle and south of the Antarctic Circle the Sun is circumpolar for part of the year. This is sometimes termed the land of the midnight Sun, where the Sun does not set during part of the summer and does not rise during part of the winter.

The increased obliquity at higher latitudes explains why days and nights are always about the same length in the tropics, and the change of length of the day becomes greater as latitude increases, and why twilight lasts longer in higher latitudes. Evening twilight begins at sunset, and morning twilight ends at sunrise. The darker limit of twilight occurs when the center of the Sun is a stated number of degrees below the celestial horizon. Three kinds of twilight are defined: civil, nautical and astronomical. See Table 1316.

The conditions at the darker limit are relative and vary considerably under different atmospheric conditions.

In Figure 1316d, the twilight band is shown, with the darker limits of the various kinds indicated. The nearly vertical celestial equator line is for an observer at latitude  $20^\circ\text{N}$ . The nearly horizontal celestial equator line is for an observer at latitude  $60^\circ\text{N}$ . The broken line for each case is the diurnal circle of the Sun when its declination is  $15^\circ\text{N}$ . The portion of the diurnal circle between the lighter and the darker limits indicates the relative duration of a particular type of twilight at the two example latitudes. But the relative duration is not directly proportional to the relative length of line shown since the projection is orthographic. Note that complete darkness will not occur at latitude  $60^\circ\text{N}$  when the declination of the Sun is  $15^\circ\text{N}$ .

### 1317. Apparent Motion due to Revolution of the Earth

If it were possible to stop the rotation of the Earth so that the celestial sphere would appear stationary, the effects of the revolution of the Earth would become more noticeable. The Sun would appear to move eastward a little

less than  $1^\circ$  per day, to make one complete trip around the Earth in a year. If the Sun and stars were visible at the same time this motion could be observed by watching the changing position of the Sun with respect to the stars. A better way is to observe the constellations at the same time each night. Each night, a star rises nearly four minutes earlier than on the previous night. The period from star rise on one night to its rise on the next night is called a **sidereal day**. Thus, the celestial sphere appears to shift westward nearly  $1^\circ$  each night, so that different constellations are associated with different seasons of the year.

Apparent motions of planets and the Moon are due to a combination of their motions and those of the Earth. If the rotation of the Earth were stopped, the combined apparent motion due to the revolutions of the Earth and other bodies would be similar to that occurring if both rotation and revolution of the Earth were stopped. Stars would appear nearly stationary in the sky but would undergo a small annual cycle of change due to aberration. The motion of the Earth in its orbit is sufficiently fast to cause the light from stars to appear to shift slightly in the direction of the Earth's motion. This is similar to the effect one experiences when walking in vertically-falling rain that appears to come from ahead due to the observer's own forward motion. The apparent direction of the light ray from the star is the vector difference of the motion of light and the motion of the Earth, similar to that of apparent wind on a moving vessel. This effect is most apparent for a body perpendicular to the line of travel of the Earth in its orbit, for which it reaches a maximum value of 21.2 seconds of arc. The effect of aberration can be noted by comparing the coordinates (declination and sidereal hour angle) of various stars throughout the year. A change is observed in some bodies as the year progresses, but at the end of the year the values have returned almost to what they were at the beginning. The reason they do not return exactly is due to proper motion and precession of the equinoxes. It is also due to nutation, an irregularity in the motion of the Earth due to the disturbing effect of other celestial bodies, principally the Moon. Polar motion is a slight wobbling of the Earth about its axis of rotation and sometimes wandering of the poles. This motion, which does not exceed 40 feet from the mean position, produces slight variation of latitude and longitude of places on the Earth.

### 1318. Apparent Motion due to Movement of other Celestial Bodies

Each celestial body makes its own contribution to its apparent motion:

The Moon revolves about the Earth each month, rising in the west and setting in the east. Its orbital plane is slightly inclined to the ecliptic (see Section 1319), and is continuously changing in response to perturbations in its motion, primarily by the Sun.

The planets revolve about the Sun (technically, the solar system barycenter, which is within the sun's interior). The inferior planets, Mercury and Venus, appear to move eastward and westward relative to the Sun. The period for Mercury's motion is 116 days and the period for Venus is 584 days (see Section 1310). The superior planets make an apparent revolution around the Earth, from west to east. The periods for their motion varies from 780 to 367 days, depending on the planet (see Section 1311).

The stars revolve about the galactic center. As they move about the galactic center, the stars, including the Sun, move with respect to one another. The component of their motion across the line of sight is called proper motion. The maximum observed proper motion is that of Barnard's Star, which is moving at the rate of 10.3 seconds of arc per year. Barnard's Star is a tenth-magnitude star, not visible to the unaided eye. Rigil Kentaurus has the greatest proper motion of the 57 stars listed on the daily pages of the almanacs, about 3.7 seconds per year. Arcturus has the greatest proper motion of the navigational stars in the Northern Hemisphere, 2.3 seconds per year. Over the course of a few years, proper motions are very small; they can be ignored when reducing celestial navigation sights. A few thousand years of proper motion is sufficient to materially alter the look of some familiar constellations.

### 1319. The Ecliptic and the Inclination of the Earth's Axis

The **ecliptic** is the mean path of the Sun through the heavens arising from the annual revolution of the Earth in its orbit and appears as a great circle on the celestial sphere. The ecliptic is currently inclined at an angle of about  $23^{\circ}26'$  to the celestial equator. This angle is called the **obliquity of the ecliptic** and is due to the inclination or tilt of Earth's rotational axis relative to its orbital plane. The perturbations of the other planets on the Earth's orbital plane decreases the obliquity of the ecliptic by about  $2/3$  of an arc minute per century. The obliquity of the ecliptic causes the Sun to appear to move north and south over the course of the year, giving the Earth its seasons and changing lengths of periods of daylight.

Refer to Figure 1319a. The **vernal equinox** occurs when the center of the Sun crosses the equator going north. It occurs on or about March 21, and is the start of astronomical spring in the northern hemisphere. At this time the Sun

is rising at the North Pole and setting at the South Pole, the Sun shines equally on both hemispheres, and day and night are approximately the same length over the entire world. The **summer solstice** occurs on or about June 21. On this date, the northern pole of the Earth's axis is tilted toward the Sun. The north polar regions are continuously in sunlight; the Northern Hemisphere is having its summer with long, warm days and short nights; the Southern Hemisphere is having winter with short days and long, cold nights; and the south polar region is in continuous darkness. The **autumnal equinox** occurs on or about September 23. On this date, the Sun is setting at the North Pole and rising at the South Pole, the Sun again shines equally on both hemispheres, and day and night are approximately the same length over the entire world. The **winter solstice** occurs on or about December 22. On this date, the Southern Hemisphere is tilted toward the Sun and conditions are the reverse of those six months earlier; the Northern Hemisphere is having its winter, and the Southern Hemisphere its summer.

The word **equinox** means "equal nights". At the equinoxes, the Sun is directly over the equator. It remains above the horizon for approximately 12 hours. The length of daylight is not exactly 12 hours because of refraction, the solar semidiameter, and the height of the eye of the observer. These cause the Sun to be above the horizon a few minutes longer than below the horizon. Following the **vernal equinox**, the Sun's declination increases (becomes more northerly), and the Sun climbs higher in the sky each day (at the latitudes of the United States), until the summer solstice, when a declination of about  $23^{\circ}26'$  north of the celestial equator is reached. The word **solstice**, meaning "Sun stands still," is used because the Sun halts its apparent northward or southward motion and momentarily "stands still" before it starts in the opposite direction. This action, somewhat analogous to the "stand" of the tide, refers to the motion in a north-south direction, not to the daily apparent revolution around the Earth.

Over the course of a year the distance between the Earth and the Sun changes by about 1.7%. The Earth is closest to the Sun during the northern hemisphere winter. To conserve angular momentum, the Earth travels faster when nearest the Sun, like a spinning ice skater pulling her arms in. As a result, the northern hemisphere (astronomical) winter is shorter than its summer by about seven days.

The distance between the Earth and Sun is not the primary source for the difference in temperature during the different seasons. Over a year, the change in Earth-Sun distance changes the solar energy flux only 3% from the average value. The tilt of the Earth's axis has a much larger affect. During the summer the rays are more nearly vertical, and hence more concentrated, as shown in Figure 1319b. At the polar circle on the summer solstice for a hemisphere, the solar flux (energy per unit area per unit time) is 73% of the flux at the tropic where the Sun is directly overhead. Winter sunlight is distributed over a larger area and shines fewer hours each day, causing less total heat energy to reach the

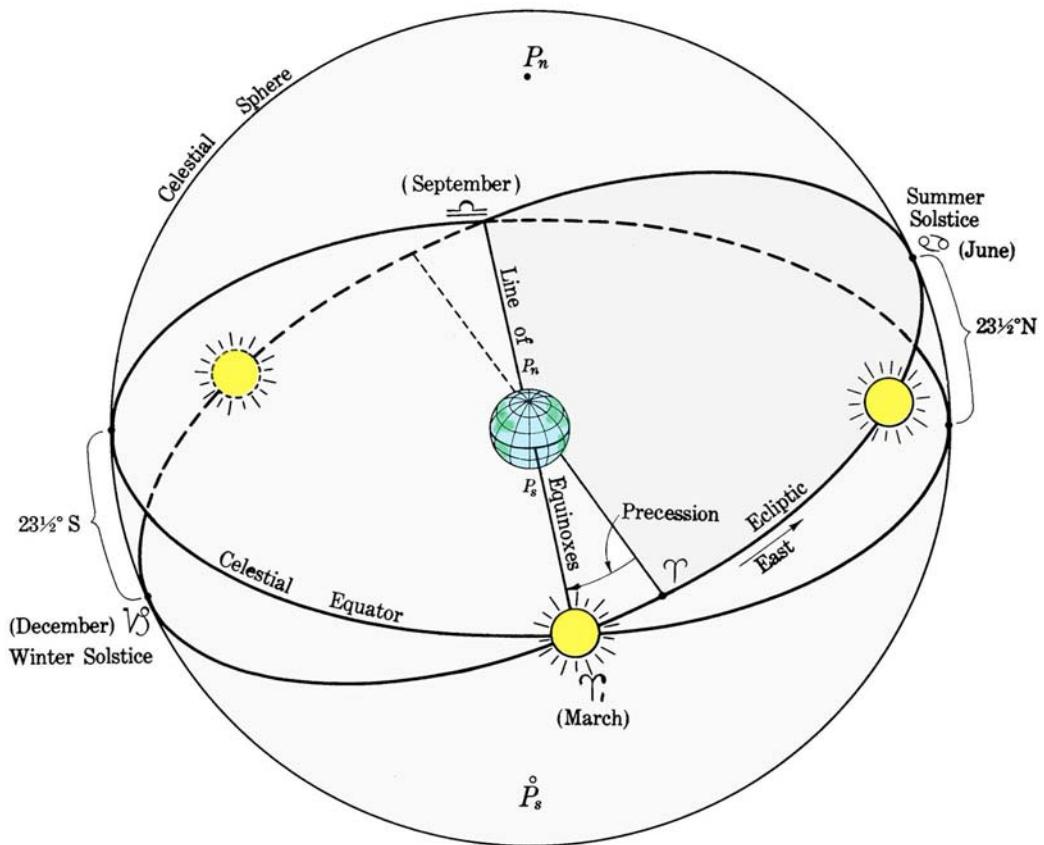


Figure 1319a. Apparent motion of the Sun in the ecliptic.

Earth. The solar flux at the polar circle on the winter solstice is nearly zero and the flux at the tropic is only 69% of the flux at the summer solstice.

Astronomically, the seasons begin at the equinoxes and solstices. Meteorologically, they differ from place to place. During the summer the Sun is above the horizon more than half the time. So, the total energy being added by absorption during a longer period than it is being lost by radiation. Following the summer solstice, the surface at a given latitude continues to receive more energy than it dissipates, but a decreasing amount. Gradually, the amount decreases until the surface is losing more energy than it gains from the Sun. This effect explains the lag of the seasons. It is analogous to the day, when the highest temperatures normally occur several hours after the Sun reaches maximum altitude at local noon.

At some time during the year, the Sun is directly overhead everywhere between the latitudes of about 23°26'N and about 23°26'S. Except at the limits, this occurs twice: once as the Sun appears to move northward, and the second time as it moves southward. The area on Earth between these latitudes is called the Tropics, or the **torrid zone**. The northern limit is the **Tropic of Cancer**, and the southern limit is the **Tropic of Capricorn**. These names come from the constellations the Sun entered at the

solstices when the names were first used more than 2,000 years ago. Today, the Sun is in the next constellation to the west because of precession of the equinoxes. The parallels about 23°26' from the poles, marking the approximate limits of the circumpolar Sun, are called **polar circles**. The polar circle in the Northern Hemisphere is called the **Arctic Circle**, and the one in the Southern Hemisphere is called the **Antarctic Circle**. The areas inside the polar circles are the north and south **frigid zones**. The regions between the frigid zones and the torrid zones are the north and south **temperate zones**.

The expression “**vernal equinox**” and associated expressions are applied both to the times and points of occurrence of these phenomena. The vernal equinox is also called the **first point of Aries** (symbol ‘ $\gamma$ ’) because, when the name was given, the Sun entered the constellation Aries, the ram, as the Sun crossed the equator going north. The vernal equinox is of interest to navigators because it is the origin for measuring **sidereal hour angle**. The terms March equinox, June solstice, September equinox, and December solstice are occasionally applied as appropriate, because the more common names are associated with the seasons in the Northern Hemisphere and are six months out of step for the Southern Hemisphere.

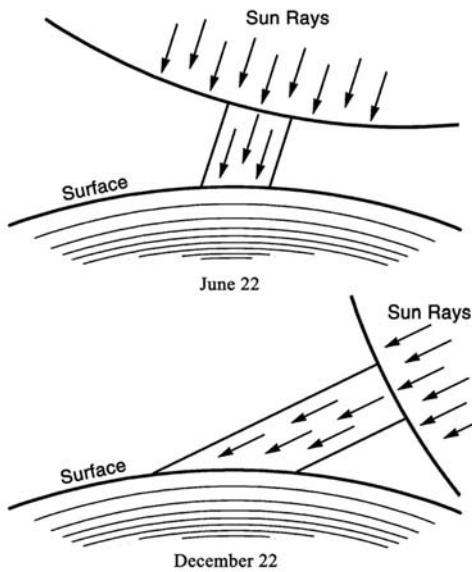


Figure 1319b. Sunlight in summer and winter. Winter sunlight is distributed over a larger area and shines fewer hours each day, causing less heat energy to reach the Earth.

### 1320. Precession and Nutation

The Earth's axis precesses: the motion of its rotation axis is similar to that of a top spinning with its axis tilted. The precession is in response to torques principally by the Sun and Moon. The spinning Earth responds to these torques in the manner of a gyroscope. The result is a slow westward movement of the equinoxes and solstices. This westward motion of the equinoxes along the ecliptic is called precession of the equinoxes. The precession has a period of about 25,800 years. There are also a series of short period motions of the Earth's axis of rotation called nutation. See Figure 1320. The nutations are all quite small. The largest nutation has an amplitude of 0.2' and a period of 18.6 years. The next largest nutation has an amplitude of just 0.01' and a period of 0.5 years.

The sidereal hour angle is measured from the vernal equinox, and declination from the celestial equator, so the coordinates of celestial bodies change because of precession. The total motion with respect to the ecliptic, called general precession, is about 50.29" per year. It may be divided into two components with respect to the celestial equator: precession in right ascension (about 46.12" per year) measured along the celestial equator, and precession in declination (about 20.04" per year) measured perpendicular to the celestial equator. The annual change in the coordinates of any given star, due to precession alone, depends upon its position on the celestial sphere.

Since precession changes the direction of Earth's pole, Polaris will not always be Earth's "Pole Star". Currently,

the north celestial pole is moving closer to Polaris because of precession. It will pass at a distance of approximately 28' about the year 2102. Afterward, the polar distance will increase, and eventually other stars, in their turn, will become the Pole Star.

### 1321. The Zodiac

The **zodiac** is a circular band of the sky extending 8° on each side of the ecliptic. The navigational planets and the Moon are within these limits. The zodiac is divided into 12 sections of 30° each, each section being given the name and symbol ("sign") of a constellation. These are shown in Figure 1321. The names were assigned more than 2,000 years ago, when the Sun entered Aries at the vernal equinox, Cancer at the summer solstice, Libra at the autumnal equinox, and Capricornus at the winter solstice. Because of precession, the zodiacal signs have shifted with respect to the constellations. Thus at the time of the vernal equinox, the Sun is said to be at the "first point of Aries," though it is in the constellation Pisces.

### 1322. Time and the Calendar

Traditionally, astronomy has furnished the basis for measurement of time, a subject of primary importance to the navigator. The **year** is associated with the revolution of the Earth in its orbit. The **day** is one rotation of the Earth about its axis.

The duration of one rotation of the Earth depends upon the external reference point used, the most common is using the Sun. One rotation relative to the Sun is called a **solar day**. However, an actual solar day varies in length. This variation is removed by using a "fictitious **mean**" Sun, leading to what we refer to as "**mean time**." For a more complete discussion see Chapter 16 - Time; Section 1600 discusses apparent and mean solar time.

**Universal Time (UT)** is a generic reference to one (of several) time scales that approximate the mean diurnal motion of the Sun. Loosely, UT is mean solar time on the Greenwich meridian. The terms "Universal Time" and "**Greenwich Mean Time**" are sometimes used interchangeably, but the latter is being deprecated. Universal Time is the standard in the application of astronomy to navigation. See Chapter 16 - Section 1602 for a more complete discussion.

If the vernal equinox is used as the reference, a **sidereal day** is obtained, and from it, **sidereal time**. This indicates the approximate positions of the stars, and for this reason it is the basis of star charts and star finders. Because of the revolution of the Earth around the Sun, a sidereal day is about 3 minutes 56 seconds shorter than a solar day, and there is one more sidereal than solar days in a year. One mean solar day equals 1.00273791 mean sidereal days. Because of precession of the equinoxes, one rotation of the Earth with respect to the stars is not quite the same as one

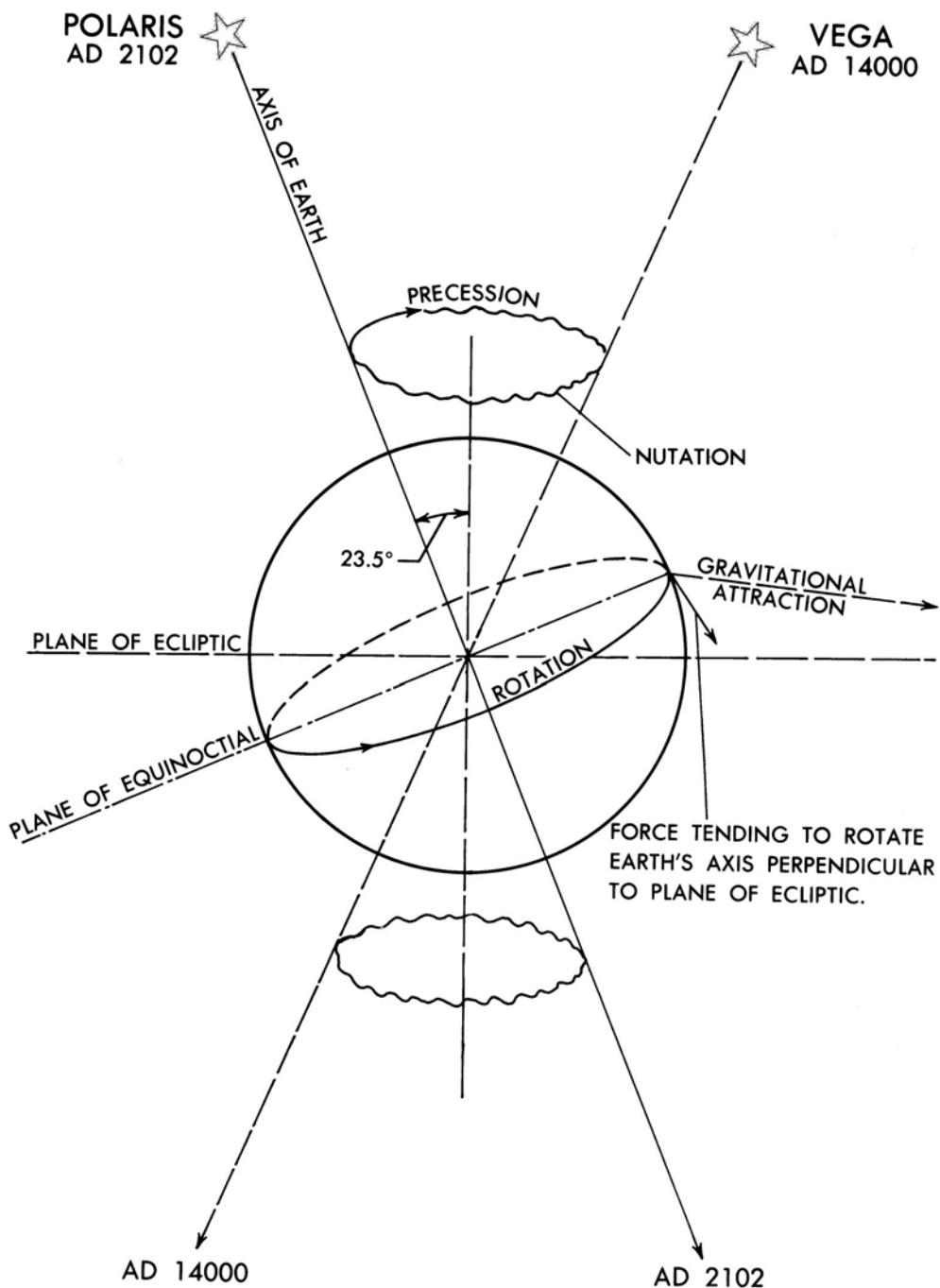


Figure 1320. Precession and nutation.

rotation with respect to the vernal equinox. One mean solar day averages 1.0027378118868 rotations of the Earth with respect to the stars.

In tide analysis, the Moon is sometimes used as the reference, producing a **lunar day** averaging 24 hours 50 minutes (mean solar units) in length, and lunar time.

Since each kind of day is divided arbitrarily into 24 hours, each hour having 60 minutes of 60 seconds, the

length of each of these units differs somewhat in the various kinds of time.

Time is also classified according to the terrestrial meridian used as a reference. **Local time** results if one's own meridian is used, **zone time** if a nearby reference meridian is used over a spread of longitudes, and **Greenwich** or **Universal Time** if the Greenwich meridian is used.

The period from one vernal equinox to the next (the

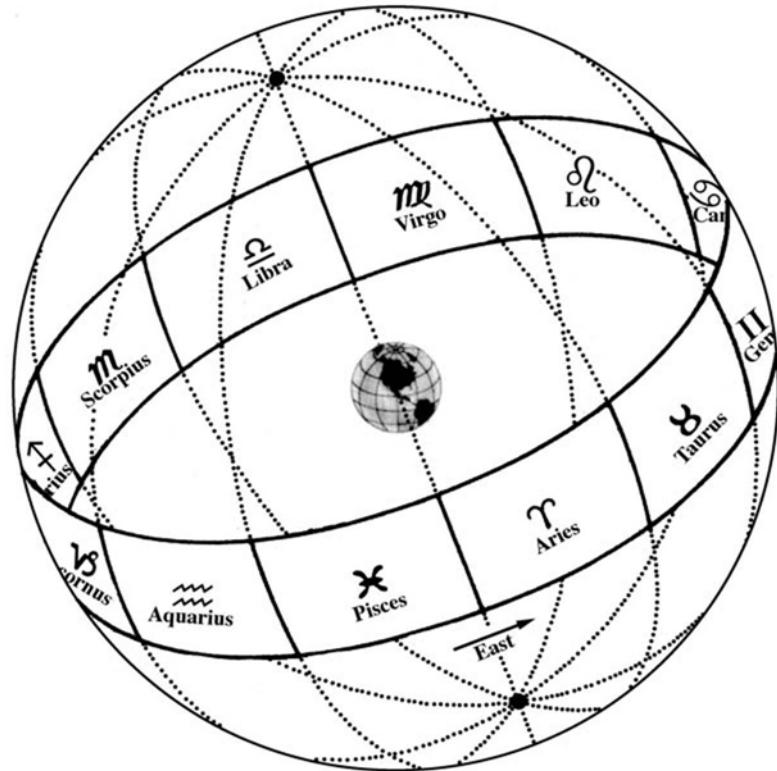


Figure 1321. The Zodiac.

cycle of the seasons) is known as the **tropical year**. It is approximately 365 days, 5 hours, 48 minutes, 45 seconds, though the length has been slowly changing for many centuries. Our calendar, the **Gregorian calendar**, approximates the tropical year with a combination of common years of 365 days and leap years of 366 days. A **leap year** is any year divisible by four, unless it is a century year, which must be divisible by 400 to be a leap year. Thus, 1700, 1800, and 1900 were not leap years, but 2000 was. A critical mistake was made by John Hamilton Moore in calling 1800 a leap year, causing an error in the tables in his book, *The Practical Navigator*. This error caused the loss of at least one ship and was later discovered by Nathaniel Bowditch while writing the first edition of *The New American Practical Navigator*.

See Chapter 16 for an in-depth discussion of time.

### 1323. Eclipses

If the orbit of the Moon coincided with the plane of the ecliptic, the Moon would pass in front of the Sun at every new Moon, causing a solar eclipse. At full Moon, the Moon would pass through the Earth's shadow, causing a lunar eclipse. Because of the Moon's orbit is inclined  $5^\circ$  with respect to the ecliptic, the Moon usually passes above or below the Sun at new Moon and above or below the Earth's shadow at full Moon. However, there are two points at

which the plane of the Moon's orbit intersects the ecliptic. These are the **nodes** of the Moon's orbit. If the Moon passes one of these points at the same time as the Sun, a **solar eclipse** takes place. This is shown in Figure 1323.

The Sun and Moon are of nearly the same apparent size to an observer on the Earth. If the Moon is near perigee (the point in its orbit closest to the Earth), the Moon's apparent diameter is larger than that of the Sun, and its umbra (darkest part of the shadow) reaches the Earth as a nearly round dot. The dot moves rapidly across the Earth, from west to east, as the Moon continues in its orbit. Within the dot, the Sun is completely hidden from view, and a **total eclipse** of the Sun occurs. The width of this dot on the Earth's surface varies from eclipse to eclipse, but can be as large as a couple hundred miles. On the **path of totality**, a **partial eclipse** occurs as the disk of the Moon appears to move slowly across the face of the Sun, hiding an ever-increasing part of it, until the total eclipse occurs. Because of the uneven edge of the mountainous Moon, the light is not cut off evenly. But several last illuminated portions appear through the Moon's valleys or passes between the Moon's mountain peaks. These are called **Baily's Beads**. For a considerable distance around the umbral shadow, part of the surface of the Sun is obscured, and a partial eclipse occurs.

A total eclipse is a spectacular phenomenon. As the last light from the Sun is cut off, the solar **corona**, or envelope of

thin, illuminated gas around the Sun becomes visible. Wisps of more dense gas may appear as **solar prominences**. The only light reaching the observer is that diffused by the atmosphere surrounding the shadow. As the Moon appears to continue on across the face of the Sun, the Sun finally emerges from the other side, first as Baily's Beads, and then as an ever widening crescent until no part of its surface is obscured by the Moon.

The duration of a total eclipse depends upon how nearly the Moon crosses the center of the Sun, the location of the shadow on the Earth, the relative orbital speeds of the Moon and Earth, and (principally) the relative apparent diameters of the Sun and Moon. The maximum length that can occur is a little more than seven minutes.

If the Moon is near apogee, its apparent diameter is less than that of the Sun, and its shadow does not quite reach the

Earth. Over a small area of the Earth directly in line with the Moon and Sun, the Moon appears as a black disk almost covering the surface of the Sun, but with a thin ring of the Sun around its edge. This is known as an **annular eclipse**; these occur a little more often than total eclipses.

If the **umbrial shadow** of the Moon passes close to the Earth, but not directly in line with it, a partial eclipse may occur without a total or annular eclipse.

An eclipse of the Moon (or **lunar eclipse**) occurs when the Moon passes through the shadow of the Earth, as shown in Figure 1323. Since the diameter of the Earth is about  $3\frac{1}{2}$  times that of the Moon, the Earth's shadow at the distance of the Moon is much larger than that of the Moon. A total eclipse of the Moon can last nearly  $1\frac{3}{4}$  hours, and some part of the Moon may be in the Earth's shadow for almost 4 hours.

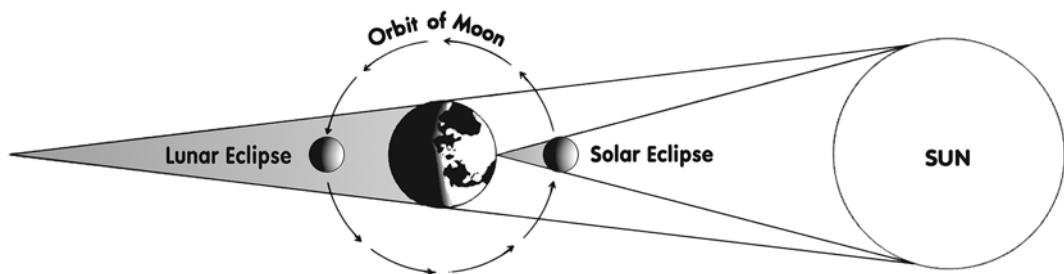


Figure 1323. Eclipses of the Sun and Moon.

During a total solar eclipse no part of the Sun is visible because the Moon is in the line of sight. But during a lunar eclipse some light does reach the Moon, diffracted by the atmosphere of the Earth, and hence the eclipsed full Moon is visible as a faint reddish disk. A lunar eclipse is visible over the entire hemisphere of the Earth facing the Moon. Anyone who can see the Moon can see the eclipse.

During any one year there may be as many as five eclipses of the Sun, and always there are at least two. There may be as many as three eclipses of the Moon, or none. The total number of eclipses during a single year does not exceed

seven, and can be as few as two. There are more solar than lunar eclipses, but the latter can be seen more often because of the restricted areas over which solar eclipses are visible.

The Sun, Earth, and Moon are nearly aligned on the line of nodes twice each “eclipse year” of 346.6 days. This is less than a calendar year because of **regression of the nodes**. In a little more than 18 years the line of nodes returns to approximately the same position with respect to the Sun, Earth, and Moon. During an almost equal period, called the **saros**, a cycle of eclipses occurs. During the following saros the cycle is repeated with only minor differences.

## COORDINATES

### 1324. Latitude and Longitude

**Latitude** and **longitude** are coordinates used to locate positions on the Earth. This section discusses three different definitions of these coordinates.

**Astronomic latitude** is the angle (ABQ, Figure 1324) between a line in the direction of gravity (AB) at a station and the plane of the equator (QQ'). **Astronomic longitude** is the angle between the plane of the celestial meridian at a station and the plane of the celestial meridian at Greenwich. These coordinates are customarily found by means of celestial observations. If the Earth were perfectly homogeneous and round, these positions would be consistent and satisfac-

tory. However, because of deflection of the vertical due to uneven distribution of the mass of the Earth, lines of equal astronomic latitude and longitude are not circles, although the irregularities are small. In the United States the east-west component of the deflection of the vertical (affecting longitude) may be a little more than  $18''$ , and the north-south component (affecting latitude) may be as much as  $25''$ .

**Geodetic latitude** is the angle (ACQ, Figure 1324) between a normal to the spheroid (AC) at a station and the plane of the geodetic equator (QQ'). **Geodetic longitude** is the angle between the plane defined by the normal to the spheroid and the axis of the Earth and the plane of the geo-

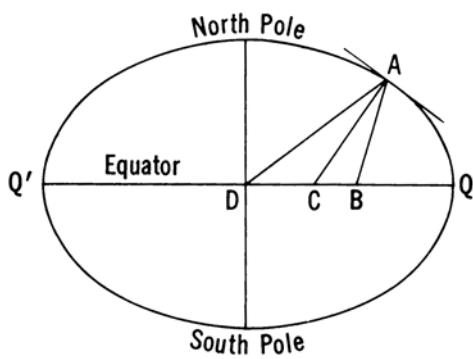


Figure 1324. Three kinds of latitude at point A.

detic meridian at Greenwich. These values are obtained when astronomical latitude and longitude are corrected for deflection of the vertical. These coordinates are used for charting and are frequently referred to as **geographic lati-**

**tude** and **geographic longitude**, although these expressions are sometimes used to refer to astronomical latitude.

**Geocentric latitude** is the angle (ADQ, Figure 1324) at the center of the ellipsoid between the plane of its equator (QQ') and a straight line (AD) to a point on the surface of the Earth. This differs from geodetic latitude because the Earth is a spheroid rather than a sphere, and the meridians are ellipses. Since the parallels of latitude are considered to be circles, geodetic longitude is geocentric, and a separate expression is not used. The difference between geocentric and geodetic latitudes is a maximum of about 11.6' at latitude 45°.

Because of the oblate shape of the ellipsoid, the length of a degree of geodetic latitude is not everywhere the same, increasing from about 59.7 nautical miles at the equator to about 60.3 nautical miles at the poles. The value of 60 nautical miles customarily used by the navigator is correct at about latitude 45°.

## MEASUREMENTS ON THE CELESTIAL SPHERE

### 1325. Elements of the Celestial Sphere

The **celestial sphere** (Section 1301) is an imaginary sphere of infinite radius with the Earth at its center (Figure 1325a). The north and south celestial poles of this sphere, PN and PS respectively, are located by extension of the Earth's mean pole of rotation. The **celestial equator** (sometimes called **equinoctial**) is the projection of the plane of the Earth's equator to the celestial sphere. A **celestial meridian** is a great circle passing through the celestial poles and the zenith of any location on the Earth.

The point on the celestial sphere vertically overhead of an observer is the **zenith**, and the point on the opposite side of the sphere vertically below him or her is the **nadir**. The zenith and nadir are the extremities of a diameter of the celestial sphere through the observer and the common center of the Earth and the celestial sphere. The arc of a celestial meridian between the poles is called the **upper branch** if it contains the zenith and the **lower branch** if it contains the nadir. The upper branch is frequently used in navigation, and references to a celestial meridian are understood to mean only its upper branch unless otherwise stated.

In order to uniquely define every point on the celestial sphere, a coordinate system must be defined. One such coordinate system uses **hour angles** and **declination**. With these two angular measurements, every position on the celestial sphere can be uniquely described.

**Hour circles** are great circles on the celestial sphere that pass through the celestial poles, and are therefore perpendicular to the **celestial equator**. An **hour angle** is the angle from a "reference" hour circle to the hour circle of a point (or object). There are three main "reference" hour circles used in celestial navigation. The first is the hour circle

through the **vernal equinox** (also known as the **first point of Aries** ( $\varphi$ )). The angular distance west of this reference circle is called the **sidereal hour angle** (SHA) (Figure 1325b). The second is using the local meridian as the reference hour circle. The angular distance west of the local meridian is known as a **local hour angle** (LHA). And the third reference is the Greenwich meridian. Measurements west from the Greenwich meridian are known as **Greenwich hour angles**, or GHA. See Figure 1325c for a depiction of how to locate a point on the celestial sphere.

Since hour circles are perpendicular to the celestial equator, hour angles can be thought of as angular measurements along the equator. This give us one of our two coordinates needed to define every point on the celestial sphere. The second coordinate, **declination**, is the angular distance from the celestial equator along an hour circle and is measured north or south of the celestial equator in degrees, from  $0^\circ$  through  $90^\circ$ , similar to latitude on the Earth. Northern and southern declinations are sometime labeled with positive or negative values, respectively if not labeled N or S. A circle parallel to the celestial equator is called a **parallel of declination**, since it connects all points of equal declination. It is similar to a parallel of latitude on the Earth.

It is sometimes more convenient to measure hour angle either eastward or westward, as longitude is measured on the Earth, in which case it is called **meridian angle** (designated "t").

A point on the celestial sphere may also be located using **altitude** and **azimuth**, which are topocentric coordinates based upon the observer's local horizon as the primary great circle instead of the celestial equator.

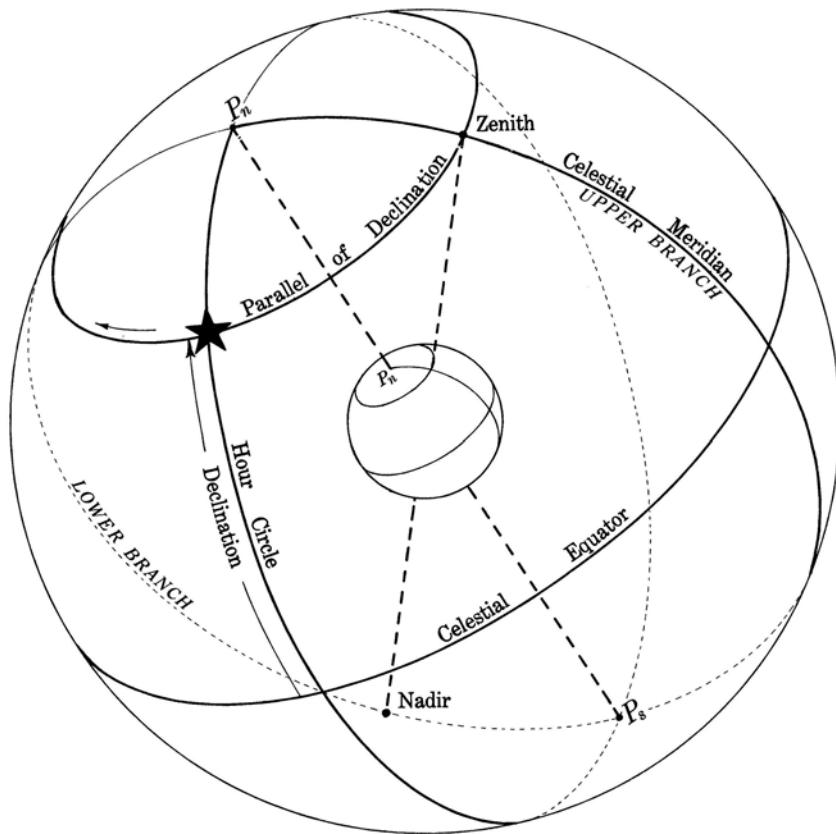


Figure 1325a. Elements of the celestial sphere.

## COORDINATE SYSTEMS

### 1326. The Celestial Equator System of Coordinates

The familiar graticule of latitude and longitude lines, expanded until it reaches the celestial sphere, forms the basis of the **celestial equator system** of coordinates. On the celestial sphere latitude becomes **declination**, while longitude becomes **sidereal hour angle**, measured from the **vernal equinox**.

**Polar distance (p)** is angular distance from a celestial pole, or the arc of an hour circle between the celestial pole and a point on the celestial sphere. It is measured along an hour circle and may vary from  $0^\circ$  to  $180^\circ$ , since either pole may be used as the origin of measurement. It is usually considered the complement of declination, though it may be either  $90^\circ - d$  or  $90^\circ + d$ , depending upon the pole used. See Figure 1326a.

**Local hour angle (LHA)** is angular distance west of the local celestial meridian, or the arc of the celestial equator between the upper branch of the local celestial meridian and the hour circle through a point on the celestial sphere, measured westward from the local celestial meridian, through  $360^\circ$ . It is

also the similar arc of the parallel of declination and the angle at the celestial pole, similarly measured. If the Greenwich ( $0^\circ$ ) meridian is used as the reference, instead of the local meridian, the expression **Greenwich hour angle (GHA)** is applied. It is sometimes convenient to measure the arc or angle in either an easterly or westerly direction from the local meridian, through  $180^\circ$ , when it is called **meridian angle (t)** and labeled E or W to indicate the direction of measurement. All bodies or other points having the same hour angle lie along the same hour circle.

Because of the apparent daily rotation of the celestial sphere, the hour angle of an object continually increases, but meridian angle increases from  $0^\circ$  at the celestial meridian to  $180^\circ$ W, which is also  $180^\circ$ E, and then decreases to  $0^\circ$  again. The rate of change in meridian angle for the mean Sun is  $15^\circ$  per hour. The rate of all other bodies except the Moon is within  $3'$  of this value. The average rate of the Moon is about  $15.5^\circ$ .

As the celestial sphere rotates, each body crosses each branch of the celestial meridian approximately once a day. This crossing is called **meridian transit** (sometimes called

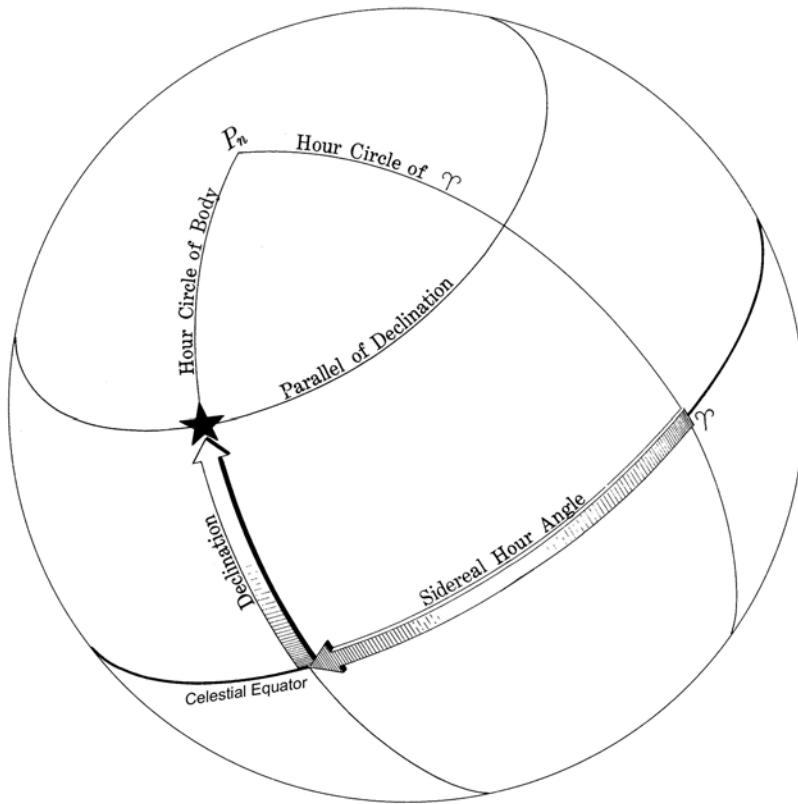


Figure 1325b. A point on the celestial sphere can be located by its declination and sidereal hour angle.

**culmination**). For circumpolar bodies, it is called **upper transit** to indicate crossing the upper branch of the meridian and **lower transit** to indicate crossing the lower branch.

The **time diagram** shown in Figure 1326b illustrates the relationship between the various hour angles and meridian angle. The circle is the celestial equator as seen from above the South Pole, with the upper branch of the observer's meridian ( $P_sM$ ) at the top. The radius  $P_sG$  is the Greenwich meridian;  $P_s\gamma$  is the hour circle of the vernal equinox. The Sun's hour circle is to the east of the observer's meridian; the Moon's hour circle is to the west of the observer's meridian. Note that when LHA is less than  $180^\circ$ , it is numerically the same and is labeled W, but that when LHA is greater than  $180^\circ$ ,  $t = 360^\circ - LHA$  and is labeled E. In Figure 1326b arc GM is the longitude, which in this case is west. The relationships shown apply equally to other arrangements of radii, except for relative magnitudes of the quantities involved.

### 1327. Atmospheric Refraction of Light

The Earth's atmosphere acts like a lens which causes light rays to bend. This bending of a light ray or path is called **refraction**. The amount of the angular change caused by refraction is primarily a function of the atmospheric density gradient. An incoming light ray is bent or

refracted towards the direction of increasing atmospheric density. The apparent path of the refracted light ray is always closer to perpendicular to the atmospheric density gradient than the path of the unrefracted ray. A light ray approaching the observer from the zenith is perpendicular to the density gradient. So, the refraction angle for the zenith is 0, and the direction of the refraction angle for other light rays is towards the zenith, making an object appear higher than if it were not refracted. In other words, an object's observed altitude is increased due to refraction. The greatest angular change occurs near the horizon where the light path is almost parallel to a given atmospheric density layer. The mean refraction angle near the horizon is approximately  $34'$ . The density gradient and refraction angle are a function of the atmospheric pressure and temperature. So, observations made near the horizon should be corrected for changes from the standard pressure (1010 mb) and temperature ( $10^\circ\text{C}$ ) used in calculating the refraction.

### 1328. The Horizons

The second set of celestial coordinates with which the navigator is directly concerned is based upon the horizon as the primary great circle. However, since several different horizons are defined, these should be thoroughly understood before proceeding with a consideration of the

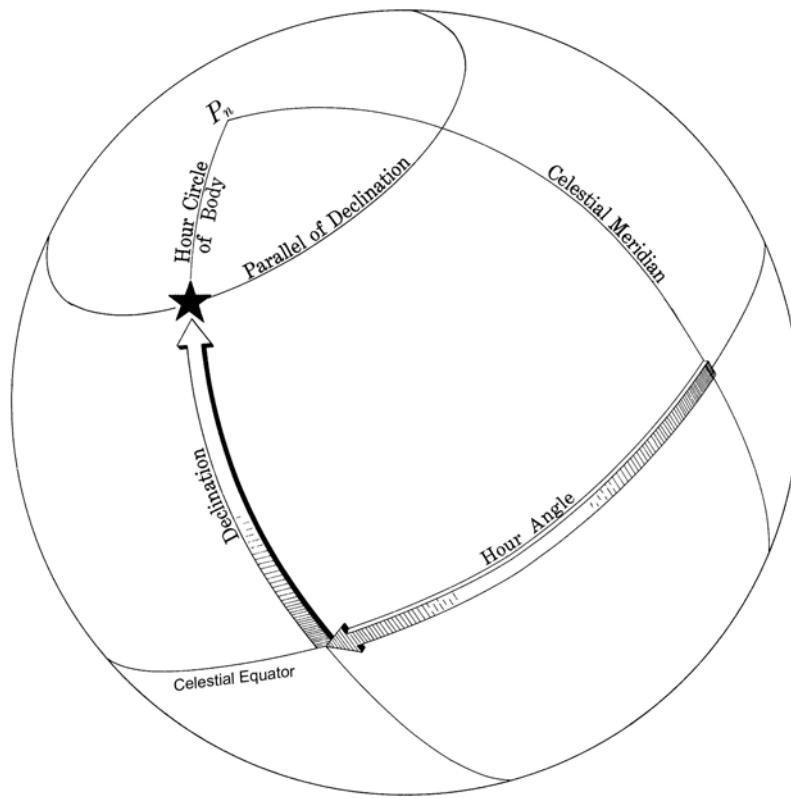


Figure 1325c. A point on the celestial sphere can be located by its declination and hour angle.

horizon system of coordinates.

The line where Earth and sky appear to meet is called the **visible** or **apparent horizon**. On land this is usually an irregular line unless the terrain is level. At sea the visible horizon appears very regular and is often very sharp. However, its position relative to the celestial sphere depends primarily upon (1) the refractive index of the air and (2) the height of the observer's eye above the surface.

In Figure 1328, the observer's eye is a height  $h$  above the Earth at A. The line through A and the center of the Earth is the vertical of the observer and contains the zenith. The plane perpendicular to the vertical is the **sensible horizon**. If the observer is at the Earth's surface,  $h = 0$ , then the plane of the sensible horizon is called the **geoidal horizon**. And if the observer is at the center of the Earth, then the plane of the sensible horizon is called the **celestial horizon**. The radius of the Earth is negligible with respect to that of the celestial sphere. Most measurements are referred only to the celestial horizon.

If the eye of the observer is at the surface of the Earth, the sensible horizon coincides with the geoidal horizon; but above the Earth's surface, at height  $h$ , the observer's eye is at the vertex of a cone, which is tangent to the Earth at the geometric horizon. The angle between the sensible and geometric horizon is the **geometric dip**. So, it is possible to

observe a body, which is above the geometric horizon but below the celestial horizon. That is, the body's altitude is negative and its zenith distance is greater than  $90^\circ$ .

The apparent (or visible) horizon, that is the horizon seen by the observer, is not identical to the geometric horizon because of the refraction of light by the atmosphere. The direction of refraction is towards the zenith, so the apparent horizon is above the geometric horizon. However, because the path of the light is bent the position of the apparent horizon, the place on the Earth's surface where the light path is tangent to the surface, is farther away than the geometric horizon. The difference between the geometric dip and the refraction angle is the **total dip**.

### 1329. The Horizon System of Coordinates

This system is based upon the celestial horizon as the primary great circle and a series of secondary vertical circles which are great circles through the zenith and nadir of the observer and hence perpendicular to his or her horizon (Figure 1329a). Thus, the celestial horizon is similar to the equator, and the vertical circles are similar to meridians, but with one important difference. The celestial horizon and vertical circles are dependent upon the position of the observer and hence move with changes position, while the

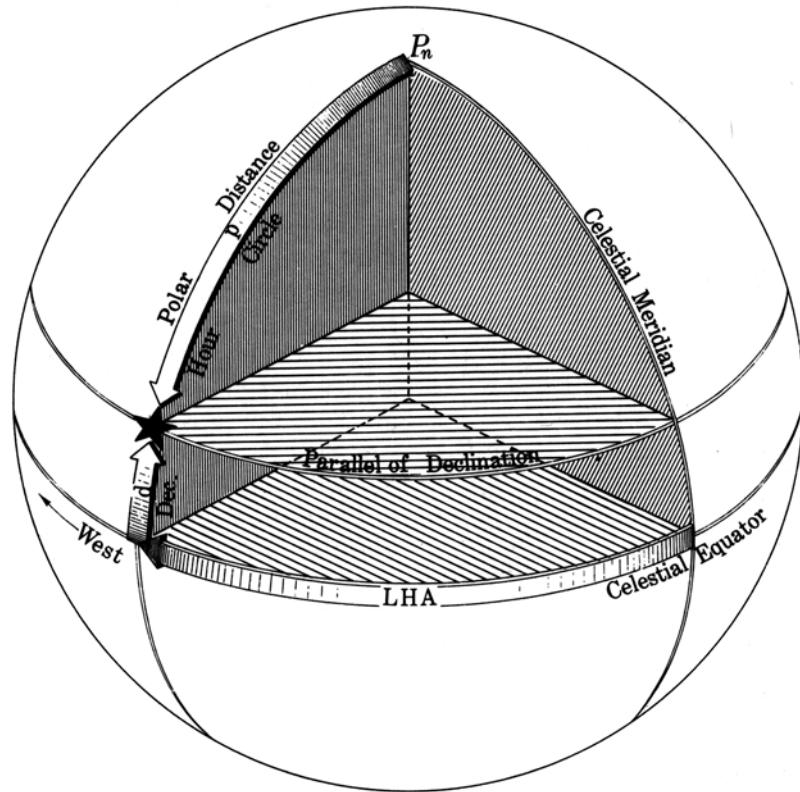


Figure 1326a. The celestial equator system of coordinates, showing measurements of declination, polar distance, and local hour angle.

primary and secondary great circles of both the geographical and celestial equator systems are independent of the observer. The horizon and celestial equator systems coincide for an observer at the geographical pole of the Earth and are mutually perpendicular for an observer on the equator. At all other places the two are oblique.

The **celestial or local meridian** passes through the observer's zenith, nadir, and poles of the celestial equator system of coordinates. As such, it passes through north and south on

As shown in Figure 1329b, altitude is angular distance above the horizon. It is measured along a vertical circle, from  $0^\circ$  at the horizon through  $90^\circ$  at the zenith. Altitude measured from the visible horizon may exceed  $90^\circ$  because of the dip of the horizon, as shown in Figure 1329a. Altitude is nominally a positive value, however, angular distance below the celestial horizon, called negative altitude, is provided for by including certain negative altitudes in some tables for use in celestial navigation. All points having the same altitude lie along a parallel of altitude.

**Zenith distance (z)** is angular distance from the zenith, or the arc of a vertical circle between the zenith and a point on the celestial sphere. It is measured along a vertical circle from  $0^\circ$  through  $180^\circ$ . It is usually considered the complement of altitude. For a body measured with respect to the

the observer's horizon. One of these poles (having the same name, N or S, as the latitude) is above the horizon and is called the **elevated pole**. The other, called the **depressed pole**, is below the horizon. In the horizon system it is called the principal vertical circle. The vertical circle through the east and west points of the horizon, and hence perpendicular to the principal vertical circle, is called the **prime vertical circle**, or simply the **prime vertical**.

celestial horizon  $z = 90^\circ - h$ .

The horizontal direction of a point on the celestial sphere, or the bearing of the geographical position, is called **azimuth** or **azimuth angle** depending upon the method of measurement. In both methods it is an arc of the horizon (or parallel of altitude). It is true azimuth ( $Z_n$ ) if measured east from north on the horizon through  $360^\circ$ , and azimuth angle ( $Z$ ) if measured either direction along the horizon through  $180^\circ$ , starting at the north for an observer in north latitudes and the south in south latitudes.

### 1330. The Ecliptic System of Coordinates

The **ecliptic system** is based upon the ecliptic as the primary great circle, analogous to the equator. The **ecliptic**

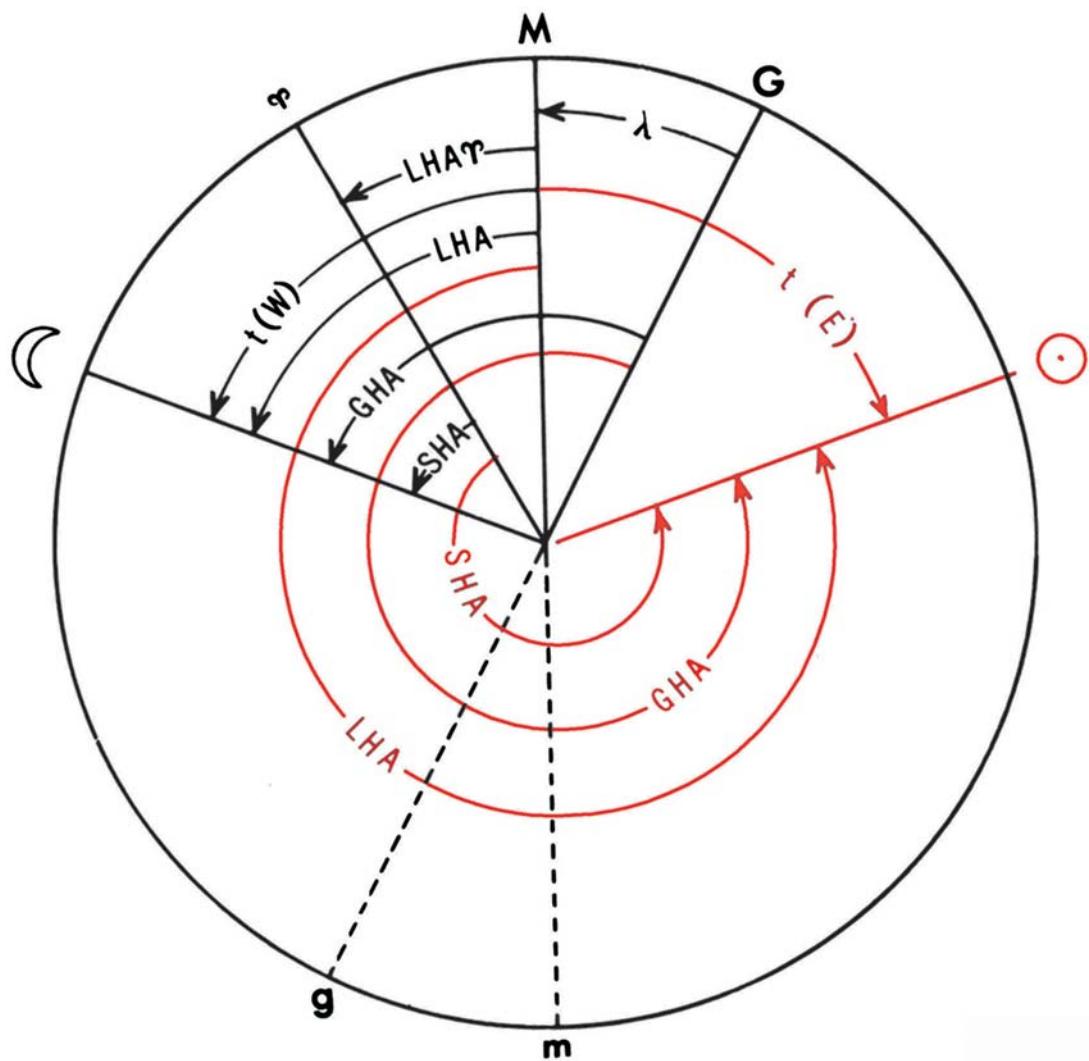


Figure 1326b. Time diagram.

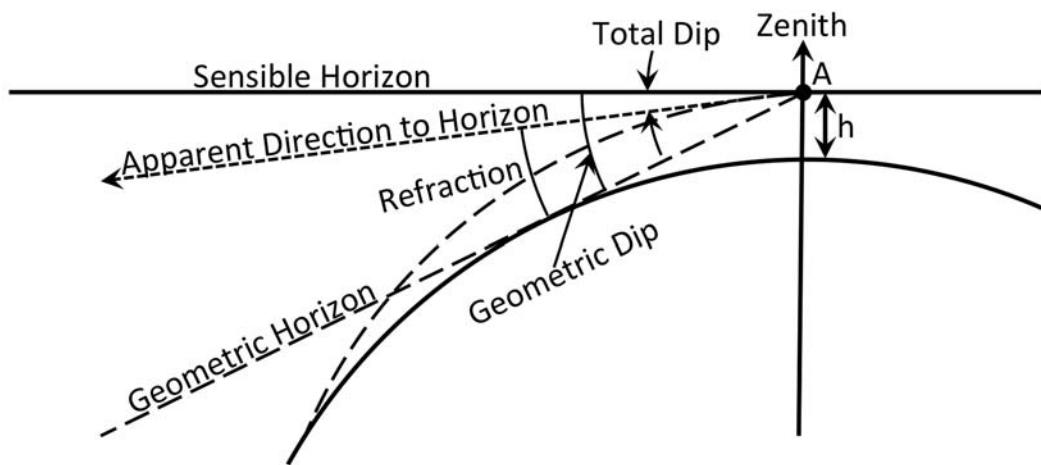


Figure 1328. The sensible horizon.

