Probabilistic Gaussian Generative Classifier

Report: 1

Generative Model & Parameter Estimation

We are using a Generative Discriminant Analysis (GDA) model, which aims to classify data by modeling the distribution of each class. To do this, we estimate three key parameters from our training data: the prior probability for each class (π_k), the mean vector for each class (μ_k), and a shared covariance matrix (Σ).

GDA assumes that the data within each class follows a Gaussian (normal) distribution. Specifically, the likelihood of observing features x given a class y is modeled as $P(x|y) \sim N(x; \mu_k, \Sigma)$, where N represents the normal distribution, μ_k is the mean vector for class k, and Σ is the covariance matrix shared across all classes.

Estimating the Parameters

We calculate these parameters from the training dataset as follows:

1. Prior Probability (π_k)

The prior probability π_k represents the probability of a data point belonging to class k. We estimate this by counting the proportion of samples belonging to each class in the training data.

$$\pi_k = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\{y^{(i)} == k\}$$

2. Mean Vector for Each Class (μ_k)

The mean vector μ_k captures the average feature values for all data points within class k. Each element of the vector corresponds to the mean of a specific feature for that class.

$$\mu_k = \frac{\sum_{i=1}^{N} \mathbf{1}\{y^{(i)} = k\} x^{(i)}}{\sum_{i=1}^{N} \mathbf{1}\{y^{(i)} = k\}}$$

3. Shared Covariance Matrix (Σ)

The covariance matrix Σ describes the relationships and spread of features across all classes, assuming a common covariance structure. It indicates how much each pair of features varies together.

$$\Sigma = \frac{1}{N} \sum_{i=1}^{N} (x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T$$

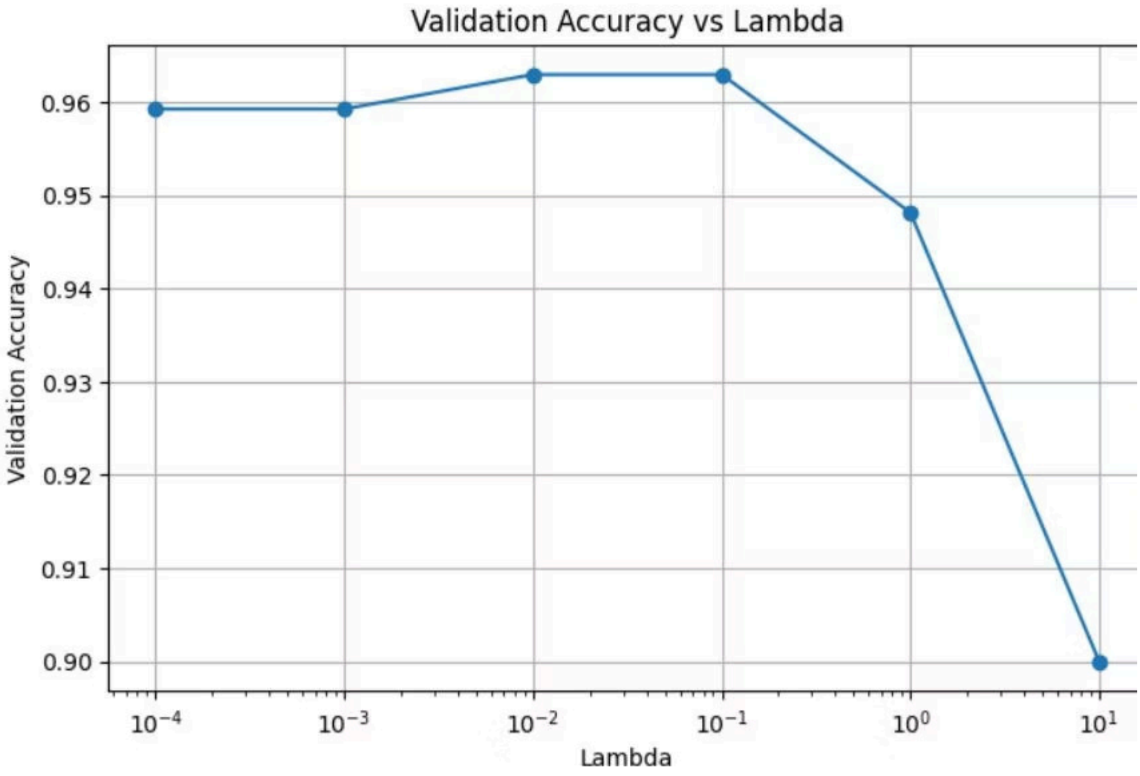
Why is covariance regularization needed?

