Modeling Kinetics of Lead in the Human Body using First Order ODEs

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Outline of Presentation

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Background of study

Lead

- Lead is a naturally occurring element found in the earth crust, its soft, blue-grey metallic element. It is very soft, highly malleable, ductile, and a relatively poor conductor of electricity.

Exposure of lead

Today almost everyone is exposed to environmental lead. Lead is taking into the body through

- Inhalation
- Ingestion
- Dermal

Problem Statement

Lead is a cumulative toxicant that affects multiple body systems and harmful to humans especially children and hence the need to study the behaviour and the concentration at the steady state to help reduce the lead content in human body through medical delivery.

Objectives

- To determine the concentration of lead in the three compartments in the steady state
- To perform the sensitivity analysis on the model parameters

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Model Assumptions

- The rate at which lead transfers from the lungs to the blood stream is proportional to the rate at which lead enters the lungs.
- The rate at which lead transfers from the digestive tract to the blood stream is proportional to the rate at which lead enters the digestive tract.
- The rate (F_{ij}) at which lead transfers from compartment i to compartment j is proportional to the amount of lead in the compartment i.
- Finally all lead transfer is measured in micrograms per day

Model Development

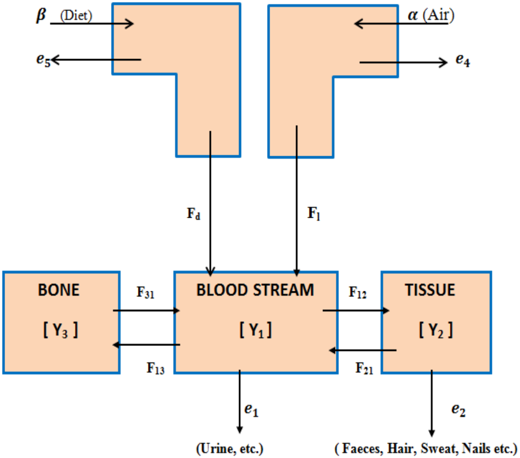

Figure 1: schematic of Lead Transfer

System of ODEs

$$
\begin{aligned}
Y_1'(t) &= -A_{11}Y_1 + A_{12}Y_2 + A_{13}Y_3 + N \\
Y_2'(t) &= A_{21}Y_1 - A_{22}Y_2 \\
Y_3'(t) &= A_{31}Y_1 - A_{33}Y_3
\end{aligned} \tag{1}
$$

$$Y'(t) = AY + F \tag{2}$$

Solution using Variation of Parameters

$$Y = Y_c + Y_p = Y_0 e^{At} + e^{At}\int_0^t e^{-At}(t)F(t)dt \tag{3}$$

$$Y = (Y_0 + A^{-1}F)e^{At} - A^{-1}F \tag{4}$$

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Analysis and Results

The system

$$Y_1'(t) = \frac{-13}{360}Y_1 + \frac{272}{21875}Y_2 + \frac{7}{200000}Y_3 + \frac{6162}{125}$$
$$Y_2'(t) = \frac{1}{90}Y_1 - \frac{1}{35}Y_2$$
$$Y_3'(t) = \frac{7}{1800}Y_1 - \frac{7}{200000}Y_3$$

Long term Solution

By Gershgorian circle theorem, the eigenvalues of A are negative or have zero real parts.
Hence the system is stable in the long run.

$$Y = (Y_0 + A^{-1}F)e^{At} - A^{-1}F \tag{5}$$

$$Y = -A^{-1}F, t \longrightarrow \infty \tag{6}$$

$Y = (1800\mu g, 700\mu g, 200000\mu g)$

Graphical analysis

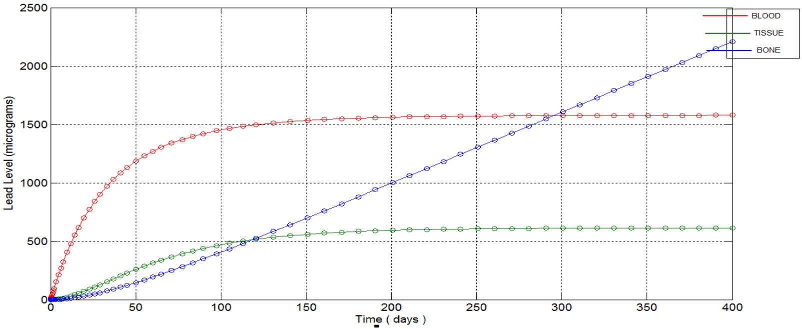

Figure 2: After 400 days, blood and tissue levels go to equilibrium, but bone levels soar