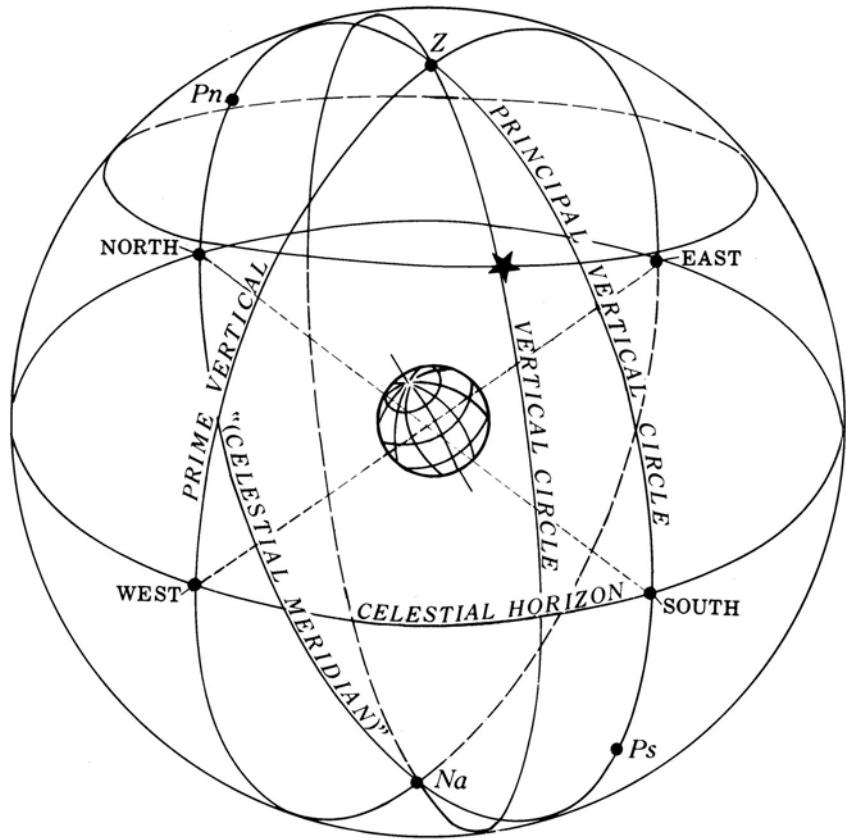


Figure 1329a. Elements of the celestial sphere. The celestial horizon is the primary great circle.

Earth	Celestial Equator	Horizon	Ecliptic
equator	celestial equator	horizon	ecliptic
poles	celestial poles	zenith; nadir	ecliptic poles
meridians	hours circle; celestial meridians	vertical circles	circles of latitude
prime meridian	hour circle of Aries	principal or prime vertical circle	circle of latitude through Aries
parallels	parallels of declination	parallels of altitude	parallels of latitude
latitude	declination	altitude	ecliptic altitude
colatitude	polar distance	zenith distance	ecliptic colatitude
longitude	SHA; RA; GHA; LHA; t	azimuth; azimuth angle; amplitude	ecliptic longitude

Table 1329. The four systems of celestial coordinates and their analogous terms.

is the apparent path of the Sun around the celestial sphere. The points  $90^\circ$  from the ecliptic are the north and south ecliptic poles. The series of great circles through these poles, analogous to meridians, are **circles of latitude**. The circles parallel to the plane of the ecliptic, analogous to parallels on the Earth, are parallels of latitude or **circles of longitude**. Angular distance north or south of the ecliptic, analogous to latitude, is ecliptic latitude. Ecliptic longitude is measured eastward along the ecliptic through  $360^\circ$ , starting at the vernal equinox. The mean plane of the Sun's orbit

lies in the ecliptic and the planes of the orbits of the Moon and planets are near the ecliptic. Because the planes of their orbits lie near the ecliptic, it is easier to predict the positions of the Sun, Moon, and planets using ecliptic coordinates.

The four systems of celestial coordinates are analogous to each other and to the terrestrial system, although each has distinctions such as differences in primary reference planes. Table 1329 indicates the analogous term or terms under each system. Also see Table 1330.

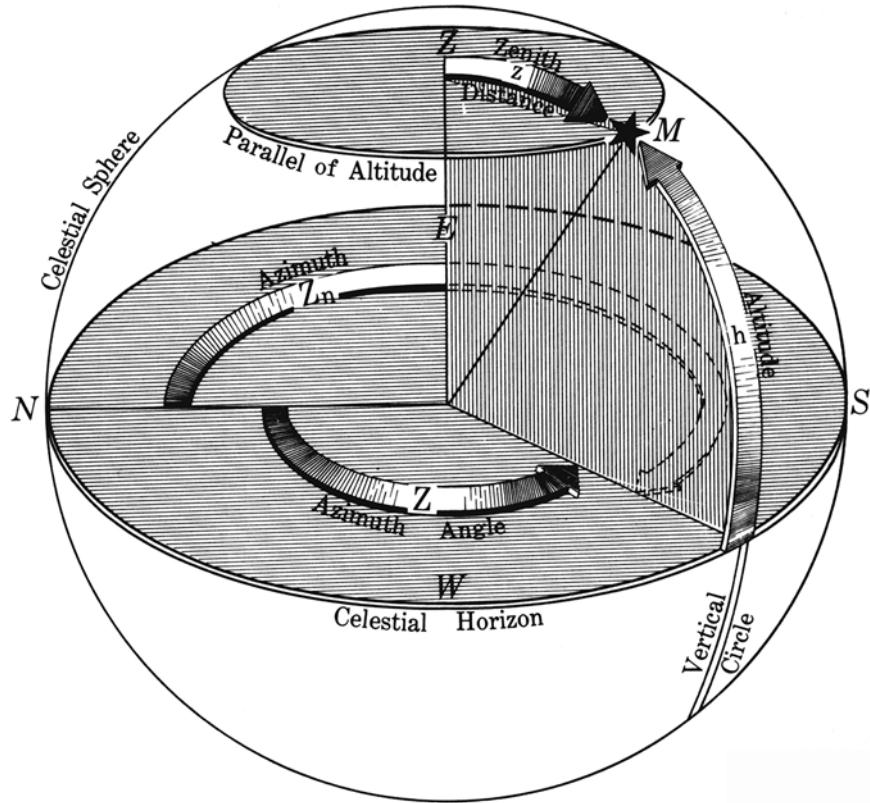


Figure 1329b. Elements of the celestial sphere. The celestial horizon.

### NAVIGATIONAL COORDINATES

Coordinate	Symbol	Measured from	Measured along	Direction	Measured to	Units	Precision	Maximum value	Labels
latitude	L, lat.	equator	meridian	N, S	parallel	$^{\circ}$ , $\text{¢}$	0¢.1	90°	N, S
colatitude	colat.	poles	meridian	S, N	parallel	$^{\circ}$ , $\text{¢}$	0¢.1	90°	—
longitude	l, long.	prime meridian	parallel	E, W	local meridian	$^{\circ}$ , $\text{¢}$	0¢.1	180°	E, W
declination	d, dec.	celestial equator	hour circle	N, S	parallel of declination	$^{\circ}$ , $\text{¢}$	0¢.1	90°	N, S
polar distance	p	elevated pole	hour circle	S, N	parallel of declination	$^{\circ}$ , $\text{¢}$	0¢.1	180°	—
altitude	h	horizon	vertical circle	up	parallel of altitude	$^{\circ}$ , $\text{¢}$	0¢.1	90°*	—
zenith distance	z	zenith	vertical circle	down	parallel of altitude	$^{\circ}$ , $\text{¢}$	0¢.1	180°	—
azimuth	Zn	north	horizon	E	vertical circle	$^{\circ}$	0°.1	360°	—
azimuth angle	Z	north, south	horizon	E, W	vertical circle	$^{\circ}$	0°.1	180° or 90°	N, S...E, W
amplitude	A	east, west	horizon	N, S	body	$^{\circ}$	0°.1	90°	E, W...N, S
Greenwich hour angle	GHA	Greenwich celestial meridian	parallel of declination	W	hour circle	$^{\circ}$ , $\text{¢}$	0¢.1	360°	—

Table 1330. Navigational Coordinates.

NAVIGATIONAL COORDINATES									
Coordinate	Symbol	Measured from	Measured along	Direction	Measured to	Units	Precision	Maximum value	Labels
local hour angle	LHA	local celestial meridian	parallel of declination	W	hour circle	°, ′, ″	0.1	360°	—
meridian angle	t	local celestial meridian	parallel of declination	E, W	hour circle	°, ′, ″	0.1	180°	E, W
sidereal hour angle	SHA	hour circle of vernal equinox	parallel of declination	W	hour circle	°, ′, ″	0.1	360°	—
right ascension	RA	hour circle of vernal equinox	parallel of declination	E	hour circle	h, m, s	1s	24h	—
Greenwich mean time	GMT	lower branch Greenwich celestial meridian	parallel of declination	W	hour circle mean Sun	h, m, s	1s	24h	—
local mean time	LMT	lower branch local celestial meridian	parallel of declination	W	hour circle mean Sun	h, m, s	1s	24h	—
zone time	ZT	lower branch zone celestial meridian	parallel of declination	W	hour circle mean Sun	h, m, s	1s	24h	—
Greenwich apparent time	GAT	lower branch Greenwich celestial meridian	parallel of declination	W	hour circle apparent Sun	h, m, s	1s	24h	—
local apparent time	LAT	lower branch local celestial meridian	parallel of declination	W	hour circle apparent Sun	h, m, s	1s	24h	—
Greenwich sidereal time	GST	Greenwich celestial meridian	parallel of declination	W	hour circle vernal equinox	h, m, s	1s	24h	—
local sidereal time	LST	local celestial meridian	parallel of declination	W	hour circle vernal equinox	h, m, s	1s	24h	—

Table 1330. Navigational Coordinates.

### 1331. The Navigational Triangle

A triangle formed by arcs of great circles of a sphere is called a **spherical triangle**. A spherical triangle on the celestial sphere is called a **celestial triangle**. The spherical triangle of particular significance to navigators is called the **navigational triangle**, formed by arcs of a celestial meridian, an hour circle, and a vertical circle. Its vertices are the elevated pole, the zenith, and a point on the celestial sphere (usually a celestial body). The terrestrial counterpart is also called a navigational triangle, being formed by arcs of two meridians and the great circle connecting two places on the Earth, one on each meridian. The vertices are the two places and a pole. In great-circle sailing these places are the point of departure and the destination. In celestial navigation they are the **assumed position (AP)** of the observer and the **geographical position (GP)** of the body (the point on the Earth's surface having the body in its zenith). The GP of the Sun is sometimes called the **subsolar point**, that of the Moon the **sublunar point**, that of a satellite (either natural or artificial) the **subsatellite point**, and that of a star its **substellar** or **subastral point**. When used to solve a celestial observation, either the celestial or terrestrial triangle may be called the **astronomical triangle**.

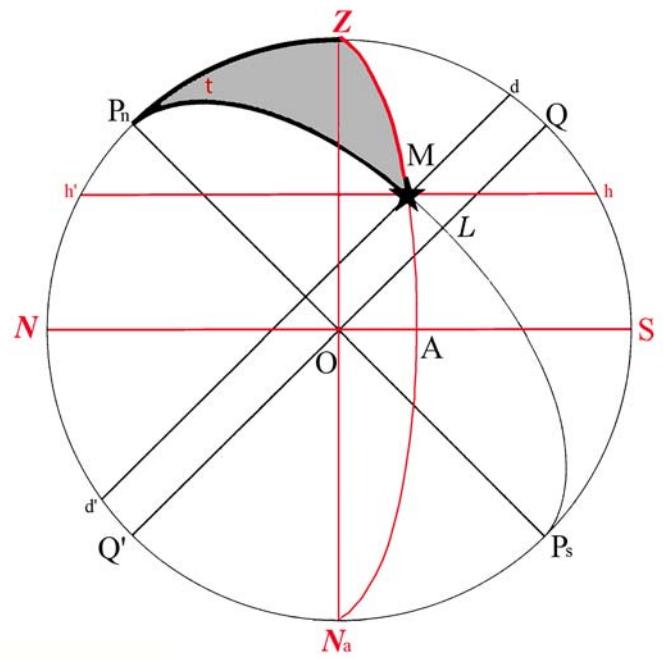


Figure 1331a. The navigational triangle.

The navigational triangle is shown in Figure 1331a on a diagram on the plane of the celestial meridian. The Earth is at the center, O. The star is at M,  $dd'$  is its parallel of declination, and  $hh'$  is its altitude circle.

In the figure, arc QZ of the celestial meridian is the latitude of the observer, and  $PnZ$ , one side of the triangle, is the colatitude. Arc AM of the vertical circle is the altitude of the body, and side ZM of the triangle is the zenith distance, or coaltitude. Arc LM of the hour circle is the declination of the body, and side  $PnM$  of the triangle is the polar distance, or codeclination.

The angle at the elevated pole,  $ZPnM$ , having the hour circle and the celestial meridian as sides, is the meridian angle,  $t$ . The angle at the zenith,  $PnZM$ , having the vertical circle and that arc of the celestial meridian, which includes the elevated pole, as sides, is the azimuth angle. The angle at the celestial body,  $ZMPn$ , having the hour circle and the vertical circle as sides, is the parallactic angle ( $q$ ) (sometimes called the position angle), which is not generally used by the navigator.

A number of problems involving the navigational triangle are encountered by the navigator, either directly or indirectly. Of these, the most common are:

1. Given latitude, declination, and meridian angle, to find of a celestial observation to establish a line of position.
2. Given latitude, altitude, and azimuth angle, to find declination and meridian angle. This is used to identify an unknown celestial body.
3. Given meridian angle, declination, and altitude, to find azimuth angle. This may be used to find azimuth when the altitude is known.
4. Given the latitude of two places on the Earth and the difference of longitude between them, to find the initial great-circle course and the great-circle distance. This involves the same parts of the triangle as in 1, above, but in the terrestrial triangle, and hence is defined differently.

Both celestial and terrestrial navigational triangles were shown in Figure 1529b of the Bowditch 2002 edition.

## IDENTIFICATION OF STARS AND PLANETS

### 1332. Introduction

A basic requirement of celestial navigation is the ability to identify the bodies observed. This is not difficult because relatively few stars and planets are commonly used for navigation, and various aids are available to assist in their identification. See Figure 1332, Figure 1333, Figure 1334a and Figure 1334b.

Identification of the Sun and Moon is straightforward, however, the planets can be mistaken for stars. A person working continually with the night sky recognizes a planet by its changing position among the relatively fixed stars. The planets are identified by noting their positions relative to each other, the Sun, the Moon, and the stars. They remain within the narrow limits of the ecliptic, but are in almost constant motion relative to the stars. The magnitude (brightness) and color may be helpful; they are some of the brightest objects in the sky. The information needed is found in the *Nautical Almanac*. The “Planet Notes” near the front of that volume are particularly useful. Planets can also be identified by planet diagram, star finder, sky diagram, or by computation.

### 1333. Stars

The *Nautical Almanac* lists full navigational information on 19 first magnitude stars and 38 second magnitude stars, plus Polaris given its proximity to the north celestial pole. These are known as “selected stars” and are listed in the Index to Selected Stars in the *Nautical Almanac*. These stars can also be seen in Figure 1333 - Distribution of Selected Stars from the *Nautical Almanac*. These are some of the

brightest stars, and span declinations from  $70^{\circ}$  south to  $89^{\circ}$  north on the celestial sphere. Abbreviated information is listed for 115 more, known as “tabulated stars.” Additional stars are listed in the Astronomical Almanac and in various star catalogs. About 6,000 stars are visible to the unaided eye on clear, dark nights across the entire sky.

Stars are designated by one or more of the following naming systems:

- **Common Name:** Most names of stars, as now used, were given by the ancient Arabs and some by the Greeks or Romans. One of the stars of the *Nautical Almanac*, Nunki, was named by the Babylonians. Only a relatively few stars and often only the brightest have common names. Several of the stars on the daily pages of the almanacs had no name prior to 1953.
- **Bayer’s Name:** Most bright stars, including those with names, have been given a designation consisting of a Greek letter followed by the possessive form of the name of the constellation. For example, the brightest star in the constellation Cygnus is known as (Greek letter “alpha”) Cygni, and also by its common name, Deneb. Roman letters are used when there are not enough Greek letters. Usually, the letters are assigned in order of brightness within the constellation; however, this is not always the case. For example, the letter designations of the stars in Ursa Major or the Big Dipper are assigned in order from the outer rim of the bowl to the end of the handle. This system of star designation was suggested by John Bayer of

## NAVIGATIONAL STARS AND THE PLANETS

Name	Pronunciation	Bayer name	Origin of name	Meaning of name	Distance*
Acamar	ä'kä-mär	θ Eridani	Arabic	another form of Achernar	120
Achernar	ä'ker-när	ε Eridani	Arabic	end of the river (Eridanus)	72
Acrux	ä'krüks	α Crucis	Modern	coined from Bayer name	220
Adhara	ä-dä'rä	ε Canis Majoris	Arabic	the virgin(s)	350
Aldebaran	äl däb'ä-rän	α Tauri	Arabic	follower (of the Pleiades)	64
Alioth	äl'oth	ε Ursa Majoris	Arabic	another form of Capella	49
Alkaid	äl-kä'd'	η Ursa Majoris	Arabic	leader of the daughters of the bier	190
Al Na'ir	äl-nä'r'	α Gruis	Arabic	bright one (of the fish's tail)	90
Alnilam	äl'ni-läm	ε Orionis	Arabic	string of pearls	410
Alphard	äl'fär'd	α Hydrae	Arabic	solitary star of the serpent	200
Alphecca	äl-fék'ä	α Corona Borealis	Arabic	feeble one (in the crown)	76
Alpheratz	äl-fé'räts	α Andromeda	Arabic	the horse's navel	120
Altair	äl-tär'	α Aquilae	Arabic	flying eagle or vulture	16
Ankaa	än'kä	α Phoenicis	Arabic	coined name	93
Antares	än-tä'rëz	α Scorpis	Greek	rival of Mars (in color)	250
Arcturus	ärk-tü'rüs	α Bootis	Greek	the bear's guard	37
Atria	ä'trë-ä	α Trianguli Australis	Modern	coined from Bayer name	130
Avior	ä'vi-ör	ε Carinae	Modern	coined name	350
Bellatrix	bë'lä'trëks.	γ Orionis	Latin	female warrior	250
Betelgeuse	bë'të-lë-zë	α Orionis	Arabic	the arm pit (of Orion)	300
Canopus	kä-nö'püs	α Carinae	Greek	city of ancient Egypt	230
Capella	kä-päl'ä	α Aurigae	Latin	little she-goat	46
Deneb	dëñ'ëb	α Cygni	Arabic	tail of the hen	600
Denebola	dë-nëb'ö-lä	β Leonis	Arabic	tail of the lion	42
Diphda	dif'dä	β Ceti	Arabic	the second frog (Fomalhaut was once the first)	57
Dubhe	düb'ë	α Ursae Majoris	Arabic	the bear's back	100
Elnath	ël'näth	β Tauri	Arabic	one butting with horns	130
Eltanin	ël-tä'nin	γ Draconis	Arabic	head of the dragon	150
Enif	ëñ'if	ε Pegasi	Arabic	nose of the horse	250
Fomalhaut	fö'mäl-öt	α Piscis Austrini	Arabic	mouth of the southern fish	23
Gacrux	gä'krüks	γ Crucis	Modern	coined from Bayer name	72
Gienah	jë'nä	γ Corvi	Arabic	right wing of the raven	136
Hadar	hä'där	β Centauri	Modern	leg of the centaur	200
Hamal	hä'mäl	α Arietis	Arabic	full-grown lamb	76
Kaus Australis	kös ös-trä'ëls	ε Sagittarii	Ar., L.	southern part of the bow	163
Kochab	kö'käb	β Ursae Minoris	Arabic	shortened form of "north star" (named when it was that, c. 1500 BC-AD 300)	100
Markab	mär'käb	α Pegasi	Arabic	saddle (of Pegasus)	100
Menkar	mëñ'kär	θ Centauri	Modern	nose (of the whale)	1,100
Menkent	mëñ'kënt	β Carinae	Ar., L.	shoulder of the centaur	55
Miaplacidus	më'ä-pläcës'-ë-düs	α Persei	Arabic	quiet or still waters	86
Mirfak	mir'fák	ε Sagittarii	Bab.	elbow of the Pleiades	130
Nunki	nün'kë	α Pavonis	Modern	constellation of the holy city (Eridu) coined from English name of constellation	150
Peacock	pë'kök	α Ursae Minoris	Greek	the pole (star)	450
Polaris	pö'lä'ris	α Ursae Minoris	Latin	Zeus' other twin son (Castor, α Geminorum, is first twin)	33
Pollux	pö'lüks	β Geminorum	Latin	before the dog (rising before the dog star, Sirius)	11
Procyon	prö'si-ön	α Canis Minoris	Greek	head of the serpent charmer	67
Rasalhague	rä'sä'lä-hä'gwë	α Ophiuchi	Arabic	the prince	67
Regulus	rëg'ü-lüs	α Leonis	Latin	foot (left foot of Orion)	500
Rigel	rë'jël	β Orionis	Arabic	foot of the centaur	4.3
Rigil Kentaurus	rë'jël këñ-tö'rüs	α Centauri	Arabic	second winner or conqueror	69
Sabik	sä'bik	η Ophiuchi	Arabic	the breast (of Cassiopeia)	360
Schedar	shëd'är	α Cassiopeiae	Arabic	cocked-up part of the scorpion's tail	200
Shaula	shö'lä	λ Scorpis	Arabic	the scorching one (popularly, the dog star)	8.6
Sirius	sir'ü-üs	α Canis Majoris	Greek	the ear of corn	155
Spica	spë'kä	α Virginis	Latin	shortened form of Al Suhail, one Arabic name for Canopus	200
Suhail	söö-häl'	λ Velorum	Arabic	the falling eagle or vulture	27
Vega	vë'gä	α Lyrae	Arabic	southern claw (of the scorpion)	66
Zubenelgenubi	zöö'bëñ'ël-jë-nü'bë	α Librae	Arabic		

## PLANETS

Name	Pronunciation	Origin of name	Meaning of name
Mercury	mür'kü-ri	Latin	god of commerce and gain
Venus	vë'nüs	Latin	goddess of love
Earth	ürt <h>h</h>	Mid. Eng.	—
Mars	märz	Latin	god of war
Jupiter	jöö'pë-tër	Latin	god of the heavens, identified with the Greek Zeus, chief of the Olympian gods
Saturn	sä'tërn	Latin	god of seed-sowing
Uranus	ü'rë-nüs	Greek	the personification of heaven
Neptune	nëp'tün	Latin	god of the sea
Pluto	plöö'tö	Greek	god of the lower world (Hades)

Guide to pronunciations:

fate, ädd, finäl, läst, äbound, ärm; bë, änd, camël, readër; ice, bli, an'imal; över, pöëtic, hët, lërd, mëön; übe, änite, tüb, cireüs, ürn

\*Distances in light-years. One light-year equals approximately 63,300 AU, or 5,880,000,000,000 miles. Authorities differ on distances of the stars; the values given are representative.

Figure 1332. Navigational stars and the planets.

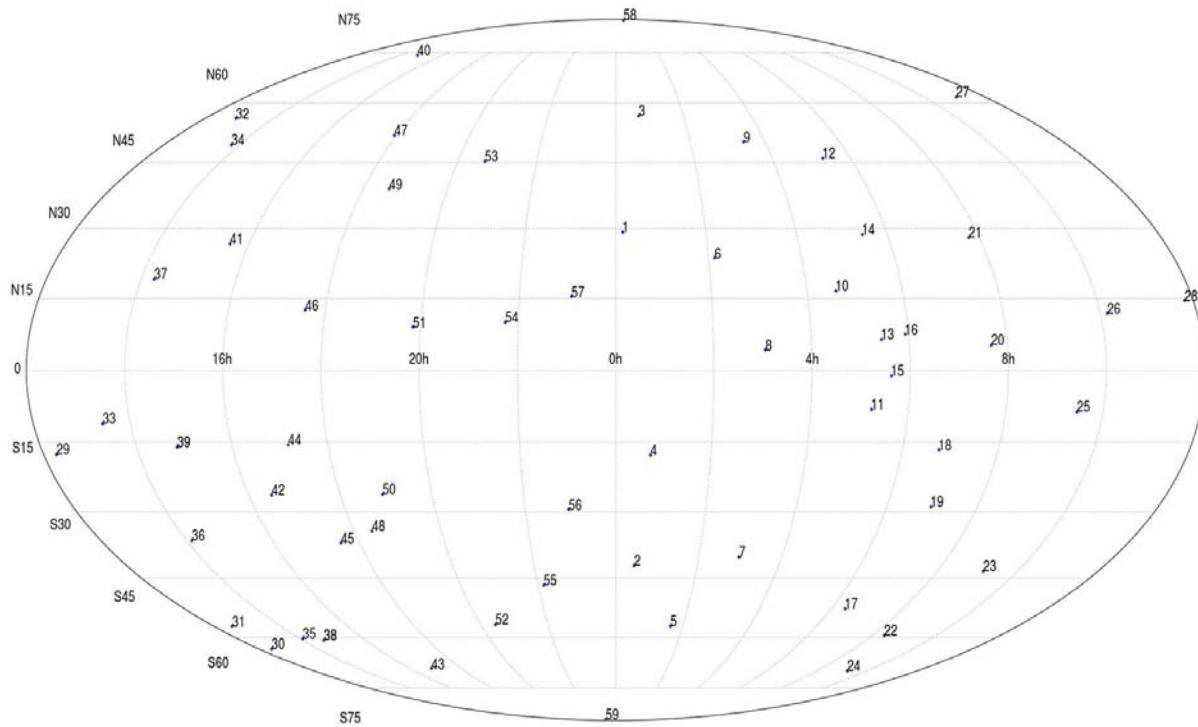


Figure 1333. Distribution of Selected Stars from the Nautical Almanac.

Augsburg, Germany, in 1603. All of the 173 stars included in the list near the back of the *Nautical Almanac* are listed by Bayer's name, and, when applicable, their common name.

- **Flamsteed's Number:** This system assigns numbers to stars in each constellation, from west to east in the order in which they cross the celestial meridian. An example is 95 Leonis, the 95th star in the constellation Leo. This system was suggested by John Flamsteed (1646-1719).
- **Catalog Number:** Stars are sometimes designated by the name of a star catalog and the number of the star as given in the catalog, such as the Henry Draper or Hipparcos catalogs. Stars are frequently listed in catalogs by increasing right ascension coordinate, without regard to constellation, for example, Polaris is known as HD 8890 and HIP 11767 in these catalogs. Navigators seldom have occasion to use this system.

#### 1334. Star Charts

It is useful to be able to identify stars by relative position. A **star chart** (Figure 1334a and Figure 1334b) is helpful in locating these relationships and others which may be useful. This method is limited to periods of relatively clear, dark skies with little or no overcast. Stars can also be identified by the Air Almanac **sky diagrams**, a **star finder**, *Pub. No. 249*, or by

computation by hand, navigational calculator, computer software or even smart phone applications.

Star charts are based upon the celestial equator system of coordinates, using declination and sidereal hour angle (or right ascension). See Figure 1334c for a graphical depiction of right ascension. The zenith of the observer is at the intersection of the parallel of declination equal to his or her latitude, and the hour circle coinciding with his or her celestial meridian. This hour circle has an SHA equal to  $360^\circ - \text{LHA } \Psi$  (or  $\text{RA} = \text{LHA } \Psi$ ). The horizon is everywhere  $90^\circ$  from the zenith.

A **star globe** is similar to a terrestrial sphere, but with stars (and often constellations) shown instead of geographical positions. The *Nautical Almanac* (page 260) includes instructions for using this device. On a star globe the celestial sphere is shown as it would appear to an observer outside the sphere. Constellations appear reversed. Star charts may show a similar view, but more often they are based upon the view from inside the sphere, as seen from the Earth. On these charts, north is at the top, as with maps, but east is to the left and west to the right. The directions seem correct when the chart is held overhead, with the top toward the north, so the relationship is similar to the sky.

The *Nautical Almanac* has four star charts, located on pages 266 and 267. Two are polar projections of each hemisphere, and two are Mercator projections from  $30^\circ\text{N}$  to  $30^\circ\text{S}$ . On any of these charts, the zenith can be located as indicated, to determine which stars are overhead. The horizon is  $90^\circ$  from the zenith. The charts can also be used to determine the location of a star relative to surrounding stars.

## STAR CHARTS

## NORTHERN STARS

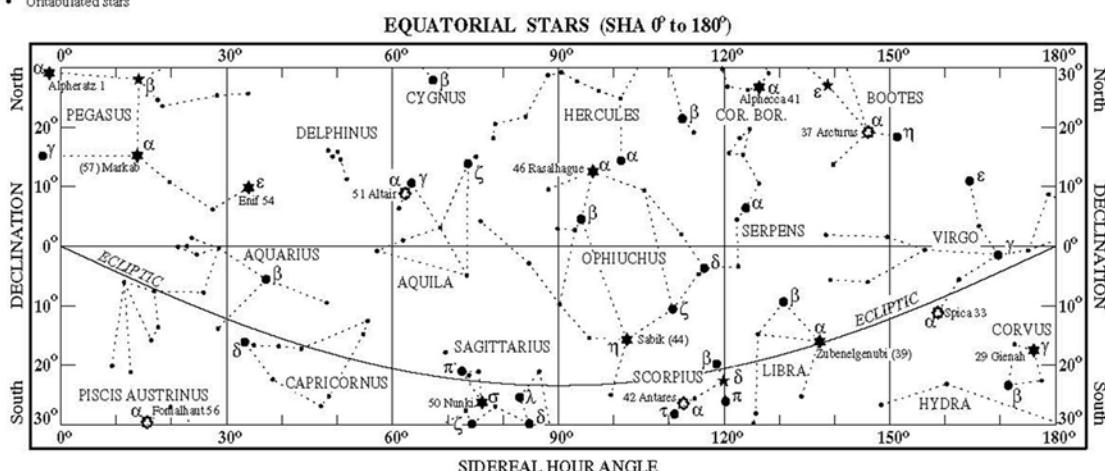
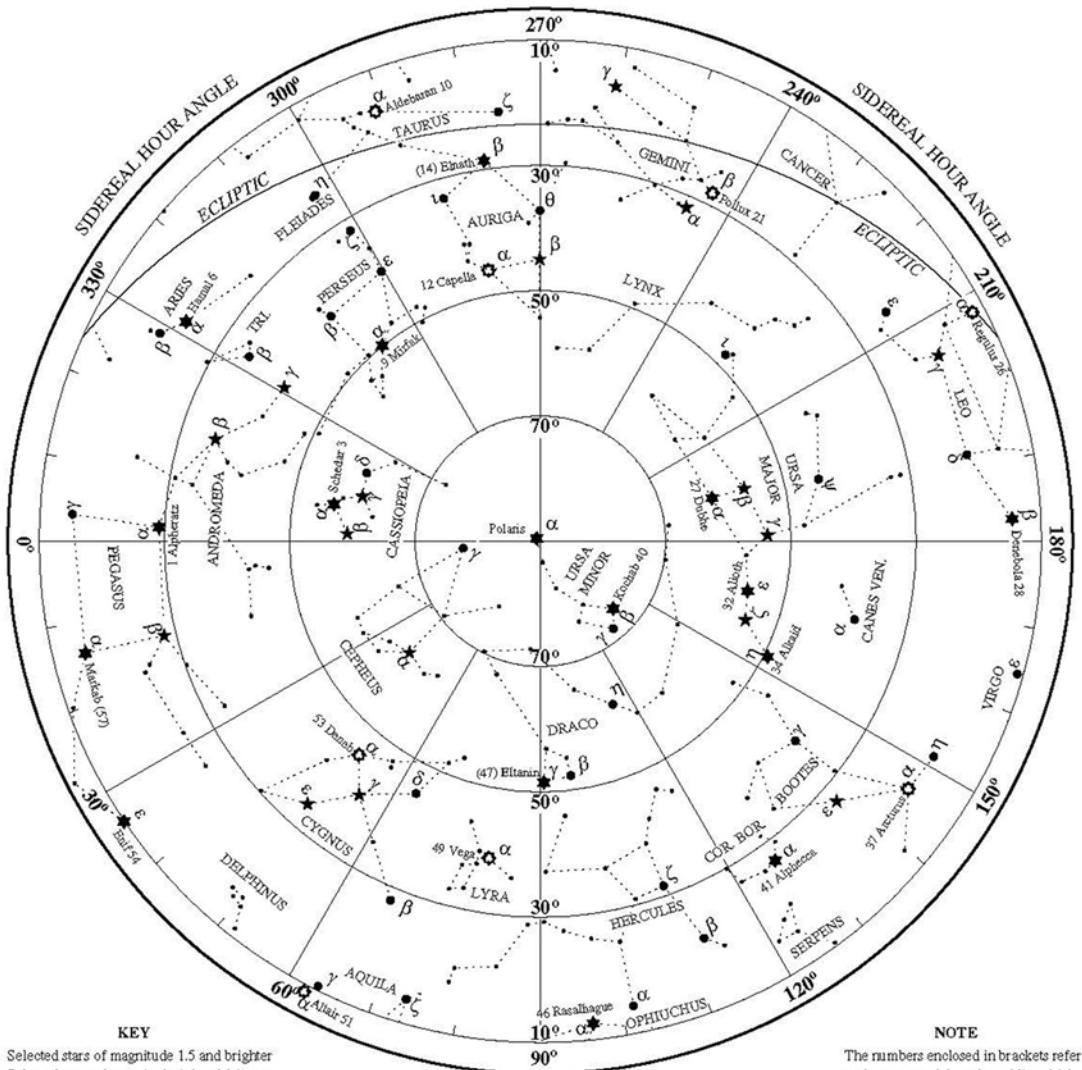
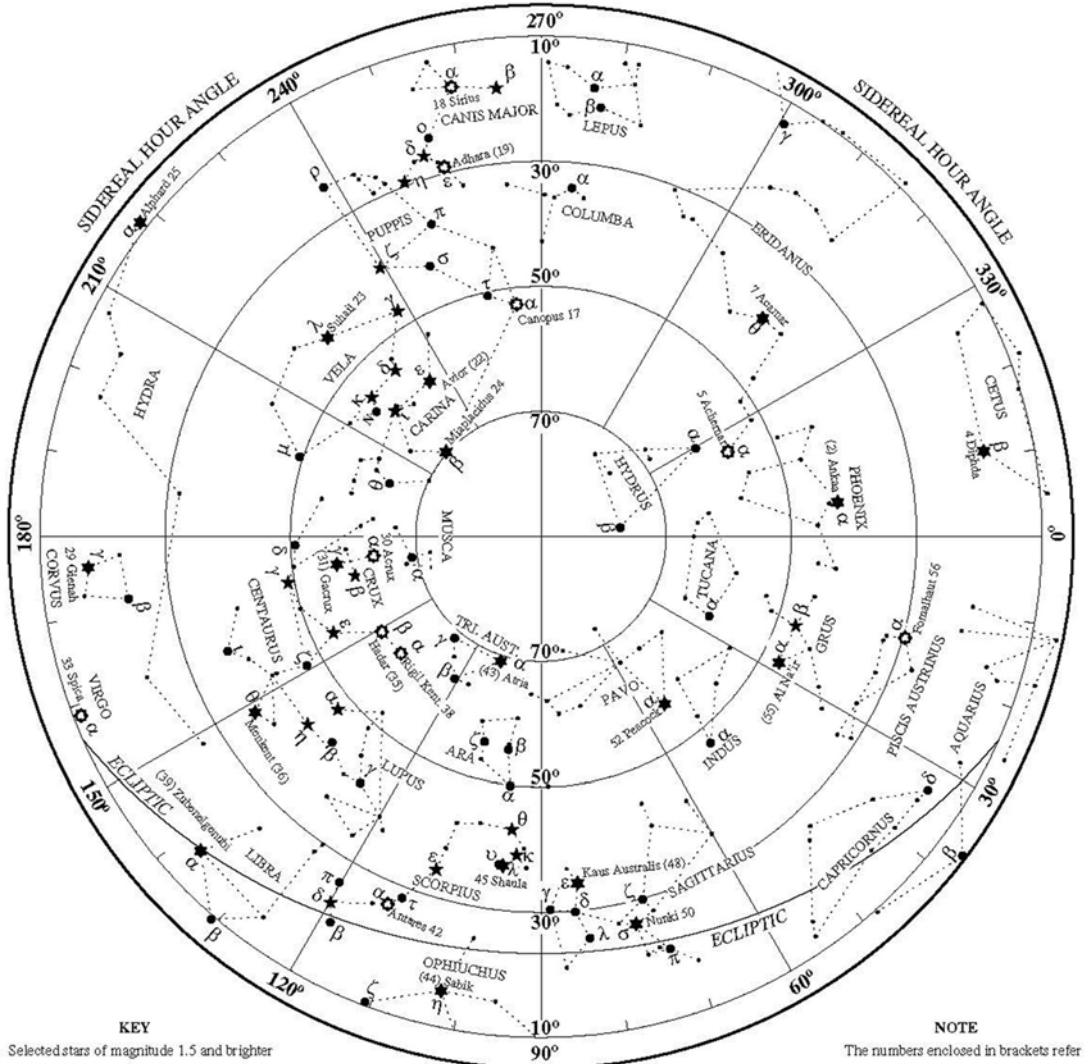


Figure 1334a. Star chart from Nautical Almanac.

## STAR CHARTS

## SOUTHERN STARS



## KEY

- Selected stars of magnitude 1.5 and brighter
  - ★ Selected stars of magnitude 1.6 and fainter
  - ☆ Other tabulated stars of magnitude 2.5 and brighter
  - Other tabulated stars of magnitude 2.6 and fainter
  - Untabulated stars

## NOTE

The numbers enclosed in brackets refer to those stars of the selected list which are not used in Sight Reduction Tables A.P. 3270, N.P. 303.

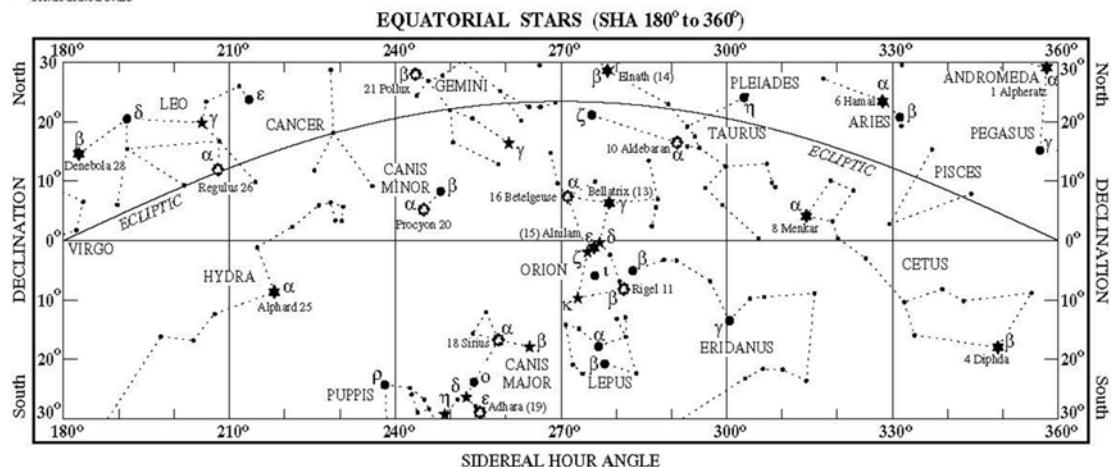


Figure 1334b. Star chart from Nautical Almanac.

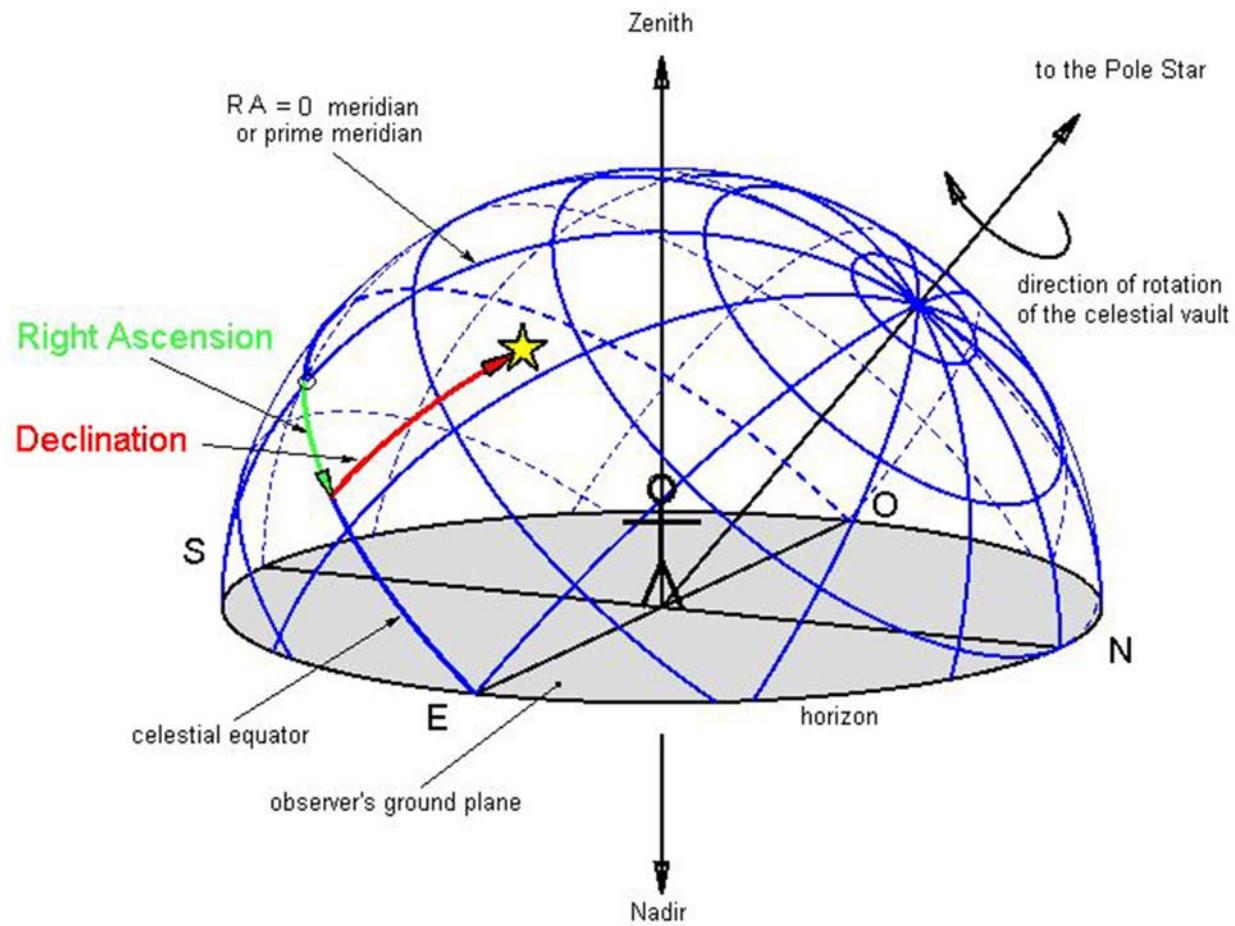


Figure 1334c. Star chart from Nautical Almanac.

	Fig. 1335	Fig. 1336	Fig. 1337	Fig. 1338
Local sidereal time	0000	0600	1200	1800
LMT 1800	Dec. 21	Mar. 22	June 22	Sept. 21
LMT 2000	Nov. 21	Feb. 20	May 22	Aug. 21
LMT 2200	Oct. 21	Jan. 20	Apr. 22	July 22
LMT 0000	Sept. 22	Dec. 22	Mar. 23	June 22
LMT 0200	Aug. 22	Nov. 21	Feb. 21	May 23
LMT 0400	July 23	Oct. 22	Jan 21	Apr. 22
LMT 0600	June 22	Sept. 21	Dec. 22	Mar. 23

Table 1334. Locating the zenith on the star diagrams.

The star charts shown in Figure 1335 through Figure 1338 on the transverse Mercator projection, are designed to assist in learning Polaris and the stars listed on the daily pages of the *Nautical Almanac*. Each chart extends about  $20^{\circ}$  beyond each celestial pole, and about  $60^{\circ}$  (four hours) each side of the central hour circle (at the celestial equator). Therefore, they do not coincide exactly with that half of the celestial sphere above the horizon at any one time or place.

The zenith, and hence the horizon, varies with the position of the observer on the Earth. It also varies with the rotation of the Earth (apparent rotation of the celestial sphere). The charts show all stars of fifth magnitude and brighter as they appear in the sky, but with some distortion toward the right and left edges.

The overprinted lines add certain information of use in locating the stars. Only Polaris and the 57 stars listed on the

daily pages of the *Nautical Almanac* are named on the charts. The almanac star charts can be used to locate the additional stars given near the back of the *Nautical Almanac* and the Air Almanac. Dashed lines connect stars of some of the more prominent constellations. Solid lines indicate the celestial equator and useful relationships among stars in different constellations. The celestial poles are marked by crosses, and labeled. By means of the celestial equator and the poles, an observer can locate the zenith approximately along the mid hour circle, when this coincides with the celestial meridian, as shown in Table 1334. At any time earlier than those shown in Table 1334. The zenith is to the right of center, and at a later time it is to the left, approximately one-quarter of the distance from the center to the outer edge (at the celestial equator) for each hour that the time differs from that shown. The stars in the vicinity of the north celestial pole can be seen in proper perspective by inverting the chart, so that the zenith of an observer in the Northern Hemisphere is up from the pole.

### 1335. Stars in the Vicinity of Pegasus

In autumn the evening sky has few first magnitude stars. Most are near the southern horizon of an observer in the latitudes of the United States. A relatively large number of second and third magnitude stars seem conspicuous, perhaps because of the small number of brighter stars. High in the southern sky three third magnitude stars and one second magnitude star form a square with sides nearly  $15^{\circ}$  of arc in length. This is Pegasus, the winged horse.

Only Markab at the southwestern corner and Alpheratz at the northeastern corner are listed on the daily pages of the *Nautical Almanac*. Alpheratz is part of the constellation Andromeda, the princess, extending in an arc toward the northeast and terminating at Mirfak in Perseus, legendary rescuer of Andromeda.

A line extending northward through the eastern side of the square of Pegasus passes through the leading (western) star of M-shaped (or W-shaped) Cassiopeia, the legendary mother of the princess Andromeda. The only star of this constellation listed on the daily pages of the *Nautical Almanac* is Schedar, the second star from the leading one as the configuration circles the pole in a counterclockwise direction. If the line through the eastern side of the square of Pegasus is continued on toward the north, it leads to second magnitude Polaris, the North Star (less than  $1^{\circ}$  from the north celestial pole) and brightest star of Ursa Minor, the Little Dipper. Kochab, a second magnitude star at the other end of Ursa Minor, is also listed in the almanacs. At this season Ursa Major is low in the northern sky, below the celestial pole. A line extending from Kochab through Polaris leads to Mirfak, assisting in its identification when Pegasus and Andromeda are near or below the horizon.

Deneb, in Cygnus, the swan, and Vega are bright, first magnitude stars in the northwestern sky. The line through the eastern side of the square of Pegasus approximates the

hour circle of the vernal equinox, shown at Aries on the celestial equator to the south. The Sun is at Aries on or about March 21, when it crosses the celestial equator from south to north. If the line through the eastern side of Pegasus is extended southward and curved slightly toward the east, it leads to second magnitude Diphda. A longer and straighter line southward through the western side of Pegasus leads to first magnitude Fomalhaut. A line extending northeasterly from Fomalhaut through Diphda leads to Menkar, a third magnitude star, but the brightest in its vicinity. Ankaa, Diphda, and Fomalhaut form an isosceles triangle, with the apex at Diphda. Ankaa is near or below the southern horizon of observers in latitudes of the United States. Four stars farther south than Ankaa may be visible when on the celestial meridian, just above the horizon of observers in latitudes of the extreme southern part of the United States. These are Acamar, Achernar, Al Na'ir, and Peacock. These stars, with each other and with Ankaa, Fomalhaut, and Diphda, form a series of triangles as shown in Figure 1335. Almanac stars near the bottom of Figure 1335 are discussed in succeeding articles.

Two other almanac stars can be located by their positions relative to Pegasus. These are Hamal in the constellation Aries, the ram, east of Pegasus, and Enif, west of the southern part of the square, identified in Figure 1335. The line leading to Hamal, if continued, leads to the Pleiades (the Seven Sisters), not used by navigators for celestial observations, but a prominent figure in the sky, heralding the approach of the many conspicuous stars of the winter evening sky.

### 1336. Stars in the Vicinity of Orion

As Pegasus leaves the meridian and moves into the western sky, Orion, the hunter, rises in the east. With the possible exception of Ursa Major, no other configuration of stars in the entire sky is as well known as Orion and its immediate surroundings. In no other region are there so many first magnitude stars.

The belt of Orion, nearly on the celestial equator, is visible in virtually any latitude, rising and setting almost on the prime vertical, and dividing its time equally above and below the horizon. Of the three second magnitude stars forming the belt, only Alnilam, the middle one, is listed on the daily pages of the *Nautical Almanac*.

Four conspicuous stars form a box around the belt. Rigel, a hot, blue star, is to the south. Betelgeuse, a cool, red star lies to the north. Bellatrix, bright for a second magnitude star but overshadowed by its first magnitude neighbors, is a few degrees west of Betelgeuse. Neither the second magnitude star forming the southeastern corner of the box, nor any star of the dagger, is listed on the daily pages of the *Nautical Almanac*.

A line extending eastward from the belt of Orion, and curving toward the south, leads to Sirius, the brightest star in the entire heavens, having a magnitude of  $-1.6$ . Only

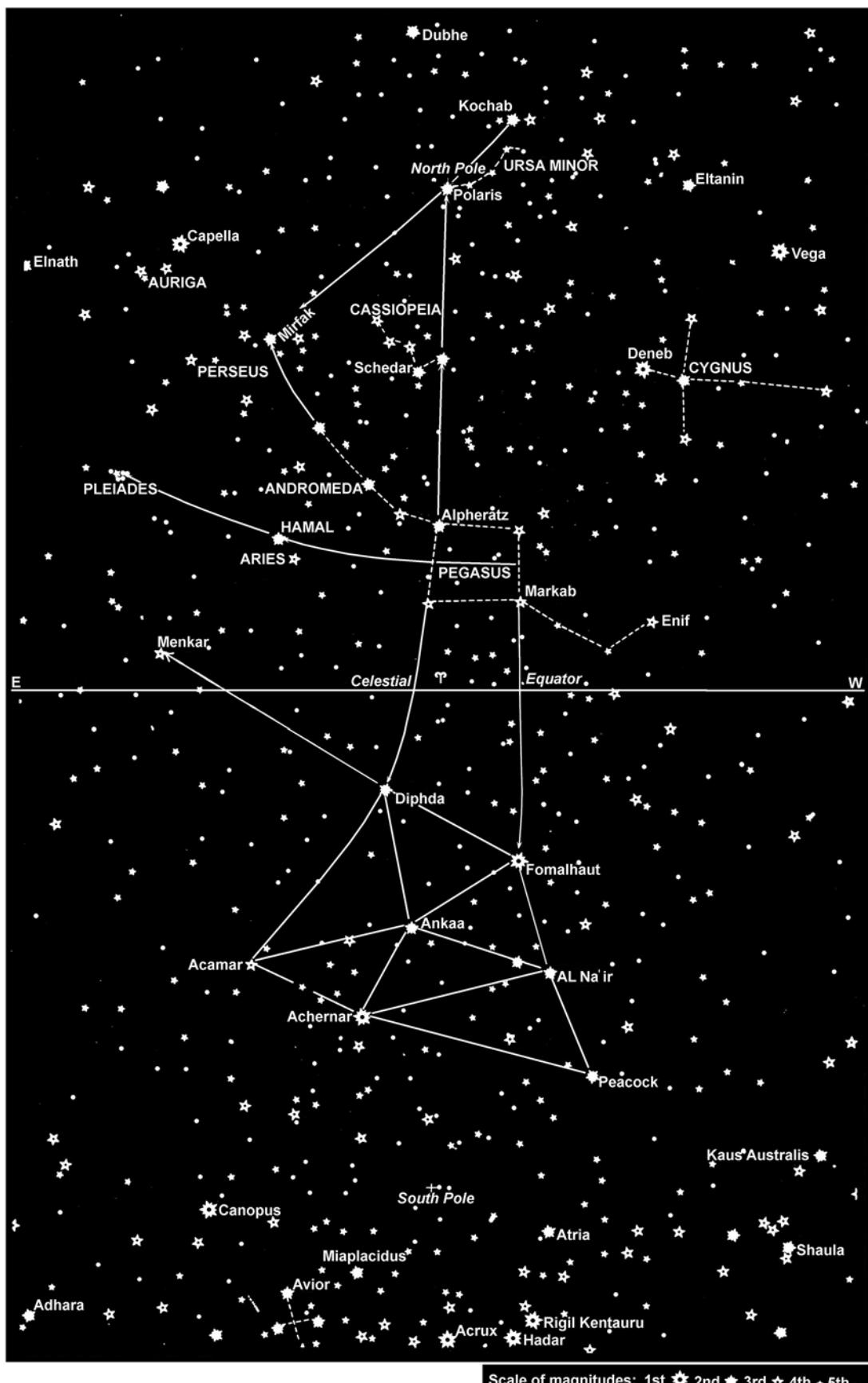


Figure 1335. Stars in the vicinity of Pegasus.

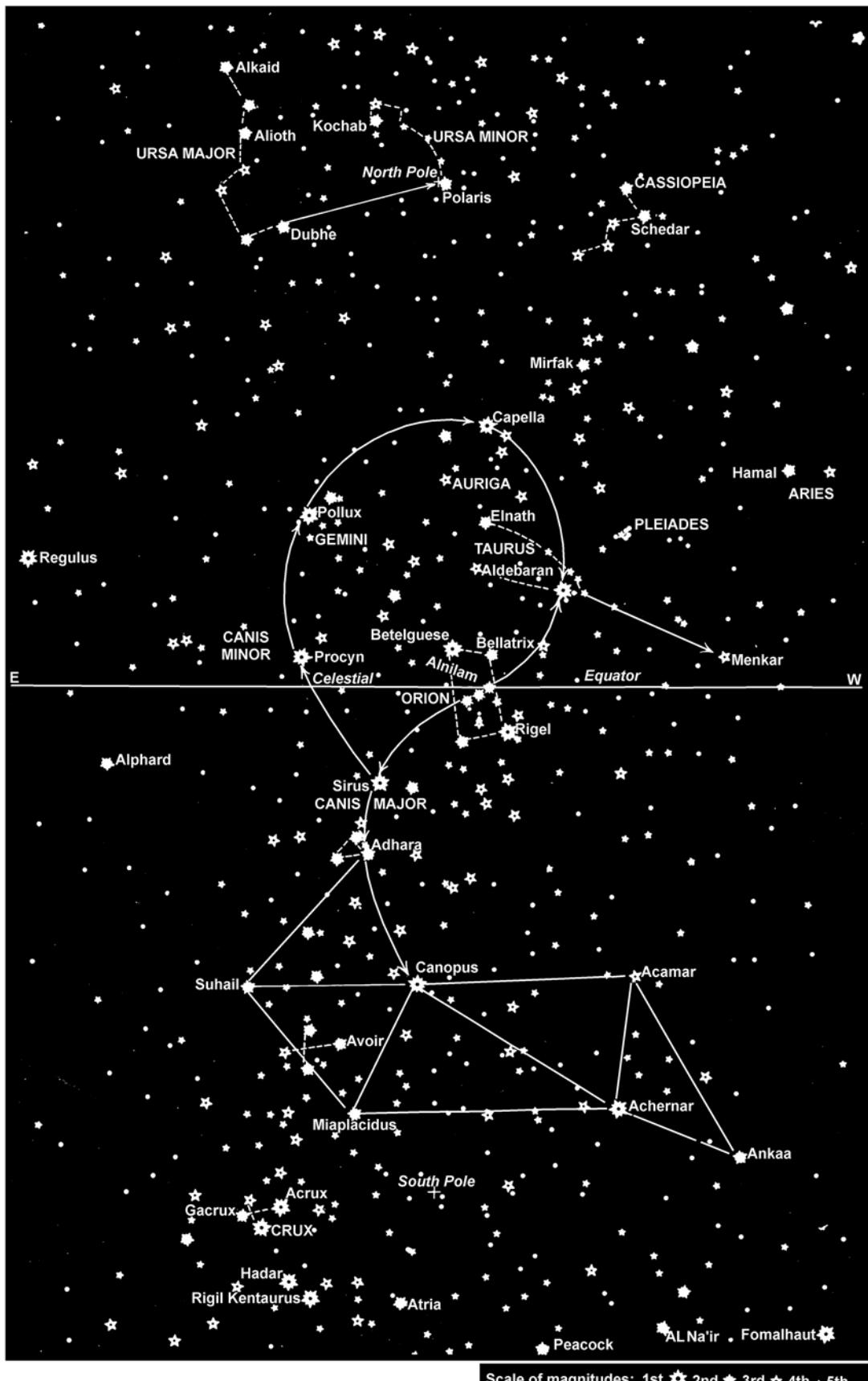


Figure 1336. Stars in the vicinity of Orion.

Mars and Jupiter at or near their greatest brilliance, the Sun, Moon, and Venus are brighter than Sirius. Sirius is part of the constellation Canis Major, the large hunting dog of Orion. Starting at Sirius a curved line extends northward through first magnitude Procyon, in Canis Minor, the small hunting dog; first magnitude Pollux and second magnitude Castor (not listed on the daily pages of the *Nautical Almanac*), the twins of Gemini; brilliant Capella in Auriga, the charioteer; and back down to first magnitude Aldebaran, the follower, which trails the Pleiades, the seven sisters. Aldebaran, brightest star in the head of Taurus, the bull, may also be found by a curved line extending northwestward from the belt of Orion. The V-shaped figure forming the outline of the head and horns of Taurus points toward third magnitude Menkar. At the summer solstice the Sun is between Pollux and Aldebaran.

If the curved line from Orion's belt southeastward to Sirius is continued, it leads to a conspicuous, small, nearly equilateral triangle of three bright second magnitude stars of nearly equal brilliancy. This is part of Canis Major. Only Adhara, the westernmost of the three stars, is listed on the daily pages of the *Nautical Almanac*. Continuing on with somewhat less curvature, the line leads to Canopus, second brightest star in the heavens and one of the two stars having a negative magnitude (−0.9). With Suhail and Miaplacidus, Canopus forms a large, equilateral triangle which partly encloses the group of stars often mistaken for Crux. The brightest star within this triangle is Avior, near its center. Canopus is also at one apex of a triangle formed with Adhara to the north and Suhail to the east, another triangle with Acamar to the west and Achernar to the southwest, and another with Achernar and Miaplacidus. Acamar, Achernar, and Ankaa form still another triangle toward the west. Because of chart distortion, these triangles do not appear in the sky in exactly the relationship shown on the star chart. Other daily-page almanac stars near the bottom of Figure 1336 are discussed in succeeding articles.