Covariance regularization in GDA is needed to ensure invertibility, numerical stability, and better generalization, especially in high-dimensional or small-sample settings.

$$\Sigma_{Reg} = \Sigma + \lambda I$$

- Model Accuracy for corresponding regularization parameter

λ	Accuracy score
0.0001	95.925 %
0.001	95.925 %
0.01	96.296 %
0.1	96.296 %
1	96.296 %
10	90 %



Validation Accuracy vs Lambda

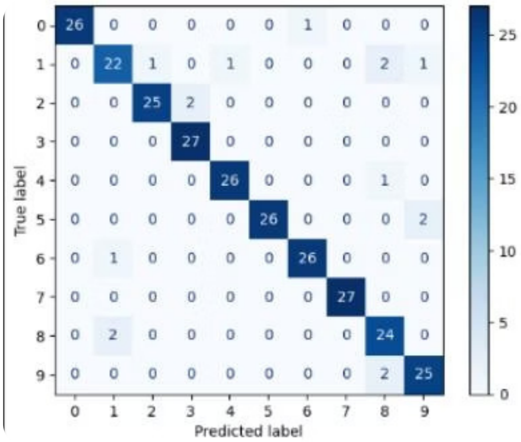
A line graph showing Validation Accuracy (Y-axis, ranging from 0.90 to 0.96) versus Lambda (X-axis, logarithmic scale, ranging from 10^{-4} to 10^{1}). The curve starts at $\lambda = 10^{-4}$ with an accuracy of approximately 0.96, remains relatively flat until $\lambda = 10^{-2}$, where the accuracy peaks at approximately 0.96. The accuracy then drops sharply, reaching approximately 0.90 at $\lambda = 10^{1}$.

Summary for Final Results

To thoroughly evaluate the performance of our GDA Classifier, we analyzed its predictions using a confusion matrix and a comprehensive classification report.

Confusion Matrix - GDA Classifier

The confusion matrix visually summarizes the performance of the classification algorithm, showing the counts of true positive, true negative, false positive, and false negative predictions for each class.



This confusion matrix for the GDA Classifier illustrates the distribution of actual versus predicted labels across 10 classes (0-9). High values on the diagonal indicate accurate classifications.

Classification Report Summary

The detailed classification report provides key performance metrics for the GDA Classifier:

- Test Accuracy: 94.07%
- Test Macro Precision: 94.19%
- Test Macro Recall: 94.07%
- Test Macro F1 Score: 94.06%

These metrics collectively demonstrate strong overall performance for the GDA Classifier across all classes.

Discussion and Analysis:

Confusion Between Digits

The confusion matrix clearly shows which digit pairs are most frequently misclassified. Digits with similar shapes, like 4 and 9, or 3 and 8, or 0 and 6, are often confused due to overlapping features. This insight highlights the model's specific failure modes and where it struggles to distinguish classes in the feature space.

Impact of Regularization Parameter λ

The choice of λ significantly impacts validation accuracy; for example, $\lambda = 0.01$ often performed best. Regularization prevents the covariance matrix from becoming ill-conditioned. Too little regularization (very small λ) can lead to numerical instability, while too much (very large λ) over-smooths the covariance estimate, affecting model performance.

Model Strengths

The Gaussian Discriminant Analysis model offers several strengths for this dataset:

- **Computational Efficiency**: It is fast to train and predict.
- **Probabilistic Predictions**: It provides probability scores for each class.
- **Few Parameters**: It requires relatively few parameters to estimate, making it less prone to overfitting with limited data.

Model Weaknesses

Despite its strengths, the GDA model has notable weaknesses:

- **Gaussian Assumption:** The assumption of class-conditional Gaussian distributions may not perfectly hold for the complex variations in handwritten digits.
- **Shared Covariance Restriction:** The assumption of shared covariance across all classes can be overly restrictive.
- **Multimodal Distributions:** The model struggles with multimodal class distributions, meaning it may not capture cases where a single digit has multiple distinct visual forms. This limits its ability to represent real-world digit complexity.