Sensitivity analysis

Assumptions

In order to improve the subject condition, we applied the following Sensitivity analysis

- Non-Lead environment
- Small-Lead environment
- Medication
- Small-Lead environment plus Medication

Sensitivity analysis (Small lead Environment)

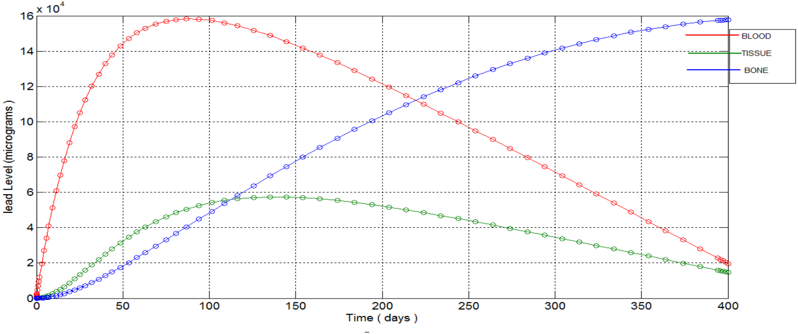

Figure 3: Lead level in the bone increases at a slower rate

Sensitivity analysis (Medication)

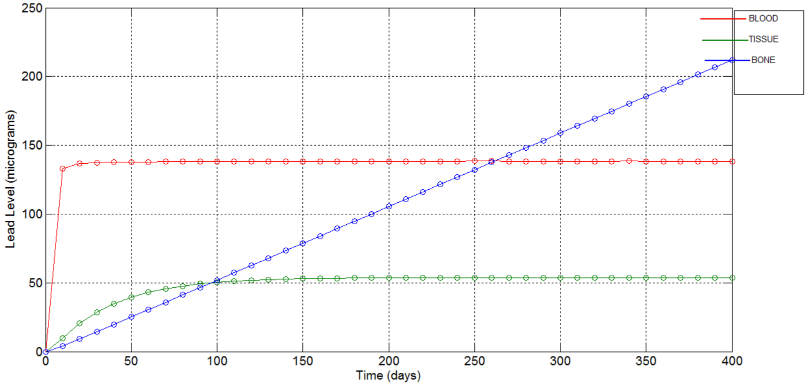

Figure 4: taking medication lowers the concentration in the blood and tissue but not in the bone.

Small Lead environment plus Medication

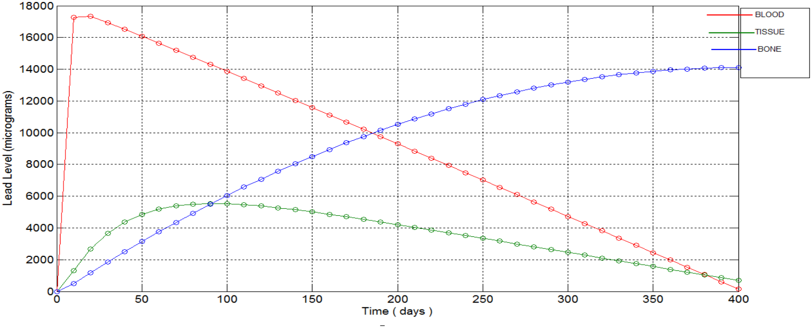

Figure 5: Reduced lead concentration even in the bone

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Conclusion

$Y(t) \longrightarrow -A^{-1}F$ as $t \longrightarrow \infty$.
Using matlab, the solution is given by $Y = -inv(A) * F$
$Y = (1800\mu g, 700\mu g, 200000\mu g)$

Finally, we observed that moving the subject from the lead environment to a small lead environment with medication reduces the lead content in the system drastically. Therefore, from the graphical analysis, it is reasonable to apply those assumptions to real life situations

Recommendation

- We recommend that further studies could be done to determine the kinetics of other metallic elements like Zinc in the human body.
- The data was obtained from the Journal of Mathematical biology. because of time and financial constraints, we therefore recommend that for any further studies, the data should be obtained from the research location.

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References

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END OF PRESENTATION

THANK YOU