In the winter evening sky, Ursa Major is east of Polaris, Ursa Minor is nearly below it, and Cassiopeia is west of it. Mirfak is northwest of Capella, nearly midway between it and Cassiopeia. Hamal is in the western sky. Regulus and Alphard are low in the eastern sky, heralding the approach of the configurations associated with the evening skies of spring.

### 1337. Stars in the Vicinity of Ursa Major

As if to enhance the splendor of the sky in the vicinity of Orion, the region toward the east, like that toward the west, has few bright stars, except in the vicinity of the south celestial pole. However, as Orion sets in the west, leaving Capella and Pollux in the northwestern sky, a number of good navigational stars move into favorable positions for observation.

Ursa Major, the great bear, appears prominently above the north celestial pole, directly opposite Cassiopeia, which appears as a "W" just above the northern horizon of most

observers in latitudes of the United States. Of the seven stars forming Ursa Major, only Dubhe, Alioth, and Alkaid are in the list of selected stars in *Nautical Almanac*. See Figure 1337.

The two second magnitude stars forming the outer part of the bowl of Ursa Major are often called the pointers because a line extending northward (down in spring evenings) through them points to Polaris. Ursa Minor, the Little Bear, contains Polaris at one end and Kochab at the other. Relative to its bowl, the handle of Ursa Minor curves in the opposite direction to that of Ursa Major.

A line extending southward through the pointers, and curving somewhat toward the west, leads to first magnitude Regulus, brightest star in Leo, the lion. The head, shoulders, and front legs of this constellation form a sickle, with Regulus at the end of the handle. Toward the east is second magnitude Denebola, the tail of the lion. On toward the southwest from Regulus is second magnitude Alphard, brightest star in Hydra, the sea serpent. A dark sky and considerable imagination are needed to trace the long, winding body of this figure.

A curved line extending the arc of the handle of Ursa Major leads to first magnitude Arcturus. With Alkaid and Alphecca, brightest star in Corona Borealis, the Northern Crown, Arcturus forms a large, inconspicuous triangle. If the arc through Arcturus is continued, it leads next to first magnitude Spica and then to Corvus, the crow. The brightest star in this constellation is Gienah, but three others are nearly as bright. At autumnal equinox, the Sun is on the celestial equator, about midway between Regulus and Spica.

A long, slightly curved line from Regulus, east-southeasterly through Spica, leads to Zubeneschamali at the southwestern corner of an inconspicuous box-like figure called Libra, the scales.

Returning to Corvus, a line from Gienah, extending diagonally across the figure and then curving somewhat toward the east, leads to Menkent, just beyond Hydra.

Far to the south, below the horizon of most northern hemisphere observers, a group of bright stars is a prominent feature of the spring sky of the Southern Hemisphere. This is Crux, the Southern Cross. Crux is about 40° south of Corvus. The "false cross" to the west is often mistaken for Crux. Acrux at the southern end of Crux and Gacrux at the northern end are selected stars, listed on the daily pages of the *Nautical Almanac*.

The triangles formed by Suhail, Miaplacidus, and Canopus, and by Suhail, Adhara, and Canopus, are west of Crux. Suhail is in line with the horizontal arm of Crux. A line from Canopus, through Miaplacidus, curved slightly toward the north, leads to Acrux. A line through the east-west arm of Crux, eastward and then curving toward the south, leads first to Hadar and then to Rigel Kentaurus, both very bright stars. Continuing on, the curved line leads to small Triangulum Australe, the Southern Triangle, the easternmost star of which is Atria.

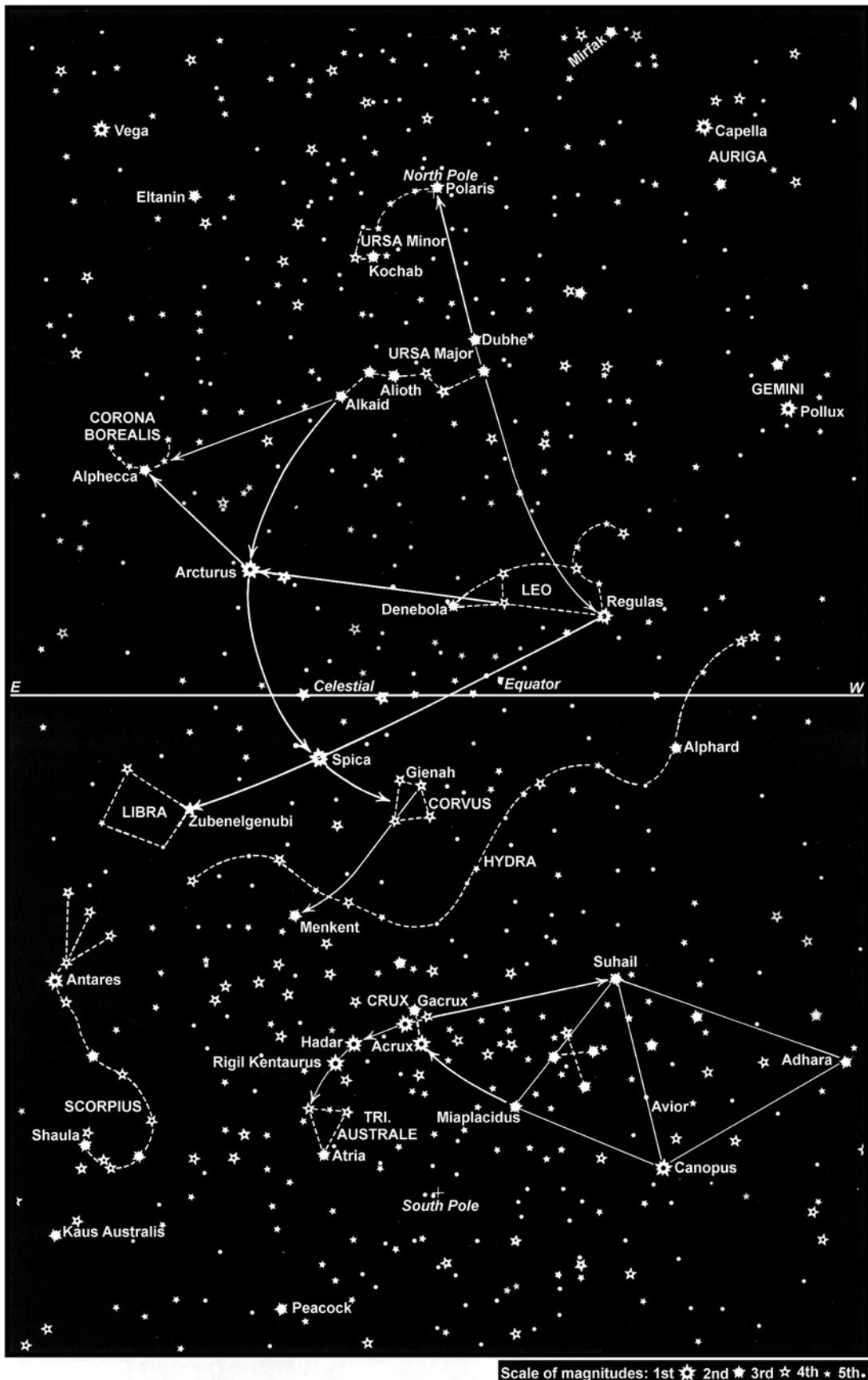


Figure 1337. Stars in the vicinity of Ursa Major.

### 1338. Stars in the Vicinity of Cygnus

As the celestial sphere continues in its apparent westward rotation, the stars familiar to a spring evening observer sink low in the western sky. By midsummer, Ursa Major has moved to a position to the left of the north celestial pole, and the line from the pointers to Polaris is nearly horizontal. Ursa Minor, is standing on its handle, with Kochab above and to the left of the celestial pole. Cassiopeia is at the right of Polaris, opposite the handle of Ursa Major. See Figure 1338.

The only first magnitude star in the western sky is Arcturus, which forms a large, inconspicuous triangle with Alkaid, the end of the handle of Ursa Major, and Alphecca, the brightest star in Corona Borealis, the Northern Crown.

The eastern sky is dominated by three very bright stars. The westernmost of these is Vega, the brightest star north of the celestial equator, and third brightest star in the heavens, with a magnitude of 0.1. With a declination of a little less than  $39^{\circ}\text{N}$ , Vega passes through the zenith along a path across the central part of the United States, from Washington in the east to San Francisco on the Pacific coast. Vega forms a large but conspicuous triangle with its two bright neighbors, Deneb to the northeast and Altair to the southeast. The angle at Vega is nearly a right angle. Deneb is at the end of the tail of Cygnus, the swan. This configuration is sometimes called the Northern Cross, with Deneb at the head. To modern youth it more nearly resembles a dive bomber, while it is still well toward the east, with Deneb at the nose of the fuselage. Altair has two fainter stars close by, on opposite sides. The line formed by Altair and its two fainter companions, if extended in a northwesterly direction, passes through Vega, and on to second magnitude Eltanin. The angular distance from Vega to Eltanin is about half that from Altair to Vega. Vega and Altair, with second magnitude Rasalhague to the west, form a large equilateral triangle. This is less conspicuous than the Vega-Deneb-Altair triangle because the brilliance of Rasalhague is much less than that of the three first magnitude stars, and the triangle is overshadowed by the brighter one.

Far to the south of Rasalhague, and a little toward the west, is a striking configuration called Scorpius, the scorpion. The brightest star, forming the head, is red Antares. At the tail is Shaula.

Antares is at the southwestern corner of an approximate parallelogram formed by Antares, Sabik, Nunki, and Kaus Australis. With the exception of Antares, these stars are only slightly brighter than a number of others nearby, and so this parallelogram is not a striking figure. At winter solstice the Sun is a short distance northwest of Nunki.

Northwest of Scorpius is the box-like Libra, the scales, of which Zubeneschamali marks the southwest corner.

With Menkent and Rigel Kentaurus to the southwest,

Antares forms a large but unimpressive triangle. For most observers in the latitudes of the United States, Antares is low in the southern sky, and the other two stars of the triangle are below the horizon. To an observer in the Southern Hemisphere Crux is to the right of the south celestial pole, which is not marked by a conspicuous star. A long, curved line, starting with the now-vertical arm of Crux and extending northward and then eastward, passes successively through Hadar, Rigel Kentaurus, Peacock, and Al Na'ir.

Fomalhaut is low in the southeastern sky of the southern hemisphere observer, and Enif is low in the eastern sky at nearly any latitude. With the appearance of these stars it is not long before Pegasus will appear over the eastern horizon during the evening, and as the winged horse climbs evening by evening to a position higher in the sky, a new annual cycle approaches.

### 1339. Planet Diagram

The planet diagram, on page 9 of the *Nautical Almanac* shows, for any date, the Local Mean Time (LMT) of meridian passage of the Sun, for the five planets Mercury, Venus, Mars, Jupiter, and Saturn, and of each  $30^{\circ}$  of SHA (Figure 1339). The diagram provides a general picture of the availability of planets and stars for observation, and thus shows:

1. Whether a planet or star is too close to the Sun for observation.
2. Whether a planet is a morning or evening star.
3. Some indication of the planet's position during twilight.
4. The proximity of other planets.
5. Whether a planet is visible from evening to morning twilight.

A band 45 minutes wide is shaded on each side of the curve marking the LMT of meridian passage of the Sun. Planets and stars lying within the shaded area are too close to the Sun for observation.

When the meridian passage occurs at midnight, the body is in opposition to the Sun and is visible all night; planets may be observable in both morning and evening twilights. When meridian passage is between 12h and 24h (that is, after the Sun's meridian passage), the object is visible in the evening sky, after sunset. When meridian passage is between 0 and 12 hours (that is, before the Sun's meridian passage) the object is visible in the morning sky, before sunrise. Graphically, if the curve for a planet intersects the vertical line connecting the date graduations below the shaded area, the planet is a morning "star"; if the intersection is above the shaded area, the planet is an evening "star".

Only about one-half the region of the sky along the ecliptic, as shown on the diagram, is above the horizon at one time. At sunrise (LMT about  $6^{\text{h}}$ ) the Sun and, hence, the

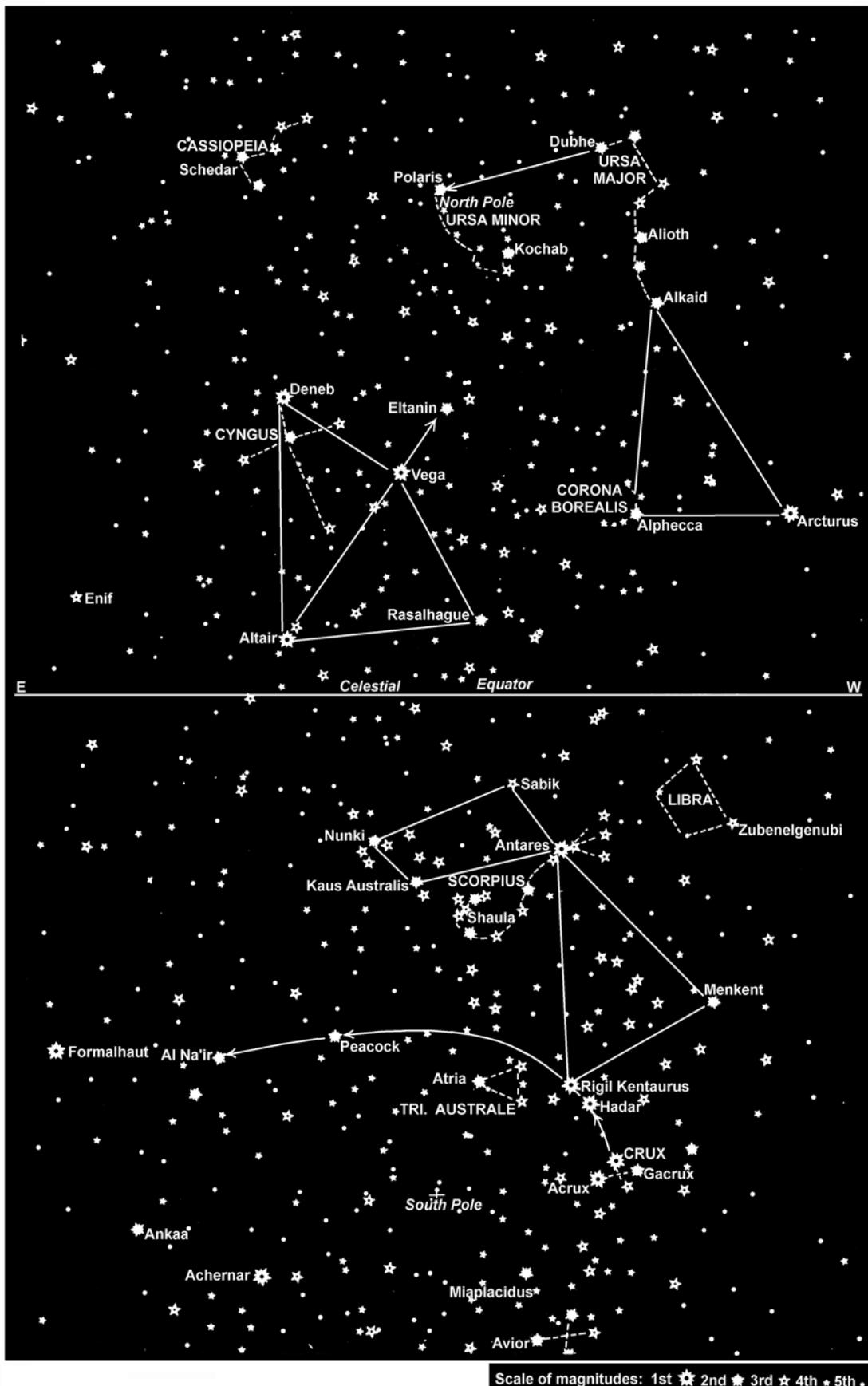


Figure 1338. Stars in the vicinity of Cygnus.

region near the middle of the diagram, are rising in the east; the region at the bottom of the diagram is setting in the west. The region half way between is on the meridian. At sunset (LMT about 18<sup>h</sup>) the Sun is setting in the west; the region at the top of the diagram is rising in the east. Marking the planet diagram of the *Nautical Almanac* so that east is at the top of the diagram and west is at the bottom can be useful to interpretation.

A similar planet location diagram in the *Air Almanac* (pages A122-A123) represents the region of the sky along the ecliptic. It shows, for each date, the Sun in the center and the relative positions of the Moon, the five planets Mercury, Venus, Mars, Jupiter, Saturn and the four first magnitude stars Aldebaran, Antares, Spica, and Regulus, and also the position on the ecliptic which is north of Sirius (i.e. Sirius is 40° south of this point). The first point of Aries is also shown for reference. The magnitudes of the planets are given at suitable intervals along the curves. The Moon symbol shows the correct phase. A straight line joining the date on the left-hand side with the same date of the right-hand side represents a complete circle around the sky, the two ends of the line representing the point 180° from the Sun; the intersections with the curves show the spacing of the bodies along the ecliptic on the date. The time scale indicates roughly the local mean time at which an object will be on the observer's meridian.

At any time only about half the region on the diagram is above the horizon. At sunrise the Sun (and hence the region near the middle of the diagram), is rising in the east and the region at the end marked "West" is setting in the west; the region half-way between these extremes is on the meridian, as will be indicated by the local time (about 6<sup>h</sup>). At the time of sunset (local time about 18<sup>h</sup>) the Sun is setting in the west, and the region at the end marked "East" is rising in the east. The diagram should be used in conjunction with the Sky Diagrams.

#### 1340. Finding Stars for a Fix

Various devices have been invented to help an observer find individual stars. The most widely used is the **Star Finder and Identifier**, also known as a **Rude Star Finder** and formerly published by the U.S. Navy Hydrographic Office as **No. 2102D**. It is no longer issued, but it is still available commercially. A navigational calculator or computer program, like the U.S. Navy STELLA program is much quicker, more accurate, and less tedious. A navigational calculator can be used to predict the best stars to observe for a fix. See Section 1900 - Computer Sight Reduction for a more thorough discussion.

*HO Publication 249, (Rapid Sight Reduction Tables for Navigation), Volume 1*, identifies the best three and seven stars for a navigational fix given an observer's latitude and LHA of Aries. This publication is also known as AP 3270.

The navigational program also solves for the LOP's for each object observed, combines them into the best fix, and displays the lat./long. position. Most navigational programs

also print out a plotted fix, just as the navigator might have drawn by hand.

Computer sight reduction programs can also automatically predict twilight on a moving vessel and create a plot of the sky at the vessel's twilight location (or any location, at any time). This plot will be free of the distortion inherent in the mechanical star finders and will show all bodies, even planets, Sun, and Moon, in their correct relative orientation centered on the observer's zenith. It will also indicate which stars provide the best geometry for a fix.

Computer sight reduction programs or celestial navigation calculators, or apps are especially useful when the sky is only briefly visible thorough broken cloud cover.

#### 1341. Identification by Computation

If the altitude and azimuth of the celestial body, and the approximate latitude of the observer, are known, the navigational triangle can be solved for meridian angle and declination. The meridian angle can be converted to LHA, and this to GHA. With this and GHA  $\Psi$  at the time of observation, the SHA of the body can be determined. With SHA and declination, one can identify the body by reference to an almanac. Any method of solving a spherical triangle, with two sides and the included angle being given, is suitable for this purpose.

Although no formal star identification tables are included in *Pub. No. 229*, a simple approach to star identification is to scan the pages of the appropriate latitudes, and observe the combination of arguments which give the altitude and azimuth angle of the observation. Thus the declination and LHA H are determined directly. The star's SHA is found from  $\text{SHA H} = \text{LHA H} - \text{LHA } \Psi$ . From these quantities the star can be identified from the *Nautical Almanac*.

Another solution is available through an interchange of arguments using the nearest integral values. The procedure consists of entering *Pub. No. 229* with the observer's latitude (same name as declination), with the observed azimuth angle (converted from observed true azimuth as required) as LHA and the observed altitude as declination, and extracting from the tables the altitude and azimuth angle respondents. The extracted altitude becomes the body's declination; the extracted azimuth angle (or its supplement) is the meridian angle of the body. Note that the tables are always entered with latitude of same name as declination. In north latitudes the tables can be entered with true azimuth as LHA.

If the respondents are extracted from above the C-S Line on a right-hand page, the name of the latitude is actually contrary to the declination. Otherwise, the declination of the body has the same name as the latitude. If the azimuth angle respondent is extracted from above the C-S Line, the supplement of the tabular value is the meridian angle,  $t$ , of the body. If the body is east of the observer's meridian,  $\text{LHA} = 360^\circ - t$ ; if the body is west of the meridian,  $\text{LHA} = t$ .

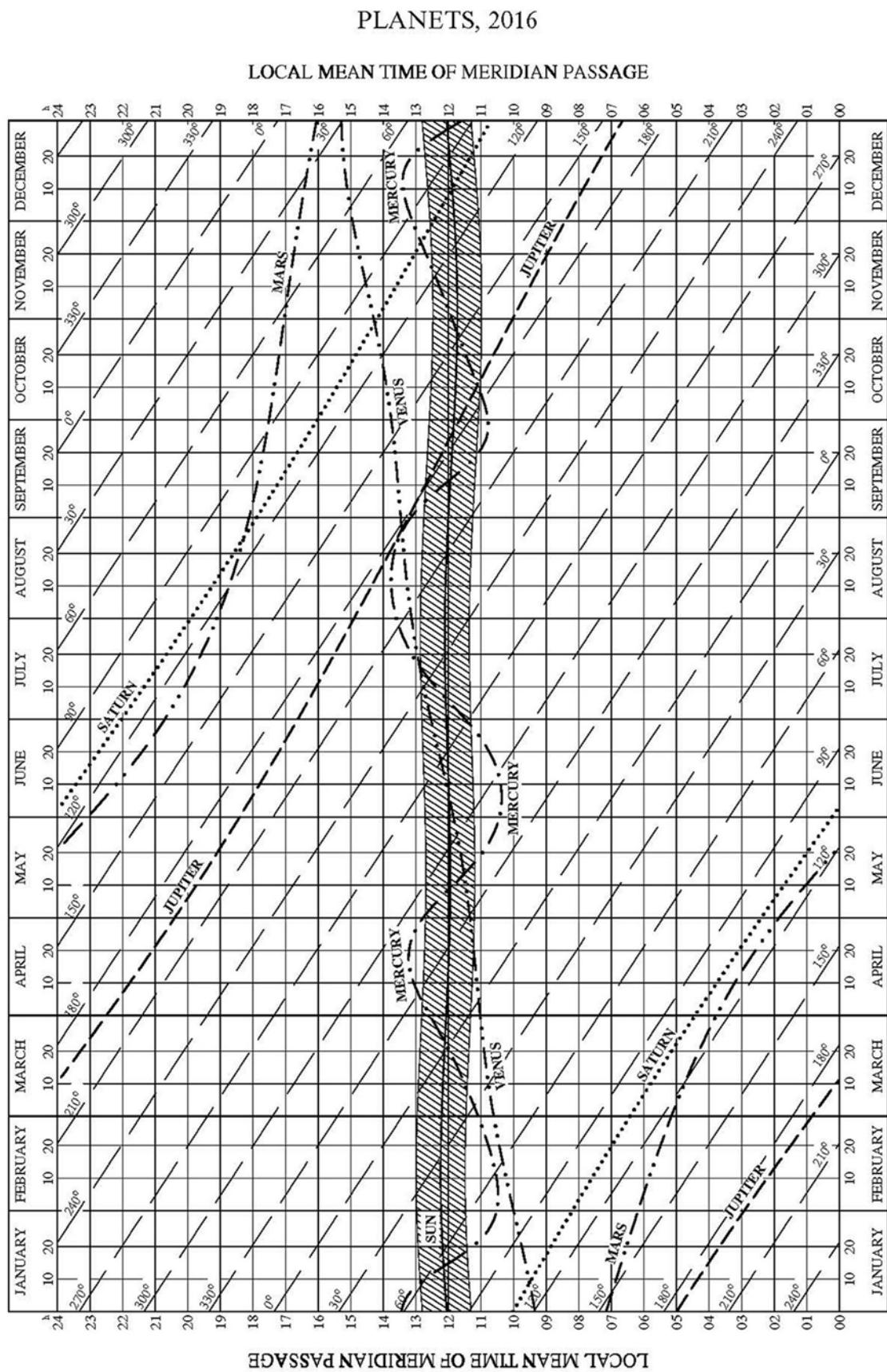


Figure 1339. Reproduction of Nautical Almanac Page 9.



Figure 1341. The Ghost of Cassiopeia. Image Credits: NASA, ESA and STScI; Acknowledgment: H. Arab (University of Strasbourg). Powerful gushers of energy from seething stars can sculpt eerie-looking figures with long, flowing veils of gas and dust. One striking example is "the Ghost of Cassiopeia" officially known as IC 63, located 550 light-years away in the constellation Cassiopeia the Queen. The constellation Cassiopeia is visible every clear night from mid-northern and higher latitudes. Its distinctive "W" asterism, which forms the queen's throne, is best seen high in the sky on autumn and winter evenings. Gamma Cassiopeiae, the middle star in the W, is visible to the unaided eye, but a large telescope is needed to see IC 63. Hubble photographed IC 63 in August 2016.

# CHAPTER 14

## INSTRUMENTS FOR CELESTIAL NAVIGATION

### THE MARINE SEXTANT

#### 1400. Description and Use

The marine sextant measures the angle between two points by bringing the direct image from one point and a double-reflected image from the other into coincidence. Its principal use is to measure the altitudes of celestial bodies above the visible sea horizon. It may also be used to measure vertical angles to find the range from an object of known height. The marine sextant can also be used to render a visual Line of Position (LOP) by turning it on its side to horizontally measure the angular distance between two terrestrial objects. See Chapter 11- Use of Sextant in Piloting.

A marine sextant can measure angles up to approximately  $120^\circ$ . Originally, the term “sextant” was applied to the navigator’s double-reflecting, altitude-measuring instrument only if its arc was  $60^\circ$  in length, or  $1/6$  of a circle, permitting measurement of angles from  $0^\circ$  to  $120^\circ$ . In modern usage the term is applied to all modern navigational altitude-measuring instruments regardless of angular range or principles of operation.

#### 1401. Optical Principles of a Sextant

When a plane surface reflects a light ray, the angle of reflection equals the angle of incidence. The angle between the first and final directions of a ray of light that has undergone double reflection in the same plane is twice the angle the two reflecting surfaces make with each other.

In Figure 1401 - *Optical principle of the marine sextant*, S to M is a ray of light from a celestial body.

The index mirror of the sextant is at M, the horizon glass at F, and the eye of the observer at A. The ray of light from S is reflected at mirror M, proceeds to mirror F, where it is again reflected, and then continues on to the eye of the observer. Geometrically, it can be shown that the altitude of the object S (angle  $\alpha$ ) is two times that of the angle between the mirrors (angle  $\beta$ ). The graduations on the arc give the altitude.

#### 1402. Micrometer Drum Sextant

Figure 1402 shows a modern marine sextant, called a **micrometer drum sextant**. In most marine sextants, brass or aluminum comprise the **frame**, A. Frames come in vari-

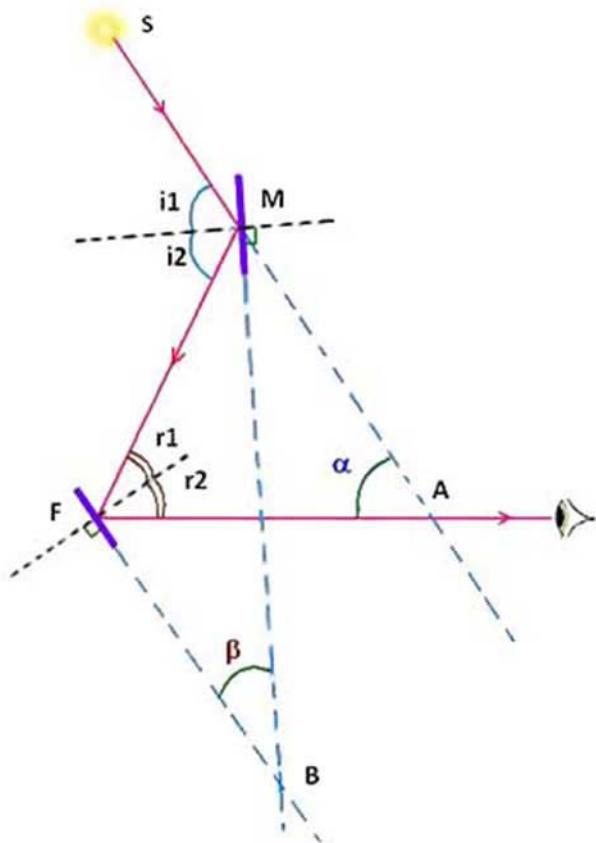


Figure 1401. Optical principle of the marine sextant.

ous designs; most are similar to this. Teeth mark the outer edge of the **limb**, B; each tooth marks one degree of altitude. The altitude graduations, C, along the limb, mark the arc. Some sextants have an arc marked in a strip of brass, silver, or platinum inlaid in the limb.

The **index arm**, D, is a movable bar of the same material as the frame. It pivots about the center of curvature of the limb. The **tangent screw**, E, is mounted perpendicularly on the end of the index arm, where it engages the teeth of the limb. Because the observer can move the index arm through the length of the arc by rotating the tangent screw, this is sometimes called an “endless tangent screw.” The **release**, F, is a spring-actuated clamp that keeps the tangent screw engaged with the limb’s teeth. The observer can disengage

the tangent screw and move the index arm along the limb for rough adjustment. The end of the tangent screw mounts a **micrometer drum**, G, graduated in minutes of altitude. One complete turn of the drum moves the index arm one degree along the arc. Next to the micrometer drum and fixed on the index arm is a **vernier**, H, that reads in fractions of a minute. The vernier shown is graduated into ten parts, permitting readings to  $1/10$  of a minute of arc (0.1'). Some sextants have verniers graduated into only five parts, permitting readings to 0.2'.

The **index mirror**, I, is a piece of silvered plate glass mounted on the index arm, perpendicular to the plane of the instrument, with the center of the reflecting surface directly over the pivot of the index arm. The **horizon glass**, J, is a piece of optical glass silvered on its half nearer the frame. It is mounted on the frame, perpendicular to the plane of the sextant. The index mirror and horizon glass are mounted so that their surfaces are parallel when the micrometer drum is set at 0°, if the instrument is in perfect adjustment. **Shade glasses**, K, of varying darkness are mounted on the sextant's frame in front of the index mirror and horizon glass. They can be moved into the line of sight as needed to reduce the intensity of light reaching the eye.

The **telescope**, L, screws into an adjustable collar in line with the horizon glass and parallel to the plane of the instrument. Most modern sextants are provided with only one telescope. When only one telescope is provided, it is

of the "erect image type," either as shown or with a wider "object glass" (far end of telescope), which generally is shorter in length and gives a greater field of view. The second telescope, if provided, may be the "inverting type." The inverting telescope, having one lens less than the erect type, absorbs less light, but at the expense of producing an inverted image. A small colored glass cap is sometimes provided, to be placed over the "eyepiece" (near end of telescope) to reduce glare. With this in place, shade glasses are generally not needed. A "peep sight," or clear tube which serves to direct the line of sight of the observer when no telescope is used, may be fitted.

Sextants are designed to be held in the right hand. Some have a small light on the index arm to assist in reading altitudes. The batteries for this light are fitted inside a recess in the **handle**, M. Not clearly shown in Figure 1402 is the **tangent screw**, E, and the three legs.

There are two basic designs commonly used for mounting and adjusting mirrors on marine sextants. On the U.S. Navy Mark 3 and certain other sextants, the mirror is mounted so that it can be moved against retaining or mounting springs within its frame. Only one perpendicular adjustment screw is required. On the U.S. Navy Mark 2 and other sextants the mirror is fixed within its frame. Two perpendicular adjustment screws are required. One screw must be loosened before the other screw bearing on the same surface is tightened.

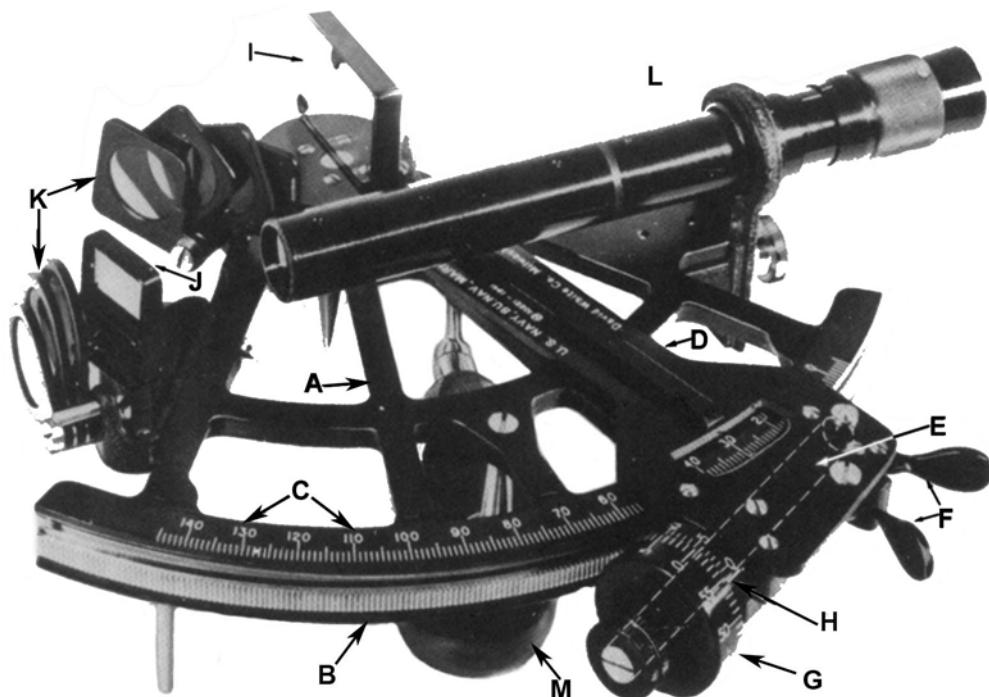


Figure 1402. U.S. Navy Mark 2 micrometer drum sextant.

### 1403. Vernier Sextant

Most recent marine sextants are of the micrometer drum type, but at least two older-type sextants are still in use. These differ from the micrometer drum sextant principally in the manner in which the final reading is made. They are called **vernier sextants**.

The **clamp screw vernier sextant** is the older of the two. In place of the modern release clamp, a clamp screw is fitted on the underside of the index arm. To move the index arm, the clamp screw is loosened, releasing the arm. When the arm is placed at the approximate altitude of the body being observed, the clamp screw is tightened. Fixed to the clamp screw and engaged with the index arm is a long tangent screw. When this screw is turned, the index arm moves slowly, permitting accurate setting. Movement of the index arm by the tangent screw is limited to the length of the screw (several degrees of arc). Before an altitude is measured, this screw should be set to the approximate midpoint of its range. The final reading is made on a vernier set in the index arm below the arc. A small microscope or magnifying glass fitted to the index arm is used in making the final reading.

The **endless tangent screw vernier sextant** is identical to the micrometer drum sextant, except that it has no drum, and the fine reading is made by a vernier along the arc, as with the clamp screw vernier sextant. The release is the same as on the micrometer drum sextant, and teeth are cut into the underside of the limb which engage with the endless tangent screw.

### 1404. Sextant Sun Sights

For a Sun sight, hold the sextant vertically and direct the sight line at the horizon directly below the Sun. After moving suitable shade glasses into the line of sight, move the index arm outward along the arc until the reflected image appears in the horizon glass near the direct view of the horizon. Rock the sextant (also known as "swinging the arc" or "to swing the arc") slightly to the right and left to ensure it is perpendicular. As you rock the sextant, the image of the Sun appears to move in an arc, and you may have to turn slightly to prevent the image from moving off the horizon glass.

The sextant is vertical when the Sun appears at the bottom of the arc. This is the correct position for making the observation. The Sun's reflected image appears at the center of the horizon glass; one half appears on the silvered part, and the other half appears on the clear part. Move the index arm with the drum or vernier slowly until the Sun appears to be resting exactly on the horizon, tangent to the lower limb. The novice observer needs practice to determine the exact point of tangency. Beginners often err by bringing the image down too far.

Some navigators get their most accurate observations by letting the body contact the horizon by its own motion, bringing it slightly below the horizon if rising, and above if

setting. At the instant the horizon is tangent to the disk, the navigator notes the time. The sextant altitude is the uncorrected reading of the sextant.

### 1405. Sextant Moon Sights

When observing the Moon, follow the same procedure as for the Sun. Because of the phases of the Moon, the upper limb of the Moon is observed more often than that of the Sun. When the terminator (the line between light and dark areas) is nearly vertical, be careful in selecting the limb to shoot. Sights of the Moon are best made during either daylight hours or that part of twilight in which the Moon is least luminous. At night, false horizons may appear below the Moon because the Moon illuminates the water below it.

### 1406. Sextant Star and Planet Sights

While the relatively large Sun and Moon are easy to find with a sextant, stars and planets can be more difficult to locate because the field of view is so narrow. One of three methods may help locate a star or planet:

**Method 1.** Set the index arm and micrometer drum on  $0^\circ$  and direct the line of sight at the body to be observed. Then, while keeping the reflected image of the body in the mirrored half of the horizon glass, swing the index arm out and rotate the frame of the sextant down. Keep the reflected image of the body in the mirror until the horizon appears in the clear part of the horizon glass; then, make the observation. When there is little contrast between brightness of the sky and the body this procedure can be difficult. If the body is "lost" while it is being brought down, it may not be recovered without starting over again.

**Method 2.** Direct the line of sight at the body while holding the sextant upside down. Slowly move the index arm out until the horizon appears in the horizon glass. Then invert the sextant and take the sight in the usual manner.

**Method 3.** Determine in advance the approximate altitude and azimuth of the body by a star finder such as No. 2102D. Set the sextant at the indicated altitude and face in the direction of the azimuth. The image of the body should appear in the horizon glass with a little searching.

When measuring the altitude of a star or planet, bring its center down to the horizon. Stars and planets have no discernible upper or lower limb; you must observe the center of the point of light. Because stars and planets have no discernible limb and because their visibility may be limited, the method of letting a star or planet intersect the horizon by its own motion is not recommended. As with the Sun and Moon, however, "swing the arc" to establish perpendicularity.

### 1407. Taking a Sight

Unless you have a navigation calculator, computer program, or app that will identify bodies automatically,

predict expected altitudes and azimuths for up to eight bodies when preparing to take celestial sights. Choose the stars and planets that will provide the best bearing spread. Try to select bodies with a predicted altitude between  $30^\circ$  and  $70^\circ$ . Take sights of the brightest stars first in the evening; take sights of the brightest stars last in the morning. See Chapter 18, Section 1810 - Sight Planning, for a more in depth discussion.

Occasionally, fog, haze, or other ships in a formation may obscure the horizon directly below a body which the navigator wishes to observe. If the arc of the sextant is sufficiently long, a **back sight** might be obtained, using the opposite point of the horizon as the reference. For this the observer faces away from the body and observes the supplement of the altitude. If the Sun or Moon is observed in this manner, what appears in the horizon glass to be the lower limb is in fact the upper limb, and vice versa. In the case of the Sun, it is usually preferable to observe what appears to be the upper limb. The arc that appears when rocking the sextant for a back sight is inverted; that is, the highest point indicates the position of perpendicularity.

If more than one telescope is furnished with the sextant, the erecting telescope is used to observe the Sun. A wider field of view is present if the telescope is not used. The collar into which the sextant telescope fits may be adjusted in or out, in relation to the frame. When moved in, more of the mirrored half of the horizon glass is visible to the navigator, and a star or planet is more easily observed when the sky is relatively bright. Near the darker limit of twilight, the telescope can be moved out, giving a broader view of the clear half of the glass, and making the less distinct horizon more easily discernible. If both eyes are kept open until the last moments of an observation, eye strain will be lessened. Practice will permit observations to be made quickly, reducing inaccuracy due to eye fatigue.

When measuring an altitude, have an assistant note and record the time if possible, with a "stand-by" warning when the measurement is almost ready, and a "mark" at the moment a sight is made. If a flashlight is needed to see the comparing watch, the assistant should be careful not to interfere with the navigator's night vision.

If an assistant is not available to time the observations, the observer holds the watch in the palm of his or her left hand, leaving his or her fingers free to manipulate the tangent screw of the sextant. After making the observation, s/he notes the time as quickly as possible. The delay between completing the altitude observation and noting the time should not be more than one or two seconds.

#### 1408. Reading the Sextant

Reading a micrometer drum sextant is done in three steps. The degrees are read by noting the position of the arrow on the index arm in relation to the arc. The minutes are read by noting the position of the zero on the vernier with relation to the graduations on the micrometer drum. The

fraction of a minute is read by noting which mark on the vernier most nearly coincides with one of the graduations on the micrometer drum. This is similar to reading the time with the hour, minute, and second hands of a watch. In both, the relationship of one part of the reading to the others should be kept in mind. Thus, if the hour hand of a watch were about on "4," one would know that the time was about four o'clock. But if the minute hand were on "58," one would know that the time was 0358 (or 1558), not 0458 (or 1658). Similarly, if the arc indicated a reading of about  $40^\circ$ , and  $58'$  on the micrometer drum were opposite zero on the vernier, one would know that the reading was  $39^\circ 58'$ , not  $40^\circ 58'$ . Similarly, any doubt as to the correct minute can be removed by noting the fraction of a minute from the position of the vernier. In Figure 1408a the reading is  $29^\circ 42.5'$ . The arrow on the index mark is between  $29^\circ$  and  $30^\circ$ , the zero on the vernier is between  $42'$  and  $43'$ , and the  $0.5'$  graduation on the vernier coincides with one of the graduations on the micrometer drum.

The principle of reading a vernier sextant is the same, but the reading is made in two steps. Figure 1408b shows a typical altitude setting. Each degree on the arc of this sextant is graduated into three parts, permitting an initial reading by the reference mark on the index arm to the nearest  $20'$  of arc. In this illustration the reference mark lies between  $76^\circ 20'$  and  $76^\circ 40'$ , indicating a reading between these values. The reading for the fraction between  $20'$  and  $40'$  is made using the vernier, which is engraved on the index arm and has the small reference mark as its zero graduation. On this vernier, 20 graduations coincide with 19 graduations on the arc. Each graduation on the vernier is equivalent to  $1/20$  of one graduation of  $20'$  on the arc, or  $0.5'$ , or  $30''$ . In the illustration, the vernier graduation representing  $6'$  most nearly coincides with one of the graduations on the arc. Therefore, the reading is  $76^\circ 20' + 6' 00''$  or  $76^\circ 26' 00''$ . When a vernier of this type is used, any doubt as to which mark on the vernier coincides with a graduation on the arc can usually be resolved by noting the position of the vernier mark on each side of the one that seems to be in coincidence.

Negative readings, such as a negative index correction, are made in the same manner as positive readings; the various figures are added algebraically. Thus, if the three parts of a micrometer drum reading are  $(- )1^\circ, 56'$  and  $0.3'$ , the total reading is  $(- )1^\circ + 56' + 0.3' = (- )3.7'$ .

#### 1409. Developing Observational Skill

A well-constructed marine sextant is capable of measuring angles with an instrument error not exceeding  $0.1'$ . Lines of position from altitudes of this accuracy would not be in error by more than about 200 yards. However, there are various sources of error, other than instrumental, in altitudes measured by sextant. One of the principal sources is the observer.

The first fix a student celestial navigator plots is likely to be disappointing. Most navigators require a great amount of practice to develop the skill necessary for consistently

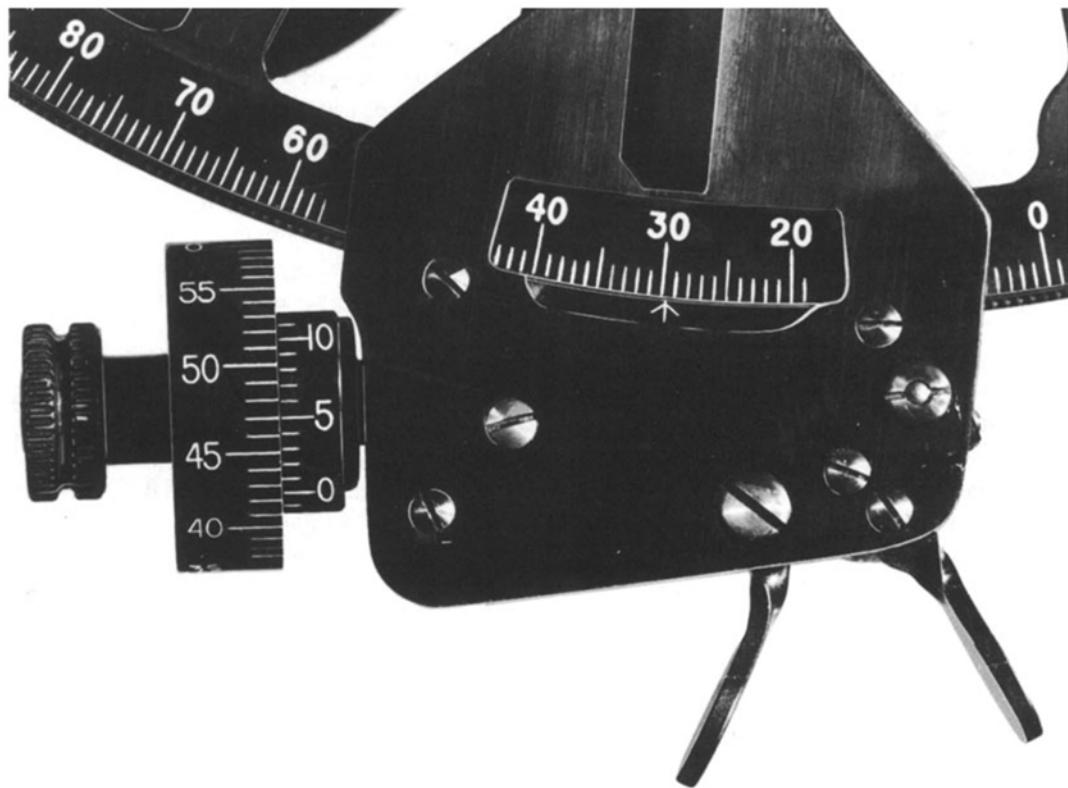


Figure 1408a. Micrometer drum sextant set at  $29^{\circ} 42.5'$ .

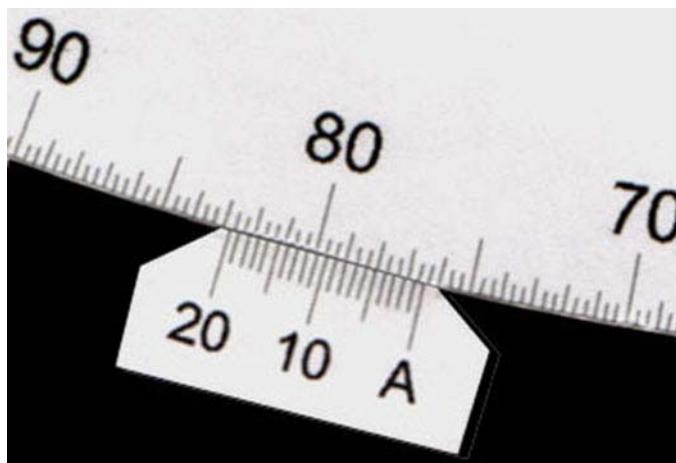


Figure 1408b. Vernier sextant set at  $76^{\circ} 26' 00''$ . Image courtesy of Omar F. Reis.

good observations. But practice alone is not sufficient. Good technique should be developed early and refined throughout the navigator's career. Many good pointers can be obtained from experienced navigators, but each student navigator must develop his or her own technique because one method proves successful for one observer may not be helpful to another. Also,

experienced navigators have a natural tendency to judge the accuracy of their observations solely by the size of the figure formed with the intersection of the plotted lines of position. Although a small area of intersection (or a "tight fix") may be present, it may not necessarily be an accurate reflection of the ship's position if individual observation errors are allowed to be introduced. There are many errors,

some of which are beyond the navigator's control. Therefore, lines of position from celestial observations should be compared often with accurate position obtained by electronics or piloting.

Common sources of error are:

1. Time errors.
2. Sextant adjustment.
3. Improper rocking of the sextant.
4. The height of eye input may be wrong.
5. Index correction computation errors.
6. Subnormal refraction (dip) might be present.
7. Inaccurate judgment of tangency.
8. Using a false horizon.
9. Other computation errors.

Generally, it is possible to correct observation technique errors, but occasionally a personal error will persist. This error might vary as a function of the body observed, degree of fatigue of the observer, and other factors. For this reason, a personal error should be applied with caution.

To obtain greater accuracy, take a number of closely-spaced observations. Plot the resulting altitudes versus time and draw a curve through the points. Unless the body is near the celestial meridian, this curve should be a straight line. Use this graph to determine the altitude of the body at any time covered by the graph. It is best to use a point near the middle of the line. Using a navigational calculator, computer program, or app to reduce sights will yield greater accuracy because of the rounding errors inherent in the use of sight reduction tables, and because many more sights can be reduced in a given time, thus averaging out errors.

A simpler method involves making observations at equal intervals. This procedure is based upon the assumption that, unless the body is on the celestial meridian, the change in altitude should be equal for equal intervals of time. Observations can be made at equal intervals of altitude or time. If time intervals are constant, the mid time and the average altitude are used as the observation. If altitude increments are constant, the average time and mid altitude are used.

If only a small number of observations is available, reduce and plot the resulting lines of position; then adjust them to a common time. The average position of the line might be used, but it is generally better practice to use the middle line. Reject any observation considered unreliable when determining the average.

#### 1410. Care of the Sextant

A sextant is a rugged instrument. However, careless handling or neglect can cause it irreparable harm. If you drop it, take it to an instrument repair shop for testing and inspection. When not using the sextant, stow it in a sturdy and sufficiently padded case. Keep the sextant away from excessive heat and dampness. Do not expose it to excessive

vibration. Do not leave it unattended when it is out of its case. Do not hold it by its limb, index arm, or telescope. Lift it only by its frame or handle. Do not lift it by its arc or index bar.

Next to careless handling, moisture is the sextant's greatest enemy. Wipe the mirrors and the arc after each use. If the mirrors get dirty, clean them with lens paper and a small amount of alcohol. Clean the arc with ammonia; never use a polishing compound. When cleaning, do not apply excessive pressure to any part of the instrument.

Silica gel kept in the sextant case will help keep the instrument free from moisture and preserve the mirrors. Occasionally heat the silica gel to remove the absorbed moisture.

Rinse the sextant with fresh water if sea water gets on it. Wipe the sextant gently with a soft cotton cloth and dry the optics with lens paper.

Glass optics do not transmit all the light received because glass surfaces reflect a small portion of light incident on their face. This loss of light reduces the brightness of the object viewed. Viewing an object through several glass optics affects the perceived brightness and makes the image indistinct. The reflection also causes glare which obscures the object being viewed. To reduce this effect to a minimum, the glass optics are treated with a thin, fragile, anti-reflection coating. Therefore, apply only light pressure when polishing the coated optics. Blow loose dust off the lens before wiping them so grit does not scratch the lens.

Occasionally, oil and clean the tangent screw and the teeth on the side of the limb. Use the oil provided with the sextant or an all-purpose light machine oil. Occasionally set the index arm of an endless tangent screw at one extremity of the limb, oil it lightly, and then rotate the tangent screw over the length of the arc. This will clean the teeth and spread oil over them. When stowing a sextant for a long period, clean it thoroughly, polish and oil it, and protect its arc with a thin coat of petroleum jelly. If the mirrors need re-silvering, take the sextant to an instrument shop.

#### 1411. Non Adjustable Sextant Errors

The non-adjustable sextant errors are prismatic error, graduation error, and centering error. The higher the quality of the instrument, the less these error will be.

**Prismatic error** occurs when the faces of the shade glasses and mirrors are not parallel. Error due to lack of parallelism in the shade glasses may be called **shade error**. The navigator can determine shade error in the shade glasses near the index mirror by comparing an angle measured when a shade glass is in the line of sight with the same angle measured when the glass is not in the line of sight. In this manner, determine and record the error for each shade glass. Before using a combination of shade glasses, determine their combined error. If certain observations require additional shading, use the colored telescope eyepiece cover. This does

not introduce an error because direct and reflected rays are traveling together when they reach the cover and are, therefore, affected equally by any lack of parallelism of its two sides.

**Graduation errors** occur in the arc, micrometer drum, and vernier of a sextant which is improperly cut or incorrectly calibrated. Normally, the navigator cannot determine whether the arc of a sextant is improperly cut, but the principle of the vernier makes it possible to determine the existence of graduation errors in the micrometer drum or vernier. This is a useful guide in detecting a poorly made instrument. The first and last markings on any vernier should align perfectly with one less graduation on the adjacent micrometer drum.

**Centering error** results if the index arm does not pivot at the exact center of the arc's curvature. Calculate centering error by measuring known angles after removing all adjustable errors. Use horizontal angles accurately measured with a theodolite as references for this procedure. Several readings by both theodolite and sextant should minimize errors. If a theodolite is not available, use calculated angles between the lines of sight to stars as the reference, comparing these calculated values with the values determined by the sextant. To minimize refraction errors, select stars at about the same altitude and avoid stars near the horizon. The same shade glasses, if any, used for determining index error should be used for measuring centering error.

The manufacturer normally determines the magnitude of all three non-adjustable errors and reports them to the user as **instrument error**. The navigator should apply the correction for this error to each sextant reading.

#### 1412. Adjustable Sextant Error

The navigator should measure and remove the following adjustable sextant errors in the order listed:

1. **Perpendicularity Error:** Adjust first for perpendicularity of the index mirror to the frame of the sextant. To test for perpendicularity, place the index arm at about  $35^\circ$  on the arc and hold the sextant on its side with the index mirror up and toward the eye. Observe the direct and reflected views of the sextant arc, as illustrated in Figure 1412a. If the two views are not joined in a straight line, the index mirror is not perpendicular. If the reflected image is above the direct view, the mirror is inclined forward. If the reflected image is below the direct view, the mirror is inclined backward. Make the adjustment using two screws behind the index mirror.

2. **Side Error:** An error resulting from the horizon glass not being perpendicular to the frame is called **side error**. To test for side error, set the index arm at zero and direct the line of sight at a star. Then rotate the tangent screw back and forth so that the reflected image passes alternately above and below the direct view. If, in changing from one position to the other, the reflected image passes directly over the unreflected image, no side error exists. If it passes to one side, side error exists. Figure 1412b illustrates observations without side error (left) and with side error (right). Whether the sextant reads zero when the true and reflected images are in coincidence is immaterial for this test. An alternative method is to observe a vertical line, such as one edge of the mast of another vessel (or the sextant can be held on its side and the horizon used). If the direct and reflected portions do not form a continuous line, the horizon glass is not perpendicular to the frame of the sextant. A third method in-

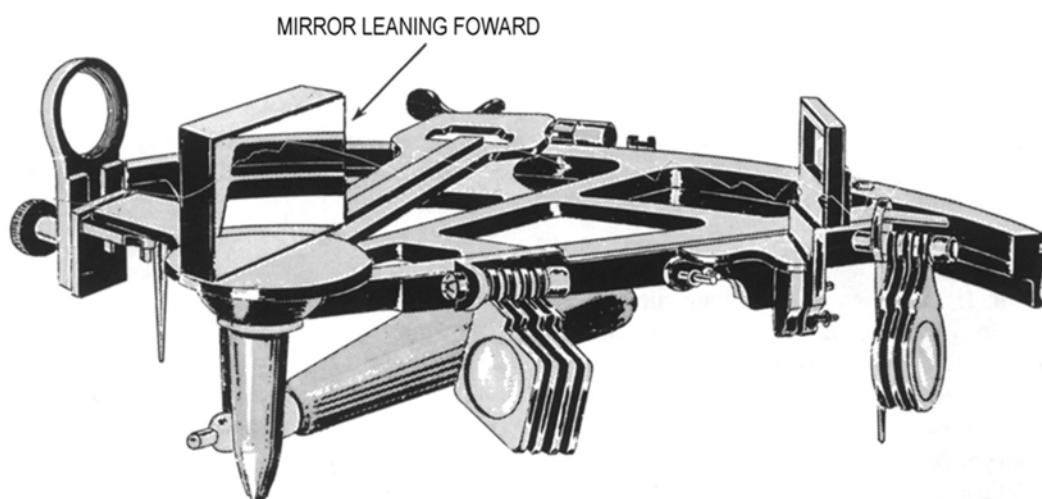


Figure 1412a. Testing the perpendicularity of the index mirror. Here the mirror is not perpendicular.

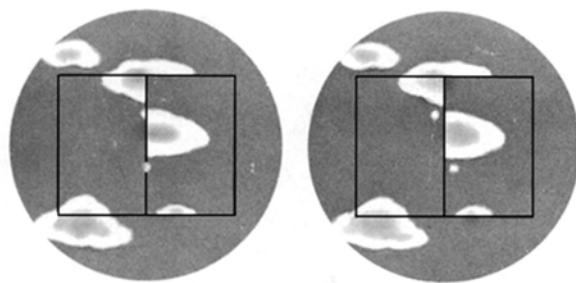


Figure 1412b. Testing the perpendicularity of the horizon glass. On the left, side error does not exist. At the right, side error does exist.

volves holding the sextant vertically, as in observing the altitude of a celestial body. Bring the reflected image of the horizon into coincidence with the direct view until it appears as a continuous line across the horizon glass. Then tilt the sextant right or left. If the horizon still appears continuous, the horizon glass is perpendicular to the frame, but if the reflected portion appears above or below the part seen directly, the glass is not perpendicular. Make the appropriate adjustment using two screws behind the horizon glass.

**3. Collimation Error:** If the line of sight through the telescope is not parallel to the plane of the instrument, a **collimation error** will result. Altitudes measured will be greater than their actual values. To check for parallelism of the telescope, insert it in its collar and observe two stars 90° or more apart. Bring the reflected image of one into coincidence with the direct view of the other near either the right or left edge of the field of view (the upper or lower edge if the sextant is horizontal). Then tilt the sextant so that the stars appear near the opposite edge. If they remain in coincidence, the telescope is parallel to the frame; if they separate, it is not. An alternative method involves placing the telescope in its collar and then laying the sextant on a flat table. Sight along the frame of the sextant and have an assistant place a mark on the opposite bulkhead, in line with the frame. Place another mark above the first, at a distance equal to the distance from the center of the telescope to the frame. This second line should be in the center of the field of view of the telescope if the telescope is parallel to the frame. Adjust the collar to correct for non-parallelism.

**4. Index Error:** Index error is the error remaining after the navigator has removed perpendicularity error, side error, and collimation error. The index mirror and horizon glass not being parallel when the index arm is set exactly at zero is the major cause of index error. To test for parallelism of the mirrors, set the instrument at zero and direct the line of sight at the horizon. Adjust the sextant reading as necessary to cause both images of the horizon to come into line. The sextant's reading when the horizon comes into line is the index error. If the index error is positive, subtract it from each sextant reading. If the index error is negative, add it to each sextant reading.

### 1413. Selecting a Sextant

Carefully match the selected sextant to its required uses. For occasional small craft or student use, a plastic sextant may be adequate. A plastic sextant may also be appropriate for an emergency navigation kit. Accurate offshore navigation requires a quality metal instrument. For ordinary use in measuring altitudes of celestial bodies, an arc of 90° or slightly more is sufficient. If back sights or determining horizontal angles are often required, purchase one with a longer arc. An experienced mariner or nautical instrument technician can provide valuable advice on the purchase of a sextant.

### 1414. The Artificial Horizon

Measurement of altitude requires an exact horizontal reference, normally provided at sea by the visible horizon. If the horizon is not clearly visible, however, a different horizontal reference is required. Such a reference is commonly termed an **artificial horizon**. If it is attached to, or part of, the sextant, altitudes can be measured at sea, on land, or in the air, whenever celestial bodies are available for observations.

An external artificial horizon can be improvised by a carefully leveled mirror or a pan of dark liquid. To use an external artificial horizon, stand or sit so that the celestial body is reflected in the mirror or liquid, and is also visible in direct view. With the sextant, bring the double-reflected image into coincidence with the image appearing in the liquid. For a lower limb observation of the Sun or the Moon, bring the bottom of the double-reflected image into coincidence with the top of the image in the liquid. For an upper-limb observation, bring the opposite sides into coincidence. If one image covers the other, the observation is of the center of the body.

After the observation, apply the index correction and any other instrumental correction. Then take *half* the remaining angle and apply all other corrections except dip (height of eye) correction, since this is not applicable. If the center of the Sun or Moon is observed, omit the correction for semidiameter.

### 1415. Artificial Horizon Sextants

Various types of artificial horizons have been used, including a bubble, gyroscope, and pendulum. Of these, the bubble has been most widely used. This type of instrument is fitted as a backup system to inertial and other positioning systems in a few aircraft, fulfilling the requirement for a self-contained, non-emitting system. On land, a skilled observer using a 2-minute averaging bubble or pendulum sextant can measure altitudes to an accuracy of perhaps 2', (2 miles). This, of course, refers to the accuracy of measurement only, and does not include additional errors such as abnormal refraction, deflection of the vertical, computing and plotting errors, etc. In steady flight through smooth air the error of a 2-minute observation is increased to perhaps 5 to 10 miles.

At sea, with virtually no roll or pitch, results should approach those on land. However, even a gentle roll causes

large errors. Under these conditions observational errors of 10-16 miles are not unreasonable. With a moderate sea, errors of 30 miles or more are common. In a heavy sea, any useful observations are virtually impossible to obtain. Single altitude observations in a moderate sea can be in error by a matter of degrees.

When the horizon is obscured by ice or haze, polar navigators can sometimes obtain better results with an artificial-horizon sextant than with a marine sextant. Some artificial-horizon sextants have provision for making observations with the natural horizon as a reference, but

results are not generally as satisfactory as by marine sextant. Because of their more complicated optical systems, and the need for providing a horizontal reference, artificial-horizon sextants are generally much more costly to manufacture than marine sextants.

Altitudes observed by artificial-horizon sextants are subject to the same errors as those observed by marine sextant, except that the dip (height of eye) correction does not apply. Also, when the center of the Sun or Moon is observed, no correction for semidiameter is required.

## CHRONOMETERS

### 1416. The Marine Chronometer

Historically, the spring-driven **marine chronometer** was a precision timepiece used aboard ship to provide accurate time for celestial observations. A chronometer differs from a spring-driven watch principally in that it contains a variable lever device to maintain even pressure on the mainspring, and a special balance designed to compensate for temperature variations. Today, many seagoing ships no longer have chronometers on board due to highly accurate time signals provided by GPS.

A spring-driven chronometer is set approximately to **Coordinated Universal Time (UTC)**, also referred to as **Greenwich Mean Time (GMT)**, or **Universal Time (UT)**, which is the international time standard used in astronomical and aviation publications, weather products, navigation, and other applications. UTC is expressed in 24-hour (military) time notation, and as with GMT it is based on the local standard time of the  $0^{\circ}$  longitude meridian which runs through Greenwich, England. A spring-driven chronometer, once set, is not reset until the instrument is overhauled and cleaned, usually at three year intervals.

The difference between UTC and chronometer time (C) is carefully determined and applied as a correction to all chronometer readings. This difference, called chronometer error (CE), is **fast (F)** if chronometer time is later than UTC, and **slow (S)** if earlier. The amount by which chronometer error changes in 1 day is called **chronometer rate**. An erratic rate indicates a defective instrument requiring repair.

The principal maintenance requirement is regular winding at about the same time each day. At maximum intervals of about three years, a spring-driven chronometer should be sent to a chronometer repair shop for cleaning and overhaul.

### 1417. Quartz Crystal Marine Chronometers

**Quartz crystal marine chronometers** have replaced spring-driven chronometers aboard many ships because of their greater accuracy. They are maintained on UTC directly from radio time signals. This eliminates chronometer error (CE) and watch error (WE) corrections. Should the second

hand be in error by a readable amount, it can be reset electrically.

The basic element for time generation is a quartz crystal oscillator. The quartz crystal is temperature compensated and is hermetically sealed in an evacuated envelope. A calibrated adjustment capability is provided to adjust for the aging of the crystal.

The chronometer is designed to operate for a minimum of 1 year on a single set of batteries. A good marine chronometer has a built-in push button battery test meter. The meter face is marked to indicate when the battery should be replaced. The chronometer continues to operate and keep the correct time for at least 5 minutes while the batteries are changed. The chronometer is designed to accommodate the gradual voltage drop during the life of the batteries while maintaining accuracy requirements.

### 1418. Watches

A chronometer should not be removed from its case to time sights. Observations may be timed and ship's clocks set with a **comparing watch**, which is set to chronometer time (UTC, GMT, also known as UT) and taken to the bridge wing for recording sight times. In practice, a wrist watch coordinated to the nearest second with the chronometer will be adequate.

A stop watch, either spring wound or digital, may also be used for celestial observations. In this case, the watch is started at a known UTC by chronometer, and the elapsed time of each sight added to this to obtain UT of the sight.

All chronometers and watches should be checked regularly with a radio time signal. Times and frequencies of radio time signals are listed in *NGA Pub. 117, Radio Navigational Aids*.

### 1419. Navigational Calculators

While not considered "instruments" in the strict sense of the word, certainly one of the professional navigator's most useful tools is the navigational calculator or computer program. Calculators eliminate several potential sources of error in celestial navigation, and permit the solution of

many more sights in much less time, making it possible to refine a celestial position much more accurately than is practical using mathematical or tabular methods.

Calculators also save space and weight, a valuable consideration on many craft. One small calculator can replace several heavy and expensive volumes of tables, and is inexpensive enough that there is little reason not to carry a spare for backup use should the primary one fail. The pre-programmed calculators are at least as robust in construction, probably more so, than the sextant itself, and when properly cared for will last a lifetime with no maintenance except, to change batteries from time to time.

If the vessel carries a computer for other ship's chores such as inventory control or personnel administration, there is little reason not to use it for celestial navigation. Free-ware or inexpensive programs are available which take up little hard disk space and allow rapid solution of all types of celestial navigation problems. Typically they will also take care of route planning, sailings, tides, weather routing, electronic charts, and numerous other tasks.

U.S. Navy and Coast Guard navigators have access to a program called **STELLA (System To Estimate Latitude and Longitude Astronomically)**; do not confuse with a sim-

ilarly named commercial astronomy program). The Astronomical Applications Department of the U.S. Naval Observatory developed STELLA in response to a Navy requirement. STELLA can perform almanac functions, position updating/DR estimations, celestial body rise/set/transit calculations, compass error calculations, sight planning, and sight reduction; on-line help and a user's guide are included. STELLA is now automatically distributed to each Naval ship; other Navy users may obtain a copy by contacting:

Superintendent  
U.S. Naval Observatory  
Code: AA/STELLA  
3450 Massachusetts Ave. NW  
Washington, DC, 20392-5420

By using a calculator or sight reduction program, it is possible to take and solve half a dozen or more sights in a fraction of the time it would normally take to shoot two or three and solve them by hand. This will increase the accuracy of the fix by averaging out errors in taking the sights. The computerized solution is always more accurate than tabular methods because it is free of rounding and arithmetic errors.

# CHAPTER 15

## AZIMUTHS AND AMPLITUDES

### INTRODUCTION

#### 1500. Checking Compass Error

The navigator must constantly be concerned about the accuracy of the ship's primary and backup compasses, and should check them regularly. A regularly annotated compass log book will allow the navigator to notice a developing error before it becomes a serious problem.

As long as at least two different types of compass (e.g. mechanical gyro and flux gate, or magnetic and ring laser gyro) are consistent with each other, one can be reasonably sure that there is no appreciable error in either system. Since different types of compasses depend on different scientific principles and are not subject to the same error sources, their agreement indicates almost certainly that no error is present.

A navigational compass can be checked against the heading reference of an inertial navigation system if one is installed. One can also refer to the ship's indicated GPS track as long as current and leeway are not factors, so that the ship's COG and heading are in close agreement.

The navigator's only completely independent directional reference (because it is extra-terrestrial and not man-made) is the sky. The primary compass should be

checked occasionally by comparing the observed and calculated azimuths and amplitudes of a celestial body. The difference between the observed and calculated values is the compass error. This chapter discusses these procedures.

Theoretically, these procedures work with any celestial body. However, the Sun and Polaris are used most often when measuring azimuths, and the rising or setting Sun when measuring amplitudes.

While errors can be computed to the nearest tenth of a degree or so, it is seldom possible to steer a ship that accurately, especially when a sea is running, and it is reasonable to round calculations to the nearest half or perhaps whole degree for most purposes.

Various hand-held calculators and computer programs are available to relieve the tedium and errors of tabular and mathematical methods of calculating azimuths and amplitudes. Naval and Coast Guard navigators will find the STELLA program useful in this regard. This program is discussed in further in Chapter 19 - Sight Reductions. STELLA stands for the System to Establish Latitude and Longitude Astronomically.

### AZIMUTHS

#### 1501. Gyro Error by Azimuth of the Sun

Mariners not having STELLA may use *Pub 229, Sight Reduction Tables for Marine Navigation* to compute the Sun's azimuth. They compare the computed azimuth to the azimuth measured with the compass to determine compass error. In computing an azimuth, interpolate the tabular azimuth angle for the difference between the table arguments and the actual values of declination, latitude, and local hour angle. Do this triple interpolation of the azimuth angle as follows:

1. Enter the *Sight Reduction Tables* with the integer values of declination, latitude, and local hour angle. For each of these arguments, extract a base azimuth angle.
2. Reenter the tables with the same latitude and LHA arguments but with the **declination** argument  $1^\circ$  greater than the base declination argument. Record the difference between the respondent azimuth

angle and the base azimuth angle and label it as the azimuth angle difference (Z Diff.).

3. Reenter the tables with the base declination and LHA arguments, but with the **latitude** argument  $1^\circ$  greater than the base latitude argument. Record the Z Diff. for the increment of latitude.
4. Reenter the tables with the base declination and latitude arguments, but with the LHA argument  $1^\circ$  greater than the base LHA argument. Record the Z Diff. for the increment of LHA.
5. Determine the increment of each of the three arguments, which is simply the actual minus the base argument. Use this to compute a correction for each of the three arguments, using the formula  $\text{Correction} = \text{Z Diff.} \times (\text{increment} \div 60)$ .
6. Sum the three corrections to obtain a total correction. Apply this to the base azimuth angle to obtain the true azimuth angle.

The gyrocompass error is the difference between the true azimuth angle ( $Z_n$ ) and the measured azimuth angle per gyrocompass ( $Z_n$  pgc). The gyro error is West if the gyro reads a larger value than true; it is East if the gyro reading is the smaller value. One way to remember this is “gyro

best, error West; gyro least, error East.” (Be careful of this mnemonic near the  $360^\circ/0^\circ$  boundary, though, where a value slightly larger than  $0^\circ$  is actually treated as slightly larger than  $360^\circ$ .)

Actual		Base Arguments	Base Z	Tab* Z	Z Diff.	Increments	Correction Z Diff x (Inc. 60)
Dec.	8°47.4' S	8°	121.3°	122.3°	+1.0°	47.4'	+0.8°
DR Lat.	23°55.0' N	23°(Contrary)	1213°	122.0°	+0.7°	55.0'	+0.6°
LHA	317°37.4'	317°	1213°	122.1°	+ 0.8°	37.4'	+0.5°
Base Z	121.3°						Total Corr. +1.9°
Corr.	(+) 1.9°						
Z	N 121.3°E						
Zn	123.2°						
Zn pgc	124.0°						
Gyro Error	0.8° W						

Figure 1501. Azimuth by Pub. No. 229.

*Example:*

*In DR latitude  $23^{\circ} 55.0'N$ , the azimuth of the Sun is  $124.0^{\circ}$  per gyrocompass (pgc). At the time of the observation, the declination of the Sun is  $8^{\circ} 47.4'S$ ; the local hour angle of the Sun is  $317^{\circ} 37.4$ . Determine gyro error.*

*Solution:*

(See Figure 1501) Enter the actual value of declination, DR latitude, and LHA. Round each argument down to the nearest whole degree. Enter the Sight Reduction Tables with these whole degree arguments and extract the base azimuth value for these rounded off arguments. Record the base azimuth value in the table.

As the first step in the triple interpolation process, increase the value of the declination by  $1^\circ$  to  $9^\circ$  because the actual declination was greater than the base declination. Enter the Sight Reduction Tables with the following arguments: (1) Declination =  $9^\circ$ ; (2) DR Latitude =  $23^\circ$ ; (3) LHA =  $317^\circ$ . Record the tabulated azimuth for these arguments.

As the second step in the triple interpolation process, increase the value of latitude by  $1^\circ$  to  $24^\circ$  because the actual DR latitude was greater than the base latitude. Enter the Sight Reduction Tables with the following arguments: (1) Declination =  $8^\circ$ ; (2) DR Latitude =  $24^\circ$ ; (3) LHA =

317°. Record the tabulated azimuth for these arguments.

As the third and final step in the triple interpolation process, increase the value of LHA to  $318^\circ$  because the actual LHA value was greater than the base LHA. Enter the Sight Reduction Tables with the following arguments: (1) Declination =  $8^\circ$ ; (2) DR Latitude =  $23^\circ$ ; (3) LHA =  $318^\circ$ . Record the tabulated azimuth for these arguments.

*Calculate the Z Difference by subtracting the base azimuth from the tabulated azimuth. Be careful to carry the correct sign.*

*Z Difference = Tab Z - Base Z*

Next, determine the increment for each argument by taking the difference between the actual values of each argument and the base argument. Calculate the correction for each of the three argument interpolations by dividing the increment by 60, and multiplying the result by the Z difference.

The sign of each correction is the same as the sign of the corresponding Z difference used to calculate it. In the above example, the total correction sums to +1.9. Apply this value to the base azimuth of  $121.3^\circ$  to obtain the true azimuth  $123.2^\circ$ . Compare this to the gyrocompass reading of  $124.0^\circ$  pgc. The compass error is  $0.8^\circ$ W.

## AZIMUTH OF POLARIS

## 1502. Gyro Error by Azimuth of Polaris

The Polaris tables in the *Nautical Almanac* list the azimuth of Polaris for latitudes between the equator and  $65^{\circ}$  N. Figure 1912b in Chapter 19 shows this table. Compare a compass bearing of Polaris to the tabular value of Polaris to determine compass error. The entering arguments for the table are LHA of Aries and observer latitude.

*Example:*

On February 23, 2016, at LAT  $29^{\circ}31.0' N$  and LON  $074^{\circ}30.0' W$ , at 04-21-15 GMT, Polaris bears  $359.9^{\circ}$  pgc. Calculate the gyro error.

*Date* 23 February 2016  
*Time (GMT)* 04-21-15

<i>GHA Aries</i>	$217^\circ 49.3'$ (from almanac daily pages & interpolation)
<i>Longitude</i>	$074^\circ 30.0'W$
<i>LHA Aries</i>	$143^\circ 19.3'$

**Solution:**

Enter the azimuth section of the Polaris table with the calculated LHA of Aries. In this case, go to the column for LHA Aries between  $140^\circ$  and  $149^\circ$ . Follow that column

down and extract the value for the given latitude. Since the increment between tabulated values is so small, visual interpolation is sufficient. In this case, the azimuth for Polaris for the given LHA of Aries and the given latitude is  $359.2^\circ$ .

<i>Tabulated Azimuth</i>	$359.2^\circ T$
<i>Gyrocompass Bearing</i>	$359.9^\circ T$
<i>Error</i>	$0.7^\circ W$

## AMPLITUDES

### 1503. Amplitudes

A celestial body's amplitude angle is the complement of its azimuth angle. At the moment that a body rises or sets, the amplitude angle is the arc of the horizon between the

body and the East/West point of the horizon where the observer's prime vertical intersects the horizon (at  $90^\circ$ ), which is also the point where the plane of the equator intersects the horizon (at an angle numerically equal to the observer's co-latitude). See Figure 1503.

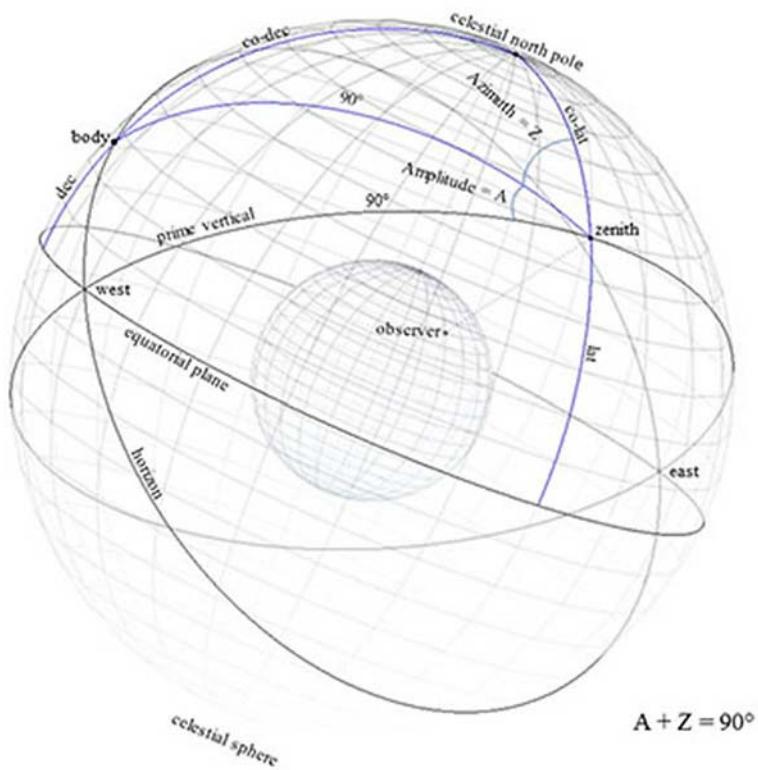


Figure 1503. The amplitude angle (A) subtends the arc of the horizon between the body and the point where the prime vertical and the equator intersect the horizon. Note that it is the compliment of the azimuth angle (Z).

In practical navigation, a bearing (psc or pgc) of a body can be observed when it is on either the celestial or the visible horizon. To determine compass error, simply convert the computed amplitude angle to true degrees and compare it with the observed compass bearing.

The angle is computed by the formula:

$$\sin A = \sin Dec / \cos Lat.$$

This formula gives the angle at the instant the body is on the celestial horizon. It does not contain an altitude term because the body's computed altitude is zero at this instant.

The angle is prefixed E if the body is rising and W if it is setting. This is the only angle in celestial navigation referenced FROM East or West, i.e. from the prime vertical. A body with northerly declination will rise and set

North of the prime vertical. Likewise, a body with southerly declination will rise and set South of the prime vertical. Therefore, the angle is suffixed N or S to agree with the name of the body's declination. A body whose declination is zero rises and sets exactly on the prime vertical.

Due largely to refraction, dip, and its disk size, the Sun is on the celestial horizon when its lower limb is approximately two thirds of a diameter above the visible horizon. The Moon is on the celestial horizon when its upper limb is on the visible horizon. Stars and planets are on the celestial horizon when they are approximately one Sun diameter above the visible horizon.

When observing a body on the visible horizon, a correction from Table 23 - *Correction of Amplitude as Observed on the Visible Horizon* must be applied. This correction accounts for the slight change in bearing as the body moves between the visible and celestial horizons. It reduces the bearing on the visible horizon to the celestial horizon, from which the table is computed.

For the Sun, stars, and planets, apply this correction to the observed bearing in the direction away from the elevated pole. For the moon, apply one half of the correction toward the elevated pole. Note that the algebraic sign of the correction does not depend upon the body's declination, but only on the observer's latitude. Assuming the body is the Sun the rule for applying the correction can be outlined as follows:

Observer's Lat.	Rising/Setting	How to Apply Correction
North	Rising	Add correction to observed bearing
North	Setting	Subtract correction from observed bearing
South	Rising	Subtract correction from observed bearing
South	Setting	Add correction to observed bearing

Table 1503. Amplitude correction for the Sun.

The following two articles demonstrate the procedure for obtaining the amplitude of the Sun on both the celestial and visible horizons.

#### 1504. Amplitude of the Sun on the Celestial Horizon

Mariners may use Bowditch Table 22 (Amplitudes) to determine the Sun's computed amplitude. The procedure is similar to that done in Section 1501. Comparing the computed amplitude to the amplitude measured with the gyrocompass determines the gyro error. In computing an amplitude, interpolate the tabular amplitude angle for the difference between the table arguments and the actual values of declination and latitude.

Do this double interpolation of the amplitude angle as follows:

- Enter Bowditch Table 22 (Amplitudes) with the nearest integral values of declination and latitude. Extract a base amplitude angle.
- Reenter the table with the same declination argument but with the latitude to the next tabulated value (greater or less than the base latitude argument, depending upon whether the actual latitude is greater or less than the base argument). Record the amplitude and the difference between it and the base amplitude angle and label it Diff.
- Reenter the table with the base latitude argument but with the declination to the next tabulated value (greater or less than the base declination argument, depending upon whether the actual declination is greater or less than the base argument). Record the amplitude and the difference between it and the base amplitude angle and label it Diff.
- Compute the corrections due to latitude and declination not being exactly at a tabular value. Apply these corrections to obtain a final amplitude. The final amplitude is then converted to a true bearing. The difference between the true bearing and the gyro bearing gives the gyro error.

#### Example:

The DR latitude of a ship is  $51^{\circ} 24.6' N$ . The navigator observes the setting Sun on the celestial horizon. Its declination is  $N 19^{\circ} 40.4'$ . Its observed bearing is  $303^{\circ} pyc$ .

#### Required:

Gyro error.

#### Solution:

Interpolate in Table 22 for the Sun's calculated amplitude as follows. See Figure 1504. The actual values for latitude and declination are  $L = 51.4^{\circ} N$  and  $dec. = N 19.67^{\circ}$ . Find the tabulated values of latitude and declination closest to these actual values. In this case, these tabulated values are  $L = 51^{\circ}$  and  $dec. = 19.5^{\circ}$ . Record the amplitude corresponding to these base values,  $32.0^{\circ}$ , as the base amplitude.

Next, holding the base declination value constant at  $19.5^{\circ}$ , increase the value of latitude to the next tabulated value:  $N 52^{\circ}$ . Note that this value of latitude was increased because the actual latitude value was greater than the base value of latitude. Record the tabulated amplitude for  $L = 52^{\circ}$  and  $dec. = 19.5^{\circ}$ :  $32.8^{\circ}$ . Then, holding the base latitude value constant at  $51^{\circ}$ , increase the declination value to the next tabulated value:  $20^{\circ}$ . Record the tabulated amplitude for  $L = 51^{\circ}$  and  $dec. = 20^{\circ}$ :  $32.9^{\circ}$ .

The latitude's actual value ( $51.4^{\circ}$ ) is 0.4 of the way between the base value ( $51^{\circ}$ ) and the value used to determine the tabulated amplitude ( $52^{\circ}$ ). The declination's actual value ( $19.67^{\circ}$ ) is 0.3 of the way between the base value ( $19.5^{\circ}$ ) and the value used to determine the tabulated amplitude ( $20.0^{\circ}$ ). To determine the total correction to base amplitude, multiply these increments (0.4 and 0.3) by the

respective difference between the base and tabulated values (+0.8 and +0.9, respectively) and sum the products. The total correction is +0.6°. Add the total correction (+0.6°) to the base amplitude (32.0°) to determine the final amplitude (32.6°) which will be converted to a true bearing.

Because of its northerly declination (in this case), the

Sun was 32.6° north of west when it was on the celestial horizon. Therefore its true bearing was 302.6° (270° + 32.6°) at this moment. Comparing this with the gyro bearing of 303° gives an error of 0.4°W, which can be rounded to 1/2°W.

Actual	Base	Base Amp.	Tab. Amp.	Diff.	Inc.	Correction
L=51.4°N dec=19.67°N	51° 19.5°	32.0° 32.0°	32.8° 32.9°	+0.8° +0.9°	0.4 0.3	+0.3° <u>+0.3°</u>
Total						+0.6°

Figure 1504. Interpolation in Table 22 for Amplitude.

## 1505. Amplitude of the Sun on the Visible Horizon

In higher latitudes, amplitude observations should be made when the body is on the visible horizon because the value of the correction is large enough to cause significant error if the observer misjudges the exact position of the celestial horizon. The observation will yield precise results whenever the visible horizon is clearly defined.

### Example:

Observer's DR latitude is 59°47'N, Sun's declination is 5°11.3'S. At sunrise the Sun is observed on the visible horizon bearing 098.5° pgc.

### Required:

Gyrocompass error.

### Solution:

Given this particular latitude and declination, the amplitude angle is 10.3°S, so that the Sun's true bearing is 100.3° at the moment it is on the celestial horizon, that is, when its Hc is precisely 0°. Applying the Table 23 correction to the observed bearing of 098.5° pgc using the rules given in Section 1503, the Sun would have been bearing 099.7° pgc had the observation been made when the Sun was on the celestial horizon. Therefore, the gyro error is 0.6°E.

## 1506. Amplitude by Calculation

As an alternative to using the amplitude tables, if a calculator is available then the amplitudes can be computed using a slightly modified version of the altitude-azimuth formula. The modification is needed because azimuth (Z) and amplitude (A) angles are complimentary, and the co-functions of complimentary angles are equal; i.e.,  $\cos Z = \sin A$ . In the following formulas, northerly latitudes and declinations are given positive values, and southerly latitudes and declinations are considered negative. If the

resulting amplitude is positive, it is north of the prime vertical; conversely, a negative amplitude is south of the prime vertical.

- a) The general case, when a body is not on the celestial horizon, the formula is:

$$\text{Amplitude} = \sin^{-1}[(\sin \text{DEC} - \sin \text{LAT} \sin \text{Hc}) / (\cos \text{LAT} \cos \text{Hc})]$$

where DEC is the celestial body's declination, LAT is the observer's latitude, and Hc is the object's computed altitude. For the Sun on the visible horizon, Hc = -0.7°.

- b) When a body is on the celestial horizon (that is, its altitude, Hc = 0), the formula becomes:

$$\text{Amplitude} = \sin^{-1} [\sin \text{DEC} / \cos \text{LAT}]$$

### Example:

Determine the gyrocompass error using the formulation instead of the tables, for the example in Section 1505.

### Required:

Gyrocompass error.

### Solution:

The observed bearing of the Sun on the visible horizon is 098.5° pgc. The computed amplitude of the Sun when it is on the visible horizon (that is, Hc = -0.7°) is found by:

$$\text{Amplitude} = \sin^{-1} [(\sin -5.19^\circ - \sin 59.78^\circ \sin -0.7^\circ) / (\cos 59.78^\circ \cos -0.7^\circ)].$$

Evaluating, we find the amplitude is 9.1°. This is 9.1° degrees away from E, in the "negative" (or southerly) direction, so the calculated azimuth is 90° + 9.1° = 99.1°. The gyrocompass error is 99.1° - 98.5° = 0.6° E. This value matches the answer obtained in Section 1505 using the tables.

# CHAPTER 16

## TIME

### TIME IN NAVIGATION

#### 1600. Apparent and Solar Time

The Earth's rotation on its axis presents the Sun and other celestial bodies to appear to proceed across the sky from east to west each day. If a navigator measures the time interval between two successive transits across the local meridian of a very distant star by the passage of time against another physical time reference such as a chronometer, he or she would be measuring the period of the Earth's rotation.

In the most practical sense, the Earth's rotation is the navigator's standard of time. When the navigator then

makes a similar measurement of the transit of the Sun, the resulting time interval would be about four minutes longer than the period determined by the Earth's rotation. This is due to the Earth's yearly orbital motion around the Sun, which continuously changes the apparent position of the Sun against the background of stars, traditionally observed as the cyclical procession of the zodiac. Thus, during the course of a day, the Sun appears to lag a little to the east with respect to the stars, and the Earth's rotation must exceed a complete rotation ( $360^\circ$ ) in order to have the Sun appear overhead at the local meridian.

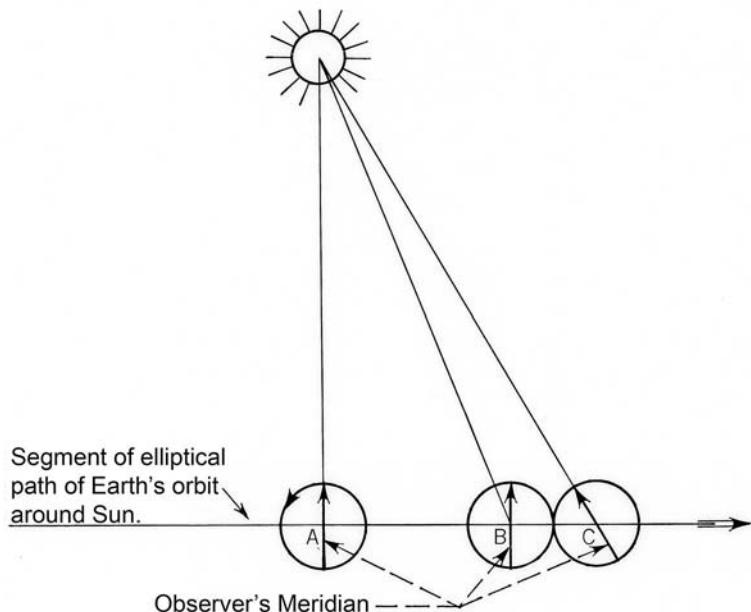


Figure 1600a. Apparent eastward movement of the Sun with respect to the stars.

**Apparent eastward lag of the Sun with diurnal observations** - when the Sun is on the observer's meridian at point A in the Earth's orbit around the Sun (see Figure 1600a), it will not be on the observer's meridian after the Earth has rotated once ( $360^\circ$ ) because the Earth will have moved along its orbit to point B. Before the Sun can again be observed on the observer's meridian, the Earth must turn a little more on its axis as shown in C. Thus, during the course of a day (as determined by the Earth's rotation period) the Sun appears to move eastward with respect to the celestial background of stars.

The apparent positions of individual stars against the celestial background are commonly determined with reference to an imaginary point called the **vernal equinox**. The vernal equinox is the intersection of the celestial equator and the ecliptic (see Figure 1600b). The full rotation of the Earth measured with respect to the vernal equinox is called a **sidereal day**, and corresponds to the Earth's rotational period. The period with respect to the Sun is called an **apparent solar day**, and includes the additional time to compensate for the Earth's orbital motion.

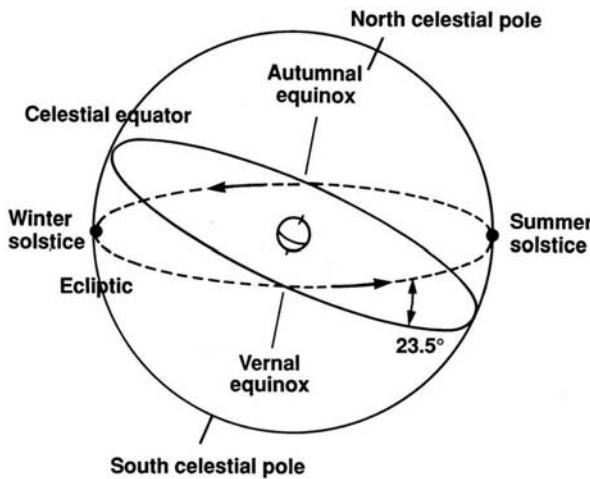


Figure 1600b. Solstices and equinoxes on the ecliptic.

A navigator using the observed position of the Sun, or the apparent Sun, to measure the passage of time from Earth's rotation results in **apparent solar time**. Apparent solar time is what a perfectly constructed and calibrated sundial would read at a given location, based on the apparent position of the Sun in the sky. In astronomical terms, apparent solar time is determined by the **local hour angle**, which is a measure of the Sun's projected angular distance east or west of the local **meridian**. Since each meridian is a line of constant longitude, at any instant of the Earth's rotation, the apparent solar time will differ for every longitude. We define apparent solar time at a specific location as  $12^h$  + the local hour angle (expressed in hours) of the apparent position of the Sun in the sky. The local hour angle is negative when presenting east of the meridian.

Apparent solar time is not a uniform time scale; the apparent Sun crosses the sky at slightly different rates at different times of the year. This means the apparent solar time runs "fast" with respect to a constant timescale, such as a chronometer, part of the year and "slow" during other parts of the year. Although the daily fractional change in the rate of the Sun's apparent motion is small, the accumulated time difference can reach as much as sixteen minutes. This effect is a function of the Earth's orbit around the Sun. It is the result of two superimposed cycles; the Earth's eccentricity (no-circular orbit) and the tilt of Earth's axis with respect to the plane of its orbit (the ecliptic).

In order to create a uniform time scale for practical use, we imagine a point in the sky called the **fictitious mean sun**, which moves at a constant rate across the sky (at the celestial equator), regardless of the time of year. That is, the fictitious mean sun averages out the variations in the position and rate of motion of the true Sun over the course of an entire year. The fictitious mean sun is never more than about 4 degrees east or west of the actual Sun, although it is only an imaginary point. We can define **mean solar time** in the same way as apparent solar time: mean solar time at a

specific location is  $12^h$  + the local hour angle (expressed in hours) of the fictitious mean sun. Of course, the fictitious mean sun is not an observable point, so we need a mathematical expression to tell us where it is with respect to the true Sun; this is the equation of time.

### 1601. Equation of Time

**Mean solar time** is sometimes ahead (fast) and sometimes behind (slow) of the apparent solar time. This difference is called the **equation of time**. The equation of time's minimum value is near  $-14^m\ 13\ s$  in mid-February, and its maximum value is near  $16^m\ 26\ s$  in early November.

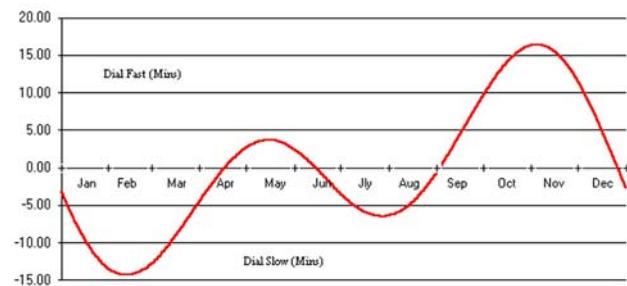


Figure 1601a. Equation of time.

The **equation of time** gives the offset in minutes applied to mean solar time, as may be determined by a chronometer, to calculate the apparent solar time; specifically at the Sun's apparent passage at the local meridian.

The navigator most often deals with the equation of time when determining the time of **upper meridian passage** of the Sun, called **Local Apparent Noon (LAN)**. Were it not for the difference in rate between the fictitious mean and apparent Sun, the Sun would always appear on the observer's meridian at  $12^h$  (noon) local time. Except for four unique times of the year related to the interaction of the Earth's eccentric orbit and inclination to the ecliptic, the LAN will always be offset from exactly noon mean solar time. This time difference, which is applied as the equation of time at meridian transit, is listed on the right hand daily pages of the *Nautical Almanac*.

The sign of the equation of time is negative if apparent time is behind mean time; it is positive if apparent time is ahead of mean time. In either case, the equation is: Apparent Time = Mean Time + (equation of time). A negative equation of time is indicated by shading in the *Nautical Almanac*.

**Example 1:** Determine the local mean time of the Sun's meridian passage (Local Apparent Noon) on June 16, 2016.

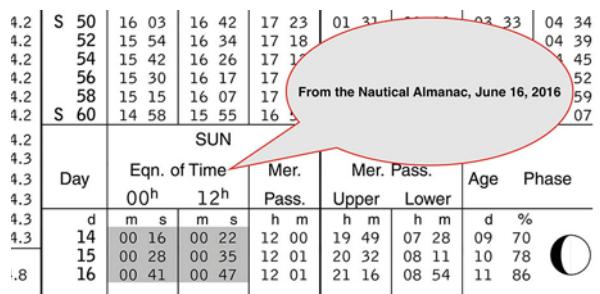
**Solution:** See the *Nautical Almanac*'s right hand daily page for June 16, 2016 (Figure 1601b). The equation of time is listed in the bottom right hand corner of the page. There are two ways to solve the problem, depending on the

accuracy required for the value of meridian passage. First, for minute accuracy, the time of the Sun at meridian passage is given to the nearest minute in the "Mer. Pass." column. For June 16, 2016, this time is 1201.

Second, to determine the nearest second of time of meridian passage, use the value given for the equation of time listed immediately to the left of the "Mer. Pass." column on the daily pages. For June 16, 2016, the value is given as negative 00 m 47 s. (Use the "12 h" column because the problem asked for meridian passage at LAN.)

Using the equation Apparent Time = Mean Time + (equation of time), we have  $1200 = \text{Mean Time} + (-0047)$ . Rearranging, we get Mean Time =  $1200 + 0047$ . The exact mean time of meridian passage for June 16, 2016, is 12 h 00 m 47 s.

To calculate latitude and longitude at LAN, the navigator seldom requires the time of meridian passage to accuracies greater than one minute (0.25 degrees of longitude). Therefore, use the time listed under the "Mer. Pass." column to estimate LAN unless extraordinary accuracy is required.



The table shows the Sun's meridian passage data for June 16, 2016. The top section is a grid of numbers, with a circled 'From the Nautical Almanac, June 16, 2016' label. The bottom section is a table with columns: Day, Eqn. of Time (00h, 12h), Mer. Pass. (Upper, Lower), Age, and Phase. The data for June 16 is: d 14, m 00, s 16, Eqn. of Time 00h, Mer. Pass. Upper 12 00, Lower 21 16, Age 09 70, Phase 11 86.

Figure 1601b. Extract from the Nautical Almanac daily pages for June 16, 2016.

## 1602. Fundamental Systems of Time

**Atomic based timekeeping** is determined by the definition of the Système International (SI) second, with duration of 9,192,631,770 cycles of electromagnetic radiation corresponding to the transition between two hyperfine levels of the ground state of cesium 133. **International Atomic Time (TAI)** is an international time scale based on the non-stationary ensemble of atomic clock observations contributed by worldwide timekeeping laboratories, qualified by the Bureau International des Poids et Mesures (BIPM).

**Universal time (UT)** is a generic reference to one of several timescales that approximate the mean **diurnal** motion of the fictitious mean sun. Loosely, UT is **mean solar time at zero longitude**, or the **Greenwich meridian** (previously known as **Greenwich Mean Time (GMT)**). The term GMT has been dropped from scientific usage. In current usage, UT either refers to **UT1** or **Coordinated**

**Universal Time (UTC)**. In the navigational publications, UT always means UT1.

UT1 is a continuous timescale precisely defined by a mathematical expression that relates it to sidereal time, or the angle and rate of Earth's rotation to fixed points (usually very distant objects) of reference on the celestial background. Thus, UT1 is observationally determined by the apparent diurnal motions of celestial bodies and is affected by irregularities and the slowing of Earth's rate of rotation.

**Coordinated Universal Time (UTC)** is a discontinuous timescale determined by TAI and maintained by the BIPM. UTC is recognized by nearly all worldwide timing centers as the standard reference clock for purposes ranging from navigation to precise time stamping of financial transactions. UTC is accurately distributed (usually better than  $\pm 1$  ms) by radiometric and optical fiber-based transmission. UTC defines the 24 hour cycle or clock as 86,400 SI seconds, not related to the rotation rate of the Earth. In this way, UTC appears to run faster than UT1, although it is UT1 that is varying because of the slowing of the Earth. To maintain the long term coordination of UTC with UT1 to within  $\pm 0.9$  seconds, a one second interval is typically added as necessary to UTC. This added interval is known as a leap second. Since the explicit synchronization of UTC and UT1 in 1958 through 2016, there have been 36 leap seconds inserted into UTC. Although the expectation of the leap second insertion should be regular, it is not, and it is this irregularity that makes the implementation of the leap second undesirable to the highly synchronized worldwide systems based on UTC. The leap second insertion is what characterizes UTC as a discontinuous time scale. The formal insertion of leap second is to expand the minute modulo by one (count the minute with a leap second as 58,59,60,00). Because the difference between UT1 and UTC are always less than 0.9 sec, navigators often do not need to account for the difference except when the highest precisions are required.

**GPS Time** is the time disseminated by the Navstar satellites of GPS, and is not UTC(USNO), meaning UTC as maintained by the United States Naval Observatory (USNO). Rather GPS Time is a continuous timescale monitored against the USNO master clock and maintained with a fixed offset of 19 seconds added to TAI. To formulate UTC, a leap second field is given within the navigation message of the GPS signal, which the receiver then uses to accordingly increment GPS Time. The need for a continuous timescale for Global Navigation Satellite Systems (GNSS), such as GPS Time, is necessary to allow for the determination of velocity and interaction with inertial navigation systems. In this way, real time system dynamics may be separated from the discrete time of day feature of GPS. See Section 1613 on dissemination systems for further details.

**Terrestrial time (TT)**, formerly known as Terrestrial Dynamical Time (TDT), is rarely used by a navigator. In practice  $TT = TAI + 32.184$  sec.

**Sidereal time** is the hour angle of the vernal equinox. If the

mean equinox is used (that is, neglecting nutation), the result is mean sidereal time; if the true equinox is used, the result is apparent sidereal time. The hour angle can be measured with respect to the local meridian or the Greenwich meridian, yielding, respectively, local or Greenwich (mean or apparent) sidereal times.

**Delta T** is the difference between Terrestrial Time and Universal Time:  $\Delta T = TT - UT1$ .

### 1603. Time and Longitude Arc

A navigator may be required to convert the measure of longitude arc to time or vice versa. The concept and math is not difficult, and calculators or tables (such as the one provided on page i in the back of the *Nautical Almanac*) can help. To illustrate, note that in this section, one day represents one complete rotation of the Earth as determined by a mean solar day. That is, one 24-hour period of 86,400 seconds is the same as the Earth rotating  $360^\circ$ . Therefore, the time of day is an indication of the phase (amount of rotation) within the Earth's orbital period, calculating how much of a mean solar day has elapsed, or what part of a rotation has been completed. For example, initialing the day at zero hours, at one hour later, the Earth has turned through  $1/24$  of its rotation, or  $1/24$  of  $360^\circ$ , or  $360^\circ \div 24 = 15^\circ$ .

Smaller intervals can also be stated in angular units; since 1 hour or 60 minutes is equivalent to  $15^\circ$  of arc, 1 minute of time is equivalent to  $15^\circ \div 60 = 0.25^\circ = 15'$  of arc, and 1 second of time is equivalent to  $15' \div 60 = 0.25' = 15''$  of arc. Therefore any time interval can be expressed as an equivalent amount of rotation, and vice versa. Conversion among these units can be aided by the relationships indicated below, summarizing in table form:

$1^\text{d}$	$= 24^\text{h}$	$= 360^\circ$
$60^\text{m}$	$= 1^\text{h}$	$= 15^\circ$
$4^\text{m}$	$= 1^\circ$	$= 60'$
$60^\text{s}$	$= 1^\text{m}$	$= 15'$
$4^\text{s}$	$= 1'$	$= 60''$
$1^\text{s}$	$= 15''$	$= 0.25'$

To convert time to arc:

If time is in hh:mm:ss format:

1. Convert to decimal hours. Take mm and divide by 60 (60 is the number of minutes per hour). Take ss and divide by 3600 (3600 is the number of seconds per hour). Add both to hh. Mathematically, decimal hours = hh + mm  $\div$  60 + ss  $\div$  3600.
2. Multiply decimal hours by 15 to obtain decimal degrees of arc.
3. If needed, convert decimal degrees of arc to deg° amin' asec" format, where deg is degree, amin is minutes of arc, and asec is seconds of arc. To do this, deg is simply the integer portion of the decimal degrees. That is, the numbers before the

decimal point. Take the remaining portion (that is, the decimal part) and multiply by 60. The minutes of arc, amin, is the integer portion of this. Take the remaining portion of this new value and again multiply it by 60. That is the seconds of arc, asec.

**Example 1:** Convert  $14^\text{h}21^\text{m}39^\text{s}$  to arc.

**Solution:**

Step 1: Convert to decimal hours.  $14 + 21 \div 60 + 39 \div 3600 = 14 + 0.35 + 0.01083 = 14.360833$  hours

Step 2: Multiply by 15.  $14.360833 \times 15 = 215.4125^\circ$

Step 3: Convert to deg° amin' asec" format. The deg equal the integer portion of 215.4125, so deg = 215. The amin is found by taking the remainder, .4125, and multiplying it by 60;  $.4125 \times 60 = 24.75$ . The amin equals the integer part, so amin = 24. The asec is found by taking the remainder of that, .75, and multiplying it by 60, which equals 45, so asec = 45. The final answer is

$$14^\text{h}21^\text{m}39^\text{s} \text{ of time} = 215^\circ 24' 45''$$

To convert arc to time, the steps are similar.

If arc is in the deg° amin' asec" format:

Step 1: Convert to decimal degrees. To do this, take amin and divide by 60 (60 is the number of minutes of arc per degree). Take asec and divide by 3600 (3600 is the number of seconds of arc per degree). Add both to deg. Mathematically, decimal degrees = deg + amin  $\div$  60 + asec  $\div$  3600.

Step 2: Divide decimal degrees of arc by 15 to obtain decimal hours of time.

Step 3: If needed, convert decimal hours to hh:mm:ss format, where hh is hour, mm is minutes of time, and ss is seconds of time. To do this, hh is simply the integer portion of the decimal hours. That is, the numbers before the decimal point. Take the remaining portion (that is, the decimal part) and multiply by 60. The minutes of time, mm, is integer portion of this. Take the remaining portion of this new value and again multiply it by 60. That is the seconds of time, ss.

Convert  $215^\circ 24' 45''$  to time units.

Step 1: Convert to decimal degrees. Decimal degrees = deg + amin  $\div$  60 + asec  $\div$  3600. In this example,  $215 + (24 \div 60) + (45 \div 3600)$ , which equals  $215.4125$  degrees.

Step 2: Divide decimal degrees of arc by 15 to obtain decimal hours of time.  $215.4125 \div 15 = 14.360833$  hours.

Step 3: Convert decimal hours to hh:mm:ss format. The hh equal the integer portion of 14.360833, so hh = 14. The mm is found by taking the remainder, .360833, and multiplying it by 60;  $.360833 \times 60 = 21.65$ . The mm equals the integer part, so mm = 21. The ss is found by taking the

remainder of that, .65, and multiplying it by 60, which equals 39, so  $ss = 39$ . The final answer is

$$215^\circ 24' 45'' = 14 \text{ h } 21 \text{ m } 39 \text{ s.}$$

Solutions can also be made using arc to time conversion tables in the almanacs. In the Nautical Almanac, the table given near the back of the volume is in two parts, permitting separate entries with degrees, minutes, and quarter minutes of arc. This table is arranged in this manner because the navigator converts arc to time more often than the reverse.

Convert  $334^\circ 18' 22''$  to time units, using the Nautical Almanac arc to time conversion table.

Convert the 22" to the nearest quarter minute of arc for solution to the nearest second of time. Interpolate if more precise results are required.

$$334^\circ 00.00 \text{ m} = 22 \text{ h } 16 \text{ m } 00 \text{ s}$$

$$000^\circ 18.25 \text{ m} = 00 \text{ h } 01 \text{ m } 13 \text{ s}$$

$$334^\circ 18' 22'' = 22 \text{ h } 17 \text{ m } 13 \text{ s}$$

#### 1604. Time Passage and Longitude

Section 1603 provides the direct coordination between the measure of longitude arc and the time passage of the mean solar day, equivalent to 24 hours equals 360 degrees of Earth's rotation. Thus, the measure of longitude between two fixed points is an angular equivalent of the time difference between these two points on the Earth. Therefore for any given time of day, places east of an observer have later time, and those places west have earlier time. The time difference observed between two places is equal to the difference of longitude between their meridians, expressed in units of time instead of arc. It is from this principle that longitude navigation through the use of a chronometer is derived. If an error free chronometer was set precisely at 12h at a given local noon, properly adjusted for the equation of time, then any longitudinal excursion (distance traveled east or west) could be determined through the interval of time passage on the chronometer, compared to the transit of the Sun across the new local (present) meridian.

#### 1605. The Date Line

Since time accumulates later when traveling toward the east and earlier toward the west, a traveler circling the Earth gains or loses an entire day depending on the direction of travel. To provide a starting place for each new mean solar day, a date line extending from Earth's poles is fixed by informal agreement, called the **International Date Line**. The International Date Line separates two consecutive

calendar days. It coincides with the 180th meridian over most of its length. In crossing this line, the date is altered by one day. The date becomes one day earlier when traveling eastward from east longitude to west longitude. Conversely the date becomes one day later when traveling westward across it. When solving celestial problems, we convert local time to UTC and then convert this to local time on the opposite side of the date line.

#### 1606. Civil Time vs. Mean Solar Time and Time Zones

Mean solar time is closely related to **civil time**, which is what our clocks read if they are set accurately. The worldwide system of civil time has historically been based on mean solar time, but in the modern system of timekeeping, there are some differences.

Civil time is based on a worldwide system of 1-hour time zone segments, which are spaced 15 degrees of longitude apart. (The time zone boundaries are usually irregular over land, and the system has broad variations; local time within a country is the prerogative of that country's government.) All places within a time zone, regardless of their longitudes, will have the same civil time, and when we travel over a time zone boundary, we encounter a 1-hour shift in civil time. The time zones are set up so that each is an integral number of hours from a time scale called Coordinated Universal Time (UTC). UTC is accurately distributed by GPS, the Internet, and radio time signals. So the minute and second "ticks" of civil time all over the world are synchronized and counted the same; it is only the hour count that is different. (There are a few odd time zones that are a  $\frac{1}{4}$  or  $\frac{1}{2}$  hour offset from neighboring zones. The minute count is obviously different in these places.)

#### 1607. Zone Time

At sea, as well as ashore, watches and clocks are normally set to some form of **zone time (ZT)**. At sea the nearest meridian exactly divisible by  $15^\circ$  is usually designated as the **time meridian** or **zone meridian**. Thus, within a time zone extending  $\pm 7.5^\circ$  on each side of the time meridian the time is the same, and time in consecutive zone increments differs by exactly one hour. The time maintained by a clock is changed as convenient, usually at a whole hour, when crossing the boundary between zones. Each time zone is identified by the number of times the longitude of its zone meridian is divisible by  $15^\circ$ , *positive in west longitude and negative in east longitude*. This number and its sign, called the **zone description (ZD)**, is the number of whole hours that are added to or subtracted from the zone time to obtain UTC. Note that the zone description does not change when Daylight Savings Time is in effect. The mean fictitious sun is the celestial reference point for zone time. See Table 1607a and Figure 1607b for more detail.

Time Zones, Zone Descriptions, and Suffixes					
ZONE	ZD	SUFFIX	ZONE	ZD	SUFFIX
7.5° W to 7.5° E	0	Z	7.5° W to 22.5° W	+ 1	N
7.5° E to 22.5° E	-1	A	22.5° W to 37.5° W	+ 2	O
22.5° E to 37.5° E	-2	B	37.5° W to 52.5° W	+ 3	P
37.5° E to 52.5° E	-3	C	52.5° W to 67.5° W	+ 4	Q
52.5° E to 67.5° E	-4	D	67.5° W to 82.5° W	+ 5	R
67.5° E to 82.5° E	-5	E	82.5° W to 97.5° W	+ 6	S
82.5° E to 97.5° E	-6	F	97.5° W to 112.5° W	+ 7	T
97.5° E to 112.5° E	-7	G	112.5° W to 127.5° W	+ 8	U
112.5° E to 127.5° E	-8	H	127.5° W to 142.5° W	+ 9	V
127.5° E to 142.5° E	-9	I	142.5° W to 157.5° W	+ 10	W
142.5° E to 157.5° E	-10	K	157.5° W to 172.5° W	+ 11	X
157.5° E to 172.5° E	-11	L	172.5° W to 7.5° W	+ 12	Y
172.5° E to 180° E	-12	M			

Note. - GMT is indicated by suffix Z. Standard times as kept in various places or countries are listed in *The Nautical Almanac* and *The Air Almanac*.

Table 1607a. Time zones, descriptions, and suffixes.

When converting ZT to UTC, a positive ZT is added and a negative one subtracted; converting UTC to ZT, a positive ZD is subtracted, and a negative one added.

**Example:** The UTC is 15<sup>h</sup>27<sup>m</sup>09<sup>s</sup>.

**Required:** (1) ZT at long. 156°24.4' W

**Solutions:**  $15\text{ h }27\text{ m }09\text{ s} - (+150/15) = 05\text{ h }27\text{ m }09\text{ s}$

In example (1), the nearest 15° increment is 150°W, leaving a remainder of less than  $\pm 7.5^\circ$  (+6.407°).

**Example:** The UTC is 15<sup>h</sup>27<sup>m</sup>09<sup>s</sup>.

**Required:** (2) ZT at long. 039°04.8' E

**Solutions:**  $15\text{ h }27\text{ m }09\text{ s} + (-45/15) = 18\text{ h }27\text{ m }09\text{ s}$

In example (2), the nearest 15° increment is 45°E, leaving a remainder of less than  $\pm 7.5^\circ$  (-5.92°).

## 1608. Chronometer Time

**Chronometer time (C)** is time indicated by a chronometer. Since a chronometer is set approximately to UTC and not reset until it is overhauled and cleaned

about every 3 years, there is nearly always a **chronometer error (CE)**, either fast (F) or slow (S). The change in chronometer error in 24 hours is called **chronometer rate**, or daily rate, and designated gaining or losing. With a consistent error in chronometer rate of +1s per day for three years, the chronometer error would accumulate 18 minutes. Since chronometer error is subject to change, it should be determined from time to time, preferably daily at sea. Chronometer error can be determined by comparison to a radio derived time signal, by comparison with another timekeeping system of known error, or by applying chronometer rate to previous readings of the same instrument. It is recorded to the nearest whole or half second. Chronometer rate is recorded to the nearest 0.1 second/day.

**Example:** At UTC 1200 on May 12 the chronometer reads 12<sup>h</sup>04<sup>m</sup>21<sup>s</sup>. At UTC 1600 on May 18 it reads 4<sup>h</sup>04<sup>m</sup>25<sup>s</sup>.

**Required:**

1. Chronometer error at 1200 UTC May 12.
2. Chronometer error at 1600 UTC May 18.
3. Chronometer rate.
4. Chronometer error at UTC 0530, May 27.

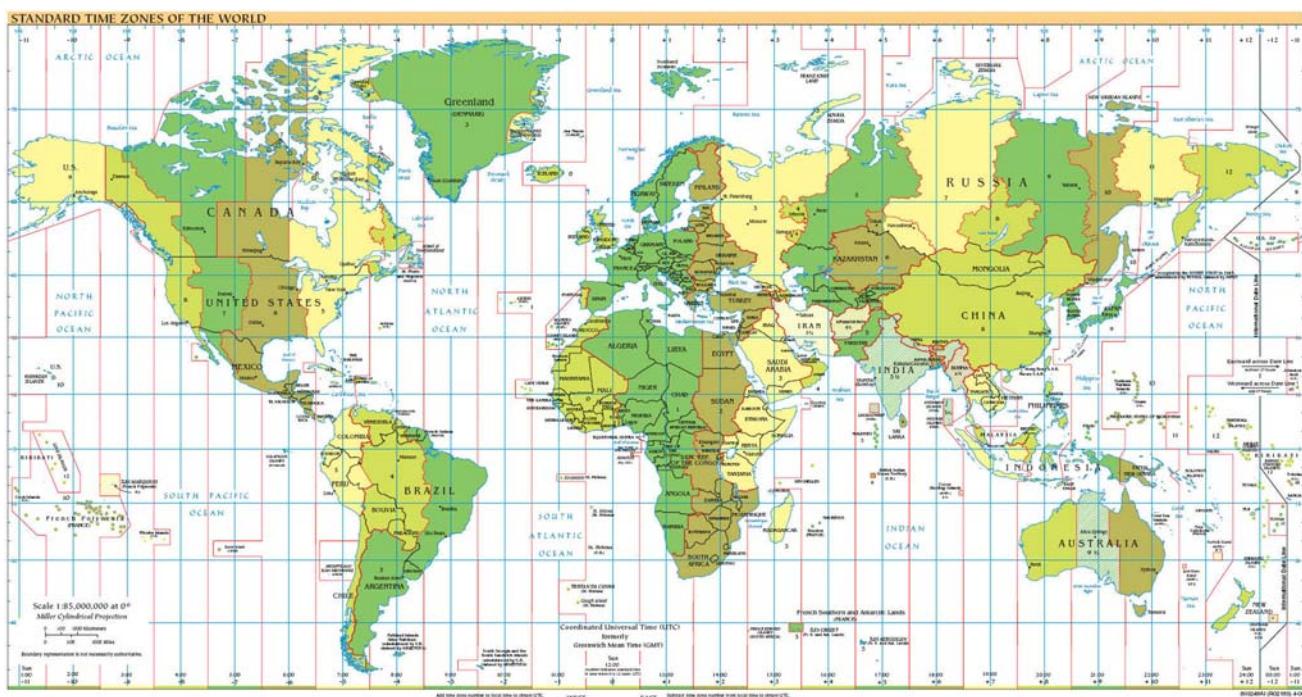


Figure 1607b. Time zones of the world.

**Solutions:**

1.	UTC C CE	12 <sup>h</sup> 00 <sup>m</sup> 00 <sup>s</sup> 12 <sup>h</sup> 04 <sup>m</sup> 21 <sup>s</sup> (F)4 <sup>m</sup> 21 <sup>s</sup>	May 12 gaining
2.	UTC C CE	16 <sup>h</sup> 00 <sup>m</sup> 00 <sup>s</sup> 04 04 25 (F)4 <sup>m</sup> 25 <sup>s</sup>	May 18 gaining
3.	UTC UTC diff. CE CE diff. daily rate	18 <sup>d</sup> 16 <sup>h</sup> 12 <sup>d</sup> 12 <sup>h</sup> 06 <sup>d</sup> 04 <sup>h</sup> = 6.2 <sup>d</sup> (F)4 <sup>m</sup> 21 <sup>s</sup> (F)4 <sup>m</sup> 25 <sup>s</sup> 4 <sup>s</sup> (gained) 0.6 <sup>s/d</sup> (gain)	present past 1200 May 12 1600 May 18 4 <sup>s</sup> /6.2 <sup>d</sup>
4.	UTC UTC diff. CE corr. CE	27 <sup>d</sup> 05 <sup>h</sup> 30 <sup>m</sup> 18 <sup>d</sup> 16 <sup>h</sup> 00 <sup>m</sup> 08 <sup>d</sup> 13 <sup>h</sup> 30 <sup>m</sup> (8.5 <sup>d</sup> ) (F)4 <sup>m</sup> 25 <sup>s</sup> (+0) <sup>m</sup> 05 <sup>s</sup> (F)4 <sup>m</sup> 30 <sup>s</sup>	present past 1600 May 18 diff. rate 0530 May 27

Because UTC is on a 24-hour basis and chronometer time on a 12-hour basis, a 12-hour ambiguity exists. This is ignored in finding chronometer error. However, if chronometer error is applied to chronometer time to find UTC, a 12-hour error can result. This can be resolved by mentally applying the zone description to local time to obtain approximate UTC. A time diagram can be used for resolving doubt as to approximate UTC with date. If

the Sun for the kind of time used (mean or apparent) is between the lower branches of two time meridians (as the standard meridian for local time, and ZT 0 or Zulu meridian for UTC, the date at the place farther east is one day later than at the place farther west.

**1609. Watch Time**

**Watch time (WT)** is usually an approximation of zone time, except that for timing celestial observations it is easiest to set a comparing watch to UTC. If the watch has a second-setting hand, the watch can be set exactly to ZT or UTC, and the time is so designated. If the watch is not set exactly to one of these times, the difference is known as **watch error (WE)**, labeled fast (F) or slow (S) to indicate whether the watch is ahead of or behind the correct time.

If a watch is to be set exactly to ZT or UTC, set it to some whole minute slightly ahead of the correct time and stopped. When the set time arrives, start the watch and check it for accuracy.

The UTC may be in offset by 12<sup>h</sup>, but if the watch is graduated to 12 hours, this will not be reflected. If a watch with a 24-hour dial is used, the actual UTC should be applied.

To determine WE, compare the reading of the watch with that of the chronometer at a selected moment. This may also be at some selected moment to UTC. Unless a watch is graduated to 24 hours, its time is designated as AM (ante meridian) before noon and PM (post meridian) after noon.

Even though a watch is set approximately to the zone time, its error to UTC can be determined and used for timing observations. In this case the 12-hour ambiguity to UTC should be resolved, and a time diagram used to avoid miscalculation. This method requires additional work, and presents a greater probability of error, and gains no greater advantage provided through WE compensation.

If a stopwatch is used for timing observations, it should be started at some convenient UTC, such as a whole 5<sup>m</sup> or 10<sup>m</sup>. The time of each observation is then the UTC plus the watch time. Digital stopwatches and wristwatches are ideal for this purpose, as they can be set from a convenient UTC and read immediately after the observation is taken.

### 1610. Local Mean Time

**Local mean time (LMT)**, like zone time, uses the mean Sun as the celestial reference point. It differs from zone time in that the local meridian is used as the terrestrial reference, rather than a zone meridian. Thus, the local mean time at each meridian differs from every other meridian, the difference being equal to the difference of longitude expressed in time units. At each zone meridian, including 0°, LMT and ZT are identical.

In navigation the principal use of LMT is in rising, setting, and twilight tables. The problem is usually one of converting the LMT taken from the table to ZT. At sea, the difference between the times is normally not more than 30<sup>m</sup>, and the conversion is made directly, without finding GMT as an intermediate step. This is done by applying a correction equal to the difference of longitude. If the observer is west of the time meridian, the correction is added, and if east of it, the correction is subtracted. If Greenwich time is desired, it is found from ZT.

Where there is an irregular zone boundary, the longitude may differ by more than 7.5° (30<sup>m</sup>) from the time meridian.

If LMT is to be corrected to daylight saving time, the difference in longitude between the local and time meridian can be used, or the ZT can first be found and then increased by one hour.

Conversion of ZT (including GMT) to LMT is the same as conversion in the opposite direction, except that the sign of difference of longitude is reversed. This problem is not normally encountered in navigation.

### 1611. Sidereal Time

**Sidereal time** uses the celestial datum of the vernal equinox (first point of Aries) as the celestial reference point instead of the apparent procession of Sun. Since the Earth revolves around the Sun, and since the direction of the Earth's rotation and revolution are the same, it completes a rotation with respect to the stars in less time (about 3 m 56.6 s of mean solar units) than with respect to the Sun, and during one revolution about the Sun (1 year) it makes one

complete rotation more with respect to the stars than with the Sun. This accounts for the daily shift of the stars nearly 1° westward each night. Hence, sidereal days are shorter than solar days, and its hours, minutes, and seconds are correspondingly shorter. Because of nutation, sidereal time is not quite constant in rate. Time based upon the average rate is called **mean sidereal time**, when it is to be distinguished from the slightly irregular sidereal time. The ratio of mean solar time units to mean sidereal time units is 1:1.00273791.

A navigator very seldom uses sidereal time.

### 1612. Time and Hour Angle

As mentioned earlier, **hour angle** is a measure of how far east or west of a meridian a celestial object appears. If the local meridian is used, this measure is called a **local hour angle (LHA)**. If the **Greenwich meridian** is used, then it is called a **Greenwich hour angle, GHA**. Hour angles are often expressed in arc units, between 0 and 360°. The hour angle is zero for an object crossing the meridian, and increases as the object moves west of the meridian (setting). In other words, an object transiting the meridian has an hour angle of 0°. Shortly after transit, its hour angle would be 1°, shortly before transit it would be 359°.

Sidereal time is the hour angle of the vernal equinox, but it is usually expressed in time units. Solar time at a specific location is also an hour angle measurement of the Sun, but since the day starts at midnight, 12 hours is added. That is, local solar time = 12 hours + local hour angle (expressed in hours) of the position of the Sun in the sky.

As with time, LHA at two places differs by their difference in longitude. In addition, it is often convenient to express hour angle in terms of the shorter arc between the local meridian and the body, that is, instead of 0° to 360°, it can be expressed 0° to 180°. This is similar to measurement of longitude from the Greenwich meridian. Local hour angle measured in this way is called meridian angle (t), which must be labeled east or west, like longitude, to indicate the direction of measurement. A westerly meridian angle is numerically equal to LHA, while an easterly meridian angle is equal to 360° - LHA. Mathematically, LHA = t (W), and LHA = 360° - t (E). Meridian angle is used in the solution of the navigational triangle.

*Find LHA and t of the Sun at for long. 118°48.2' W if the GHA of the Sun is 231° 04.0'.*

*LHA = GHA - west longitude, and LHA = GHA + east longitude, thus, for our example*

$$LHA(\text{Sun}) = 231^\circ 04.0' - 118^\circ 48.2' = 112^\circ 15.8'$$

$$t = 112^\circ 15.8' W$$

## RADIO DISSEMINATION OF TIME SIGNALS

### 1613. Dissemination Systems

Of the many systems for time and frequency dissemination, the majority employ some type of radio transmission, either in dedicated time and frequency emissions or established systems such as radionavigation systems. The most accurate means of time and frequency dissemination today are by the mutual exchange of round-trip time signals through communication (commonly called Two-Way) and by the mutual observation of one-way signals from navigation satellites (such as Common View, All-in-View, and Differential GPS). One-way direct access to Global Navigation Satellite Systems (GNSS) is an excellent way to obtain UTC if many satellite observations are averaged.

Radio time signals can be used either to perform a clock's function or to set clocks. When using a radio wave several factors must be considered. One is the delay time of approximately 3 microseconds per kilometer it takes the radio wave to propagate and arrive at the reception point. Thus, a user 1,000 kilometers from a transmitter receives the time signal about 3 milliseconds later than the on-time transmitter signal. If time is needed to better than 3 milliseconds, a correction must be made for the time it takes the signal to pass through the receiver.

In most cases standard time and frequency emissions as received are more than adequate for ordinary needs. However, many systems exist for the more exacting scientific requirements, such as Precise Point Positioning using GNSS carrier phase.

### 1614. Characteristic Elements of Dissemination Systems

A number of common elements characterize most time and frequency dissemination systems. Among these elements, the most important are accuracy, ambiguity, repeatability or precision, coverage, availability of time signal, reliability, ease of use, cost to the user, and the number of users served. No single system optimizes all desired these characteristics. The relative importance of these characteristics will vary by application, and the solution for one user may not be satisfactory to another. These trade among these common elements are discussed in the following examination of a hypothetical radio signal.

Consider a very simple system consisting of an unmodulated 10-kHz signal as shown in Figure 1614. This signal, leaving the transmitter at 0000 UTC, will reach the receiver at a later time due to the propagation delay. The user must know this delay because the accuracy of the recovered time from the transmitted signal can be no better than the certainty in this delay. Since all cycles of the signal waveform are identical, the signal is ambiguous and

the user must resolve which cycle is the "on time" cycle, in this case the cycle leaving at 0000 UTC. This means, with respect to a 10-kHz signal waveform, the user must already know the propagation delay to within  $\pm 50$  microseconds (half the period of the signal). The calibration of the waveform cycle over cycle phase (zero crossings as defined in the figure) to resolve ambiguity in time dissemination is called the "tick to phase" determination. Further, the user may desire to periodically use the time-transfer system, say once a day, to check their clock or frequency standard. However, if the travel delay and instrument repeatability vary from one day to the next without the user knowing or correcting, the accuracy will be limited by the amounts attributed to these uncertainties.

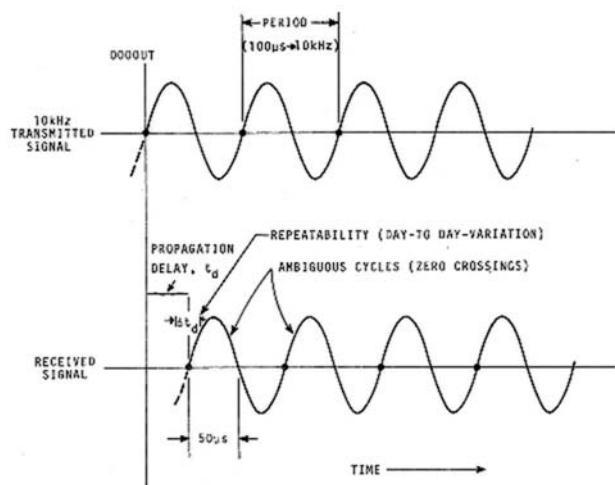


Figure 1614. Single tone time dissemination.

Many users are interested in making time-coordinated measurements over large geographic areas. They would like all measurements to be traceable to one master reference time to make corrections for the offsets between geographically distributed timekeeping systems. In addition, traceability to a master reference system increases confidence that all time measurements are related to each other in a consistent manner. Thus, the accuracy over the range of geographic coverage of a dissemination system is an important characteristic. Another important characteristic of a time dissemination system is the percentage of availability.

For most social uses of time, people who have to keep an appointment usually need to know the time of day to within a few minutes. Although requiring only coarse time information, people keeping a social schedule want it on demand, and thus carry a wristwatch or other portable device with a clock function that gives the time with continuous availability. People who have access to the internet can set the time of their personal computers to an accuracy of UTC

of  $\pm 100$  milliseconds, or considerably better, through the Network Time Protocol (NTP), with near continuous availability, dependent on the network's reliability. On the other hand, a person with a scientific interest may possess a very good clock capable of maintaining a few microseconds with only an occasional need for an accuracy update, perhaps only once or twice a day. However, in this distinguishing case, the scientific user requires much greater precision and accuracy in time dissemination than the social user, when available. This leads to the characteristic of time dissemination reliability, i.e., the likelihood that a time signal will be available when scheduled. In the case of the scientific user, the availability of time dissemination may be a critical operational need, and reliability may be as important as precision. Propagation fade-out or user location (such as in a basement or the woods) can sometimes prevent or distort signal reception. Thus, the quality and cost of time dissemination services contrast accuracy, availability and reliability against the application needs of the user community and the capability of their local clocks.

### 1615. Radio Wave Propagation Factors

Radio has been used to transmit standard time and frequency signals since the early 1900's. As opposed to the physical transfer of time via portable clocks, the transfer of timing information by radio involves the use of electromagnetic propagation from a transmitter, usually carrying the master reference time, to a navigator's receiver at long distance.

In the broadcast of frequency and time over radio, the transmitted signals are directly related to some master clock and are usually received with some degradation in accuracy. In a vacuum and with a noise-free background, timing signals should be received at a distant receiver essentially undistorted, with a constant path delay due to the propagation of the radio wave at the speed of light (299,773 kilometers per second). However the propagation media, including the Earth's atmosphere and ionosphere, reflections and refractions caused by man-made obstructions and geographic features, and space weather (solar-activity), as well as the inherent characteristics of transmitters and receivers, degrade the fidelity and accuracy of timing derived from the received radio signals. The amount of degradation in timing recovered from the signals is also dependent upon the frequency of the transmitted radio wave (carrier frequency), and the length of signal path. In many cases the application of propagation models or supplementary information must be used to correct for the distorting effects. For example, GPS receivers, which only use the L1 frequency, have correction models built into their systems to correct for propagation through the ionosphere from space

Radio dissemination systems can be classified in a number of different ways. One way is to separate those carrier frequencies low enough to be reflected by the ionosphere (below 30 MHz) from those sufficiently high to

penetrate the ionosphere (above 30 MHz). The former can be observed at great distances from the transmitter but suffer from ionospheric propagation distortion that limits accuracy; the latter are restricted to line-of-sight applications but show less signal degradation caused by propagation effects. The most accurate systems tend to be those which use the higher, line-of-sight frequencies, and with the advent of space-based satellite navigation, such as GPS, these also have promoted the most users and applications for radio time dissemination.

### 1616. Standard Time Broadcasts

The World Radiocommunication Conference (WRC), is the means by which the International Telecommunications Union (ITU), allocates certain frequencies in five bands for standard frequency and time signal emission. For such dedicated standard frequency transmissions, the ITU Radiocommunication Sector (ITU-R) recommends that carrier frequencies be maintained so that the average daily fractional frequency deviations from the internationally designated standard for measurement of time interval should not exceed  $\pm$  ten parts per trillion.

### 1617. Time Signals

The modern method of determining chronometer error and daily rate is by comparison to time recovered from radionavigation signals. The most accurate and readily available method for vessels is from navigation receivers of GPS, or other GNSS, and/or, where available, Enhanced Long Range Navigation (eLORAN) signals. Also, many maritime nations broadcast time signals on short-wave frequencies, such as the U.S. station (WWV), or German station (DCF77). Further discussion can be found in NGA Pub. 117, *Radio Navigational Aids* and the British Admiralty *List of Radio Signals*. A list of signals transmitted by timing labs is published in the Annual Report of the International Bureau of Weights and Measures (BIPM). The BIPM report is currently available on the Internet (see Figure 1617a). An important reason for employing more than one technique is to guard against both malfunction in equipment or malicious interference, such as spoofing.



Figure 1617a. BIPM Annual Report on Time Activities.  
<http://www.bipm.org/en/bipm/tai/annual-report.html>

If a vessel employs a mechanically actuated (main-spring) chronometer or even an atomic clock, the time should nonetheless be checked daily against a time derived from radio signals, beginning at least three days prior to de-

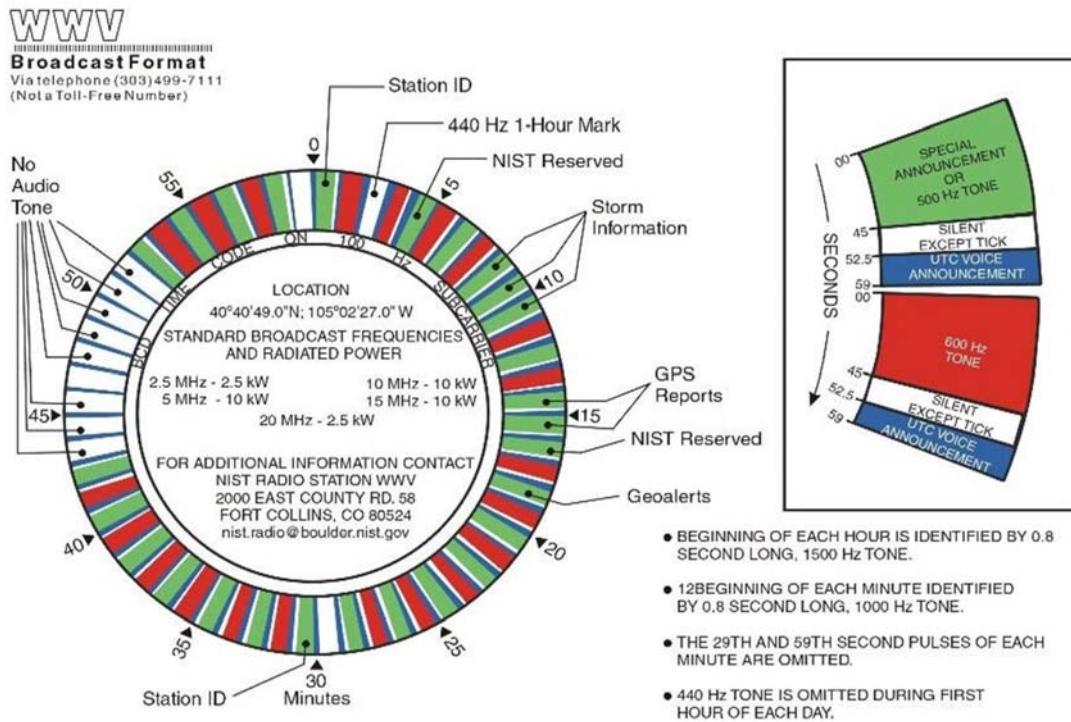


Figure 1617b. Broadcast format of station WWV.

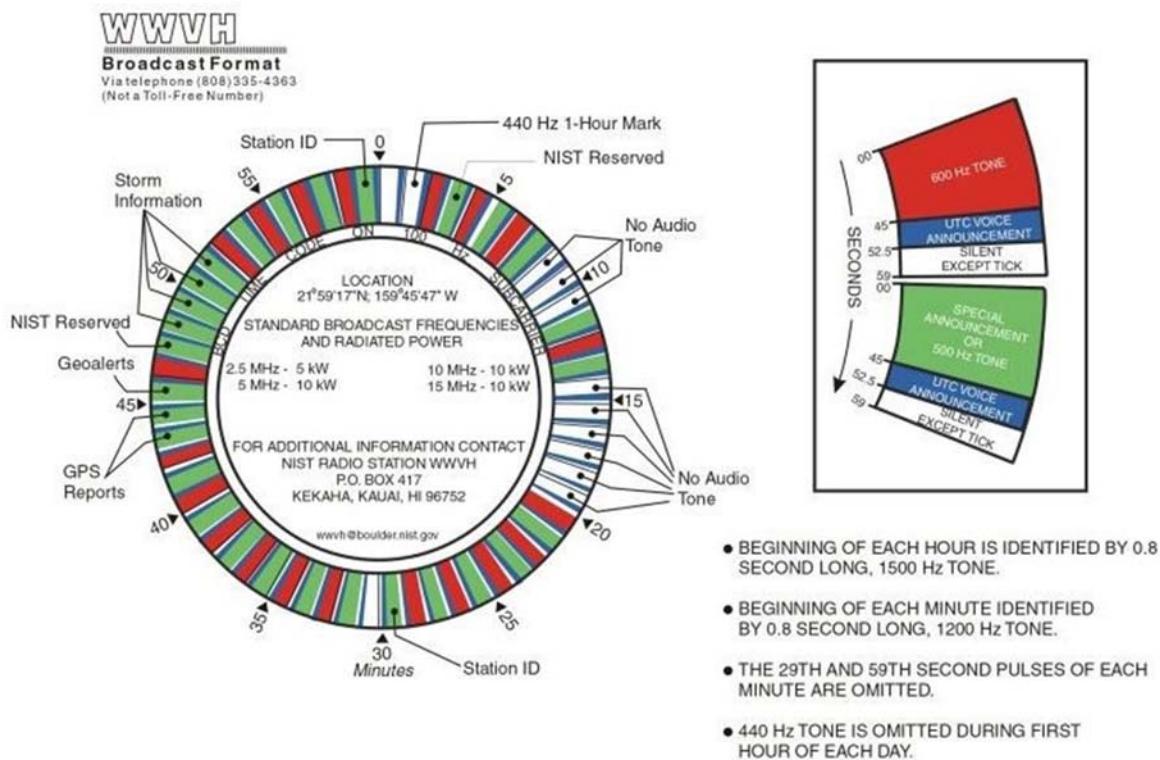


Figure 1617c. Broadcast format of station WWVH.

parture. The offset and computed rate should be entered in the chronometer record book (or record sheet) each time they are determined, although for an atomic clock the main concern is catastrophic or end of life failure. For example, cesium-beam tube atomic clocks have a limited life due to the consumption of the cesium metal during extended operation, typically 5 to 7 years.

For the U.S. the National Institute of Standards and Technology (NIST) broadcasts continuous time and frequency reference signals from WWV, WWVH, and WWVB. Because of their wide coverage and relative simplicity, the HF services from WWV and WWVH are used extensively for navigation. Station WWV broadcasts from Fort Collins, Colorado at the internationally allocated frequencies of 2.5, 5.0, 10.0, 15.0, and 20.0 MHz; station WWVH transmits from Kauai, Hawaii on the same frequencies with the exception of 20.0 MHz. The broadcast signals include standard time and frequencies, and various voice announcements. Details of these broadcasts are given in NIST Special Publication 432, NIST Frequency and Time Dissemination Services. Both HF emissions are derived from cesium beam atomic frequency standards with traceable reference to the NIST atomic frequency and time standards.

The time ticks in the WWV and WWVH emissions are shown in Figure 1617b and Figure 1617c. The 1-second UTC markers are transmitted continuously by WWV and WWVH, except for omission of the 29th and 59th marker each minute. With the exception of the beginning tone at each minute (800 milliseconds) all 1-second markers are of 5 milliseconds duration and at a tone of 440 Hz. Each pulse is preceded by 10 milliseconds of silence and followed by 25 milliseconds of silence. Time voice announcements are given also at one minute intervals. All time announcements are UTC.

As explained in the next section, Coordinated Universal Time (UTC) may differ from (UT1) by as much as 0.9 second; the actual difference can be found at IERS web pages Bulletin A, which published on the internet at <http://datacenter.iers.org/eop/-/somos/5Rgv/latest/6>. NGA

*Pub. No. 117, Radio Navigational Aids*, should be referred to for further information on time signals.

### 1618. Leap-Second Adjustments

By international agreement, UTC is maintained to be no more than  $\pm 0.9$  seconds from agreement with the continuous celestial timescale, UT1. The introduction of leap seconds allows a clock maintaining UTC to stay approximately coordinated with mean solar time or stay near the procession of the fictitious mean sun across the sky. The insertion of leap seconds makes UTC a discontinuous timescale. The main contributor to the need for a leap second adjustment is the slowing of the Earth's rotation at about 1.7 ms/century. However, because of irregular variations in the yearly rate of the rotation of the Earth, year over year occurrences of the insertion of a leap seconds is not predictable.

The Central Bureau of the International Earth Rotation and Reference Frames Service (IERS) decides upon and announces the introduction of a leap second. The IERS announces the leap second insertion at least eight weeks in advance. Because of the irregularity of the Earth's rotation, the IERS provides that a second may be advanced or retarded, positive or negative leap second, though a negative leap second has never been required since its institution in 1972. The leap second is introduced as the last second of a UTC month, but first preference is given to the end of December and June, and second preference is given to the end of March and September. A positive leap second begins at 23 h 59 m 60 s and ends at 00 h 00 m 00 s of the first day of the following month. In the case of a negative leap second, 23 h 59 m 58 s is followed one second later by 00 h 00 m 00 s (skipping 23 h 59 m 59 s) of the first day of the following month. Leap second adjustments of UTC are performed uniformly, and in synchrony (per interval of a SI second) across the world.

The dating of events in the vicinity of a leap second is effected in the manner indicated in Figure 1618a and Figure 1618b.

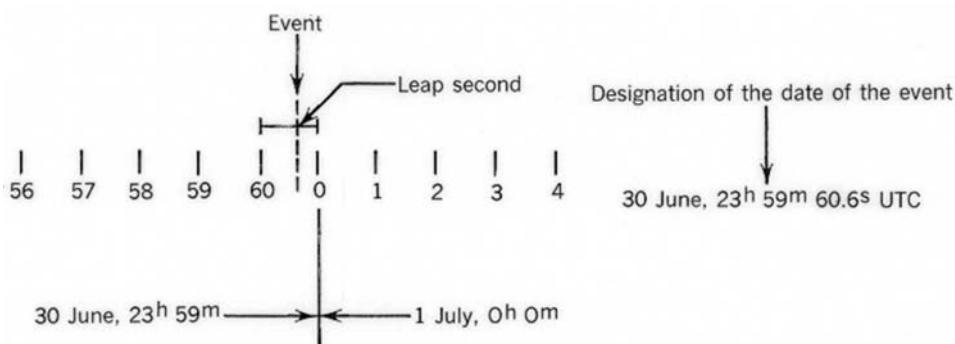


Figure 1618a. Dating of event in the vicinity of a positive leap second.

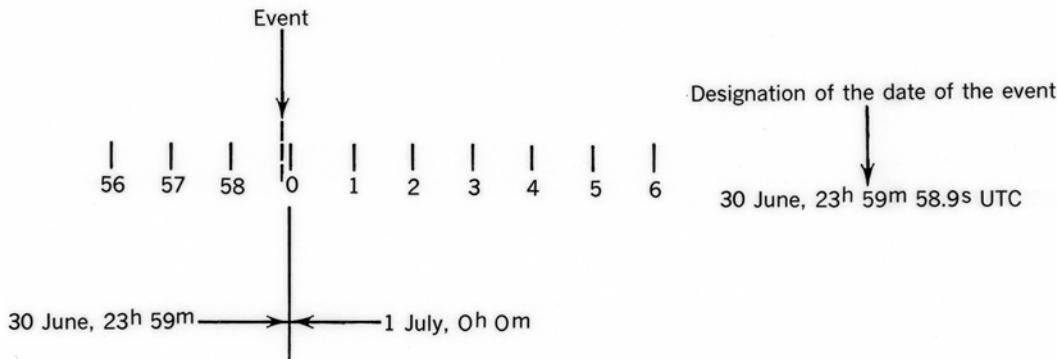


Figure 1618b. Dating of event in the vicinity of a negative leap second.



Figure 1618c. USNO leap second data between January 1972 and January 2017. Link:  
<http://tycho.usno.navy.mil/leapsec.html>

Whenever a leap second adjustment is to be made to UTC, navigators are advised by information presented on the web pages of the United States Naval Observatory, USNO, IERS Bulletin C and the International Bureau of Weights and Measures (BIPM). Additional information is available on the USNO and IERS webpages (see Figure 1618c and Figure 1618d).

#### 1619. Use of Time, Time-interval, and other Novel Techniques for Approximate Determination of Chronometer Time, Latitude, and Longitude.

There may arise situations in which a mariner needs to address the problem of determining date, time, latitude, and longitude using only minimal resources and with little, if any, prior knowledge of the values of these parameters. Given this, it is useful to consider the value of using simple time, “time-interval”, azimuth, “azimuth interval”, and “instrument-free” or “instrument-limited” measurements, performed in conjunction with table look-up of data from the Air or Nautical Almanacs and/or back-of-the-envelope computations. The term “instrument-limited”, in this context, applies when azimuth readings are made with a simple compass, and elevation readings are accomplished using a handheld inclinometer rather than a sextant or tripod-mounted surveying transit.



Figure 1619. Combined compass/inclinometer with internal lighting and automatic leveling.

Figure 1619 illustrates a convenient instrument, which is a combined inclinometer/compass that can be used on land without a clearly defined horizon, and at night using internal illumination. One of the user's eyes reads the internal scales while the other eye lines the internal graticule up with the star or other object being measured. The human ability to merge the different optical images into one perceived image is not universal. Up to 15% of individuals are unable to merge the different visual images. Although not of sextant accuracy, the device is rugged and portable, and is precise to about 1 degree for handheld use without a tripod.

Note that a modern smart phone, with its built in clock, camera, inclinometer, and compass can be used for the same purpose if GPS is denied, and can also be programmed with a star atlas, almanac data, and navigation algorithms. However, the successful use of a smartphone as

a combined sextant, chronometer, and navigation computer depends critically on battery life.

The level of precision of an inclinometer and compass can be compared with the celestial measurements described previously as follows. Sextant measurements typically have a best-case precision of 0.2 minutes of arc. Related time measurements are typically accomplished with a resolution of one second. Note that 1 minute of arc at the equator corresponds to a distance of one nautical mile and equates to four seconds of clock time. Thus, it takes 4 seconds for the earth to rotate one arc-second around its axis.

With respect to precision measurement of time, knowledge of Greenwich or Universal time is typically specified to less than quartz clock accuracy (i.e., to about one second resolution). When there is a clock offset bias, its value and drift rate are typically known. It will often be the case that local time is synchronized to Greenwich time within one second, even for everyday consumer applications, and far better than this for time signals disseminated from a wireless network to one's cell phone.

Note that the poor relative precision of a magnetic compass with respect to that of a sextant precludes the combined use of sextant and azimuth measurements. However, when an inclinometer of limited precision is the best available instrument, it can also be beneficial to include compass-derived azimuth measurements of comparable precision.

With this background, some useful examples of relatively simple, but in certain situations of great value, navigation techniques are presented.

- 1. Quick observation of Polaris and the northern sky to approximate latitude and longitude.** To estimate latitude simply make an observation (when in the northern hemisphere) of Polaris, the north star. If Greenwich or universal time is available using a simple quartz watch or cell phone, longitude can be inferred. This can be done with the help of a star chart, by observing the “clock angles” of the constellation Cassiopeia and Canis Major (the big dipper). Experienced viewers of the night sky routinely estimate time by unassisted observations of the moon and of the constellations of the Zodiac.
- 2. Noon observation of the sun to compare with an observation of Polaris to determine solar declination, and hence to determine approximate date and time.** During daylight hours, the maximum angle of the sun above the horizon at *local apparent noon* can be determined by a series of measurements made at time intervals of a few minutes. The highest elevation angle of the sun,  $Elevation_{sun}$ , occurs at local noon when the sun is due south of the observer. This measurement, combined with the estimate of latitude from

measurement of the north star, Polaris, yields the declination of the sun. Specifically, the latitude value obtained from measurement of Polaris is related to the solar declination by the equation:

$$90 \text{ degrees} - Elevation_{sun} + \text{declination} = \text{Latitude}$$

The declination depends on time, but not on the observer's position. An approximate measurement of the declination can be matched to the daily tables in the Nautical Almanac to yield the date, and within a few hours, a value for Universal Time (which in this context can be regarded as being equivalent to Greenwich Mean Time, or GMT). For example, the elevation of the sun on September 30, 2016 measured at 1700 hours GMT is computed, from the Nautical Almanac, to be 47:50:30 deg:min:sec with an azimuth of 180.8 degrees, indicating that the measurement is made at a time that is very close to local apparent noon. Using the equation above, we deduce that the declination is  $\text{latitude} + \text{elevation} - 90 = 39:00:00 + 47:50:30 - 90:00:00 = -3:09:30$ , in very close agreement with the Nautical Almanac lookup value of -3:09:18 deg:min:sec.

Using this value of declination to identify a table entry in the Nautical Almanac takes one immediately to the daily entry for September 30, 2016 at 1700 hours universal time (e.g., GMT), thus illustrating the causal relationship between solar declination and date and time. Once GMT is known, the traditional methods of determining latitude and longitude using the stars, planets, and/or sun can be implemented.

Of course, if one knows Greenwich time to high precision from, for example, a digital watch, this same measurement, in conjunction with another measurement of the sun at a different point in time, yields the traditional *running fix*, which lies in the purview of the earlier sections of this chapter.

- 3. Observations of sunrise and sunset to determine longitude.** If Greenwich time is known from a digital watch and an intelligent estimate of the relevant time zone, a simpler implementation of the running fix is easily accomplished. In this case, one measures only the times of sunrise and sunset, neither of which requires a sextant or artificial horizon when a clear horizon is available (i.e., on or near the ocean or other large body of water). The value for local noon is given as the midpoint in local time of the sunrise and sunset measurements. When this value is corrected to Greenwich time by the appropriate time zone corrections, the longitude is estimated by multiplying the time of local noon by 15 degrees per hour.

A better estimate of longitude is then obtained by adding/subtracting the requisite correction for the *equation of time*. This is found on the daily page

of the Nautical Almanac for the date and time of the observation, and is added/subtracted to the value of time that is then multiplied by the factor of 15 degrees per hour.

For example, the Washington Post newspaper provides daily values for local sunrise and sunset. On September 30, 2016, these are given as 7:03 A.M. and 6:52 P.M. EDT. Subtracting 1 hour to change to standard time, then taking the midpoint time yields a value for local noon of 11:57:30 h:m:s. Adding 10 minutes as the approximate correction for the equation of time (taken from the Nautical Almanac daily page for September 30th) corrects the time of local noon to GMT/UT, resulting in a value of 12:07:30 h:m:s.

If Washington was precisely 5 times zones away from Greenwich, then local noon in Washington would occur at 12:00:00 local time, after correcting for the equation of time. Five time zones, at 15 degrees per hour, is 75 degrees of longitude. Adding the additional 7.5 minutes corresponds to an additional 1.9 degrees of longitude, yielding a putative value for the longitude of Washington D.C. of 77 degrees West. (Note that the Naval Observatory, USNO, is at 38.9217° N, 77.0669° W).

4. **Compass measurement of the azimuth to Polaris to determine latitude and magnetic variation in order to determine position when latitude is already known.** Measurements made using a simple compass can be surprisingly useful. Measurement of the bearing of Polaris can be used to determine the local value of magnetic variation. Combined with an observation of latitude using an inclinometer or sextant, a map of magnetic variation versus latitude can then be used to generate an approximate position measurement.

In cases where magnetic variation is not known, relative bearing measurements yield “azimuth interval” measurements which remove the common mode error due to magnetic variation. In any case, the approach described herein is used routinely for pointing certain types of portable satellite telephone terminals at the appropriate satellite location in the geostationary arc.

5. **“Guess and Test” using simple Nautical Almanac equations in order to take advantage of combined elevation and azimuth measurements.** *Nautical Almanac* computations can be quite com-

plicated. For the purposes of this section, a convenient path forward is to use the straightforward equations for computing the calculated values of elevation angle  $H_c$  and azimuth  $Z$  from assumed values of the time, Greenwich Hour Angle (GHA), Sidereal Hour Angle (SHA), and declination for the celestial objects of interest. The relevant equations are given in the *Nautical Almanac* and are readily implemented using a calculator or perhaps a smartphone “App”.

Rather than use traditional iterative computations, this approach requires one to *guess* an “assumed position” and *test* the computed values of elevation  $H_c$  and azimuth  $Z$  against their measured values. One utility is that this provides a convenient way to integrate compass measurements of azimuth, corrected for magnetic variation as described above, into the data stream. The benefit is that a single sighting of the sun, if an azimuth measurement is included, provides the two data points needed to compute a latitude and longitude fix. There simply may not be time, or suitable weather conditions, to compute a running fix. The running fix, as described above, requires multiple measurements of the sun at widely spaced intervals of time.

6. **Cloudy night celestial navigation.** On a cloudy night, when only a single star is visible through a break in the clouds, a single measurement of the elevation and azimuth to a star lets one compute a location fix. Even if the identity of the star is not known, it is possible to perform the  $H_c$  and  $Z$  computations, for the assumed position, for several stars. Then the star whose measurement yields the most plausible position fix can often be reliably be assumed to be the star that was actually observed. Note that even a poor measurement of azimuth can be used to help identify the name, and hence the correct declination and sidereal hour angle values, to be used in the position computation.

There are many variations and extensions of these techniques and methods. The combination of a precision time reference and an accurate sextant is regaining favor after decades of single-system dependence on GPS, and more recently, E-LORAN. In extremis, and with little practice, even a combination of a protractor with a home-made plumb bob and a simple pendulum of length  $L$  and period  $2\pi\sqrt{L/g}$  might bring one safely home.

# CHAPTER 17

## THE ALMANACS

### PURPOSE OF ALMANACS

#### 1700. Introduction

Celestial navigation requires accurate predictions of the geographic positions of the celestial bodies observed. These predictions are available from three almanacs published annually by the U.S. Naval Observatory and H.M. Nautical Almanac Office (part of the U.K. Hydrographic Office) in England.

The *Astronomical Almanac* precisely tabulates celestial data for the exacting requirements found in scientific fields. Its precision is far greater than that required by celestial navigation. Even if the *Astronomical Almanac* is used for celestial navigation, it will not necessarily result in more accurate fixes due to the limitations of other aspects of the celestial navigation process. This printed book is available in the U.S. through the Government Publishing Office eBookstore and resellers, and elsewhere via U.K. Hydrographic Office distributors. There is also an Astronomical Almanac Online complementary website.

The *Nautical Almanac* contains astronomical information specifically needed by marine navigators. Information is tabulated to the nearest 0.1' of arc and, with interpolation, to 1 second of time. GHA and declination are available for the Sun, Moon, planets (Venus, Mars, Jupiter and Saturn), and 173 stars, as well as corrections necessary to reduce the observed values to true. Also included are Sun rise/set, equation of time, Moonrise/set, moon phase, twilight times, time zones, and star charts. Explanations, examples, and sight reduction procedures are also given. This printed book is available in the U.S. through the Government Publishing Office eBookstore and resellers, and elsewhere via U.K. Hydrographic Office distributors.

The *Air Almanac* was originally intended for air navigators, but is used today mostly by the maritime community. In general, the information is similar to the *Nautical Almanac*, but is given to a precision of 1' of arc and 1 second of time, at intervals of 10 minutes (values for the Sun and Aries are given to a precision of 0.1'). Unique to the *Air Almanac* are its monthly sky diagrams, used to find

navigational stars, planets, Sun and Moon at various latitudes. This publication is suitable for ordinary navigation at sea, but lacks the precision of the *Nautical Almanac*, and provides GHA and declination for only the 57 commonly used navigation stars. The *Air Almanac* is available on CD or as a free download through the Government Publishing Office eBookstore, The CD and download contain the same information as was found previously in the annual publications, with page images in PDF files.

The US Naval Observatory also provides celestial navigation data via the web at <http://aa.usno.navy.mil/data/>.



Figure 1700. USNO Data Services.

This robust website includes a navigational star chart and other data services, which provide GHA, declination, computed altitude, azimuth and altitude correction information for the navigational objects above the horizon at a given assumed position and time. Additional data services found on this website includes Rise/Set/Transit/Twilight data, Phases of the Moon, Eclipses and Transits, Positions of Selected Celestial Bodies, Synthetic Views of Selected Solar System Bodies and Dates & Times.

The Navy's **STELLA** program (**S**ystem **T**o **E**stimate **L**atitude and **L**ongitude **A**stronomically), found aboard all seagoing Navy and Coast Guard vessels, contains an interactive almanac as well; this product is restricted to DoD and DoD contractors. A variety of privately produced electronic almanacs are available as computer programs or applications (apps). These invariably are associated with sight reduction software which replaces tabular and mathematical sight reduction methods.

### FORMAT OF THE NAUTICAL AND AIR ALMANACS

#### 1701. *Nautical Almanac*

The major portion of the *Nautical Almanac* (pages 10 to 253) is devoted to hourly tabulations of Greenwich

Hour Angle (GHA) and declination, to the nearest 0.1' of arc. On each set of facing pages, information is listed for three consecutive days. On the left-hand page, successive columns list GHA of Aries ( $\varphi$ ), and both GHA and

declination of Venus, Mars, Jupiter, and Saturn, followed by the Sidereal Hour Angle (SHA) and declination of 57 stars. The GHA and declination of the Sun and Moon, and the horizontal parallax of the Moon, are listed on the right-hand page. Where applicable, the quantities  $v$  and  $d$  are given to assist in interpolation. The quantity  $v$  is the difference between the actual change of GHA in 1 hour and a constant value used in the interpolation tables, while  $d$  is the change in declination in 1 hour. Both  $v$  and  $d$  are listed to the nearest 0.1'.

On the left hand pages, the magnitude of each planet at Universal Time (UT) 1200 of the middle day of the three listed on a given page is found at the top of the column. The UT of transit across the Greenwich meridian is listed as "Mer. Pass.". The value for the first point of Aries for the middle of the three days is listed to the nearest 0.1' at the bottom of the Aries column. The time of transit of the planets for the middle day is given to the nearest whole minute, with SHA (at UT 0000 of the middle day) to the nearest 0.1', below the list of stars.

On the right hand pages, to the right of the Moon data is listed the Local Mean Time (LMT) of sunrise, sunset, and beginning and ending of nautical and civil twilight for latitudes from 72°N to 60°S. These times, which are given to the nearest minute, are UT of the phenomena on the Greenwich meridian. They are given for every day for moonrise and moonset, but only for the middle day of the three on each page for solar phenomena. For the Sun and Moon, the time of transit to the nearest whole minute is given for each day. For the Moon, both upper and lower transits are given. Also listed, are the equation of time for 0<sup>h</sup> and 12<sup>h</sup> UT, without sign, to the nearest whole second, with negative values shaded. The age and phase of the Moon is listed; age is given to the nearest whole day and phase is given by symbol. The semidiameters of both the Sun and Moon are also listed.

The main tabulation is preceded by a list of religious and civil holidays, phases of the Moon, a calendar, information on eclipses occurring during the year, and notes and a diagram giving information on the planets.

The main tabulation is followed by explanations and examples (pages 254 to 261). Next are four pages of standard times (zone descriptions). Star charts are next (pages 266-267), followed by a list of 173 stars in order of increasing SHA. This list includes the 57 stars given on the daily pages, identified by a number in the "Name and Number" field. It gives the SHA and declination each month, and the magnitude.

Stars are listed by Bayer's name, a designation that originated from Johann Bayer, a German uranographer (celestial cartographer), who in 1603 published an atlas that named the entire celestial sphere. The Bayer's name is used to identify these stars and also the popular name is listed where applicable. The brightest stars have been given a designation consisting of a Greek letter followed by the possessive form of the name of the constellation to which they belong.

Following the star list are the Polaris tables (pages 274-276). These tables give the azimuth and the corrections to be applied to the observed Polaris altitude to find one's latitude.

Following the Polaris table is the "Sight Reduction Procedures" section, divided into two subsections. The first, "Methods and Formula for Direct Computation" (pages 277 to 283) gives formulas and examples for the entry of almanac data, the calculations that reduce a sight, and a method of solution for position, all for use with a calculator or computer. The second, "Use of Concise Sight Reduction Tables" (pages 284 to 319), gives instructions and examples of how to use the provided concise sight reduction tables and a sight reduction form. Tabular precision of the concise tables is one minute of arc.

The next pages (pp. 320-325) contain data on polar phenomena. Examples and graphs are given to estimate times of sunrise, sunset, civil twilight, moonrise, and moonset at extreme northern latitudes for each month of the year.

Next is a table for converting arc to time units (page i). This is followed by a 30-page table (pages ii - xxxi) called "Increments and Corrections," used for interpolation of the hourly GHA and declination to get to the nearest second of the sextant observation. This table is printed on tinted paper for quick location. Then come tables for interpolating for times of rise, set, and twilight (page xxxii); followed by two indices of the 57 stars listed on the daily pages, one index in alphabetical order, and the other in order of decreasing SHA (page xxxiii).

Altitude corrections are given at the front and back of the almanac. Tables for the Sun, stars, and planets, and a dip table, are given on the inside front cover and facing page, with an additional correction for nonstandard temperature and atmospheric pressure on the following page. Tables for the Moon, and an abbreviated dip table, are given on the inside back cover and facing page. Corrections for the Sun, stars, and planets for altitudes greater than 10°, and the dip table, are repeated on one side of a loose bookmark. The star indices are repeated on the other side.

## 1702. Air Almanac

The *Air Almanac*, formerly a printed publication, is now available as a CD-ROM, and also as a free download from either the US Naval Observatory or Government Printing Office websites. The electronic version contains the same information as was previously found in the printed version, but with PDFs of the page images. Navigation through the e-book is done via a web interface and two options are given. The default option is a "logical" layout and a second is a "book layout", which maintains the same page order as the printed book. The description below follows the book layout.

As in the Nautical Almanac, the major portion of the Air Almanac is devoted to a tabulation of GHA and declination. However, in the Air Almanac values are listed at

intervals of 10 minutes, to a precision of 0.1' for the Sun and Aries, and to a precision of 1' for the Moon and the planets. Values are given for the Sun, first point of Aries (GHA only), the three navigational planets most favorably located for observation, and the Moon. The magnitude of each planet listed is given at the top of its column, and the percentage of the Moon's disc illuminated, waxing (+) or waning (-), is given at the bottom of each page. The magnitude of each planet listed is given at the top of its column. Values for the first 12 hours of the day are given on the right-hand page, and those for the second half of the day on the left-hand page. Each daily page includes the UT of moonrise and moonset on the Greenwich meridian for latitudes from 72° N to 60° S; a "half-day" difference column provides data to find the time of moonrise and moonset at any longitude. In addition, each page has a critical table of the Moon's parallax in altitude, and below this the semidiameter of the Sun and Moon, and the percentage of the Moon's disc illuminated and whether it is waxing (+) or waning (-).

Critical tables for interpolation for GHA are given on the inside front cover, which also has an alphabetical listing of the 57 navigational stars, with the number, magnitude,

yearly mean SHA, and yearly mean declination of each. The same interpolation table and star list are printed on a flap which follows the daily pages. This flap also contains a star chart, a star list with the same data as the other, but in increasing navigational number order, and a table for interpolation of the UT of moonrise and moonset for longitude.

Following the flap are instructions for the use of the almanac; a list of symbols and abbreviations in English, French, and Spanish; a list of time differences between Greenwich and other places; monthly sky diagrams by latitude and time of day; planet location diagrams; star recognition diagrams for periscopic sextants; sunrise, sunset, and civil twilight tables; rising, setting, and depression graphs; semiduration graphs of Sunlight, twilight, and Moonlight in high latitudes; a single Polaris correction table; a list of 173 stars by number and Bayer designation (also popular name where there is one), giving the SHA and declination each month (to a precision of 0.1'), and the magnitude; tables for interpolation of GHA Sun and GHA  $\gamma$ ; a table for converting arc to time; a refraction correction table; a Coriolis correction table; and on the inside back cover, an aircraft standard dome refraction table; a correction table for dip of the horizon.

## USING THE ALMANACS

### 1703. Entering Arguments

The time used as an entering argument in the almanacs is UT, (formerly referred to as GMT), which is equivalent to  $12^h + \text{GHA}$  of the mean Sun. This scale may differ from the broadcast time signals by an amount of 0.9<sup>s</sup> which, if ignored, will introduce an error of up to 0.2' in longitude determined from astronomical observations. The difference arises because the time argument depends on the variable rate of rotation of the Earth while the broadcast time signals are now based on atomic time. Leap seconds, that is step adjustments of exactly one second are made to the time signals as required (primarily at 24h on December 31 and June 30) so that the difference between the time signals and UT, as used in the almanacs, may not exceed 0.9<sup>s</sup>. If observations to a precision of better than 1<sup>s</sup> are required, corrections must be obtained from coding in the signal, or from other sources. The correction may be applied to each of the times of observation. Alternatively, the longitude, when determined from observations, may be corrected by the corresponding amount shown in Table 1703.

Correction to time signals	Correction to longitude
-0.9 <sup>s</sup> to -0.7 <sup>s</sup>	0.2' to east
-0.6 <sup>s</sup> to -0.3 <sup>s</sup>	0.1' to east
-0.2 <sup>s</sup> to +0.2 <sup>s</sup>	no correction

Table 1703. Corrections to time.

Correction to time signals	Correction to longitude
+0.3 <sup>s</sup> to +0.6 <sup>s</sup>	0.1' to west
+0.7 <sup>s</sup> to +0.9 <sup>s</sup>	0.2' to west

Table 1703. Corrections to time.

The main contents of the almanacs consist of data from which the GHA and the declination of all the bodies used for navigation can be obtained for any instant of UT. The LHA can then be obtained with the formula:

$$\begin{aligned} \text{LHA} &= \text{GHA} + \text{east longitude} \\ \text{LHA} &= \text{GHA} - \text{west longitude} \end{aligned}$$

For the Sun, Moon, and the four navigational planets, the GHA and declination are tabulated directly in the *Nautical Almanac* for each hour of UT throughout the year; in the *Air Almanac*, the values are tabulated for each whole 10 m of UT. For the stars, the SHA is given, and the GHA is obtained from:

$$\text{GHA Star} = \text{GHA } \gamma + \text{SHA Star.}$$

The SHA and declination of the stars change slowly and may be regarded as constant over periods of several days or even months if lesser accuracy is required. The SHA and declination of stars tabulated in the *Air Almanac* may be considered constant to a precision of 1.5' to 2' for the period covered by each of the volumes providing the

data for a whole year, with most data being closer to the smaller value. GHA  $\varphi$ , or the GHA of the first point of Aries (the vernal equinox), is tabulated for each hour in the *Nautical Almanac* and for each whole 10<sup>m</sup> in the *Air Almanac*. Permanent tables list the appropriate increments to the tabulated values of GHA and declination for the minutes and seconds of time.

In the *Nautical Almanac*, the permanent table for increments also includes corrections for  $v$ , the difference between the actual change of GHA in one hour and a constant value used in the interpolation tables; and  $d$ , the average hourly change in declination.

In the *Nautical Almanac*,  $v$  is always positive unless a negative sign (-) is shown. This occurs only in the case of Venus. For the Sun, the tabulated values of GHA have been adjusted to reduce to a minimum the error caused by treating  $v$  as negligible; there is no  $v$  tabulated for the Sun.

No sign is given for tabulated values of  $d$ ; whether to add or subtract a correction to the declination must be done via inspection of the increasing or decreasing trend of the declination values.

In the *Air Almanac*, the tabulated declination values, except for the Sun, are those for the middle of the interval between the time indicated and the next following time for which a value is given, making interpolation unnecessary. Thus, it is always important to take out the GHA and declination for the time immediately before the time of observation.

In the *Air Almanac*, GHA  $\varphi$  and the GHA and declination of the Sun are tabulated to a precision of 0.1'. If these values are extracted with the tabular precision, the "Interpolation of GHA" table on the inside front cover (and flap) should not be used; use the "Interpolation of GHA Sun" and "Interpolation of GHA Aries" tables, as appropriate. These tables are found on pages A164 and A165.

#### 1704. Finding GHA and Declination of the Sun

**Nautical Almanac:** Enter the daily page table with the whole hour before the given GMT, unless the exact time is a whole hour, and take out the tabulated GHA and declination. Inspect the trend in the following declination value to determine if declination is increasing or decreasing; this is needed to know whether to add or subtract the  $d$  correction. Also record the  $d$  value given at the bottom of the declination column. Next, enter the increments and corrections table for the number of minutes of GMT. If there are seconds, use the next earlier whole minute. On the line corresponding to the seconds of GMT, extract the value from the Sun-Planets column. Add this to the value of GHA from the daily page. This is GHA of the Sun. Next, enter the correction table for the same minute of GMT with the  $d$  value and take out the correction. Apply the  $d$  correction, either adding or subtracting (as determined earlier by inspection of the tabulated declination values), to the declination from the daily page. This is the declination.

The correction table for GHA of the Sun is based upon a rate of change of 15° per hour, the average rate during a year. At most times the rate differs slightly. The slight error is minimized by adjustment of the tabular values. The  $d$  value is the average hourly amount that the declination changes on the middle day of the three shown.

**Air Almanac:** Enter the daily page with the whole 10<sup>m</sup> preceding the given GMT, unless the time is itself a whole 10<sup>m</sup>, and extract the GHA. The declination is extracted without interpolation from the same line as the tabulated GHA or, in the case of planets, the top line of the block of six. If the values extracted are rounded to the nearest minute, enter the "Interpolation of GHA" table on the inside front cover (and flap), using the "Sun, etc." entry column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction listed half a line above the entry time. Add this correction to the GHA taken from the daily page. This is GHA. No adjustment of declination is needed. If the values are extracted with a precision of 0.1', the table for interpolating the GHA of the Sun to a precision of 0.1' must be used (page A164). Again no adjustment of declination is needed.

#### 1705. Finding GHA and Declination of the Moon

**Nautical Almanac:** Enter the daily page table with the whole hour before the given GMT, unless this time is itself a whole hour, and extract the tabulated GHA and declination. Record the corresponding  $v$  and  $d$  values tabulated on the same line, and determine whether the  $d$  correction is to be added or subtracted, by inspecting the trend in the next tabular declination value. The  $v$  value of the Moon is always positive (+) but it is not marked in the almanac. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT, take the GHA correction from the Moon column. Then, enter the correction table for the same minute with the  $v$  value, and extract the correction. Add both of these corrections to the GHA from the daily page. This is the GHA of the Moon. Then, enter the same correction table page with the  $d$  value and extract the corresponding  $d$  correction. Apply the  $d$  correction, either adding or subtracting (as determined earlier by inspection of the trend of the tabulated declination values), to the declination from the daily page. This is the declination of the Moon.

The correction table for GHA of the Moon is based upon the minimum rate at which the Moon's GHA increases, 14°19.0' per hour. The  $v$  correction adjusts for the actual rate. The  $v$  value is the difference between the minimum rate and the actual rate during the hour following the tabulated time. The  $d$  value is the amount that the declination changes during the hour following the tabulated time.

**Air Almanac:** Enter the daily page with the whole 10<sup>m</sup> next preceding the given GMT, unless this time is a whole 10<sup>m</sup>, and extract the tabulated GHA and the declination

without interpolation. Next, enter the “Interpolation of GHA” table on the inside front cover, using the “Moon” entry column, and extract the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line above the entry time. Add this correction to the GHA taken from the daily page to find the GHA at the given time. No adjustment of declination is needed.

The declination given in the table is correct for the time 5 minutes later than tabulated, so that it can be used for the 10-minute interval without interpolation, to an accuracy to meet most requirements. Declination changes much more slowly than GHA. If greater accuracy is needed, it can be obtained by interpolation, remembering to allow for the 5 minutes.

## 1706. Finding GHA and Declination of a Planet

**Nautical Almanac:** Enter the daily page table with the whole hour before the given GMT, unless the time is a whole hour, and extract the tabulated GHA and declination. Record the  $v$  and  $d$  values given at the bottom of each of these columns; determine whether the  $d$  correction is to be added or subtracted by inspecting the trend in the declination. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT, take the GHA correction from the Sun-planets column. Next, enter the correction table with the  $v$  value and extract the correction, giving it the sign of the  $v$  value. Add the first correction to the GHA from the daily page, and apply the second correction in accordance with its sign. This is the GHA of the planet. Then enter the increments and correction table for the same minute with the  $d$  value, and extract the correction. Apply the  $d$  correction, either adding or subtracting (as determined earlier by inspection of the tabulated declination values), to the declination from the daily page to find the declination of the planet at the given time.

The correction table for GHA of planets is based upon the mean rate of the Sun,  $15^\circ$  per hour. The  $v$  value is the difference between  $15^\circ$  and the average hourly change of GHA of the planet on the middle day of the three shown. The  $d$  value is the average hourly amount the declination changes on the middle day. Venus is the only body listed which ever has a negative  $v$  value.

**Air Almanac:** Enter the daily page with the whole  $10^m$  before the given GMT, unless this time is a whole  $10^m$ , and extract the tabulated GHA and declination, without interpolation. The tabulated declination is correct for the time  $30^m$  later than tabulated, so interpolation during the hour following tabulation is not needed for most purposes. Next, enter the “Interpolation of GHA” table on the inside front cover, using the “Sun, etc.” column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction half a line above the entry time. Add this correction to the GHA

from the daily page to find the GHA at the given time. No adjustment of declination is needed.

## 1707. Finding GHA and Declination of a Star

If the GHA and declination of each navigational star were tabulated separately, the almanacs would be several times their present size. But since the sidereal hour angle and the declination are nearly constant over several days (to the nearest  $0.1'$ ) or months (to the nearest  $1'$ ), separate tabulations are not needed. Instead, the GHA of the first point of Aries, from which SHA is measured, is tabulated on the daily pages. In the *Nautical Almanac*, a single listing of SHA and declination for the 57 navigational stars is given for each double page (computed at 12 UT1 of the middle day); monthly values are given for 173 bright stars (pages 268 through 273). In the *Air Almanac*, the yearly mean SHA and declinations are listed on the inside cover and flap; for higher accuracy, monthly values are tabulated for the 173 navigation stars (pages A158 through A163). Finding the GHA is similar to finding the GHA of the Sun, Moon, and planets.

**Nautical Almanac:** Enter the daily page table with the whole hour before the given GMT, unless this time is a whole hour, and extract the tabulated GHA of Aries. Also record the tabulated SHA and declination of the star from the listing on the left-hand daily page. Next, enter the increments and corrections table for the minutes of GMT, and, on the line for the seconds of GMT, extract the GHA correction from the Aries column. Add this correction and the SHA of the star to the GHA  $\varphi$  on the daily page to find the GHA of the star at the given time. Subtraction of  $360^\circ$  may be necessary to keep GHA between  $0^\circ$  and  $360^\circ$ . No adjustment of declination is needed.

The SHA and declination of 173 stars, including Polaris and the 57 listed on the daily pages, are given for the middle of each month. For a star not listed on the daily pages, this is the only almanac source of this information. Interpolation in this table is not necessary for ordinary purposes of navigation, but is sometimes needed for precise results.

**Air Almanac:** Enter the daily page with the whole  $10^m$  before the given GMT, unless this is a whole  $10^m$ , and extract the tabulated GHA  $\varphi$ . Next, enter the “Interpolation of GHA” table on the inside front cover, using the “Sun, etc.” entry column, and extract the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line above the entry time. From the tabulation at the left side of the same page, extract the SHA and declination of the star. Add the GHA from the daily page and the two values taken from the inside front cover to find the GHA at the given time. No adjustment of declination is needed. Should higher precision be needed, use the SHA and declination values on pages A158 to A163, and the interpolation of GHA Aries table on A165.

## RISING, SETTING, AND TWILIGHT

### 1708. Rising, Setting, and Twilight

In both *Air* and *Nautical Almanacs*, the times of sunrise, sunset, moonrise, moonset, and twilight information, at various latitudes between 72°N and 60°S, is listed to the nearest whole minute. By definition, rising or setting occurs when the upper limb of the body is on the visible horizon, assuming standard refraction for zero height of eye. Because of variations in refraction and height of eye, computation to a greater precision than 1 minute of time is not justified.

In high latitudes, some of the phenomena do not occur during certain periods. Symbols are used in the almanacs to indicate:

1. Sun or Moon does not set, but remains continuously above the horizon, indicated by an open rectangle.
2. Sun or Moon does not rise, but remains continuously below the horizon, indicated by a solid rectangle.
3. Twilight lasts all night, indicated by 4 slashes (///).

Both the *Nautical Almanac* and the *Air Almanac* provide graphs for finding the times of rising, setting, or twilight in polar regions.

In the *Nautical Almanac*, sunrise, sunset, and twilight tables are given only once for the middle of the three days on each page opening. Moonrise and moonset tables are given for each day. For many purposes this information can be used for all three days. For high precision needs, interpolation tables are provided (page xxxii). In the *Air Almanac*, sunrise, sunset, and twilight tables are given every three days (pages A130-A145). Graphs and tables are provided to compute phenomena at altitudes up to 60,000 feet. Moonrise and moonset tables are given daily in the main table.

The tabulations are in UT on the Greenwich meridian. They are approximately the LMT of the corresponding phenomena on other meridians; they can be formally interpolated if desired. The conversion of UT to LMT and vice versa of a phenomenon is obtained by the formula:

$$UT = LMT + W \text{ Longitude}$$

$$UT = LMT - E \text{ Longitude}$$

To use this formula, convert the longitude to time using the table on page i or by computation, and add or subtract as indicated.

### 1709. Finding Times of Sunrise and Sunset

To find the time of sunrise or sunset in the *Nautical Almanac*, enter the table on the daily page, and extract the LMT for the latitude next smaller than your own (unless it is exactly the same). Apply a correction from Table I on almanac page xxxii to interpolate for latitude, determining

the sign by inspection. Then convert LMT to ZT using the difference of longitude between the local and zone meridians.

For the *Air Almanac*, the procedure is the same as for the *Nautical Almanac*, except that the LMT is taken from the tables of sunrise and sunset instead of from the daily page, and the latitude correction is by linear interpolation.

The tabulated times are for the Greenwich meridian. Except in high latitudes near the time of the equinoxes, the time of sunrise and sunset varies so little from day to day that no interpolation is needed for longitude. In high latitudes interpolation is not always possible. Between two tabulated entries, the Sun may in fact cease to set. In this case, the time of rising and setting is greatly influenced by small variations in refraction and changes in height of eye.

### 1710. Twilight

Morning twilight ends at sunrise, and evening twilight begins at sunset. The time of the darker limit can be found from the almanacs. The time of the darker limits of both civil and nautical twilights (center of the Sun 6° and 12°, respectively, below the celestial horizon) is given in the *Nautical Almanac*. The *Air Almanac* provides tabulations of civil twilight from 60°S to 72°N. The brightness of the sky at any given depression of the Sun below the horizon may vary considerably from day to day, depending upon the amount of cloudiness, haze, and other atmospheric conditions. In general, the most effective period for observing stars and planets occurs when the center of the Sun is between about 3° and 9° below the celestial horizon. Hence, the darker limit of civil twilight occurs at about the mid-point of this period. At the darker limit of nautical twilight, the horizon is generally too dark for good observations.

At the darker limit of astronomical twilight (center of the Sun 18° below the celestial horizon), full night has set in. The time of this twilight is given in the *Astronomical Almanac*. Its approximate value can be determined by extrapolation in the *Nautical Almanac*, noting that the duration of the different kinds of twilight is proportional to the number of degrees of depression for the center of the Sun. More precise determination of the time at which the center of the Sun is any given number of degrees below the celestial horizon can be determined by a large-scale diagram on the plane of the celestial meridian, or by computation. Duration of twilight in latitudes higher than 65°N is given in a graph in both the *Nautical* and the *Air Almanac*.

In both *Nautical* and *Air Almanacs*, the method of finding the darker limit of twilight is the same as that for sunrise and sunset.

Sometimes in high latitudes the Sun does not rise but twilight occurs. This is indicated in the almanacs by a solid

black rectangle symbol in the sunrise and sunset column. To find the time of beginning of morning twilight, subtract half the duration of twilight as obtained from the duration of twilight graph from the time of meridian transit of the Sun; and for the time of ending of evening twilight, add it to the time of meridian transit. The LMT of meridian transit never differs by more than 16.4<sup>m</sup> (approximately) from 1200. The actual time on any date can be determined from the almanac.

### 1711. Moonrise and Moonset

Finding the time of moonrise and moonset is similar to finding the time of sunrise and sunset, with one important difference. Because of the Moon's rapid change of declination, and its fast eastward motion relative to the Sun, the time of moonrise and moonset varies considerably from day to day. These changes of position on the celestial sphere are continuous and complex. For precise results, it would be necessary to compute the time of the phenomena at any given place by lengthy complex calculation. For ordinary purposes of navigation, however, it is sufficiently accurate to interpolate between consecutive moonrises or moonsets at the Greenwich meridian. Since apparent motion of the Moon is westward, relative to an observer on the Earth, interpolation in west longitude is between the phenomenon on the given date and the following one. In east longitude it is between the phenomenon on the given date and the preceding one.

To find the time of moonrise or moonset in the *Nautical Almanac*, enter the daily pages table with latitude and extract the LMT for the tabulated latitude next smaller than the observer's latitude (unless this is an exact tabulated value). Apply a correction from table I of almanac page xxxii to interpolate for latitude, determining the sign of the correction by inspection. Repeat this procedure for the day following the given date, if in west longitude; or for the day preceding, if in east longitude. Using the difference between these two times, and the longitude, enter table II of the almanac on the same page and take out the correction. Apply this correction to the LMT of moonrise or moonset at the Greenwich meridian on the given date to find the LMT at the position of the observer. The sign to be given the correction is such as to make the corrected time fall between the times for the two dates between which interpolation is being made. This is nearly always positive (+) in west longitude and negative (-) in east longitude. Convert the corrected LMT to ZT.

To find the time of moonrise or moonset by the *Air Almanac* for the given date, determine LMT for the observer's latitude at the Greenwich meridian in the same manner as with the *Nautical Almanac*, except that linear interpolation is made directly from the main tables, since no interpolation table is provided. Extract, also, the value from the "Diff." column to the right of the moonrise and moonset column, interpolating if necessary. This "Diff." is the half-daily difference. The error introduced by this approxi-

mation is generally not more than a few minutes, although it increases with latitude. Using this difference, and the longitude, enter the "Interpolation of moonrise, moonset" table on flap F4 of the *Air Almanac* and extract the correction. The *Air Almanac* recommends taking the correction from this table without interpolation. The results thus obtained are sufficiently accurate for ordinary purposes of navigation. If greater accuracy is desired, the correction can be taken by interpolation. However, since the "Diff." itself is an approximation, the *Nautical Almanac* or computation should be used if accuracy is a consideration. Apply the correction to the LMT of moonrise or moonset at the Greenwich meridian on the given date to find the LMT at the position of the observer. The correction is positive (+) for west longitude, and negative (-) for east longitude, unless the "Diff." on the daily page is preceded by the negative sign (-), when the correction is negative (-) for west longitude, and positive (+) for east longitude. If the time is near midnight, record the date at each step, as in the *Nautical Almanac* solution.

As with the Sun, there are times in high latitudes when interpolation is inaccurate or impossible. At such periods, the times of the phenomena themselves are uncertain, but an approximate answer can be obtained by the Moonlight graph in the almanacs. With the Moon, this condition occurs when the Moon rises or sets at one latitude, but not at the next higher tabulated latitude. It also occurs when the Moon rises or sets on one day, but not on the preceding or following day. This latter condition is indicated in the *Air Almanac* by the symbol \* in the "Diff." column.

Because of the eastward revolution of the Moon around the Earth, there is one day each synodical month (29<sup>1</sup>/<sub>2</sub> days) when the Moon does not rise, and one day when it does not set. These occur near last quarter and first quarter, respectively. This day is not the same at all latitudes or at all longitudes, thus the time of moonrise or moonset found from the almanac may occasionally be the preceding or succeeding one to that desired (indicated by a time greater than 23<sup>h</sup> 59<sup>m</sup>). When interpolating near midnight, caution will prevent an error.

The effect of the revolution of the Moon around the Earth, generally, is to cause the Moon to rise or set later from day to day. The daily retardation due to this effect does not differ greatly from 50<sup>m</sup>. However, the change in declination of the Moon may increase or decrease this effect. This effect increases with latitude, and in extreme conditions it may be greater than the effect due to revolution of the Moon. Hence, the interval between successive moonrises or moonsets is more erratic in high latitudes than in low latitudes. When the two effects act in the same direction, daily differences can be quite large. When they act in opposite directions, they are small, and when the effect due to change in declination is larger than that due to revolution, the Moon sets earlier on succeeding days.

This condition is reflected in the *Air Almanac* by a negative “Diff.” If this happens near the last quarter or first quarter, two moonrises or moonsets might occur on the same day, one a few minutes after the day begins, and the other a few minutes before it ends. Interpolation for longitude is always made between consecutive moonrises or moonsets, regardless of the days on which they fall.

Beyond the northern limits of the almanacs the values can be obtained from a series of graphs given near the back of the books (pages 322-325 for *Nautical*, A153-A157 for *Air*). For high latitudes, graphs are used instead of tables because graphs give a clearer picture of conditions, which may change radically with relatively little change in position or date. Under these

conditions interpolation to practical precision is simpler by graph than by table. In those parts of the graph which are difficult to read, the times of the phenomena's occurrence are uncertain, being altered considerably by a relatively small change in refraction or height of eye.

On all of these graphs, any given latitude is represented by a horizontal line and any given date by a vertical line. At the intersection of these two lines the duration is read from the curves, interpolating by eye between curves; see Figure 1711a for an example of a Semiduration of Moonlight plot for the month of January 2016.

### SEMIDURATION OF MOONLIGHT 2016

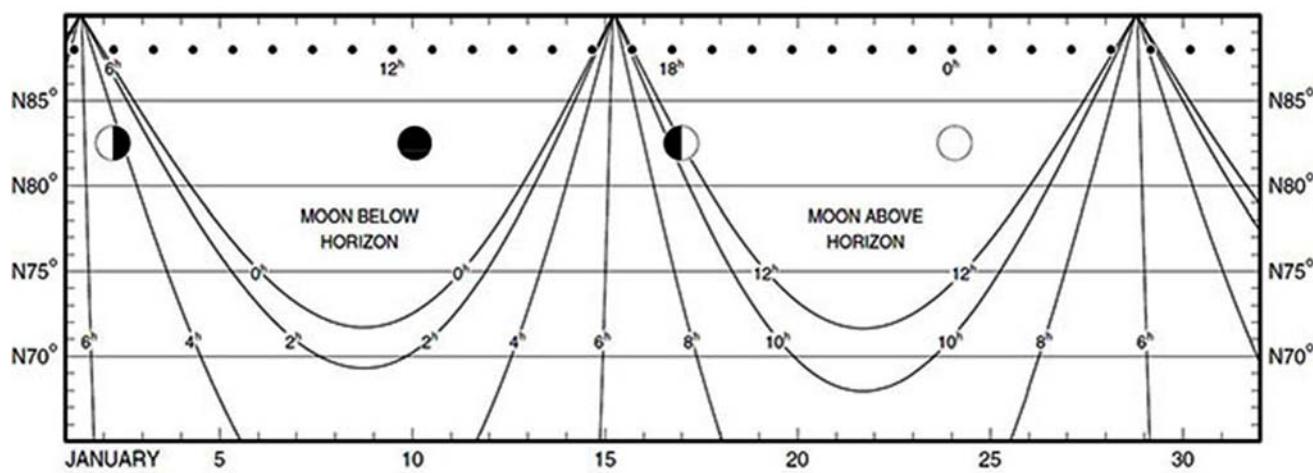


Figure 1711a. Semiduration of moonlight for high latitudes in January 2016.

The “Semiduration of Sunlight” graph gives the number of hours between sunrise and meridian transit or between meridian transit and sunset. The dot scale near the top of the graph indicates the LMT of meridian transit, the time represented by the minute dot nearest the vertical dateline being used. If the intersection occurs in the area marked “Sun above horizon,” the Sun does not set; and if in the area marked “Sun below horizon,” the Sun does not rise.

The “Duration of Twilight” graph gives the number of hours between the beginning of morning civil twilight (center of Sun 6° below the horizon) and sunrise, or between sunset and the end of evening civil twilight. If the Sun does not rise, but twilight occurs, the time taken from the graph is half the total length of the single twilight period, or the number of hours from beginning of morning twilight to LAN, or from LAN to end of evening twilight. If the intersection occurs in the area marked “continuous twilight or Sunlight,” the center of the Sun does not move more than 6° below the horizon, and if in the area marked “no twilight nor Sunlight,” the Sun remains more than 6°

below the horizon throughout the entire day.

The “Semiduration of Moonlight” graph gives the number of hours between moonrise and meridian transit or between meridian transit and moonset. The dot near the top of the graph indicates the LMT of meridian passage, and the spacing between each dot is approximately 50 minutes. The phase symbols indicate the date on which the principal Moon phases occur, the open circle indicating full Moon and the dark circle indicating new Moon. If the intersection of the vertical dateline and the horizontal latitude line falls in the “Moon above horizon” or “Moon below horizon” area, the Moon remains above or below the horizon, respectively, for the entire 24 hours of the day.

If approximations of the times of moonrise and moonset are sufficient, the semiduration of Moonlight is taken for the time of meridian passage (dots along top scale) and can be used without adjustment. For example, to estimate moonrise on 19 January 2016 at latitude 70°N and the following moonset, see Figure 1711b. Using the dot along the top scale, the semiduration of

moonlight is 10h at 70°N. The meridian passage itself is about at 20:30 LMT, found by adding 50 minutes to each successive dot after the 18h one. Approximate moonrise is the semiduration minus meridian passage, 10h - 20:30, or at 10:30 LMT. The following moonset is semiduration plus meridian passage, 10h + 20:30, or at 06:30 the following day. For more accurate results

(seldom justified), the times on the required date and the adjacent date (the following date in W longitude and the preceding date in E longitude) should be determined, and an interpolation made for longitude, as in any latitude, since the intervals given are for the Greenwich meridian.

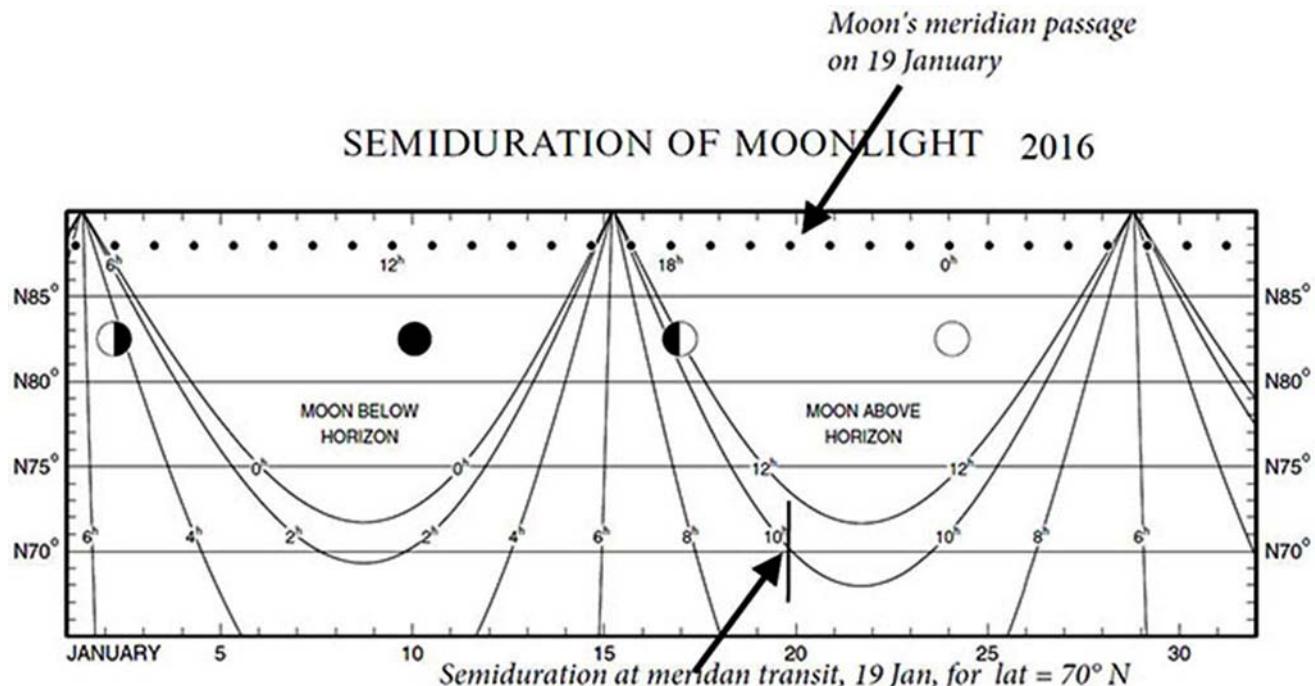


Figure 1711b. Moon's meridian passage on 19 January 2016.

Sunlight, twilight and Moonlight graphs are not given for south latitudes. Beyond latitude 65°S, the northern hemisphere graphs can be used for determining the semiduration or duration, by using the vertical dateline for a day when the declination has the same numerical value but opposite sign. The time of meridian transit and the phase of the Moon are determined as explained above, using the correct date. Between latitudes 60°S and 65°S, the solution is made by interpolation between the tables and the graphs.

Semiduration or duration can be determined graphically using a diagram on the plane of the celestial meridian, or by computation. When computation is used, solution is made for the meridian angle at which the required negative altitude occurs. The meridian angle expressed in time units is the semiduration in the case of sunrise, sunset, moonrise, and moonset; and the semiduration of the combined Sunlight and twilight, or the time from meridian transit at which morning twilight begins or evening twilight ends. For sunrise and sunset the altitude used is (-)50'. Allowance for height of eye can be made by algebraically subtracting (numerically adding) the dip correction from this altitude. The altitude used for twilight is (-)6°, (-)12°, or (-)18°

for civil, nautical, or astronomical twilight, respectively. The altitude used for moonrise and moonset is  $-34' - SD + HP$ , where SD is semidiameter and HP is horizontal parallax, from the daily pages of the *Nautical Almanac*.

Other methods of solution of these phenomena are available. If an internet connection is available, the US Naval Observatory website provides calculators ([aa.usno.navy.mil/data/](http://aa.usno.navy.mil/data/)). Sunrise and sunset for latitudes from 76°N to 60°S can be derived using Table 4 of NOAA's *Tide Tables* publications.

## 1712. Rising, Setting, and Twilight on a Moving Craft

Instructions to this point relate to a fixed position on the Earth. Aboard a moving craft the problem is complicated somewhat by the fact that time of occurrence depends upon the position of the craft, which itself depends on the time. The US military can use STELLA, which calculates phenomena from a moving platform (see Section 1900), for others, at ship speeds, it is generally sufficiently accurate to make an approximate mental solution and use the position of the vessel at this time to make a more accurate solution. If

greater accuracy is required, the position at the time indicated in the second solution can be used for a third solution. If desired, this process can be repeated until the same answer is obtained from two consecutive solutions. However, it is generally sufficient to alter the first solution by  $1^{\text{m}}$  for each  $15'$  of longitude that the

position of the craft differs from that used in the solution, adding if west of the estimated position, and subtracting if east of it. In applying this rule, use both longitudes to the nearest  $15'$ . The first solution is the **first estimate**; the second solution is the **second estimate**.

# CHAPTER 18

## SIGHT PLANNING

### NEED FOR SIGHT PLANNING

#### 1800. The Need for Sight Planning

One of the challenges of celestial navigation is sight planning. Good sight planning is essential to acquiring a good fix.

A single sight produces a **line of position (LOP)**. A fix, the determination of the observer's most likely position, requires at minimum two LOPs. The fix is the intersection of the LOPs. If the sights were perfectly accurate, then no further work would be required. However, no observation is perfectly accurate. A navigator experienced in taking sights with a sextant can, under ideal conditions, take sights accurate to a few tenths of an arc minute. Under typical shipboard conditions sights are expected to be accurate to about an arc minute. A navigator not experienced at taking sights can expect an accuracy of a few arc minutes. Sight planning is one tool for reducing errors in producing a fix. Good sight planning will reduce the effect of the errors from both taking the sights and on the derived fix.

There are several considerations that go into sight planning. Among them are:

- When should sights be taken?
- What bodies will be visible?
- What distribution of celestial bodies will produce the best fix?
- What is the best order in which to take the sights?

The process of sight planning can be broken down into three broad categories: general sight planning, daytime sight planning, and twilight sight planning.

#### 1801. General Principles

Experienced navigators through history have come to understand that under normal conditions only about 70% of the visible sky is ideal for taking celestial observations. When it comes to sight planning it is important to appreciate that celestial body pre-selection will yield the best chance for achieving an accurate celestial fix of position when they come from bodies observed within certain altitude ranges and at certain times. For example, it is useful to understand that when selecting celestial bodies for observation, it can be difficult to accurately determine the true altitude of bodies lying low near the horizon due to refraction, while it can be equally daunting to accurately

determine a celestial LOP from a body near zenith because the assumption that a straight line of position approximates the body's circle of equal altitude begins to breakdown.

Except for sights of the Sun, Moon, and sometimes Venus and Jupiter, all other bodies used in celestial navigation sights can be measured only during nautical twilight, the period during which the center of the Sun is between 6° and 12° below the horizon. During this period the sky is dark enough to make out the celestial bodies used for sights, but bright enough that the horizon is well enough defined to take an accurate sight.

The process of taking sights is weather dependent. An accurate sight requires that both the body and the horizon below it be visible at the time the sight is taken. If either the body or the horizon is obscured, but still visible, a reduced accuracy sight may still be taken. Such a sight should be taken if it is the only option. A better option is to have an extended list of possible bodies to observe. The navigator can then select those bodies that are clearly visible with a well defined horizon below them.

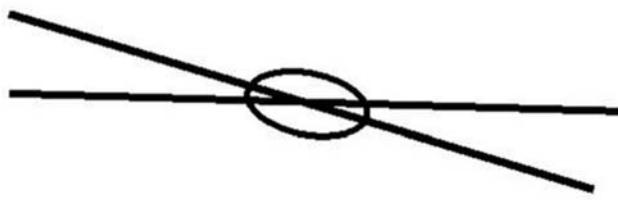
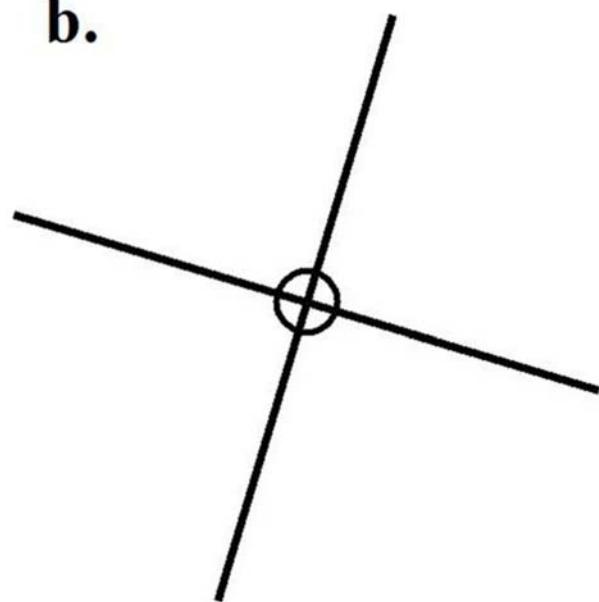
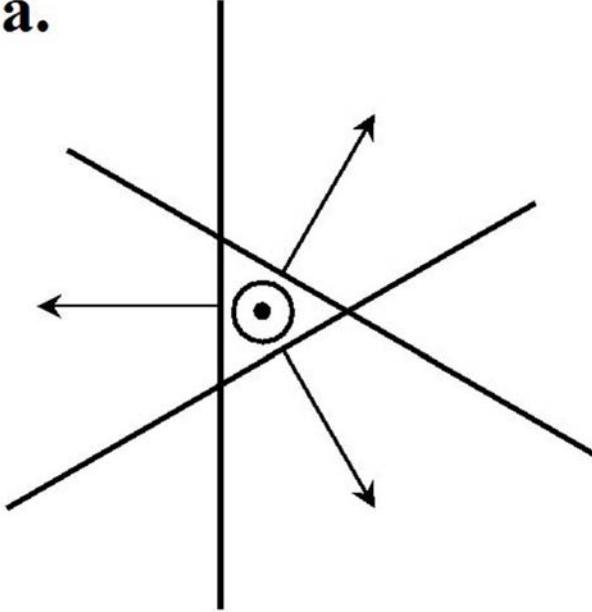
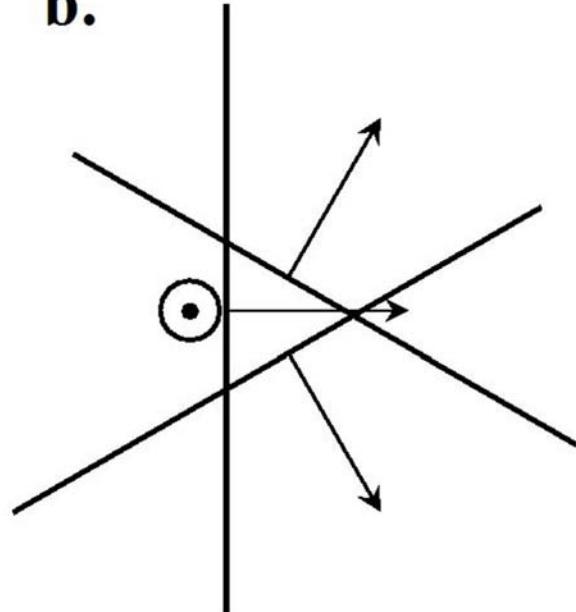
The process of sight planning can be broken down into three broad categories: general sight planning, daytime sight planning, and twilight sight planning.

#### 1802. Distribution of Bodies in Azimuth

The *Nautical Almanac* contains data for reducing sights of 179 bodies: the Sun, Moon, four planets, 57 navigational stars, and 116 supplemental stars (pp. 268-273). On average, there is one body for every 230 square degrees of sky, and these bodies are unevenly distributed on the sky. An accurate fix requires the observed bodies to be well distributed in azimuth.

Figure 1802a shows two LOPs for two objects whose azimuths are separated by 15°. The LOPs also intersect with an acute angle of 15°. The result is: it is difficult to determine where the two LOPs cross along the axis bisected by the acute angle. That is, there is a large uncertainty in the fix position in that direction. The uncertainty in the fix along the axis bisected by the oblique angle is approximately the same as it would be if the LOPs met at right angles.

Figure 1802b the two LOPs are perpendicular to each other. The result is that the uncertainty in the fix is the same in all directions. The closer the separation of the azimuths of two sights comes to perpendicularity the better the chance a fix will have minimum uncertainty. Finding two

**a.***LOPs separated by 15°.***b.***LOPs separated by 90°.**Figure 1802a. The change of error ellipse with angle of intersection.***a.***Azimuths separated by 120°.***b.***Azimuths separated by 60°.**Figure 1802b. Effects of the azimuthal distribution of bodies and a systematic error on the most likely positions.*

bodies with azimuths separated by exactly  $90^\circ$  is unlikely, so an acute angle of at least  $30^\circ$  is recommended to reduce the uncertainty along the axis that bisects the acute angle of two LOPs.

Sights well distributed in azimuth also act to cancel out systematic errors in determining  $H_o$  such as an incorrect index correction (IC) or error in dip. For example, Figure 1802a shows three LOPs made from bodies separated by  $120^\circ$  in azimuth. A systematic error in determining  $H_o$  will move the LOPs in a direction perpendicular to the LOPs themselves, indicated by the arrows. A systematic error will move all of the LOPs the same amount in directions distributed  $120^\circ$  in azimuth. The result is the most likely position for the fix remains at the center of the “cocked hat”. In Figure 1802b, the three LOPs were plotted from bodies distributed by  $60^\circ$  in azimuth. The resulting “cocked hat” looks identical to the one in Figure 1802a. A systematic error, however, will move all of the LOPs the same amount in directions distributed in azimuth by  $60^\circ$  on either side of the center of the distribution. The result is that the most likely position for the fix is no longer at the center of the “cocked hat”. The most likely position may even lie outside of the “cocked hat” altogether if the systematic error is more than a few tenths of an arc minute.

### 1803. Altitude of Bodies

Bodies at high altitudes are difficult to observe. They can be a challenge to acquire, to “bring to the horizon” with a sextant, and to determine their approximate azimuth to measure an accurate  $H_s$ . As the body gets closer to the zenith the assumption that the circle of equal distances can be approximated by an LOP breaks down. Sights of a body taken at high altitudes may also require the use of more complicated procedures, such as the use of second differences when calculating  $H_c$ . Taking sights of body at high altitudes, greater than  $75^\circ$ , should be avoided for these reasons.

Refraction affects all observations. Refraction forms part of the corrections for both dip and apparent altitude. Refraction is larger and the correction becomes more uncertain for bodies near the horizon. The correction for non-standard air temperature and pressure can be more than  $1'$  for a sight made within  $5^\circ$  of the horizon and can still be several tenths of an arc minute for a sight made within  $10^\circ$  of the horizon.

The amount of atmosphere the light has to pass through for a body observed near the horizon is greater than for a body observed at a greater altitude. A body viewed near the horizon will appear dimmer and redder because the light is absorbed or scattered by the atmosphere. Taking sights of bodies at low altitudes, less than approximately  $15^\circ$ , should be avoided for these reasons.

Correcting for non-standard air pressure and temperature does not guarantee that a sight will have no refraction error. The apparent position of the horizon itself is subject

to phenomena such as temperature inversions. There are three things a navigator can do to reduce any systematic errors caused by uncorrected refraction:

1. Make sure the observations are well distributed in azimuth. At sea, it is usually the case that the factors that contribute to refraction are similar in all directions. Taking sights well distributed in azimuth will cause the systematic errors to cancel out.
2. Take the sights from a place close to the sea surface, if possible. Almost all of the abnormal refraction encountered is caused by that part of the atmosphere between the observer's eye and the surface of the sea. Reducing the observer's height decreases the distance to the horizon. An observer close to the sea surface will have a nearby horizon, which is more likely to have similar refraction conditions in all directions.
3. Observe celestial bodies with similar altitudes, all greater than  $15^\circ$ . Bodies at the same altitude have the same total values for refraction. So, the systematic effect of errors in computed refraction will tend to cancel out if the bodies are well distributed in azimuth. The change in refraction angle is small, except near the horizon, so relative altitude is a minor consideration when choosing which bodies to use.

### 1804. Brightness of Bodies

One source of systematic error is the **personal equation**, that is how the individual judges the position of an object that does not appear to be a perfect point. This judgment of position varies from individual to individual. One person might tend to favor an “upper edge”, while another favors a “lower edge”, etc.

A bright object always appears somewhat larger than a dim one with a similar apparent size, seen against the same background. This property is called *irradiation*, and is a result of the way the brain interprets what it sees. Irradiation depends more on the difference in brightness between object and background than on the apparent size of the body. So, it is particularly striking for point sources such as stars. If possible, select bodies that are approximately the same brightness, to minimize the effect of personal error arising from irradiation. This effect is usually small, so it is of less importance than other considerations in the selection of bodies for sights.

### 1805. Number of Sights and Number of Bodies

One method to reduce the random error in determining an LOP is to take a number of observations of the same body over a short period of time. Averaging these observations together into a single sight, taking into account the

change in  $H_s$  with time, reduces random error with the square root of the number of observations. Averaging four observations into a sight reduces the random error to one-half that of a single observation sight and averaging nine observations into a sight reduces the random error to one-third that of a single observation. The incremental reduction in the random error quickly diminishes with the number of sights. Averaging together four or five observations into a single sight is about optimum.

The common method of averaging sights is the fit-slope method, e.g. Burch, D. 2015, *Celestial Navigation, Second Edition* (Seattle, Starpath Pub.) pp. 176-177. The fit-slope method is a graphical approximation of a linear least-squares method, e.g. Bevington, P.R. and Robinson, D.K. 1992, *Data Reduction and Error Analysis for the Physical Sciences*, (Boston, McGraw-Hill), Chapter 6. These methods assume that the rate of change of  $H_s$  is constant. Usually, this is a good assumption over a single sight session. But if the body is near transit,  $Zn = 0^\circ$  or  $Zn = 180^\circ$ , the rate of change of  $H_s$  may be changing quickly. In this case these linear methods will fail. But if the observed body does transit during the sighting session, the vessel's position can be determined using the same method to determine LAN (Sections 1910 and 1911).

Another way to perform this task is to reduce all the observations made of the same object at an observing session as individual sights, and then average together the resulting values for  $a$  and  $Zn$ . This second method consumes more time in the reduction of the individual observations, but it removes the difficulty of accounting for the change in  $H_s$  with time required for averaging together the observations.

It is preferable to take observations in a round-robin fashion when taking sights of more than one body at a session. Taking consecutive observations of different bodies helps assure that all the bodies are observed at least once should a sudden change in weather put an end to the observation of one or more bodies. Taking non-consecutive observations of a body helps to remove systematic errors in its observations by adding a randomizing factor to the sight taking.

Taking sights of more than two bodies can significantly reduce the random error of a fix just as taking more than a single observation can reduce the random error in an *LOP*.

There are two parameters to be determined, latitude and longitude, involved in a fix. So the random error of a fix is reduced by the square root of the number of sights minus two. Determining a fix from five or six sights is about optimum to reduce the random error in a fix

By averaging a number of observations into a single sight and then combining it with other sights into a single fix, the navigator can significantly reduce the uncertainty of the vessel's position.

The difference of a course of advance and the track made good for a running fix results in a less accurate fix than one made from taking sights of two bodies at a single observing session. The best method for reducing error in a running fix is to average together multiple observations, particularly those of the latter observing session, to improve the accuracy of the *LOP*.

## 1806. Precomputation

Precomputation is the practice of determining the predicted values of phenomena using estimated values for the time and position and data from the almanac. Precomputed values usually include times of rise and set of the Sun and Moon, the time of local apparent noon, the times and duration of twilight, and the  $H_c$ 's and  $Zn$ 's of those bodies being considered for sight observations.

Precomputing the  $H_c$  and  $Zn$  of a body for a sight serves two purposes:

1. It determines if the selected bodies provide a good distribution in azimuths. For a running fix using a single body, it determines how much time must elapse between sights to get an acceptable minimum change in azimuth of the body.
2. It eases the process of identification. Set the sextant to the precomputed  $H_c$  and face the precomputed  $Zn$ . The chosen object will usually stand out in the reflection of the index mirror when the horizon is viewed through the horizon glass. This practice is particularly helpful in a crowded star field at twilight or when trying to pick out Venus, or occasionally Jupiter, against the bright daytime background.

## DAYLIGHT SIGHT PLANNING

### 1807. Sun Sights

The principal activity of daylight celestial navigation is sighting the Sun to determine a vessel's position from running fixes and latitude from  $H_o$  at local apparent noon. Precomputing the Sun's expected  $H_c$  and  $Zn$  at various times throughout the day makes it possible to determine the optimum times to take sights for both of these activities.

For example, in the Torrid Zone (tropics) the Sun's azimuth changes slowly for most of the morning and most of

the afternoon switching rapidly from east to west around local apparent noon. To achieve a good running fix, sights need to be obtained before, near-to, and after local apparent noon. Near the equator, the change in azimuth is within  $30^\circ$  of  $180^\circ$  from February through April and August through October. During these periods a sight near local apparent noon (when the Sun's azimuth is near  $0^\circ$  or  $180^\circ$ ) is essential for a good running fix. At high latitudes (north or south), on the other hand, the motion of the Sun is mostly in azimuth, at approximately  $15^\circ/\text{hr}$ . So, a good running fix from the

Sun can be made from two sights as long as at least two and fewer than ten hours have elapsed between sights and the Sun is high enough above the horizon to take an accurate sight.

Occasionally, it is necessary to take a Sun sight when it is near the horizon, to make a compass check for example. Precomputing the time and  $Zn$  of sunrise or sunset are useful to provide an approximate time and azimuth for making such an observation. If the Sun is more than one or two degrees above the horizon, an accurate sight for  $Hs$  as well as  $Zn$  can be determined as long as corrections are made for the change in refraction from non-standard temperature and air pressure. The upper limb of the Sun can be observed to further reduce possible complications from non-standard refraction.

### 1808. Moon Sights

When the Moon is more than a few days from New Moon it is bright enough to be easily visible during the daytime. It is also well separated from the Sun. It is best situated for daytime sights around the times of First Quarter (age 6 to 8 days) and Last Quarter (age 21 to 23 days). Near Full Moon the Sun and Moon are opposite each other in the

sky, so the resulting *LOPs* may be nearly parallel and the resulting fix would be poor. Instead, sights of the Full Moon should be combined with sights of celestial bodies other than the Sun.

It is more difficult to observe and make accurate measurements of the dark side of the Moon than of its bright side. Select the lighted limb when taking sights, and avoid taking sights when the “horns” of either the lighted or unlighted side point parallel to the horizon as in Figure 1808.

The local times of moonrise and moonset at  $0^\circ$  longitude are tabulated as a function of latitude for each day in the daily pages of the *Nautical Almanac*. The tabulation interval is  $10^\circ$  from the equator to latitude  $30^\circ$ ,  $5^\circ$  from latitude  $30^\circ$  to latitude  $50^\circ$ , and  $2^\circ$  from latitude  $50^\circ$  to the limit for each hemisphere. Times of moonrise and moonset at high northern latitudes,  $65^\circ$  N to the North Pole, can be estimated using the semiduration of moonlight graph on pages 323 through 325 of the *Nautical Almanac*. Interpolation in both latitude and change in time of the phenomenon with longitude need to be performed to determine the LMT of moonrise or moonset. The Moon's phase and age at  $12^h$  UT for each day are also tabulated on the daily pages.

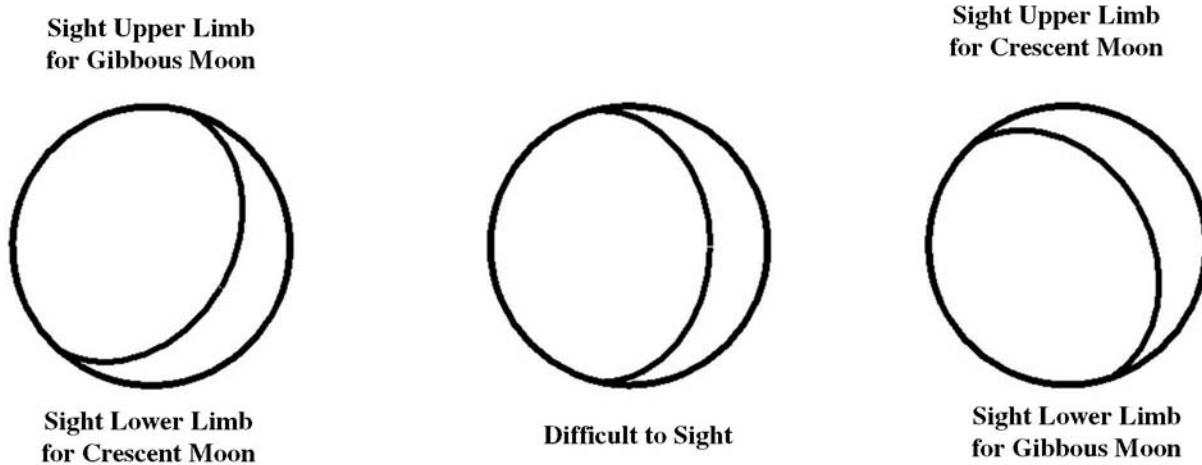


Figure 1808. Where to sight Moon with phase.

### 1809. Planet Sights

Venus can be observed during the daytime when it is well separated from the Sun, particularly when its altitude is greater than the Sun's. Jupiter can also occasionally be observed during the daytime. Both planets can be observed immediately after sunset or before sunrise rather than waiting for nautical twilight. The best way to find Venus against the bright daytime sky is to precompute its  $Hc$  and  $Zn$ , set the sextant for the expect-

ed altitude, and then use a compass to view along the expected azimuth.

The navigational planets move against the backdrop of the “fixed” stars from night to night, but their motions are small enough that they can be found in the same general area of the sky for several weeks. Also, they are bright enough to be easily identifiable. One way to take advantage of these properties when using an aid such as a star finder is to mark the planets positions at the expected middle of a voyage.

## TWILIGHT SIGHT PLANNING

### 1810. Determining the Period of Twilight

Good sight planning is essential to make good use of the short period of nautical twilight for taking sights and minimize errors. Sight planning for twilight observations consists of three tasks:

1. Determine the period of nautical twilight.
2. Select the celestial bodies to be observed.
3. Determine the order in which to observe the bodies.

The length of the period of nautical twilight is a function of latitude and time of year. For most practical celestial navigation work, it lasts between 24 minutes in the tropics to an hour or more at high latitudes (near the poles, twilight can last days or weeks). Local weather conditions such as clouds and fog may significantly modify the period during which sights may be taken. During the period of nautical twilight only the brightest celestial bodies are visible.

The daily pages of the *Nautical Almanac* tabulate the LMT of beginning of morning nautical and civil twilight and the ending of evening civil and nautical twilights, to the nearest minute, for the middle of each three-day period from N 72° to S 60°. The tabulation interval is 10° from the equator to latitude 30°, 5° from latitude 30° to latitude 50°, and 2° from latitude 50° to the limit for each hemisphere. Times of twilight at high northern latitudes, 65° N to the North Pole, can be estimated using the semiduration of sunlight graph on page 322 of the *Nautical Almanac*. These intervals are adequate to interpolate the LMT twilight times to the DR latitude. It is advisable to also interpolate the times of twilight between the values on the current page and either the preceding or subsequent page if:

1. the latitude is greater than 20°,
2. the time of the phenomenon is more than 18 hours from the UT of the middle of the three-day interval, and
3. the date is within two months of either the vernal equinox (March 21) or the autumnal equinox (September 23).

### 1811. Twilight Moon Sights

When the Moon is between about 5 and 24 days old it is bright enough that it visibly lights the sea surface near the Moon's azimuth. Confusion between the horizon and the glint of moonlight off of the sea surface closer to the observer may occur at these times. A sight taken where the lighted sea is mistaken for the horizon will result in a value of  $H_s$  that is too high. To reduce this problem, twilight sights of the Moon or other bodies with a similar azimuth should be taken, if possible, shortly after sunset or before sunrise when the horizon is easily distinguishable and the

glare of moonlight is minimal. If a sight must be taken when there is significant glare:

- Observe from a position near the sea surface. A sight taken near the sea surface has a closer horizon, so the effect of the glare off the sea surface is minimized.
- Check the horizon under the Moon with a powerful pair of binoculars to determine if the glare extends to the apparent horizon.

### 1812. Selection of the Celestial Bodies for Sights

The most important consideration in selecting bodies for a fix is to ensure that the bodies are well distributed in azimuth. A fix from twilight observations alone requires sights of a minimum of two celestial bodies. Separating the bodies by at least 30° degrees in azimuth is desired to improve the acute angle of the intersection between LOPs. A fix made from at least three bodies that are well distributed in azimuth minimizes systematic errors in determining  $H_o$ . Observing four to six bodies significantly reduces the uncertainty of a fix. Precomputing the approximate altitudes and azimuths for eight to ten bodies will provide a sufficient buffer for weather and other obstructions to observing.

Another important factor to consider is that bright bodies are much easier to identify during early twilight when the horizon is still sharp. Venus and Jupiter, when available, are among the brightest objects in the sky, so they should be among the first bodies chosen. The Moon is also easy to identify, but is not always a good target. It should be used when either the upper or lower limb is well defined (the Moon's "horns" are not parallel to the horizon) and the glint of moonlight on the sea surface is not bright enough to cause a problem in determining the location of the horizon.

A third consideration is to select bodies with an altitude greater than 15° to minimize systematic errors in refraction, and with an altitude less than 75° to prevent errors arising from the break down in the approximation that an LOP is equivalent to a circle of equal altitude. Select bodies that are at a similar altitude and of a similar brightness to further minimize systematic errors in taking sights

### 1813. Order of Observation

Take sights in a round-robin fashion, when possible. A number of individual observations of each body is desirable, but taking consecutive observations of different bodies helps assure that at least one observation is made of each body in case there is a sudden change in the weather or the horizon becomes obscured. Taking non-consecutive sights of a body adds an element of randomness preventing systematic errors from creeping into the observations.

Brighter bodies are visible earlier during evening twilight and later during morning twilight. The Moon, Venus, and Ju-

piter can be observed before sunset or after sunrise, and the brightest stars can be observed during civil twilight. Sights of these objects made during these periods are more likely to have a well defined horizon, and allows more time for taking sights of dimmer stars and navigational planets during nautical twilight. Making observations of the brighter bodies during civil twilight can be particularly helpful in the Torrid Zone (tropics) where the length of nautical twilight is less than half an hour.

During twilight, the horizon remains well defined near the azimuth of sunset or sunrise for a longer period of time than it does 180° away from that azimuth. Plan to take sights closer to that azimuth later during evening twilight and earlier during morning twilight. Precomputing the approximate azimuth of sunrise or sunset from the data in daily pages of the *Nautical Almanac* can aid in planning.

## AIDS TO SIGHT PLANNING

### 1814. Aids to Sight Planning

There are a number of aids to help the navigator in sight planning:

The *Nautical Almanac* contains a planet location diagram on pp. 8 and 9, and star charts on pp. 266 and 267.

The *Air Almanac* contains a set of sky diagrams on pp. A26-A121. These diagrams show the altitudes and bearings of the Sun, Moon, navigational planets and stars at selected hours of the day, throughout the year, and for various latitudes. Each set includes diagrams for the North Pole and latitudes from 75° N to 50° S at an interval of 25°. A complete explanation of the sky diagrams is found on pages A24 and A25. The *Air Almanac* also includes a moonlight interference diagram on page A125 and star recognition diagrams for 40 (22 in the northern hemisphere and 18 in the southern hemisphere) of the 57 navigational stars on pp. A126-A129. Both sets of diagrams include instructions for their use.

**STELLA** (System To Estimate Latitude and Longitude Astronomically) is a software application for Windows computers that automates the sight reduction process. It includes a sight planning utility. STELLA also automatically logs all data entered for future reference. It is an allowance list requirement for U.S. Navy ships, and is also utilized by the U.S. Coast Guard. It is available for Navy or DoD components from the U.S. Naval Observatory.

**MICA** (Multiyear Interactive Computer Almanac) can, for a given location and time, compute the apparent altitude and azimuth of celestial bodies. It can compute the times and azimuths of rise and set and time and altitude of transit for a given location and date. For circumpolar bodies it computes the times and altitudes of both upper and lower transit. It can also compute the times of civil and nautical twilight. A catalog of the 57 navigational stars is included with MICA, and other catalogs can be added. MICA is produced by the Astronomical Applications Department of the U.S. Naval Observatory (USNO). It is available from Willmann-Bell, <http://www.willbell.com>, for the general public, and from the USNO for Department of Defense Components.

The **Data Services** section of the USNO - Astronomical Applications Department [website](http://aa.usno.navy.mil/data/index.php) includes several calculators for use in sight planning (see Figure 1814a for the link):



Figure 1814a. USNO Data Services  
<http://aa.usno.navy.mil/data/index.php>

1. The **Complete Sun and Moon Data for One Day** page computes the times and azimuths of rise, set for the Sun and Moon, and the times and altitudes of the transits and times of civil twilight.
2. The **Rise/Set/Transit Times for Major Solar System Bodies and Bright Stars** page computes the times and azimuths of rise, set and the times and altitudes of the transits for the Sun, Moon, planets and 22 of the navigational stars.
3. The **Celestial Navigation Data for Assumed Position and Time** page computes the *Hc*, *Zn*, *GHA*, and *Dec* of the Sun, Moon, planets and navigational stars. It also calculates the standard correction for refraction for all bodies and the corrections for the semi-diameter and parallax for the Sun, Moon, and planets. This service determines which bodies are available at a given time and place and color-codes the results for ease of use. See the Notes on the Data Services web page for details.

**UK Rapid Sight Reduction Tables for Navigation NP 303 / AP/3270 (formerly Pub. 249 Vol. 1, Sight Reduction Tables for Air Navigation Vol I (Selected Stars))** provides a list of the seven navigational stars by LAT (latitude) and LHA. It also marks the three stars most appropriate for making a fix from stars well distributed on the sky. This publication has the advantage that it can be used in situations where electric power is not available and values of  $Hc$  and  $Zn$  can be determined swiftly near the epoch of the edition. Its main disadvantage is that values of  $Hc$  and  $Zn$  are sensitive to precession and can change by up to 0.8 per year. So,  $Hc$  and  $Zn$  must be interpolated for precession for dates more than one or two years from the epoch of the edition used. (See the Correction for Precession and Nutation table in *Pub. NP 303/AP3270* for instructions on its use.) The correction table is designed only for observations made

within an eight-year span (four years of the epoch of a particular edition), so a new edition of this volume is published every five years.

The **RUDE 2102-D** star finder is device designed to estimate the approximate  $Hc$  and  $Zn$  of the 57 navigational stars given the observer's Lat and LHA of Aries. It can be used to find the positions of the planets and Moon as well with some additional effort. See Cutler, T.J. 2004, *Dutton's Nautical Navigation, Fifteenth Edition* (Annapolis, MD: Naval Institute Press) articles 2101-2105 for details of its description and use. The advantage of the star finder is that it can be used in situations where electric power is not available. Its principle disadvantage is it can take a while to use and interpret its data for a navigator not practiced in its use.

# CHAPTER 19

## SIGHT REDUCTION

### BASIC PROCEDURES

#### 1900. Computer Sight Reduction

The purely mathematical process of sight reduction is an ideal candidate for computerization, and a number of different hand-held calculators, apps, and computer programs have been developed to relieve the tedium of working out sights by tabular or mathematical methods. The civilian navigator can choose from a wide variety of hand-held calculators and computer programs that require only the entry of the DR position, measured altitude of the body, and the time of observation. Even knowing the name of the body is unnecessary because the computer can identify it based on the entered data. Calculators, apps, and computers can provide more accurate solutions than tabular and mathematical methods because they can be based on precise analytical computations rather than rounded values inherent in tabular data.

U.S. Navy and Coast Guard navigators have access to a U.S. Government program called **STELLA (System To Estimate Latitude and Longitude Astronomically)**; do not confuse with a similarly named commercial astronomy program). The Astronomical Applications Department of the U.S. Naval Observatory developed STELLA in response to a Navy requirement. The algorithms used in STELLA provide an accuracy of one arc-second on the Earth's surface, a distance of about 30 meters. While this accuracy is far better than can be obtained using a sextant, it does support possible naval needs for automated navigation systems based on celestial objects. These algorithms take into account the oblateness of the Earth, movement of the vessel during sight-taking, and other factors not fully addressed by traditional methods.

STELLA can perform almanac functions, position updating/DR estimations, celestial body rise/set/transit calculations, compass error calculations, sight planning, and sight reduction; on-line help and a user's guide are included. STELLA is now automatically distributed to each naval ship; other Navy users may obtain a copy by contacting:

Superintendent  
U.S. Naval Observatory  
Code: AA/STELLA  
3450 Massachusetts Ave. NW  
Washington, DC, 20392-5420

#### 1901. Tabular Sight Reduction

The process of deriving from celestial observations the information needed for establishing a **line of position, LOP**, is called **sight reduction**. The observation itself consists of measuring the altitude of the celestial body above the visible horizon and noting the time.

This chapter concentrates on sight reduction using the *Nautical Almanac* and *Pub. No. 229: Sight Reduction Tables for Marine Navigation*. *Pub 229* is available on the NGA website. The method described here is one of many methods of reducing a sight. Use of the *Nautical Almanac* and *Pub. 229* provide the most precise sight reduction practical, 0.1' (or about 180 meters).

The *Nautical Almanac* contains a set of concise sight reduction tables and instruction on their use. It also contains methods and formulae for direct computation that may be used with a calculator or programmable computer.

The *Air Almanac* and NGA's *Pub. 249, Sight Reduction Tables for Air Navigation*, may also be used to reduce sights. Use of the *Nautical Almanac*'s concise reduction tables, the *Air Almanac*, and *Pub. 249* may all be used to reduce sights to a precision of 1'. The *Nautical Almanac*'s concise reduction tables allow sight reduction by providing all celestial data in a single publication.

Reducing a celestial sight to obtain a line of position consists of six steps:

1. Correct the **sextant altitude,  $H_s$** , to obtain **observed altitude,  $H_o$** , (sometimes called true altitude).
2. Determine the body's **Greenwich Hour Angle,  $GHA$**  and **declination,  $Dec$** .
3. Select an **assumed position,  $AP$**  and find its **Local Hour Angle,  $LHA$** .
4. Compute **altitude,  $H_c$**  and **azimuth,  $Z_n$** , for the  $AP$ .
5. Compare the  $H_c$  and  $H_o$ .
6. Plot the **line of position,  $LOP$** .

The introduction to each volume of *Pub. 229* contains information discussing:

1. The use of the publication for a variety of special celestial navigation techniques;
2. Interpolation:

- a. Explaining the second difference interpolation required in some sight reductions;
- b. Providing tables to facilitate the interpolation process; and,
3. The publication's use in solving problems of great circle sailings.

Prior to using *Pub. 229*, carefully read this introductory material.

The goal of celestial navigation is to determine the navigator's position from the intersection of two or more *LOPs* from observations of the altitudes of one or more celestial bodies and their **geographic positions**, *GP*, at the time of observation. The *GP* is the point on the Earth's surface that intersects the line from a celestial body to the Earth's center. In other words, at the *GP*, the celestial body is directly overhead. The locus of points on the Earth's surface where a body appears at a constant altitude forms a circle centered on the *GP*. So, when navigators observe the altitude of a body, they are determining the circle of constant altitude on which they are located. Except when a body's *Ho* approaches 90°, the portion of the circle of constant altitude plotted on a chart can be approximated by a line called the *LOP*. Assuming the *AP* is within 30' of the true position, the maximum error accrued from estimating the circle of constant altitude with an *LOP* is 3/8 mile for an *Ho* of 70° and 3/4 mile for an *Ho* of 80°.

In sight reduction, navigators choose an assumed position, *AP*, near, but usually not coincident with, their *DR* position. The *AP*'s latitude and longitude are chosen to correspond with the Local Hour Angle, *LHA*, and latitude which are entering arguments of the sight reduction tables in *Pub. 229*. From *Pub. 229*, the navigator determines the body's *Hc*, its computed altitude, and its true azimuth, *Zn*, at the *AP* at the time of observation. The difference between *Hc* and *Ho* is the **altitude intercept**, *a*. The value of *a*, in minutes of arc, is equal to the distance, in nautical miles, between the circle of equal altitude on which the *AP* is located and the one from which the body was observed.

The values of *a* and *Zn* are used to plot an *LOP*. First, the *AP* is plotted on the chart or plotting sheet. A line is drawn through *AP* at the angle *Zn* with respect to true North. The *LOP* is drawn perpendicular to this line. If *Ho* is greater than *Hc*, then the *LOP* is plotted in nautical miles from the *AP* in the direction of *Zn*. If *Ho* is less than *Hc*, then the *LOP* is plotted in nautical miles from the *AP* in the direction away from *Zn*.

## 1902. Selection of the Assumed Position (AP)

As mentioned above, the *AP* is chosen so the navigator can use the tabular values of a publication minimizing the need to interpolate. Thus, the *AP* is typically not your *DR*, and it is different for each object observed, even if the ship is stationary. The *AP* latitude is chosen to be the nearest whole degree in latitude to the *DR* latitude. The *AP*

longitude is that nearest the *DR* longitude resulting in a whole degree of *LHA* for the observed body. The tabular interval in the sight reduction tables of *Pub. 229* for both latitude and *LHA* is one degree. Selecting the *AP* in this manner eliminates interpolation in *LHA* and latitude.

## 1903. Comparison of Computed and Observed Altitudes

The **altitude intercept** (sometimes just called **intercept**), *a*, is the difference between the radii of the circles of equal altitude passing through the *AP* and the observer's true position. The position with the greater altitude is on the circle of smaller radius and closer to the observed body's *GP*. In Figure 1904, the *AP* is shown on the inner circle. Therefore, *Hc* is greater than *Ho*. One minute of arc is equal to one nautical mile. Therefore, *a* is expressed in nautical miles toward, *T*, or away, *A*, from the *GP*, as measured from the *AP*. If *Ho* is greater than *Hc*, the *LOP* intersects at right angles the line drawn from the *AP* Towards the *GP* at a distance of *a* miles. If *Hc* is greater than *Ho*, the line of position intersects at right angles the line drawn from the *AP* Away from the *GP* at a distance of *a* miles. Useful mnemonics for remembering the relation between *Ho*, *Hc*, and *a* are: **HoMoTo** for *Ho More Towards*, and **C-G-A** or *Coast Guard Academy* for *Computed Greater Away*.

## 1904. Plotting a Line of Position (LOP) and Fixing the Position

Plotting an *LOP* is done in four steps:

1. Plot the *AP*.
2. Draw a light dashed line at true azimuth *Zn* through the *AP*.
3. Measure *a* along this line.
4. Draw a line perpendicular to the azimuth line through this point.

This line is the *LOP*, which represents that segment of the circle of equal altitude passing through the navigator's true position. See Figure 1904. The navigator's true position is somewhere along this line, but with only one sight performed, we do not know where.

A celestial navigation fix is the place where two or more *LOPs* intersect. When three or more *LOPs* are used to make a fix, they will usually not all intersect at the same point due to limitations in the precision of the observations. Instead, they will form a 'cocked hat.' A fix may be obtained from plotting the *LOPs* of multiple objects observed at approximately the same epoch (time) or from plotting the *LOPs* of observations of one or more objects made at multiple epochs.

The position of the vessel will have changed significantly between the first and last observations for a fix from observations made at multiple epochs. This type of fix is referred to as a

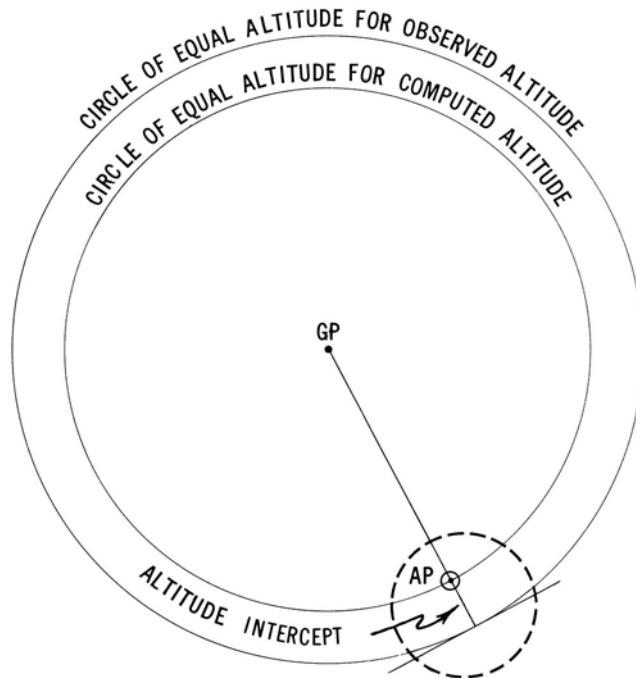


Figure 1904. The basis for the line of position from a celestial observation.

**running fix.** To determine the ship's position at a particular epoch, *LOPs* from earlier epochs must be advanced along the *DR* track and *LOPs* from later epochs must be retarded along the *DR* track. To advance or retard an *LOP*:

1. Draw a light line segment from the *AP* at the epoch of observation so that it intersects the *LOP* at right angles.
2. Measure this line segment's *Zn* and length.
3. At the *AP* of the common epoch, draw a light line segment of the same length and at the same *Zn*.
4. At the non-*AP* end of this second line segment, draw a line perpendicular to it.

This new line is the advanced or retarded *LOP*. A running fix is not as accurate as a fix made from observations made at about the same epoch due to accumulated errors from sources such as steering errors and currents.

If a running fix is made from multiple observations of the same body, then enough time between the observations should be allowed to elapse so that the *Zn* of the body has precessed by at least 30°. A change in *Zn* is required to determine where two *LOPs* cross. A difference of 90° is ideal, but a variance of 30° is adequate to provide a good fix.

## 1905. Sight Reduction Procedures

It is important to develop a practical procedure to reduce celestial sights consistently and accurately. Sight reduction involves several steps. An accurate sight

reduction requires that each step be concisely and accurately performed. Sight reduction tables reduce the mathematics involved as much as possible to addition and subtraction. Careless errors, however, can render the *LOP* deduced from even the most skillfully measured sights inaccurate. The navigator must work methodically to avoid errors.

Naval navigators will most likely use OPNAV 3530/1, *U.S. Navy Navigation Workbook*, which contains "strip forms" to aid in the reduction of sights using either NGA *Pub. 229* or *Pub. 249* with either the *Nautical Almanac* or the *Air Almanac*. OPNAV 3530/1 also contains strip forms to aid in determining ship's latitude by *Polaris* and the local times of sunrise, sunset, moonrise, and moonset using data from either the *Nautical Almanac* or the *Air Almanac*. The *Nautical Almanac* includes a strip form designed specifically for use with its concise sight reduction tables. Use of other strip forms is authorized with the proviso that they become an official part of the record for the workbook being used.

Figure 1905 is a reproduction of the OPNAV 3530/1 strip form for sight reduction using the *Nautical Almanac* and *Pub. 229*. Working from top to bottom the entries are:

**Date:** The UT date of the sight.

**Body:** The name of the body whose altitude was measured. Indicate whether the upper or lower limb was measured if the body was the Sun or the Moon.

**GMT (Greenwich Mean Time):** The UT (GMT is an outdated name for UT) of the observation. The UT is the *Watch Time* of the observations adjusted for the *Watch*

OPNAV 3530/40 (4-73) H.O. 229 NAUT ALM		NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 8-01)
		DATE/DR POSIT
Date		
Body		
GMT		
IC		
D		
Sum		
hs		
ha		
Alt Corr		
Add'l Corr Moon HP/Corr		
Ho		
GHA (h)		
Incre (m/s)		
v/v Corr SHA		
Total GHA		
± 360°		
aλ (+E, -W)		
LHA		
Tab Dec		
d#/ d cor		
True Dec		
a LAT Same Contrary		
Dec Inc / d		
Tens / DSD		
Units / DSD Corr		
Total Corr		
Hc (Tab)		
Hc (Comp)		
Ho		
a		
Z		
Zn		
Fix Lat		
Long		
Fix Time		
Sounding		
Signature		

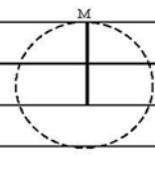


Figure 1905. Sight reduction strip form for use with the Nautical Almanac and Pub 229.

Correction and Zone Description (see Chapter 20).

**IC (Index Correction):** The instrumental correction for the sextant used. Chapter 18 discusses determining its magnitude and sign.

**D (Dip):** Dip correction is a function of the height of eye of the observer and atmospheric refraction. Its magnitude is determined from the Dip Table on the inside front cover of the *Nautical Almanac*.

**Sum:** The sum of *IC* and *D*.

**Ho (Sextant Altitude):** The altitude of the body measured by the sextant.

**Ha (Apparent Altitude):** The sum of *Ho* and the *IC* and *D* corrections.

**Alt. (Altitude) Correction:** Every observation requires an altitude correction. This correction is a function of the apparent altitude of the body. The *Nautical Almanac* contains tables for determining these corrections. The tables for the Sun, planets, and stars are located on the inside front cover and facing page, pages A2 and A3. The tables for the Moon are located on the back inside cover and preceding page, pages xxxiv and xxxv.

These tables are based on observations taken under "standard" weather conditions; that is, temperatures near 50° F and air pressures near 1010mb. If observations are taken in conditions that deviate much from this, an additional altitude correction is needed; see the *Nautical Almanac* table on page A4.

Note that the correction found on A4 is to be applied in addition to the corrections found on pages A2, A3, xxxiv or xxxv.

**Add'l Corr/Moon HP Corr.:** An additional correction is required for sights of Mars, Venus and the Moon. It adjusts for the phase and parallax of these bodies. The correction is a function of the body observed, the epoch of observation, and *Ha*. The corrections for Venus and Mars are listed inside front cover of the *Nautical Almanac*. These corrections change from year to year. The correction for the Moon is a function of the Moon's *Ha*, its *HP*, and whether the upper, *U*, or lower, *L*, limb was observed. The tables for this correction are located inside the back cover and on the preceding page. Enter the table at the appropriate values for *Ha* and *HP*, and then choose the value associated with the *L* or *U* column as appropriate. If the upper limb was observed subtract 30', as well.

**Ho (Observed Altitude):** Add together *Ha*, the Altitude Correction, and the Additional Correction or Moon *HP* Correction, as appropriate. The result is the observed altitude, *Ho*.

**GHA (Tabulated GHA):** The tabulated value for the whole hour immediately preceding the time of the sight observation. For the Sun, the Moon, or a planet, extract the tabulated value for the Greenwich Hour Angle, GHA, of that body from the daily pages of the *Nautical Almanac*. For example, if the sight was obtained at 13<sup>h</sup>50<sup>m</sup>45<sup>s</sup> UT, extract the GHA value for 13<sup>h</sup>. For a star sight reduction, extract the tabulated value of the GHA of Aries  $\varphi$ .

**Incre:** The GHA increment is an interpolation factor, correcting for the time that the sight differed from the whole hour.

For example, if the sight was obtained at  $13^{\text{h}}50^{\text{m}}45^{\text{s}}$  UT, this increment correction accounts for the 50 minutes and 45 seconds after  $13^{\text{h}}$ . The increment value is tabulated in the Increments and Corrections tables on pages ii through xxxi in the *Nautical Almanac*. The entering arguments are the minutes and seconds after the hour. Select the correction from the appropriate column. Use the column labeled *Aries* for sights of stars.

**v / v Corr. / SHA:** The true rate of motion in hour angle for the Moon and planets usually varies from the mean motion used to determine the increments. The parameter *v* is the difference between the mean and true motion in arc minutes per hour. The change in hour angle arising from *v* of the body at the time of the sight observation is accounted for with the *v* correction. The value of *v* for a planet sight is found at the bottom of the planet's column in the daily pages of the *Nautical Almanac*. The value of *v* for the Moon is located directly beside the tabulated hourly GHA values in the daily pages of the *Nautical Almanac*. A body's *v* is positive unless it is listed with a negative sign. Enter the value for *v* on the left-hand side of the strip form. The *v* correction is found using the Increments and Correction tables on pages ii through xxxi in the back of the *Nautical Almanac*. Enter the table using the minutes of the time of observation. Find the value in the “*v* or *d*” columns corresponding to the value of *v* for the time of observation. Enter the corresponding correction on the right-hand side of the strip form with the same sign as *v*.

The Sidereal Hour Angle (*SHA*) is the difference between the GHA of a star and the GHA of Aries. The *SHA* of a star changes slowly. The *SHA*'s of the 57 navigational stars are listed, in alphabetical order of the stars names, in the star column of the daily pages of the *Nautical Almanac*. The mean monthly *SHA*'s of 173 stars, including the 57 navigational stars, are listed in order of *SHA* on pages 268 through 273 of the *Nautical Almanac*. Enter the *SHA* in place of the *v* correction in the strip form if reducing a star sight.

**Total GHA:** The total GHA is the sum of the tabulated GHA, the GHA increment, and either the *v* correction or the star's *SHA*.

**$\pm 360^{\circ}$ :** Since the LHA will be determined from subtracting or adding the assumed longitude to the GHA, adjust the GHA by  $360^{\circ}$  if needed to facilitate the addition or subtraction.

Rule of Thumb:

- In East Longitudes,  $\text{LHA} = \text{GHA} + \text{Longitude}(-360^{\circ}$  as necessary)
- In West Longitudes,  $\text{LHA} = \text{GHA} - \text{Longitude}(+360^{\circ}$  as necessary)

*Example: For a longitude of  $90^{\circ}$  East and GHA of  $300^{\circ}$*

$\text{LHA} = \text{GHA} + \text{Longitude} - 360^{\circ}$  ( $300^{\circ} + 90^{\circ} = 390^{\circ} - 360^{\circ} = 30^{\circ}$ ).

***a λ* (Assumed Longitude):** Choose an assumed longitude, a *λ*. If the vessel is west of the prime meridian,  $\text{LHA} = \text{GHA} - a \lambda$ , where LHA is the Local Hour Angle. If the vessel is east of the prime meridian,  $\text{LHA} = \text{GHA} + a \lambda$ . The *a λ* is chosen so that it is the longitude closest to that DR longitude where the LHA is a whole degree.

**LHA (Local Hour Angle):** The LHA is the hour angle of the observed body at a *λ*. The LHA is  $\text{GHA} - a \lambda$ , for west longitudes and  $\text{GHA} + a \lambda$  for east longitudes. Note that this should be a whole degree, else you have chosen the *a λ* incorrectly.

**Tab. Dec.:** Obtain the tabulated declination for the Sun, Moon, stars, or planets from the daily pages of the *Nautical Almanac*. The declination values for the Sun, Moon, and planets are listed in hourly increments. Enter the declination value for the whole hour immediately preceding the sight for these bodies. For example, if the sight is at  $12^{\text{h}}58^{\text{m}}40^{\text{s}}$ , enter the tabulated declination for  $12^{\text{h}}$ . The declinations of the 57 navigational stars are listed, in alphabetical order of the stars names, in the star column of the daily pages of the *Nautical Almanac*. The mean monthly declinations of 173 stars, including the 57 navigational stars, are listed in order of *SHA* on pages 268 through 273 of the *Nautical Almanac*.

***d / d* Corr.:** The declinations of the Sun, Moon, and planets change with time. The parameter *d* is the amount of change in declination in arc minutes per hour. The change in declination of the body at the time of the sight observation is accounted for with the *d correction*. The value of *d* for a planet sight is found at the bottom of the planet's column in the daily pages of the *Nautical Almanac*. The value of *d* for the Moon is located directly beside the tabulated hourly declination values in the daily pages of the *Nautical Almanac*. Enter the value for *d* on the left-hand side of the strip form. The sign of the *d* correction is determined by the trend of declination value. For example, for a sight taken at  $12^{\text{h}}30^{\text{m}}00^{\text{s}}$ . Compare the declination values for  $12^{\text{h}}$  and  $13^{\text{h}}$  and determine if the declination value has increased or decreased. If it has increased, the *d* correction is positive, and if it has decreased, the *d* correction is negative. The magnitude of the *d* correction is found using the Increments and Correction tables on pages ii through xxxi in the *Nautical Almanac*. Enter the table using the minutes of the time of observation. Find the value in the “*v* or *d*” columns corresponding to the value of *d* for the time of observation. Enter the corresponding correction on the right-hand side of the strip form. The rate of change in declination of the stars is so small that their sights do not require a *d* correction.

**True Dec.:** The sum of the tabulated declination and the *d* correction is the true declination.

***aLAT* (Assumed Latitude):** Choose the whole degree of latitude closest to the vessel's DR latitude as the assumed latitude, *aLAT*. If the assumed latitude and declination are both north or both south, label the assumed latitude *Same*. If one is north and the other is south, label the assumed latitude *Contrary*.

**Dec. inc. / d:** Two of the three arguments used to enter the main table of *Pub. 229*, LHA and *aLAT*, are whole degree values. The third argument, declination, is interpolated in *Pub. 229*. The method for interpolating declination is described on pages X-XIV of each volume of *Pub. 229*. The interpolation tables are located on the inside of the front cover and following page and inside of the back cover and preceding page of each volume of *Pub. 229*.

Interpolation is done using the **declination increment**, *d*. (This *d* is not the same as the *d* factor in the *Nautical Almanac*.) From the main table of *Pub. 229* extract the value of *d* for the tabular declination value preceding the body's declination. For example, if the body's declination is  $30^\circ 35.1'$ , then record the tabular values in the row for Dec. =  $30^\circ$ . If the value for *d* is printed in italics and followed by a dot, then second-difference interpolation is required to maintain precision. In this case record the preceding and following entries for the value of *d* as well. For example, for LHA =  $28^\circ$ , *aLat* =  $15^\circ$  *Same*, Dec. =  $16^\circ$ , the entry for *d* is  $+0.8\bullet$ . The preceding entry is  $+2.8\bullet$ , and the following entry is  $-1.3\bullet$ . Record all three entries, in order.

**Tens / DSD:** If *d* is greater than  $10'$ , then extract the interpolated value for the tens of *d* from the interpolation tables in *Pub. 229*. Refer to the description for use of the interpolation tables on pages XI-XII of any volume of *Pub. 229* for details.

**Units / DSD Corr:** Extract the interpolated value for the units of *d* from the interpolation tables in *Pub. 229*. Refer to the description for use of the interpolation tables on pages XI-XII of any volume of *Pub. 229* for details. If a second difference correction is required: subtract the value of the following entry for *d* from the preceding value. For example, if the preceding entry is  $+2.8$ , and the following entry is  $-1.3$ , then the result is  $2.8 - (-1.3) = 4.1$ . Use this value to enter in the appropriate part of the second difference portion of the interpolation table in *Pub. 229*. Refer to the description for the use of the second difference interpolation on page XIV of any volume of *Pub. 229* for details. Add the second difference correction to the units correction before entering it in the strip form. Failure to include the second difference correction may result in an error of as much as  $2.1'$  in the final value of *Hc*.

**Total Corr.:** The sum of the tens and units corrections is the total correction.

**Hc (Tab.):** The tabulated value of *Hc* from the same entry in *Pub. 229* from which *d* was extracted.

**Hc (Comp.):** The sum of *Hc* (Tab.) and the total corrections is *Hc* (Comp.).

**Ho:** The observed altitude calculated above.

**a (Altitude Intercept):** The absolute value of the difference between *Hc* and *Ho* is the altitude intercept, *a*. If *Ho* is greater than *Hc*, then label *a* as *Toward*. If *Hc* is greater than *Ho*, then label *a* as *Away*. Remember, “computed greater away.”

**Z (Azimuth Angle):** The tabulated value of the azimuth angle, *Z*, is extracted from the same entry in *Pub. 229* from which *Hc* and *d* were extracted. Interpolation is not required.

**Zn (True Azimuth):** The azimuth, *Z*, is the angular distance between the direction towards the observed body and the direction towards the elevated pole. The true azimuth, *Z<sub>n</sub>*, is the angular distance measured eastward from the direction towards the North Pole to the direction towards the observed body. The value of *Z<sub>n</sub>* is a function of *Z*, LHA, and whether the observer is located north or south of the equator.

In northern latitudes:

$$Z_n = \begin{cases} Z & \text{LHA} > 180^\circ \\ 360^\circ - Z & \text{LHA} < 180^\circ \end{cases}$$

In southern latitudes:

$$Z_n = \begin{cases} 180^\circ - Z & \text{LHA} > 180^\circ \\ 180^\circ + Z & \text{LHA} < 180^\circ \end{cases}$$

**Fix, Lat., Long., Time:** Enter the point of intersection and time when two or more *LOPs* are plotted to determine a fix. The time of the fix is not necessarily the time of the sight because *LOPs* may be advanced or retired. The diagram may be used to sketch the *Z<sub>n</sub>*'s and *a*'s used to determine the fix.

**Sounding:** If a sounding is available its value may be entered here.

**Signature:** The sight reduction is a part of the ship's official record and must be signed by the navigator.

## SIGHT REDUCTION

The section above discussed the basic theory of sight reduction and presented a method to be followed when reducing sights. This section puts that method into practice in reducing sights of a star, the Sun, the Moon, and planets.

### 1906. Reducing Star Sights to a Fix

In this section, we will reduce three star sights using the *Nautical Almanac* and *Pub 229*. The first of the three

sights will be reduced in step-by-step detail; the remaining two will not, since the technique is exactly the same. A strip form is utilized to aid in the reduction. A completed strip form showing the values from the three examples presented is shown in Figure 1906a.

On March 09, 2016, at the times indicated, the navigator takes and records the following sights (see the three column table with sextant altitude and *zone time* for Deneb, Antares and Alkaid):

OPNAV 3530/40 (4-73) H.O. 229 NAUT ALM		NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 7-74)		
Date	DATE/DR POSIT 9 Mar 2016 39° N 045° 26.0' W	DATE/DR POSIT 9 Mar 2016 39° N 045° 26.0' W	DATE/DR POSIT 9 Mar 2016 39° N 045° 26.0' W	DATE/DR POSIT
Body	Deneb	Antares	Alkaid	
GMT	08-58-27	09-02-14	09-06-32	
IC	+0.2'	+0.2'	+0.2'	
D	-8.0'	-8.0'	-8.0'	
Sum	-7.8'	-7.8'	-7.8'	
hs	50° 34.4'	23° 57.2'	52° 33.9'	
ha	50° 26.6'	23° 49.4'	52° 26.1'	
Alt Corr	-0.8'	-2.2'	-0.7'	
Add'l Corr Moon HP/Corr	—	—	—	
Ho	50° 25.8'	23° 47.2'	52° 25.4'	
GHA (h)	287° 26.6'	302° 29.1'	302° 29.1'	
Incre (m/s)	14° 39.2'	33.6'	1° 38.3'	
v/v Corr SHA	49° 30.5'	112° 23.9'	152° 57.1'	
Total GHA	351° 36.3'	415° 26.6'	457° 4.5'	
± 360°	—	055° 26.6'	097° 4.5'	
aλ (+E, -W)	045° 36.3' W	045° 26.6' W	045° 4.5' W	
LHA	306°	10°	52°	
Tab Dec	N 45° 20.2'	S 26° 27.8'	N 49° 13.8'	
d#/d cor	—	—	—	
True Dec	N 45° 20.2'	S 26° 27.8'	N 49° 13.8'	
a LAT Same Contrary	N 39° same	N 39° cont	N 39° same	
Dec Inc / d	20.2' / +10.8'	27.8' / -59.4'	13.8' / + 4.4'	
Tens / DSD	—	—	—	
Units / DSD Corr	—	—	—	
Total Corr	+3.6'	-27.5'	+1.0'	
Hc (Tab)	50° 10.5'	24° 19.9'	52° 04.7'	
Hc (Comp)	50° 14.1'	23° 52.4'	52° 05.7'	
Ho	50° 25.8'	23° 47.2'	52° 25.4'	
a	11.7' toward	5.2' away	19.7' toward	
Z	63.3°	170.1°	57.3°	
Zn	63.3°	189.9°	302.7°	
Fix Lat				
Long	M	M	M	M
Fix Time				
Sounding				
Signature				USN, NAVIGATOR

Figure 1906a. Strip form for sight reduction of stars.

Height of eye is 68 feet and index correction (IC) is  $+0.2'$ . The DR latitude for all sights is  $39^\circ$  N. The DR longitude for all sights is  $045^\circ 26.0'W$ . See Figure 1906b. Reduce the Deneb sight first.

Start by converting the sextant altitudes to the observed altitudes.

Determine the sum of the index correction and the dip correction. Go to the inside front cover of the *Nautical Almanac* to the table entitled "DIP." See Figure 1906c. This table lists dip corrections as a function of height of eye measured in either feet or meters. The dip table is a "critical table"; that is it is evaluated in intervals. To use it, find the interval of height of eye and read the corresponding correction. In the above problem, the observer's height of eye is 68 feet, which lies between the tabulated values of 67.1 to 68.8 feet; the corresponding correction for this interval is  $-8.0'$ . Add the IC and the dip correction, being careful to carry the correct sign. The sum of the corrections here is  $-7.8'$ . Apply this correction to the sextant altitude ( $h_s$ ) to obtain the apparent altitude ( $h_a$ ) of  $50^\circ 26.6'$ .

Next, apply the altitude correction. Find the altitude correction table on the inside front cover of the *Nautical Almanac* next to the dip table. See Figure 1906c. This is also a critical table. The altitude correction varies as a function of both the type of body sighted (Sun, star, or planet) and the body's apparent altitude. For the problem above, enter the star altitude correction table;  $h_a$  in this case was  $50^\circ 26.6'$ . This value lies between the tabulated endpoints  $48^\circ 45.0'$  and  $52^\circ 16.0'$ . The correction corresponding to this interval is  $-0.8'$ . Applying this correction to  $h_a$  yields an observed altitude of  $50^\circ 25.8'$ .

Having calculated the observed altitude, *determine the time and date of the sight in UT1 (Universal Time) or GMT (Greenwich Mean Time): Date = 09 March 2016, DR Latitude =  $39^\circ$  N, DR Longitude =  $045^\circ 26.0'W$ , Observation Time = 05-58-27, Watch Error = 0, Zone Time = +3, GMT = 08-58-27, and GMT Date = 09 March 2016.*

Record the observation time and then apply any watch error to determine zone time. Then, use the DR longitude at the time of the sight to determine time zone description. In this case, the DR longitude indicates a zone description of  $+3$  hours. Add the zone description to the zone time to obtain UT/GMT. It is important to carry the correct date when applying this correction (Note: this step, other than recording the DR, UT/GMT, and UT/GMT date, is not on the example strip form (Figure 1906a).

After calculating both the observed altitude and the UT/GMT time, calculate the star's Greenwich Hour Angle (GHA) and declination using the daily pages of the *Nautical Almanac*.

First, record the GHA of Aries from the March 09, 2016 daily page:  $287^\circ 26.6'$ .

Next, determine the incremental addition for the minutes and seconds after 0800 from the Increments and Corrections table in the back of the *Nautical Almanac*. The increment for 58 minutes and 27 seconds is  $14^\circ 39.2'$ .

Then, calculate the GHA of the star. Remember:

$$\text{GHA (star)} = \text{GHA } \Psi + \text{SHA (star)}$$

The *Nautical Almanac* lists the SHA of selected stars on each daily page. The SHA of Deneb on March 09, 2016:  $49^\circ 30.5'$ . The Total GHA is  $287^\circ 26.6' + 14^\circ 39.2' + 49^\circ 30.5' = 351^\circ 36.3'$ . If this were  $360^\circ$  or larger, then subtract  $360^\circ$ .

Initially you must choose an assumed longitude,  $a\lambda$ , rather than using the ship's actual longitude. This is done because on of *Pub 229*'s entering arguments is in whole degrees of LHA of the observed body. In order to get a whole degree of LHA, we will have to slightly change our DR longitude to a new, assumed longitude,  $a\lambda$ . The  $a\lambda$  is chosen so that it is the longitude closest to that DR longitude where the LHA is a whole degree. Remember that if the vessel is west of the prime meridian,  $\text{LHA} = \text{GHA} - a\lambda$ , and if the vessel is east of the prime meridian,  $\text{LHA} = \text{GHA} + a\lambda$ . In this example the vessel is in west longitude, so subtract its assumed longitude from the GHA of the body to obtain the LHA. The assumed longitude must end in  $36.3'$ , so that when subtracted from the Total GHA, a whole degree of LHA will result. Since the DR longitude was  $045^\circ 26.0'$ , the assumed longitude ending in  $36.3'$  closest to the DR longitude is  $045^\circ 36.3'$ . Subtracting this assumed longitude from the calculated GHA of the star yields an LHA of  $306^\circ$ .

The next value of concern is the star's true declination. This value is found on the March 09 daily page next to the star's SHA (see Figure 1906b). Deneb's declination is  $N 45^\circ 20.2'$ . There is no  $d$  correction for a star sight, so the star's true declination equals its tabulated declination.

The assumed latitude is determined from the whole degree of latitude closest to the DR latitude at the time of the sight. In this case, the assumed latitude is  $N 39^\circ$ . It is marked "same" because both DR latitude and star's declination are north.

In order to find an object's altitude and azimuth using *Pub 229*, we need the ship's assumed latitude, the object's LHA, and its declination. We have this information for our example sight of Deneb. To recap, we have: the ship's assumed longitude ( $045^\circ 36.3'W$ ) and assumed latitude ( $39^\circ$  N same); Deneb's LHA at  $a\lambda$  ( $306^\circ$ ); and Deneb's declination ( $N 45^\circ 20.2'$ ). We are now ready to use *Pub 229* to find Deneb's altitude and azimuth.

Find the page in the Sight Reduction Table (using *Pub 229*, for our example) corresponding to an LHA of  $306^\circ$  and an assumed latitude of  $N 39^\circ$ , with latitude same to declination. Enter this table with the body's whole degree of declination. In this case, the body's whole degree of declination is  $45^\circ$ . This declination corresponds to a tabulated altitude ( $h_c$  Tab) of  $50^\circ 10.5'$ . This value is for a declination of  $45^\circ$ ; the true declination is  $45^\circ 20.2'$ . Therefore, we need to interpolate to this increment ( $20.2'$ ) to obtain the computed altitude.

54 2016 MARCH 7, 8, 9 (MON., TUES., WED.)

UT	ARIES	VENUS	-3.8	MARS	+0.1	JUPITER	-2.5	SATURN	+0.5	STARS		
										GHA	GHA	Dec
7 00	165 08.6	199 13.8	S14 36.2	286 42.1	S18 58.9	355 08.6	N 5 54.3	270 01.4	S21 00.2	Acamar	315 17.2	S40 14.8
01	180 11.1	214 13.2	35.2	301 43.6	59.1	10 11.3	54.5	285 03.8	00.2	Achernar	335 26.0	S57 09.6
02	195 13.5	229 12.6	34.3	316 45.1	59.3	25 14.1	54.6	300 06.2	00.2	Acru	173 06.4	S63 11.3
03	210 16.0	244 12.0	. 33.4	331 46.7	. 59.6	40 16.8	. 54.7	315 08.6	. 00.2	Adhara	255 10.9	S29 00.1
04	225 18.5	259 11.4	32.4	346 48.2	18 59.8	55 19.6	54.9	330 11.0	00.2	Aldebaran	290 47.3	N16 32.3
05	240 20.9	274 10.8	31.5	1 49.7	19 00.0	70 22.4	55.0	345 13.3	00.2			
06	255 23.4	289 10.3	S14 30.5	16 51.3	S19 00.2	85 25.1	N 5 55.1	0 15.7	S21 00.2	Alioth	166 18.7	N55 52.2
07	270 25.8	304 09.7	29.6	31 52.8	00.4	100 27.9	55.3	15 18.1	00.2	Alkaid	152 57.1	N49 13.8
08	285 28.3	319 09.1	28.7	46 54.3	00.6	115 30.7	55.4	30 20.5	00.2	Al Na'ir	27 41.9	S46 52.9
M 09	300 30.8	334 08.5	. 27.7	61 55.9	. 00.8	130 33.4	. 55.5	45 22.9	. 00.2	Alnilam	275 44.5	S 1 11.8
O 10	315 33.2	349 07.9	26.8	76 57.4	01.0	145 36.2	55.7	60 25.3	00.3	Alphard	217 54.0	S 8 44.0
N 11	330 35.7	4 07.4	25.8	91 58.9	01.3	160 39.0	55.8	75 27.6	00.3			
D 12	345 38.2	19 06.8	S14 24.9	107 00.5	S19 01.5	175 41.7	N 5 55.9	90 30.0	S21 00.3	Alphecca	126 09.3	N26 39.5
A 13	0 40.6	34 06.2	23.9	122 02.0	01.7	190 44.5	56.0	105 32.4	00.3	Alpheratz	357 41.9	N29 10.7
Y 14	15 43.1	49 05.6	23.0	137 03.5	01.9	205 47.3	56.2	120 34.8	00.3	Altair	62 06.6	N 8 54.7
15	30 45.6	64 05.1	. 22.0	152 05.1	. 02.1	220 50.0	. 56.3	135 37.2	. 00.3	Ankaa	353 14.3	S42 13.3
16	45 48.0	79 04.5	21.1	167 06.6	02.3	235 52.8	56.4	150 39.6	00.3	Antares	112 23.9	S26 27.8
17	60 50.5	94 03.9	20.2	182 08.2	02.5	250 55.6	56.6	165 41.9	00.3			
18	75 53.0	109 03.3	S14 19.2	197 09.7	S19 02.7	265 58.3	N 5 56.7	180 44.3	S21 00.3	Arcturus	145 53.8	N19 05.8
19	90 55.4	124 02.7	18.3	212 11.2	03.0	281 01.1	56.8	195 46.7	00.3	Atria	107 24.0	S69 02.9
20	105 57.9	139 02.2	17.3	227 12.8	03.2	296 03.8	57.0	210 49.1	00.3	Avior	234 16.7	S59 34.1
21	121 00.3	154 01.6	. 22.0	242 14.3	. 03.4	311 06.6	. 57.1	225 51.5	. 00.3	Bellatrix	278 30.0	N 6 21.5
22	136 02.8	169 01.0	15.4	257 15.8	03.6	326 09.4	57.2	240 53.9	00.3	Betelgeuse	270 59.2	N 7 24.3
23	151 05.3	184 00.5	14.4	272 17.4	03.8	341 12.1	57.3	255 56.3	00.3			
8 00	166 07.7	198 59.9	S14 13.5	287 18.9	S19 04.0	356 14.9	N 5 57.5	270 58.6	S21 00.3	Canopus	263 55.1	S52 42.8
01	181 10.2	213 59.3	12.5	302 20.5	04.2	11 17.7	57.6	286 01.0	00.3	Capella	280 31.7	N46 00.8
02	196 12.7	228 58.7	11.6	317 22.0	04.4	26 20.4	57.7	301 03.4	00.3	Deneb	49 30.5	N45 20.2
03	211 15.1	243 58.2	. 10.6	332 23.6	. 04.6	41 23.2	. 57.9	316 05.8	. 00.3	Denebola	182 31.5	N14 28.8
04	226 17.6	258 57.6	09.7	347 25.1	04.8	56 26.0	58.0	331 08.2	00.3	Diphda	348 54.4	S17 54.1
05	241 20.1	273 57.0	08.7	2 26.7	05.1	71 28.7	58.1	346 10.6	00.3	Dubhe	193 48.9	N61 39.7
06	256 22.5	288 56.5	S14 07.8	17 28.2	S19 05.3	86 31.5	N 5 58.3	1 13.0	S21 00.3	Elnath	278 10.2	N28 37.1
07	271 25.0	303 55.9	06.8	32 29.7	05.5	101 34.3	58.4	16 15.4	00.3	Eltanin	90 45.4	N51 29.1
T 08	286 27.5	318 55.3	05.9	47 31.3	05.7	116 37.0	58.5	31 17.7	00.3	Enif	33 45.6	N 9 56.9
09	301 29.9	333 54.7	. 04.9	62 32.8	. 05.9	131 39.8	. 58.6	46 20.1	. 00.3	Fomalhaut	15 22.3	S29 32.3
E 10	316 32.4	348 54.2	03.9	77 34.4	06.1	146 42.6	58.8	61 22.5	00.3	Gacrux	171 58.2	S57 12.2
S 11	331 34.8	3 53.6	03.0	92 35.9	06.3	161 45.3	58.9	76 24.9	00.3	Gienah	175 50.1	S17 38.0
D 12	346 37.3	18 53.0	S14 02.0	107 37.5	S19 06.5	176 48.1	N 5 59.0	91 27.3	S21 00.3	Hadar	148 44.7	S60 26.8
A 13	1 39.8	33 52.5	01.1	122 39.0	06.7	191 50.9	59.2	106 29.7	00.3	Hamal	327 58.9	N23 32.2
Y 14	16 42.2	48 51.9	14 00.1	137 40.6	06.9	206 53.6	59.3	121 32.1	00.3	Kaus Aust.	83 41.5	S34 22.3
15	31 44.7	63 51.3	13 59.1	152 42.1	. 07.1	221 56.4	. 59.4	136 34.5	. 00.3	Kochab	137 19.6	N74 05.2
16	46 47.2	78 50.8	58.2	167 43.7	07.4	236 59.2	59.5	151 36.9	00.3	Markab	13 36.8	N15 17.5
17	61 49.6	93 50.2	57.2	182 45.2	07.6	252 01.9	59.7	166 39.2	00.3	Menkar	314 13.3	N 4 08.9
18	76 52.1	108 49.6	S13 56.2	197 46.8	S19 07.8	267 04.7	N 5 59.8	181 41.6	S21 00.3	Menkent	148 05.1	S36 26.8
19	91 54.6	123 49.1	55.3	212 48.3	08.0	282 07.4	5 59.9	196 44.0	00.3	Miaplacidus	221 38.3	S69 47.3
20	106 57.0	138 48.5	54.3	227 49.9	08.2	297 10.2	6 00.1	211 46.4	00.3			
21	121 59.5	153 47.9	. 53.3	242 51.4	. 08.4	312 13.0	. 00.2	226 48.8	. 00.3	Kochab	137 19.6	N74 05.2
22	137 01.9	168 47.4	52.4	257 53.0	08.6	327 15.7	00.3	241 51.2	00.3	Markab	13 36.8	N15 17.5
23	152 04.4	183 46.8	51.4	272 54.6	08.8	342 18.5	00.5	256 53.6	00.3	Menkar	314 13.3	N 4 08.9
9 00	167 06.9	198 46.3	S13 50.4	287 56.1	S19 09.0	357 21.3	N 6 00.6	271 56.0	S21 00.3	Mirfak	308 37.8	N49 55.1
01	182 09.3	213 45.7	49.5	302 57.7	09.2	12 24.0	00.7	286 58.4	00.3	Nunki	75 56.2	S26 16.4
02	197 11.8	228 45.1	48.5	317 59.2	09.4	27 26.8	00.8	302 00.8	00.4	Peacock	53 16.8	S56 40.7
03	212 14.3	243 44.6	. 47.5	333 00.8	. 09.6	42 29.6	. 01.0	317 03.1	. 00.4	Pollux	243 25.3	N27 59.0
04	227 16.7	258 44.0	46.6	348 02.3	09.8	57 32.3	01.1	332 05.5	00.4	Procyon	244 57.6	N 5 10.7
05	242 19.2	273 43.5	45.6	3 03.9	10.0	72 35.1	01.2	347 07.9	00.4	Rasalhague	96 04.8	N12 32.9
W 06	257 21.7	288 42.9	S13 44.6	18 05.5	S19 10.2	87 37.9	N 6 01.4	2 10.3	S21 00.4	Regulus	207 41.3	N11 53.1
E 07	272 24.1	303 42.3	43.6	33 07.0	10.4	102 40.6	01.5	17 12.7	00.4	Rigel	281 10.3	S 8 11.4
E 08	287 26.6	318 41.8	42.7	48 08.6	10.7	117 43.4	01.6	32 15.1	00.4	Rigil Kent.	139 48.7	S60 53.8
D 09	302 29.1	333 41.2	. 41.7	63 10.1	. 09.9	132 46.2	. 01.8	47 17.5	. 00.4	Sabik	102 10.4	S15 44.5
N 10	317 31.5	348 40.7	40.7	78 11.7	11.1	147 48.9	01.9	62 19.9	00.4	Schedar	349 38.8	N56 37.5
N 11	332 34.0	3 40.1	39.7	93 13.3	11.3	162 51.7	02.0	77 22.3	00.4	Shaula	96 19.4	S37 06.6
E 12	347 36.4	18 39.5	S13 38.8	108 14.8	S19 11.5	177 54.5	N 6 02.1	92 24.7	S21 00.4	Sirius	258 32.0	S16 44.7
S 13	2 38.9	33 39.0	37.8	123 16.4	11.7	192 57.2	02.3	107 27.1	00.4	Spica	158 29.0	S11 14.8
D 14	17 41.4	48 38.4	36.8	138 18.0	11.9	208 00.0	02.4	122 29.5	00.4	Suhail	222 50.7	S43 30.2
A 15	32 43.8	63 37.9	. 35.8	153 19.5	. 12.1	223 02.8	. 02.5	137 31.9	. 00.4	Vega	80 37.8	N38 47.8
Y 16	47 46.3	78 37.3	34.8	168 21.1	12.3	238 05.5	02.7	152 34.2	00.4	Zuben'ubi	137 03.2	S16 06.4
17	62 48.8	93 36.8	33.9	183 22.7	12.5	253 08.3	02.8	167 36.6	00.4	SHA	Mer. Pass.	
18	77 51.2	108 36.2	S13 32.9	198 24.2	S19 12.7	268 11.1	N 6 02.9	182 39.0	S21 00.4	Venus	32 52.1	h m
19	92 53.7	123 35.7	31.9	213 25.8	12.9	283 13.8	03.0	197 41.4	00.4	Mars	121 11.2	4 50
20	107 56.2	138 35.1	30.9	228 27.4	13.1	298 16.6	03.2	212 43.8	00.4	Jupiter	190 07.2	0 15
21	122 58.6	153 34.5	. 29.9	243 28.9	. 13.3	313 19.3	. 03.3	227 46.2	. 00.4	Saturn	104 50.9	5 55
22	138 01.1	168 34.0	29.0	258 30.5	13.5	328 22.1	03.4	242 48.6	00.4			
23	153 03.5	183 33.4	28.0	273 32.1	13.7	343 24.9	03.6	257 51.0	00.4			

Figure 1906b. Left hand daily page of the Nautical Almanac for March 7, 8, &amp; 9, 2016.

A2 ALTITUDE CORRECTION TABLES  $10^{\circ}$ - $90^{\circ}$ —SUN, STARS, PLANETS

OCT.—MAR. SUN APR.—SEPT.				STARS AND PLANETS				DIP							
App.	Lower	Upper	Limb	App.	Lower	Upper	Limb	App.	Corr <sup>n</sup>	App.	Additional	Ht. of	Corr <sup>n</sup>	Ht. of	Corr <sup>n</sup>
Alt.	Limb	Limb		Alt.	Limb	Limb		Alt.		Alt.	Corr <sup>n</sup>	Eye		Eye	
° /	/	/	/	° /	/	/	/	2016		m		ft.	/	m	/
9 33	+ 10.8	- 21.5		9 39	+ 10.6	- 21.2		9 55	- 5.3	VENUS		2.4	/	8.0	1.0 — 1.8
9 45	+ 10.9	- 21.4		9 50	+ 10.7	- 21.1		10 07	- 5.2		Jan. 1-Dec. 3	2.6	- 2.8	8.6	1.5 — 2.2
9 56	+ 11.0	- 21.3		10 02	+ 10.8	- 21.0		10 20	- 5.1			2.8	- 2.9	9.2	2.0 — 2.5
10 08	+ 11.0	- 21.2		10 14	+ 10.8	- 21.0		10 32	- 5.1			3.0	- 3.0	9.8	2.5 — 2.8
10 20	+ 11.1	- 21.2		10 27	+ 10.9	- 20.9		10 46	- 4.9			3.2	- 3.2	10.5	3.0 — 3.0
10 33	+ 11.2	- 21.1		10 40	+ 11.0	- 20.8		10 59	- 4.8	Dec. 4-Dec. 31		3.4	- 3.3	11.2	See table
10 46	+ 11.3	- 21.0		10 53	+ 11.1	- 20.7		11 14	- 4.7			3.6	- 3.4	11.9	←
11 00	+ 11.4	- 20.9		11 07	+ 11.2	- 20.6		11 29	- 4.6			3.8	- 3.4	12.6	
11 15	+ 11.5	- 20.8		11 22	+ 11.3	- 20.5		11 44	- 4.5			4.0	- 3.5	13.3	m /
11 30	+ 11.6	- 20.7		11 37	+ 11.4	- 20.4		12 00	- 4.5			4.3	- 3.6	14.1	20 — 7.9
11 45	+ 11.7	- 20.6		11 53	+ 11.5	- 20.3		12 17	- 4.4			4.5	- 3.8	14.9	22 — 8.3
12 01	+ 11.8	- 20.5		12 10	+ 11.6	- 20.2		12 35	- 4.2		Jan. 1-Mar. 11	4.7	- 3.9	15.7	24 — 8.6
12 18	+ 11.9	- 20.4		12 27	+ 11.7	- 20.1		12 53	- 4.1	Sept. 16-Dec. 31		5.0	- 4.0	16.5	26 — 9.0
12 36	+ 12.0	- 20.3		12 45	+ 11.8	- 20.0		13 12	- 4.0			5.2	- 4.1	17.4	28 — 9.3
12 54	+ 12.1	- 20.2		13 04	+ 12.0	- 19.9		13 32	- 3.9			5.5	- 4.2	18.3	
13 14	+ 12.2	- 20.1		13 24	+ 12.1	- 19.8		13 53	- 3.8			5.8	- 4.3	19.1	30 — 9.6
13 34	+ 12.3	- 20.0		13 44	+ 12.2	- 19.6		14 16	- 3.7			6.1	- 4.3	20.1	32 — 10.0
13 55	+ 12.4	- 19.9		14 06	+ 12.3	- 19.5		14 39	- 3.6	Mar. 12-Apr. 30		6.3	- 4.4	21.0	34 — 10.3
14 17	+ 12.5	- 19.8		14 29	+ 12.4	- 19.4		15 03	- 3.5	July 5-Sept. 15		6.6	- 4.5	22.0	36 — 10.6
14 41	+ 12.6	- 19.7		14 53	+ 12.5	- 19.3		15 29	- 3.4			6.9	- 4.6	22.9	38 — 10.8
15 05	+ 12.7	- 19.6		15 18	+ 12.6	- 19.2		15 56	- 3.3			7.2	- 4.7	23.9	40 — 11.1
15 31	+ 12.8	- 19.5		15 45	+ 12.7	- 19.1		16 25	- 3.2	May 1-July 4		7.5	- 4.8	24.9	42 — 11.4
15 59	+ 12.9	- 19.4		16 13	+ 12.8	- 19.0		16 55	- 3.1			7.9	- 5.0	26.0	44 — 11.7
16 27	+ 13.0	- 19.3		16 43	+ 12.9	- 18.9		17 27	- 3.0			8.2	- 5.1	27.1	46 — 11.9
16 58	+ 13.1	- 19.2		17 14	+ 13.0	- 18.8		18 01	- 2.9			8.5	- 5.2	28.1	48 — 12.2
17 30	+ 13.2	- 19.1		17 47	+ 13.1	- 18.7		18 37	- 2.8			8.8	- 5.3	29.2	ft. /
18 05	+ 13.3	- 19.0		18 23	+ 13.2	- 18.6		19 16	- 2.7			9.2	- 5.4	30.4	2 — 1.4
18 41	+ 13.4	- 18.9		19 00	+ 13.3	- 18.5		19 56	- 2.6			9.5	- 5.5	31.5	4 — 1.9
19 20	+ 13.6	- 18.7		19 41	+ 13.4	- 18.4		20 40	- 2.5			9.9	- 5.6	32.7	6 — 2.4
20 02	+ 13.7	- 18.6		20 24	+ 13.5	- 18.3		21 27	- 2.4			10.3	- 5.7	33.9	8 — 2.7
20 46	+ 13.8	- 18.5		21 10	+ 13.6	- 18.2		22 17	- 2.3			10.6	- 5.8	35.1	10 — 3.1
21 34	+ 13.9	- 18.4		21 59	+ 13.7	- 18.1		23 11	- 2.2			11.0	- 5.9	36.3	See table
22 25	+ 14.0	- 18.3		22 52	+ 13.8	- 18.0		24 09	- 2.1			11.4	- 6.0	37.6	←
23 20	+ 14.1	- 18.2		23 49	+ 13.9	- 17.9		25 12	- 2.0			11.8	- 6.1	38.9	
24 20	+ 14.2	- 18.1		24 51	+ 14.0	- 17.8		26 20	- 1.9			12.2	- 6.2	40.1	ft. /
25 24	+ 14.3	- 18.0		25 58	+ 14.1	- 17.7		27 34	- 1.8			12.6	- 6.3	41.5	70 — 8.1
26 34	+ 14.4	- 17.9		27 11	+ 14.2	- 17.6		28 54	- 1.7			13.0	- 6.4	42.8	75 — 8.4
27 50	+ 14.5	- 17.8		28 31	+ 14.3	- 17.5		30 22	- 1.6			13.4	- 6.5	44.2	80 — 8.7
29 13	+ 14.6	- 17.7		29 58	+ 14.4	- 17.4		31 58	- 1.5			13.8	- 6.6	45.5	85 — 8.9
30 44	+ 14.7	- 17.6		31 33	+ 14.5	- 17.3		33 43	- 1.4			14.2	- 6.7	46.9	90 — 9.2
32 24	+ 14.8	- 17.5		33 18	+ 14.6	- 17.2		35 38	- 1.3			14.7	- 6.8	48.4	95 — 9.5
34 15	+ 14.9	- 17.4		35 15	+ 14.7	- 17.1		37 45	- 1.2			15.1	- 6.9	49.8	
36 17	+ 15.0	- 17.3		37 24	+ 14.8	- 17.0		40 06	- 1.1			15.5	- 7.0	51.3	100 — 9.7
38 34	+ 15.1	- 17.2		39 48	+ 14.9	- 16.9		42 42	- 1.0			16.0	- 7.1	52.8	105 — 9.9
41 06	+ 15.2	- 17.1		42 28	+ 15.0	- 16.8		45 34	- 0.9			16.5	- 7.2	54.3	110 — 10.2
43 56	+ 15.3	- 17.0		45 29	+ 15.1	- 16.7		48 45	- 0.8			16.9	- 7.3	55.8	115 — 10.4
47 07	+ 15.4	- 16.9		48 52	+ 15.2	- 16.6		52 16	- 0.7			17.4	- 7.4	57.4	120 — 10.6
50 43	+ 15.5	- 16.8		52 41	+ 15.3	- 16.5		56 09	- 0.6			17.9	- 7.5	58.9	125 — 10.8
54 46	+ 15.6	- 16.7		56 59	+ 15.4	- 16.4		60 26	- 0.5			18.4	- 7.6	60.5	
59 21	+ 15.7	- 16.6		61 50	+ 15.5	- 16.3		65 06	- 0.4			18.8	- 7.7	62.1	130 — 11.1
64 28	+ 15.8	- 16.5		67 15	+ 15.6	- 16.2		70 09	- 0.3			19.3	- 7.8	63.8	135 — 11.3
70 10	+ 15.9	- 16.4		73 14	+ 15.7	- 16.1		75 32	- 0.2			19.8	- 7.9	65.4	140 — 11.5
76 24	+ 16.0	- 16.3		79 42	+ 15.8	- 16.0		81 12	- 0.1			20.4	- 8.0	67.1	145 — 11.7
83 05	+ 16.1	- 16.2		86 31	+ 15.9	- 15.9		87 03	0.0			20.9	- 8.1	68.8	150 — 11.9
90 00				90 00				90 00				21.4	70.5	155 — 12.1	

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.

Figure 1906c. A2 Altitude Correction Tables  $10^{\circ}$  -  $90^{\circ}$  for Sun, Stars, Planets.

The difference between the tabulated altitudes for  $45^\circ$  and  $46^\circ$  is given in *Pub. 229* as the value  $d$ ; in this case,  $d = +10.8'$ . Note it is not in italics and followed by a dot, which makes the interpolation easier. There are two ways to do this interpolation. First, we could use the Interpolation Table on the inside front and back covers of *Pub 229*. We find the value of  $+3.6'$ . Alternatively, we can easily compute the value. Express as a ratio the declination increment (in this case,  $20.2'$ ) and the total interval between the tabulated declination values (in this case,  $60'$ ) to obtain the 'distance' between the tabulated declination values represented by the declination increment. Next, multiply that by  $d$ , paying attention to the sign. In this case:  $(20.2'/60') (+10.8') = +3.6'$ .

As expected, the two methods (using the Interpolation Table or computation) give the same result. Add  $3.6'$  to the tabulated altitude to obtain the final computed altitude:  $H_c = 50^\circ 25.8' - 50^\circ 14.1' = +11.7'$ .

It will be valuable here to review exactly what  $h_o$  and  $h_c$  represent. Recall the methodology of the altitude-intercept method. The navigator first measures and corrects an altitude for a celestial body. This corrected altitude,  $h_o$ , corresponds to a circle of equal altitude passing through the navigator's actual position whose center is the geographic position (GP) of the body. The navigator then determines an assumed position (AP) near, but not coincident with, his or her actual position; they then calculate an altitude for an observer at that assumed position (AP). The circle of equal altitude passing through this assumed position is concentric with the circle of equal altitude passing through the navigator's actual position. The difference between the body's altitude at the assumed position ( $h_c$ ) and the body's observed altitude ( $h_o$ ) is equal to the differences in radii length of the two corresponding circles of equal altitude.

In the above problem, therefore, the navigator knows that the equal altitude circle passing through his or her actual position is understood as: the difference between  $h_o$  and  $h_c$  is  $50^\circ 25.8' - 50^\circ 14.1' = 11.7'$  (which is 11.7 nautical miles) away from the equal altitude circle passing through his or her actual position. Since  $h_o$  is greater than  $h_c$ , the navigator knows that the radius of the equal altitude circle passing through his or her actual position is less than the radius of the equal altitude circle passing through the assumed position. The only remaining question is what direction from the assumed position is the body's actual GP. *Pub. 229* provides this final piece of information. This is the value for  $Z$  which is tabulated with the  $h_c$  and  $d$  values discussed above. In this case, enter *Pub. 229* as before, with LHA, assumed latitude, and declination. Extract the value  $Z = 063.3^\circ$ . Interpolation is not required. The relation between  $Z$  and  $Z_n$ , the true azimuth, is as follows:

In northern latitudes:

$$\begin{aligned} \text{LHA} > 180^\circ, \text{ then } Z_n &= Z \\ \text{LHA} < 180^\circ, \text{ then } Z_n &= 360^\circ - Z \end{aligned}$$

In southern latitudes:

$$\begin{aligned} \text{LHA} > 180^\circ, \text{ then } Z_n &= 180^\circ - Z \\ \text{LHA} < 180^\circ, \text{ then } Z_n &= 180^\circ + Z \end{aligned}$$

In this case,  $\text{LHA} > 180^\circ$  and the vessel is in northern latitude. Therefore,  $Z_n = Z = 063.3^\circ$ . The navigator now has enough information to plot a line of position.

Using the same technique, reduce the sights of Antares and Alkaid. The values for the sight reductions are shown in Figure 1906a.

## 1907. Reducing a Sun Sight

The example below points out the similarities between reducing a sun sight and reducing a star sight. It also demonstrates the additional corrections required for low altitude ( $<10^\circ$ ) sights and sights taken during non-standard temperature and pressure conditions.

On March 09, 2016, at 07-00-24 local time, at DR position  $L 39^\circ 11.0' N$   $145^\circ 22.0' W$ , a navigator takes a sight of the Sun's lower limb. The navigator has a height of eye of 68 feet, the temperature is  $88^\circ F$ , and the atmospheric pressure is 982 mb. The sextant altitude is  $6^\circ 37.5'$ . There is an index correction of  $+0.2'$ . Determine the observed altitude, computed altitude, and azimuth.

A completed strip form showing the values from this example is shown in Figure 1907a.

Apply the index and dip corrections to  $h_s$  to obtain  $h_a$ . Because  $h_a$  is less than  $10^\circ$ , use the special altitude correction table for sights between  $0^\circ$  and  $10^\circ$  located on the right inside front page of the *Nautical Almanac*.

Enter the table with the apparent altitude, the limb of the Sun used for the sight, and the period of the year. Interpolation for the apparent altitude is not required. In this case, the table yields a correction of  $+8.4'$ . The correction's algebraic sign is found at the head of each group of entries and at every change of sign.

An additional correction is required because of the non-standard temperature and atmospheric pressure under which the sight was taken. The correction for these non-standard conditions is found in the *Additional Corrections* table located on page A4 in the front of the *Nautical Almanac*.

First, enter the *Additional Corrections* table with the temperature and pressure to determine the correct zone letter: in this case, zone M. Then, locate the correction in the M column corresponding to the apparent altitude of  $6^\circ 29.7'$ . Interpolate between the table arguments of  $6^\circ 00.0'$  and  $7^\circ 00.0'$  to determine the additional correction:  $+0.8'$ .

OPNAV 3530/40 (4-73) H.O. 229 NAUT ALM		NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 7-74)	
Date	DATE/DR POSIT 9 Mar 2016 39° 11' N, 045° 22.0 W	DATE/DR POSIT 22 Mar 2016 DR not given in example	DATE/DR POSIT 9 Mar 2016 DR not given in example
Body	Sun LL	Moon LL	Mars
GMT	10-00-24	21-01-04	08-58-34
IC	+0.2'	+0.2'	+0.2'
D	-8.0'	-8.0'	-8.0'
Sum	-7.8'	-7.8'	-7.8'
hs	6° 37.5'	3° 55.0'	29° 43.0'
ha	6° 29.7'	3° 47.2'	29° 35.2'
Alt Corr	+8.4'	+56.1'	-1.7'
Add'l Corr Moon HP/Corr	+0.8'	+0.7'	+0.1'
Ho	6° 38.9'	4° 44.0'	29° 33.6'
GHA (h)	327° 23.6'	319° 28.2'	48° 8.6'
Incre (m/s)	6.0'	+15.3'	14° 38.5'
v/v Corr SHA	—	15.0 / +0.4'	1.5'
Total GHA	327° 29.6'	319° 43.9'	62° 48.6'
± 360°	—	—	—
aλ (+E, -W)	045° 29.6' W		
LHA	282°		
Tab Dec	S 4° 14.9'	N 1° 59.8'	S 19° 10.7'
d# / d cor	-1.0 / 0.0'	-9.4 / -0.2'	+0.2 / +0.2'
True Dec	S 4° 14.9'	N 1° 59.6'	S 19° 10.9'
a LAT Same Contrary	39° cont		
Dec Inc / d	14.9' / -38.6'		
Tens / DSD	—		
Units / DSD Corr	—		
Total Corr	-9.6'		
Hc (Tab)	6° 44.1'		
Hc (Comp)	6° 34.5'		
Ho	6° 38.9'		
a	4.4' toward		
Z	100.7°		
Zn	100.7°		
Fix Lat			
Long	M	M	M
Fix Time			
Sounding			
Signature			

USN, NAVIGATOR

Figure 1907a. Strip for showing example values for Sun, Moon, and Mars.

The total correction to the apparent altitude is the sum of the altitude and additional corrections:  $+9.2'$ . This results in an  $h_o$  of  $6^\circ 38.9'$ .

Date	09 March 2016
DR Latitude	N $39^\circ 11.01'$
DR Longitude	W $045^\circ 22.0'$
Observation Time	07-00-24
Watch Error	0
Zone Time	07-00-24
Zone Description	+3
UT/GMT	10-00-24

Next, determine the Sun's GHA and declination. Again, this process is similar to the star sights reduced above. See Figure 1907b. Notice, however, that SHA, a quantity unique to star sight reduction, is not used in Sun sight reduction.

Determining the Sun's GHA is less complicated than determining a star's GHA. The *Nautical Almanac*'s daily pages list the Sun's GHA in hourly increments. In this case, the Sun's GHA at 1000 GMT on March 09, 2016 is  $327^\circ 23.6'$ . The  $v$  correction is not applicable for a Sun sight; therefore, applying the increment correction yields the Sun's GHA. In this case, the GHA is  $327^\circ 29.6'$ .

Determining the Sun's LHA is similar to determining a star's LHA; see Section 1906 if more details are needed. In determining the Sun's declination, however, an additional correction not encountered in the star sight, the  $d$  correction, must be considered. This is because the Sun moves in declination over the course of an hour, unlike that of the stars; the  $d$  value is an interpolation factor for the Sun's declination. The bottom of the Sun column on the daily pages of the *Nautical Almanac* lists the  $d$  value. The sign of the  $d$  factor must be determined by noting from the *Almanac* whether the Sun's declination value is increasing or decreasing around the time of observation. If it is increasing, the factor is positive; if it is decreasing, the factor is negative. In the above problem, the Sun's declination value is decreasing throughout the day. Therefore, the  $d$  factor is  $-1.0$ .

Having obtained the  $d$  factor, enter the 00 minute increment and correction table. Under the column labeled " $v$  or  $d$  corr," find the value for  $d$  in the left hand column. The corresponding number in the right hand column is the correction. Apply the correction to the tabulated declination. In this case, the correction corresponding to a  $d$  value of  $-1.0$  is  $0.0'$ .

The final step will be to determine  $h_c$  and Zn. Enter *Pub. 229* with an LHA of  $282^\circ$ , a declination of  $S4^\circ 14.9'$ , and an assumed latitude of  $39^\circ N$ . The remaining values to determine  $h_c$  and Zn are shown on the strip form (Figure 1907a).

## 1908. Reducing a Moon Sight

The Moon is easy to identify and is often visible during the day. However, the Moon's proximity to the Earth requires

applying additional corrections to  $h_a$  to obtain  $h_o$ . This section will cover Moon sight reduction.

At 21-01-04 UT, March 22, 2016, under standard meteorological conditions, the navigator obtains a sight of the Moon's lower limb.  $h_s$  is  $3^\circ 55.0'$ . Height of eye is 68 feet and the index correction is  $+0.2'$ . Determine  $h_o$ , the Moon's GHA, and the Moon's declination.

A completed strip form showing the values from this example is shown in Figure 1907a. Also see Figure 1908a.

This example demonstrates the extra corrections required for obtaining  $h_o$  for a Moon sight. Apply the index and dip corrections in the same manner as for star and Sun sights.

The altitude correction for the Moon comes from tables located on the inside back covers of the *Nautical Almanac*. See Figure 1908b. In this case, the apparent altitude was  $3^\circ 47.2'$ . Enter the altitude correction table for the Moon with the above apparent altitude. Interpolation is not required. The correction is  $+56.1'$ . An arrow shows the correct location of this correction.

The altitude correction due to horizontal parallax (HP) is unique to Moon sights. The table for determining this correction is directly under the main altitude correction table for the Moon, on the back inside cover of the *Nautical Almanac* (see Figure 1908b). First, go to the daily page for March 22 at 21-00-00 UT/GMT (see Figure 1908a). In the column for the Moon, find the HP corresponding to 21-00-00. Its value is  $54.3'$  (no interpolation is required). Take this value to the Altitude Correction Tables for the Moon on the inside back cover of the *Almanac*. Notice that the HP correction columns line up vertically with the apparent altitude correction table columns. Find the HP correction column directly under the apparent altitude correction table heading corresponding to the apparent altitude. Enter that column with the HP from the daily pages. The column has two sets of figures listed under "L" and "U" for lower and upper limb respectively. In this case, trace down the "L" column until it intersects with the HP of  $54.3'$ . The altitude correction due to the Moon's horizontal parallax is  $+0.7'$  in our example; an arrow shows the location of this correction. Interpolation is not required.

An additional correction based on non-standard weather condition is not applicable in this case because the sight was taken under standard temperature and pressure conditions.

The total correction to  $h_a$  is the sum of all the corrections; in this case, this total correction is  $+56.8'$ .

To obtain the Moon's GHA, enter the daily pages in the Moon column and extract the applicable data just as for a star or Sun sight. Determining the Moon's GHA requires an additional correction, the  $v$  correction. The  $v$  correction is needed because the Moon's motion is not close to uniform throughout the year.

First, record the GHA of the Moon for 21-00-00 on March 22, 2016, from the daily pages of the *Nautical Almanac*. The increment correction is done as in the previous

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UT	SUN		MOON					Lat.	Twilight		Sunrise	Moonrise					
	GHA	Dec	GHA	$\nu$	Dec	$d$	HP		Naut.	Civil		7	8	9	10		
7 00	d h	o /	o /	o /	/	o /	/	N 72	h m	h m	h m	h m	h m	h m	h m		
	01	177 14.7 S 5 11.5	205 09.1	7.7	S12 30.7	8.3	59.5	68	04 31	05 51	06 58	07 13	07 08	07 03	06 58		
	02	192 14.9	219 35.8	7.8	12 22.4	8.4	59.5	66	04 48	05 53	06 49	06 38	06 47	06 55	07 01		
	03	207 15.0	234 02.6	7.8	12 14.0	8.5	59.5	64	04 54	05 54	06 45	06 25	06 40	06 52	07 03		
	04	222 15.2 .	248 29.4	7.8	12 05.5	8.5	59.6	62	04 59	05 55	06 42	06 15	06 33	06 49	07 04		
M 00	05	237 15.3 .	262 56.2	7.7	11 57.0	8.7	59.6	60	05 04	05 55	06 39	06 06	06 27	06 47	07 05		
	06	252 15.5 .	277 22.9	7.8	11 48.3	8.7	59.6	58	05 07	05 55	06 37	05 58	06 22	06 44	07 06		
	07	267 15.6 S 5 05.7	291 49.7	7.7	S11 39.6	8.8	59.7	56	05 10	05 56	06 35	05 51	06 18	06 43	07 06		
	08	282 15.8 .	306 16.4	7.8	11 30.8	8.9	59.7	54	05 13	05 56	06 33	05 45	06 14	06 41	07 07		
	09	297 15.9 .	320 43.2	7.7	11 21.9	9.0	59.7	52	05 15	05 56	06 31	05 39	06 11	06 40	07 08		
M 10	10	312 16.1 .	335 09.9	7.8	11 12.9	9.1	59.8	50	05 17	05 56	06 29	05 34	06 07	06 38	07 08		
	11	327 16.2 .	349 36.7	7.7	11 03.8	9.1	59.8	48	05 18	05 56	06 28	05 30	06 05	06 37	07 09		
	12	342 16.4 .	5 00.8	4 03.4	7.7	10 54.7	9.2	59.8	45	05 22	05 56	06 25	05 20	05 58	06 34	07 10	
	13	357 16.5 S 4 59.8	18 30.1	7.8	S10 45.5	9.3	59.9	40	05 24	05 55	06 22	05 12	05 53	06 32	07 11		
	14	372 16.7 .	32 56.9	7.7	10 36.2	9.4	59.9	35	05 25	05 55	06 20	05 05	05 48	06 30	07 12		
M 20	15	37 16.8 .	47 23.6	7.7	10 26.8	9.4	59.9	30	05 26	05 54	06 18	04 59	05 44	06 29	07 12		
	16	42 17.0 .	61 50.3	7.8	10 17.4	9.6	60.0	20	05 26	05 52	06 14	04 48	05 37	06 26	07 14		
	17	57 17.1 .	55.9	76 17.1	7.7	10 07.8	9.6	60.0	10	05 25	05 50	06 11	04 39	05 31	06 23	07 15	
	18	72 17.3 .	55.0	90 43.8	7.7	9 58.2	9.6	60.0	0	05 23	05 47	06 07	04 30	05 25	06 21	07 16	
	19	87 17.4 S 4 54.0	105 10.5	7.7	S 9 48.6	9.8	60.0	10	05 18	05 43	06 04	04 21	05 19	06 18	07 17		
M 23	20	102 17.6 .	119 37.2	7.8	9 38.8	9.8	60.1	20	05 12	05 38	06 00	04 12	05 13	06 16	07 18		
	21	117 17.7 .	134 04.0	7.7	9 29.0	9.9	60.1	30	05 04	05 32	05 56	04 01	05 06	06 13	07 20		
	22	132 17.9 .	148 30.7	7.7	9 19.1	10.0	60.1	35	04 58	05 28	05 53	03 55	05 02	06 11	07 21		
	23	147 18.0 .	162 57.4	7.7	9 09.1	10.0	60.2	40	04 51	05 23	05 50	03 48	04 58	06 09	07 22		
	24	162 18.2 .	49.1	177 24.1	7.7	8 59.1	10.1	60.2	45	04 42	05 17	05 47	03 39	04 52	06 07	07 23	
8 00	25	177 18.3 S 4 48.1	191 50.8	7.8	S 8 49.0	10.1	60.2	50	04 31	05 10	05 43	03 29	04 46	06 04	07 24		
	26	192 18.5 .	47.2	206 17.6	7.7	8 38.9	10.2	60.2	52	04 25	05 07	05 41	03 25	04 43	06 03	07 25	
	27	207 18.7 .	46.2	220 44.3	7.7	8 28.7	10.3	60.3	54	04 19	05 03	05 39	03 20	04 39	06 02	07 26	
	28	222 18.8 .	45.2	235 11.0	7.7	8 18.4	10.4	60.3	56	04 12	04 58	05 37	03 14	04 36	06 01	07 27	
	29	237 19.0 .	44.2	249 37.7	7.7	8 08.0	10.4	60.3	58	04 04	04 54	05 34	03 08	04 32	05 59	07 27	
T 00	30	252 19.1 .	43.3	264 04.4	7.8	7 57.6	10.5	60.3	60	03 54	04 48	05 31	03 00	04 27	05 57	07 28	
	31	267 19.3 S 4 42.3	278 31.2	7.7	S 7 47.1	10.5	60.4	Lat.	Sunset		Twilight		Moonsel				
	32	282 19.4 .	41.3	292 57.9	7.7	7 36.6	10.6	60.4	Civil	Naut.	7	8	9	10			
	33	297 19.6 .	40.3	307 24.6	7.7	7 26.0	10.6	60.4	h m	h m	h m	h m	h m	h m			
	34	312 19.7 .	39.3	321 51.3	7.7	7 15.4	10.7	60.4	17 25	18 33	19 53	14 32	16 33	18 34	20 34		
T 08	35	327 19.9 .	38.4	336 18.0	7.7	7 04.7	10.8	60.5	70	17 30	18 31	19 44	14 50	16 42	18 35	20 28	
	36	342 20.0 .	37.4	350 44.7	7.8	6 53.9	10.8	60.5	68	17 35	18 30	19 36	15 04	16 49	18 36	20 24	
	37	357 20.2 S 4 36.4	5 11.5	7.7	S 6 43.1	10.8	60.5	66	17 38	18 29	19 29	15 16	16 55	18 37	20 20		
	38	372 20.3 .	35.4	19 38.2	7.7	6 32.3	10.9	60.5	64	17 41	18 28	19 24	15 25	17 01	18 38	20 16	
	39	37 20.5 .	34.5	34 04.9	7.7	6 21.4	11.0	60.6	62	17 44	18 28	19 20	15 33	17 05	18 39	20 14	
T 10	40	42 20.7 .	33.5	48 31.6	7.7	6 10.4	11.0	60.6	60	17 46	18 27	19 16	15 40	17 09	18 40	20 11	
	41	57 20.8 .	32.5	62 58.3	7.7	5 59.4	11.0	60.6	N	17 48	18 27	19 13	15 46	17 12	18 40	20 09	
	42	72 21.0 .	31.5	77 25.0	7.7	5 48.4	11.1	60.6		17 50	18 27	19 10	15 52	17 15	18 41	20 07	
	43	87 21.1 S 4 30.6	91 51.7	7.8	S 5 37.3	11.1	60.6	17 51	18 27	19 08	15 57	17 18	18 41	20 05			
	44	102 21.3 .	29.6	106 18.5	7.7	5 26.2	11.2	60.7	17 53	18 27	19 06	16 01	17 20	18 42	20 03		
T 16	45	117 21.4 .	28.6	120 45.2	7.7	5 15.0	11.2	60.7	58	17 54	18 26	19 04	16 05	17 23	18 42	20 02	
	46	132 21.6 .	27.6	135 11.9	7.7	5 03.8	11.3	60.7	45	17 57	18 27	19 01	16 13	17 27	18 43	19 59	
	47	147 21.7 .	26.6	149 38.6	7.7	4 52.5	11.3	60.7	N	18 00	18 27	18 58	16 20	17 31	18 44	19 56	
	48	162 21.9 .	25.7	164 05.3	7.7	S 4 41.2	11.3	60.7		18 02	18 27	18 57	16 26	17 35	18 44	19 54	
	49	177 22.1 S 4 24.7	A total eclipse of the Sun occurs on this date. See page 5.						18 04	18 28	18 56	16 32	17 38	18 45	19 52		
W 00	50	192 22.2 .	23.7	Naut.	294 05.7	7.7	2 58.1	11.6	60.9	30	18 08	18 30	18 55	16 41	17 43	18 45	19 48
	51	207 22.4 .	22.7		308 32.4	7.7	2 46.5	11.6	60.9	35	18 27	18 53	19 23	17 25	18 08	18 49	19 31
	52	222 22.5 .	21.7		322 59.1	7.7	2 34.9	11.6	60.9	40	18 30	18 57	19 30	17 31	18 11	18 50	19 28
	53	237 22.7 .	20.8		337 25.8	7.7	2 23.3	11.7	60.9	45	18 33	19 03	19 38	17 38	18 15	18 50	19 26
	54	267 23.0 S 4 18.8	265 12.3	7.7	S 3 21.2	11.5	60.8	S	18 37	19 10	19 49	17 45	18 19	18 51	19 22		
W 08	55	282 23.2 .	17.8	279 39.0	7.7	3 09.7	11.6	60.8	35	18 39	19 13	19 55	17 49	18 21	18 51	19 21	
	56	297 23.3 .	16.9	294 11.6	7.7	2 58.1	11.6	60.9	40	18 41	19 17	20 01	17 53	18 23	18 51	19 19	
	57	312 23.5 .	15.9	308 32.4	7.7	2 46.5	11.6	60.9	45	18 43	19 21	20 08	17 57	18 25	18 52	19 18	
	58	327 23.6 .	14.9	322 59.1	7.7	2 34.9	11.6	60.9	50	18 46	19 26	20 16	18 02	18 28	18 52	19 16	
	59	342 23.8 .	13.9	337 25.8	7.7	2 23.3	11.7	60.9	55	18 49	19 31	20 25	18 08	18 31	18 52	19 13	
W 16	60	357 23.9 S 4 12.9	351 52.5	7.7	S 2 11.6	11.7	60.9	Day	SUN		MOON		Moonrise				
	61	372 24.1 .	12.0	6 19.2	7.7	1 59.9	11.6	60.9	Eqn. of Time	Mer.	Mer. Pass.	Upper	Lower	Age	Phase		
	62	27 24.3 .	11.0	20 45.9	7.7	1 48.3	11.7	60.9	00 <sup>h</sup>	12 <sup>h</sup>	Pass.						
	63	42 24.4 .	10.0	35 12.6	7.7	1 36.6	11.8	60.9	m s	m s	h m	h m	d %				
	64	57 24.6 .	09.0	49 39.3	7.7	1 24.8	11.7	60.9	11 01	10 54	12 11	10 43	23 11	28 4			
W 24	65	72 24.7 .	08.0	64 06.0	7.7	1 13.1	11.7	60.9	10 47	10 40	12 11	11 38	24 06	29 1			
	66	87 24.9 S 4 07.1	78 32.7	7.7	S 1 01.4	11.8	60.9	10 49	10 25	12 10	12 34	00 06	00 0				
	67	102 25.1 .	06.1	92 59.4	7.6	0 49.6	11.8	61.0	11 02	10 57	12 11	11 41	23 12	28 4			
	68	117 25.2 .	05.1	107 26.0	7.7	0 37.8	11.7	61.0	10 47	10 40	12 11	11 38	24 06	29 1			
	69	132 25.4 .	04.1														

Figure 1907b. Right hand daily page of the Nautical Almanac for March 7, 8, & 9, 2016.

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UT	SUN		MOON				Lat.	Twilight		Sunrise	Moonrise			
								Naut.	Civil		22	23	24	25
	d	h	o	/	o	/	o	/	o	/	o	/	o	/
22	00	178	16.7	N	0	42.9	13	39.7	14.5	N	5	13.5	9.1	54.6
	01	193	16.9		43.9		28	13.2	14.5		5	04.4	9.1	54.5
	02	208	17.1		44.9		42	46.7	14.6		4	55.3	9.2	54.5
	03	223	17.3	.	45.9		57	20.3	14.6		4	46.1	9.1	54.5
	04	238	17.5		46.9		71	53.9	14.6		4	37.0	9.1	54.5
	05	253	17.6		47.9		86	27.5	14.7		4	27.9	9.2	54.5
	06	268	17.8	N	0	48.8	101	01.2	14.6	N	4	18.7	9.2	54.5
	07	283	18.0		49.8		115	34.8	14.7		4	09.5	9.2	54.5
	08	298	18.2		50.8		130	08.5	14.7		4	00.3	9.2	54.5
	09	313	18.4	.	51.8		144	42.2	14.8		3	51.1	9.2	54.4
23	10	328	18.6		52.8		159	16.0	14.7		3	41.9	9.3	54.4
	11	343	18.8		53.8		173	49.7	14.8		3	32.6	9.2	54.4
	12	358	19.0	N	0	54.8	188	23.5	14.8	N	3	23.4	9.3	54.4
	13	13	19.2		55.7		202	57.3	14.8		3	14.1	9.2	54.4
	14	28	19.3		56.7		217	31.1	14.8		3	04.9	9.3	54.4
	15	43	19.5	.	57.7		232	04.9	14.9		2	55.6	9.3	54.4
	16	58	19.7		58.7		246	38.8	14.8		2	46.3	9.3	54.4
	17	73	19.9	.	59.7		261	12.6	14.9		2	37.0	9.3	54.4
	18	88	20.1	N	1	00.7	275	46.5	14.9	N	2	27.7	9.3	54.4
	19	103	20.3		01.6		290	20.4	14.9		2	18.4	9.3	54.3
24	20	118	20.5		02.6		304	54.3	14.9		2	09.1	9.3	54.3
	21	133	20.7	.	03.6		319	28.2	15.0		1	59.8	9.4	54.3
	22	148	20.9		04.6		334	02.2	14.9		1	50.4	9.3	54.3
	23	163	21.0		05.6		348	36.1	15.0		1	41.1	9.3	54.3
	00	178	21.2	N	1	06.6	3	10.1	15.0	N	1	31.8	9.4	54.3
	01	193	21.4		07.6		17	44.1	15.0		1	22.4	9.3	54.3
	02	208	21.6		08.5		32	18.1	15.0		1	13.1	9.4	54.3
	03	223	21.8	.	09.5		46	52.1	15.0		1	03.7	9.3	54.3
	04	238	22.0		10.5		61	26.1	15.1		0	54.4	9.3	54.3
	05	253	22.2		11.5		76	00.2	15.0		0	45.1	9.4	54.2
25	06	268	22.4	N	1	12.5	90	34.2	15.1	N	0	35.7	9.3	54.2
	07	283	22.5		13.5		105	08.3	15.0		0	26.4	9.4	54.2
	08	298	22.7		14.5		119	42.3	15.1		0	17.0	9.3	54.2
	09	313	22.9	.	15.4		134	16.4	15.1	N	0	07.7	9.4	54.2
	10	328	23.1		16.4		148	50.5	15.1	S	0	01.7	9.3	54.2
	11	343	23.3		17.4		163	24.6	15.1		0	11.0	9.3	54.2
	12	358	23.5	N	1	18.4	177	58.7	15.1	S	0	20.3	9.4	54.2
	13	13	23.7		19.4		192	32.8	15.1		0	29.7	9.3	54.2
	14	28	23.9		20.4		207	06.9	15.1		0	39.0	9.3	54.2
	15	43	24.1	.	21.3		221	41.0	15.2		0	48.3	9.3	54.2
26	16	58	24.3		22.3		236	15.2	15.1		0	57.6	9.3	54.2
	17	73	24.4		23.3		250	49.3	15.1		1	06.9	9.3	54.2
	18	88	24.6	N	1	24.3	265	23.4	15.2	S	1	16.2	9.3	54.1
	19	103	24.8		25.3		279	57.6	15.1		1	25.5	9.3	54.1
	20	118	25.0		26.3		294	31.7	15.2		1	34.8	9.3	54.1
	21	133	25.2	.	27.2		309	05.9	15.1		1	44.1	9.3	54.1
	22	148	25.4		28.2		323	40.0	15.1		1	53.4	9.2	54.1
	23	163	25.6		29.2		338	14.1	15.2		2	02.6	9.3	54.1
27	00	178	25.8	N	1	30.2	352	48.3	15.2	S	2	11.9	9.2	54.1
	01	193	26.0		31.2		7	22.5	15.1		2	21.1	9.3	54.1
	02	208	26.1		32.2		21	56.6	15.2		2	30.4	9.2	54.1
	03	223	26.3	.	33.2		36	30.8	15.1		2	39.6	9.2	54.1
	04	238	26.5		34.1		51	04.9	15.2		2	48.8	9.2	54.1
	05	253	26.7		35.1		65	39.1	15.1		2	58.0	9.1	54.1
	06	268	26.9	N	1	36.1	80	13.2	15.2	S	3	07.1	9.2	54.1
	07	283	27.1		37.1		94	47.4	15.1		3	16.3	9.1	54.1
	08	298	27.3		38.1		109	21.5	15.1		3	25.4	9.2	54.1
	09	313	27.5	.	39.0		123	55.6	15.2		3	34.6	9.1	54.1
28	10	328	27.7		40.0		138	29.8	15.1		3	43.7	9.1	54.1
	11	343	27.8		41.0		153	03.9	15.1		3	52.8	9.0	54.1
	12	358	28.0	N	1	42.0	167	38.0	15.2	S	4	01.8	9.1	54.0
	13	13	28.2		43.0		182	12.2	15.1		4	10.9	9.0	54.0
	14	28	28.4		44.0		196	46.3	15.1		4	19.9	9.1	54.0
	15	43	28.6	.	44.9		211	20.4	15.1		4	29.0	9.0	54.0
	16	58	28.8		45.9		225	54.5	15.1		4	38.0	8.9	54.0
	17	73	29.0		46.9		240	28.6	15.1		4	46.9	9.0	54.0
	18	88	29.2	N	1	47.9	255	02.7	15.1	S	4	55.9	8.9	54.0
	19	103	29.4		48.9		269	36.8	15.0		5	04.8	9.0	54.0
29	20	118	29.6		49.9		284	10.8	15.1		5	13.8	8.9	54.0
	21	133	29.7	.	50.8		298	44.9	15.1		5	22.7	8.8	54.0
	22	148	29.9		51.8		313	19.0	15.0		5	31.5	8.9	54.0
	23	163	30.1		52.8		327	53.0	15.1	S	5	40.4	8.8	54.0

ALTITUDE CORRECTION TABLES  $0^\circ$ – $35^\circ$ —MOON

App. Alt.	$0^\circ$ – $4^\circ$		$5^\circ$ – $9^\circ$		$10^\circ$ – $14^\circ$		$15^\circ$ – $19^\circ$		$20^\circ$ – $24^\circ$		$25^\circ$ – $29^\circ$		$30^\circ$ – $34^\circ$		App. Alt.
	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	Corr <sup>n</sup>	
00	0 34'5	5 58'2	10 62'1	15 62'8	20 62'2	25 60'8	30 58'9	00							
10	36'5	58'5	62'2	62'8	62'2	60'8	58'8	10							
20	38'3	58'7	62'2	62'8	62'1	60'7	58'8	20							
30	40'0	58'9	62'3	62'8	62'1	60'7	58'7	30							
40	41'5	59'1	62'3	62'8	62'0	60'6	58'6	40							
50	42'9	59'3	62'4	62'7	62'0	60'6	58'5	50							
00	1 44'2	6 59'5	11 62'4	16 62'7	21 62'0	26 60'5	31 58'5	00							
10	45'4	59'7	62'4	62'7	61'9	60'4	58'4	10							
20	46'5	59'9	62'5	62'7	61'9	60'4	58'3	20							
30	47'5	60'0	62'5	62'7	61'9	60'3	58'2	30							
40	48'4	60'2	62'5	62'7	61'8	60'3	58'2	40							
50	49'3	60'3	62'6	62'7	61'8	60'2	58'1	50							
00	2 50'1	7 60'5	12 62'6	17 62'7	22 61'7	27 60'1	32 58'0	00							
10	50'8	60'6	62'6	62'6	61'7	60'1	57'9	10							
20	51'5	60'7	62'6	62'6	61'6	60'0	57'8	20							
30	52'2	60'9	62'7	62'6	61'6	59'9	57'8	30							
40	52'8	61'0	62'7	62'6	61'6	59'9	57'7	40							
50	53'4	61'1	62'7	61'6	61'5	59'8	57'6	50							
00	3 53'9	8 61'2	13 62'7	23 61'5	28 59'7	33 57'5	00								
10	54'4	61'3	62'7	62'5	61'4	59'7	57'4	10							
20	54'9	61'4	62'7	62'5	61'4	59'6	57'4	20							
30	55'3	62'5	62'8	62'5	61'3	59'5	57'3	30							
40	55'7	62'6	62'8	62'4	61'3	59'5	57'2	40							
50	56'1	61'6	62'8	62'4	61'2	59'4	57'1	50							
00	4 56'4	9 61'7	14 62'8	19 62'4	24 61'2	29 59'3	34 57'0	00							
10	56'8	61'8	62'8	62'4	61'1	59'3	56'9	10							
20	57'1	61'9	62'8	62'3	61'1	59'2	56'9	20							
30	57'4	61'9	62'8	62'3	61'0	59'1	56'8	30							
40	57'7	62'0	62'8	62'3	61'0	59'1	56'7	40							
50	58'0	62'1	62'8	62'2	60'9	59'0	56'6	50							
HP	L	U	L	U	L	U	L	U	L	U	L	U	HP		
54'0	0 3 0'9	0 3 0'9	0 4 1'0	0 5 1'1	0 6 1'2	0 7 1'3	0 9 1'5	54'0							
54'3	0 7 1'1	0 7 1'2	0 8 1'2	0 8 1'3	0 9 1'4	1 1 1'5	1 2 1'7	54'3							
54'6	1'1	1 1'4	1'1 1'4	1'2 1'5	1'3 1'6	1'4 1'7	1'5 1'8	54'6							
54'9	1'4 1'1	1'5 1'6	1'5 1'6	1'6 1'7	1'6 1'8	1'8 1'9	1'9 2'0	54'9							
55'2	1'8 1'8	1'8 1'8	1'9 1'8	1'9 1'9	2'0 2'0	2'1 2'1	2'2 2'2	55'2							
55'5	2'2 2'0	2'2 2'0	2'2 2'1	2'3 2'1	2'4 2'2	2'4 2'3	2'5 2'4	55'5							
55'8	2'6 2'2	2'6 2'2	2'6 2'2	2'7 2'3	2'7 2'4	2'8 2'4	2'9 2'5	55'8							
56'1	3'0 2'4	3'0 2'5	3'0 2'5	3'0 2'5	3'1 2'6	3'1 2'6	3'2 2'7	56'1							
56'4	3'3 2'7	3'4 2'7	3'4 2'7	3'4 2'7	3'4 2'8	3'5 2'8	3'5 2'9	56'4							
56'7	3'7 2'9	3'7 2'9	3'8 2'9	3'8 2'9	3'8 3'0	3'8 3'0	3'9 3'0	56'7							
57'0	4'1 3'1	4'1 3'1	4'1 3'1	4'1 3'1	4'2 3'2	4'2 3'2	4'2 3'2	57'0							
57'3	4'5 3'3	4'5 3'3	4'5 3'3	4'5 3'3	4'5 3'3	4'5 3'4	4'6 3'4	57'3							
57'6	4'9 3'5	4'9 3'5	4'9 3'5	4'9 3'5	4'9 3'5	4'9 3'5	4'9 3'6	57'6							
57'9	5'3 3'8	5'3 3'8	5'2 3'8	5'2 3'7	5'2 3'7	5'2 3'7	5'2 3'7	57'9							
58'2	5'6 4'0	5'6 4'0	5'6 4'0	5'6 4'0	5'6 3'9	5'6 3'9	5'6 3'9	58'2							
58'5	6'0 4'2	6'0 4'2	6'0 4'2	6'0 4'2	6'0 4'1	5'9 4'1	5'9 4'1	58'5							
58'8	6'4 4'4	6'4 4'4	6'4 4'4	6'3 4'4	6'3 4'3	6'3 4'3	6'2 4'2	58'8							
59'1	6'8 4'6	6'8 4'6	6'7 4'6	6'7 4'6	6'7 4'5	6'6 4'5	6'6 4'4	59'1							
59'4	7'2 4'8	7'1 4'8	7'1 4'8	7'1 4'8	7'0 4'7	7'0 4'7	6'9 4'6	59'4							
59'7	7'5 5'1	7'5 5'0	7'5 5'0	7'5 4'9	7'4 4'9	7'3 4'8	7'2 4'8	59'7							
60'0	7'9 5'3	7'9 5'3	7'9 5'2	7'8 5'2	7'8 5'1	7'7 5'0	7'6 4'9	60'0							
60'3	8'3 5'5	8'3 5'5	8'2 5'4	8'2 5'4	8'1 5'3	8'0 5'2	7'9 5'1	60'3							
60'6	8'7 5'7	8'7 5'7	8'6 5'7	8'6 5'6	8'5 5'5	8'4 5'4	8'2 5'3	60'6							
60'9	9'1 5'9	9'0 5'9	9'0 5'9	8'9 5'8	8'8 5'7	8'7 5'6	8'6 5'4	60'9							
61'2	9'5 6'2	9'4 6'1	9'4 6'1	9'3 6'0	9'2 5'9	9'1 5'8	8'9 5'6	61'2							
61'5	9'8 6'4	9'8 6'3	9'7 6'3	9'7 6'2	9'5 6'1	9'4 5'9	9'2 5'8	61'5							

DIP			
Ht. of Eye	Corr <sup>n</sup>	Ht. of Eye	Corr <sup>n</sup>
m	ft.	m	ft.
2'4	2'8	8'0	9'5
2'6	2'9	8'6	9'9
2'8	3'0	9'2	10'3
3'0	3'1	9'8	10'6
3'2	3'2	10'5	11'0
3'4	3'3	11'2	11'4
3'6	3'4	11'9	11'8
3'8	3'4	12'6	12'2
4'0	3'5	13'3	12'6
4'2	3'6	14'1	13'0
4'5	3'8	14'9	13'4
4'7	3'9	15'7	13'8
5'0	4'0	16'5	14'2
5'2	4'1	17'4	14'7
5'5	4'2	18'3	15'1
5'8	4'3	19'1	15'5
6'1	4'4	20'1	16'0
6'3	4'5	21'0	16'5
6'6	4'6	22'0	16'9
6'9	4'6	22'9	17'4
7'2	4'7	23'9	17'9
7'5	4'8	24'9	18'4
7'9	4'9	26'0	18'8
8'2	5'0	27'1	19'3
8'5	5'1	28'1	19'8
8'8	5'2	29'2	20'4
9'2	5'3	30'4	20'9
9'5	5'4	31'5	21'4
			70'5

## MOON CORRECTION TABLE

The correction is in two parts; the first correction is taken from the upper part of the table with argument apparent altitude, and the second from the lower part, with argument HP, in the same column as that from which the first correction was taken. Separate corrections are given in the lower part for lower (L) and upper (U) limbs. All corrections are to be added to apparent altitude, but  $30'$  is to be subtracted from the altitude of the upper limb.

For corrections for pressure and temperature see page A4.

For bubble sextant observations ignore dip, take the mean of upper and lower limb corrections and subtract  $15'$  from the altitude.

App. Alt. = Apparent altitude  
= Sextant altitude corrected for index error and dip.

Figure 1908b. Altitude Correction Tables  $0^\circ$ – $35^\circ$  – Moon (from the Nautical Almanac).

examples. In this case, it is  $15.3'$  because the sight was taken one minute and four seconds after the hour. From the daily page, record also the  $v$  correction factor, it is  $+15.0$ . The  $v$  correction factor for the Moon is always positive. To obtain the  $v$  correction, go to the tables of increments and corrections. In the 1 minute table in the  $v$  or  $d$  correction columns locate the correction that corresponds to  $v = 15.0'$ . The table yields a correction of  $+0.4'$ . Adding this correction to the tabulated GHA and increment gives the final GHA as  $319^{\circ} 43.9'$ .

Finding the Moon's declination is similar to finding the declination for the Sun or stars. The tabulated declination and the  $d$  factor come from the *Nautical Almanac*'s daily pages. Go to the daily pages for March 22, 2016; extract the Moon's declination and  $d$  factor.

Record the declination and  $d$  correction and go to the increment and correction pages to extract the proper correction for the given  $d$  factor. In this case, go to the correction page for 1 minute. The correction corresponding to a  $d$  factor of  $-9.4$  is  $-0.2'$ . It is important to extract the correction with the correct algebraic sign. The  $d$  correction may be positive or negative depending on whether the Moon's declination value is increasing or decreasing in the interval covered by the  $d$  factor. In this case, the Moon's declination value at 21-00-00 GMT on 22 March was N  $01^{\circ} 59.8'$ ; at 22-00-00 on the same date the Moon's declination was N  $01^{\circ} 50.4'$ . Therefore, since the declination value was decreasing over this period, the  $d$  correction is negative. Do not assume to determine the sign of this correction by noting the trend in the  $d$  factor. For this problem, had the  $d$  factor for 21-00-00 been a value more than 22, it would not indicate that the  $d$  correction should be positive. Remember that the  $d$  factor is analogous to an interpolation factor; it provides a correction to declination. Therefore, the trend in declination values, not the trend in  $d$  values, controls the sign of the  $d$  correction. Combine the tabulated declination and the  $d$  correction factor to determine the true declination. In this case, the Moon's true declination is N  $01^{\circ} 59.6'$ .

Having obtained the Moon's GHA and declination, calculate LHA and determine the assumed latitude. Enter the *Sight Reduction Table* with the LHA, assumed latitude, and calculated declination. Calculate the intercept and azimuth in the same manner used for star and Sun sights. (Note, for this example a DR was not provided, so these last steps are not shown on the strip form).

### 1909. Reducing a Planet Sight

There are four navigational planets: Venus, Mars, Jupiter, and Saturn. Reducing a planet sight is similar to reducing a Sun or star sight, but there are a few important differences. This section will cover the procedure for determining  $h_o$ , the GHA and the declination for a planet sight.

On March 09, 2016, at 08-58-34 GMT, under standard meteorological conditions, the navigator takes a sight of

Mars. H s is  $29^{\circ} 43.0'$ . The height of eye is 68 feet, and the index correction is  $+0.2'$ . Determine  $h_o$ , GHA, and declination for Mars. A completed strip form showing the values from this example is shown in Figure 1907a.

The values on the filled-in strip form demonstrate the similarity between reducing planet sights and reducing sights of the Sun and stars. Calculate and apply the index and dip corrections exactly as for any other sight. Take the resulting apparent altitude and enter the altitude correction table for the stars and planets on the inside front cover of the *Nautical Almanac*.

In this case, the altitude correction for  $29^{\circ} 35.2'$  results in a correction of  $-1.7'$ . The additional correction for refraction is not applicable because the sight was taken at standard temperature and pressure; the horizontal parallax correction is not applicable to a planet sight, only for the Moon. All that remains is the correction specific to Mars or Venus. The altitude correction table on the inside front cover of the *Nautical Almanac* also contains this correction; once again see Figure 1906c. Its magnitude is a function of the body sighted (Mars or Venus), the time of year, and the body's apparent altitude. Entering this table with the data for this problem yields a correction of  $+0.1'$ . Applying these corrections to  $h$  results in an  $h_o$  of  $29^{\circ} 33.6'$ .

Determine the planet's GHA in the same manner as with the Sun. That is, extract the tabular GHA for the hour the sight was taken ( $48^{\circ} 08.6'$ ). Then determine the incremental addition for the minutes and seconds after the hour from the Increments and Corrections table in the back of the *Nautical Almanac* ( $14^{\circ} 38.5'$ ). The only difference between determining the Sun's GHA and a planet's GHA lies in applying the  $v$  correction. Recall that the  $v$  correction was also needed for the Moon. As mentioned earlier, the  $v$  correction is needed because the Moon and planets' motion are not close to uniform throughout the year.

Find the  $v$  factor at the bottom of the planets' GHA columns on the daily pages of the *Nautical Almanac*; see Figure 1906b. For Mars on March 09, 2016, the  $v$  factor is 1.5. If no algebraic sign precedes the  $v$  factor, add the resulting correction to the tabulated GHA. Subtract the resulting correction only when a negative sign precedes the  $v$  factor. Entering the  $v$  or  $d$  correction table corresponding to 58 minutes yields a correction of  $1.5'$ . Remember, because no sign preceded the  $v$  factor on the daily pages, add this correction to the tabulated GHA. The final GHA is  $62^{\circ}48.6'$ .

Read the tabulated declination directly from the daily pages of the *Nautical Almanac*; in our example it is S  $19^{\circ} 10.7'$ . The  $d$  correction factor is listed at the bottom of the planet column; in this case, the factor is 0.2. Note the trend in the declination values for the planet; if they are increasing around the time the sight was taken, the correction factor is positive. If the planet's declination value is decreasing around this time, the correction factor is negative. Next, enter the  $v$  or  $d$  correction table corresponding to 58 minutes and extract the correction for a  $d$  factor of 0.2. The correction in this case is  $+0.2'$ . The true declination is S  $19^{\circ} 10.9'$ .

From this point, reducing a planet sight is exactly the

same as reducing a Sun sight. Having obtained the planet's GHA and declination, the navigator can calculate LHA and determine the assumed latitude. Enter the Sight Reduction Table with the LHA, assumed latitude, and true declination.

## MERIDIAN PASSAGE

This section covers determining both latitude and longitude at the meridian passage of the Sun, or Local Apparent Noon (LAN). Latitude at LAN is a special case of the navigational triangle where the Sun is on the observer's meridian and thus the triangle becomes a straight north/south line. Should the navigator wish to plot the resulting LOP, it would be a straight east/west line running along the navigator's latitude. However, no complete sight reduction is necessary; the navigator need only to combine the Sun's zenith distance and its declination according to the rules discussed below. A new strip form, different from the one used in a sight reduction, is usually used (see Figure 1910).

Longitude at LAN is a function of the time elapsed since the Sun passed the Greenwich meridian. The navigator must determine the time of LAN and calculate the GHA of the Sun at that time. The following examples demonstrate determining latitude and longitude at LAN.

### 1910. Latitude at Meridian Passage of the Sun (Local Apparent Noon)

At 1208 ZT, March 09, 2016, a vessel's DR position is L 39° 49.0'N and 1 044° 33.0' W. The ship is on course 045°T at a speed of ten knots. (1) Calculate the first and second estimates of Local Apparent Noon. (2) The navigator actually observes LAN at 12-08-04 zone time. The sextant altitude at LAN is 45° 54.0'. The index correction is +0.2' and the height of eye is 68 feet. Determine the vessel's latitude. A completed strip form showing the values from this example is shown in Figure 1910.

First, determine the time of meridian passage from the daily pages of the Nautical Almanac. It is found in the lower right corner of the right hand daily pages in Figure 1907b. In this case, the meridian passage for March 09, 2016, is 1210. That is, the Sun crosses the central meridian of the time zone at 1210 ZT. Next, determine the vessel's DR longitude for the time of meridian passage. In this case, the vessel's 1208 DR longitude is 044° 33.0' W. Determine the time zone in which this DR longitude falls and record the longitude of that time zone's central meridian. In this case, the central meridian is 45° W. Enter the Conversion of Arc to Time table in the *Nautical Almanac* (page i, near the back) with the difference between the DR longitude and the central meridian longitude. The conversion for 27' of arc is 1<sup>m</sup> 48<sup>s</sup>, which can be rounded to 2<sup>m</sup> of time. Sum these two times. If the DR position is east of the central meridian (as it is in this case), subtract this time from the time of tabulated meridian passage. If the longitude difference is to the west of the central meridian, add this time to the

Calculate the intercept and azimuth in the same manner used for star and Sun sights. (Note, for this example a DR was not provided, so these last steps are not shown on the strip form).

tabulated meridian passage. In this case, the DR position is east of the central meridian. Therefore, subtract 2 minutes from 1210, the tabulated time of meridian passage. The estimated time of LAN is 12-08-00 ZT.

This first estimate for LAN took into account the vessel's movement. Therefore, it is unnecessary to conduct second estimate of LAN.

Solving for latitude requires that the navigator calculate two quantities: the Sun's declination and the Sun's zenith distance. First, calculate the Sun's true declination at LAN. The problem states that LAN is observed at 12-08-04. (Determining the exact time of LAN is covered in Section 1911.) Enter the time of observed LAN and add the correct zone description to determine GMT. Determine the Sun's declination in the same manner as in the sight reduction problem in Section 1906. In this case, the tabulated declination was S 4° 10.0', and  $d$  is -1.0'; the  $d$  correction (from the increments and corrections table) is -0.1'. The true declination, therefore, is S 4° 09.9'.

Next, calculate zenith distance. Recall from Chapter 17 - *Navigational Astronomy* that zenith distance is simply 90° - observed altitude,  $h_o$ . Therefore, correct  $h_s$  to obtain  $h_a$ ; then correct  $h_a$  to obtain  $h_o$ . This is done the same way as was done for reducing a Sun sight in Section 1907. Then, subtract  $h_o$  from 90° to determine the zenith distance. For our example, after applying the appropriate corrects,  $h_o$  is 46° 01.5', and the zenith distance is 43° 58.5'.

Your latitude is determined by applying the Sun's true declination to the zenith distance using the following rules:

- If DR lat and Dec (Sun) are same (e.g., both North or both South), then ADD
- If DR lat and Dec (Sun) are contrary (e.g., one North, one South), then SUBTRACT
- If DR lat and Dec (Sun) are same, and Dec is greater, then SUBTRACT

In this case, the DR latitude is N 39° 49.0' and the Sun's declination is S 4° 09.9', which are contrary. Therefore, subtract the true declination from the zenith distance. The latitude for our example is N 39° 48.6'.

### 1911. Longitude at Meridian Passage

Determining a vessel's longitude at LAN is straightforward. In the western hemisphere, the Sun's GHA at LAN equals the vessel's longitude. In the eastern hemisphere, subtract the Sun's GHA from 360° to determine longitude. The difficult part lies in determining the precise moment of meridian passage.

OPNAV 3530/35 (4-73) LAN		NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 7-74)
Date		DATE/DR POSIT 9 Mar 2016 39° 49.0' N, 044° 33.0' W
DR $\lambda$	044° 33.0' W	
STD Meridian	045° W	
d $\lambda$ (arc)	27.0' E	
D $\lambda$ (time)	-2m (rounded)	
LMT Mer Pass	1210	
ZT LAN (1 <sup>st</sup> . Est.)	1208	
Rev DR $\lambda$	—	
STD Meridian	—	
d $\lambda$ (arc)	—	
d $\lambda$ (time)	—	
LMT Mer Pass	—	
ZT LAN (2 <sup>nd</sup> est.)	—	
LAT BY LAN:		
ZT LAN (OBS)	12-08-04	
ZD	+3	
GMT	15-08-04	
Tab Dec	S 4° 10.0'	
d# / d Corr	-1.0 / -0.1'	
True Dec	S 4° 9.9'	
IC	+0.2'	
D	-8.0'	
Sum	-7.8'	
hs	45° 54.0'	
ha	45° 46.2'	
Alt Corr	+15.3'	
Ho	46° 1.5'	
89° 60'	89° 60.0'	
HO (-)	-46° 1.5'	
Z Dist	43° 58.5'	
True Dec	S 4° 9.9'	
LAT	N 39° 48.6'	
Time		
Sounding		
Signature		

Figure 1910. Sight reduction strip form for Local Apparent Noon (LAN).

Determining the time of meridian passage presents a problem because the Sun appears to hang for a finite time at its local maximum altitude. Therefore, noting the time of maximum sextant altitude is not sufficient for determining the precise time of LAN. Two methods are available to obtain LAN with a precision sufficient for determining longitude: (1) the graphical method and (2) the calculation method. The graphical method is discussed first below.

For about 30 minutes before the estimated time of LAN, measure and record several sextant altitudes and their corresponding times. See Figure 1911. Continue taking sights for about 30 minutes after the Sun has descended from the maximum recorded altitude. Increase the sighting frequency near the meridian passage. One sight every 20-30 seconds should yield good results near meridian passage; less frequent sights are required before and after.

Plot the resulting data on a graph of sextant altitude versus time and draw a fair curve through the plotted data. Next, draw a series of horizontal lines across the curve formed by the data points. These lines will intersect the faired curve at two different points. The x coordinates of the points where these lines intersect the faired curve represent the two different times when the Sun's altitude was equal (one time when the Sun was ascending; the other time when the Sun was descending). Draw three such lines, and ensure the lines have sufficient vertical separation. For each line, average the two times where it intersects the faired curve. Finally, average the three resulting times to obtain a final value for the time of LAN. From the *Nautical Almanac*, determine the Sun's GHA at that time; this is your longitude in the western hemisphere. In the eastern hemisphere, subtract the Sun's GHA from 360° to determine longitude. For a quicker but less exact time, simply drop a perpendicular from the apex of the curve and read the time along the time scale.

The second method of determining LAN is similar to the first. Estimate the time of LAN as discussed above. Measure and record the Sun's altitude as the Sun approaches its maximum altitude. As the Sun begins to descend, set the sextant to correspond to the altitude recorded just before the Sun's reaching its maximum altitude. Note the time when the Sun is again at that altitude. Average the two times. Repeat this procedure with two other altitudes recorded before LAN, each time presetting the sextant to those altitudes and recording the corresponding times that the Sun, now on its descent, passes through those altitudes. Average these corresponding times. Take a final average among the three averaged times; the result will be the time of meridian passage. Determine the vessel's longitude by determining the Sun's GHA at the exact time of LAN.

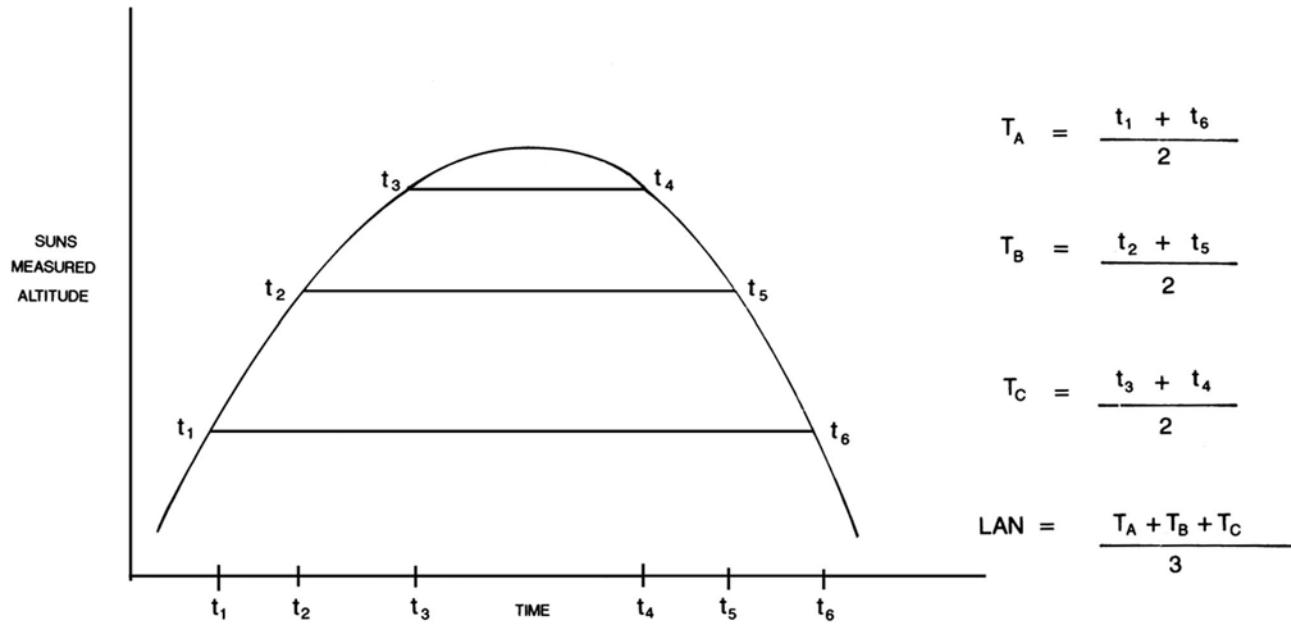


Figure 1911. Time of LAN.

## LATITUDE BY POLARIS

### 1912. Latitude by Polaris

Because Polaris is always within about  $1^\circ$  of the North Pole, the altitude of Polaris, with a few minor corrections, equals the latitude of the observer. This relationship makes Polaris an extremely important navigational star in the northern hemisphere.

The corrections are necessary because over the course of 24 hours, Polaris appears to move in a small circle around the pole. When Polaris is at the exact same altitude as the pole, the correction is zero. When on the observer's meridian, that is at upper or lower culmination, the corrections are maximum. A special table, The Polaris Table, in the *Nautical Almanac* is used to determine the correction. The following example illustrates converting a Polaris sight to latitude. A new strip form, different from the one used in a sight reduction and LAN, is often used (see Figure 1912b).

At 23-18-56 GMT, on March 22, 2016, at DR Lat.  $40^\circ 46.0' N$ ,  $l = 043^\circ 22.0' W$ , the observed altitude of Polaris ( $h_o$ ) is  $40^\circ 52.1'$ . Find the vessel's latitude (see Figure 2310c).

To solve this problem, use the equation:



Figure 1912a. Polaris time lapse illustration.

$$\text{Latitude} = h_o - 1^\circ + A_0 + A_1 + A_2$$

where  $h_o$  is the sextant altitude ( $h_s$ ) corrected as in any other star sight;  $1^\circ$  is a constant; and  $A_0$ ,  $A_1$ , and  $A_2$  are correction factors from the Polaris tables found on pages 274-276 of the *Nautical Almanac* (see Figure 1912c). These three

OPNAV 3530/40 (4-73) LAT BY POLARIS		NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 7-74)
Date		DATE/DR POSIT 22 Mar 2016 40° 46' N, 043° 22.0' W
Body	Polaris	
GMT	23-18-56	
IC		
D		
Sum		
Hs		
Ha		
Alt Corr		
Add'l Corr		
Ho	40° 52.1'	
GHA $\gamma$ (h)	165° 52.3'	
Incre (m/s)	4° 44.8'	
Total GHA $\gamma$	170° 37.1'	
± 360°	—	
DRλ (+E -W)	-43° 22.0'	
LHA $\gamma$	127° 15.1'	
A <sub>0</sub>	54.9'	
A <sub>1</sub>	0.5'	
A <sub>2</sub>	0.9'	
Sum	56.3'	
1° 00' (-)	-1° 00.0'	
Total Corr	-3.7'	
Ho	40° 52.1'	
LAT	N 40° 48.4'	
Time		
Sounding		
Signature		

Figure 1912b. Sight reduction strip form for longitude by Polaris.

correction factors are always positive. One needs the following information to enter the tables: LHA of Aries, DR latitude, and the month of the year. Therefore, LHA of Aries is determined similarly to the LHA of any other object. That is, first determine the GHA, then either add or subtract the DR longitude to compute LHA. Determining the GHA is done as with any other body. Extract the tabular GHA for the hour the sight was taken (165° 52.3'). Then determine the incremental addition for the minutes and seconds after the hour from the Increments and Corrections table in the back of the *Nautical Almanac* (4° 44.8'). The total GHA of Aries is 170° 37.1'. As described above, LHA is GHA - 1 for west longitudes and GHA + 1 for east longitudes. Because our example DR is in the western hemisphere, subtract the DR longitude from the GHA to obtain the LHA of Aries, 127° 15.1'.

Next, enter the Polaris table with the calculated LHA of Aries at the time of observation (127° 15.1'). The first correction, A<sub>0</sub>, is a function solely of the LHA of Aries. Enter the table column indicating the proper range of LHA of Aries; in this case, enter the 120°-129° column. The numbers on the left hand side of the A<sub>0</sub> correction table represent the whole degrees of LHA  $\gamma$ ; interpolate to determine the proper A<sub>0</sub> correction. In this case, LHA  $\gamma$  was 127° 15.1'. The A<sub>0</sub> correction for LHA = 127° 54.7' and the A<sub>0</sub> correction for LHA = 128° is 55.4'. The A<sub>0</sub> correction for 127° 15.1' is 54.9'.

To calculate the A<sub>1</sub> correction, enter the A<sub>1</sub> correction table with the DR latitude, being careful to stay in the 120°-129° LHA column. There is no need to interpolate here; simply choose the latitude that is closest to the vessel's DR latitude. In this case, L is 40°N. The A<sub>1</sub> correction corresponding to an LHA range of 120°-129° and a latitude of 40°N is + 0.5'.

Finally, to calculate the A<sub>2</sub> correction factor, stay in the 120°-129° LHA  $\gamma$  column and enter the A<sub>2</sub> correction table. Follow the column down to the month of the year; in this case, it is March. The correction for March is + 0.9'.

Sum the corrections, remembering that all three are always positive (56.3'). Subtract 1° from the sum to determine the total correction (-3.7'), then apply the resulting value to the observed altitude of Polaris. The result is the vessel's latitude, N 40° 48.4'.

## THE DAY'S WORK IN CELESTIAL NAVIGATION

### 1913. Celestial Navigation Daily Routine

The navigator need not follow the entire celestial routine if celestial navigation is not the primary navigation method. It is appropriate to use only the steps of the celestial day's work that are necessary to provide a meaningful check on the primary fix

source and maintain competency in celestial techniques.

The list of procedures below provides a complete daily celestial routine to follow. This sequence works equally well for all sight reduction methods, whether tabular, mathematical, computer program, or celestial navigation calculator. See Figure 1913 for an example of a typical day's celestial plot.

**POLARIS (POLE STAR) TABLES, 2016**  
 FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

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## ILLUSTRATION

ILLUSTRATION  
On 2016 April 21 at  
23<sup>h</sup> 18<sup>m</sup> 56<sup>s</sup> UT in longitude  
W 37° 14' the apparent altitude  
(corrected for refraction),  $H_0$ , of  
*Polaris* was 49° 31' 6

From the daily pages:

From the daily pages.	°	/	$H_0$	49	31°
GHA Aries (23 <sup>h</sup> )	195	26·5	$a_0$ (argument $162^\circ 57'$ )	1	18·6
Increment (18 <sup>m</sup> 56 <sup>s</sup> )	4	44·8	$a_1$ (Lat $50^\circ$ approx.)		0·6
Longitude (west)	-37	14	$a_2$ (April)		0·9

## LHA Aries

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Figure 1912c. Excerpt from the Polaris Tables.

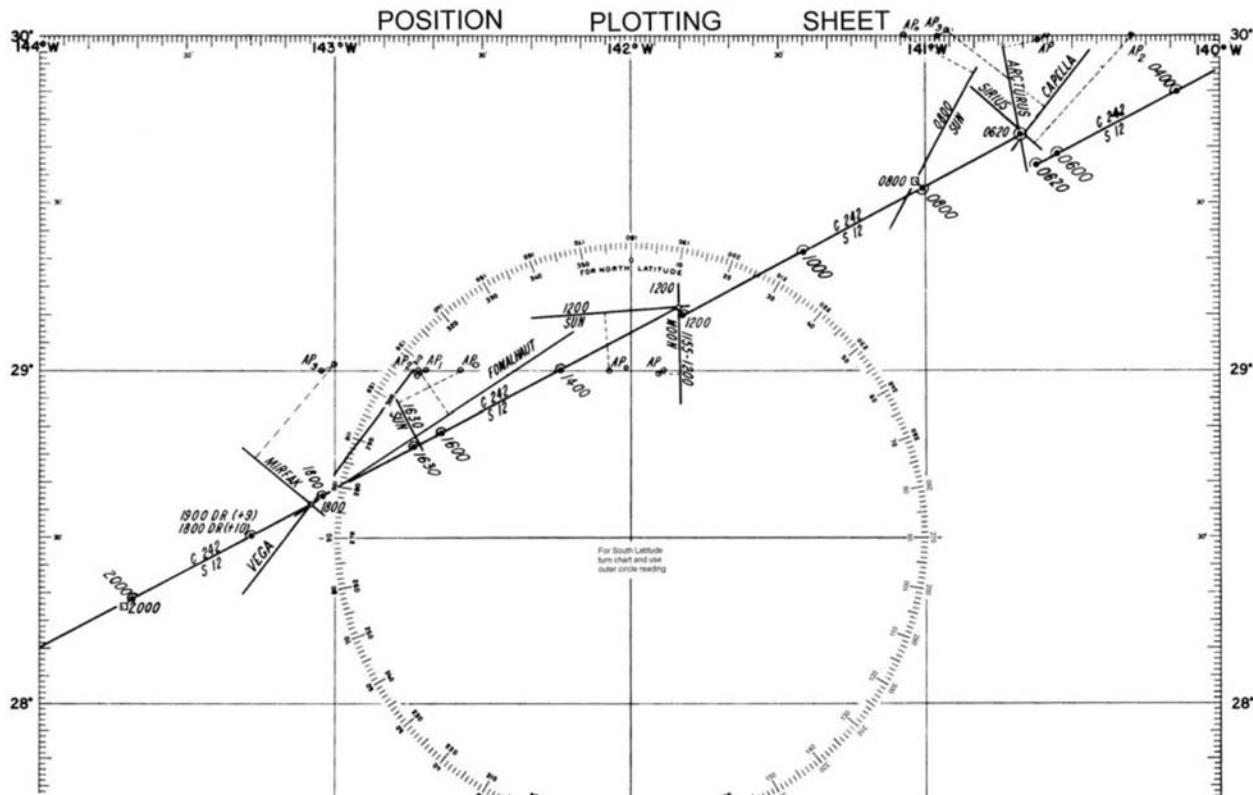


Figure 1913. Typical celestial plot at sea.

1. Before dawn, compute the time of morning twilight and plot the dead reckoning position for that time. (See Chapter 21 The Almanacs - Section 2109 Finding Times of Sunrise and Sunset.)
2. At morning twilight, take and reduce celestial observations for a fix. At sunrise take an amplitude of the Sun to obtain gyro error. (See Chapter 19 Azimuths and Amplitudes.)
3. Mid-morning, compare the chronometer with UT to determine chronometer error using a radio time tick. (See Chapter 20 Time.)
4. Mid-morning, reduce a Sun sight for a morning Sun line. (See section 1907 in this chapter.)
5. Calculate an azimuth of the Sun to obtain gyro error, if no amplitude was taken at sunrise. (See Chapter 19 Azimuths and Amplitudes.)
6. At LAN, obtain a Sun line and advance the morning Sun line for the noon fix. Compute a longitude deter-

mined at LAN for an additional LOP. (See Section 1910 and Section 1911 in this chapter.)

7. Mid-afternoon, again take and reduce a Sun sight. This is primarily for use with an advanced noon Sun line, or with a Moon or Venus line if the skies are overcast during evening twilight.
8. Calculate an azimuth of the Sun to obtain gyro error at about the same time as the afternoon Sun observation. The navigator may replace this azimuth with an amplitude observation at sunset. (See Chapter 19 Azimuths and Amplitudes.)
9. During evening twilight, reduce celestial observations for a fix. (See Section 1906 in this chapter.)
10. Be alert at all times for the moon or brighter planets which may be visible during daylight hours for additional LOP's, and Polaris at twilight for a latitude line.